Gold Discoveries During the US Classical Gold Standard Era: Monetary News Shocks and Adaptive Learning in a DSGE Model

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter I

Bimetallism and the "Crime of 1873"

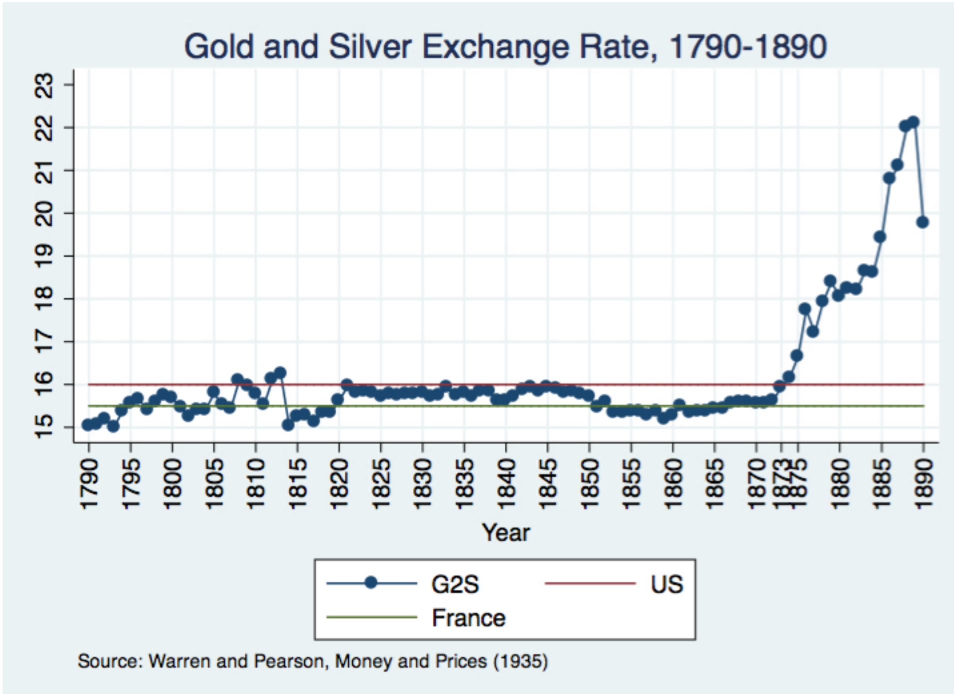
1.1 The beginning of United States Bimetallism and the ratio change in 1834

The Congress, following the recommendation of Alexander Hamilton, passed the Coinage Act of April 2, 1792. This Act (or Mint Act) not only established the United States Mint but also defined the basic monetary unit of the United States as the dollar and defined subsidiary coinage on a decimal basis. It further defined the dollar as equal to 371.25 grains of pure silver and the $10 eagle as equal to 24.70 grains of pure gold. Additionally this Act authorized free coinage of both silver and gold at the specified ratio of 15 to 1, and specified the fraction of alloy to be combined with pure metal in striking the coins.This marked that United states has formally entered the Bimetallic era. i.e., both silver and gold coins are legal tenders in payment of debts. At the instance of Alexander Hamilton, the ratio of fifteen was adopted and there is no room to doubt that this was very close to the true market ratio at the time. While from Figure 1, we can see the real market ratio since 1794 was well above 15. Gold began to grow scarce in circulation as early as 1810, and had wholly disappeared in 1817 because of the bimetallic arbitrage. After 1817 and until the ratio change 1834 United States was on de facto silver standard.

To understand the bimetallic arbitrage mechanism, a simple get-rich arbitrage scheme is intuitive: Anyone who had one unit gold and wanted to convert it to money would do better by exchanging the one unit gold for 15.5 units of silver at the market ratio, pocketing the 0.5 unit of silver and taking the 15 units of silver to the mint to fetch one unit of gold, rather than by taking the one unit of gold directly to the mint. In other words the 0.5 unit of silver is the arbitrage profit. Therefore the US Mint will be filled with silver coins, and gold will be traded more as a type of commodity or raw materials.

This Legal ratio of 15:1 between gold and silver remained unchanged until another Coinage Act in 1834, and this time it was changed to 16:1 (see Table.1), which was slightly above the market ratio. Therefore the bimetallic arbitrage again kicked in, drove silver out of the circulation, and shifted United States to a de facto gold standard.

Figure 1: Gold and Silver Exchange Rate, 1790-1890



It is also noteworthy that US was undergoing a constant shortage of domestic species supply before the major discovery of gold (CA, 1848) and silver (Nevada, 1858). In fact before the Coinage Act of 1857 (see Table.1), certain foreign coins are legal tenders with a fluctuating conversion rate to fulfill the domestic demand. The only silver money in circulation consisted exclusively of foreign coins, mostly Spanish and Mexican, but with a considerable sprinkling of English, French, German, and Scandinavian pieces. Every merchant kept a coin chart manual for handy reference to determine the value of these pieces as they were offered in trade. Therefore, Under the money market clearance condition, the supply shock is extremely important to the money market. And it is reasonable to assume the ratio change in 1834 was a one time unexpected and exogenous monetary shock. A question would naturally follow: how does this exogenous money supply shock affect the US economy, both qualitatively and quantitively?

1.2 Reasons for the Ratio Change in 1834

As White (1893) accounted:

“In 1834 our people had become tired of lugging silver around. They had by this time found out

Table 1: Official Value of U.S. dollar and its related Act

Year	Value of dollars in fine grains		Mint Ratio	Price per Fine Troy Ounce		New Features in Coinage Act
	Gold	Silver		Gold	Silver	
1792	24.75	371.25	15	19.39	1.29	Established the 15:1 ratio
1834	23.2	371.25	16	20.69	1.29	Changed the ratio to 16:1
1837	23.22	371.25	15.98	20.67	1.29	Changed the ratio to 15.98:1
1853	23.22	371.25	15.98	20.67	1.29	Made the silver coins subsidiary legal tender (up to 5 USD)
1857	23.22	371.25	15.98	20.67	1.29	Forbade the use of foreign coins as legal tender
1873	23.22	371.25	15.98	20.67	1.29	Suspended free silver coinage
1879[1]	23.22	-	-	20.67	-	Resumption of a single gold standard
1913	23.22	-	-	20.67	-	Fed was founded

what was the matter. They, determined to have some gold in their pockets, but it can not be affirmed that Congress had reached a scientific conclusion in favor of the single gold standard. What is certain is that Congress adopted the ratio of sixteen to one in 1834 by very large majorities in spite of proofs urgently presented that this ratio would drive silver out of circulation altogether, as it did. This bill was called the “Gold Bill” in the discussions of the time. As reported by the special committee, it provided for a ratio of fifteen and sixty one-hundredths to one, but when it came up for discussion Mr. Campbell P. White, the chairman of the committee, who favored the single gold standard, moved to amend by making the ratio sixteen to one, and his amendment was adopted without a division. On the main question, the debate was long and animated. An amendment to the amendment was offered making the ratio 15.625 to one, and it was supported on the ground that this was the true market ratio and that it would enable the country to keep both silver and gold in concurrent circulation. That was what the House did not want. This amendment was voted down; yeas 52, nays 127. The bill was then passed in the House by 145 to 36, and in the Senate by 35 to 7.

There was a variety of motives leading to the passage of the gold bill, but among these the desire of having gold in place of silver was the most influential. Thomas H. Benton, one of the strongest advocates of the measure, declared that the object of his endeavors was: To enable the friends of gold to go to work at the right place to effect the recovery of that precious metal *which their fathers once possessed*, which the subjects of European kings now possess, which the citizens of the young republics to the south all possess, which even the free negroes of San Domingo possess, but which the yeomanry of this America have been < deprived of for more than twenty years and will be deprived of forever unless they discover the cause of the evil and apply the remedy to its root." (Speech of Senator Benton of Missouri, quoted by Louis R. Ehrich in his " Question of Silver.")”

On the other hand, Friedman (1990) believe the political reasons behind this ratio change played an important roll. This was at the height of the famous "bank war" between President Andrew Jackson and Biddle, which finally resulted in the failure of the bank to obtain a new charter when its original federal charter expired in 1836. the select committee for the Gold bill of 1834, As O’Leary (1937) put it, the ratio of 16 to 1 was "a golden club ... used by Jackson and his supporters to belabor their hated enemy, the Bank" (p.84). The unsatisfactory state of the currency-a mixture of U.S. and foreign silver coins plus paper money issued by state banks, some of doubtful quality-had made the notes issued by the bank a favored medium of exchange. The act of 1834 was expected to weaken the bank by making gold coins an effective substitute for its notes.

1.3 The “Crime of 1873” and Its International Background

1.3.1 The “Crime of 1873”

As much as the populist or inflationist loved the Bimetallism, it faces some inherent problems. Most importantly, the bullion market price is changing quickly, but the legal ratio adjustment is relatively slow and costly under the assumptions of nominal rigidity and adjustment cost.

As mentioned above, the Coinage Act of 1834 that switched the legal ratio between gold and silver to 16:1 had virtually driven silver dollars out of circulation. Friedman & Schwartz (1963) explained that because the market price of silver had been higher than the mint price, the silver dollar had not been in circulation since 1836, and was an unknown coin to Americans. Moreover, during the Civil War, the depreciation of greenbacks caused not only the disappearance of gold from circulation but also the disappearance of subsidiary silver coinage. As greenbacks depreciated in value, the holders of half dollars, quarters, and dimes began to hoard them or ship them to Canada in exchange for gold that could be used to buy depreciated greenbacks. As a result, subsidiary coins over a penny virtually disappeared from circulation. Therefore, when the Office of the Comptroller of the currency reviewed the coinage and mint laws in 1870, it recommended dropping the silver dollar from the coinage: a change equivalent to declaring that silver was no longer usable as money since the silver dollar was the major silver coin being minted. It seems to have been generally accepted that the demonetization of standard silver dollars simply gave legal recognition to the fact that US had been on de facto Gold standard for a long time, and silver spokesmen in congress did not oppose the legislation. As a result, the proposal was adopted by Congress in the Coinage Act of 1873. However, in the next following 2 to 3 decades, the US government was facing constant pressures of "free silver", and made several compromises. (Bland-Allison Act, Sherman Silver Purchase Act etc.)

Originally, the mint valued silver at \$1.292 an ounce, ever since the first coinage act of 1792 (see Table.1). However, the price of silver began to fall in 1872, a fall which turned out to be the beginning of a sharp secular decline. By 1874 the price of silver had fallen to \$1.238 per ounce. The reasons for the price decline seem fairly clear: on the supply side, rich new mines were opened in the American West, and there was a world-wide increase in productivity; on the demand side, number of European countries shifted from a sliver or bimetallic to a gold standard and sharply reduced their monetary use of silver.

By 1875, U.S. silver producers discovered to their distress that, while it would then have been profitable to bring silver to the mints for coinage under earlier legislation , they were debarred from doing so by the Act of 1873. They therefore branded the the Coinage Act of 1873 as the "Crime of 1873", alleging that the provision for dropping the standard silver dollar had been secretly introduced into the Act as a result of a conspiracy of Eastern bankers and legislators.

A second question can be raised here: is the Coinage Act of 1873 really a "crime" in a economic sense? How did the demonetization affect the output and price level? This paper will study this question both in a closed economy DSGE model and in an international setting.

1.3.2 “Scramble for Gold”

Internationally, in 1870’s, most industrialized nations had moved to gold: Germany first switched from silver to gold standard in 1872 . In 1873, France, the largest bimetallic country in the world, and its alleys in Latin Monetary Union (Belgium, Italy, and Switzerland), limited silver coinage in a bid to avoid the consequences of Gresham’s Law (i.e., “swallowing" German silver and losing its entire gold circulation). This was a departure from strict adherence to bimetallism. Though from 1873 until 1876 French officials said that they were very likely to return to full-fledged bimetallism, that hope hinged on the possibility of reviving bimetallism in the face of the subsequent move to the gold standard by many other countries. Then Germany was followed by Norway (1873), Sweden (1873), Denmark (1873), Holland (1875), Finland(1877), and United states (1879).

1.3.3 Reasons of Preference for Gold standard

Three major “structural reasons” for an international preference for gold standard can be summarized as follow: (1) a growing ideological attraction to gold and aversion to silver, (2) industrialization and economic development, and (3) changes in political power structures. Schumpeter (1954) first explained the international “scramble for gold” phenomenon in 19th century with a “non-economic” factor: the quest for monetary “prestige”. Gallarotti (1993) summarized this “non-economic” factor as “ideology for gold” and argued with evidences that this is one main reason for the international phenomenon of “scramble for gold”.

Schumpeter (1954) argued, when we realized that gold monometallism as a standard had become a “symbol of sound practice and badge of honor and decency,” and that national monetary authorities were compelled by the “admired example of England.” The ideological status of gold derived disproportionately from the British example, who had adopted the single gold standard in 1798. That Great Britain was not only a monetary role model but also an economic-policy role model in the 19th century is generally acknowledged in the historical literature on the period. On all dimensions of monetary policy, nations would “ turn to England for financial wisdom.” Nations prevailingly believed that the gold bloc were characterized by nations that were “civilized, rich, and active” and a silver bloc characterized by “less advanced” nations.

Laughlin (1886); White (1893); Helfferich (1927) all accounted for the scramble of the 1870s as part of a general monetary evolution that continually causes inconvenient monies to be replaced by more convenient monies.

As the economy developed, people are in greater needs of using metals of superior “portability and density” to fit their increasing daily transaction amount. Silver was “bulky and inconvenient”. Also the enormous growth of trade in the middle decades of 19th century naturally conferred a greater attractiveness onto the superior trade-clearing metal. The growth in trade also served to compel nations toward Great Britain’s standard because most of the world’s trade was cleared in London. Clough & Cole (1946) believed the growing attraction of gold over silver party reflected changing political power structures across the nineteenth century. A rising urban-capitalist class (professionals, business, banking) was displacing an agricultural class (farmers and landowners) in the political hierarchy, and the monetary victory of gold over silver and bimetallism was in many ways coterminous with the political victory of the bourgeoisie.

1.4 The hope of resurrection of Bimetallism in 1880’s and 1890’s

The international ideological preference for gold as the basis for the currency that culminated in the universal call for adoption of the gold standard at the International Monetary Conference of 1867 waned in the deflationary days of the late 1870s and 1880s. Between the time of the suspension of free silver in the Latin Union and the end of the nineteenth century several international conferences were convened that tried to revive the international bimetallic system under a uniform currency system (Paris 1878 and 1881, Brussels 1892). Even though many countries favored a return to free silver in some form or another, no agreement could be reached on the size, weight, and value of the standard coin, or the gold-silver ratio to be adopted in the system. No country by itself- including the Latin Union as a group - dared to go at it alone to restore free silver, for fear of losing its gold. Only a universal shift to bimetallism was perceived as feasible.

Table 2: Gold and Silver In and Outside the Treasury

Year	Held in Treasury Gold	Silver	In Circulation Gold	Silver	Total Gold	Silver
1879	120	28	90	8	210	36
1880	118	39	198	25	316	64
1881	157	23	282	68	439	91
1882	143	33	319	87	462	120
1883	138	39	359	108	497	147
1884	134	39	365	136	499	175
1885	120	64	421	140	541	204
1886	157	93	384	140	541	233
1887	187	69	414	198	601	267
1888	194	43	457	256	651	299
1889	187	22	438	311	625	333
1890	190	16	449	353	639	369
1891	118	17	469	365	587	382
1892	114	5	483	384	597	389
1893	95	6	424	383	519	389
1894	65	15	482	378	547	393
1895	108	29	439	372	547	401
1896	102	36	400	383	502	419
1897	141	31	450	410	591	441

Source: Friedman & Schwartz (1963) Page 130 Table.5

One can only speculate as to the possible effect of continued free silver in the Latin Union after 1873 on the monetary policies of the other main financial powers. Perhaps many of the "followers" that took the lead of France and Belgium and demonetized silver after 1873, such as Switzerland, Italy, the Netherlands, and Spain, would not have switched to gold. Continued bimetallism in a major part of the Western world might even have enticed the United States at some point after 1878 to reverse its 'Crime of 1873' that demonetized silver.

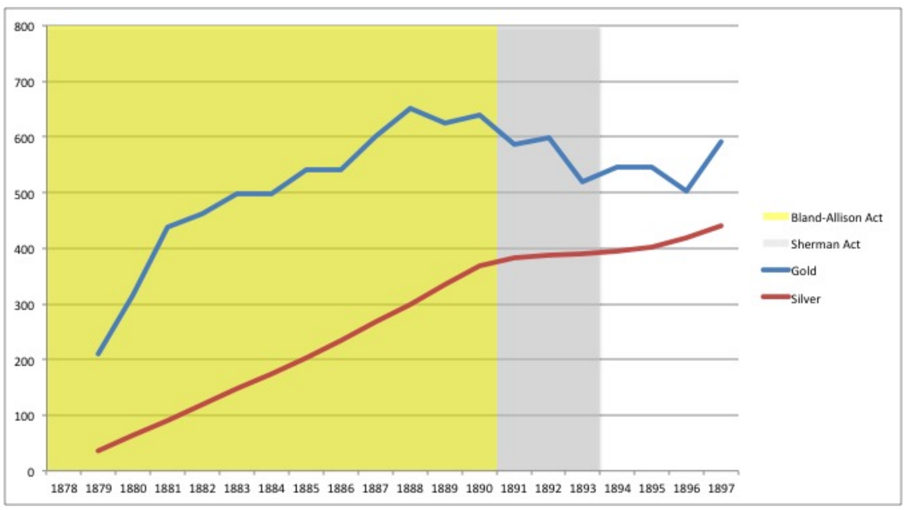
Domestically, while silver producers demanded as a remedy the free and unlimited coinage of silver at the ratio of sixteen to one, debtor farmers in the Middle West and South, who had no interest in a higher price for silver, joined the silver producers, in the belief that "free coinage " or "free silver", as they termed it, would increase the money supply and thereby lower the real burden of their debt. Greenback party adherents also accepted this argument with the belief that putting new silver dollars into circulation would be just as effective in increasing the money stock as issuing more greenbacks.

Atack & Passell (1994) pointed out considerable political pressure, described in previous section, was put on Congress and the executive to restore bimetallism. These pressures were resisted, but there were compromises. In 1878 Congress overrode a presidential veto approving the purchase of between $2 million and $4 million of silver bullion per month at market prices for coinage into silver dollars at the 16:1 ratio (the Bland-Allison Act). The silver dollars so minted were to be full legal tender, but the silver certificates that were also issued are not, although they were receivable by the government. During the twelve years that this law was in force, the Treasury bought 291.3 million ounces of silver for $308.3 million and coined $378.2 million. The difference between the expenditure and the value of the coinage represents the seigniorage received by the Treasury.

Then The Sherman Silver Purchase Act of 1890 directed the Treasury to buy 4.5 million ounces of silver a month at market prices using specially printed Treasury notes (" Treasury Notes of 1890"). These were full legal tender and redeemable in silver or gold. but the continuous decreasing silver price jeopardized the US with "silver risk". US soon found itself facing a run on gold. In early 1890 the treasury's gold balance had been almost $200 million, but on April 22, 1893, the gold reserves dipped below $100 million for the first time since payment in specie had been resumed in 1879 (2). In August President Cleveland called Congress back for a special session at which he pleaded for the repeal of the Sherman silver purchase Act, warning that it jeopardized go gold as the de facto monetary standard, reinforcing fears of the "silver risk".

Figure.2 showed the effect of these two Silver Purchase Acts. As one can easily found, even after 1879, when the silver was forever demonetized, the total silver in and outside treasury increased drastically due to the Silver Purchase Acts. Gold in and outside treasury was staggering increasing in1880's, and when the Sherman Act kicked in, it even decreased.

Figure 2: Total Gold and Silver in and outside Treasury (Friedman & Schwartz (1963))



Chapter II

Closed Economy DSGE Model

2.1 Baseline Model

I adopted the DSGE framework to study the two questions I raised earlier in order to find out the impact of these two monetary policy changes on other macro variables. The first model I used is the Gali (2008) model. There is neither government nor capital in this model. In addition to the baseline model, I modified Bernanke, Gertler & Gilchrist (1998) model to fit my research needs. Detailed modifications will be specified in the following section.

The baseline model is the conventional trinity model but substituting the Taylor Rule by an AR(1) Money Sequence with exogenous money supply shock. Here, only the log-linearized equilibrium equations are listed. For a detailed derivation, please refer to Gali (2008, Chapter.3).

- New Keynesian Phillips Curve

$$\pi_t = \beta * E_t\{\pi_{t+1}\} + \kappa * \widetilde{y_t} \tag{II.1}$$

- Consumer's Euler's Equation Combined with Market clearing condition ($y_t = c_t$)

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} + log(\beta)) \tag{II.2}$$

- Real Balance Definition ($l_t \equiv m_t - p_t$)

$$l_{t-1} \equiv l_t + \pi_t - \Delta m_t \tag{II.3}$$

- Money demand ($y_t = c_t$)

$$m_t - p_t = y_t - \eta i_t \tag{II.4}$$

, together with two AR(1) processes of tech (z_t) and money growth (Δm_t) we can close the model.

2.2 BGG Model

For the auxiliary model based on Bernanke Gentler and Gilchrist (1999), two modifications have been made. First I change the household utility function and allowed household to derive utility from holding real balance.

$$U = \frac{c^{1-\sigma}}{1-\sigma} + \frac{(M/P)^{1-\phi}}{1-\phi} - \frac{h^{1+\zeta}}{1+\zeta} \tag{II.5}$$

The other change that I made was substituting the Taylor Rule originally in their model, by an AR(1) money supply sequence with exogenous money supply shock. The log linearized equilibrium is listed below. For detailed derivation please refer to Bernanke, Gertler & Gilchrist (1998).

1. Aggregate Demand

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c_t^e + \cdots + \phi_t^y \tag{II.6}$$

$$\sigma c_t = -r_{t+1} + \sigma E_t\{c_{t+1}\} \tag{II.7}$$

$$c_t^e = n_{t+1} + \cdots + \phi_t^{c^e} \tag{II.8}$$

$$E_t\{r_{t+1}^k\} - r_{t+1} = -\nu[n_{t+1} - (q_t + k_{t+1})] \tag{II.9}$$

$$r_{t+1}^k = (1-\epsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \epsilon q_{t+1} - q_t \tag{II.10}$$

$$q_t = \psi(i_t - k_t) \tag{II.11}$$

2. Aggregate Supply

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{II.12}$$

$$y_t - h_t - x_t - \sigma c_t = \eta^{-1} h_t \tag{II.13}$$

$$\pi_t = E_{t-1}\{\kappa(-x_t) + \beta\pi_{t+1}\} \tag{II.14}$$

3. Evolution of State Variables:

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{II.15}$$

$$n_{t+1} = \frac{\gamma RK}{N}(r_t^k - r_t) + r_t + n_t + \cdots + \phi_t^n \tag{II.16}$$

4. Demand of Real Balance and Exogenous Shocks

$$m_t - p_t = \frac{\sigma}{\phi} c_t - \varepsilon r_{t+1}^n \tag{II.17}$$

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \tag{II.18}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{II.19}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{II.20}$$

2.3 Data

For the ratio change in 1834, I used the annual data of output, inflation rate and money supply from 1834 to 1872. As to the question of demonetization of silver in 1873, the quarterly data are available from Fagan, Lothian & D. (2013) for year 1873 to 1913. However to keep consistent and comparable with pre-1873 data, I still mainly used the annual data of these same variables for the post-1873 period. I also used the quarterly data to estimate the parameters, and the results are very similar to those with the annual data. All the annual data is from Historical Statistics of United States: Millennial Edition. The reason I choose to let my study period stop at 1913 is that the exogenous money supply assumption would be no longer reasonable after 1914, when Fed reserve was established.

2.4 The Effect of Ratio Change in 1834

Firstly I used the data available to estimate both models and then used the posterior mode of the estimated parameters to plot the impulse response function on a 35 years horizon (See Appendix). What is the effect of the legal ratio change in 1834? Two models give us consistent predictions. Quantitively, the ratio change will increase the money supply, and this should cause an increase in GDP, investment, and inflation, and a decrease in real interest rate. Table.3 shows the full information Bayesian estimation results for the Baseline Model Using annual data from 1834-1872. The purpose is to estimate during this periods the long run parameters. The posterior standard deviation of exogenous monetary shock is 10.38%. As shown in Figure.11, the baseline model predicts that one standard deviation 10.38% of exogenous monetary shock would boost the GDP growth by 4.3%, inflation by about 5.5%, and decrease real capital's rate of return by 3% in the short run. The Auxiliary model gives us similar prediction. Table.4 shows the full information Bayesian estimation results for auxiliary model, As shown in Figure.15, the Auxiliary model predicts that one standard deviation (10.48%) of exogenous monetary shock would boost the GDP growth by 3%, investment by about 6%, and inflation by about 5.3% in the short run.

Therefore to estimate how the legal ratio changing from 15:1 to 16:1 affected the economy, we just need to estimate this policy equivalent money supply change in 1834. The total species in circulation of year 1833 was 30, 650, 000, and the total money in circulation (including all species and notes) was 122,150, 000. For year 1834, the numbers are 41, 000, 000 and 135, 839, 570, respectively. Because the coinage act of 1834 was passed by the United States Congress on June 27, 1834. I simply take the average of these two years numbers. Also I have the data for US Gold and silver Coinage from 1789. The average ratio of silver to all species coinage from 1789 to 1833 is 76.33%. Therefore I can estimate that changing the legal ratio from 15 to 16 is equivalent to a 0.4382% monetary shock. The baseline model predicts it will cause a 0.1815% in GDP growth, and a 0.1266% increase in inflation. The Auxiliary model predicts it will cause a 0.1266% in GDP growth, and a 0.2237% increase in inflation. As we can see the direct effect of the ratio change in 1834 on macroeconomic performance is relatively small.

2.5 Counterfactual Analysis for the demonetization of silver in 1873

A counterfactual analysis was performed to examine the effect of demonetization of silver by comparing the actual macroeconomic variable versus the hypothetical ones if the silver were not demonetized. To achieve this goal, I first estimated the baseline and Auxiliary model for the period from 1879 to 1913 using both quarterly and annual data. (Table.6 and 7). For baseline model, both quarterly and annual

Table 3: Baseline Model Bayesian Estimation (1834-1872)

	Prior			Post.	
Para.	Dist.	Prior Mean	Std.	Post. Mode	Std.
α	Normal	1/3	0.1	0.3333	0.1000
β	Normal	0.95	0.02	0.9571	0.0201
θ	Beta	0.5	0.1	0.5	0.1066
η	Normal	4	1	4.5310	0.9501
ϵ	Normal	6	1	6	1.0000
ψ	Normal	2	0.1	2	0.1000
σ	Normal	1.25	0.1	1.4006	0.0960
ρ_z	Beta	0.5	0.2	0.6539	0.1122
ρ_m	Beta	0.5	0.2	0.0676	0.0468
σ_z	Inv. G	0.01	2	0.0751	0.0090
σ_m	Inv. G	0.01	2	0.1038	0.0109

Table 4: Auxiliary Model Bayesian Estimation (1834-1872)

	Prior			Post.	
	Dist.	Mean	SD	Mode	S.D
α	beta	0.4	0.1	0.4325	0.1078
β	beta	0.95	0.02	0.9586	0.0169
η	norm	4	1	3.7513	1.0580
ε	norm	4	1	5.1536	0.9544
Ω	beta	0.98	0.02	1	0.0254
δ	beta	0.5	0.2	0.8748	0.1177
θ	beta	0.5	0.1	0.4885	0.1042
ρ_A	beta	0.5	0.2	0.4313	0.1283
ρ_G	beta	0.5	0.2	0.4297	0.1310
ρ_M	beta	0.5	0.2	0.1327	0.0672
ν	beta	0.5	0.2	0.4650	0.1948
σ	norm	1.25	0.1	1.3914	0.0826
ϕ	norm	1.25	0.1	1.0738	0.0924
ψ	beta	0.25	0.1	0.3277	0.1081
γ	beta	0.973	0.01	0.9723	0.0119
ε_G	invg	0.01	2	0.1808	0.0306
ε_A	invg	0.01	2	0.1836	0.0443
ε_M	invg	0.01	2	0.1048	0.0111

data give similar Bayesian estimations except the autocorrelation for TFP (0.7210 versus 0.1758), which makes sense because the shorter the period get, TFP tends to be more autocorrelated. The standard deviations of monetary shock are 0.0401 and 0.0439 respectively. For auxiliary model, both quarterly and annual data give similar Bayesian estimations except the auto-coefficients for government spending and money supply (0.7833 versus 0.1797, 0.0152 versus 0.1497). Bayesian estimation using quarterly data is probably more accurate and make more economic sense because government policy should be more correlated in a shorter period, and money supply should be less correlated, because by assumption, money supply is exogenous and not affected by gov. policy. Nevertheless, both models give similar estimations (about 4%) for the standard deviation of exogenous monetary shock either using quarterly data or annual data.

Table.5 compares the two different periods' (pre-1873 and post-1873) annual variance decompositions. we can observe that the TFP shock (ϵ_a) are the first contributor to both variations of output (46.36% for pre-1873, 63.1% for post-1873) and inflation (86.65% for pre-1873, 91.33% for post-1873) in both periods. However, monetary shock explained for the post-1873 output variance (32.90%) about two times of that in post-1873 period (17.52%). And it explained for the pre-1873 inflation variance (12.42%) about two times of that in post-1873 period (6.42%) as well. This delivers a message that the post-1873 period output was mostly caused by a rapid technological development rather than monetary policy shocks, and indirectly proves that monotonic gold standard can provide a much steadier output growth and inflation.

Counterfactually, I substituted the post-1873 annual money supply with the pre-1873 one to simulate a scenario that free coinage of silver was not suspended hence money supply was unchanged. Table.8 compared actual variance v.s. simulated and counterfactual variances of the two models. From 1879 to 1913, the actual variance of output is 3.64% and variance of inflation is 3.06%. By comparing the baseline model (abbreviated to "Base. Bayes. Est.") and auxiliary model estimations (abbreviated to "Aux Bayes. Est."), we can tell the former one is closer to the fact. Because the simulated variance of output (inflation) from auxiliary model is almost 2 (3) times of the actual one. While the simulated variances in output and inflation are 3.63% and 3.17% respectively.

Table 5: Two Periods Variance Decomposition Comparison

	Pre-1873			Post-1873		
	ϵ_m	ϵ_g	ϵ_a	ϵ_m	ϵ_g	ϵ_a
y	32.90	20.74	46.36	17.52	19.38	63.1
c	18.89	0.72	80.39	12.88	1.29	85.83
i	74.14	5.80	20.07	42.37	1.56	56.07
ce	32.42	7.43	60.15	30.14	6.42	63.44
n	32.42	7.43	60.15	30.14	6.42	63.44
rk	26.03	5.99	67.98	17.03	11.01	71.96
ir	11.29	3.05	85.67	2.15	4.74	93.11
q	51.28	5.49	43.23	48.7	6.22	45.08
k	75.62	5.82	18.57	41.56	0.97	57.47
x	8.21	1.32	90.48	3.42	2.78	93.8
h	3.68	5.19	91.13	1.96	7.91	90.13
infl	12.42	0.93	86.65	6.42	2.25	91.33
rn	20.08	0.43	79.50	15.67	0.58	83.74

Table 6: Baseline Model Bayesian Estimation (1879-1913)

	Prior			Quarterly	Post.	Annually	Post.
	Dist.	Mean	SD	Mode	S.D	Mode	S.D
α	norm	0.333	0.1	0.3333	0.0907	0.3333	0.1010
β	norm	0.95	0.02	0.9589	0.0207	0.9524	0.0204
θ	beta	0.5	0.1	0.5	0.1011	0.5	0.1015
η	norm	4	1	5.1784	0.8199	4.1293	1.0292
ϵ	norm	6	1	6	1.0279	6	1.0300
ϕ	norm	2	0.1	2	0.1003	2	0.1009
σ	norm	1.25	0.1	1.5242	0.0919	1.3051	0.0954
ρ_z	beta	0.5	0.2	0.7210	0.0678	0.1758	0.1216
ρ_m	beta	0.5	0.2	0.0120	0.0119	0.0995	0.0607
e_z	invg	0.01	2	0.0398	0.0029	0.0418	0.0059
e_m	invg	0.01	2	0.0401	0.0024	0.0439	0.005

Table 7: Auxiliary Model Bayesian Estimation (1879-1913)

	Prior			Quarterly	Post.	Annually	Post.
	Dist.	Mean	SD	Mode	S.D	Mode	S.D
α	beta	0.4	0.1	0.4671	0.0624	0.5281	0.1026
β	beta	0.95	0.02	0.9517	0.0229	0.9594	0.0189
η	norm	4	1	3.7923	0.9954	3.72	1.0783
ε	norm	4	1	5.9432	0.8642	4.4702	0.9945
Ω	beta	0.98	0.02	1	0.0237	1	0.0248
δ	beta	0.5	0.2	0.9011	0.0713	0.5985	0.1441
θ	beta	0.5	0.1	0.4958	0.0880	0.5134	0.0969
ρ_A	beta	0.5	0.2	0.2157	0.0738	0.2253	0.1113
ρ_G	beta	0.5	0.2	0.7833	0.0488	0.1797	0.1166
ρ_M	beta	0.5	0.2	0.0152	0.0175	0.1497	0.0769
ν	beta	0.5	0.2	0.7292	0.1329	0.69	0.173
σ	norm	1.25	0.1	1.5018	0.0828	1.3391	0.0856
ϕ	norm	1.25	0.1	1.0041	0.0850	1.1443	0.0864
ψ	beta	0.25	0.1	0.2177	0.1014	0.1902	0.088
γ	beta	0.973	0.01	0.978	0.0086	0.9756	0.0103
ε_G	invg	0.01	2	0.1177	0.0079	0.1674	0.0243
ε_A	invg	0.01	2	0.1073	0.0155	0.1356	0.033
ε_M	invg	0.01	2	0.04	0.0025	0.0433	0.0056

The counterfactual practice using baseline model predicts output and inflation variances would be 5.97% and 6.57% respectively. Compared to the actual ones, both volatilities approximately increased by one-fold. In other words, the economy would be more volatile and price level would be more unstable if the silver were not demonetized. Although the auxiliary model tends to overestimate the variances, two models described here give counterfactual estimation in same direction. This again verifies that single gold standard is better for United States economic development in 1879 to 1913 in that it provided smoother economic growth and steadier price level.

Whether the Bimetallism is stable and desirable is under constant debate. Defenders of bimetallism maintain that concurrent circulation of gold and silver is possible for long periods of time under a wide range of circumstances and that the setting of a legal ratio between gold and silver coins acts to stabilize the market price of the (uncoined) metals around this ratio. Velde & Weber (2000) compute welfare and the variance of the price level under a variety of regimes (bimetallism, monometallism with and without trade money) and find that bimetallism can significantly stabilize the price level, depending on

Table 8: Counterfactual Analysis vs. Actual Data

	$\sigma_{\hat{y}_t}$	σ_{π}	$\sigma_{\Delta m}$
Actual	0.0364	0.0306	0.0460
Base. Bayes. Est.	0.0363	0.0317	0.0441
B.B.E. Counter.	0.0597	0.0657	0.1040
Aux Bayes. Est..	0.0695	0.0989	0.0433
A.B.E. Counter.	0.0937	0.1127	0.1048

the covariance between the shocks to the supplies of metals. Moreover, The proponents of Bimetallism claim that bimetallism reduces fluctuations in the price level due to shocks to the supplies of the metals compared to monometallic standards. On the other hand, bimetallism might be theoretically inefficient when compared to a gold or silver standard if both precious metals have non-monetary uses (Velde & Weber (2000)).

However, according to these two models, My counterfactual analysis demonstrates that if the silver were not demonetized, both of the volatilities in output and inflation would be much higher than the actual ones. In other words, gold standard is better than Bimetallism for the United States economic development in post-1873 period. The Coinage Act of 1873 is not a crime, at least in an economic sense.

Chapter III

US Bimetallism in an international setting

3.1 International Bimetallism

In the previous part, by adopting Gali (2008, Chapter.3) as baseline model and *BGG's model and Bayesian method, also by assuming the ratio change and demonetization are exogenous to this closed economy, I was able to depict the impulse responses of the aggregate real variables to these two monetary shocks and reached to a conclusion that if the silver was not demonetized, the US economy would have experienced a much bumpier path (higher volatilizes in bot output and price level). However, both model are closed economies therefore unable to put the U.S. historical study in an international context. Though opening up the model is theoretically viable, but this strategy has its limitations. Unsurprisingly, even in 1830s, US cannot be assumed as a simple "small" open economy. Although "big" open economy models are well studied (referring to the Euro Zone monetary and fiscal policies literature), a relative large number of free parameters to be calibrated/estimated compared to the limited numbers of available observables would make the model of little forecasting power.

With that being said, there is a rich literature of bimetallic general equilibrium models which allows me to study United States' bimetallism in an international setting. Flandreau (1996a) and Oppers (1996) developed similar general equilibrium models to study whether the global switch to gold standard was inevitable in 1870s. Later, Oppers (2000) and Velde & Weber (2000) worked on dynamic bimetallism models. And most recently Meissner (2015) used a different dataset and a simpler version of Flandreau (1996a)'s model and re-estimated the "limits of bimetallism".

3.2 The Existence of Bimetallism

The direct evidence of the existence of bimetallism in 19th century is limited. As mentioned earlier, United States had clearly experienced alternating effective monometallism according to her mint ratios and Congress records. France, who was the center of 19th century bimetallic system bears the biggest

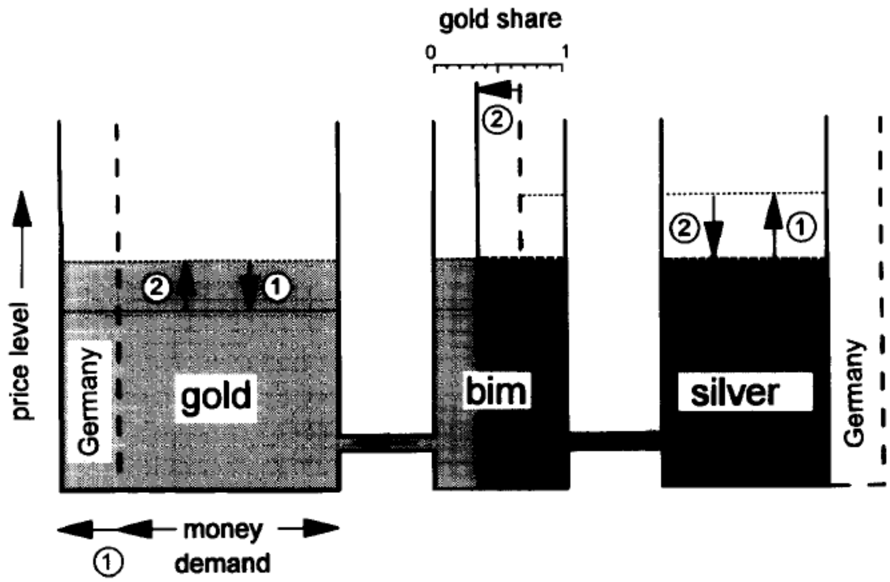


Figure 3: Mechanism of the counterfactual French free silver coinage after Germany demonetized silver

hope, if the de facto bimetallism ever did exist.

With that being said, Economists started to develop theoretical models for this long ago. Fisher (1894) first formally modeled Gresham Law and show the process of "good money driving bad money out of circulation" as follow: When the relative price of gold in the commodity markets dropped below the fixed relative price in the monetary system, arbitrageurs would transfer gold coins from the commodity markets to the monetary system. The resulting decrease in the supply of gold in the commodity markets tended to raise the relative price of gold there, pushing it back towards the mint ratio. An equivalent mechanism operated when the market ratio rose above the mint ratio.

Figure.3 Oppers1 shows the triggered bimetallic arbitrage mechanism when Germany switched to Gold Standard in 1872 if the Latin Union (consisting of France, Belgium, Switzerland, and Italy) continued unlimited free silver coinage. The three containers represent nominal money demand in three areas: a gold standard area with primarily gold coins in the money supply, a silver-standard area with mostly silver coins in the money supply, and a bimetallic area with both gold and silver coins in the money supply. The width of the containers represents real money demand in each area, the height the price level. The light-shaded contents of the containers represent gold, the dark-shaded contents silver.

The containers are connected with pipes that permit bullion to flow between the bimetallic area and the gold and silver areas. When Germany adopt Gold monometallism, France with enough gold coins in circulation therefore would serve as the “buffer stock” and be able to peg the relative gold price around her mint ratio 15.5 through bimetallic arbitrage. The silver supply increased and gold supply decreased in bullion market. Gold coin share in France will decrease and silver share will increase. However, if the buffer stock got exhausted during this adjusting process, France would be rendered de facto gold standard, and the market ratio would start to float. Observing that the market ratio remained in a 6% band around the French mint ratio from 1823 to 1873 (See Figure.1), Friedman (1990) argued gold and silver coins must be circulating side by side,despite the major fluctuations in the relative supplies of gold and silver. Therefore a continuously adjusting relative circulation of gold and silver coins in France acted as an equilibrating mechanism.

Flandreau (1995) examined a coin survey in the 1870s, which showed gold and silver coins with mint dates covering the full periods since 1820 surviving in the money supply at the time. He then concludes these coins must have circulated since their mint dates, proving that Gresham’s Law did not operated and bimetallism was continuously being equilibrated by France. Although his argument is vulnerable to the criticism that the coins may have been hoarded or traded at a premium for periods of time, But his work is the first one and by now the only one with direct evidence to prove that Bimetallism ever existed in France.

The “existence of bimetallism” can be alternatively proved by the “limits of bimetallism” model from Flandreau (1996a) and Meissner (2015). Meissner (2015) concludes that, for the concurrent circulation to happen (i.e. both monetary silver and gold supply are positive for bimetallic bloc), the relative value of world monetary stock of gold and silver has to be limited within a band.

I modify Meissner (2015)’s model by assuming gold bloc and silver bloc output price level are different from bimetallic bloc’s price level. This enables me to study the price effect of monetary policy change in one bloc on itself as well on other blocs.

3.3 Counterfactual Analysis for the Viability of Bimetallism

As I introduced earlier, since 1870's, industrialized countries started to "scrambled for gold". the fate of the bimetallism seemed to rest in the hand of the biggest de facto bimetallic standard country, France. Previously, several counterfactual analysis have been done to check whether France can peg the gold to silver ratio at 15.5:1 and change the deadly fate of Bimetallism if she hadn't quantitatively limited the silver coinage in 1873. Flandreau (1996a) and Oppers (1996) asked if German de-monetization of silver could have sealed bimetallism's fate. Both answers were negative. Meissner (2015) verified their conclusion and maintained that if France hadn't quantitatively limited its silver coinage, it would have faced a complete drain on its gold circulation at the historical mint ratio of 15.5 to one around 1875, and became a de facto silver regime. I did another counterfactual analysis for the case that US and France formed a coalition to endorse the bimetallism and see whether the bimetallism would have survived and how long it could sustain.

3.4 Model

I modified the Flandreau (1996a) and Meissner (2015)'s models and take price level into consideration. The major economies are classified into three blocs by their monetary standards: gold, silver, and bimetallic blocs. Note that all de facto gold standard countries are included in gold bloc, and de facto silver standard countries are included in silver bloc. The only country in the bimetallic bloc is France in that as discussed above the bullion market ratio was pegged around France's legal ratio before 1872 (see Figure.1).The money demand for these three blocs are given by the following equations respectively:

$$p_G M_G^g = k^g p^g Y^g \tag{III.1}$$

$$M_S^s = k^s p^s Y^s \tag{III.2}$$

$$p_G M_G^b + M_S^b = k^b p^b Y^b \tag{III.3}$$

where M_j^i is the monetary demand for metal j (G or S) for standard i, k^i is bloc i's Cambridge coefficient, i equals g for gold, s for silver, and b for bimetallism. p^i is the price level in bloc i, and p_G is the price of one unit of gold in terms of silver. In addition, assume that the nominal outputs in gold and silver blocs had a constant relation to that in bimetallic bloc such that:

$$Y^g = \beta^g Y^b \quad and \quad Y^S = \beta^s Y^b \tag{III.4}$$

Where β_g and β_s are the parameters reflecting relative sizes. In addition as a simplified version of Flandreau (1996a), I only consider the monetary use of gold and silver. The model is closed by equating world gold and silver supply.

$$G = M_G^g + M_G^b \; and \; S = M_S^s + M_S^b \tag{III.5}$$

Where G and S are the total world supplies of gold and silver respectively. Define some compound parameters which can simplify the representations:

$$m_G^m = k^g \beta^g \; and \; m_S^m = k^s \beta^s \tag{III.6}$$

Finally, because we use the bimetallic bloc as numeraires, I set $k^b = k$, $Y^b = Y$.

$$p_G M_G^g = k^g p^g Y^g = k^g p^g \beta^g Y^b = m_G^m p^g Y \tag{III.7}$$

$$M_S^s = k^s p^s Y^s = k^s p^s \beta^s Y^b = m_S^m p^s Y \tag{III.8}$$

$$p_G M_G^b + M_S^b = k^b p^b Y^b = k p^b Y \tag{III.9}$$

plug equation (27) and (28) into (25),

$$p_G G = p_G M_G^g + p_G M_G^b = m_G^m p^g Y + p_G M_G^b \tag{III.10}$$

$$S = M_S^s + M_S^b = m_S^m p^s Y + M_S^b \tag{III.11}$$

Combining (30) and (31) together with equation (29) we have a system of equations:

$$\begin{cases} p_G G = m_G^m p^g Y + p_G M_G^b \\ S = m_S^m p^s Y + M_S^b \\ k p^b Y = p_G M_G^b + M_S^b \end{cases}$$

I further define m_G as the share of the gold bloc's money demand in the world money demand, and m_S as the share of the silver bloc's money demand in the world money demand as expressed as follow

$$m_G = \frac{m_G^m p^g Y}{k p^b Y + m_G^m p^g Y + m_S^m p^s Y} = \frac{m_G^m p^g Y}{p_G G + S} \quad \text{(III.12)}$$

$$m_S = \frac{m_S^m p^s Y}{k p^b Y + m_G^m p^g Y + m_S^m p^s Y} = \frac{m_S^m p^s Y}{p_G G + S} \quad \text{(III.13)}$$

plug them into equation (30) and (31), we have:

$$p_G G = p_G M_G^b + m_G (p_G G + S) \quad \text{(III.14)}$$

$$S = M_S^b + m_S (p_G G + S) \quad \text{(III.15)}$$

rearrange them, we have:

$$\begin{cases} p_G M_G^b = (1 - m_G) \cdot p_G G - m_G \cdot S \\ M_S^b = -m_S \cdot p_G G + (1 - m_S) \cdot S \end{cases}$$

With constrained regression, m_G and m_S can be estimated given the world monetary gold (G) and silver supply (S) and France monetary gold (M_G^b) and silver supply (M_S^b). Notice that:

$$m_G + m_S = \frac{p_G G + S - kpY}{p_G G + S} = 1 - \frac{kpY}{p_G G + S} \quad \text{(III.16)}$$

Therefore given additional information on the France output level (Y), kp can be estimated. Also, $m_G^m p^g$ and $m_S^m p^s$ are the ratios of gold bloc's and silver bloc's money demand to the france's output respectively such that:

$$m_G^m p^g = \frac{M_G^g p_G}{Y} = \frac{m_G \cdot (p_G G + S)}{Y} \tag{III.17}$$

$$m_S^m p^s = \frac{M_S^s}{Y} = \frac{m_S \cdot (p_G G + S)}{Y} \tag{III.18}$$

So $m_G^m p^g$ and $m_S^m p^s$ can be estimated as well.

A proper bimetallic equilibrium requires that both metals circulate in the bimetallic bloc or $p_G M_G^b > 0$, and $M_S^b > 0$, therefore we can derive the upper and lower bounds for the relative gold price in silver $\frac{p_G G}{S}$ as expressed in the following inequalities:

$$\frac{m_G}{1-m_G} < \frac{p_G G}{S} < \frac{1-m_S}{m_S} \tag{III.19}$$

3.5 Comparative Statics for Germany's switching from Silver to Gold Standard

Before 1872 Germany had the single silver standard. However to facilitate commerce and international trade, She used a heterogeneous assortment of gold coins, partly domestic and partly foreign, including napoleons, pistoles, guineas, eagles, Russian commercial money. the question of a reform of the currency had been under discussion by the economists and publicists of Germany for nearly ten years, but until 1868 the question under debate was a question of uniformity of money rather than of the metallic standard.It was not until after the Paris monetary conference of 1867 that the commercial classes began to take an active interest in the question. Nearly all the European Governments were represented in this conference, plus United States. One of the earliest questions to be decided was that of a standard. The first vote was on the question of adopting the single standard of silver. This was rejected unanimously. Then the single standard of gold was adopted with only one dissenting vote-that of Holland. Nobody proposed bimetallism.

On the 5th of November, 1871, the Finance Minister of the new German Empire, Herr Delbruck, presented to the Imperial Diet a brief report of the "motives" which had led the Government to propose a measure for the unification of the German coinage. The measure said because that silver was "bulky and inconvenient" and that it brought about a forced circulation of paper and prevented any wise regulation of bank issues-the single gold standard was recommended, with a silver subsidiary coinage. It was passed on November 23ed, 1871, but was provisional only. And a second and more detailed one was enacted in 1873, since when the Germany started to sell silver in the international bullion market.

When Germany started to sell silver and buy gold from the international bullion market, it increase the gold bloc's gold demand and decreased the silver bloc's silver demand. Assuming the ratio of defection country's total money demand to the bimetallic bloc's money demand is α, then the new values (indicated with $'$) should be:

$$m_G^{m\prime} \cdot p^{g\prime} = m_G^m \cdot p^g + \frac{\alpha kpY}{Y} = m_G^m \cdot p^g + \alpha kp \tag{III.20}$$

$$m_S^{m\prime} \cdot p^{s\prime} = m_S^m \cdot p^s - \frac{\alpha kpY}{Y} = m_S^m \cdot p^s - \alpha kp \tag{III.21}$$

$$m_G{}' = \frac{m_G^{m\prime} \cdot p^{g\prime}}{\frac{(p_G G+S)}{Y}} = m_G + \frac{\alpha kpY}{p_G G + S} = m_G + \alpha(1 - m_G - m_S) \tag{III.22}$$

$$m_S{}' = \frac{m_S^{m\prime} \cdot p^{s\prime}}{\frac{(p_G G+S)}{Y}} = m_S - \frac{\alpha kpY}{p_G G + S} = m_S - \alpha(1 - m_G - m_S) \tag{III.23}$$

and

$$\frac{m_G^{m\prime} \cdot p^{g\prime}}{m_G^m \cdot p^g} = \frac{k^{g\prime} p^{g\prime} \beta^{g\prime}}{k^g p^g \beta^g} = \frac{k^{g\prime} p^{g\prime}}{k^g p^g} \tag{III.24}$$

Assume again in the short run the Cambridge coefficients don't change, then we have:

$$\frac{m_G^{m\prime} \cdot p^{g\prime}}{m_G^m \cdot p^g} = \frac{p^{g\prime}}{p^g} \quad (Gold\ blocs'\ price\ level\ percentage\ change) \tag{III.25}$$

Using data from year 1849 to 1871, we can estimates the parameters as below: $m_G = 0.3429$ and $m_S = 0.1977$, the corresponding upper $\frac{1-m_S}{m_S}$ and lower limits $\frac{m_G}{1-m_G}$ for the bimetallism are 4.0582 and 0.5218. And the the world ratio of value of gold to silver production ($\frac{p_G G}{S}$) was in $[1.3804, 2.6542]$ interval from year 1849 to 1890, which are clearly within the bimetallic limits. $kp = \frac{(1-m_S-m_G)}{\frac{Y}{(p_G G+S)}} = (1 - 0.1977 - 0.3429)/1.140225 = 0.4029$. Adopting estimated $\alpha = 0.31$ (Oppers (1996); Jones & Obstfeld (2000)), we get the new $m_G' = 0.4853$ and $m_S' = 0.0553$, the new corresponding upper $\frac{1-m_S'}{m_S'}$ and lower limits $\frac{m_G'}{1-m_G'}$ are 17.08 and 0.9429. Notice because the increase gold demand the lower limit of bimetallism is pushed up, however, $\frac{p_G G}{S}$ would still be contained in the limits.

If US didn't suspend the species payment in 1873, is would still be on de facto gold, then her price level change when Germany started to sell silver in 1873 can be calculated by equation (45). $\frac{m_G^{m\prime} \cdot p^{g\prime}}{=} \frac{m_G'}{\frac{Y}{(p_G G+S)}} = \frac{0.4853}{1.140225} = 0.4256$ and $\frac{m_G^{m} \cdot p^{g}}{=} \frac{m_G}{\frac{Y}{(p_G G+S)}} = \frac{0.3429}{1.140225} = 0.3007$. Therefore $\frac{m_G^{m\prime} \cdot p^{g\prime}}{m_G^{m} \cdot p^{g}} = \frac{p^{g\prime}}{p^{g}} = \frac{0.4256}{0.3007} =$ 41.54%. This Seems to be unbelievably huge, but after investigate into the London Labour's average wage, it makes more sense. The average labour nominal wage increased about 25% from 1871 to 1873 as a typical gold standard countries ever since 1798. The effect of this German defection is obvious! And if United States didn't suspend the species payment in the same year, she would have probably experienced an inflation as well instead of the actual deflation. In fact, from 1873 to 1979, US experienced a 56% sharp declined in whole sale prices.

3.6 Counterfactual Analysis of the hypothetical coalition between US and France

Previous works *MeissnerBook, Flandreau, Oppers1 have only used French monetary stock as the bimetallism reservoir, and studied if France had firmly endorsed bimetallism and honor her legal ratio, whether the bimetallism could have survived.

It is true that compared to US, Gold as a legal tender in France is less appreciated relative to silver, therefore it's natural for one to imagine the France would be the arbitrage frontier once the Germany switched from bimetallism to Gold Standard in 1872. However, By the time Germany switched, US was still on bimetallism by legislation, (only until 1873, did US switched from bimetallism to fiat standard by suspending the free coinage of silver). Therefore if US committed not to switch or even formed a coalition with France, Bimetallism would have a much bigger chance to survive!

Suppose that United States didn't suspend the free coinage in 1873, and formed a coalition with France to endorse the Bimetallism. Even though in case all the major silver standard countries fled to gold standard and forced France to honor its 15.5:1 legal ratio, and eventually this might deplete France's gold, then gold to silver ratio will be subjected to bullion market supply and demand. When the gold stock in France was depleted, the prices of gold and silver will start to float. When market ratio increased to 16:1, United States as it remained its legal ratio between silver and gold at a 16:1, will play the roll of the second to last resort of this arbitrage. Which makes one wonder if this coalition ever happened, will the fate of bimetallism be rewritten?

To find out this, one only needs to imagine the bimetallic bloc now had two members: France and US with their own legal ratio unchanged. All the parameters specified in section 2 need to be re-estimated with the pooled bimetallic monetary supply. we denoted the hypothetical counterfactual new parameters and variables with asterisks.

$$\begin{cases} p_G \cdot M_G^{b*} = (1-m_G^*) \cdot p_G \cdot G - m_G^* \cdot S \\ M_S^{b*} = -m_S^* \cdot p_G \cdot G + (1-m_S^*) \cdot S \\ m_G^{m*} \cdot p^{g*} = \frac{M_G^{g*} \cdot p_G}{Y^*} = \frac{m_G^* \cdot (p_G \cdot G + S)}{Y^*} \\ m_S^{m*} \cdot p^{s*} = \frac{M_S^{s*}}{Y^*} = \frac{m_S^* \cdot (p_G \cdot G + S)}{Y^*} \end{cases}$$

Therefore all counterfactual m_G^* m_S^* $m_G^{m*} \cdot p^{g*}$ and $m_S^{m*} \cdot p^{s*}$ can be estimated using pooled money supply of US and France from year 1949 to 1871.

$m_G^* = 0.1436$ and $m_S^* = 0.1769$, the corresponding upper $\frac{1-m_S^*}{m_S^*}$ and lower limits $\frac{m_G^*}{1-m_G^*}$ for the bimetallism are 4.6529 and 0.1677. In the event of an international fleeing from Silver to Gold standard featured with Scandinavian Countries, silver bloc's monetary silver demand will decrease which should equal to increased gold demand of gold bloc. Name the ratio of the increased gold demand to bimetallic bloc total monetary demand as β, After the bulky switches, new counterfactual parameters (indicated with ') can be expressed as:

$$\begin{cases} m_G^{m*\prime} \cdot p^{g*\prime} = m_G^{m*} \cdot p^{g*} + \frac{\alpha k^* p^* Y^*}{Y^*} = m_S^{s*} \cdot p^{s*} + \beta k^* p^* \\ m_S^{m*\prime} \cdot p^{s*\prime} = m_S^{s*} \cdot p^{s*} - \frac{\alpha k^* p^* Y^*}{Y^*} = m_S^{s*} \cdot p^{s*} - \beta k^* p^* \\ m_G^{*\prime} = \frac{m_G^{m*\prime} \cdot p^{g*\prime} \cdot Y^*}{p_G \cdot G + S} = m_G^* + \beta(1 - m_G^* - m_S^*) \\ m_S^{*\prime} = \frac{m_S^{m*\prime} \cdot p^{s*\prime} \cdot Y^*}{p_G \cdot G + S} = m_S^* - \beta(1 - m_G^* - m_S^*) \end{cases}$$

$m_G^{m*\prime} \cdot p^{g*\prime}$ increased and $m_S^{m*\prime} \cdot p^{s*\prime}$ decreased, hence $m_G^{*\prime}$ increased, and $m_S^{*\prime}$ decreased. Recall equation (39) depicted the necessary condition for bimetallic to exist: $\frac{m_G^*}{1-m_G^*} < \frac{p_G G}{S} < \frac{1-m_S^*}{m_S^*}$, the change would push up the lower limit $\frac{m_G^*}{1-m_G^*}$ and the upper limit $\frac{p_G G}{S} < \frac{1-m_S^*}{m_S^*}$. The decease of bimetallism (i.e. both France and US's Gold depleted) would be inevitable if β is large enough so that $\frac{p_G G}{S}$ fell below the new lower limit $\frac{m_G^{*\prime}}{1-m_G^{*\prime}}$. From 1873 to 1885, $\frac{p_G G}{S}$ monotonically decreased from 2.1879 to 1.6854. Even with a most conservative estimation with the smallest world gold to silver ratio 1.6854, $\frac{m_G^{*\prime}}{1-m_G^{*\prime}}$ needs to be at least 0.6276. And $1-m_G^*-m_S^* = .6795$. Which implies the smallest β to make global gold standard inevitable is 71.23%. From Oppers (1996); Jones & Obstfeld (2000)'s estimation, All the countries that switched to Gold standard in 1870s, including Germany (1872), Norway (1873), Sweden (1873), Denmark (1873), Holland (1875), and Finland (1877), only took up 29.84% of the total species holding of United states and France altogether. Therefore under this hypothetical coalition, the switched wouldn't jeopardize the existence of bimetallism.

3.7 Conclusion

In this paper, I started from the United States bimetallism. In a general equilibrium setting, two critical events are studied here. The first one is the legal ratio change in 1834, which is treated as a one-time unexpected monetary shock. The second event is the coinage act of 1873 which suspended the species payment and demonetized silver. I modified Gali (2008); Bernanke, Gertler & Gilchrist (1998)'s model by revising household utility function and making the money supply an exogenous sequence.

Table.3 shows the full information Bayesian estimation results for the Baseline Model Using annual data from 1834-1872. The purpose is to estimate during this periods the long run parameters. The posterior standard deviation of exogenous monetary shock is 10.38%. As shown in Figure.11, the baseline model predicts that one standard deviation 10.38% of exogenous monetary shock would boost the GDP growth by 4.3%, inflation by about 5.5%, and decrease real capital's rate of return by 3%. The Auxiliary model gives us similar prediction. Table.4 shows the full information Bayesian estimation results for auxiliary model, As shown in Figure.15, the Auxiliary model predicts that one standard deviation (10.48%) of exogenous monetary shock would boost the GDP growth by 3%, investment by about 6%, and inflation by about 5.3%. Changing the legal ratio from 15 to 16 is equivalent to a 0.4382% monetary shock. The baseline model predicts it will cause a 0.1815% in GDP growth, and a 0.1266% increase in inflation. The Auxiliary model predicts it will cause a 0.1266% in GDP growth, and a 0.2237% increase in inflation. Therefore besides switching US money standard from de facto silver to de facto gold, the effect of the Coinage Act of 1834 on real economy activity is relatively small.

Secondly I studied the effect of the "Coinage Act of 1873" on economy and whether it is a crime in an economic sense. Counterfactually, I substituted the post-1873 annual money supply with the pre-1873 one to simulate a scenario that free coinage of silver was not suspended hence money supply was unchanged. Table.8 compared actual variance v.s. simulated and counterfactual variances by two models. From 1879 to 1913, the actual variance of output is 3.64% and variance of inflation is 3.06%. The more accurate simulated variances from baseline model in output and inflation are 3.63% and 3.17% respectively. The counterfactual practice using baseline model predicts output and inflation variances would be 5.97% and 6.57% respectively. It demonstrates that if the silver were not demonetized, both of the volatilities in output and inflation would be much higher than the actual ones.

In the Second part of this paper, I studied the US bimetallism in an international setting. In the high tide of "scramble for gold", United states' experience was unique. Unlike other industrialized countries that shifted from silver to gold, US first switched from nominal bimetallism to paper standard in 1873, then switched from paper to gold 6 years later. I modify Flandreau (1996a); Meissner (2015)'s model, and found out that if United States didn't enact the "Coinage Act of 1873", its price level could have experiences an sharp increase as much as 40%. This finding can be corroborated by the fact that London's Labour wage had increased almost 30% in 1873.

Furthermore, a hypothesis of bimetallic coalition between US and France is tested. If United States and France had collaborated to endorse bimetallism, to make bimetallism obsolete, we would need a group of defection countries from silver to gold bloc, whose total money demands are as big as 71.23% of United States and France's total money stock. At the same time, the US output price level should be steadier. In fact, the model predicts same price level if this bimetallic coalition was formed.

Both DSGE's and general equilibrium's counterfactual analyses give us same conclusions for the "coinage act of 1873": If US didn't have this Act, and detached its money standard from bimetallism in 1873, the price level (both first moment and second moment) could have sharply increased. And the output growth could have been more volatile. In other words, the “Coinage Act of 1873” is definitely not a crime in an economic sense!

Chapter IV

Gold Discoveries During the US Classical Gold Standard Era

4.1 Introduction

4.1.1 Historical Background for Classical Gold Standard Era

Ever since the Coinage Act of 1834, which had changed the legal ratio of 15:1 between gold and silver to 16:1, legal price of gold was lower than the prevailing gold price on the market. Therefore, even before the silver was demonetized by the Coinage Act of 1873, gold was in effect already the major commodity currency in circulation. Coinage act of 1873 demonetized silver and terminated bimetallism that had been unchanged for ninety years since first Coinage Act of 1792. However, due to the civil war, species payment had been suspended [1] and United States was under a fiduciary standard (greenbacks period) from 1862 to 1878. When the U.S. federal government resumed species payment at prewar parity on January 1st, 1879, American monetary history had formally entered the classical gold standard era. Looking at general economic conditions during the classical gold standard era, we can divide this period into two sub-periods for both international and domestic reasons.

1879 to 1896 can be named “secularly declining period". This sub-period is included in a longer deflation period which historical economists call “The Great Deflation" in 19th Century (1870-1896). Deflation on average was nearly 3% per year; productivity advance was rapid; Real income, money income and money stock were growing[2]. A numerous reasons are accountable for this secular decline in price level: Firstly, domestic gold production slowed down after the initial gold rush started from 1848.

Gold production slowed down from 1878 and didn’t resume to its pre-1878 level until 1898. More prominently, gold discovery experienced a sharp decline in 1876. It increased drastically with a series of gold discoveries in Indiana and Colorado during late 1880s and early 1890s.

Second reason for this secular decline was that an international "scramble for gold"[3] trend started

[1] For details about the species payment suspension and resumption, see spe (2003)

[2] see Page. 94-95 Friedman & Schwartz (1963)

[3] This notion was first put forward by Schumpeter (1954), later advanced by Laughlin (1886); White (1893); Helfferich

in early 1870s, boosted the world demand for gold rapidly. Germany first switched from silver to gold standard in 1871. In 1873, France, the arguably largest[4] bimetallic country coetaneously, and its alleys in Latin Monetary Union (Belgium, Italy, and Switzerland), limited silver coinage to avoid the consequences of Gresham's Law (i.e.,absorbing all German silver and jeopardizing its entire gold circulation). This was the France's first departure from her strict adherence to bimetallism. Although from 1873 until 1876 French officials said that they were very likely to return to full-fledged bimetallism[5]. The signaling effect on worldwide policy makers' confidence in bimetallism was devastating and profound. Following French monetary policy change in 1873, the "flight" to gold began. United States demonetized silver in 1873, followed by Norway (1873), Sweden (1873), Denmark (1873), Holland (1875), Netherlands (1875-1876), and Finland (1877). The world gold demand hence ascended and the aggravated the effect of slowdown in gold production. In addition, rapid increase in productivity and economic outputs also contributed to the price decline in the first sub-period. The logic is simple: more demand and less supply in gold made gold more expensive relative to other commodities. In other words, other commodities are less expensive in unit of gold dollars, which means deflation. The second sub-periods is from the end of first sub-period, 1897 to 1914, the year in which Federal Reserved system was established. Price level in this period rose nearly 50%. Friedman & Schwartz (1963) pointed out "The proximate cause of the world price rise was clearly the tremendous outpouring of gold after 1890 that resulted from discoveries in South Africa, Alaska, and Colorado and from the development of improved methods of mining and refining. The gold stock of the world is estimated to have more than doubled from 1890 to 1914". Bordo et al. (2004) accede to Friedman & Schwartz (1963)'s aforementioned argument that the gold production is an important factor contributing to the second period inflation. Furthermore, Bordo et al. (2004) believe the "V-shaped" price level series over the whole classical gold standard period can be partially explained by the fluctuations in gold production and discoveries. He explained that the first sub-period's long deflation was caused by the slowdown of gold production, which in turn was a result of decline in new gold discoveries. After the gold rush in 1850s, domestic gold discovery was constant and relatively

(1927); Clough & Cole (1946).They analyzed the reasons behind international gold standards from various trade, political and ideological reasons.

[4]Flandreau (1995) shows direct proof of both silver and gold coins in circulations.

[5]See Flandreau (1996b).

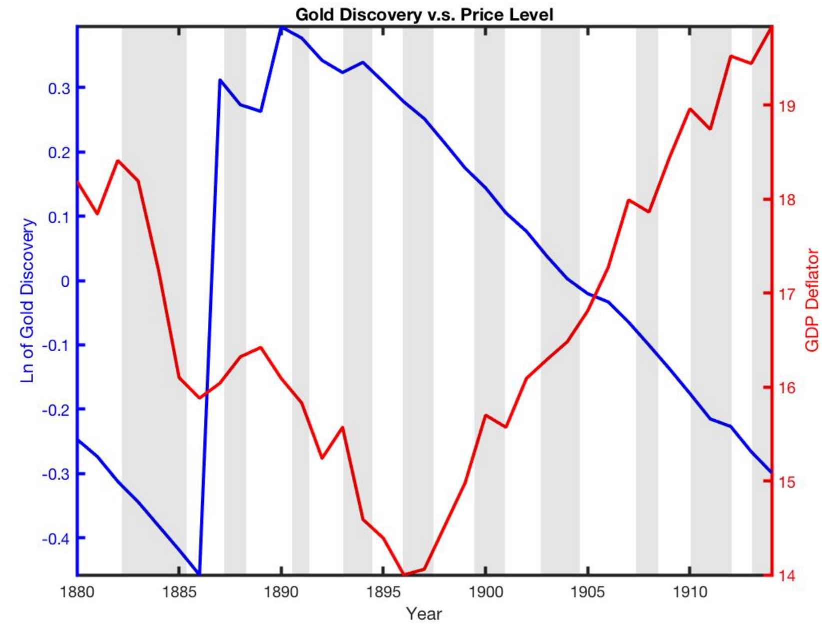


Figure 4: Gold Discovery v.s. Price Level

low from 1870 to late 1880 until it spiked in the early 1890s. Data verifies their hypotheses.Figure 4 depicts the natural logarithm of gold discovery v.s. the GDP deflator in level. V-shape is prominent; negative correlation (with a lag) is significant.

Figure 5 on the next page plots the de-trended gold production growth v.s. the filtered inflation rate over 1880-1914. For the most part, these two series move together except for the early 1890s period. Gold production was steadily increasing. On contrast, inflation was really volatile due to a myriad of economic events such as Silver Purchase Acts, recurrent financial panics, Runs on banks and treasury gold reserves, and international trade conditions. To understand how these external influences caused extra fluctuations in money and price levels, I want to first briefly explain the international environment. For the most time under the classical gold standard, United states was in an “international" gold standard environment[6]. Most countries finished their transitions to gold standard by the end of 19th century except for China, Persia, and parts of Latin America who never joined Gold Standard party. Under such an environment, countries like US that were strictly committed to convertibility with a fixed mint prices

[6]for a more detailed explanation for the mechanism please see Officer (2008)

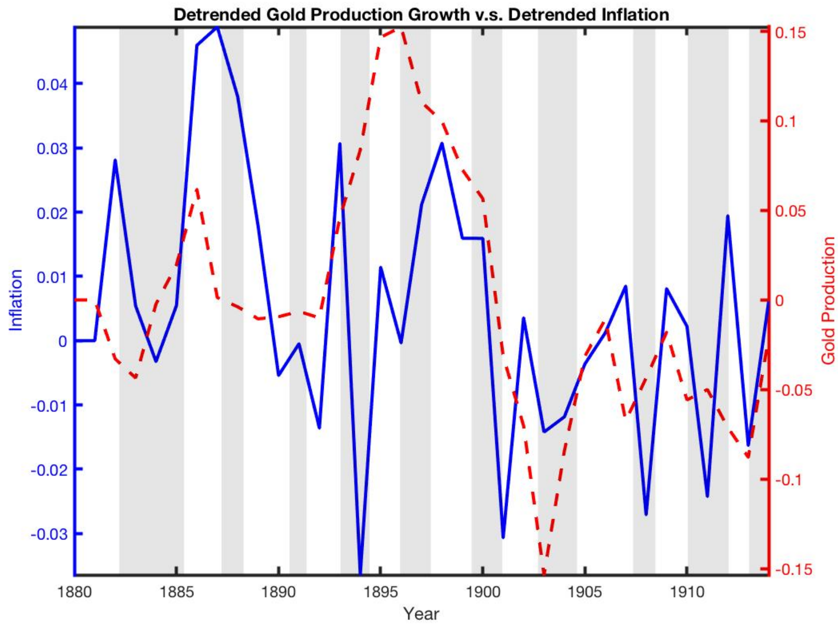


Figure 5: Gold Production v.s. Inflation

of their national currencies implicitly kept "mint parity"[7] and made their domestic stock of money[8] and price level largely determined by a lot of external influences. The influence channel can be demonstrated by the following example: European crops failed in summer of 1891, and American production was the largest that year. This short change in trading conditions created a surplus in balance of payment and gains in gold for United States. Money supply increased, money income expanded, price level rose, exports decreased and imports increased. Eventually balance-of-payments equilibrium would be restored via the current account. This short reversed inflation was reflected in figure 5. It plots the de-trended gold production growth v.s. the filtered inflation rate over 1880-1914. This short reversed inflation caused small ripples around year 1891-1892 during "the Great Depression" when the bigger trend was long deflation.

Whether gold standard was destabilizing? Chernyshoff et al. (2009) considered this question in a open economy setting. They argued that hard regimes like the gold standard limit monetary shocks by

[7]For example, the dollar-sterling mint parity was $4.8665635 per pound sterling (the British pound).

[8]In their book, Friedman & Schwartz (1963) defined "money stock" as the total of currency held by the public, demand deposit and time deposit of commercial banks. This definition is very close to the current M2 definition, and this is also the time series I used as M2 in this study.

tying policymakers' hands; but exchange-rate inflexibility compromises shock absorption in a world of real disturbances and nominal stickiness. They showed how lack of flexibility affects the transmission of terms-of-trade shocks. they also showed evidence from the late nineteenth and early twentieth century US was exposed a dramatic change and concluded that the classical gold standard did absorb shocks, but the interwar gold standard did not, supporting the view that the interwar gold standard was a poor regime choice.

Another monetary policy shock that worth mentioning is the silver agitation and legislation after species payment resumption. Bland-Allison Act of 1878[9] required the Treasury to purchase between $2 million and $4 million worth of silver bullion each month at market prices and put it into circulation as silver dollar until it was replaced by the Sherman Silver Purchase Act of 1890. Under the Bland-Allison Act, silver dollars and silver certificates share in "High-powered money stock"[10] grew relatively slowly and reached its highest value of 17% at 1886. Although there were no significant gold discovery (see figure 6 on page 39) until this date, gold dollar and gold certificates rose steadily to meet domestic money demand. What really hurt the economy was the Sherman Purchase Act that was enacted on July 14th, 1890. It required the Treasury to purchase 4.5 million ounces of silver using Treasury notes of 1890, in addition to the purchase amount stipulated by Bland-Allison Act. From 1890 to 1893, silver dollars and Treasury Notes of 1890 increased $168 millions in total. Ironically, this law made U.S. on gold standard jumped to second place worldwide in silver purchase, right behind India who was on silver standard coevally. These two Silver Purchase Acts definitely affected the stock of money because the silver dollar and Treasury Notes of 1890 were full-fledged legal tender. It actually alleviates the contemporaneous critical money demand. However, the increases in money supply due to the silver purchases didn't affect domestic price level directly like that caused by gold production increase. Because price level is relative to gold price, without affecting world's gold supply and demand, increased silver dollars and Treasury notes will not directly affect price level. However, it shook foreign investors' confidence in United States' maintenance of gold standard. Foreigners were less willing to hold US dollars. The prevailing fear in

[9] Passed on by the February 28th, 1878.

[10] The term was originally named by Friedman & Schwartz (1963) to refer to the total amount of hand-to-hand currency held by the public plus value cash before 1914.

U.S. would be forced off gold standard discouraged capital inflows and caused gold exports, and further enforced the deflation. Within the same three years, monetary gold decreased $168 millions.

Fluctuations in gold discoveries and gold productions undoubtedly played an very important role in affecting the price level. However, its relative importance compared to the positive productivity shock is yet to be determined in a more rigorous structural framework. To my best knowledge, there hasn't been a dynamic stochastic general equilibrium model incorporating gold production and gold discoveries as monetary shocks for the classical gold standard periods. I now want to argue that gold discoveries can be modeled as monetary news shock in a DSGE setting.

4.1.2 Gold Discoveries as Monetary News Shocks

The unique "dual identity" of gold makes gold discoveries different from other natural resources discoveries. On one hand, gold is money with intrinsic value (compared to fiat money system). Any perturbation from gold production would have a real effect on the macroeconomy even without imposing nominal rigidity. On the other hand, gold is also a commodity good, being consumed for decorative, manufactures' and art needs. When the relative price of gold to other consumption goods is cheaper, household will desire more commodity gold. It put upward pressure on the relative gold price. More output will flood into commodity gold, until the relative price returns to its steady state price. In addition, free coinage makes the flow between commodity gold and monetary gold almost frictionless domestically.

I constructed a small-scale DSGE model with a gold production sector in addition to the conventional final good production sector. Conventionally, money supply is treated as exogenous in literature that studying the gold standard era before the Federal Reserve was established. However, in my model, money supply is an endogenous optimal production decision. The gold production sector uses readily producing gold resources and labor as inputs. Output gold goes both into monetary gold stock and non-monetary gold stock. Transition from discovery of new gold deposits to increased gold production typically takes some time, due to regulations and equipment installation and other various reason. Therefore gold discoveries affect real economy with a lag. Figure 6 on the following page verifies the existence of the lag. The gold resource jumped in 1887 when rich deposits were found at assorted sites in Indiana and Colorado, but the production hadn't increased until 1895-1896, corroborating an approximate 5 to 6 years of gap. This feature makes the gold discovery a paradigm of news shocks. More importantly this monetary news shock can be directly observed and quantified. Hence, This is a useful period to study monetary news shocks and their transmission mechanism within a Bayesian DSGE framework.

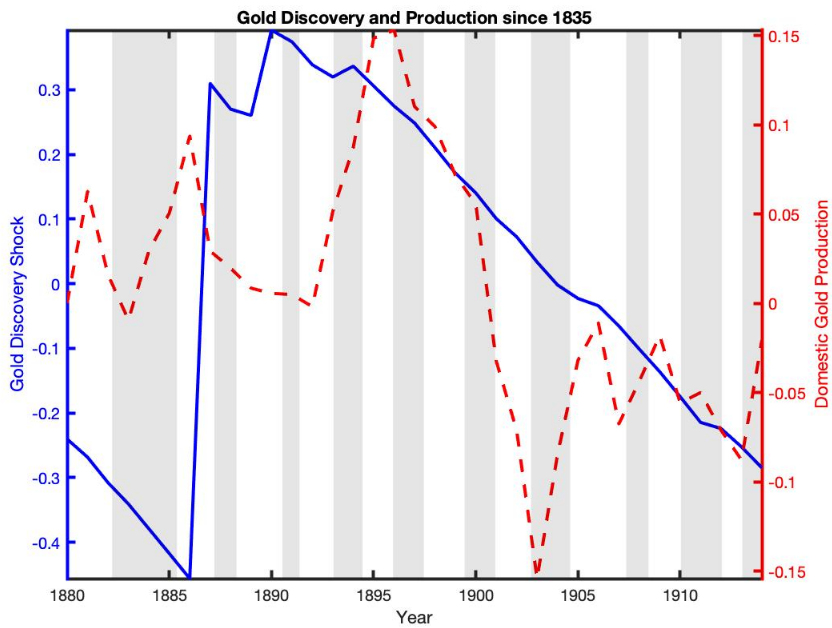


Figure 6: Gold Discovery Time Lag[11]

The news-driven business cycle hypothesis was originally advanced by Pigou (1927) and reincarnated in its modern form chiefly in Beaudry & Portier (2004). They posit that business cycles might arise on the basis of expectations of future fundamentals. If favorable news about future productivity can trigger a boom today, then a realization of productivity which is worse than expected can induce a bust without any actual reduction in productivity itself ever occurring.

News shocks are anticipated shocks with lags as oppose to unanticipated contemporaneous shocks. A news shock denoted as $\epsilon^{j}_{m,t-j}$ means the news arrived at period $t-j$ stochastically. But it is anticipated

[11] Annual gold production in fine troy ounces (in red dash line, using right Y-axis) comes from **?** and has been taken natural logarithm. Detrended growth rate of gold discovery sequence in fine troy ounces (in blue line, using left Y-axis) is compiled and cleaned by myself. Details can be found in data appendix. Original data came from Long et al. (2000). Gold production data is from **?**.

to be zero from period t-j to period t-1, until it is materialized at period t. Notice that there might be other news shocks that arrive during the [t-j, t] period, but these shocks would only affect the economy at a later date. News shock materializes at period t with a size of $\epsilon^{j}_{m,t-j}$.

As the revival of anticipated shocks have gained more attentions from economists, different kinds of news shocks have been introduced to the DSGE model, including monetary news shock. However, most papers study calibrated DSGE models or a bayesian DSGE model without identifying the news shocks directly and examining its empirical importance (e.g., Schmitt-Grohé & Uribe (2012); Gomes et al. (2017); Arezki et al. (2017)).[12]. One work by Arezki et al. (2017) is worth mentioning because the authors construct an oil discovery series, and argued the oil discoveries can by thought of as a directly observable measure of news shock about future output and created a two-sector model including an oil production sector. However they relied on a calibrated DSGE with a deterministic shock series that describes the depletion rate of a single oil discovery normalized to 1% of GDP, and their empirical analysis is restricted to a dynamic panel regression.

Employing a similar methodology to Arezki et al. (2017)'s, I took advantage of a unique dataset of gold and silver deposits in the US that includes the date of discovery and the estimated total deposit size by Long et al. (2000). Zhang (2016) put forward that, after including a TFP news shock, contemporaneous monetary shocks fail to account for an appreciable amount of macroeconomic variation. Instead of only considering the contemporaneous component in monetary shocks as Zhang (2016) does, we will also account for a news component.

With fully expected monetary news shock, economics agents still respond to this anticipated shock. Using the posterior mode value of parameters, one standard deviation of gold discovery explained about 57% of the variance in labor. 65% of variance in gold production; 53% of the variance in real interest rate.54% of the variance in real price of gold relative to other goods.However, it made little contribution to the price level, which is largely explained by the unanticipated contemporary monetary shock. Moreover, because the gold production only calibrated to constitute 1% of the aggregate production, gold discovery doesn't affect the output level very much(0.04%). Yet it significantly affects money supply,

[12] Beaudry & Portier (2006) study the empirical implications of the news shock using the VAR approach but not in a Bayesian DSGE setting

interest rate, relative price level, and employment.

This paper also looses up the "Rational Expectation Assumption" which is an important foundation fo modern DSGE model. I studied two types of adaptive learnings: Constant Gain Learning (CG) and Kalman Filter Learning (KF). Data prefers KF over RE, and CG is the least favorable model.

4.2 Model

4.2.1 Gold

Following Arezki et al. (2017), gold production uses labor and producing gold resource rather than known gold resource. Known resource increases as soon as new gold is discovered. However, it takes time for known resource to become readily producing resource. Gold production function is Cobb-Douglas:

$$Y_{2,t} = A_{2,t} N_{2,t}^{\alpha_2} E_t^{1-\alpha_2} \tag{IV.1}$$

$Y_{2,t}$ is the domestic gold production. $A_{2,t}$ is the gold production sector's Total Factor Productivity (TFP). $N_{2,t}$ is labor. E_t is the producing resource. The stock of producing resource evolves as follows:

$$E_t = X + E_{t-1} - Y_{2,t} + \epsilon_{e,t-j} \tag{IV.2}$$

X is a fixed value of external gold inflows, $\epsilon_{e,t-j}$ is the gold discovery shocks in $t-j$ period. j reflects this time lag between discovery and first production.

4.2.2 Consumption

Production function of other non-durable final goods is as follows:

$$Y_{1,t} = A_{1,t} N_{1,t} \tag{IV.3}$$

Household derives utility from a composite of durable consumption gold stock and other non-durable final goods consumption, defined as:

$$H \equiv (1-\alpha_1)^{\frac{1}{\eta}} C_1^{\frac{\eta-1}{\eta}} + \alpha_1^{\frac{1}{\eta}} D_2^{\frac{\eta-1}{\eta}} \tag{IV.4}$$

Each period's gold consumption is the change in commodity gold's stock minus depreciation:

$$C_{2,t} = D_{2,t} - (1-\delta_D) D_{2,t-1} \tag{IV.5}$$

4.2.3 Money

Following Barro (1979)'s practice, I also assume that the total money supply (M_t^s) is proportional to monetary gold ($G_{m,t}$):

$$M_t^s = \frac{1}{\tau} P_g G_{m,t} + \epsilon_{m,t} \tag{IV.6}$$

where the parameter τ, which satisfies $0 \leqslant \tau \leqslant 1$, measures the gold "backing" of the monetary issue.

4.2.4 Exogenous Shocks

productivity shocks follow AR(1) processes with contemporaneous unanticipated shocks $\epsilon_{a_h,t}(h = 1,2)$.

$$\ln A_{1,t} = \rho_{a_1} \ln A_{1,t-1} + \epsilon_{a1,t} \tag{IV.7}$$

TFP shock for household gold production:

$$\ln A_{2,t} = \rho_{a_2} \ln A_{2,t-1} + \epsilon_{a2,t} \tag{IV.8}$$

Together with $\epsilon_{m,t}$ and ϵ_{t-j}^R, there are four exogenous shocks in my model.

4.2.5 Household Utility Maximization Problem

Social planner's maximization problem can be written as:

$$\max_{\substack{C_t, D_t, N_{1,t}, N_{2,t},\\ B_{t+1}, R_{t+1}, M_t}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Bigg[\frac{\Big[H^{\frac{\eta}{\eta-1}} - \psi N_t^{\theta}\Big]^{1-\sigma} - 1}{1-\sigma} + \gamma_m \Bigg(\frac{(\frac{M_t}{P_t})^{1-\delta} - 1}{1-\delta} \Bigg) \Bigg] \tag{IV.9}$$

subject to Budget constraint:

$$\begin{aligned} Y_{1,t} + \frac{P^g}{P_t} Y_{2,t} + R_{t-1} \frac{B_t}{P_t} - N_{1,t} \frac{\gamma_n}{2} \Big(\frac{N_{1,t}}{N_{1,t-1}} - 1 \Big)^2 - \frac{P^g}{P_t} N_{2,t} \frac{\gamma_n}{2} \Big(\frac{N_{2,t}}{N_{2,t-1}} - 1 \Big)^2 & \\ \geq C_{1,t} + \frac{P^g}{P_t} (D_{2,t} + (1-\delta_D) D_{2,t-1}) + \frac{B_{t+1}}{P_t} + \Big(\frac{M_t - M_{t-1}}{P_t} \Big) & \end{aligned} \tag{IV.10}$$

And Resource constraint:

$$E_t = X + E_{t-1} - Y_{2,t} + \epsilon_{e,t-j} \tag{IV.11}$$

After substituting in $D_{2,t} = C_{2,t} + (1-\delta_D) D_{2,t-1}$, the Lagrangian with budget constraint (equation (IV.10)) and resource constraint (equation (IV.11)) can be written as:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Bigg[\frac{\Big[H^{\frac{\eta}{\eta-1}} - \psi N_t^{\theta}\Big]^{1-\sigma} - 1}{1-\sigma} + \gamma_m \Bigg(\frac{(\frac{M_t}{P_t})^{1-\delta} - 1}{1-\delta} \Bigg) \Bigg] & \\ + \lambda_t \Bigg[Y_{1,t} + \frac{P^g}{P_t} Y_{2,t} + R_{t-1} \frac{B_t}{P_t} - N_{1,t} \frac{\gamma_n}{2} \Big(\frac{N_{1,t}}{N_{1,t-1}} - 1 \Big)^2 - \frac{P^g}{P_t} N_{2,t} \frac{\gamma_n}{2} \Big(\frac{N_{2,t}}{N_{2,t-1}} - 1 \Big)^2 & \\ - C_{1,t} - \frac{P^g}{P_t} (D_{2,t} - (1-\delta_D) D_{2,t-1}) - I_{1,t} - I_{2,t} - \frac{B_{t+1}}{P_t} - \Big(\frac{M_t - M_{t-1}}{P_t} \Big) \Bigg] & \\ + \zeta_t \Bigg[E_t + X - Y_{2,t} - E_{t+1} \Bigg] \Bigg\} & \end{aligned}$$

Full set of Equilibrium and steady state conditions can be found in section A: Appendix.

4.3 Data

All gold discovery data comes from Long et al. (1998).This dataset contains all significant gold deposits in United States, with their discovery years, production spans, total output, and estimated remaining resource. Each new gold discovery can be thought of a monetary news shock. Due to the limitation of maximum extraction (Depletion) capacity, news shock is gradually released to the economy.

Table 9 shows an example of the gold discovery data frame.

Gold Deposit ID	Gold Deposit Name	Discovery Year	Ultimate Recoverable Resource in Ounces	Max Depletion Rate	First Production Year	Last Production Year	Total Production Years
361	Reymert	1876	1,900	1.11%	1887	1977	90
362	Homestake	1876	44,423,996	0.83%	1876	1996	120
363	Keystone	1876	128,500	5.88%	1876	1893	17
364	Copper Queen	1876	2,223,552	0.86%	1880	1996	116
365	Greenhorn	1877	168,000	2.22%	1892	1937	45
366	Gibbonsville	1877	96,000	1.64%	1877	1938	61
367	Golden Reward	1877	1,182,426	0.92%	1887	1996	109
368	Beveridge	1877	1,114,400	2.04%	1877	1926	49
369	Osceola	1877	92,093	1.22%	1877	1959	82
370	Mogul (Horseshoe)	1877	461,250	4.17%	1893	1917	24

Table 9: Example of Estimated URR of Gold Mines

Here I simplify Arezki et al. (2017)'s method to estimate the maximum depletion rate and depletion profile. Based on a general tendency that smaller deposits have higher depletion rate, maximum depletion rate (d_m) is assumed to be:

$$d_m = \gamma \text{URR}^{\delta} \tag{IV.12}$$

γ is estimated to be 0.17, δ is estimated to be 0.85.

Let Q_t represent cumulative production. Remaining recoverable reserves (RRR) is defined as:

$$RRR_t = RRR_{t-1} - Q_t \tag{IV.13}$$

$$\text{Annual Depletion Rate} = \begin{cases} 0 & \text{if } t \leq 5, \\ d_m RRR_t & \text{if } t > 5 \end{cases}$$

Deposit Name	Discovery Year	1st Year RRR	2nd Year RRR	3rd Year RRR	4th Year RRR	5th Year RRR	6th Year RRR	7th Year RRR	8th Year RRR	9th Year RRR	10th Year RRR
Reymert	1876	1.90E+03	1.90E+03	1.90E+03	1.90E+03	1.90E+03	1.88E+03	1.86E+03	1.84E+03	1.82E+03	1.80E+03
Homestake	1876	4.44E+07	4.44E+07	4.44E+07	4.44E+07	4.44E+07	4.41E+07	4.37E+07	4.33E+07	4.30E+07	4.26E+07
Keystone	1876	1.29E+05	1.29E+05	1.29E+05	1.29E+05	1.29E+05	1.21E+05	1.14E+05	1.07E+05	1.01E+05	9.49E+04
Copper Queen	1876	2.22E+06	2.22E+06	2.22E+06	2.22E+06	2.22E+06	2.20E+06	2.19E+06	2.17E+06	2.15E+06	2.13E+06
Greenhorn	1877	1.68E+05	1.68E+05	1.68E+05	1.68E+05	1.68E+05	1.64E+05	1.61E+05	1.57E+05	1.54E+05	1.50E+05
Gibbonsville	1877	9.60E+04	9.60E+04	9.60E+04	9.60E+04	9.60E+04	9.44E+04	9.29E+04	9.14E+04	8.99E+04	8.84E+04
Golden Reward	1877	1.18E+06	1.18E+06	1.18E+06	1.18E+06	1.18E+06	1.17E+06	1.16E+06	1.15E+06	1.14E+06	1.13E+06
Beveridge	1877	1.11E+06	1.11E+06	1.11E+06	1.11E+06	1.11E+06	1.09E+06	1.07E+06	1.05E+06	1.03E+06	1.01E+06
Osceola	1877	9.21E+04	9.21E+04	9.21E+04	9.21E+04	9.21E+04	9.10E+04	8.99E+04	8.88E+04	8.77E+04	8.66E+04
Mogul (Horseshoe)	1877	4.61E+05	4.61E+05	4.61E+05	4.61E+05	4.61E+05	4.42E+05	4.24E+05	4.06E+05	3.89E+05	3.73E+05

Table 10: Annual Remaining

Remaining resource (RR) can be derived as:

$$RR_t = \begin{cases} URR & \text{if } t \leq 5, \\ URR(1-d_m)^{t-5} & \text{if } t > 5 \end{cases}$$

Readily producing resource (RPR) can be derived as:

$$RPR_t = \begin{cases} 0 & \text{if } t \leq 5, \\ d_m * URR(1-d_m)^{t-6} & \text{if } t > 5 \end{cases}$$

Table 10 gives an example of the remaining resource profile for each gold discovery.

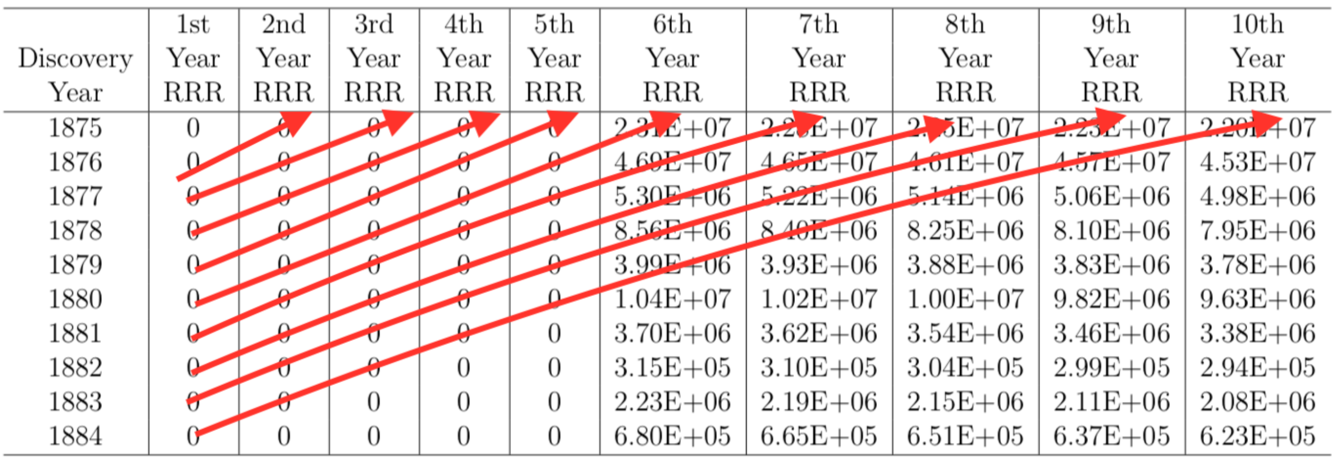
As discussed earlier, gold discovery new shocks happened earlier, will only affect gold production after 6 years. Therefore the first five years RRR have no effect on the economy. Collapse all discoveries by year, 11 demonstrates the remaining resource profile for each year's total gold discoveries. Summing up each years' remaining resource for all previously discovered and producing deposits, we got the annual gold discovery sequence. This process is demonstrated in Figure 7.

4.3.1 Other Sources of Data

Two sets of dataset with different frequencies (quarterly and annual) were used in this study. When I choose the lag by marginal data density, I used the quarterly data because it contained more information. While in the following section about learning, I used annual data. All sequences (Real GNP, GNP

Discovery Year	1st Year RRR	2nd Year RRR	3rd Year RRR	4th Year RRR	5th Year RRR	6th Year RRR	7th Year RRR	8th Year RRR	9th Year RRR	10th Year RRR
1875	0	0	0	0	0	2.31E+07	2.28E+07	2.25E+07	2.23E+07	2.20E+07
1876	0	0	0	0	0	4.69E+07	4.65E+07	4.61E+07	4.57E+07	4.53E+07
1877	0	0	0	0	0	5.30E+06	5.22E+06	5.14E+06	5.06E+06	4.98E+06
1878	0	0	0	0	0	8.56E+06	8.40E+06	8.25E+06	8.10E+06	7.95E+06
1879	0	0	0	0	0	3.99E+06	3.93E+06	3.88E+06	3.83E+06	3.78E+06
1880	0	0	0	0	0	1.04E+07	1.02E+07	1.00E+07	9.82E+06	9.63E+06
1881	0	0	0	0	0	3.70E+06	3.62E+06	3.54E+06	3.46E+06	3.38E+06
1882	0	0	0	0	0	3.15E+05	3.10E+05	3.04E+05	2.99E+05	2.94E+05
1883	0	0	0	0	0	2.23E+06	2.19E+06	2.15E+06	2.11E+06	2.08E+06
1884	0	0	0	0	0	6.80E+05	6.65E+05	6.51E+05	6.37E+05	6.23E+05

Table 11: Production Profile for Gold Mines Discovered Each Year



Discovery Year	1st Year RRR	2nd Year RRR	3rd Year RRR	4th Year RRR	5th Year RRR	6th Year RRR	7th Year RRR	8th Year RRR	9th Year RRR	10th Year RRR
1875	0	0	0	0	0	2.31E+07	2.28E+07	2.25E+07	2.23E+07	2.20E+07
1876	0	0	0	0	0	4.69E+07	4.65E+07	4.61E+07	4.57E+07	4.53E+07
1877	0	0	0	0	0	5.30E+06	5.22E+06	5.14E+06	5.06E+06	4.98E+06
1878	0	0	0	0	0	8.56E+06	8.40E+06	8.25E+06	8.10E+06	7.95E+06
1879	0	0	0	0	0	3.99E+06	3.93E+06	3.88E+06	3.83E+06	3.78E+06
1880	0	0	0	0	0	1.04E+07	1.02E+07	1.00E+07	9.82E+06	9.63E+06
1881	0	0	0	0	0	3.70E+06	3.62E+06	3.54E+06	3.46E+06	3.38E+06
1882	0	0	0	0	0	3.15E+05	3.10E+05	3.04E+05	2.99E+05	2.94E+05
1883	0	0	0	0	0	2.23E+06	2.19E+06	2.15E+06	2.11E+06	2.08E+06
1884	0	0	0	0	0	6.80E+05	6.65E+05	6.51E+05	6.37E+05	6.23E+05

Figure 7: Illustration of Creating Annual Aggregate Production Profile

deflator, commercial paper rate, M2) are from Balke & Gordon (1986). More detailed original data sources can be found in Appendix B.4. Notice that the gold discoveries series I compiled has an annual frequency. Therefore, in the quarterly model I created, I also constructed an annual gold discovery aggregate, which can be observed every 4 quarters. Missing value will be handled by Kalman filter.

4.3.2 Calibration and Steady-state Values

- τ is the ratio of the monetary gold stock to the high powered money, $\tau = 0.51$
- α_1 is the share of durable good (Gold) in the consumption index. It decides the relative size of Y1

and Y2.

- α_2 is the exponent on labor in gold production sector.

- δ_D is the depreciation rate for the durable good;

τ	0.1637	δ	1.1
α_1	0.02	α_2	0.62
σ	1.2	η	1.1
γ_M	0.01	γ_N	2
ψ	0.5	θ	1.4
β	0.9852	ρ_{a1}	0.5
δ_D	0.05	ρ_{a2}	0.5
X	0.0505		

Table 12: Calibration

VARIABLE	MEAN	STD
C1	0.9099	2.4051
C2	0.007	1.5287
D2	0.435	4.106
Y1	0.9099	2.9169
Y2	0.007	0.1548
Y	0.917	2.9834
N1	0.9099	0.0933
N2	0.0039	0.0839
dN1	1	0.0628
dN2	1	11.3233
Z	0.6894	2.4395
N	0.9138	0.0402
G	0.1796	4.55
m	1.3291	35.8942
Pi	0	2.5407
a1	0	3.2603
a2	0	3.2972
Res	0.1457	4.4739
r	0.015	0.0418
pg	1.0095	7.2621

Table 13: Steady-state Value

4.3.3 Lag Selection

As mentioned earlier, the data shows a time lag of six years, So I started from six years, or 24 quarters and compared the log data density with models with five years lag and seven years lag, and the bayesian estimation results favor the six year lag. See table 14.

	Posterior		
	20Q Lags	24Q Lags	28Q Lags
Log Data Density	265.28	413.95	385.41

Table 14: Lag Selection

4.3.4 Bayesian Estimation Result

Figure 8,9,10 are the Bayesian Impulse Response for one standard deviation of gold discovery shock with a lag of 6 years. The unit impulse response are real. Compared to the steady state values showed in Table13.

When one standard deviation of gold discovery shock(e_d) hit the economy, agents form the expectation that the relative gold price (p_g) is going to decreasing in six years. That means the relative price of other commodities is going to increase (π). With this expectation in mind, agents would immediately substitute inter-temporally. They will increase current consumption on other commodities (C_1) and decrease it when the news is materialized. Because the prices are forward looking variables, the relative price of gold (p_g) decrease and general price level for other commodities goods (π) increase immediately. Because the price of gold decrease at current period 0, the consumption gold (C_2) also increases at current period 0. With the prospect of gold price drop, agent would also choose to hold less real balance (m) at hand. Constrained by the gold market clearance condition, the decrease in real balance overcomes the increase in gold consumption, therefore gold production (y_2) slightly decrease at first. and gold production labor demand (N_2) also decreases at period 0. cost labor adjustment is costly, therefore, N_1 increases at period 0. Y_1 increases because N_1 increases at period 0. But at period 6, when all the

	Dist.	Prior Mean	Prior Stdev	Post.Mode	Post.Stdev
σ	norm	1.592	0.2	1.5661	0.0368
τ	beta	0.255	0.05	0.2584	0.0018
α_2	norm	0.734	0.2	0.9088	0.0067
α_1	norm	0.026	0.01	0.0277	0.0006
δ	norm	1.335	0.2	1.3427	0.0145
δ_g	beta	0.021	0.02	0.0322	0
η	norm	1.01	0.75	1.003	0.0001
ρ_{a_1}	beta	0.5	0.1	0.9529	0.0005
ρ_{a_2}	beta	0.5	0.1	0.9528	0.0005
θ	gamm	3.623	0.5	4.1256	0.0613
γ_M	beta	0.019	0.01	0.0359	0.0008
γ_N	beta	0.041	0.01	0.0749	0.0011
ψ	gamm	0.358	0.01	0.3577	0.0005
ϵ^d	invg	0.01	2	0.7809	0.0024
ϵ^{ME}	invg	0.01	2	4.9996	0.148
ϵ^{a1}	invg	0.01	2	1.0249	0.0399
ϵ^{a2}	invg	0.01	2	2.8279	0.2733
ϵ^m	invg	0.01	2	1.0395	0.0572

Table 15: Bayesian Estimation Result with six years News Lag

newly discovered gold hit the economy both N_2 and y_2 significantly increase. N_1 and y_1 significantly increase.because the gold production sector is calibrated as 1% of total GNP, so overall, the composite consumption (H)[13] increases, shadow price of consumption (λ) decrease. from equation (B.12), the decrease in λ overpowered increase in π, interest rate has a really tiny increase first.

[13]For calculation simplicity, I denote $H \equiv ((1-\alpha_1)^{\frac{1}{\eta}} \cdot C_1^{\frac{\eta-1}{\eta}} + \alpha_1^{\frac{1}{\eta}} \cdot D_2^{\frac{\eta-1}{\eta}}$ and $Z \equiv H^{\frac{\eta}{\eta-1}} - \psi \cdot (N^{\theta})$.

Figure 8: Bayesian IRF of One Standard Deviation of Gold discovery (1)

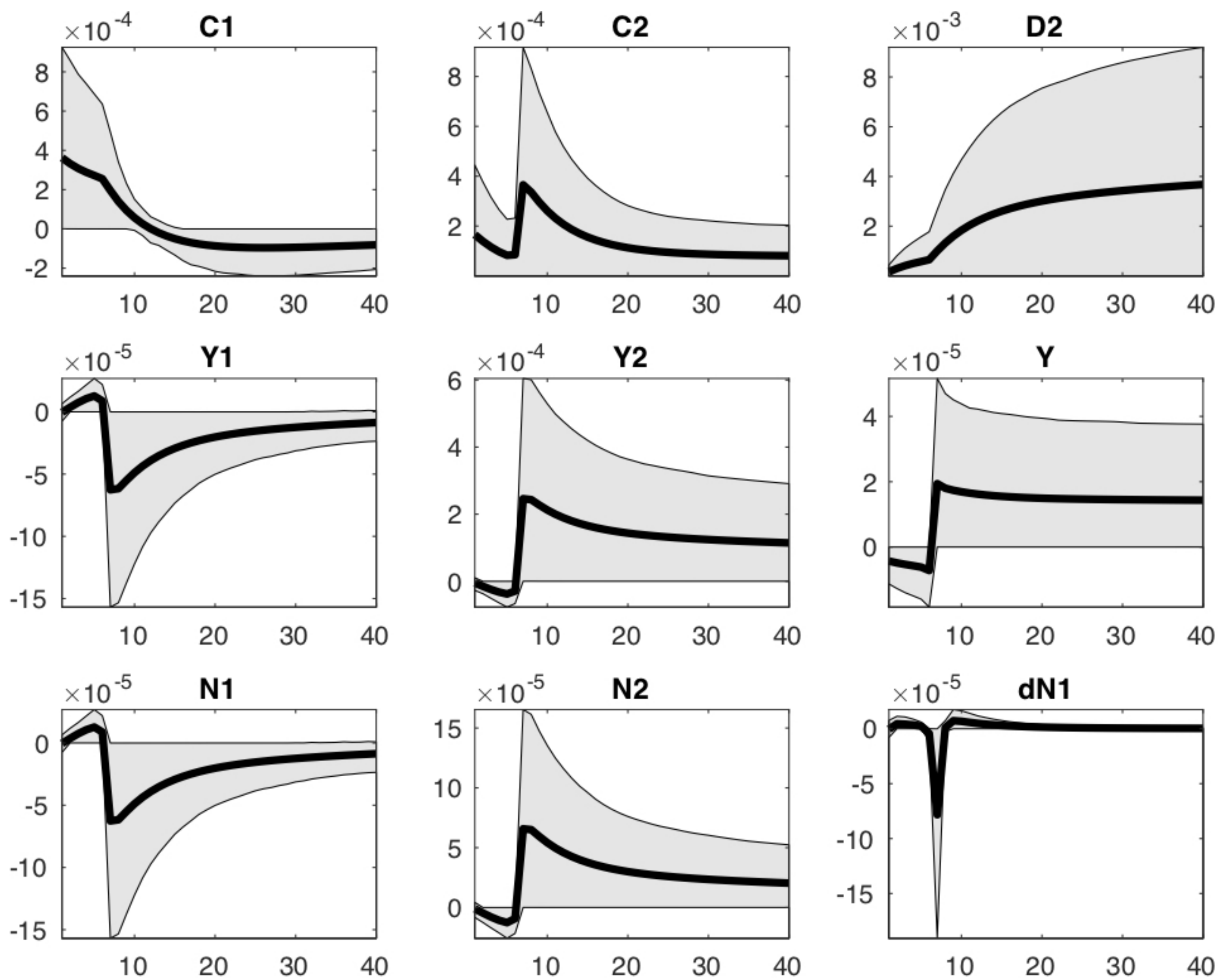


Figure 9: Bayesian IRF of One Standard Deviation of Gold discovery (2)

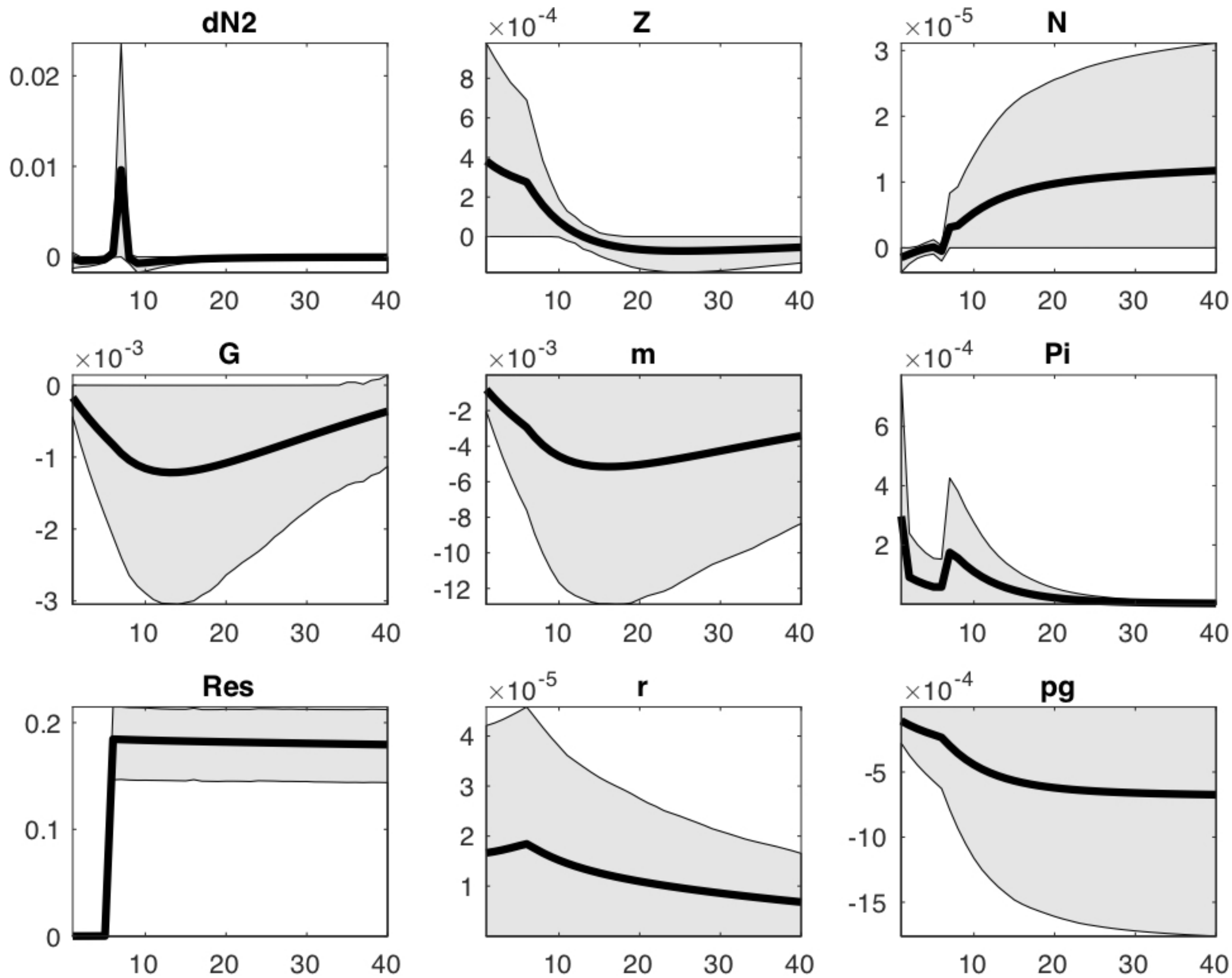


Figure 10: Bayesian IRF of One Standard Deviation of Gold discovery (3)

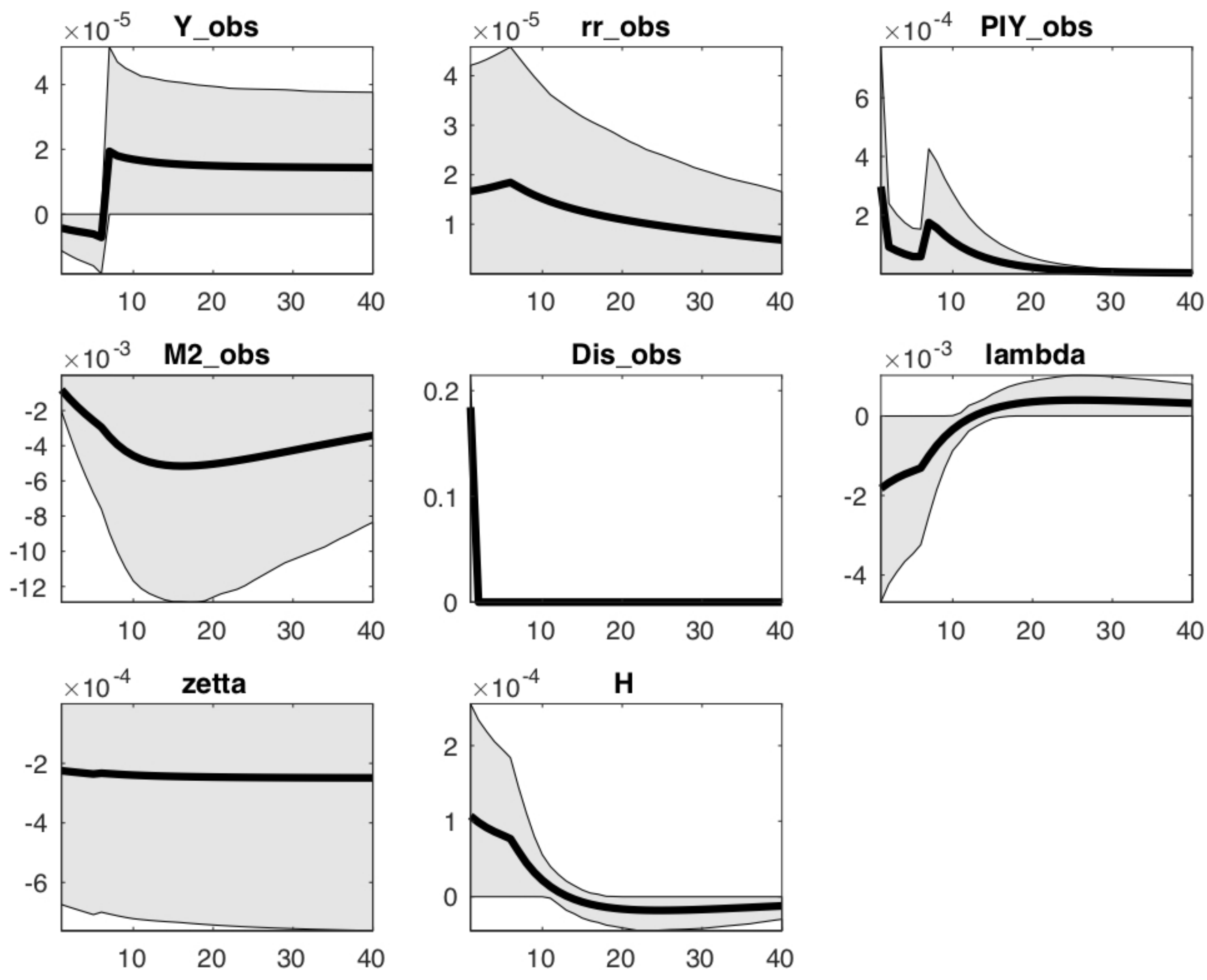


Table 16: Illustration of creating production profile

	ϵ^{a1}	ϵ^{a2}	ϵ^{d}	ϵ^{ME}	ϵ^{m}
C1	89.62	0.37	2.58	0	7.43
C2	1.7	0.09	0.55	0	97.66
D2	19.59	1.23	72.84	0	6.34
Y1	99.94	0.01	0.03	0	0.02
Y2	15.32	12.07	65.17	0	7.43
Y	99.95	0.01	0.04	0	0
N1	44.43	6.55	32.46	0	16.55
N2	23.43	9.01	56.61	0	10.96
dN1	2.14	3.72	9.21	0	84.93
dN2	2.69	6.6	16.77	0	73.94
Z	90.09	0.34	2.42	0	7.14
N	35.29	1.17	57.03	0	6.52
G	69.97	1.3	23.36	0	5.38
m	87.04	1.1	11.1	0	0.77
Pi	2.68	0.1	0.64	0	96.58
a1	100	0	0	0	0
a2	0	100	0	0	0
Res	10.91	6.15	82.91	0	0.03
r	23	5.9	53.14	0	17.97
pg	38.93	0.82	54.72	0	5.54

Table 17: Variance Decomposition (in Percent)

4.4 Adaptive Learning

4.4.1 Introduction

Dynamic Stochastic general equilibrium (DSGE) models relies on two major assumptions among others. One is the intertemporal optimizing behaviors by economic agents. The other one is the rational expectation. RE assumes that people's expectations are always formed consistently with the underlying model and policy. and all information is used efficiently. In other words, every economic agent needs to be an econometrician not only knowing the state space model representation, but also need know all the structural parameters in this model and form their expectation of the future accordingly.

However, DSGE has difficulties in explaining the persistence in macro variables like output and inflation. Economists have tried to add various different features to DSGE model to explain the persistence including habit formation, lagged inflation in price setting and various adjustment cost. Milani (2007) loose the rational expectation assumption, and assume agents learn the model via constant gain learning and Bayesian estimated structure parameters along with constant learning rate.

In my study, learning deserved more consideration because it is a historical period and the major shock of interest in my model is news shocks. In 19th centuries, average education level is much lower than that today. Assuming an average person in 19th century actually knew the underlying economics model is too strict to be realistic. Also, news about gold discovery needs more times to travel and diffuse throughout the whole countries and learning process actually take much more than back then than present.

Two types of learnings are considered here. Constant Gain Learning (CGL) and Kalman Filter Learning (KFL). I will discuss them separately. First, I will introduce a general DSGE representation I will use throughout this paper. I borrowed this representation format from Evans & Honkapohja (2012). Let w_t represents all exogenous shocks and lagged innovations. y_t stands for a vector of all endogenous variables. A general DSGE framework can be written as follows:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \epsilon_t = const. \tag{IV.14}$$

Under rational expectations, the solution of the model is provided by

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu^{RE} + T^{RE} \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R^{RE} \epsilon_t, \tag{IV.15}$$

4.4.2 Kalman Filter learning

The Kalman Filter learning is first advanced by Marcet & Sargent (1989); Evans & Honkapohja (2012). It is Dynare package is modified by Slobodyan & Wouters (2012). It assumes that the agents forecast the values of the forward variables as a reduced-formed linear functions of the state variables. Because these forward variables are also typically show up in Euler equations, therefore it is also named as Euler equation learning. Following Slobodyan & Wouters (2012)'s practice, I assume that among all endogenous variables y_t, there are state variables y^s and forward variables y^f. All forward variables y^f have a simple univariate AR(2) perceived Law of motion. That is for y^f, I assume that for each period, economic agents form expectations of forward variables y^f not through rational expectation from the DSGE model, but via the simple forecast model outside the DSGE system as follow:

$$y_j^f = X_j^T \beta_j \tag{IV.16}$$

Where X_j^T contains a constant and two lags of y_j^f. This forecast model (IV.16) is also called Perceived Law of Motion (PLM). Agents update the $AR(2)$ model at each period given the information available at that point. Slobodyan & Wouters (2012) assume that the agents use an efficient Kalman filter updating mechanism. Agents are assumed to update the coeffiencts β follow a Vector Autoregressive process around $\bar{\beta}$.

$$vec(\beta_t - \bar{\beta}) = F \cdot vec(\beta_{t-1} - \bar{\beta}) + v_t \tag{IV.17}$$

where F is a diagonal matrix with $\rho \leq 1$ on the main diagonal.Errors v_t are assumed to be independently and identically distributed with variance-covariance matrix V. Equation (IV.16) can be rewritten in a SURE format:

$$\begin{bmatrix} y_{1t}^f \\ y_{2t}^f \\ \vdots \\ y_{mt}^f \end{bmatrix} = \begin{bmatrix} X_{1,t-1} & 0 & \dots & 0 \\ 0 & X_{2,t-1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix} \tag{IV.18}$$

Agent update their parameter belief as soon as they receive new information at each period via Kaman Filter.

$$\begin{aligned} \beta_{t|t} &= \beta_{t|t-1} + P_{t|t-1} X_{t-1} [\Sigma + X_{t-1}^T P_{t|t-1} X_{t-1}]^{-1} x (y_t^f - X_{t-1}^T \beta_{t|t-1}), \\ &\text{with } (\beta_{t+1|t} - \bar{\beta}) = F \cdot (\beta_{t|t} - \bar{\beta}) \end{aligned} \tag{IV.19}$$

$$\begin{aligned} P_{t|t} &= P_{t|t-1} - P_{t|t-1} X_{t-1} [\Sigma + X_{t-1}^T P_{t|t-1} X_{t-1}]^{-1} \times X_{t-1}^T P_{t|t-1}, \\ &\text{with } P_{t+1|t} = F \cdot P_{t|t} \cdot F^T + V \end{aligned} \tag{IV.20}$$

The initial value is from OLS estimation. $\beta_{1|0} = \bar{\beta} = (E[X^T X])^{-1} \cdot E[X^t y]$. Error matrix $\Sigma = E[(y_t^f - X_{t-1}^T \beta_{1|0}) \times (y_t^f - X_{t-1}^T \beta_{1|0})^T]$ all the expectations including $(E[X^T X])^{-1}$, $E[X^t y]$ and $E[(y_t^f - X_{t-1}^T \beta_{1|0}) \times (y_t^f - X_{t-1}^T \beta_{1|0})^T]$ are from the RE solution. and the initial guess about the mean square forecast error $P_{1|0}$ and the variance-covariance matrix of shocks V are both taken proportional to $(X^T \Sigma^{-1} X)^{-1}$.

After agents update their believes about β at each period, they substitute β_t into equation (IV.18) and get their expectations of all forward-looking variables, $E_t y_{t+1}^f$. plug $E_t y_{t+1}^f$ back into equation (IV.14) and get the pure back-looking solution of the model under the Kalman Filter learning, also called as Actual Law of Motion (ALM) in learning literature:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t^{KF} + T_t^{KF} \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t^{KF} \epsilon_t, \tag{IV.21}$$

Comparing the kaman filter learning solution (IV.21) with rational expectation solution(IV.15), we notice the parameters in (IV.21) are updated each period.

4.4.3 Constant Gain Learning

I refer heavily on Milani (2006, 2007)'s work on CG learning in DSGE. Compared to the PLM for Kalman Filter learning IV.16, CGL's PLM is more sophisticated. CG assumes that agents know the true underlying econometric model when they form the expectation for next period. To illustrate how the CG works, we again start with the DSGE representation following Evans & Honkapohja (2012) as in equation (IV.15). The PLM has the same structural form of the rational expectations solution of the system, i.e., it includes the same regressors that appear in the minimum state variable (MSV) solution under rational expectations (IV.15). Define CG's PLM as:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t^{CG} + T_t^{CG} \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t^{CG} \epsilon_t, \tag{IV.22}$$

However, agents learn the structural parameters($\mu_t^{CG}, T_t^{CG}, R_t^{CG}$) at each period via Constant Gain

Learning, as described by the following formulas:

$$\hat{\phi}_t = \widehat{\phi_{t-1}} + \bar{g}R_{t-1}^{-1}X_t(Z_t - X_t^T\widehat{\phi_{t-1}}) \tag{IV.23}$$

$$R_t = R_{t-1} + \bar{g}(X_{t-1}X_{t-1}^T - R_{t-1}) \tag{IV.24}$$

where $\hat{\phi}_t = (\mu_t^{CG^T}, vec(T_t^{CG}, R_t^{CG})^T)^T$, $X_t \equiv \{1, y_{t-1}, w_{t-1}\}_0^{t-1}$ and R_T is the second moment's matrix of X_t. all the parameters are estimated via Bayesian estimation methods.

4.4.4 Learning Result

	Prior Mean	Posterior Mean		
		CG	KF	RE
σ	1.5	1.4169	1.4911	1.5679
τ	0.2	0.0762	0.0931	0.1735
α_2	0.7	0.8069	0.816	0.8124
α_1	0.04	0.0427	0.0257	0.0118
δ	1.4	1.7759	1.7811	1.1191
δ_D	0.03	0.0321	0.0615	0.0466
η	1.5	1.0022	0.9578	1.0087
ρ_{a_1}	0.5	0.9526	0.9939	0.9506
ρ_{a_2}	0.5	0.9511	0.9446	0.9486
θ	4	3.1207	3.1873	3.3876
γ_M	0.03	0.0705	0.0407	0.0362
γ_N	0.04	0.0534	0.0525	0.0362
ψ	0.35	0.3767	0.4158	0.3536
Log Density		-2339.23	-176.68	-259.81

Table 18: Learning Result Comparison

Table 18 shows the comparison of key differences between different models. First thing come to notice is the log data density for the models under KF(-176.68), CG (-2339.23) and RE (-259.81). The result support Kalman Filter Learning over rational expectation solution. However, CG is the least favorable. The result is mixed. It is intuitive that the Kalman filter learning is preferred by the data over the RE solution. As we reasoned earlier, the assumption that all agents are econometrician is too strict. Also it takes time for the news to spread, the learning process is gradual. However, this result argues that if agents already understand the true economic model, i.e. the underlying MSV representations of DSGE model, then they are much more likely to form rational expectation rather than constantly learn the parameters. This result is different from Slobodyan & Wouters (2012). In their paper, both learning methods are preferred over the RE estimation.

Also notice that, under the data-preferred Kalman filter learning assumption, several parameter estimations are drastically different from RE estimation. Under KFL, τ (gold backing ratio) is only one half of that under RE. Also α_1 (share of gold consumption in total consumption bundle) doubled under KFL. These two differences collaboratively show that agents notice more of the gold discovery impact on direct consumption, less of its impact on money supply transmission mechanism. Hence gold discoveries under learning has really little Impact on general price level. δ under KF is greater than that under RE. From equation (B.13) we can learn that real balance (m) will decrease less under KF than that it decrease under RE when shadow price λ drops.

4.5 Conclusion

This paper studies the real effect of gold discoveries as a form of monetary news shock during the classical gold standard era (1880-1914). I took advantage of a unique dataset of gold and silver deposits in the US that includes the date of discovery and the estimated total deposit size. I first built a simple DSGE model with a gold production sector and found that even the monetary news shock is full expected, economic agents still responded to this anticipated shock. It significantly affects money supply, interest rate, relative price level, and employment. Because the gold production only calibrated to constitute 1% of the aggregate production, gold discovery doesn't affect the output level very much.

When one standard deviation of gold discovery shock(e_d) hit the economy, agents form the expectation that the relative gold price (p_g) is going to decreasing in six years. That means the relative price of other commodities is going to increase (π). With this expectation in mind, agents would immediately substitute inter-temporally. They will increase current consumption on other commodities (C_1) and decrease it when the news is materialized. Because the prices are forward looking variables, the relative price of gold (p_g) decrease and general price level for other commodities goods (π) increase immediately. Because the price of gold decrease at current period 0, the consumption gold (C_2) also increases at current period 0. With the prospect of gold price drop, agent would also choose to hold less real balance (m) at hand. Constrained by the gold market clearance condition, the decrease in real balance overcomes the increase in gold consumption, therefore gold production (y_2) slightly decrease at first. and gold production labor demand (N_2) also decreases at period 0. cost labor adjustment is costly, therefore, N_1 increases at period 0. Y_1 increases because N_1 increases at period 0. But at period 6, when all the newly discovered gold hit the economy both N_2 and y_2 significantly increase. N_1 and y_1 significantly increase.because the gold production sector is calibrated as 1% of total GNP, so overall, the composite consumption (H) increases, shadow price of consumption (λ) decrease. from equation (B.12), the decrease in λ overpowered increase in π, interest rate has a really tiny increase first.

This paper also loosened up the "Rational Expectation (RE) Assumption" which is an important foundation fo modern DSGE model. I studied two types of adaptive learnings: Constant Gain Learning (CG) and Kalman Filter Learning (KF). Kalman filter learning is preferred by the data over the RE solution. It seems that the assumption that all agents know the true underlying DSGE model is a bit strict. There are also a lot anecdotal records showed that it takes time for the news to spread. However, CG is the least favorable among these three. This result argues that if agents already understand the true economic model, i.e. the underlying MSV representations of DSGE model, then they are much more likely to form rational expectation rather than constantly learn the parameters. The data is in favor of a story under KFL assumption that agents notice more of the gold discovery impact on direct consumption, less of its impact on money supply transmission mechanism.

Appendix A

Impulse Response Functions for Chapter 3

Figure 11: IRF to Monetary shock (1834-1872 Baseline Model)

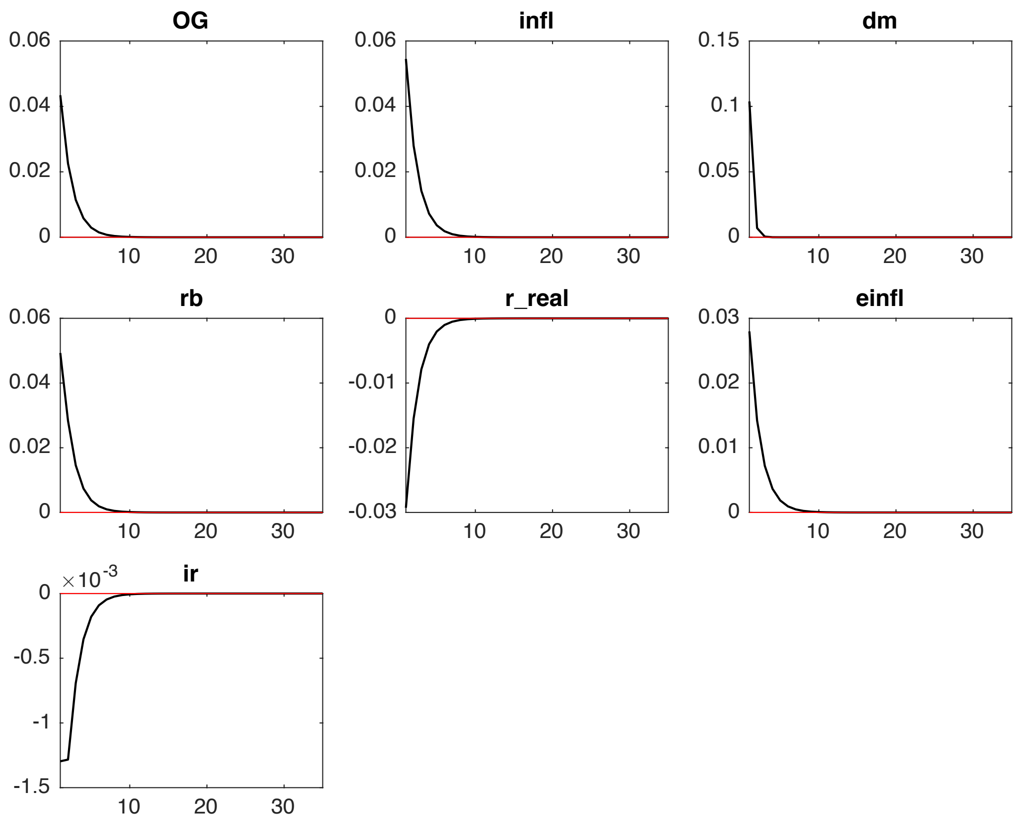


Figure 12: IRF to TFP shock (1834-1872 Baseline Model)

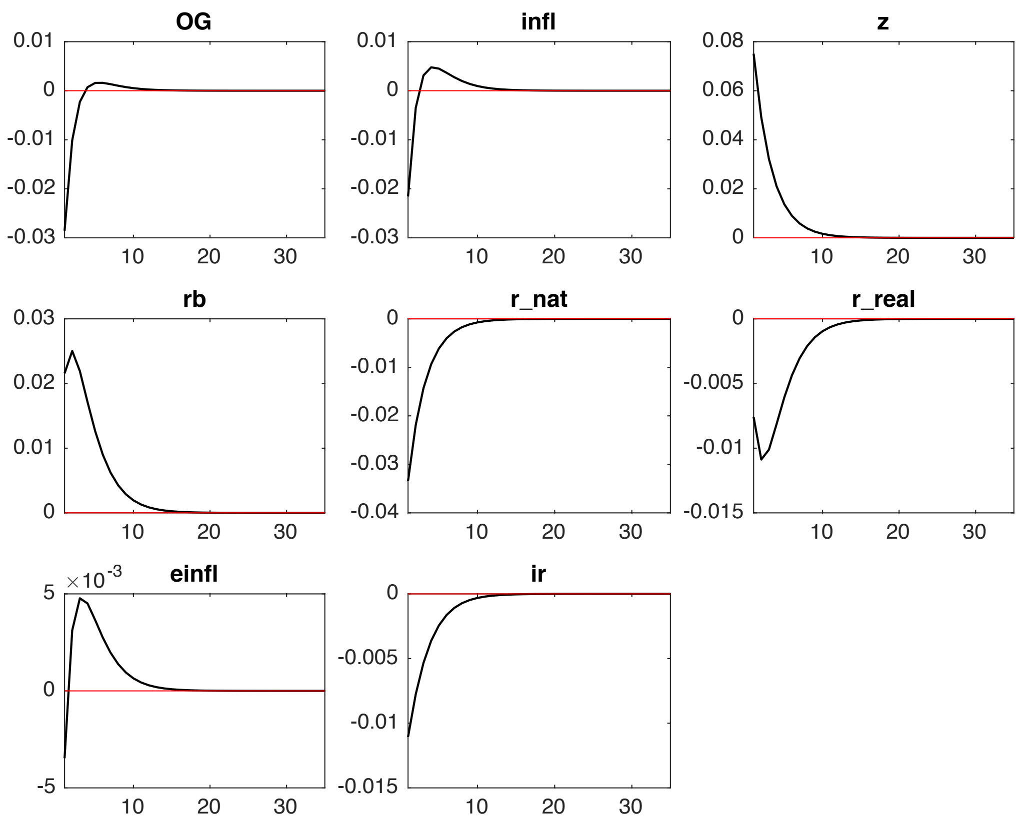


Figure 13: IRF to TFP shock (1834-1872 Auxiliary Model)

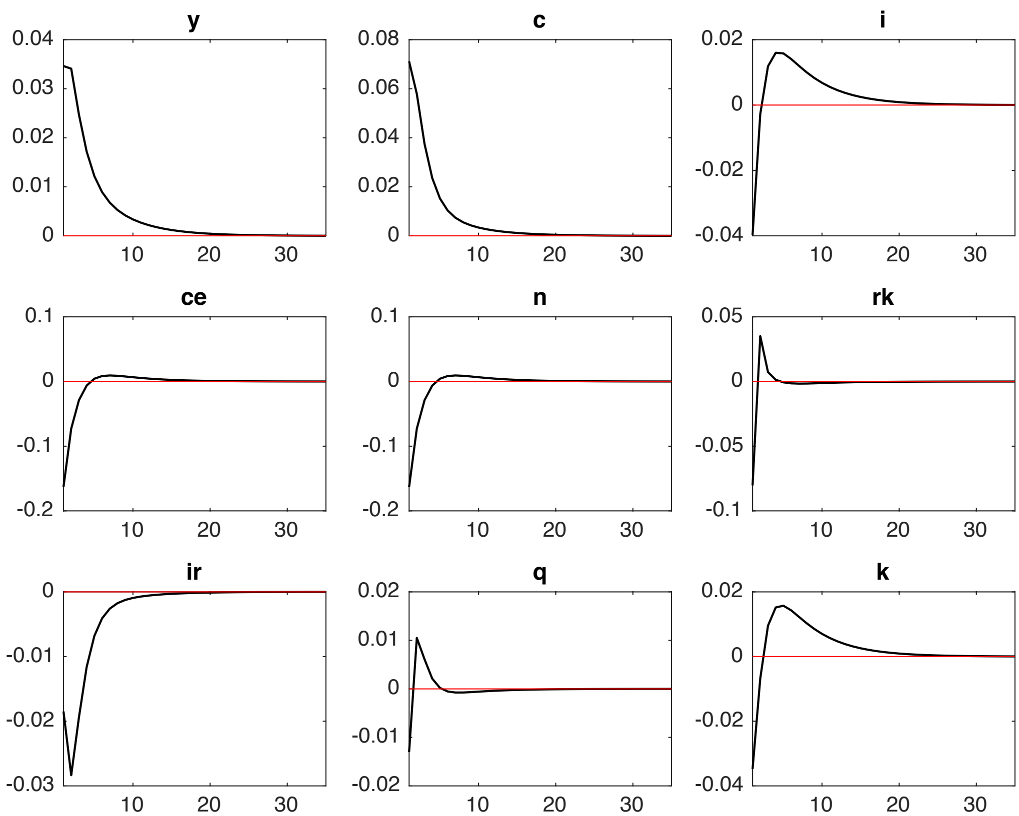


Figure 14: IRF to TFP shock Cont'd (1834-1872 Auxiliary Model)

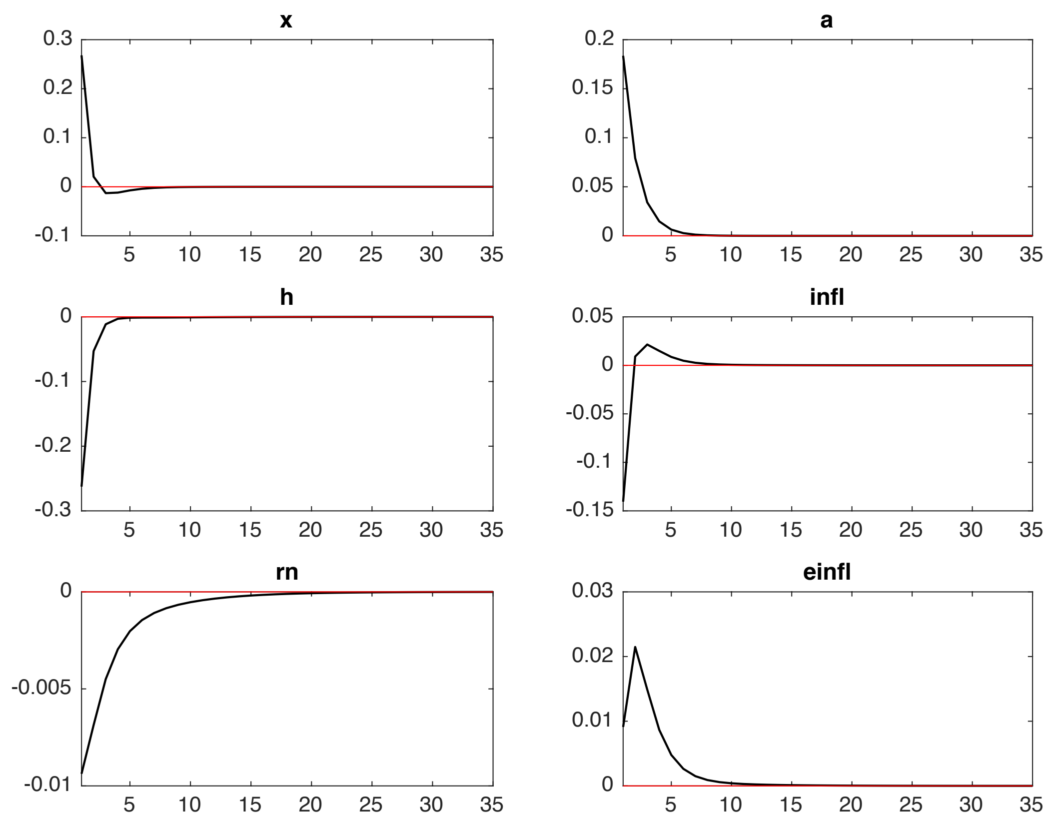


Figure 15: IRF to Monetary shock (1834-1872 Auxiliary Model)

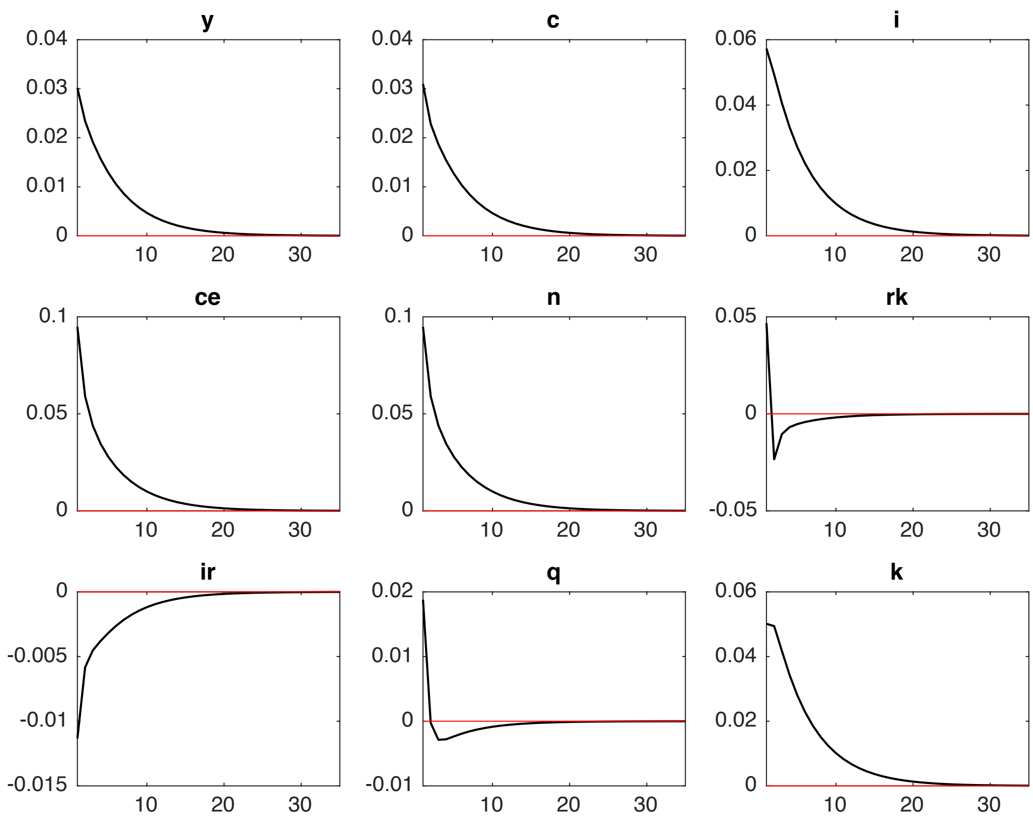


Figure 16: IRF to Monetary shock Cont'd (1834-1872 Auxiliary Model)

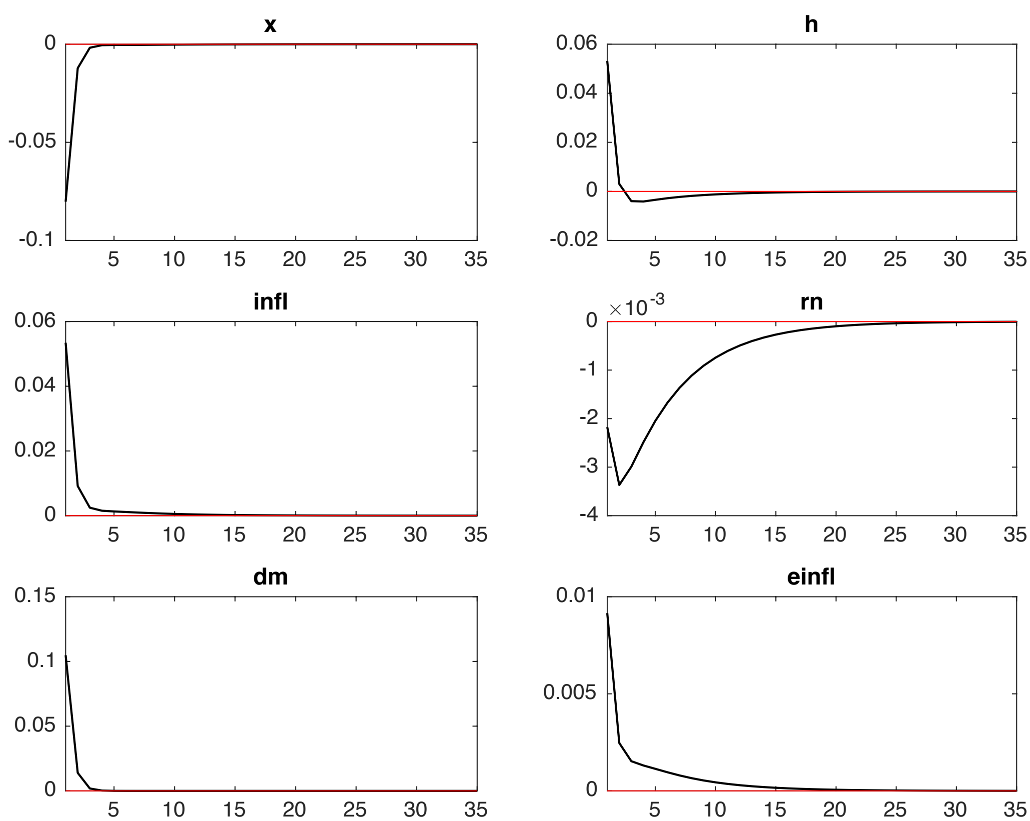


Figure 17: IRF to Gov. spending shock (1834-1872 Auxiliary Model)

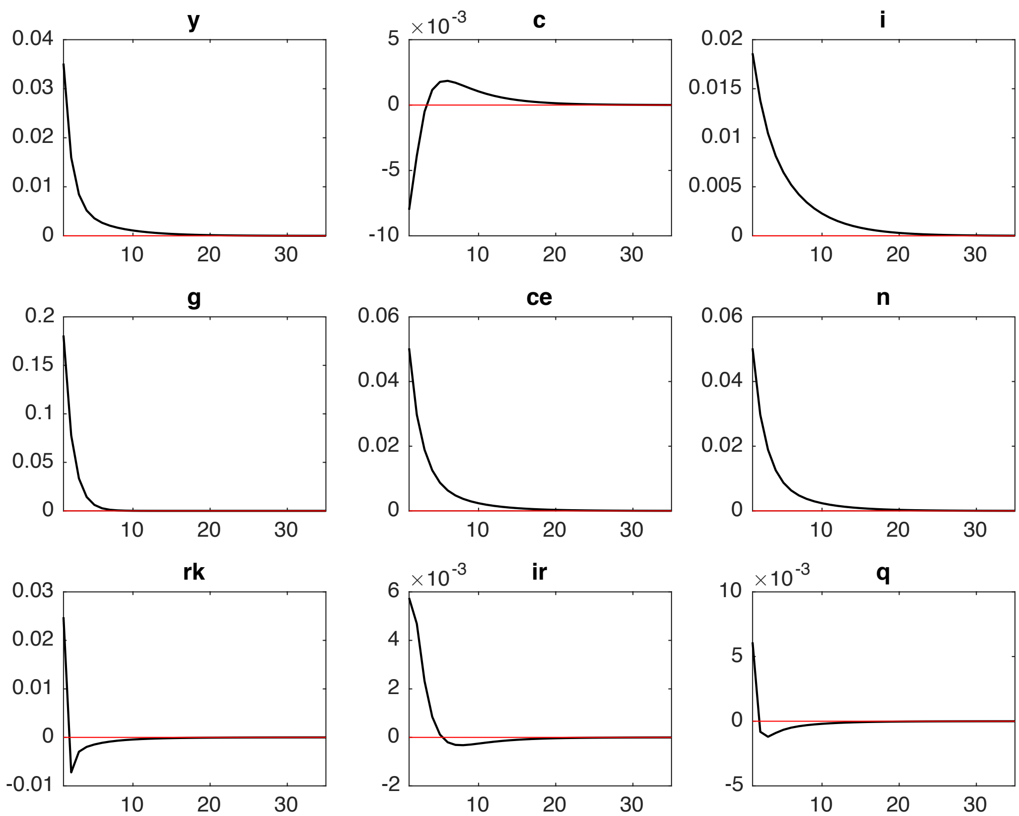
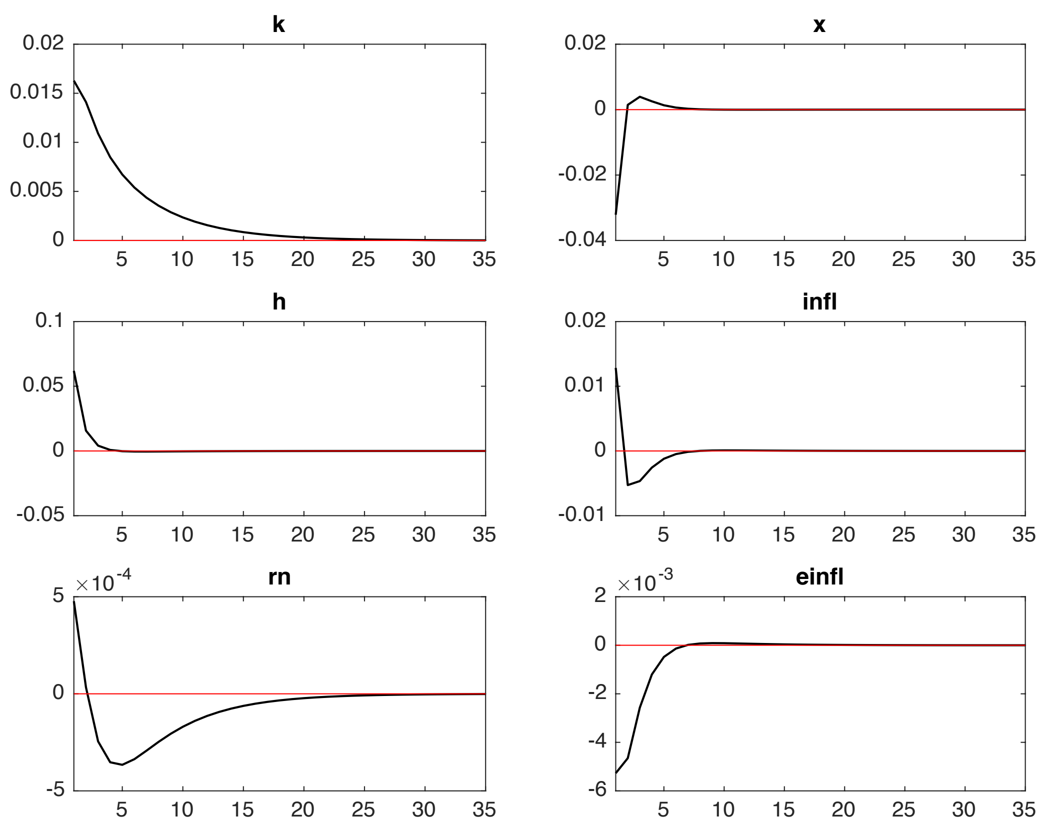


Figure 18: IRF to Gov. spending shock Cont'd (1834-1872 Auxiliary Model)



Appendix B

Model Derivation for Chapter 4

B.1 FOCs

Let $Z_t = \left[H^{\frac{\eta}{\eta-1}} - \psi N_t^{\theta}\right]$ and $(1+r_t) = R_t$. The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow (Z_t)^{-\sigma} H^{\frac{1}{\eta-1}} (1-\alpha_1)^{\frac{1}{\eta}} C^{\frac{-1}{\eta}} = \lambda_t \tag{B.1}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial D_t} = 0 \Leftrightarrow &(Z_t)^{-\sigma} H^{\frac{1}{\eta-1}} \alpha_1^{\frac{1}{\eta}} D^{\frac{-1}{\eta}} + \lambda_t \kappa_t (1-\chi)(1-\delta_D) P^g \frac{P_{t+1}}{P_t} \\ &= \frac{P^g}{P_t}\lambda_t - \beta \frac{P^g}{P_{t+1}} \lambda_{t+1}(1-\delta_D)\end{aligned} \tag{B.2}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_{1,t}} = 0 \Leftrightarrow (Z_t)^{-\sigma} \psi \theta N_t^{\theta-1} =& \lambda_t \alpha_1 \frac{Y_{1,t}}{N_{1,t}} - \lambda_t \frac{\gamma_n}{2}(dn_{1,t}-1)^2 \\ &- \lambda_t dn_{1,t} \gamma_n (dn_{1,t}-1) + \beta \lambda_{t+1} dn_{1,t+1}^2 \gamma_n (dn_{1,t+1}-1)\end{aligned} \tag{B.3}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_{2,t}} = 0 \Leftrightarrow (Z_t)^{-\sigma} \psi \theta N_t^{\theta-1} =& (\lambda_t \frac{P^g}{P_t} - \zeta_t)\alpha_2 \frac{Y_{2,t}}{N_{2,t}} - \lambda_t \frac{P^g}{P_t}\frac{\gamma_n}{2}(dn_{2,t}-1)^2 \\ &- \lambda_t \frac{P^g}{P_t} dn_{2,t} \gamma_n (dn_{2,t}-1) + \beta \lambda_{t+1} \frac{P^g}{P_{t+1}} dn_{2,t+1}^2 \gamma_n (dn_{2,t+1}-1)\end{aligned} \tag{B.4}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Leftrightarrow (1+\pi_{t+1})\lambda_t = \beta \lambda_{t+1}(1+i_t) \tag{B.5}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = 0 \Leftrightarrow \lambda_t = \gamma_m m_t^{-\delta} + \frac{\beta \lambda_{t+1}}{1+\pi_{t+1}} \tag{B.6}$$

$$\frac{\partial \mathcal{L}}{\partial E_{t+1}} = 0 \Leftrightarrow \zeta_t = \beta \zeta_{t+1} + \beta(\lambda_{t+1} p_{t+1}^g - \zeta_{t+1})(1-\alpha_2-\alpha_k)\frac{Y_{2,t+1}}{R_{t+1}} \tag{B.7}$$

B.2 Equilibrium

Define $Z \equiv H^{\frac{\eta}{\eta-1}} - \psi(N^{\theta})$. The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow (Z_t)^{-\sigma} H^{\frac{1}{\eta-1}} (1-\alpha_1)^{\frac{1}{\eta}} C^{\frac{-1}{\eta}} = \lambda_t \tag{B.8}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial D_t} = 0 \Leftrightarrow (Z_t)^{-\sigma} H^{\frac{1}{\eta-1}} \alpha_1^{\frac{1}{\eta}} D^{\frac{-1}{\eta}} + \lambda_t \kappa_t (1-\chi)(1-\delta_D) P^g \frac{P_{t+1}}{P_t} \\ = \frac{P^g}{P_t}\lambda_t - \beta \frac{P^g}{P_{t+1}} \lambda_{t+1}(1-\delta_D)\end{aligned} \tag{B.9}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_{1,t}} = 0 \Leftrightarrow (Z_t)^{-\sigma} \psi \theta N_t^{\theta-1} = \lambda_t \alpha_1 \frac{Y_{1,t}}{N_{1,t}} - \lambda_t \frac{\gamma_n}{2}(dn_{1,t}-1)^2 \\ -\lambda_t dn_{1,t}\gamma_n (dn_{1,t}-1) + \beta \lambda_{t+1} dn_{1,t+1}^2 \gamma_n (dn_{1,t+1}-1)\end{aligned} \tag{B.10}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_{2,t}} = 0 \Leftrightarrow (Z_t)^{-\sigma} \psi \theta N_t^{\theta-1} = (\lambda_t \frac{P^g}{P_t} - \zeta_t)\alpha_2 \frac{Y_{2,t}}{N_{2,t}} - \lambda_t \frac{P^g}{P_t}\frac{\gamma_n}{2}(dn_{2,t}-1)^2 \\ -\lambda_t \frac{P^g}{P_t} dn_{2,t}\gamma_n (dn_{2,t}-1) + \beta \lambda_{t+1} \frac{P^g}{P_{t+1}} dn_{2,t+1}^2 \gamma_n (dn_{2,t+1}-1)\end{aligned} \tag{B.11}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Leftrightarrow (1+\pi_{t+1})\lambda_t = \beta \lambda_{t+1}(1+i_t) \tag{B.12}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = 0 \Leftrightarrow \lambda_t = \gamma_m m_t^{-\delta} + \frac{\beta \lambda_{t+1}}{1+\pi_{t+1}} \tag{B.13}$$

$$\frac{\partial \mathcal{L}}{\partial R_{t+1}} = 0 \Leftrightarrow \zeta_t = \beta \zeta_{t+1} + \beta(\lambda_{t+1} p_{t+1}^g - \zeta_{t+1})(1-\alpha_2-\alpha_k)\frac{Y_{2,t+1}}{R_{t+1}} \tag{B.14}$$

B.3 Steady State

Let variables with no time subscript denote the steady state value. We have $dn_1 = dn_2 = 1, \pi = 0, A_1 = A_2 = 1, p^g = 1$, and the equilibrium conditions can be written as:

$$1 + i = \frac{1}{\beta}$$

$$N = (\psi\theta)^{\frac{1}{1-\theta}}$$

$$C_2 = Y_2 = X$$

$$D_2 = \frac{Y_2}{\delta_D}$$

$$C1 = (1-\beta(1-\delta_D))^{\eta}\frac{1-\alpha_1}{\alpha_1}D2;$$

$$N_2 = \alpha_2 Y_2$$

$$N_1 = N - N_2$$

$$Y_1 = N_1$$

$$Y = Y_1 + Y_2$$

$$H = (1-\alpha_1)^{\frac{1}{\eta}} C_1^{\frac{\eta-1}{\eta}} + \alpha_1^{\frac{1}{\eta}} D_2^{\frac{\eta-1}{\eta}};$$

$$Z = H^{\frac{\eta}{\eta-1}} - \psi(N^{\theta})$$

$$R = \left(\frac{Y_2}{N_2^{\alpha_2}}\right)^{\frac{1}{1-\alpha_2}}$$

$$\lambda = Z^{-\sigma} H^{\frac{1}{\eta-1}} (1-\alpha_1)^{\frac{1}{\eta}} C1^{\frac{-1}{\eta}};$$

$$m = \left(\frac{\lambda - \beta\lambda}{\gamma_M}\right)^{-\frac{1}{\delta}};$$

$$G = \frac{\tau m}{p^g}$$

$$\zeta = \lambda - \frac{\lambda N_2}{\alpha_2 Y_2}$$

B.4 Other Data Sources

- Nominal GNP (Billions of dollars)
  - 1869-88: nominal income (net national product) from Milton Friedman and Anna J. Schwartz, Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867-1975 (Chicago: University of Chicago Press, 1982), 122-29, added to capital consumption from Simon Kuznets, Capital in the American Economy; Its Formation and Financing (Princeton: University of Princeton Press, 1961), table R8, 499, and unraveled five- year moving average table R29, 572-73. Linked in 1889 to
  - 1889-1908: series A7 from Long Term Economic Growth, 1860-1970 (Washington, D.C.: Department of Commerce, 1973). Linked in 1909 to
  - 1909-14: National Income and Product Accounts of the United States, 1929-76 Sta- tistical Tables (Washington, D.C.: Department of Commerce, 1981), table 1.22.
- Real GNP (Billions of dollars)
  - 1869-88: real income (real net national product) from Friedman and Schwartz, Monetary Trends, 122-29, added to capital consumption from Kuznets (1961), table R8, 499, and unraveled five-year moving average table R29, 572- 73. Linked in 1889 to
  - 1889-1908: series Al from Long Term Economic Growth, 1860-1970. Linked in 1909 to
  - 1909-14: National Income and Product Accounts o f the United States, 1929-76 Statistical Tables, table 1.22.
- GNP deflator

  - 1869-1914: nominal GNP divided by real GNP, then multiplied by 100.

- Commercial paper rate

  - 1869-89: Friedman and Schwartz, Monetary Trends, 122-29. Linked in 1890 to 1890-1941: 4-6-month prime commercial paper from Banking and Monetary Statistics, 1919-1941, 448.

- Money supply, M2

  - 1869-74: money stock from Friedman and Schwartz, Monetary Trends, 122-29. Linked in 1875 to

  - 1875-1983: averaged from the quarterly M2 series described in section 2 source notes.

Table 19 shows the fine grains of gold in one dollar coin. It hadn't changed since 1834, therefore I didn't give P_G a time subscript. Therefore, the change in relative gold price to general price level of other commodities is in effect the inverse inflation rate.

Table 19: Official Value of U.S. dollar and its related Act

Year	Value of dollars in fine grains		Mint Ratio	Price per Fine Troy Ounce		New Features in Coinage Act
	Gold	Silver		Gold	Silver	
1792	24.75	371.25	15	19.39	1.29	Established the 15:1 ratio
1834	23.2	371.25	16	20.69	1.29	Changed the ratio to 16:1
1837	23.22	371.25	15.98	20.67	1.29	Changed the ratio to 15.98:1
1853	23.22	371.25	15.98	20.67	1.29	Made the silver coins subsidiary legal tender (up to 5 USD)
1857	23.22	371.25	15.98	20.67	1.29	Forbade the use of foreign coins as legal tender
1873	23.22	371.25	15.98	20.67	1.29	Suspended free silver coinage
1879[1]	23.22	-	-	20.67	-	Resumption of a single gold standard
1913	23.22	-	-	20.67	-	Fed was founded

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