

CHANGING LOCAL PRACTICE FOR GOOD: WALKING SCALE GEOMETRY AS  
DESIGNED DISRUPTIONS FOR PRODUCTIVE HYBRIDITY

By

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## TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS .....	ii
LIST OF FIGURES .....	vi
Chapter	
I. INTRODUCTION.....	1
II. WALKING SCALE GEOMETRY AS DESIGNED DISRUPTIONS FOR PRODUCTIVE HYBRIDITY .....	7
Designing disruptions for productive hybridity .....	8
Hybridity .....	10
Productive Hybridity.....	19
Designing Disruptions for Hybridity .....	24
Designed Disruptions in Walking Scale Geometry.....	28
Design Principles .....	34
Research Questions .....	36
III. RESEARCH DESIGN .....	39
Participants.....	39
Study Design.....	41
Instructional Design .....	44
Methods of Analysis .....	53
IV. WALKING SCALE GEOMETRIES IN PRACTICE.....	57
Students Engaging in Walking Scale Geometries.....	58
Typical Material, Embodied, and Mathematical Engagements: “How You Wanna Do It?” (KCMS, Day 9) .....	59
Intersections of Play and Mathematics: “We Can’t Lose Dean’s Height” (KCMS, Day 2).....	81
Development of a New Representational Practice: “What We Needed the Yardsticks For” (KCMS, Day 2) .....	91
Assessment via Distribution of Labor and Materials: “So the Middle’s Here” (SEC).....	102
Walking Scale Geometries Produce Trouble .....	110

	Assembling Materials and Bodies for an Argument: “Each Person Holds Two Ropes” (KCMS, Day 7).....	111
	Shifting Mathematical Meanings: “How Did You Decide on this Green Thing?” (SEC).....	133
	Summary .....	141
V.	RESOURCES FOR ACCESS TO PARTICIPATION AND SENSE-MAKING IN WALKING SCALE GEOMETRY.....	144
	Access to Participation and Sense-Making.....	145
	Material, Spatial, and Social Resources for Developing a Strategy: “Four Points Over There” (SEC) .....	145
	Material and Physical Manipulations as Access to Participation: “I Get It Now” (SEC).....	157
	Coordinating Bodies and Mathematics: “Each Person Holds Two Ropes” (KCMS, Day 7).....	171
	Summary .....	174
VI.	DISCUSSION .....	176
	Disruptions, Engagements, and Opportunities to Learn .....	177
	Space .....	177
	Tools .....	179
	Perspective .....	182
	Division of Labor.....	184
	Comparisons Between KCMS and SEC .....	185
	Design Considerations .....	188
	The Pragmatics Of Walking Scale Geometry .....	188
	Materials and Mathematics to Support Productive Hybridity .....	192
	Ensemble Mathematics for Future Iterations of Walking Scale Geometry .....	195
	Generalizing Across Walking Scale Geometry and Classroom Instruction.....	200
	Appendix	
A.	KCMS LESSON PLANS.....	203
B.	SEC LESSON PLANS.....	228
C.	KCMS INTERVIEW QUESTIONS .....	230
D.	SEC INTERVIEW QUESTIONS .....	231
	REFERENCES .....	232

## LIST OF FIGURES

Figures	Page
1-1. Ben, Dean, Dwayne, Eddy, and Harry make a large rectangle out of rope .....	1
1-2. Ben, Dean, Dwayne, Eddy, and Harry’s diamond .....	3
2-1. Models for promoting hybridity in classroom settings.....	15
2-2. Designing WSG as a disruption to support productive hybridity .....	17
2-3. How WSG disrupts typical classroom mathematics.....	29
2-4. Constructing an isosceles triangle at walking scale.....	31
3-1. Summary of KCMS lessons.....	49
3-2. SEC WSG tasks .....	51
4-1. Task and students in “How You Wanna Do It?” .....	59
4-2. Excerpt 1, “How You Wanna Do It?” .....	61
4-3. Examples of how students used materials and bodies in WSG .....	64
4-4. Excerpt 2a, “How You Wanna Do It?” .....	65
4-5. Excerpt 2b, “How You Wanna Do It?” .....	67
4-6. Excerpt 2c, “How You Wanna Do It?” .....	69
4-7. Excerpt 2d, “How You Wanna Do It?” .....	70
4-8. Excerpt 3, “How You Wanna Do It?” .....	74
4-9. Kaitlin’s description of the group’s WSG rectangle. ....	78
4-10. Ms. H’s revoicing of the group’s strategy for making a WSG rectangle.....	79
4-11. Task and students in “We Can’t Lose Dean’s Height” .....	81
4-12. Excerpt 1, “We Can’t Lose Dean’s Height” .....	83

4-13.	Excerpt 2, “We Can’t Lose Dean’s Height” .....	86
4-14.	Task and students in “What We Needed the Yardsticks For” .....	90
4-15.	Excerpt 1, “What We Needed the Yardsticks For” .....	92
4-16.	Excerpt 2a, “What We Needed the Yardsticks For” .....	94
4-17.	Excerpt 2b, “What We Needed the Yardsticks For” .....	97
4-18.	Excerpt 2c, “What We Needed the Yardsticks For” .....	99
4-19.	Task and students in “So the Middle’s Here” .....	102
4-20.	Excerpt 1, “So the Middle’s Here” .....	105
4-21.	Excerpt 2, “So the Middle’s Here” .....	108
4-22.	Task and students in “Each Person Holds Two Ropes” .....	111
4-23.	Instructional video for “Each person holds two ropes” .....	113
4-24.	Excerpt 1a, “Each Person Holds Two Ropes” .....	116
4-25.	Excerpt 1b, “Each Person Holds Two Ropes” .....	118
4-26.	Excerpt 1c, “Each Person Holds Two Ropes” .....	119
4-27.	Excerpt 1d, “Each Person Holds Two Ropes” .....	121
4-28.	Excerpt 1e, “Each Person Holds Two Ropes” .....	123
4-29.	Excerpt 2a, “Each Person Holds Two Ropes” .....	128
4-30.	Excerpt 2b, “Each Person Holds Two Ropes” .....	130
4-31.	Natalie’s dilation strategy, completed .....	134
4-32.	Excerpt 1a, “How Did You Decide on this Green Thing?” .....	135
4-33.	Excerpt 1b, “How Did You Decide on this Green Thing?” .....	136
4-34.	Excerpt 2a, “How Did You Decide on this Green Thing?” .....	137

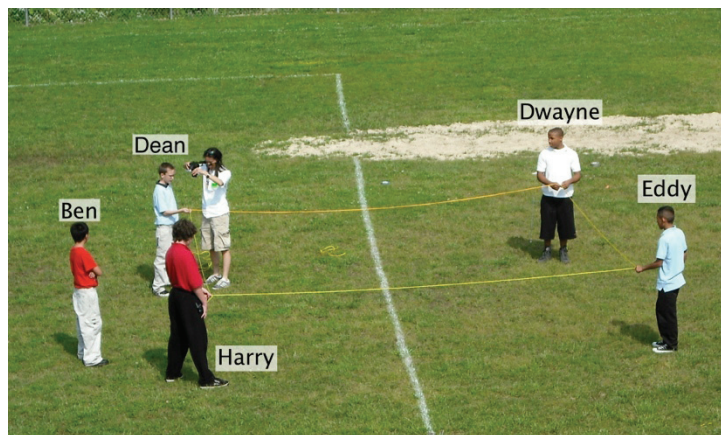
4-35.	Excerpt 2b, “How Did You Decide on this Green Thing?” .....	139
5-1.	Task and students in “Four Points Over There” .....	145
5-2.	Excerpt 1, “Four Points Over There” .....	148
5-3.	Excerpt 2a, “Four Points Over There” .....	150
5-4.	Excerpt 2b, “Four Points Over There” .....	151
5-5.	Excerpt 2c, “Four Points Over There” .....	153
5-6.	Excerpt 1a, “I Get It Now” .....	159
5-7.	Excerpt 1b, “I Get It Now” .....	160
5-8.	Excerpt 1c, “I Get It Now” .....	161
5-9.	Excerpt 2a, “I Get It Now” .....	164
5-10.	Excerpt 2b, “I Get It Now” .....	166
6-1.	Desktop Geometry .....	194



## CHAPTER I

### INTRODUCTION

On a hot May morning, five seventh grade students, Ben, Dean, Dwayne, Eddy, and Harry stand in a soccer field during math class (Figure 1-1). Four of them are holding together a large rectangle made of rope, standing at the vertices so that each grasps the ends of two sides. Ben stands to the side and watches. The group has just had photos of their rectangle and each of the rectangle's angles taken from ground level and from atop the hill next to the field, so that the whole class can discuss what they made once they go back inside.



*Figure 1-1.* Ben, Dean, Dwayne, Eddy, and Harry make a large rectangle out of rope.

Once the pictures have been taken, Harry tells everyone to “swing,” and he begins to twirl the ropes he is holding like jump ropes. A researcher walks by, telling the group to turn their square into a parallelogram that has no right angles. Eddy says, “Alright

that's easy, just make it into a diamond." Dwayne quietly repeats, "Diamond." Then more loudly, he proclaims, "Like this!" He points at the three other students standing at the vertices of the square, and tells them, "Look, stand right there, it's a diamond! Look from this angle. Look dead at me, you look dead at him, that is a diamond." He then says, "They just gotta take it from this side, like here." In other words, if the photograph is taken from where he is standing, it will look like a diamond.

Harry, standing at the vertex opposite Dwayne, takes a step to his right to look at the quadrilateral, while keeping the ropes in his left hand in place. He does not see a diamond. He says to "scoot in," and Dwayne and Dean both take a step forward toward the middle of the square. Even as he steps in Dwayne objects, "It's gonna look wrinkled." Dean agrees, saying, "Wrinkled like an old lady." The ropes that represent the sides of the quadrilateral are no longer pulled taut now that two of the vertices have moved inward, making it "wrinkled." The students laugh at this, and Dean and Dwayne step back to their original positions.

Eddy declares that he has an idea, and asks Dwayne and Harry (at opposite vertices) to each step in. He then asks Ben to come hold his ropes, and he walks over to take Dean's vertex. While he waits, Dean whips his ropes up and down, accidentally pulling one out of Dwayne's hands. Dwayne says, "This does not look like no diamond." Eddy says, "It's gonna be." Dean responds, "It looks more like a wrinkled Superman sign."

The group continues to shift their bodies-as-vertices in and out and argue about whether the quadrilateral is a diamond and whether it has right angles. In the end, Dwayne and the opposite vertex move in, and the other two vertices move out. Dean takes Dwayne's vertex so that Dwayne can walk around and look at the quadrilateral

from different angles. As Dwayne walks around, he adjusts his groupmates' bodies to his liking, and they get their photographs taken (Figure 1-2).



Figure 1-2. Ben, Dean, Dwayne, Eddy, and Harry's diamond.

This group of students was engaged in Walking Scale Geometry (WSG), a designed setting for solving geometry problems at large scale. WSG problems *disrupt typical classroom instruction* by taking students outside, giving them everyday materials (including their bodies) as tools, placing them within the inscriptions rather than looking over them on paper from a bird's-eye-view, and requiring them to work together to draw and manipulate representations too big for any one person to work with at a time. This constitutes an experience that necessitates students' engagement of novel resources not typical of school mathematics. The tasks that I designed were similar to activities that students had encountered in their typical mathematics instruction, but the disruptions in space, tools, perspective, and division of labor invited them to see and engage in geometry in new ways, inventing new tools (both material and conceptual) and strategies for solving problems.

Ben, Dean, Dwayne, Eddy, and Harry had learned about quadrilaterals, rectangles, and parallelograms in math class before, in the sixth grade and in discussion earlier that morning. Does this experience changing this large rectangle into a parallelogram indicate that these students do not know what a parallelogram is? For their first task that morning in the classroom all five of these students correctly categorized parallelograms drawn on paper, and participated in whole class discussion about the properties of parallelograms. Then, why was this task non-trivial for the group? Alternatively, why bother doing the task if the students can already identify parallelograms and their properties?

In this episode, the students grappled with a geometry problem not terribly different from problems that they have seen before, to draw a parallelogram. However, there were significant differences to the setting of the task. The group started with a rectangle, and had to manipulate its parts to draw the new figure. It took place outdoors, at very large scale, with ropes and multiple students' bodies. How do these features of the problem solving setting change how students participate in the task, what they have to do to accomplish it, and what mathematics becomes relevant?

Just in this brief scenario we can see that the students negotiated their problem solving strategies and argued about their solutions. They manipulated the materials and their bodies to try out solutions, and to convince themselves and each other. They also took on each other's perspectives and embodied roles inside the quadrilateral. They used everyday terms and understandings to describe what they were doing and what they saw. They played with the materials in "off task" ways, twirling and whipping the ropes. In the end, the group converged on a solution satisfactory to all, six minutes after they began.

The disruptions of WSG tasks were designed to drastically alter the setting of problem solving so as to support more diverse opportunities to learn by inviting students to engage in geometry activity in novel, sensible (to them) ways. For this dissertation, I conducted the first iteration of a design experiment around WSG tasks. I build on studies of hybridity and hybrid learning contexts research (e.g., Calabrese Barton, Tan, & Rivet, 2008; Gutierrez, Rymes, & Larson, 1995; Jurow, Hall, & Ma, 2011; Moje et al., 2004), as well as research on multi-modal and multi-party engagements in and resources for mathematics learning (e.g., Enyedy, 2005; Hall, 1996; Hall & Ma, 2011; Hall & Nemirovsky, 2008; Nemirovsky, Tierney, & Wright, 1998; Radford, 2009). I investigate how the disruptions in the WSG setting influence student engagement in problem solving. I then ask how they might promote emergent hybridity in classroom instruction by providing a shared geometry experience for students that makes available and relevant resources that are not typically salient in the classroom.

I begin by framing the design study in terms of research on hybridity in classroom learning, and introduce an alternative to the typical way of designing for hybridity. I then describe the WSG design and my conjectures about how it can support hybridity in classroom instruction and be productive for learning. The third chapter reports on the design and analytical methods of the dissertation study. The next two chapters contain the data analysis and findings for the study. Chapter four explores how students engage in WSG tasks and the resources they recruit for problem solving and negotiating shared understandings. Chapter five discusses different modes of engagement possible in the WSG setting, and investigates how these might expand or constrain opportunities to learn, or access to participation, for a variety of students. Finally, in the last chapter, I

summarize the findings from the previous two chapters in terms of the four disruptions of WSG. I then make some comparisons between the two contexts in which the study was implemented, and make some comments about design conjectures for future iterations of the design study based on the findings described in this dissertation, and some practical considerations of implementing WSG tasks in school. These design conjectures include a suggestion for bridging between the WSG setting and the classroom setting, as well as one for designing WSG tasks as ensemble performance. I conclude with some thoughts about how this study can add to principles for the design of learning contexts.

## CHAPTER II

### WALKING SCALE GEOMETRY AS DESIGNED DISRUPTIONS FOR PRODUCTIVE HYBRIDITY

This dissertation takes a sociocultural, or situative (Greeno & Middle School Mathematics through Applications Project Group, 1998), view of learning. That is, learning is a trajectory of participation in activities, within communities of practice, mediated by individuals' engagement within and across different settings as well as historically developed aspects of practice, such as participation structures, tools and technologies, and representational systems. As individuals learn, they engage in developing pragmatic goals rooted in contexts of activity, and form identities within (although sometimes in opposition to) practice. Individuals' participation is not only mediated by but also influences their interpretation of contexts of practice, and this participation often has consequences for the conduct of practice. The development of the activity itself is also influenced by learners' participation. Trajectories of learning may differ significantly from person to person, but successful learning necessarily means that an individual has increasing capabilities and responsibilities within the community of practice, as defined by that community. This view of learning is influenced mainly by research in the traditions of social practice theory (Lave & Wenger, 1991), cultural historical activity theory, (Cole & Engeström, 1993; Engeström, 1993), and distributed cognition (Hutchins, 1995). This perspective on learning does not preclude the more traditional psychological view of changing mental representations, but takes as the focus of analysis larger contexts of

activity, including bodies, groups of people, interactions, material and cultural resources, and histories of participation.

The WSG tasks described in this study were designed to disrupt aspects of typical paper and pencil school mathematics activity in ways that afford new modes of student engagement and an expanded set of available and relevant resources for varying forms of participation and for learning. By interrogating students' WSG problem solving activities in two different settings, I will discuss these modes of engagement and expanded resources in terms of productive hybridity, and explore how the designed disruptions supported them.

In this chapter, I begin by discussing what I mean by designing disruptions for productive hybridity. I then describe the disruptions offered by WSG, and how this kind of design could promote hybridity and learning in the classroom. Lastly, I present research questions that guided the design of the study and the analysis.

### **Designing Disruptions for Productive Hybridity**

School mathematics (and, indeed, schooling itself) is a socially and historically constructed cultural system, or practice, in and of itself that privileges certain cultural and socioeconomic groups (Boaler, 1993; Lee, 2001; Moschkovich, 2002; Stigler & Hiebert, 1998; Walkerdine, 1997). In other words, the out-of-school resources that some (for example, upper-middle class White) Americans bring to school are well-aligned with the practices typically involved with succeeding in school, while those that others (for example, working class Latinos) bring are not only different from, but often contrary to, school practices. Therefore, learning environments that provide opportunities for more



students to participate in a variety of ways, valuing and leveraging students' out-of-school knowledge and practices of meaning making in productive ways, and expanding the set of relevant, available resources, could serve to improve learning for everyone. The idea of designing for productive hybridity comes from the desire to create mathematics learning environments that can foster learning in this way.

I take a relational view of diversity and equity here, which “sees issues of diversity as emerging from the relations between communities in which students participate rather than as characteristics of students and their communities” (Cobb & Hodge, 2002, p. 257). Cobb and Hodge emphasize that diversity emerges from students' *participation* in a variety of local communities (e.g., farming or dominoes) and broader groups within wider society (e.g., African American or Haitian immigrant communities), not as a result of simply *belonging* to certain groups or categories. Gutierrez and Rogoff (Gutiérrez & Rogoff, 2003) make a similar argument, pointing out that neither cultural communities nor individuals' participation in them is static. They propose that, instead of assuming and referring to traits of individuals belonging to particular cultural groups, we focus on their *repertoires of practice*, developed as a result of their histories of participation in various communities.

From this point of view, equitable instructional design takes into account the variety of ways that students participate in local and broader communities, and seeks to ensure that all students have access to learning despite (or better yet, because of) their diverse repertoires of practice and histories of engagement in cultural practices. Productive hybrid learning environments, in principle, promote equity in mathematics education. They invite and leverage students' diverse repertoires of practice in relation to valued

school and disciplinary practices. In this chapter, I begin with just a few of the many ways that equity in education research has been addressed. In particular, I highlight studies that have explored hybridity both as a promising lens for analyzing classroom activity and as a goal for instructional design, and speculate about different possible forms of hybridity. I then lay out my commitments to what *productive* hybridity would include, and contrast this study with designs that attempt to bridge classroom practices with known student practices or funds of knowledge. Instead, this dissertation investigates a setting that leverages *designed disruptions* to typical school mathematics, occurs alongside classroom instruction, constitutes a shared experience for students, and supports the *emergence of* productive hybridity in the classroom.

## **Hybridity**

Many education studies and research agendas have advocated for attention to students' out-of-school resources, including home, community, and other cultural discourses, knowledge, skills, and experiences. For example, in her work with teachers of African American students, Gloria Ladson-Billings (1995; 1998) argued for culturally relevant pedagogy, which promotes academic excellence through culturally grounded learning environments that support critical sociopolitical thinking. By "culturally grounded," Ladson-Billings meant that students' culture is intentionally positioned to support learning. As an example, she described a teacher who encouraged students to express themselves using their home language before translating into "standard" English. Similarly, in a high school class Carol Lee (2001) called "response to literature," she used discursive practices familiar to students as resources for reading and interpreting texts.

Moll and his colleagues (1992) took a slightly different approach, collaborating with teachers to research and document the “funds of knowledge” available to students in their home and community networks, and particularly through families’ labor histories. They describe funds of knowledge as “historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being” (p. 133). They and others have since been investigating designs that leverage these resources in the classroom (Calabrese Barton & Tan, 2009; Gonzalez, Andrade, Civil, & Moll, 2001; Moje et al., 2004).

These designs produce “hybridity,” bridging home and community practices with school practices, thus transforming the learning environment and creating new opportunities for children’s learning. They also validate a spectrum of practices, experiences, and identities as visible in and relevant to school topics and learning. Therefore, it is not just children’s learning that is transformed, but also our understandings of what counts as legitimate school content, activity, and success. Hybridity theory has its roots in postcolonial and other postmodern work dealing with culture and identity under the conditions of globalization, migration, and oppression (for more detail, see Gutierrez, Baquedano, & Tejeda, 1999; Kamberelis, 2001; Lucey, Melody, & Walkerdine, 2003). Using Bakhtin’s (1981) notions of heteroglossia and dialogism, literacy scholars have taken up hybridity as a way to understand classroom discourse and learning. Heteroglossia, or the multiple voices in a social space, can be acknowledged and fostered so that dialogic meanings can be negotiated and developed. These multiple voices include not just those of the individuals present, but also the many identities and histories of experience participants may choose to present in an interaction. This is in contrast to monologic and authoritative

meanings that can be imposed in classrooms and other settings by those in power positions (e.g., teachers).

For example, representational conventions like contour maps have been developed over large spans of time by societal groups for the purposes of their practices. One way of teaching these conventions to students is to present them, explain what they are for and how they work, and expect students to use them appropriately given relevant tasks. This type of instructional practice relies mainly on an authoritative discourse, treating representations as information in and of themselves, rather than as interpretable (Greeno & Hall, 1997). On the other hand, Enyedy (2005) describes a classroom where students are given a task which leads to their “discovery” or “invention” of contour maps. This instructional setting relied on students generating their own meanings of the problem (representing a city made of blocks from a bird’s eye view) and making sense of the different representational strategies offered by the class. While Enyedy did not describe his study this way, the voices and ideas of individual students, based on their various experiences in and out of class, in combination with historically developed mathematical meanings, developed over the course of the lesson unit into a shared representational practice (contour mapping) for the class.

Kris Gutierrez and her colleagues (Gutierrez et al., 1995; Gutierrez et al., 1999; Gutierrez, Baquedano-López, & Alvarez, 1999) have written about hybrid discourse practices that animate what they call the third space, “in which alternative and competing discourses and positionings transform conflict and difference into rich zones of collaboration and learning” (Gutierrez et al., 1999, pp. 286-287). They point out that there is hybridity, and therefore transformation, in all learning environments, as a result of the diverse

resources and experiences participants necessarily bring to and deploy in an activity. In addition to officially sanctioned social and discursive spaces, classrooms are always also made up of unofficial spaces (or activity systems). Hybridity, for Gutierrez and her colleagues, is made up of official and unofficial spaces, and can be resisted or leveraged for learning opportunities. Third spaces arise when unofficial spaces, in interaction with official spaces and discourses, are included and supported.

Gutierrez and her colleagues argue that these third spaces, animated by hybrid practices, can be productive for equitable school learning (Gutierrez et al., 1999). In their study, the teacher intentionally used diversity as a resource. In other words, student engagement in unauthorized or unofficial behaviors and language practices were acknowledged and transformed into opportunities for learning. For example, in a combined second and third grade class, a unit on human reproduction was developed as a result of a moment of conflict (some children were calling each other names, including “homo”). Then during the unit, the teacher reframed students’ giggling as a legitimate and understandable expression of anxiety. Teachers’ responses to students’ emergent behaviors and contributions during classroom instruction can be designed at multiple levels to support unofficial practices and produce hybridity.

Building on the hybrid language practices and funds of knowledge work, Angela Calabrese Barton and Edna Tan (2009) conducted a design experiment where researchers, teachers, and students collaboratively planned science lessons that would explicitly allow and encourage students to draw from their everyday funds of knowledge related to cooking and eating. They found that students brought to bear a wide variety of family, community, peer, and popular culture funds of knowledge in relation to this topic, that

the classroom was transformed, and hybrid spaces were created in three different ways: physically, politically, and pedagogically. Calabrese Barton and Tan's study is one of few that attempts to detail aspects of the learning context, other than language, that might be productively hybrid. The transformations of physical space (e.g. classroom as kitchen), political space (e.g. students were positioned with more authority), and pedagogical space (e.g. students helped design the lesson) were a result of a deliberate design which targeted existing lesson content that students enjoyed, and employed pedagogical strategies to support students in making connections between home knowledge and practices and school.

The lessons in both Gutierrez et al.'s (1999) and Calabrese Barton and Tan's (2009) studies were responses to student participation and engagement. For Gutierrez et al., the human reproduction lesson emerged from a brief yet conflict-ridden interaction among students. For Calabrese Barton and Tan, the food and cooking lesson was designed in response to past student interest in the topic and in collaboration with a small group of current students. However, the emergence of hybrid learning environments in each setting was characterized very differently.

For Gutierrez et al. (1999), hybrid third space arose from conflict and tension, or the interaction between an official and unofficial script. An unofficial script, assumed to always be present, was recognized, legitimized, and incorporated into instruction. When hybrid discourse practices were supported, the activity system of the classroom was transformed, providing new rich opportunities for learning at the levels of instructional design as well as moment-to-moment pedagogy (Figure 2-1a). In the Calabrese Barton and Tan (2009) study, which was a design experiment, hybridity was more deliberately

planned and at a longer timescale. While co-designing the lesson with students, researchers actively sought to discover and anticipate the funds of knowledge that the class could bring to their learning, in combination with their school science funds of knowledge. Instead of emerging from conflict, the hybrid discourse and space in this study was designed with the cooperation of students (Figure 2-1b).

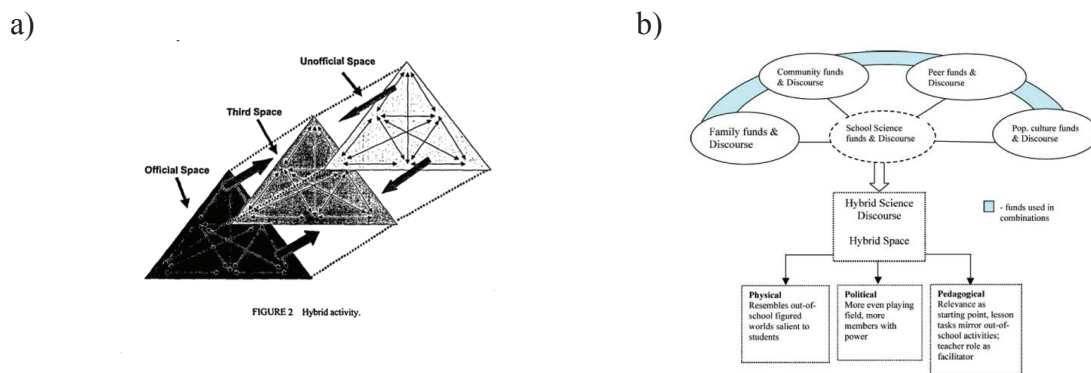


Figure 2-1. Models for promoting hybridity in classroom settings: a) from Gutierrez, Baquedano, & Tejada, 1999, p. 292; b) from Calabrese Barton & Tan, 2009, p. 68.

Despite this difference, both groups of researchers documented productive hybridity in their respective classrooms. The studies had three important commonalities. First, the researchers took as assumed that diversity is a resource in teaching and learning, and can be leveraged. They did this by incorporating typically invisible or undervalued student resources into classroom activity. Second, the cooperating teachers in each classroom actively invited and supported students' diverse contributions, whether they were discourse practices or other forms of funds of knowledge. A third commonality is that in both studies the researchers and teachers deliberately used aspects of instruction—topics, participation structures, instructional strategies—to bridge between school and out-of-

school practices. These three aspects of design are common across many studies that seek to promote hybridity (Gonzalez et al., 2001; Gutierrez et al., 1999; Moje et al., 2004; Wager, 2012).

The design of this dissertation study shares with the two examples a commitment to treating diversity as a resource and supporting and leveraging student contributions, including those that might be characterized as counterscript. Settings that embrace these two instructional goals position students with conceptual agency, granting them opportunities to make sense of problems and concepts and to author strategies and solutions in ways meaningful to them (Boaler & Greeno, 2000; Hull & Greeno, 2006).

The dissertation study differs from the two examples in that the designed context is not meant to *produce* hybridity, but to provide resources to *support* it. By this I mean that I view the designed WSG setting as a *mediating setting* that bridges between school and out-of-school practices. Rather than attempting to make direct connections between students' mathematical funds of knowledge and classroom mathematics, WSG tasks provide opportunities for students to recruit a variety of funds of knowledge and other resources for problem solving (including resources from the classroom).

The WSG tasks are a setting separate from but tightly related to typical classroom mathematics. These tasks require a new set of goals, tools, rules, and division of labor, and can be thought of as constituting a new activity system. However, they take place during the time of typical classroom mathematics (although not in the same space), and have the same top level goal of learning geometry. The problems have close family resemblances to typical classroom geometry problems, often using the same language. They are introduced in the setting of classroom mathematics, and unsurprisingly, students



draw from their knowledge of and experiences with classroom mathematics in order to solve the problems. At the same time, WSG tasks also require students to recruit and adapt other resources. I treat the WSG tasks as a designed, shared experience that both contains disruptions and serves as a disruption to typical classroom mathematics. This shared experience provides opportunities for students to draw from their out-of-school experiences and practices as resources for participation and learning, and ultimately itself becomes a resource for classroom learning (Figure 2-2).

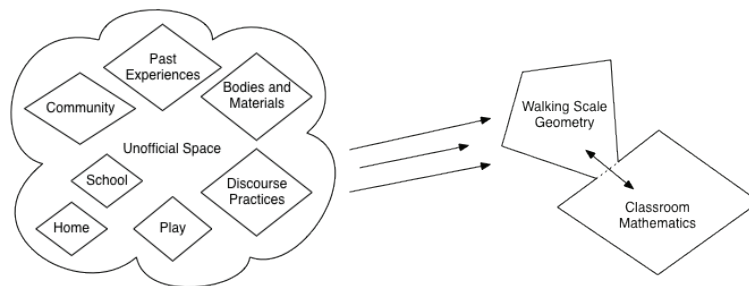


Figure 2-2. Designing WSG as a disruption to support productive hybridity.

My design also deviates from other examples of designing for hybridity in that *sources of hybridity* were designed to be emergent. Gutierrez and her colleagues wrote about hybridity that emerged from instruction that opportunistically leveraged unofficial student discourse. Calabrese Barton and Tan’s design relied on students’ funds of knowledge in relation to a particular topic as a source of hybridity. In the WSG design, the possibility for students to recruit resources of all kinds was left open. Of course, the materials and tasks in the design afforded particular kinds of resources to be recruited. For example, I provided ropes for drawing. However, I did not assume that students would draw from any particular practices in using the ropes or even that their use of the ropes

would evoke any one context or fund of knowledge. In analysis, I attended to evidence that students recruited resources not typically relevant to classroom mathematics.

The idea of designing *for* hybridity (rather than designing the hybridity itself) emphasizes the agency of students along with teachers in developing, then regenerating or transforming the classroom culture and norms. Students' diverse practices are not necessarily known to designers or teachers, and how recruited resources will get taken up, adapted, or rejected by the developing and evolving classroom culture cannot be completely anticipated. These resources may originate from counterscript, students' home, community, or cultural funds of knowledge, or their many repertoires of practice. There may also be other sources from which students draw in participating in emergent hybrid learning settings. Once students begin to engage in the tasks, teachers and designers (in interaction and between sessions) have many opportunities to shape emerging hybridity. This responsive, ongoing instruction and design can support the continued emergence of new contributions to hybridity through both legitimizing students' recruitment of resources and continuing to promote conceptual agency.

Characterizing the resulting hybridity as emergent acknowledges that students and teachers play active roles in creating teaching and learning contexts; bring diverse histories of participation in diverse practices to each setting; have dynamic interactions and developing relationships that can/should not be scripted. Erickson's (2004) description of face to face social interaction is also apt for the dynamic character of participating (as students and as teachers) in an emergent hybrid learning context: it's "like climbing a tree that climbs you back at the same time" (p. 110). The transformed(ing), hybrid learning context emerges from (ongoing) design, is determined by students' and teachers' ongoing

participation, and, given the regular incorporation of unofficial and unexpected student contributions, never stabilizes.

This study describes a designed disruption, WSG, that deliberately invited diverse student ways of making and negotiating meaning. Instead of designing around and leveraging known funds of knowledge or discourses, this designed disruption granted students conceptual agency in problem solving activities by expecting them to invent tools and strategies for problem solving. The disruption created an ongoing shared experience for students with the potential to support emergent hybridity for classroom learning. This dissertation focuses on student engagement in the WSG setting, with an eye toward how classroom instruction could be designed to leverage students' experiences in WSG as resources for classroom mathematics.

The lesson design explored in this study was not meant to be an exemplar of an instructional sequence of geometry tasks. Instead, it was treated as a design experiment (Cobb, Confrey, Disessa, Lehrer, & Schauble, 2003), where the idea of designing a particular setting for geometry tasks was explored through the theoretical lenses of hybridity by way of conceptual agency and the recruitment of novel resources, and disruptions. In the next two sections, I describe what I take to be productive hybridity, what I mean by disruptions, and how the study contributes to existing research about principles for designing learning contexts.

### **Productive Hybridity**

Any learning environment can be seen as hybrid, given the diverse histories of experiences and cultural practices that participants inevitably bring to any context (Guti-

errez et al., 1999). Whether this diversity is acknowledged or rejected, it nevertheless shapes the space and influences the engagement and learning of all students. Additionally, as diverse resources are introduced and integrated into classroom practices, the hybrid spaces produced are dynamic and emergent. Old ways of learning, doing, and knowing mathematics are transformed, and develop over time into new ways. How then, can we determine if the hybridity that emerges from our design is productive, either in the short term or in the long term, and in ongoing or retrospective analysis?

One way to gauge the productivity of a learning context is to see if there are *opportunities to learn* for all students. Typically, opportunities to learn have been described in terms of students' interaction with relevant disciplinary content and available resources for learning (Pullin & Haertel, 2008). From a sociocultural perspective, opportunities to learn are opportunities to participate in more and more central ways in a community of practice (Greeno & Gresalfi, 2008; Gresalfi, 2009). Broadly, this means opportunities to make contributions to the goals of the community. In a mathematics classroom, these goals include solving mathematical problems, and improving students' and the group's capacities to solve more and more sophisticated problems involving content matter delineated by standards or other course objectives. This involves:

affordances for changing participation and practice. In this view, understanding a learner's trajectory involves hypotheses about affordances that are available to the learner to participate in particular ways. Affordances (the term was contributed by Gibson 1979) are relational. An affordance for an individual in an activity system includes the resources and practices of the system, that individual's access to those resources and practices, and the dispositions and abilities of the individual to participate in a way that supports her or his activity and learning in some way (Norman, 1988). Affordances are not all-or-none; rather, they vary along a continuum. When we say that an activity affords some aspect of participation for some individuals, we mean that it makes it relatively easy for those individuals to participate in that way. (Greeno & Gresalfi, 2008, p. 172)

In particular, tasks have affordances through their mathematical content, their cognitive demand, and the kinds of agency they position students with. For example, if a formula is given to students along with instructions to plug in certain values, then the mathematical content may not be relevant, the cognitive demand of the task has been diminished, and the students have little or no agency in the problem solving process. If students are too advanced, problems become trivial and offer no opportunities to learn. Alternatively, if students do not have (or are not allowed) resources to engage in those same problems, then the problems offer no opportunities to learn for them either. This is especially true if problems (or other aspects of the instructional setting, for example the teacher) constrain students to engage in particular ways. A productively hybrid learning context provides more, and more diverse opportunities to learn for all students in the mathematics class, in part by expanding the possibilities for what counts as legitimate resources for making sense of problems. Students have opportunities to participate in various ways, at different levels of sophistication, in all parts of the task.

Of course, there are many other factors that influence whether or not these affordances, or opportunities to learn, are actually taken up, and how they are taken up. A designed disruption for productive hybridity not only provides opportunities to learn, but also provides a context in which students are likely to participate in productive ways.

Engle and Conant (Engle, 2011; Engle & Conant, 2002) proposed an approach that takes student participation as a goal of instruction, and suggested a framework for understanding students' participation in learning environments. They call their framework *productive disciplinary engagement* (PDE), which has three aspects. The first is simply *engagement*. While slippery to measure due to students' diverse ways of engaging and

researchers' varying commitments to forms of participation, this construct is an important first building block to the productive disciplinary engagement framework. Engle and Conant suggest a variety of ways to measure the intensity of engagement (e.g., involvement in “off-task” activities and making substantive contributions to discussion), and recommend looking for converging evidence of engagement. The second construct is *disciplinary engagement*, which, for my study, adds mathematics to the idea of engagement. They do not make claims about what counts as disciplinary, but, for now, leave that to researchers discuss and to decide for their respective studies. The third construct is *productive disciplinary engagement*. The addition of “productive” means there is disciplinary progress, depending on what disciplinary elements have been identified as relevant.

I make the initial assumption that, in order for emergent hybridity in the classroom context to be productive, the mediating WSG setting must also be, to some extent, productive as a learning environment in and of itself. In order for the WSG experience to be a resource for learning in the classroom, students must take up opportunities to participate in WSG problem solving activities. This is not to say that failures or trouble encountered in the WSG setting cannot be valuable resources for learning. However, WSG tasks should be designed so that they have affordances for the participation of all students, and for engagement to be productive and disciplinary.

Engle and Conant (Engle, 2011; Engle & Conant, 2002) also offer four interrelated design principles for fostering PDE. First, content must be problematized in ways that engender uncertainty in students, is relevant to students' commitments, and is consequential for disciplinary learning. Second, students must be granted intellectual authority, which includes agency, authorship, contributorship, and the ability to be recognized as a

local authority on some topics. Third, students and their ideas must be held accountable to others', including their classmates' ideas and concepts in the discipline. Last, students must have access to the resources necessary for the kinds of participation and learning expected of them.

The concept of productive disciplinary engagement and the associated design principles are consistent with the kind of learning environment that my conceptualization of a design for productive hybridity aims for. However, the PDE framework, as written, does not take into account the possibility for students' unofficial contributions to be recognized as relevant and valued learning opportunities (engagement). The very idea of engagement assumes "appropriate" or "on task" participation in some ratified official space, defined a priori by researchers, teachers, administrators, and the like, as an indicator of individual learning. In order to determine whether emergent hybrid learning environments are productive, we must allow for unexpected and unofficial student contributions that may result in disciplinary learning that might not appear to fit into any category of engagement. Additionally, in the disrupted mediating setting, what counts as engagement that will best support PDE in the classroom later? It may be a question of timescales (Lemke, 2000). What looks to a teacher or an analyst like disengagement now may result in productive disciplinary engagement, later. The outcome may depend on any number of aspects of the student's participation in the activity, including his or her interactions with the materials, other students, or the teacher.

Given the emergent nature of hybridity, the notion of what makes hybridity and the ongoing mediating setting productive becomes problematic, especially at small timescales. The analysis in this study will continue to build on what might count as productive,

and contribute ideas for designing classroom instruction to support emergent hybridity that leverages the WSG setting as a resource.

### **Designing Disruptions for Hybridity**

Students cannot be expected to spontaneously make connections with out-of-school practices in classroom settings, and designing tasks to support or elicit these practices and funds of knowledge, even when they are researched and known, is challenging (Gonzalez et al., 2001; Moje et al., 2004; Wager, 2012). In order to create opportunities for students to recruit out-of-school resources for their participation in and contributions to an emergent hybrid learning environment, my instructional design involved a separate, ongoing, mediating geometry setting that created a major disruption to typical school practices. My treatment of the WSG design as a disruption to typical classroom geometry follows an extended analogy to studies of conceptual change at work conducted by Hall and colleagues (Hall, Stevens, & Torralba, 2002; Hall, Wieckert, & Wright, 2010). While their work found and analyzed naturally occurring cases of disruption in technical or design-oriented workplaces, I deliberately disrupted structural aspects of mathematical activity in (and out of) school classrooms. In the following section, I describe how I use the idea of “disruption” in the context of my design experiment for this dissertation study.

In a comparative case study of two workplaces, Hall, Stevens, and Torralba (2002) described disruptions that were meant to change representational infrastructure, which support conceptual systems (Hall & Greeno, 2008). These disruptions were characterized as a result of interactions between participants from different disciplines. In the first case, entomologists invited a statistician to help them reconceptualize a representational device



for classifying termites. In the second, architects, structural engineers, and historical preservationists negotiated the renovation of a town library. In these two cases, conflict arose “when discipline-specific perception and action from one discipline threaten[ed] to destabilize existing practice in another (i.e., statistics vs. entomology at the BugHouse) or to undermine the prevailing interests of another (i.e., structural engineering vs. historical preservation at JCArchitects)” (p. 205).

The many in- and out-of-school practices in which students participate can be thought of as different disciplines in constant interaction. Therefore, we might expect that disruptions to representational infrastructure of not just professional practices, but also day-to-day activities, might be commonplace. However, in most contexts individuals are positioned as, and participate with, particular identities. Moreover, in typical school settings only a few types of identities are legitimized or welcomed. From a figured worlds perspective, participants in schooling learn to interpret the actors, events, artifacts, and goals of schooling in particular ways, and come to understand appropriate and inappropriate ways to interact and behave in relation to others as well as events, artifacts, and goals (Holland, Lachicotte, Skinner, & Cain, 1998). In most classrooms, practices that are misaligned with typical valued school practices are ignored or rejected as part of the unofficial space (Gutierrez et al., 1995; Gutierrez et al., 1999). Conflicts such as those documented by Hall et al. (2002) are less likely to occur, since students are (presumably) in school for the very purpose of having their existing practices destabilized (i.e., learning), and if they have interests that are not aligned with those of schooling, they do not have the rights to promote them.

Still, the analogy does provide some leverage for an instructional design meant to make space for unofficial practices and contributions. If all in- and out-of-school practices have the potential to be legitimized and seen as beneficial for student learning, then the idea that the interaction between disciplinary specialists can be productive sites for change is a powerful one. The assumption that I make, along with Hall and his colleagues (2002), is that disciplinary specialists have particular ways of seeing and animating representational materials for the purposes of problem solving and communication (Goodwin, 1994; Stevens & Hall, 1998). Let us take school mathematics as a discipline, and our enterprise to be training students to become conversant, if not specialists, in school mathematics. Then what is it that students come to classrooms as? They are not *just* novice school mathematicians, and they are certainly not blank slates waiting to be filled. They have particular (and probably not homogenous) ways of seeing, as well as particular ways of animating representational materials for the purposes of problem solving and communication. Likely, they have many different ways of doing these things, depending on the contexts and pragmatic goals of the task. In this case, we can consider them to be specialists in at least one or more disciplines, or practices. Their interactions with school mathematics and the disciplinary specialists of school mathematics (teachers) have the potential to be fertile ground for change. They also have the potential to be fertile ground for resistance and conflict.

Taking into account the ways that students are specialists is a necessary precondition to promoting change rather than resistance. This is not new idea. Constructivist theories of learning and associated pedagogical methods, for example, including reform mathematics education, have long promoted making student thinking visible and build-

ing on students' prior knowledge and experiences (Confrey, 1990; Gravemeijer, 2004; Simon, 1995). However, the concept of disruptions, with its situative perspective, takes into account not just students' prior knowledge, but also their histories of participation in contexts where they have deployed this knowledge and the relevant responsibilities and goals of their engagements. Additionally, the idea of designing disruptions for productive hybridity emphasizes an expansion of what might *count* as relevant prior knowledge and experiences. Designing disruptions for productive hybridity foregrounds and suggests a possibility for the transformation of existing, often invisible school mathematics teaching and learning practices by providing meaningful contexts and goals that support sense-making and the recruitment of students' diverse resources to do so.

In the case comparison offered by Hall et al. (2002), disruptions occurred in the interaction between two or more disciplines, either where incumbents agreed to develop new infrastructure or where existing infrastructure were challenged. They noted another possibility, analyzed in Hutchins's (1995) study of a Naval aircraft carrier, where infrastructure breaks down. Given the difficulty in previous studies to engage students' out-of-school resources (e.g., Moje et al., 2004), designs cannot rely simply on offering students opportunities to make connections to out-of-school practices. However, it is possible that by "breaking" or disrupting existing norms and resources, it may become likely that students would be more willing, or even compelled to make unofficial contributions. This kind of design would be concerned not just with disrupting representational infrastructure, but ways of learning, knowing, and doing mathematics in general.

## Designed Disruptions in Walking Scale Geometry

For the WSG lessons designed in this study, I disrupted the spaces, tools, perspective, and division of labor of typical mathematics learning by changing the scale of the problem solving activities. Mathematics is generally taught in classrooms where students sit at desks and their written work takes place on the paper in front of them. WSG, on the other hand, involves geometry tasks solved at very large scale, with the ground as a “drawing” surface. This typically takes place outside, but we have also had groups engage in these tasks in large, open areas indoors. Participants are not allowed to engage in problem-solving using paper and pencil, although we have negotiated uses of paper and pencil (or similar items like cardboard and markers) as tools deployed in the service of walking scale problem solving. For example, one group traced the vertex of a walking scale angle that they had “drawn” with rope on a piece of cardboard, then used that piece of cardboard to help construct congruent angles.

The large scale of WSG tasks disrupts students’ typical engagement with mathematics in four ways (see Figure 2-3 for a summary). First, there is a *spatial* disruption—they take students outside, away from the classroom and desks and chalkboards, into large, open spaces more commonly used for athletics and play. Second, they require students to adopt new *tools* for drawing, representing, and completing mathematical procedures. They no longer have paper, pencils, rulers, protractors, or compasses, and can no longer engage by filling out solutions to problems on, for example, worksheets. Instead, students are asked to reason with and about everyday material objects and their bodies, and find ways to mathematize them. Third, students’ *perspective* on the geometric objects is radically different. Not only are students much smaller in comparison to the

objects, they must view them from within, rather than from a birds-eye-view as they do when working at paper and pencil scale. This means a figure looks significantly different to any individual, and students do not generally have the same view as others. As a consequence, judging whether constructed lines are straight or parallel, or whether angles have familiar degree measures (e.g., 90 degrees) becomes problematic, inviting or requiring new techniques for making or judging these mathematical properties. Fourth, students are no longer able to draw and manipulate the geometric representations individually. At walking scale, the *division of labor* of geometry problem solving is distributed so that individual solutions on tasks are difficult, if not impossible. Students must work together and communicate effectively to develop, negotiate, and accomplish their goals.

	Typical Classroom Mathematics	Walking Scale Geometry
Space	In the classroom, individually at desks, focused on whiteboards.	Outside, wide open space, athletic field or lawn.
Tools	Paper, pencil, rulers, protractors, manipulations of the hand.	Ropes, lawn flags, tape, uses of whole bodies.
Perspective	Extrinsic, bird's-eye-view on representations, can see whole figure at once.	Intrinsic, inside the figure, each student sees something different.
Division of Labor	Often individual, most tasks can (or must) be completed by one person.	Requires more than one person to solve.

Figure 2-3. How WSG disrupts typical classroom mathematics with respect to space, tools, perspective, and division of labor.

For example, we asked students, in small groups, to construct an isosceles triangle whose sides were at least twice the length of one group member's body, and then justify

that it was isosceles. We provided students with materials that had been useful to others in pilot studies of the tasks, as well as others that they asked for. This instructional design could be thought of as using Vygotsky's method of double stimulation (Engeström, 2007; Engeström, 2008; Vygotsky, 1978), in which learners are provided with a problem context beyond their abilities, and also a "neutral" object that they may recruit and reorganize in meaningful ways to help them solve the problem. I do not consider the materials that we provided students to be neutral, since the assumption was that, even if students did not request all of the materials, they have had experiences with those or similar items in everyday contexts. Students develop these materials (including their own and each other's bodies) into mathematical tools (they mathematize them), over the course of problem solving. In making mathematical meanings for the familiar materials and inventing conceptual and representational tools with them, students are granted rich conceptual agency.

Students solved this isosceles triangle task in a variety of ways. In one instance, a group constructed the triangle with rope by doubling up some length to obtain the two congruent sides, then stretching the rope out to create straight line segments, and determining the third side based on some arbitrary angle between the two equal sides. In order to accomplish strategies like this, students had to share some sort of understanding about the solution strategy so they could jointly implement it. Different students had to take charge of holding each end of rope in order to make it taut and, therefore, straight (see Figure 2-4 for an image of three seventh grade students making an isosceles triangle). They had to negotiate how hard they pulled (enough to create a shared agreement of straightness, but not hard enough to pull the rope out of another student's hands) and how to hold their rope ends (the boy in the red shirt crossed the ends to create an angle rather

than simply joining them, influencing the length of the triangle's sides). This new kind of engagement with a mathematical task and problem solving supported students' recruitment of resources, including tools, strategies, and discourse practices, from out-of-school activities (e.g., students may have had experiences playing tug of war, or ways of coordinating multi-body activity playing team sports).



*Figure 2-4. Constructing an isosceles triangle at walking scale.*

There is evidence that these kinds of disruptions in space, tools, perspective, and division of labor to typical school activity do expand learning possibilities for the whole class. Hall and Nemirovsky (2008) used a large scale geometry task with pre-service teachers, and discovered that teachers with experience in competitive marching bands used ropes and coordinated pacing to build and transform the drawing of a G-clef, understood and performed as a creative interleaving of musical and mathematical objects. In an analysis of a high school cabin construction project which was situated spatially and socially in both school-like and non-school-like ways, Leander (2002) found that building

a cabin during school altered the physical location of the class as well as the paper-and-pencil-at-desk scale of typical schooling. As a result, students encountered new opportunities to contribute and participate in the activity. Leander's analysis demonstrated the importance of how material and social space contributes to hybridity on the one hand, and is experienced as a result of hybridity on the other.

Additionally, research has also demonstrated that students' recruitment of unexpected, non-classroom resources can also serve to support students' sense-making in problem solving activities and transform learning. In Roth's (1996) study of a student design unit involving building weight-bearing structures, students' innovative uses of a glue gun provided opportunities as well as constraints for their learning. In a productive way, students had the opportunity to learn to exploit the glue gun in flexible ways, discovering purposes for it other than the traditional gluing use. In a less productive way, some uses of the glue gun allowed students to avoid discovering a strong bracing structure that teachers were hoping they would learn.

Roseberry, Ogonowski, DiSchino, and Warren (2010) describe how an experience during a fire drill on a frigid Massachusetts school day became an unexpected resource for the class in talking about and understanding the transfer of heat. This shared experience became a rich resource in the context of a participation structure the class called "Sherlock," which was "a space for listening, understanding, and exploring possible meanings, not for evaluating or correcting students' ideas" (p. 330).

In Roth's (1996) analysis, students flexibly made use of the newly recruited tool (glue gun) in the context of their *emerging design goals*. In the Roseberry et al. (2010) study, a *participation structure* and discursive practice gave students opportunities to



make sense of an unexpected experience outside in the cold. In my design for productive hybridity, the disruptions associated with WSG promote both emergent goals and participation structures that will support these kinds of student responses. The disruptions in tools and perspective push students to recruit and invent new tools for drawing and manipulating geometric figures (e.g., folding rope in half in order to produce two congruent line segments), as well as new ways of seeing and recognizing categories of geometric objects (e.g., walking around a quadrilateral and viewing it from different angles to determine whether it is, indeed, a parallelogram). Like in Roth's study, goals emerging from problem solving activity support the recruitment and flexible adaptation of unexpected resources.

The newly configured division of labor of WSG tasks takes an alternative approach to Roseberry et al.'s study where a designed participation structure, Sherlock, was in place to support students' negotiation of meanings. WSG tasks provide a goal-oriented need for students develop ways to communicate and coordinate their understandings and bodies, at least to the extent that they can jointly solve the problems together. The in-class lessons accompanying WSG tasks should also be structured to support students' learning. These were less intensively designed in this study, but my experiences with the classroom activities that I designed and helped implement alongside the WSG inform the analysis here. Additionally, findings in this dissertation will support conjectures for designing the classroom instruction in future WSG iterations.

## **Design Principles**

Returning to Engle and Conant's (Engle, 2011; Engle & Conant, 2002) four design principles of problematizing content, intellectual authority, accountability, and access to resources, how might the designed disruption in WSG contribute to fostering PDE?

The disruption to the spaces and places of learning and doing geometry both problematizes mathematics content and changes the kinds of resources students have access to. WSG takes students out of their classroom, away from their seated configurations in desks and the predominant orientation toward the teacher and whiteboard, into large open spaces. Children and adults often do not recognize everyday or out-of-school activity as mathematical or as doing mathematics, although mathematicians and educators might classify it as such (Goldman & Booker, 2009; Lave, 1988). The change in space and scale of WSG problematizes the idea of mathematics as being contained inside classrooms, on desks, on paper and pencil, and explicitly challenges students to make connections between their school understandings of geometric figures and relationships with whole body, multi-party activity outside.

Additionally, Nespor (1997) wrote of the bodily meanings that school had for kids in the school that he studied. School meant being inside, constrained, and controlled, whereas their drawings of and stories about their neighborhoods emphasized spaces where they were engaged bodily. Nespor critiqued schooling as devaluing the body and removing it as a primary mediational tool for experiencing the world. Drawing from Nespor's ethnography of children's bodies in school and community space, the change in space and scale of WSG seeks to reconstitute the body, as well as embodied ways of being in the world, as resources for doing mathematics.

The disruption in material and conceptual tools of geometry affects all four of the PDE principles, but most saliently this disruption provides learners a large degree of intellectual authority. In WSG, students mathematize everyday materials in the service of solving problems and completing tasks. They represent mathematical objects like points, lines, and angles with rope, flags, and bodies. They invent routines for copying, scaling, and performing transformations. They develop ad hoc units of measurement and strategies for comparison. Providing students with the opportunities to make their own mathematical sense, grapple with strategies, and invent representations contributes significantly to conceptual agency, authorship, contributorship, and the ability to be a local authority (Boaler, 2000; Enyedy, 2005; Hall & Greeno, 2008; Lehrer & Pritchard, 2002).

The change in perspective from extrinsic to intrinsic views of geometric objects in WSG problematizes geometry concepts in a number of ways. The ways that students have developed of “seeing” concepts (Goodwin, 1994; Stevens & Hall, 1998) like straight, acute, and congruent are disrupted when standing inside the representations, since entities described by these relations look different, depending on where the viewer is standing. The intrinsic perspective often also means that students don’t have visual or physical access to the entire representation or problem at once. These partial views, a radical shift from the typical bird’s eye view of school mathematics, affords new approaches for coordinating vision, representation, talking, and inscription during problem solving.

Lastly, the disruption to the typical division of labor of school mathematics promotes accountability. Because individual students do not have complete physical access to the materials and representations of WSG, they must coordinate and negotiate their ideas and actions in order to accomplish their problem solving goals. As will be clear in

the analyses that follow, a pressing need for coordination puts new demands on learners to explain what they are doing or are proposing that others do, in coordination with them.

In a design for emergent productive hybridity, it is also important to monitor and support student contributions. Ironically, the large scale nature of WSG tasks makes it more difficult than usual for teachers (and researchers) to get access to student thinking during problem solving activities. Students work far away from each other, so moving within and between groups to ask questions takes time, and overhearing conversations is less common than in a typical classroom. For this reason, the WSG tasks were always accompanied by classroom discussions, during which students shared and compared their strategies and solutions, and the teacher highlighted and developed mathematical concepts that were in the lesson plan or that had emerged (through student work) as being salient. This was accomplished with the help of photographs taken by researchers during WSG activities. While students worked, researchers took still images from on the ground as well as from elevated viewing positions above (and adjacent to) the surface of the field. Students were also told to pause for on the ground and overhead photographs after the completion of each task. This provided the teacher with portable, classroom scaled representations of student work that could be used in class discussion, and occasionally for students' own annotations.

### **Research Questions**

Given the designed disruptions of WSG and the proposal that experiences in this setting might support productive hybridity in the classroom, this study asks the following questions.

1. How do the disruptions of WSG invite new ways of problem solving? What forms of engagement emerge in practice? What resources are available or get recruited for problem solving in WSG?
2. Do the designed disruptions of WSG provide more (and more diverse) opportunities to learn? What are the resources for participation and sense-making, and how do they vary across students?

These two questions are similar in that they investigate resources associated with participating in the WSG setting. However, they each point to a different aspect of participation and learning. The first question focuses on the relationship between participation and the design of the setting. Here, I look to see how students solve problems in the WSG setting, and what resources they recruit for doing so. The next question explores possibilities for different forms of engagement in problem solving. This can be seen as a resource for problem solving, but for the purposes of organizing the dissertation, I separate it in order to consider the kinds of opportunities to learn, or access to participation in the WSG setting. For this question I am asking about the different ways of engaging meaningfully and consequentially for the purposes of accomplishing the goals of problem solving.

In this dissertation I explore the particular disruptions of one WSG design, the forms of participation and engagement it supports, and how participating in WSG might support hybridity in the classroom setting. In Chapter Four, I respond to the first question by presenting episodes of students engaging in WSG tasks to illustrate ways in which materials and bodies were used in coordination for mathematical problem solving. I highlight resources that are made available as a result of the four disruptions, or that students

recruit because of them. I also point to different aspects of participation and learning, including how students mathematize materials, ongoing assessment, and argumentation.

In Chapter Five, I respond to the second question by examining episodes that highlight opportunities to learn in the form of access to participation and sense-making in WSG tasks. I consider how the disruptions of WSG provide resources to make access available and diverse so that students at varying levels of classroom mathematical sophistication, and students with varying out-of-school experiences, might have opportunities to participate in problem solving.

In the concluding discussion I make comments, based on the findings from the presented episodes, about how the experience of participating in this WSG setting might provide resources for hybridity and learning in the classroom. I also make some conjectures about how to design classroom instruction to leverage these resources in future iterations of WSG and in other forms of design that purposely disrupt existing school mathematics instruction.

## CHAPTER III

### RESEARCH DESIGN

This study was conducted as part of a design experiment (Brown, 1992; Cobb et al., 2003; Sandoval, 2004). Design experiments are concerned with the design of learning settings and, at the same time, theoretical understanding of learning within those settings. Conjectures are formulated about how design will support learning, then revised and refined over iterative design cycles. My tasks, featuring the designed disruptions described above, were created and implemented in this way, with careful attention to the design process and cycles of conjecture and revision as the lessons unfolded. Ongoing design as well as final analysis were conducted with future iterations of the study in mind. The lesson tasks (with adjustments) were implemented in two different settings, although I will not treat them here as different iterations, since the second setting was abbreviated, and it occurred directly after the first so there was not time for thorough retrospective analysis of the first setting before the second began. The WSG tasks were not changed substantially between the two settings, except to account for changes in grade level and available instruction time. They were not changed based on my experiences at the first setting.

#### **Participants**

The first setting for the study was a seventh grade mathematics class at an urban public middle school (KCMS). There were 16 students in the class, although there were rarely 16 students present. The class was taught by a Teach for America fellow, Ms.

N, who was in her second year of teaching and had an interest in ambitious, high quality teaching. During the study, Ms. H, the numeracy coach and a former middle school mathematics teacher of five years, took the lead on instruction when she was there. Ms. N acted as teacher's aide. Other members of the research team and I occasionally did as well. Mostly researchers answered student questions when necessary, especially when they were working in groups.

KCMS was of average size in the district, with 500 students in grades 5 through 8. According to the state Department of Education annual report, the student population during the year of the study was 63% Black, 18% Hispanic, and 18% White. Over 95% of students at KCMS were eligible for free and reduced lunch. It was a "fresh start" school in 2008 (meaning that all staff had to reapply for their positions), and it typically performed poorly on the state's standardized tests. In the year following my study, the school underwent another reorganization, replacing the principal and becoming the district's first teacher-led school. KCMS was a Title 1 school, and had many more resources even than most other Title 1 middle schools in the district. These resources included staff, technology, and programs. The class in which I conducted the study was an Advancement Via Individual Determination (AVID) class. This program was "designed for students in the academic middle who have the desire to graduate from high school and go to college" (from the district's website). The students in the program were generally placed in their classes together, and took an additional AVID elective that included study skills and tutoring. There were non-AVID students in the class as well, who had been placed there because they performed and behaved better in that setting. All students in this class tended to be higher performing and better behaved relative to other students at KCMS,



although not more than half scored as “proficient” on the state’s standardized mathematics assessment.

The second setting for the study was a Summer Enrichment Course (SEC) that took place at a university. The students were 12 rising 9<sup>th</sup> and 10<sup>th</sup> graders who had been accepted to the summer program and had signed up to take an intensive two week June course about spatial reasoning. The students lived on campus and spent 6 hours each day for 11 days in class. All of the students were in the SEC program voluntarily, and some were there for the second time (having taken a different course the previous year). Coursework was not graded, although instructors did send home brief evaluations of participation and learning, and parents of students had the opportunity to meet with instructors at the end of the program.

The SEC students had not only taken more years of school mathematics than the KCMS group, but all had experienced far greater success with it. Some of them had completed a full course in high school geometry, but not all. However, based on interview data and our experiences with them, all students in SEC had strong mathematical and general learning identities.

### **Study Design**

The lessons at KCMS occurred over the course of 7 weeks from early April until the end of May. The study began with observations, video recordings, and field notes of “typical” class sessions. At least one researcher took field notes during observations, and classroom artifacts (including student work) were collected. We observed class sessions where similar content (spatial reasoning/geometry) was covered, as well as instruction of

other mathematical topics. Some of these class sessions were review classes in preparation for the state standardized assessment, and some of these class sessions took place after the tests were over. These classes were not typical in the sense that test review is often different from the introduction of new concepts, and that lessons after state tests are often less “high stakes” than those before. However, based on informal visits that I made prior to this data collection, Ms. N’s instruction did not change drastically, and neither did students’ engagement. These observations helped me get a sense of how these students were typically taught in math class, what kinds of tasks they were given, how they engaged in them individually and in group work, and what resources they used.

This knowledge of the students’ typical math class experience was important for designing lessons and tasks that were disruptive in the ways that I intended, but also congruent in other ways with class routines. For example, on one of the days that I visited the class, the students went out into the hall to collect time data on walks of a certain distance for a lesson on rate, time, and distance. While this could be considered a disruption of space and tools, students’ movements and actions were tightly controlled by Ms. N, and the data collected, methods of collecting them, and their later use were predetermined. In terms of routine, each day began with a “Do Now” and ended with an “Exit Slip,” which Ms. N and Ms. H felt were important for managing student behavior and for formative assessment. My own design used these existing activity structures as well. However, I left the implementation of whole class discussions to Ms. H, giving her just guiding questions and objectives to address in relation to WSG tasks and activities.

The class used my instructional designs during the final five weeks of school in the late spring of 2011, after the state standardized assessment was over. The class met

for 70 minutes a day five days a week, although on Wednesdays the class met twice, for approximately 35 minutes each time. Class did not meet every day of the five weeks due to field trips and other events. In addition, we missed some class time as a result of “lock-downs,” which occurred whenever there were real, perceived, or “practice” threats to the safety of the school. More often than not these were either drills or routine searches of student property. In total, we had 18 full class days, of which 10 were devoted to WSG. We were able to go outside for 6 of these days. On the days that we stayed inside (mainly because of rain), we implemented activities and held discussions directly related to their previous or upcoming WSG tasks.

Ms. H and Ms. N collaborated with me on the design of the lessons. Before instruction began, Ms. H, Ms. N, and other research team members met to discuss my ideas about WSG, my research goals, the geometry concepts we wanted to cover, and the best ways to implement tasks like these with their students. I provided them with a rough outline of instruction for the 5 weeks beforehand, then emailed formal lesson plans each night before the next day’s instruction once we started. Ms. H and I, sometimes joined by Ms. N or other research team members, met after class each day (with a few exceptions) to discuss what happened and what we should do the next day. This helped to assure that my designs were sensible and appropriate in this setting, for these students.

The SEC students spent just 2.5 total hours with WSG over the course of 2 days. Two of those hours were spent outside working on WSG tasks, and 30 minutes were spent in the classroom, discussing their strategies and looking at photos of their constructions. I was the lead instructor for these lessons, and other research team members

answered questions and probed students for their thinking. The other lessons in the course were taught by different instructors, and were not as explicitly mathematics oriented.

Because the WSG lessons in the SEC context were so abbreviated and took place outside of a typical school mathematics classroom setting, I do not treat these two sites symmetrically in this dissertation. The KCMS context is the primary setting, and the SEC data provides additional data about how students might engage in WSG tasks. The SEC setting also provides some comparative leverage, so that I can consider how a different population of students participated in this context. I make some comments about this in the final chapter.

In both contexts we videotaped, captured still images, took fieldnotes when possible, and collected student work artifacts. In general we had three video cameras operating on the ground, and one capturing the activity from a spot above the lawn. At KCMS the soccer field was set below the school's parking lot, so the upper camera was placed at the top of the hill between the lot and the field. At SEC we used a large rectangle of lawn behind an academic building, and the upper camera was placed at the top of that building. In addition, we conducted semi-structured interviews with the students (in small groups at KCMS and individually at SEC) to examine their experiences of the lessons, as well as their developing mathematical understandings. (See Appendices C and D for interview questions in each setting).

### **Instructional Design**

Walking Scale Geometry (WSG) began as simply “Walking Geometry” (WG) (Hall & Nemirovsky, 2008), a way to explore how changing scale and modality influ-

ences mathematical experience and understanding. WG tasks used location aware devices, usually Global Positioning System (GPS) devices, to draw on the surface of the earth. Students carried GPS units that recorded their locations in space as they walked in planned paths. Their tracks could then be uploaded into software so that they could be viewed superimposed on maps of the areas in which they had walked. GPS drawing (see <http://www.gpsdrawing.com> for an example) in this way served as a technical infrastructure for creating a different way of doing constructive geometry.

WSG is a version of WG that takes on everyday materials as tools of inscription and representation rather than walking bodies and track logging devices. The two types of tasks have many properties in common, but also many divergent features. The disruptions in space, tools, perspective, and division of labor occur in both, but sometimes in different ways. In WG, one individual's body (whoever is carrying the track logger) is the drawing device, tracing representations on the ground. Others may assist in helping to make sure that individual's path is on target, and the logger may be handed off, but only one person draws at a time. In WSG, multiple bodies might be parts of representations. There is no experience of drawing in the sense that an object or body moves over a surface leaving a trace or track behind. Instead, existing materials are manipulated (cut, folded, stretched taut, tied together, etc.) to take the form of the object to be represented. These materials may only maintain the form of the object for a short while, and can be gathered up to create the next object or thrown away or saved for another project. However, the space where WG occurs is the same as WSG, and they both share the property of placing students inside large geometric objects, providing intrinsic views of geometric figures and problem solving.

The sequence of WSG tasks for both settings were based on the general design conjecture that students' understandings of geometry concepts (e.g., straightness, angular rotation, and sameness) can be improved if their diverse out-of-school resources are elicited, engaged, and leveraged in classroom instruction. These resources include not just experiences with and knowledge of the everyday materials, but ways of making mathematical sense and of communicating and negotiating meanings. The disruptions in space, tools, perspective, and division of labor grant students conceptual and discursive agency, providing them with opportunities to engage out-of-school resources in productive disciplinary ways.

The following is a basic description of the design for the WSG lessons for KCMS and SEC. More detailed lesson plans for each context can be found in Appendices A and B. The lessons were meant to build on each other, and each highlight a small set of geometry concepts.

In the KCMS setting I was able to conduct ongoing analysis during instructional design and implementation (Cobb et al., 2003). Revising and refining my instructional design during the KCMS study involved reviewing student work, (videotaped) meetings with Ms. H, Ms. N, and research team members, writing summarizing and analytic memos (Schatzman & Strauss, 1973), and finalizing lesson plans. In all of these activities the main questions were:

- What happened today? What did students learn? What did they have trouble with? What was too easy? (Did the class seem productive?)
- How did students engage in the tasks?
- Did anything unexpected happen? Why do we think it happened? Is it something

we should fix? Capitalize on? (Was there trouble?)

- Are there new learning objectives or sub-goals it would be important to address now?
- How are students responding to the disruptions? What is hybrid about their engagement?

Unsurprisingly, many of these questions were pedagogical in nature. Additionally, I also asked how the ongoing design experiment was helping me answer my research questions.

One major result of this ongoing analysis was the discovery that expecting students to recruit material resources from out-of-school settings to help solve WSG problems was unrealistic. When we asked if they would want other materials to use, they almost always declined. I also realized that it was unlikely that, if students were making connections to out-of-school experiences with and understandings of the materials, I would be able to find out what these connections were. Most likely, students were not themselves making explicit connections to specific practices. While on occasion students did reveal that they were making and using these connections (for example, Dean comparing the quadrilateral sides to a wrinkled old lady in the introductory episode), it was rare. However, in retrospective microanalysis I was able to discover other resources that students were recruiting to solve the problems that are not available or seen as relevant in their typical mathematics classroom experiences. These resources include whole body and material coordinations, as well as assemblages of classroom mathematical concepts and WSG manipulations.

The lessons summarized below represent the lessons as implemented. These lessons were revised or reworked from original lesson plans not just based on ongoing analysis, but also in order to respond to external contingencies, like weather and lock-downs. Each task problematized one or more mathematical concepts explored in previous tasks.

The lessons began with trying to get students to think about what tools they might need when drawing at walking scale (Day 1). Students then drew WSG line segments and triangles as introductory tasks (Day 2). Next, they viewed WSG angles from different perspectives to explore whether or not “eyeballing it” was a good strategy to characterize angle properties at walking scale (Days 3,4). Next, students made instructional videos for making a WSG isosceles triangle, and used each other’s videos to make triangles (Days 5-7). After making the triangles specified in video tutorials, they also changed isosceles into equilateral triangles. The next day I provided congruent lengths of rope and asked them to make isosceles triangles with these, then to transform the figure by either doubling their top angle if it was acute, or halving if it was obtuse (Day 8). Here I wanted students to explore how changing angle measures in a triangle affected other parts of the triangle (both angles and sides). We also wanted them to develop strategies for halving and doubling angles, although all groups just approximated. Over the next two days students drew WSG quadrilaterals. First, they drew a rectangle, and changed it into a parallelogram with no right angles (Day 9). The objective in this task was to introduce the concepts of right angles and parallel lines in WSG, and also to have students experience how they could move the sides of a rectangle but keep the sides parallel. The next day they drew a quadrilateral that was not a rectangle, and then one congruent to it (Day 10). This



was to promote the development of some strategies for producing congruent angles (see Figure 3-1 for a full summary of the lessons at KCMS).

- Day 1 (Tuesday, April 26): Problematizing classroom tools at large scale (all indoors)
1. DO NOW: “Draw a straight line segment and a triangle.” Discuss different strategies.
  2. DISCUSSION: Norms for discussion
  3. LARGE SCALE GEOMETRY on floor with classroom tools. Draw a LARGE line segment and a triangle (no pencils, pens, markers, etc.).
  4. DISCUSSION: What did you do? What happened? What tools do you need?
  5. EXIT SLIP: The difference between doing geometry at your desk on paper and doing it at walking scale.
- Day 2 (Thursday, April 28): WSG line segment and triangle
1. DO NOW: Explain plan for creating triangle outside.
  2. WSG: Draw a straight line segment and a triangle. Researchers take pictures from above and at ground level.
  3. DISCUSSION: What materials they used, how they used them, how they knew their lines/triangle sides were straight, what other tools could they use.
  4. LABELLING PHOTOS (also EXIT SLIP): Label WSG photos with parts and strategies.
- Day 3 (Friday, April 29): Viewing angles on the ground
1. DO NOW: “Describe two ways to make sure a walking scale obtuse angle is really obtuse.”
  2. WSG Angles and Windows: Make an obtuse angle, then trace it with plexiglass window from different positions around the angle.
  3. WORKSHEET: Trace angles onto the worksheet, and answer questions.
  4. DISCUSSION: Standing at ground level with large objects vs looking down onto objects drawn on paper. What makes it obtuse if it looks acute from certain positions?
  5. EXIT SLIP: “If an obtuse angle *looks* acute when you’re standing in a certain location, do you think that makes it an acute angle? Why or why not?”
- Day 4 (Monday, May 2): Perspective (all indoors)
1. DO NOW: Look at a triangle from above, and predict what it might look like on the ground from different positions.
  2. PHOTOS DISCUSSION: Look at photos from WSG activities last week, and talk about straightness, length, and different strategies and tools.
  3. VOCABULARY TASK: Ms. H made a list of vocab that was coming up in discussion (Ground level; Air level; Straight; Dipped line; Dimension) and had the kids, in groups, explain what they meant and why they were important on easel paper.
  4. DISCUSSION: Comparing different groups’ definitions and descriptions.
  5. EXIT SLIP: “Give explicit plans to a group of 3 students for making a large ISOSCELES TRIANGLE on the field. Assume that they have never done geometry on the field before, but are very good at following instructions. You can have them use any of the tools that we have had. You can use words and pictures in your instructions. Tomorrow, another group will use your instructions to try and make an isosceles triangle. We’ll see which group has the instructions that produce the best triangle.”

*Figure 3-1.* Summary of KCMS lessons

- Day 5 (Tuesday, May 3): Isosceles triangle instructional videos (all indoors)
- DO NOW: "Jasmine has done her best to give you instructions for drawing her secret shape. Follow her instructions closely to draw the shape on paper with your pencil.
    - Draw a straight line, and label the endpoints A and B.
    - Draw a straight line upward and to the left from A. Label the new endpoint C.
    - Draw a straight line to the right, the same length as AB. Label the new endpoint D.
    - Connect D and B."
  - DISCUSSION: Show students' different solutions. Point out reasons the instructions might have led to different answers. (unspecified direction, length, angles)
  - INSTRUCTIONAL VIDEO: Students plan for and create an instructional video for another group to make a large isosceles triangle outside.
  - EXIT SLIP: "Explain how you can check to see if a LARGE triangle using our outside tools is ISOSCELES. What will you DO to make sure two sides are the same length?"
- Day 6 (Wednesday, May 4): Isosceles triangle instructional video revisions (all indoors)
- DO NOW: "List 3 important things you talked about in your instructional video"
  - Watch and revise instructional video.
  - Watch videos and compare, using chart highlighting how to determine side lengths, how to make 2 congruent sides, how to make base, how to determine where vertices are, how to determine angle measures.
- Day 7 (Thursday, May 5): Making isosceles triangles using instructional videos
- DO NOW: We give them six triangles, and ask them to circle the isosceles triangles (different orientations, angles).
  - VIDEO VIEWING: Groups watch a video from the other class, take notes in preparation for making the triangle.
  - WSG ISOSCELES TRIANGLE: Groups follow the instructions from the video to make the isosceles triangle. Then, make it into an equilateral triangle.
  - DISCUSSION: How were the instructions; How did you make your isosceles into an equilateral?
  - EXIT SLIP: We give them an acute isosceles triangle, and ask them to draw an equilateral triangle with same side lengths as the legs. What happens to the base? What happens to the angles?
- Day 8 (Friday, May 6): Isosceles triangles and changing angles
- DO NOW: We give them an obtuse isosceles triangle, and ask them to draw an equilateral triangle with same side lengths as the legs. What happens to the base? What happens to the angles?
  - WSG ISOSCELES WITH GIVEN CONGRUENT SIDES: We give them 2 congruent lengths of string (every group gets the same), and ask them to make an isosceles triangle. The base can be any material and any length. We then ask them to change it so that the top angle is twice as big or small (depending on if it's acute or obtuse). We then ask them to move the top vertex in, and adjust the triangle without moving the base.
  - DISCUSSION: How did the sides and angles change for each task?
  - EXIT SLIP: We give them some triangles with the angle measures labeled. Ask them to identify the longest side of the triangle.
- Day 9 (Monday, May 9): Quadrilaterals (rectangle and parallelogram)
- DO NOW: Students categorize quadrilaterals that we give them.
  - WSG: Students go outside and make a rectangle, then change the rectangle to a parallelogram with no right angles.
  - DISCUSSION: They compared strategies for each part.
  - EXIT SLIP: Three new things that you learned today in class.
- Day 10 (Tuesday, May 10): Quadrilaterals and congruence
- DO NOW: Draw a quadrilateral congruent to the given (it was a rhombus), and explain your strategy.
  - WSG: Make a quadrilateral that is not a rectangle, then make a congruent quadrilateral.
  - DISCUSSION: Compare strategies, then look at pictures to see how congruent their figures were.
  - EXIT SLIP: None.

*Figure 3-1, continued.* Summary of KCMS lessons.

The SEC tasks (Figure 3-2) were more drawing focused, and targeted students' development of different WSG representational strategies. For example, drawing congruent triangles might involve inventing a tool for making congruent line segments, while drawing a similar quadrilateral might compel students to find a way to copy (or at least compare) angles.

As a group, draw each geometric object with the given materials, using the lawn as your paper. Each object should have sides *at least* the length of 2 of your bodies. When you are finished, let us know so that we can take a picture from ground level and from the top of Wyatt.

1. A line segment
2. A triangle
3. A triangle congruent to one that another group has drawn
4. A quadrilateral
5. A quadrilateral 1.5 times the size of the one you just drew

*Figure 3-2. SEC WSG tasks.*

In general, students worked in small groups of three or four to solve the tasks. Each task had a preferred number of group members, based on how they were often solved in pilot studies with student volunteers at the university. I wanted to provide groups with the minimal number of students that I imagined possible for them to solve the tasks relatively efficiently. For example, since participants often stood at the vertices when drawing WSG triangles, I wanted three students in each group when the task involved drawing triangles. Having a fourth group member often led to one student disengaging (see “How You Wanna Do It?” in Chapter Four for an extreme case of this). In KCMS Ms. H often adjusted group numbers based on absences and social relationships. In SEC we kept the group sizes at three students each, since the students did most of the tasks in one session. For each task or small set of tasks, students individually responded to questions about their strategies and solutions before and after completing the tasks. In KCMS students did this on paper in writing and/or drawing. In SEC I asked students to

just think individually or together about the questions. Students also participated in whole class discussions during which students shared and compared their ideas.

In KCMS, the lessons began with a whole class discussion inviting students to imagine themselves constructing geometric figures and solving geometric problems outside, on the athletic field. This discussion served as a brainstorming session for students to suggest tools they would need for this new problem solving context. We began by engaging students in thinking about how WSG might be the same as or different from solving the kinds of geometry problems that they have encountered in the past. This helped to highlight the tools and strategies that have been disrupted by moving to walking scale, so that students had an opportunity to consider what they might need to work with geometric figures at a much larger scale.

I wanted for students to have the opportunity to come up with the materials they would like to adapt for this purpose. In our pilot trials of these types of tasks, useful materials have included rope and flagging tape. Because the students did not come up with very many ideas for materials, I provided these “standard” items the first day that we went outside. I also did my best to obtain the tools requested by students (hammers, tent stakes, a cardboard box) during the five weeks as they participated in WSG tasks.

Back in the classroom, Ms. H would typically start discussion by asking students what they did to complete the task, and how their strategies were different. Meanwhile, I uploaded the photos that had been taken while the class was outside, so that we could view and discuss them as a whole class. This was how I chose to make the WSG activity visually available in the classroom. When they looked at photos, it was generally to see how their drawings turned out. While Ms. H led discussion to explore students’ strategies,

she did not use these times to guide students toward any convergent understandings about the relevant geometry concepts. Nor did I design tasks or assessments to do this kind of work. At SEC we had very little time for in-class instruction, so we reviewed the photographs and each group shared their solutions strategies for each task.

Given the short amount of time available for WSG in SEC, I provided these students with the standard set of materials immediately, encouraging students to ask for others if they needed any. No students requested additional materials in that setting.

### **Methods of Analysis**

Retrospective analysis of design and instruction followed methods for the development of grounded theory (Charmaz, 1983; Glaser & Strauss, 1967). Video data was treated in accordance with the methods of interaction analysis (Jordan & Henderson, 1995). All video was content logged based on initial viewing and any available field notes and memos written during data collection. Three target groups of students were chosen, two from KCMS and one from SEC. The groups were chosen based on consistent video data (not every group was videotaped every day, and not every student was in attendance every day) and diversity of strategies. I began by writing descriptive memos for each segment of problem solving activities of the focal groups, divided by task.

The analysis of each segment of problem solving activity began with the question, “How did the group complete the task (or subgoal at hand)?” The engagement of each group member was tracked as best as possible given the data, which generally included one or two video angles, between one and four separately positioned audio sources, and a smattering of still images taken from on the ground and from above. Additional sources

of data, like students' contributions in class discussion, student work artifacts, and interviews were used to confirm interpretations of student participation in the WSG tasks. Given the sociocultural lens of the study, student participation during tasks and across tasks as well as the mathematical meanings socially constructed by groups was an appropriate analytical focus.

I then chose "hot spot" episodes from each group to analyze closely. Because my research questions deal with forms of engagement, resources for participation, and opportunities to learn mathematics with respect to the four disruptions of space, tools, perspective, and distribution of labor, hot spots were oriented toward phenomena that I expected these disruptions to support. I looked for uses of the space and tools that led to mathematical sense-making. I looked for instances of uncertainty and trouble due to the intrinsic perspective of students, and also moments where this perspective was leveraged for sense-making and coordination. Similarly, I looked for instances of trouble due to the new division of labor and moments where this distribution seemed significant for a group's accomplishment of a task. In micro-analysis, I asked how trouble was resolved or how the disruptive aspect(s) themselves became resources for each of these, and how students were supported in participating in the activity. In other words, I looked for the resources that students recruited for the purposes of problem solving, and how they took up or rejected opportunities to learn.

Micro-analysis of hot spot episodes began with transcripts organized by turns at talk. These transcripts were then augmented with embodied and material phenomena that were judged relevant to students' activity. Some of the episodes were viewed in interaction analysis working group sessions (Jordan & Henderson, 1995), where members of the

research team or other researchers watched the episode repeatedly and discussed what they noticed. This served to help expand my analysis beyond my own, preconceived notions driven by theoretical commitments and my experiences designing the instruction and collecting data. In addition, the working group sessions offered opportunities for me to hear, consider, and discuss alternative explanations, holding interpretations accountable to the video record and other sources of data.

In general I began micro-analysis with KCMS, although analysis became an iterative process as I compared findings across episodes and sites. As I found initial answers to these “how” questions during content logging and writing descriptive memos, I began to look for similar instances in other segments, for the same group and in other groups.

In the next two chapters, I will use example episodes to share analysis and related findings that appeared consistently across episodes, but also comment on nuances and differences that I noticed. I chose these episodes to most clearly illustrate my findings. My analysis and conclusions do not include the full diversity of forms of engagement and resources for participation, or the frequency of phenomena that I found in the retrospective analysis. Instead, I investigate some of the events that occurred in the two implementations of WSG with an eye toward possible forms of engagement and resources for participation, and future design of WSG tasks and accompanying classroom instruction.

The analysis chapters are organized thematically to convey the leading implications of the findings, either for modes of student engagement and resources for problem solving, or for resources for participation and sense-making (opportunities to learn). While these two themes are necessarily intertwined and embedded in group problem

solving activity, for the purposes of sharing a large amount of complicated analysis and findings, I have organized the findings in this way.



## CHAPTER IV

### WALKING SCALE GEOMETRIES IN PRACTICE

The major disruptions of WSG—space, tools, perspective, and division of labor—make for a unique context for geometry. The disruptions remove or render useless many resources for participating successfully in classroom geometry, as discussed in Chapter 2. At the same time, the WSG context provides new resources for learning and doing geometry. In this chapter, I present some episodes of WSG from both the KCMS and SEC settings in order to illustrate some of the particularities of the WSG tasks in my design and how they affect the resources available to and recruited by students in problem solving. These episodes demonstrate the variety of ways that students engaged in these tasks, and have implications both for understanding how these disruptions may affect mathematics learning, how to design for the interplay between the classroom context and the WSG context, and for future iterations of WSG design.

The first four episodes in this chapter are examples of how students engaged in WSG, highlighting how the disruptions to typical classroom mathematics and the newly available and recruited resources for problem solving result in particularities of the WSG context that can become resources in the classroom context. The last two episodes represent two instances of how the disruptions resulted in trouble for students' problem solving, and how the trouble was resolved.

In this chapter and the next I present each episode by first introducing the group of students and the task, describing the events leading up to the episode, and summariz-

ing the episode. I then separate each episode into short excerpts in order to make the microanalysis of long sequences of talk and activity more manageable for readers. Each excerpt begins with a detailed description of the talk and action, followed by a figure that includes the transcript and relevant still images<sup>1</sup>. Following the description, I provide some discussion of the excerpt, providing analysis of the excerpt and explaining findings. For the last excerpt of each episode, I also summarize the findings for the entire episode in the discussion section.

### **Students Engaging in Walking Scale Geometries**

The first episode in this section represents a typical case of drawing an object in WSG, and illustrates how students worked together, and how they used the materials and their bodies in the service of producing mathematical entities and relations. It also serves to demonstrate properties of WSG tasks that are distinctly different from paper and pencil versions, how these properties change students' mathematical engagement, can make salient new mathematical concepts, and can become obstacles for learning. The second and third episodes show how students integrate outdoors forms of play into their WSG activity, and how subgoals of mathematics and play mutually influence each other. The final episode in this section illustrates how the distribution of materials and labor in WSG

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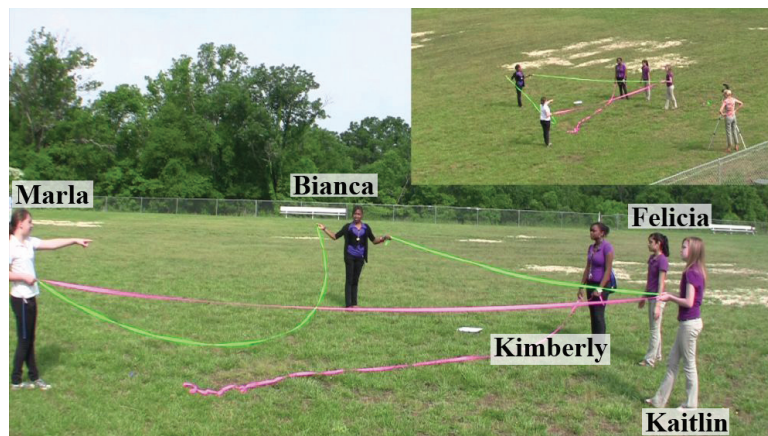
<sup>1</sup> Transcripts follow a modified version of Jefferson's transcription convention (Atkinson & Heritage, 2006). Turns at talk and new lines, determined by topic of talk or activity, are labeled with [Line#], based on numbering from transcripts of video files (not all episodes begin with [1]). Frame stills at select moments are labeled with <#>, and placed below the relevant transcript. Non-talk activity is enclosed in *((italicized, double parentheses))*. Overlapping talk across turns is signified by vertically-aligned [. Emphasis is underlined, louder utterances are CAPITALIZED. Breathing and laughter are indicated with (hh), and drawn out speech with co::olons.

tasks provides students with productive forms of ongoing assessment of shared understanding during problem solving activity.

All of the episodes in this section highlight students' invented, mathematical uses of WSG materials and their bodies. These uses often must be negotiated between group members so that understandings of solution strategies are shared, at the very least, enough so that the tasks can be completed. These engagements (students' invention of mathematical tools and processes of negotiation) become resources for making sense of mathematical concepts and participating in classroom mathematics.

**Typical Material, Embodied, and Mathematical Engagements: “How You Wanna Do It?” (KCMS, Day 9)**

- a)
  - 1. *Make a large rectangle. When you are finished, ask for a picture from above and at ground level.*
- b)



*Figure 4-1.* Task and students in “How You Wanna Do It?” a) The rectangle task. b) Bianca, Felicia, Kaitlin, Kimberly, and Marla sort out the sides of their rectangle.

This episode represents a relatively typical case of drawing a figure in WSG in terms of how materials and bodies are used for mathematical representation, and how the

group negotiated their solution. It took place at KCMS in the third week. The group of students in the episode include Bianca, Felicia, Kaitlin, Kimberly, and Marla. The task was to make a large rectangle (Figure 4-1a). In the episode, the group first produces pairs of congruent sides for the quadrilateral, using pieces of flagging tape. They then sort the pieces out so that eventually one student stands at each vertex, holding the tape ends, and the congruent sides are across from each other.

As the episode opens, Kaitlin and Marla have just made two congruent sides of the rectangle by pulling out one green and one pink piece of tape from their respective rolls, which Kaitlin is holding, at the same time. Marla holds the end of the two pieces of tape. Bianca walks along the stretched out length of tape, running her hand along it. After Kaitlin cuts the two pieces of tape, they decide what to do about the other two sides.

**Excerpt 1 description.** Although Kaitlin and Marla have already begun to make two of the sides of the rectangle, the group has not yet come to consensus on what they will do. Kaitlin asks Bianca to check if the two sides they just made are the same length [9]. Bianca confirms [11], and then the group discusses what to do next. They agree that the next two should be shorter than the first two [14-17]. When Marla asks, “How you wanna do it?” [18], Kaitlin leads the implementation of this without speaking [19]. The strategy for creating two congruent sides is the same as how they made the first two. Kaitlin holds the two rolls of tape in her left hand, and pulls the two ends out at the same time <1>, like a machine for making congruent sides. She then hands the ends to Marla <2>, and backs away, continuing to unroll the two pieces of tape while keeping them taut <3>. In this case, when she creates two sides that are a bit shorter than the first two, she stops

<4>, and cuts the tape. Note that now there are two pink and two green pieces of tape, but the pairs of congruent sides consist of one pink and one green.

- [9] Kaitlin: Oh they're not the same length (are they?)  
[10] Felicia: Oh my bad.  
[11] Bianca: Yeah. They are.  
[12] Kaitlin: Ok.  
[13] Bianca: [( )]-  
[14] Kaitlin: [Now what do we have to do, make two of em?  
[15] Bianca: Yeah we (make them both much shorter)  
[16] Marla: We have to make two that are, what, smaller than this? Or what?  
[17] Bianca: Yeah.  
[18] Marla: How you wanna do it? *((Backs up and holds the two pieces of tape in her hands out by her sides so that they are taut and separate.))*  
[19] *((Kaitlin walks toward Marla, holding the two rolls of tape in her left hand and pulling both pieces of tape out with her right hand at the same time <1>. She hands the ends to Marla <2>, and walks backward toward Bianca <3>. She stops and cuts the tape when she is just short of Bianca.<4>))*

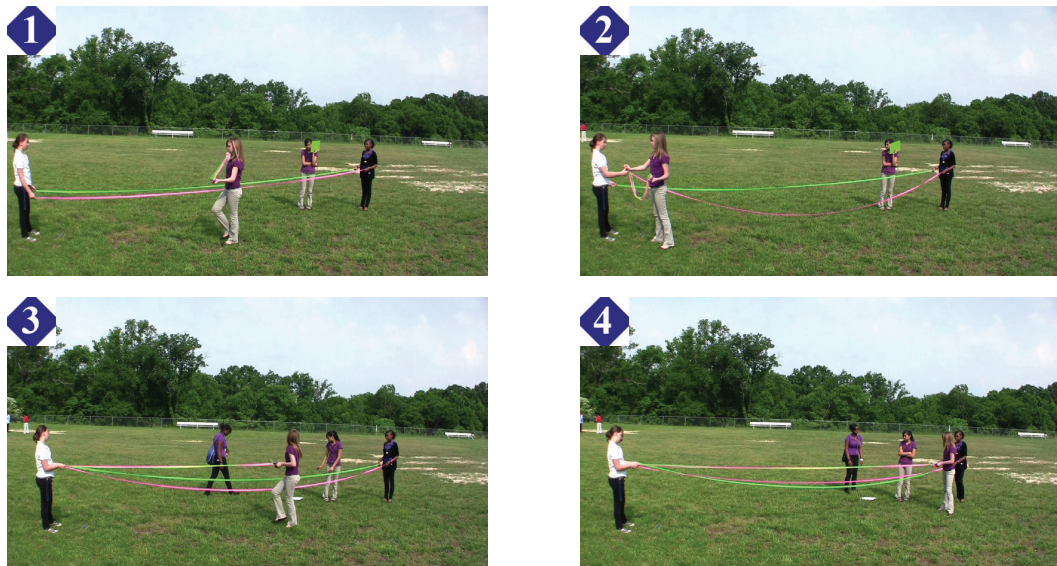


Figure 4-2. Excerpt 1, “How You Wanna Do It?”

**Excerpt 1 discussion.** Although the five students work together (for the most part) to make the quadrilateral (here and later in the episode), the group does not explic-

itly articulate a plan for making it. In this excerpt, there is no talk about how they will make congruent sides, or even that pairs of sides need to be congruent. The lengths of the sides is not discussed, nor how they will sort the sides out and place them. As the group engages in the task, some of these decisions are made by individuals, and some negotiated by the group as they work. Kaitlin's "congruent side machine" is a multi-party, whole bodied, material device that is quietly accepted by the group (although Kaitlin does ask Bianca to check if the lengths of her first two sides are the same [9]). Kaitlin, Bianca, and Marla have a brief discussion about the next two sides being shorter [14-17], but when Marla asks, "How you wanna do it?" [18] Kaitlin takes on the task without comment. The rest of the group helps when necessary, but in general stands and watches.

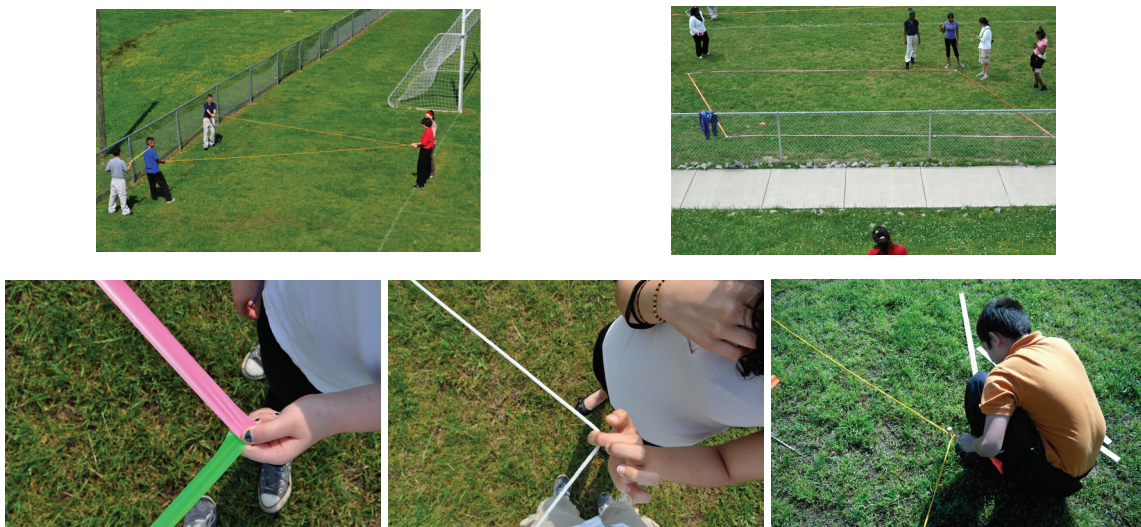
This is typical for how groups accomplished the WSG tasks. Very little was discussed before students began working, often one or two students took the lead (although it was not uncommon for the leading role to shift between students), and talk directly related to the task only ramped up when there was trouble of some kind (Excerpt 2 in this episode is an example of this). In general little was said, but students engaged by watching each other, manipulating materials, and moving their bodies with large actions and small adjustments. These physical engagements, large and small, were often in concert with or in response to the activity of other group members.

As for students' mathematical uses of materials and bodies, Kaitlin's congruent side machine is a good example. She matched the loose ends of two rolls of flagging tape, and pulled it out to a desired length before cutting both at the same time in the same place. While it would probably be difficult for Kaitlin to articulate where this idea came from—we did not ask in this particular case, but we did ask about other uses of the mate-

rials, and Kaitlin and the other students almost always said that they did not know—it is clearly related to an everyday practice recruited for the context of drawing a WSG rectangle. The closest analogy to this strategy in classroom mathematics would be to hold two pencils at a fixed distance apart, and draw with them at the same time. Typically, in the classroom at KCMS, the students did not do this to draw congruent line segments. They traced, used rulers, or devised other ways of “remembering” the length (e.g., placing a finger at either end of the first segment, then moving them to represent endpoints of the next, and drawing a line segment in between them).

In general, students in both the KCMS and SEC settings used the materials and their bodies in similar ways while solving the WSG problems (see Figure 4-3 for examples). They used flagging tape and rope to represent line segments (including angle and polygon sides), either laying them on the ground or holding them. Students sometimes used long, continuous pieces of rope for all sides of an angle or polygon, and sometimes used cut pieces. If they held the tape or rope, they often used their own bodies or hands as vertices of the figures. They also used flags and tent stakes as vertices. Bodies, flags, and tent stakes also served the purpose of holding ropes and angles in place.

Students also developed routines for producing mathematical properties and relations. For example, “pull it tight!” became a commonly heard call at KCMS for getting straightness. Students rolled or stretched out lengths of rope beside each other to compare lengths or to produce same or shorter lengths. They often used their bodies as units of measure. They folded rope in order to halve it, or to produce doubles or other multiples. They rarely developed strategies for comparing or duplicating angles, instead preferring to “eyeball it.”



*Figure 4-3.* Examples of how students used materials and bodies in WSG. A WSG triangle held up by students; a WSG quadrilateral laid on the ground; two pieces of tape held at a vertex; one continuous rope pulled back at a vertex; a piece of rope wrapped around a tent stake at a vertex.

In the classroom at KCMS, these physical representations and manipulations became common language for students to use in combination with mathematical language. (We did not have a formal class discussion at SEC except to share students' experiences.) "Pulling tight" was synonymous with "straight," and words like "dip," "sink," and "wrinkled" became synonymous with "not straight." This language, in combination with the class's shared experiences of solving WSG tasks became an additional resource for reasoning about the geometric concepts. For example, in this classroom "not straight" might also mean "not vertical" to students, whereas "dipped" very clearly implied that the inscription did not represent the shortest distance between two points.

The WSG disruption of materials and tools invited the emergence of new strategies for producing mathematical objects (line segments) and relationships (congruence). The shift in the properties of the materials, embodied engagements with the materials,

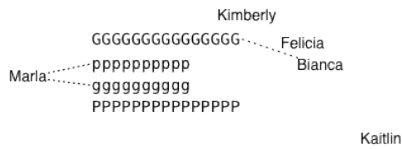


and inscription in WSG supported the invention of novel strategies for completing the tasks and associated subgoals.

**Excerpt 2 description.** Once the two shorter sides have been cut, Kaitlin declares, “Everybody’s gonna have to ha, hold a side” [20]. She recruits the others to participate, calling Kimberly “lazy” in the process [22]. Meanwhile, Bianca, Felicia, and Marla have begun to sort out the four sides. At [21], Marla holds the ends of all four sides, but drops the ends of the two longer sides (one pink and one green). Bianca, who was holding the other ends of the two longer sides, drops the pink end so that she is only holding the green end <5>. Felicia picks up the opposite ends of the two short pieces (also one pink and one green) that Marla is holding just as Marla drops them and picks up the end of the green long side <6>. Then Felicia then drops the pink end that she is holding [24], and starts to gather the green side up. Kaitlin picks up the short pink piece and hands Marla one end of it <7>. Kaitlin and Marla begin to spread apart. Kimberly picks up an end of the long pink tape <8>.

- [20] Kaitlin: Everybody’s gonna have to ha, hold a side. Felicia, (put your        ).  
[21] ((*Marla dropps the two long sides, and holds the two shorter sides. Bianca drops the pink end and holds only the end of the long green long <5>. Felicia walks over and picks up the ends of the two short sides on the ground. Marla lets go of them and picks up the end of the long green side, and reaches toward Kaitlin <6>.*))

*Figure 4-4. Excerpt 2a, “How You Wanna Do It?”*



[22] Kaitlin: C'mon Kimberly, stop being lazy.

[23] Kimb: I'm not being lazy.

[24] ((Felicia drops her pink end, and starts gathering up the short green. Kaitlin picks up the short pink side, and hands Marla an end <7>. Kaitlin and Marla walk away from each other. Kimberly comes and picks up the long pink end <8>..))

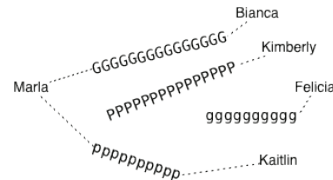


Figure 4-4, continued. Excerpt 2a, “How You Wanna Do It?” The diagrams below each still image provide a simple view of what each student is holding. Because each pair of congruent sides consists of one green and one pink piece of tape, the longer green piece of tape is represented by “GGGGGGGGGGGGGGGG” and the longer pink piece by “PPPPPPPPPPPPPPPP.” The shorter pieces are represented by “gggggggggg” and “pppppppppp.” Dotted line segments indicate that a student is holding that piece of tape.

At this point, Bianca, who has been watching the exchange between Kaitlin and Marla, says that “that’s one of the sides” [25], meaning that the pink tape between them is

one of the short sides of the rectangle. She then declares, “I need a side” [27]. Since she is holding the end of one of the long pieces, the other piece she will hold in her other hand should be a short one. Felicia says, “This is a side,” and hands one end of the green piece that she is holding to Bianca.

Marla is discovering a similar idea, asking whether the pink tape between her and Kaitlin is “one of the things that go like this,” holding the pink side out perpendicular to the green that she is holding in the other hand <9> [28]. She then answers her own question, saying “Yeah cause this one ( ),” pointing at the other, longer pink piece of tape <10>. It is difficult to hear what she says, but she contrasts her pink tape, “one of the things that go like this,” with the other pink tape, which is longer and thus will not be adjacent to the long green tape. Marla has been meticulous about sorting the four pieces and keeping track of which she is holding. She has two options for what will be adjacent to her long green tape, a short piece and a long piece. She also has access to two pink pieces, and one of which is short, and one long. She identifies the short piece as the one that should be adjacent to the long green piece.

[25] Bianca: *((Watching Marla and Kaitlin))* Ok that’s one of the sides.

[26] Kaitlin: ( )

[27] Bianca: I need a side. *((<9> Walks toward Felicia.))*

[28] Felicia: [This is a side *((Handing Bianca an end of the short green that she is holding. <10> Bianca backs away.))*

[29] Marla: Oh- ain’t this one [of the things that go like this? *((<9> Holds pink side out perpendicular to green))* (And this one-) Yeah cause this one ( ) *((<10> Pointing at pink long side on the ground))*.

*Figure 4-5. Excerpt 2b, “How You Wanna Do It?”*



Figure 4-5, continued. Excerpt 2b, “How You Wanna Do It?”

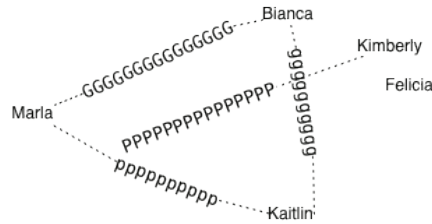
Kaitlin asserts that “This is one of the sides” [30] as she walks backward, pulling the short piece of pink tape between her and Marla taut. Bianca begins to give Kimberly and Felicia directions to rearrange their bodies, based on which pieces of tape they are holding [32]. Bianca and Marla are holding the long green tape, and Marla and Kaitlin are holding the short pink tape, and Bianca and Felicia are holding the short green tape. Felicia and Kaitlin stand together though, so the three are essentially forming a triangle, with Kimberly standing and holding the long pink tape off to the side. Bianca would like for Felicia to move to Bianca’s left, so that the green side she is holding is parallel to Marla and Kaitlin’s pink side. She then tells Kimberly to go over to where Kaitlin is. As Bianca speaks, Kaitlin reaches for the tape that Felicia is holding, which is the other end of the short green piece that Bianca is holding.

Kaitlin is now holding the ends of both of the short pieces of tape. Felicia says, “Hm?” and walks in the direction that Bianca directed her, even though she is no longer holding tape. Kimberly takes a step forward toward Kaitlin, in the direction Bianca

pointed to. She stops when Marla says, “No, the gr- the pink <14> one goes to that <15> one I think” [34], pointing to the longer pink tape on the ground and the shorter green tape between Bianca and Kaitlin.

[30] Kaitlin: This is one of the sides.

[31] ((Kaitlin reaches over and takes the end of the green short side from Felicia.))



[32] Bianca: Let Kimberly get ov- like, you ((<11> Points at Kimberly)) get over there ((<12> Pointing toward Kaitlin)) and Felicia get over ((<13> Pointing to her own left)) here.



[33] Felicia: Hm? ((Walks toward Kimberly.))

[34] Marla: No, the gr- the pink ((<14> Pointing to the long pink side on the ground that Kimberly is holding the end of.)) one goes to ((<15> Points to the green short side that Kaitlin and Bianca are holding.)) that one I think.



Figure 4-6. Excerpt 2c, “How You Wanna Do It?”

At this point Kaitlin realizes that there are too many students if each of them is to stand in for a vertex, and she dismisses Kimberly [35]. Kimberly hands Felicia the end of

the long pink tape <16>, and goes to stand a few steps back from the group. Felicia then walks over to Bianca, who hands her the end of the short green tape <17>, the other end of which has just slipped out of Kaitlin’s hand [36]. Kaitlin objects, saying “This is one of the sides” [37], pointing to the short green tape that Felicia has just taken. Marla agrees [38]. As they talk, Felicia picks up the other side of the long pink tape and starts to bring it to Bianca <18>. Kaitlin tells her, “Give me the pink one. NO, give me the pink one. Bianca, don’t take it. Give her the green one” [40]. Felicia turns and hands Kaitlin the pink tape <19>, and brings the green one to Bianca <20>. Bianca grasps it in the middle of the length, and Felicia then begins to walk to the right, pulling the green tape out and taut <21-24>.

- [35] Kaitlin: We only need four, we got five. Kimberly you can go. ((<16> Kimberly hands Felicia her end and walks a few steps away.))
- [36] ((Felicia brings the long pink side over to Bianca, who is holding her two pieces out to her sides. Kaitlin drops her end of the green short side. <17> Felicia takes the end of the short green side from Bianca.))
- [37] Kaitlin: Oh my god. ((Points to the green short side on the ground.)) This is one of the sides.
- [38] Marla: Yeah, that’s that- the gr- yeah.
- [39] ((<18> Felicia picks up the end of the pink long side and starts to hand it to Bianca.))

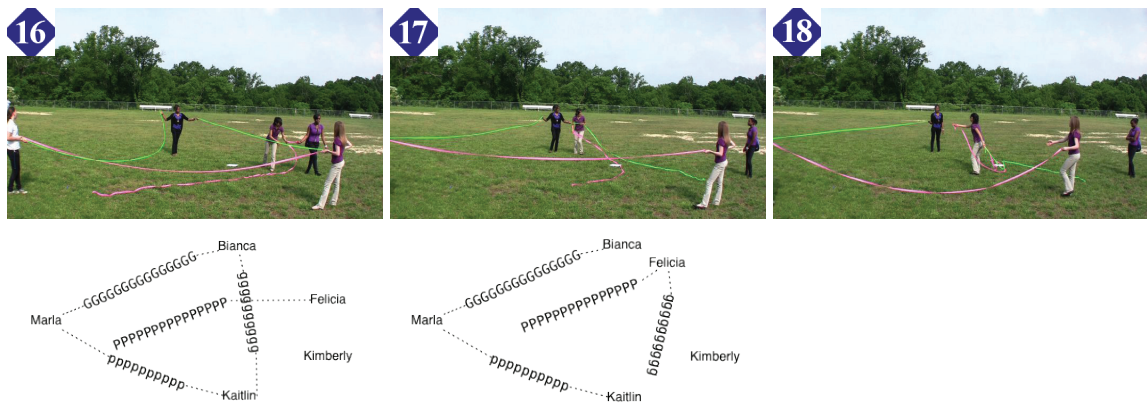


Figure 4-7. Excerpt 2d, “How You Wanna Do It?”

[40] Kaitlin: Give me the pink one. NO, ((<19> Felicia turns and brings the end of the long pink side to Kaitlin.)) give me the pink one. Bianca don't take it. Give her the green one.

[41] ((<20> Felicia hands the end of the short green side to Bianca, then <21-24> walks over to pull the sides she has taut. Each student now has two ends, one in each hand.))

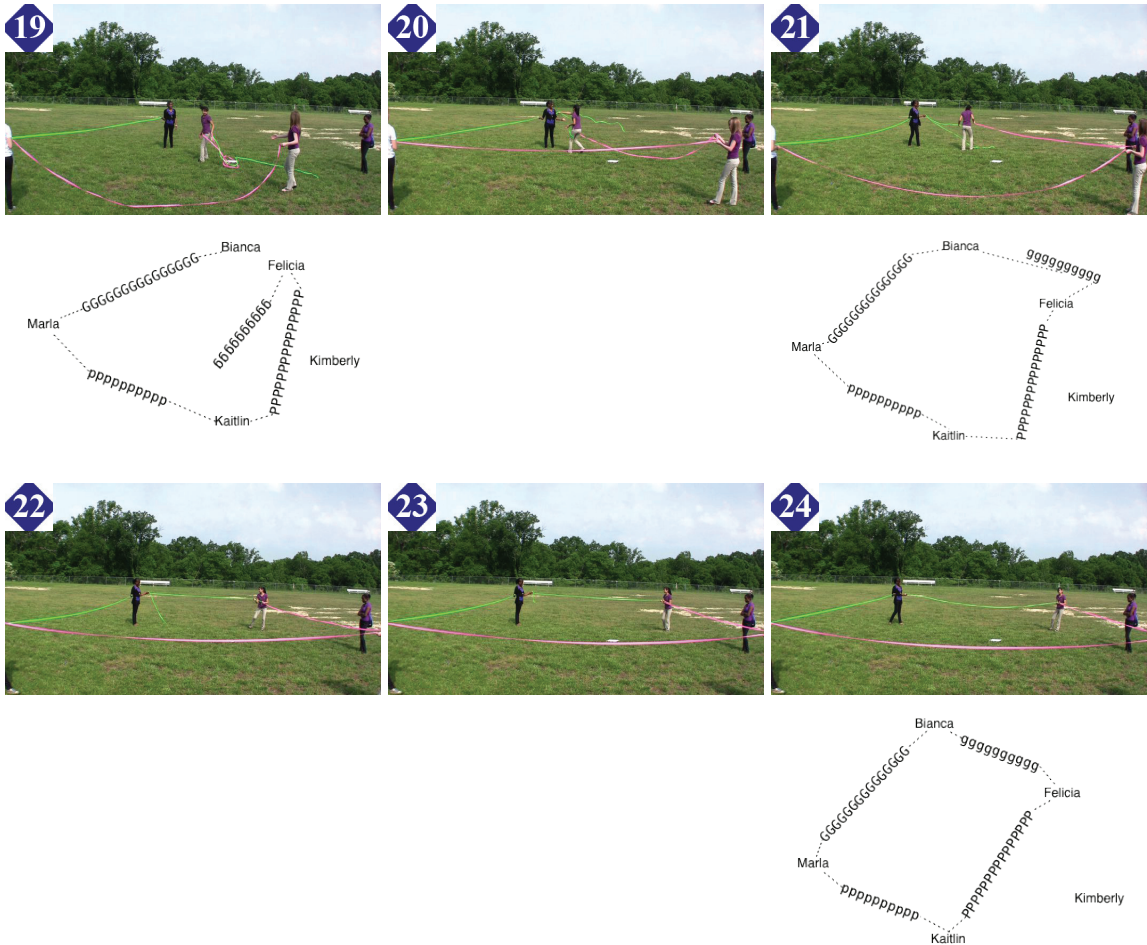


Figure 4-7, continued. Excerpt 2d, “How You Wanna Do It?”

**Excerpt 2 discussion.** As the students begin to pick up sides in order to arrange them into a rectangle in this excerpt, trouble arises and there is more talk and more verbal and physical repairs. Unsurprisingly, as mentioned above, when more students were needed to accomplish a subgoal (e.g., arranging the sides), there was often more trouble and more discussion. These discussions had the possibility of making the mathematics

come to the forefront for students, and fostering sense-making grounded in the problem solving context. In this episode the students grappled with the concept of which sides are adjacent in a rectangle that has two shorter sides. As Bianca, Felicia, Kaitlin, and Marla picked up, traded, dropped, and walked around with the four pieces of tape, they made claims about which were the “sides,” who “needed” a side, where they should put their bodies, and how they could get their ends to connect.

This excerpt also illustrates how the mathematical concepts that WSG tasks highlight can be quite different from the “same” paper and pencil task. The students in this group drew their rectangle by first producing the two long sides at the same time, then the two short sides, independently of their eventual spatial relations. They then, as a group, sorted and arranged the four sides and their four bodies so that each student held one end of a short side and one end of a long side. Once this was accomplished, the sides were pulled taut (made straight), and their locations and rotations were adjusted so that the ends all connected. At that time, it was accepted that they had drawn a rectangle. (In class discussion before going outside, Ms. H had reviewed the properties of different quadrilaterals, noting that a rectangle had four sides, opposite sides equal, and four right angles. In making their rectangle, and in class later, this group did not talk about angles.)

When drawing at paper and pencil scale, length, straightness, and location are produced at the same time, as the pencil tip moves and leaves a trace. Changing these properties of the inscription requires erasing and redrawing. Here, lengths were predetermined, while straightness and placement were not permanent and could be adjusted (or accidentally lost) at any time. In fact, arrangement and adjustment of the spatial locations of the geometric objects (line segments, angles, polygons) were often important elements



of the tasks. The sorting and arranging that these students were engaged in highlighted the mathematical relationship between congruent sides in a rectangle, and which sides are adjacent to a given vertex. The next excerpt illustrates more of the inscriptional and representational properties of WSG, some of which could be viewed as obstacles to learning.

**Excerpt 3 description.** As Felicia finishes walking and pulling the green tape so that Bianca is holding the end of it and it is taut, Kaitlin calls out, “Hold on” [42], and Felicia loses her hold on her end of the pink tape that is between them. As she steps over and bends to pick it up, she pulls the green tape out of Bianca’s hand [43]. Kaitlin tells them, “You all gotta hold it tight” [46]. She then declares, “This is not equal,” and Marla confirms, saying “This isn’t, look at this!” [47]. Bianca says it is “Because the wind blowin’” [48], but Kaitlin responds, “I know but look, mine don’t connect <25>” [49]. When she says this, Marla (off screen) and Bianca both step and adjust so that their hands and the tape ends that they are holding come together <26>.

Felicia, Bianca, and Marla then each announce that their ends connect. Bianca loses her end of the green tape that she shares with Marla [55], and Kaitlin yells, “Yo! Pull it tight!” [56] while the others laugh. Katie, the researcher operating the camera, points out to Kaitlin that she’s “got a- tail, on one side, see?” [57]. In other words, she is not holding the piece of tape at the end, so a “tail” is hanging out the back of her hand, and the length of that side is shorter than it should be. Kaitlin adjusts, then accidentally drops the pink side that she shares with Marla. The group laughs, and as Kaitlin picks it back up she says that “We need to take our picture” [64], and Marla responds that “You keep dropping it” [65].

I walk over to take pictures of their angles, and they prepare to have one taken from above. Marla and Felicia declare that their rectangle looks like a square [67-69], which given the task is not problematic, but their goal was to make two sides shorter. Bianca and Kaitlin believe that the picture from above will look different [72-75]. They will be discussing what they drew when they go back inside (the slideshow), and so what is important is what the pictures look like, not what it looks like to them while they hold it together.

[42] Kaitlin: Hold on.

[43] *((Felicia loses her pink end. She bends down, and pulls the green out of Bianca's hand.))*

[44] Bianca: Felicia!

[45] Felicia: You let it go!

[46] Kaitlin: You all gotta hold it tight. This is not equal.

[47] Marla: This isn't, look at this!

[48] Bianca: Because the wind blowin'

[49] Kaitlin: I know but look, mine don't connect <25>.

[50] *((<26> Bianca takes a step to her left, and brings her hands together. Marla takes a step forward, and brings her two ends together.))*



Figure 4-8. Excerpt 3, "How You Wanna Do It?"

- [51] Felicia: Mine's connect!  
 [52] Bianca: Mine's connect!  
 [53] Marla: Mine is.  
 [54] Felicia: Mine's is.  
 [55] *((Bianca drops one of hers.))*  
 [56] Kaitlin: Yo! Pull it tight!  
 [57] Katie: Kaitlin you still got a- tail, on one side, see?  
 [58] Felicia: Yeah you have a tail at the bottom  
 [59] Kaitlin: I know. Kay.  
 [60] Katie: (That's why.)  
 [61] Kaitlin: There we go!  
 [62] Bianca: Yeah.  
 [63] *((Kaitlin drops her pink short end, and the others snicker.))*  
 [64] Kaitlin: We need to take our picture.  
 [65] Marla: You keep dropping it.  
 [66] Katie: Jazz, are you ready to take a picture?  
 [67] Marla: This looks like a,  
 [68] Felicia: Like a [square!  
 [69] Marla: [square!  
 [70] *((Jasmine comes over and starts taking pictures of the angles.))*  
 [71] Jasmine: I'm gonna take a picture of your angle.  
 [72] Bianca: Probably up high it will probably look right.  
 [73] Felicia: What?  
 [74] Bianca: (From up high it will probably look right.)  
 [75] Kaitlin: She said from up high it probably won't look like that. Are we doing a slideshow?

*Figure 4-8, continued. Excerpt 3, "How You Wanna Do It?"*

**Excerpt 3 discussion.** The trouble that these students face in assembling and then maintaining their quadrilateral are typical examples of how WSG tasks produce new problems for drawing and representation. Students' movements caused line segments to curve, shift, and rotate, and sometimes blow away in the wind. When a piece of tape was held by more than one student, and as the students became increasingly connected to each other through the materials, the actions of one student had effects on the others, sometimes resulting in adjustments and other times resulting in dropped (or, occasion-

ally, ripped) tape. The system as a whole was fragile, but only had to be maintained long enough to be photographed.

The fragility of the system is not just problematic in maintaining the completed figure, but also in production. The highlighted mathematical relations in drawing a WSG rectangle (and other polygons) include not just the sorting and arrangement of parts (sides and vertices), but also adjustments that allow all the parts to connect. On pencil and paper, creating connections is trivial, as a student needs only to change the direction of his or her pencil and continue drawing, or place the tip in a certain place. There are, of course, times when creating connections does become salient in classroom mathematics (e.g., constructing an equilateral triangle). However, in WSG, with predetermined side lengths, there is always a need to coordinate spatial locations and movements of sides and vertices. In this case, Kaitlin [46, 49] worried that the reason her ends did not connect was because the side lengths were unequal. The issue was resolved as other members of the group (acting as vertices) shifted their bodies, and as Katie helped her realize that one of her sides had a “tail.” Given a different WSG inscriptional procedure, like placing four flags in the ground then wrapping the tape around them, this issue would remain hidden, although different mathematical issues might arise with flag placement.

While I did not make this feature of the predetermined lengths strategy an explicit topic of class discussion or design in KCMS or SEC, it is a mathematically important one for understanding properties of polygons, such as the minimal number of defined sides and angles are needed to determine a unique triangle or quadrilateral.

In addition to the fragility of the inscriptions, given students’ positions inside the representations and their intrinsic perspective, they did not have access to the whole

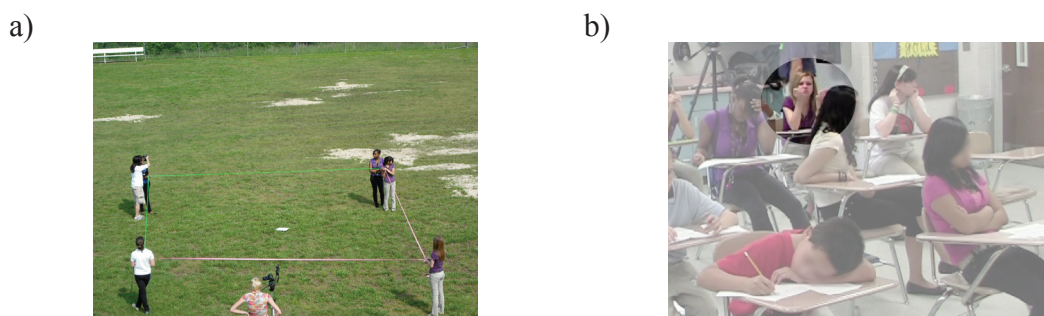
object at once, and they could not see it in plan view (from above, as is typical in Euclidean geometry). They could not trust simply what they could see to be what they had made, in terms of the Euclidean geometry they were learning. The photographs from above were a way to approximate a plan view for students (although they were not taken from directly above) to be used when we returned to the classroom. We also took pictures from on the ground, sometimes to approximate students' perspectives from inside the figures, sometimes to contrast the ground level view with the aerial view. Often we took pictures from just above each angle, so that students would have a chance, as a whole class, to discuss how this aspect of their figures turned out.

The photographing of the representation was also important because this was the established way for students to show and share their work with the teachers and the rest of the class. The WSG representations were neither immutable nor mobile (Latour, 1999). Photographs of the representations provided these inscriptional qualities, but also left behind the large scale, three dimensional qualities, material and embodied qualities of the representations.

This episode has provided an example of how groups of students typically engaged in WSG tasks in both the KCMS and SEC settings, and of how students used the materials and their bodies in the course of problem solving, both to represent mathematical objects (line segments, vertices, rectangles) and produce mathematical relations (congruent, shorter). It also served as an illustration of how the WSG disruptions in tools, perspective, and division of labor to typical classroom mathematics afford a very different representational and inscriptional system with properties. This system makes salient previously unproblematic geometry concepts (arrangements of sides, what spatial

configurations allow sides to “connect,” what does it mean to “see” congruent sides or a rectangle from a non-planar perspective). At the same time, its fragility causes trouble for some important classroom mathematics learning practices (pragmatics of drawing should not get in the way of learning, having a durable and portable inscription to discuss).

Back in the classroom, when Ms. H asked the students how they made their rectangles, Kaitlin spoke for her group (Figure 4-9). In recounting the process, Kaitlin’s description of their strategy centered around making the pairs of sides equal and one pair shorter than the other pair. The trouble that the group experienced sorting the pieces of tape, arranging them physically, and getting them to connect at the ends was all collapsed into the statement, “And then we put it- our sides together and made the rectangle.” Kaitlin had framed her account their outdoor WSG activity around the mathematical topics of classroom geometry, that rectangles have pairs of equal sides, and their rectangle had one pair of sides shorter than the other. That WSG tasks and materials make salient new (for typical classroom activity) geometry questions about the spatial relationships and connections between sides and angles is a feature to be leveraged and explored in future iterations of the design study.



*Figure 4-9.* Kaitlin’s description of the group’s WSG rectangle. a) The group’s rectangle, photographed from above; b) Kaitlin shows with her hands how they held two sides together;

c)

Ok. We took two of our sides and we put em together by- beside each other and made them the same. Like- And then we put those down. And then for the other two sides we- we knew we was gonna have to make them a little bit smaller so we pulled those two beside each other and made them a little bit smaller. But they was the same length. And then we cut it. And then we put it- our sides together and made the rectangle.

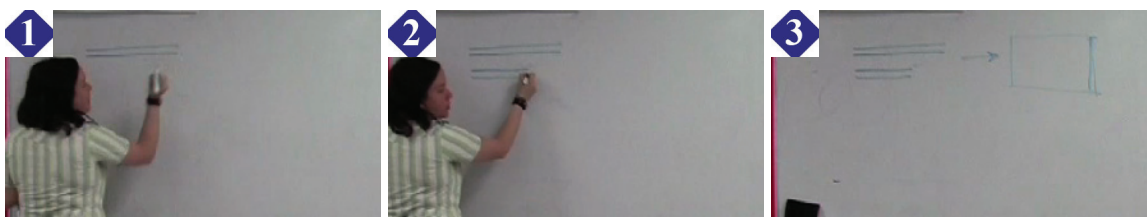
*Figure 4-9, continued.* Kaitlin’s description of the group’s WSG rectangle. c) Kaitlin’s explanation.

When Ms. H revoiced the group’s strategy (Figure 4-10), she highlighted the physical and material manipulations. She noted that the group “measured” two pairs of sides, one pair smaller than the other, and put four pieces together. She then asked if they connected them by holding them “at the corners.” With this last question, she does highlight the material and embodied manipulations required to produce the rectangle to some extent, but does not continue to investigate why this might be mathematically significant.

Ms. H: So they had then, let me- tell me if this is what you did. They measured two sides, and when I say measured I don’t mean with a measuring tape, I mean they made them the same <1>, and then she said they made two other sides that were the same but they made them smaller <2>, so how many pieces did they have?

Students: Four.

Ms. H: Four. And then they put them together, and did you guys hold them at the corners, did you ta- what did you do? <3>



*Figure 4-10.* Ms. H’s revoicing of the group’s strategy for making a WSG rectangle.

Depending on designers' and teachers' instructional goals, different mathematical relations affected by these material concerns could be highlighted. For example, asking how the students holding the tape at the corners managed to get the ends to connect might bring up issues concerning the relationship between side lengths and angles in a quadrilateral. Or, asking how the students at each corner actually held the tape could bring up questions regarding scale and precision (if a student bunches some tape up in her hand, that side length will be shorter).

This episode illustrates typical material uses and complications in WSG tasks. These uses and complications include the ways materials and bodies get recruited and manipulated to produce geometric relations and entities, as well as properties of the inscriptional system. They can be ignored, treated simply as trouble for the pragmatics of completing the tasks, or leveraged to highlight particular mathematical concepts.

In the next episode, the fragile nature of the inscriptional system and the need for these aerial photographs results in an activity that might be viewed as off-task, but heavily influenced by the mathematical task. The researcher taking pictures is working with another group, and so these students cannot move on to the next task. They take the opportunity to engage in some play while they wait, all the while maintaining the mathematical integrity of the line segment that they have produced.

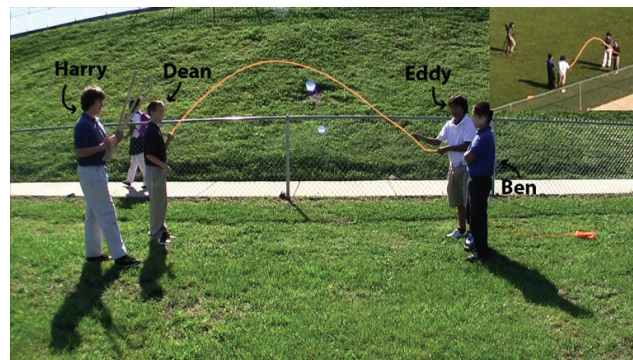


**Intersections of Play and Mathematics: “We Can’t Lose Dean’s Height” (KCMS, Day 2)**

a)

1. Using the soccer field as your “paper,” draw a straight line segment. It should be AT LEAST as long as two of you put together are tall. You may use any of the tools that we have brought out. **Tell a research team member when you are finished so we can take a picture.**

b)



*Figure 4-11.* Task and students in “We Can’t Lose Dean’s Height” a) The task. b) Students wait to get a photograph of their WSG line segment taken. Dean and Eddy turn their line segment like a jump rope while Harry and Ben look on.

In this episode, a group of four KCMS students engage in the very first WSG task (Figure 4-11a). The second sentence of the prompt, that the line segment “should be AT LEAST as long as two of you put together are tall,” was included to encourage students to make their line segments large. In pilots of WSG tasks, participants used the materials, but were often tempted to draw very small figures, thus avoiding the disruptions of perspective and division of labor. However, this group of students (as well as others in both KCMS and SEC, though not all groups) took very seriously the two bodies constraint, but did not attend as carefully to the “AT LEAST” modifier. This led to the students using their heights, and therefore bodies, to measure the lengths of their line segments.

The students in this group, Ben, Dean, Eddy, and Harry, have made a line segment by measuring Dean twice on a continuous stretch of the rope. They did not cut the rope at the spot that corresponded with twice Dean's height. In the episode, as the four students wait for their picture to be taken, they shift from taking turns at jumping rope to limbo to jumping over the rope stretched taut. As their play develops, the mathematical properties that they have introduced to the rope remain intact, influencing the kinds of play they try, and their successes and failures. One particularly salient property of the rope-as-line-segment is its measure, determined by Dean's height.

At the opening of the episode Dean is holding the end of the rope, and Eddy is holding the part that marks twice Dean's height, holding that particular spot carefully in his hand. While they wait for Kevin (a researcher) to take the picture, they mess around. They are turning the line segment like a jump rope, and they talk about using it as one.

**Excerpt 1 description.** Dean and Eddy turn the rope line segment like a jump rope, and Dean tells Harry and Ben to try to jump in. Harry wants Dean and Eddy to make the rope longer so it'll be a better jump rope, but Eddy refuses, saying "we can't lose Dean's height," because the researchers are about to take a picture [3]. Ben and Dean both try to jump in, and fail. Ben thinks it's because they are holding it too high up. Eddy thinks they should have made it longer. He says they should have "done Harry" [8], meaning that, since Harry is taller than Dean, if they had measured Harry then their rope would be longer.

When Eddy takes his turn to jump in, he asks Dean to hold his end for him "right <1> here <2>" [12]. He is very careful to fold the rope in the exact spot that corresponds to twice Dean's height, and he hands it carefully to Dean. This is evident from the 1.5

seconds of silence during the transaction, and the way the two boys' heads are bent over the piece of rope <3>.

As they finish the handoff, Ben says that he wishes they could use scissors [14]. At the same time, Harry suggests that they use his height. Ben's comment about scissors suggests that, if they cut the rope, they wouldn't have to worry about losing Dean's height. (They are allowed to use scissors, but he/they might not know that at this point). Harry's comment is taken to mean that jumping rope would be easier if they had a longer rope. Dean responds that they "could just be closer together" [15]. As he says this, he takes a step toward Ben so that as they turn the rope, it is both much higher at its peak and closer to the ground at its lowest. As they begin to turn it again, Ben takes a couple of steps forward too, so that it touches the ground when it swings down.

- [1] Dean: Alright. Now you all try to do it.  
[2] Harry: No- Bring it out more.  
[3] Eddy: No, we can't lose Dean's height. They're gonna bring the camera ( )  
[4] *((Ben jumps in, and gets tangled up in the rope.))*  
[5] Dean: Here. Hold it. Ben. Let me try.  
[6] Eddy: You all playing with my ( ) *((wagging the rope back and forth))*  
[7] Ben: You guys have it like too, too high up.  
[8] Eddy: We should have done Harry so it can be longer.  
[9] *((Dean jumps in, and mis-times it.))*  
[10] Dean: Ah.  
[11] Harry: ( )  
[12] Eddy: Dean here hold this right<1> here<2>. (1.5s) *((<3>He folds the rope where he was holding it. Dean takes it from him.))*

*Figure 4-12. Excerpt 1, "We Can't Lose Dean's Height"*



- [13] Harry: [Let's use me.  
 [14] Ben: [I wish we could use scissors.  
 [15] Dean: *((Taking a step forward))* Or we could just be closer together.  
 [16] Ben: Yeah, or that. (2s)  
 [17] *((Dean and Ben turn the rope. Ben takes two steps forward. Eddy vocalizes a beat along with the turning.))*  
 [18] (2s)

Figure 4-12, continued. Excerpt 1, “We Can’t Lose Dean’s Height”

**Excerpt 1 discussion.** Eddy refers to the length of the rope as “Dean’s height.” Even though the rope is twice Dean’s height, the unit measure is Dean’s height, and it is enough for them to call it Dean’s height. When they consider changing the length of the rope by using Harry’s height instead, Eddy says “should have done Harry” [8] (in the next excerpt Ben and Eddy use similar language [20-21]). The measure of the length of the line segment is inextricably tied to members of the group and their respective heights, and their talk about changing the length is in relation to Dean and Harry rather than the length of the rope itself. The students’ knowledge of their own heights has become a resource for thinking about the length and changing the length of the line segment. In fact, even as they discuss which student to use as the unit measure for the line segment, Harry is holding and playing with three yardsticks, ignoring their use as conventional tools of length measure.

Students’ bodies as readily available, intimately familiar objects were often used as mathematical resources in WSG activity. They were used to stand in for mathematical

objects, usually as points, vertices, line segments, or angles. They were used as units of measure, or as recording devices for fixed measure (e.g., holding fingers or arms apart to remember a length or angle). They were also used, individually and in coordination with each other, to make mathematical sense of material manipulations (e.g., each person standing at the vertex of a rectangle should hold one short length of tape and one long length). In the next episode, “So the Middle’s Here,” bodies also provide feedback for joint coordinations and ongoing negotiations of problem solving strategies.

Although the students are “messing around,” they are aware of and careful to maintain some properties of the rope’s mathematical identity as line segment. In other words, the rope becomes a hybrid mathematical and play object. As subgoals for play emerge, they are constrained by the mathematical properties of the material in the context of the mathematical task the group is engaged in. In this case, the salient properties of the line segment are its measured length and endpoints. This is evident in this excerpt (as well as the group’s activity before and after the excerpt) from their careful, coordinated maintenance of agreed upon length (twice Dean’s height) of the line segment as well as the general locations of the two endpoints. Activities like jumping rope are possible because they have a length of rope. The success of these activities, as well as possibilities for other activities, are constrained by the students’ accomplishment of stable properties of the rope-as-line-segment, which include a person at each endpoint and a length based on Dean’s height. Jumping rope fails because the line segment is not long enough, and in order to make it longer they have to “do Harry,” or measure the rope based on Harry. In the next excerpt, the group continues their play as they wait for their picture. As the subgoal of their WSG activity shifts (getting ready to take a picture), so does their play.

**Excerpt 2 description.** The new configuration, with Dean and Ben standing closer together, does not satisfy Ben though. He suggests again that they should have used Harry. Eddy, preparing to jump in, agrees, and asks if they want to do that. Harry says “Yes!” [22], punctuating his exclamation by slamming the ends of his three yardsticks into the ground. Eddy changes his mind, saying “let’s just go ahead and do Dean” [23]. Eddy jumps in, but then the rope hits his shoulders on its way up and around. Ben reminds him, “It don’t go up that high, I told you” [27]. He switches places with Dean again.

Dean backs up as if to jump rope again, but then steps forward next to the rope. Harry hollers up that they are ready for their picture. Eddy backs up, making the rope taut, making it into a (straight) line segment again. As he does this, he makes a discovery about what else they can do with the rope, telling Dean, “Hey, try to jump over it” [32]. He then coordinates with Ben to hold the rope low and parallel to the ground. As they do this, Harry sings, “Limbo limbo lim:bo!” [33], but the activity turns to jumping over the rope at different heights. The boys take turns doing this until their photo is taken.

[19] Harry: Yo-

[20] Ben: We shoulda used Harry.

[21] Eddy: Yeah we should have. You all wanna do Harry?

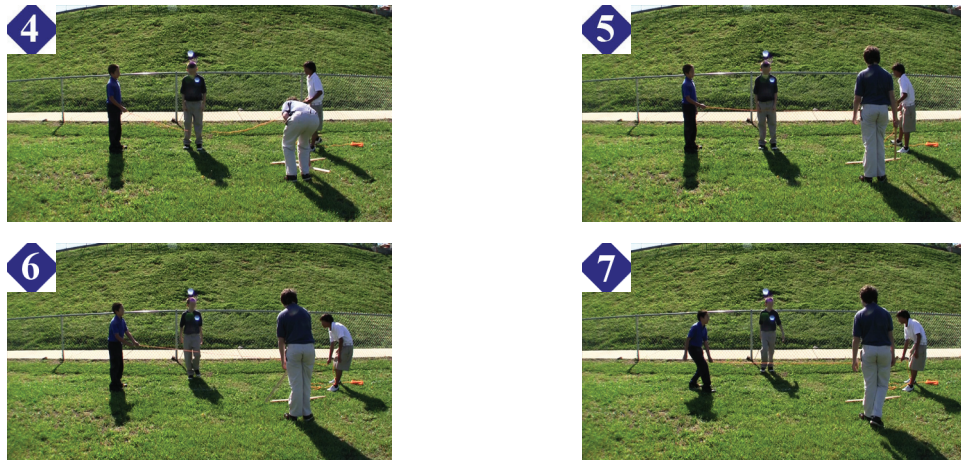
[22] Harry: Yes! *((Poking the yardsticks he’s holding into the ground emphatically, twice. He then starts to try and push the ends of the yardsticks into the ground.))*

[23] Eddy: No no, let’s just go ahead and do Dean. *((Gets ready to jump in again.))*  
Not fast, du::udes. *((slapping at the rope with his hands))*

[24] Dean: I’m tryin’. That way it won’t mess up ( )

*Figure 4-13. Excerpt 2, “We Can’t Lose Dean’s Height”*

- [25] *((Eddy jumps in, and the rope hits his back half a turn in.))*  
 [26] Eddy: Aww come on!  
 [27] Ben: It don't go up that high, I told you.  
 [28] *((Dean and Eddy trade spots.))*  
 [29] Eddy: Yeah it doesn't go that high. [30] *((Dean walks up to the rope and stands right at the middle))*  
 [31] Harry: We want a picture!  
 [32] Eddy: *((Backing up, and pulling the rope taut))* Hey<4>, try to jump over it.  
 <5> Hold tha- De- Ben. <6> (0.5s) <7> *((Puts his end of the rope closer to the ground. Ben does the same.))*



- [33] Harry: Hey look- Limbo limbo lim:bo! *((in a sing-song voice))*  
 [34] *((Dean starts to jump over it, but lands with one foot on either side.))*  
 [35] Eddy: No, completely jump over it. Ok, I'll try.  
 [36] Ben: No no, I'll do it. I can jump over it. *((He hands his end of the rope to Dean, walking over. He continues holding his hand where he was standing though, until Dean gets the rope from him.))*

Figure 4-13, continued. Excerpt 2, "We Can't Lose Dean's Height"

**Excerpt 2 discussion.** At the beginning of this excerpt, it is suggested again that they use Harry's height as the base measure for the length of the line segment and jump rope. Harry's enthusiastic response, in combination with his earlier request ("Let's use me," [13]), shows that he has an investment in having his body be the unit of measure for the group's line segment. This is an opportunity for Harry's body to take on mathematical meaning. To "use Harry" is also to allow him to participate in the activity in math-

ematically significant ways, even if he is not physically one of the endpoints of the line segment.

As the group's turn to have their picture taken approaches, they restore and preserve the straightness of the line. The rope is not a line segment if it is not straight, even though the students have carefully maintained its length. After Harry calls for their picture to be taken, they shift their game to jumping over the rope (Harry suggests limbo, which they do later). Even as the students engage in play rather than the mathematical task of making a line segment, their activity is both constrained and afforded by these mathematical properties of the object they are working with.

This episode demonstrates the flexible ways in which students recruited the materials and their own bodies as resources for mathematics and for different activities (in this case, play), occasionally at the same time. In the context of WSG, the students' uses of the rope and their bodies for play were both afforded and constrained by the demands of the task and salient mathematical properties of the materials.

The disruption of space and tools in WSG affords new forms of interactions with the materials of problem solving. For these students, being outside positioned them to initiate play that included running and jumping. These were not activities in which they indulged inside the classroom, even when not required to sit at their desks. In fact, on informal visits to KCMS, I observed that even in gym class these students's bodies were tightly controlled, allowed to run and jump only in certain spaces in the gymnasium and under the direction of the teacher.

The play seen here, emerging during moments in between task obligations, was not activity separate from that of WSG. Instead the mathematization of the rope as mea-



sured line segment and students' bodies as endpoints remained deliberately intact even as students repurposed these tools for jumping rope, limbo, and jumping over the rope. Typically, in both the KCMS and SEC sites, play emerged in this way, during in between moments, under the constraints of the mathematical activity. Play occurred less often in SEC than in KCMS.

Note that the discoveries that the students make about what they can do with the rope emerge in moment-to-moment interaction as subgoals for play and problem solving are formed. As the episode begins, the students are waiting for researchers to take a picture of their line segment. As they wait, they start using the line segment as a jump rope. The failure of three of the group members to jump in and their conjecture about why they can not do it (rope is too short) does not in and of itself deter them from continuing to try. However, when Dean stands in front of the middle of the rope, Eddy voices his new idea to simply try to jump over the rope, and the group takes this up. As the episode ends, Harry is singing "Limbo limbo lim:bo!" and the students start to do a combination of jumping over and limbo-ing under the line segment, while the endpoint students raise and lower it. The students' bodies and talk in interaction with the materials as objects of play and as mathematical objects, and the mathematical task, are all resources for their emergent play. Later on during this class session (described next in "What We Needed the Yardsticks For"), play also becomes a resource for mathematizing the materials.

**Development of a New Representational Practice: “What We Needed the Yardsticks For” (KCMS, Day 2)**

a)

2. Using the soccer field as your “paper,” draw a **LARGE** triangle. Each side of the triangle should be **AT LEAST** as long as two of you put together are tall. You may use any of the tools that we have brought out. **Tell a research team member when you are finished so we can take a picture.**

b)



*Figure 4-14.* Task and students in “What We Needed the Yardsticks For” a) The task. b) Ben, Dean, Eddy, and Harry wait to get a photograph of their WSG line segment taken. Dean and Eddy turn their line segment like a jump rope while Harry and Ben look on.

I discovered early during the KCMS implementation of the design experiment that it was generally difficult to determine how or why students came to use materials in particular ways. This is likely because students don’t usually know what practices or experiences they might be drawing from, and when using everyday materials like rope, there are probably many relevant practices and experiences. However, I occasionally had access to an event that clearly led to an innovation. In the following episode, a new idea for representing vertices using a yardstick is developed during the students’ play.

In this episode, which begins a little over 7 minutes after “We Can’t Lose Dean’s Height,” the task was to draw a large triangle (Figure 4-14a). Once the triangle was made, researchers would take photographs from “ground level” and from above. Ben, Dean,

Eddy, and Harry have made their triangle. Their triangle is made of one continuous piece of rope, with each side length measured to be either twice Dean's height or four times his height. They have been keeping track of where the rope should be bent to create vertices by holding on to the right spot or making a mark on the rope with mud from the ground. The group has just had their picture taken from above by Kevin, and are waiting for me to take their ground level photo. As they wait, they play around. Right before the episode begins, Harry has just run through one of the sides of the triangle from the middle, "messing it up." The rest of the group is angry with him, because they now need to remake the triangle to get their photograph taken.

**Excerpt 1 description.** As the episode opens, Harry tells Ben that it's his turn to try and run through the triangle [1-3]. It is ok now because the picture has been taken [3], and they no longer have to keep the triangle intact. Eddy wants to play tug of war [4], another activity that will "break" the triangle. Ben objects, saying that I have not taken the picture from ground level yet [5]. The rope must remain in triangle configuration (with its associated measurements) until that happens. Eddy suggests that they all step into the triangle [6], while Harry impatiently yells at me to take the picture [7]. Ben, who has stepped into the triangle, charges the side Harry and Dean are holding, with a dramatic yell [8]. At the last minute he ducks under it, and laughs. Harry laughs along with him.

Eddy also pretends he is breaking the triangle, picking up the yardstick lying at his feet <1> and stabbing at the inside of the triangle, yelling "Die!" <2> [10]. The stabbing action <3> becomes something new, however, when Eddy places the yardstick inside the vertex of the triangle, perpendicular to it. He holds it there, with one hand on

the yardstick and one hand on both the stick and the rope, holding them together <4>.

Although Eddy does not yet replace himself as the vertex with this yardstick, this is the action will eventually lead to that innovation. Harry, on seeing this use of the yardstick, calls the assemblage a slingshot [12].

- [1] Harry: Alright, Ben, your turn.  
[2] Ben: What?  
[3] Harry: You have to try to run through. He got the picture.  
[4] Eddy: [Hey we can play tug of war. Tug of war. Wha what? Tug of war. Tug of war.  
[5] Ben: [No we gotta get the one from ground level. *((Steps into the triangle.))*  
When Jasmine comes over here and she takes the photo ( )  
[6] Eddy: Hey you know what we should do? We should go all of us three (in there) together.  
[7] Harry: Jasmine we need your picture! Alright, pull. *((Steps back, and pulls rope closer to himself. Dean and Eddy adjust and pull tighter.))*  
[8] Ben: *((Breathes in))* Uuuuaahh *((Runs toward the side between Dean and Harry, then ducks under the side.))* Ha ha. *((Laughs))*  
[9] Harry: (Seriously, it's all it's all like) aaah *((pulling his vertex upward sharply))*  
[10] Eddy: *((Picking up a yardstick))* <1> (0.5s) <2> Die! *((He stabs the inside of the triangle, then puts the yardstick in his vertex))* <3>  
[11] Dean: (Don't move. Shoot, don't move)  
[12] Harry: *((<4> Eddy uses the yardstick to pull back on his vertex of the triangle, putting his right hand on the stick and his left hand below, holding the yardstick and the rope.))* Hey, slingshot!

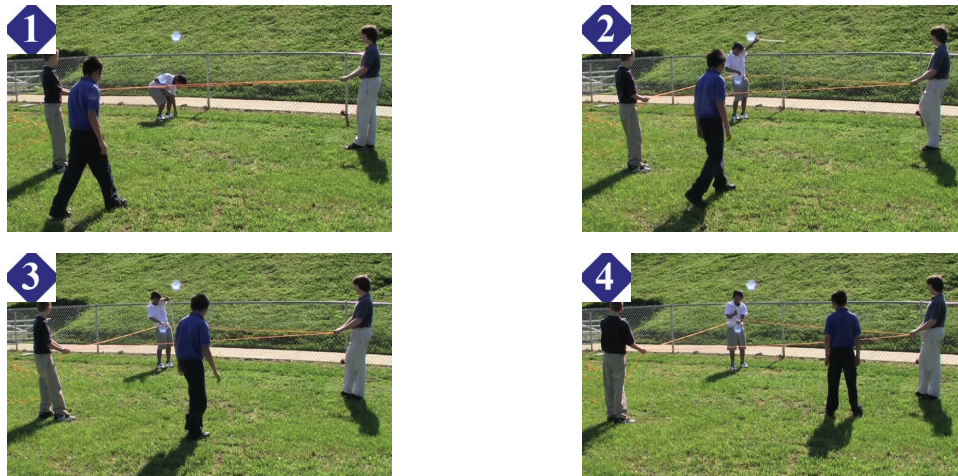


Figure 4-15. Excerpt 1, “What We Needed the Yardsticks For”

**Excerpt 1 discussion.** As in “We Can’t Lose Dean’s Height,” the students’ play is very much constrained by the mathematical activity in which it is contained. Games like running through and tug of war will ruin the triangle in ways that will make it difficult for them to shift back to the mathematical activity of having drawn a triangle. Ben can pretend to run through, and Eddy can pretend to kill the triangle by stabbing the empty space inside it. However, throughout this exchange, the triangle remains intact, and the play that is discussed, rejected, or taken up is influenced by this goal.

At the end of the excerpt, when Eddy picks up the yardstick and stabs the triangle, the action initiated for the purposes of play fluidly becomes an inscriptional action. The motion of the yardstick shifts mid-stab, and Eddy moves it back into his vertex, keeping it vertical to the triangle. He still keeps his left hand on the rope at that spot though, so the yardstick does not at this time replace his hand as a mechanism for holding the vertex in place (and for pulling the sides of the triangle taut) but supplements it. At this point in time, the yardstick is already an inscriptional innovation for this class of students, as no one has yet used any of the material objects to hold a vertex.

**Excerpt 2 description.** After Harry yells, “Hey, slingshot,” Ben plays along, pretending to run out of the way of the projectile [13]. Eddy does not like this idea, however, declaring, “It’s not a slingshot!” [15]. The idea of the yardstick pulling the rope back is relevant to the innovation that emerges, however. Dean has been participating, but not making active contributions to the play. He has been adjusting his body to the movements of the others (as they pull on the rope), and his head has been turning back and forth from his group members to something to his left. He could have been looking at me, walking toward the group, or one of two other groups who were working in that general direction.

As Eddy completes his assertion that it's not a slingshot, Dean interrupts by saying, "Hey THAT's what we needed the yardsticks for!" [17].

Harry, about to reassert that it is a slingshot, interrupts himself and asks, "What?" Dean asks Ben to get him one of the two yardsticks lying on the ground behind Harry<5>. Ben runs over, and as he does Harry bends down to try and grab them as well. When he does, he pulls the rope away from Eddy. Eddy shifts his hands and his yardstick, to compensate <6-7>. Ben reaches the yardsticks on the ground first, and picks them up. Harry stands back up, and as he does, he creates some slack in the rope, and Eddy keeps the rope between them taut by pulling his hands and his yardstick toward his body <8>. After Eddy pulls the yardstick and rope back toward his body, he puts his left hand on the rope and adjusts where the yardstick makes contact with it <9>. Harry's movements bending down and then standing back up causes changes in tension on the rope between him and Eddy. Eddy both experiences and creates tension on the rope with his yardstick in this embodied exchange with Harry.

[13] Ben: Aah! (Get out of the way)

[14] (1s)

[15] Eddy: It's not a slingshot! =

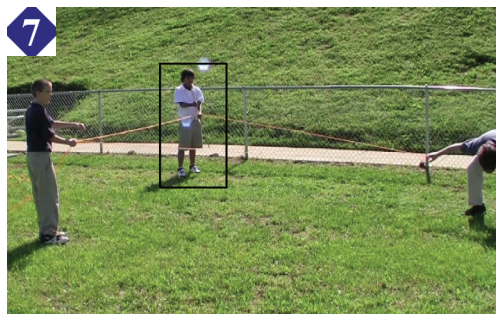
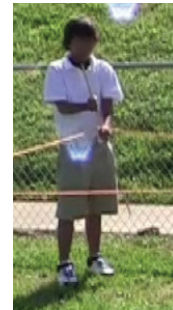
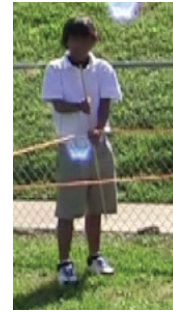
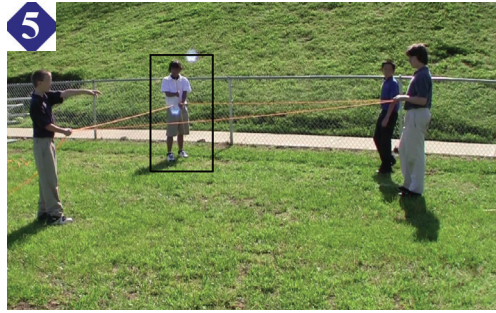
[16] ((Dean looks at Eddy, then to his left. As he starts to look back over at the other group members, he starts to speak))

[17] Dean: =[Hey THAT's what we needed the yardsticks for!

[18] Harry: =[Yeah it i- ((To Dean)) What?

[19] Dean: Here, gimme one <5> (1.5s) ((Ben runs to get a yardstick. Harry bends down to get them, pulling the rope away from Eddy. Eddy compensates by holding his hands, and yardstick, slightly away from his body)) <6> (0.5s) <7> (0.5s)

Figure 4-16. Excerpt 2a, "What We Needed the Yardsticks For"



[20] ((<8> As Harry stands back up, Eddy's hands, and his yardstick, move back toward his body))

[21] (0.5s) ((Eddy puts his left hand under the rope, steadying it.)) <9>

Figure 4-16, continued. Excerpt 2a, "What We Needed the Yardsticks For"

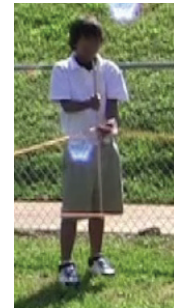
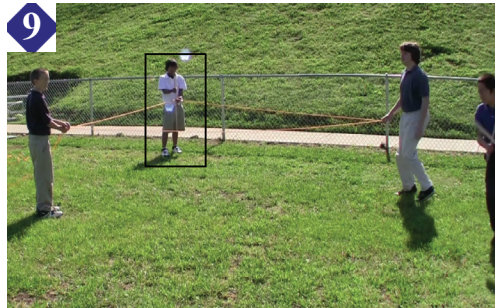
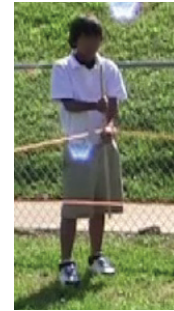
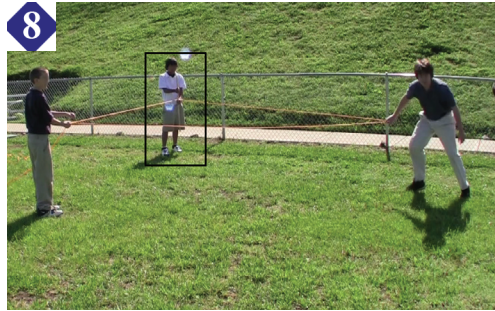


Figure 4-16, continued. Excerpt 2a, “What We Needed the Yardsticks For”

Eddy then puts his hand back on the yardstick, but this time above the rope, so that both of his hands are on the stick above the rope <10-12>. The vertex is now being held up and in place by the pressure of the yardstick pulling against it, rather than Eddy’s hand. As Eddy does this, he experiences more changes in tension from Dean. While Eddy moves his left hand from the rope to holding just the yardstick, Dean is reaching back to get a yardstick from Ben. He is careful not to move his hands, although as he accepts the yardstick and places it in his own vertex, they do move in toward Eddy some <10-12>, creating a little bit of slack in the rope between them. Therefore, Eddy’s ability to hold his vertex in place using just the yardstick is limited by Dean’s actions, and lasts only a moment <13>. He is soon getting too much slack from Dean’s side, and he puts his left hand back on the rope <14>.



[22] Ben: Move! ((He runs over and hands one to Dean))

[23] Dean: Here. Ok.

[24] ((Dean reaches for one of the yardsticks that Ben is carrying, keeping his right hand, which is holding the rope, in approximately the same position)) <10> (0.75s)  
<11> (0.5s) <12>

((Eddy puts his left hand back on his yardstick, this time above the rope.)) <10> (0.5s)  
<11> (0.5s) <12>.



Figure 4-17. Excerpt 2b, “What We Needed the Yardsticks For”

[25] ((Dean begins to put the yardstick in his vertex. As he reaches for and grabs the yardstick, his hands move slightly toward Eddy, giving some slack to the rope.))  
 <12> (0.5s) <13> (0.5s) <14>.

((Eddy adjusts his hands and the stick slightly as the rope dips <13>, then puts his left hand back on the rope <14>..))

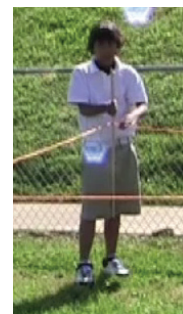


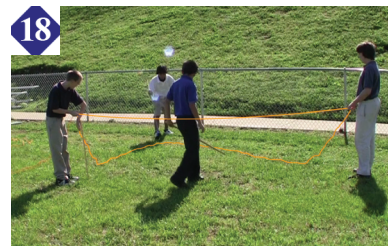
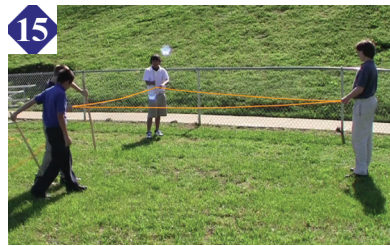
Figure 4-17, continued. Excerpt 2b, “What We Needed the Yardsticks For”

Dean starts to show them what he’s thinking, but his actions, which have the unintentional effect of loosening the rope’s tension between him and Eddy, are making Eddy’s (similar) idea fail. Eddy tells Dean, “Hey don’t move it cause look it can go like thi:i-!” [26]. As he says “move” the rope has already lost a lot of tension <16>, as evidenced by how far out he is pulling the yardstick and rope with his right hand. By the time he says “like” the rope is already beginning to fall to the ground <17>. Eddy’s talk trails off, and he interrupts himself with “Oh snap” and bends to pick his vertex back up <18>.

The students laugh, but Eddy is undeterred as he takes up what Dean is doing. Dean stands with his vertex against the top of his yardstick, which is propped perpen-

dicular to the ground. He is trying to push the yardstick into the ground, the yardstick bending as he does this repeatedly. As Eddy stands, he places his yardstick similarly, but his vertex is looped around his yardstick, and is held only by the stick. Eddy announces, “Oh look we can stab it in the ground!” [28]. Eddy then also tries to push the end of his yardstick on the ground. Harry takes this idea up, saying to Jasmine that they “need a hammer” [30].

[26] Eddy: Hey <15> don’t move <16> it cause look it can go like <17> thi:i- ((Eddy lets go of the rope, and only holds the yardstick, with the rope bent around it. The rope falls)) Oh snap <18>. ((Eddy bends over to pick up the rope. They laugh.))



- [27] Harry: Not any more.
- [28] Eddy: Oh look we can stab it in the ground!
- [29] Ben: ( )
- [30] Harry: Hey we need a hammer!
- [31] Dean: No we don’t.

Figure 4-18. Excerpt 2c, “What We Needed the Yardsticks For”

**Excerpt 2 discussion.** In this episode, the group’s play produced an innovation for what might constitute a vertex at walking scale. The students went inside soon after the end of this episode, but at the end of class, when we asked what other materials they

wanted for future WSG tasks, Harry and Dean asked for hammers and tent stakes (instead of yardsticks), and this group began to use these materials to construct their objects. As a practical matter, this invention could free students from holding the geometric figures in order to do other things. It could also help avoid the continual moving and adjusting that results in slippage or accidentally dropped rope (as we saw causing the students trouble in “How You Wanna Do It?”).

As the episode unfolded, the students discovered advantages of using a stick to act as vertex. Eddy negotiated the position of his yardstick-in-vertex to maintain rope tension while Harry and Dean pulled and gave on their shared sides. The stick, pulled back with the appropriate amount of tension on the rope, kept the triangle side taut and at a constant height. Dean’s observations of Eddy’s not-slingshot gave him a similar idea. His vertex, however, was not made of one continuous length of rope as Eddy’s was. Instead, he had the cut end of the rope held together with a measured spot on the other side of the rope, which was looped around Harry then Eddy and back to Dean to create the triangle. Dean’s idea began with holding the vertex at the top of the yardstick, and pushing the yardstick into the ground. Eddy’s idea began with holding the vertex using the yardstick pulled back. By the end of the episode, Eddy had also incorporated Dean’s idea and was trying to push his yardstick into the ground.

Each member of the group had a different set of material resources, physical positions in, and visual perspectives of the space. This variation, in addition to the different past experiences of each student, made for distinct trajectories of participation which, in the end, converged on the vertex innovation for the group.

This vertex innovation emerged from the play of the whole group, constrained by the goal of keeping the triangle intact. The development of the innovation was supported by students' talk and their actions as they tried/demonstrated ideas, and the effects of these actions on the entire rope-bodies-triangle system. Students' thinking was made available to other group members not just through talk, but also through observable and feel-able physical actions (more on this in the next episode, "So the Middle's Here"). Additionally, the fact that movements by one student holding a vertex inevitably caused changes in tension for the others positioned those others to compensate in some (active) way. The interconnectedness of the inscriptional system in this case supported an innovation, rather than simply resulting in trouble for the group (dropped ropes).

Lastly, subgoals important for the development of the innovation emerged at the intersection of the group's goals of play and of keeping the triangle intact for the mathematical task. For example, Eddy's dramatization of stabbing the triangle fluidly transformed into holding the yardstick in the vertex instead of his hand. As Harry bent down and stood back up, the changes in tension that Eddy experienced and his attempts to keep his vertex intact led to his discovery that he could hold the vertex in place with just the yardstick and enough pressure on the rope. These subgoals shifted back and forth between improving the group's play and completing the mathematical task, one often leading to the next.

As was noted in "We Can't Lose Dean's Height," WSG's disruption to the spaces of learning and doing mathematics affords certain kinds of play, involving students' whole bodies and WSG materials. Play is influenced by the mathematical properties of the materials and subgoals of the task at hand. In this case, this play produced an innova-

tion for treating a yardstick as a vertex. In particular, as each student pursued subgoals for play and for the WSG task, the coordinations between their bodies and materials provided resources for the innovation to emerge. In the next episode, coordinations between bodies and materials in the midst of problem solving become a resource for ongoing assessment and negotiation of shared understandings of an emerging strategy.

### Assessment via Distribution of Labor and Materials: “So the Middle’s Here” (SEC)

a)

**As a group, draw each geometric object with the given materials, using the lawn as your paper. Each object should have sides at least the length of 2 of your bodies. When you are finished, let us know so that we can take a picture from ground level and from the top of Wyatt.**

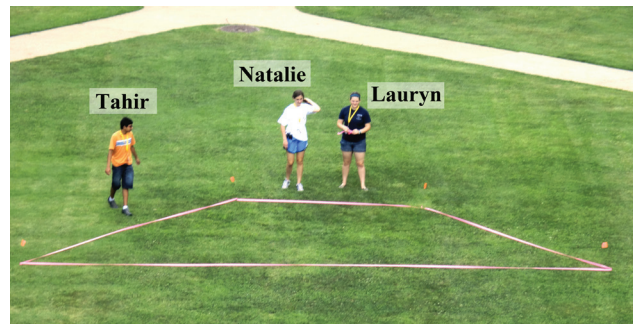
...

5. A **quadrilateral 1.5 times** the size of the one you just drew

To think about:

- What stays the same?
- What changes?
- How can you be sure that the new quadrilateral is 1.5 times the size of the original? What could you do to check?

b)



*Figure 4-19.* Task and students in “So the Middle’s Here” a) The task. b) Lauryn, Natalie, and Tahir stand by their WSG quadrilateral.

This episode takes place in SEC, with a group of three students, composed of Lauryn, Natalie, and Tahir (Figure 4-19b). Natalie was a rising tenth grader, and had

already completed a year of high school level Geometry before SEC. Lauryn and Tahir were rising ninth graders, and had not. The group had made a quadrilateral, and was at the beginning of solving a problem involving scaling it 1.5 times (Figure 4-19a). The wording was intentionally vague about whether or not students are meant to draw similar quadrilaterals, but this group takes this as the task. Before the episode begins, Natalie has developed and proposed a strategy involving dilating from an arbitrary point in the middle of the original quadrilateral to find corresponding vertices, then connecting them (see the episode entitled “Four Points Over Here” in the next chapter). Lauryn and Tahir seem committed to a strategy that involves scaling each side of the original quadrilateral, then spreading them out around the original and connecting them. They are not aware of the mathematical property that, in addition to four sides being proportional, one corresponding angle must be confirmed to be congruent in order for two quadrilaterals to be similar.

The three are about to measure 1.5 times the length of one side of the quadrilateral. This is not a part of Natalie’s strategy, but rather the beginnings of the implementation of Lauryn and Tahir’s idea. Natalie does not like this strategy, since it does not guarantee that the angles in the new quadrilateral will be congruent with the corresponding angles in the original quadrilateral.

However, Natalie goes along with Lauryn and Tahir’s strategy as they scale one side of the quadrilateral. As the group does this together, their strategies for accomplishing this scaling task seem to be aligned. However, when their ideas diverge the students encounter trouble. This trouble, or misunderstanding, is readily identifiable through the visual and physical feedback available in the problem solving environment.

**Excerpt 1 description.** This is the first time the group attempts to scale any line segment by a factor of 1.5. Initially, Lauryn and Tahir take charge of this subgoal. Lauryn initiates the process of finding “one and a half” of the side of the quadrilateral, pulling out a piece of green tape with Natalie [208-209]. Tahir suggests that they need to “find a half point” [210]. His use of “a” rather than “the” implies that there could be more than one half point, or that any number of points would probably be close enough to count as “a half point.” His willingness at [219] for Lauryn to rip the tape where she is holding it is further evidence of this. Once Natalie reaches one vertex of the quadrilateral and Lauryn reaches an adjacent vertex, Lauryn declares, “Here’s one” [211]. Natalie is still reluctant to go along with Lauryn and Tahir’s idea for making the new quadrilateral, looking over at me and saying she’s “still a little confused” [212], but participates in the task <1> (by the end of the episode she will be directing the procedure).

Once they have laid the green tape over the side of the quadrilateral and found “one,” Tahir holds the tape down at the vertex and tells Lauryn to “fold it over and bring it back” [213]. In order to make 1.5, Lauryn will roll the tape out, doubling over toward Natalie, until she reaches the midpoint of the side. Neither Tahir nor Lauryn have specified yet how Lauryn will determine where to stop. Lauryn walks back toward the middle of the side <2>, allowing the green tape to fold where Tahir is holding it down at the vertex where she was just standing. Lauryn starts to eyeball where the halfway point might be (“So would you say” [217]), when Natalie starts to implement her own plan. Natalie stands up with her end of the tape, and starts walking toward Lauryn <3>. Tahir then stands up <4>, letting his end of the tape go. The tape segment that is the length of “one” quadrilateral side is no longer stretched out taut on the ground. Tahir no longer marks its



endpoint by holding it in his hands, and the tape no longer has the properties of a line segment (straight, immobile) that would allow Lauryn to “find a half point” by just looking. Tahir tells Lauryn to rip the tape where she has it pulled out, “I guess” [219]. She did not complete the plan of doubling back the tape to an approximate halfway point, but what she did is good enough.

- [208] Lauryn: *((Pulls green tape out from roll. Natalie takes the end.))* We need to figure out one and a half though.
- [209] Natalie: Okay.= *((Lauryn backs up to the back left vertex of the quadrilateral, while Natalie steps toward the front left vertex.))*
- [210] Tahir: =Just like, find a half point,
- [211] Lauryn: Here's one. =
- [212] Natalie: =I'm s::s *((<1> looks over at Jasmine. At the same time, she bends down to put her end on the ground.))* [s::still a little confused
- [213] Tahir: [And then, (you can, like, fold it), one- *((Lauryn bends down to put her end of the tape down at her vertex.))* like, I'll hold it here, *((Bends down and puts his hand on the tape at the vertex.))* fold it over and bring it back.



- [214] *((<2> Lauryn starts walking back toward Natalie, rolling out more green tape.))*
- [215] Natalie: [Okay- so,= *((Stands up with her end of the tape.))*
- [216] Tahir: [(b- )
- [217] Lauryn: =So would you say=
- [218] Natalie: =*((Starts walking toward Lauryn.))* let's just <3> hold it right there, *((Tahir lets go of his end and stands up. The tape flutters in the wind.))* and <4>, um,
- [219] Tahir: Ok, rip it off right there, I guess, I (don't think)

Figure 4-20. Excerpt 1, “So the Middle’s Here”

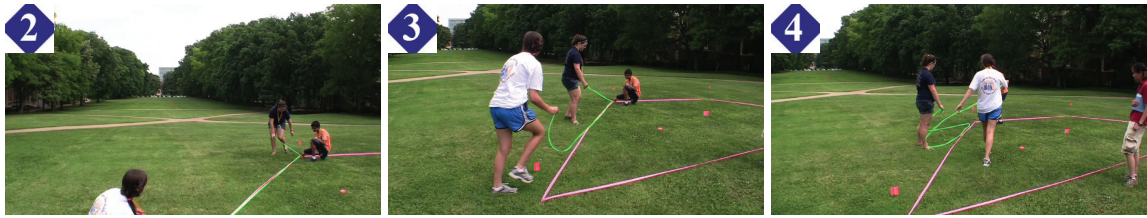


Figure 4-20, continued. Excerpt 1, “So the Middle’s Here”

**Excerpt 1 discussion.** Initially, Lauryn and Tahir led the group’s solution for the task of scaling one side of the quadrilateral 1.5 times. The group seemed to be all in agreement about this solution (for the subgoal of scaling the side, if not for the scaling of the whole quadrilateral,) as they rolled out the green tape to overlap the pink side, creating “one.” However, their ideas about how to get “half” differed. Tahir and Lauryn were satisfied with folding the tape over at the “one” mark and approximating “a half.” Natalie’s idea involves doubling the “one” over to find the “exact” half point that Lauryn would use to measure out the rest of the green tape (this will become apparent in the next excerpt).

This is an example of how the requirements of the distribution of materials across individuals and embodied activity, and the division of labor in WSG gives students access to each other’s thinking in significant and consequential ways. This (mis)coordination of the material manipulations between Lauryn, Natalie, and Tahir in this excerpt served as a resource for the group to identify conflicting understandings of their strategy for scaling the line segment. Natalie and Tahir’s release of the “one” so that it blew up into the air, stopped overlapping the original quadrilateral side [218], in fact stopped constituting the geometric object it once was, was a clear visual indicator of disagreement within the group. For Tahir, Natalie’s standing up marked the destruction of the “one,” and he stood

up too and released his end. For Natalie, Tahir's release of his end, losing the length of "one" side length made it impossible for her to double the tape up to find an exact half-way point. This was highly visible material and embodied feedback for their activity. The next excerpt is another example of this, with the addition of haptic and kinesthetic feedback as a resource.

**Excerpt 2 description.** Natalie takes over. She tells them to "Wait," and that she wants their scaling to be "exact" [220]. She tells Tahir to go back and hold the tape down at the corner again. He does, and Natalie bends down with him to establish their "one" <5>. She then stands up to walk toward Tahir, in order to fold the "one" in half to find the midpoint. As she walks, Lauryn bends down to put her end of the tape down on the ground <6>. Natalie tells her to "Just wait there" [220]. However, Lauryn tries to anticipate Natalie's strategy, and puts her hand down to hold down the tape that Natalie is in the process of folding in half <7>. Lauryn has held the tape down in a way that prevents Natalie from walking all the way to Tahir (closer to Natalie's end than to Tahir's end), and as the tape between them pulls taut Natalie stops walking. The unexpected physical feedback makes it clear that Lauryn and Natalie are not in agreement about this procedure. Natalie turns <8> and says, "Wait- no" [222]. Lauryn immediately lets go of the tape enough to allow Natalie to bring her end over to Tahir's vertex <9-10>. Natalie then directs Lauryn to "pull it- tight so you find the middle. ExACTly" [224], and points to the spot where Lauryn is holding the fold <11>. Lauryn now demonstrates her understanding by pulling it tight <12> (Natalie confirms [224]) and saying, "So the middle's here" [225] (Natalie confirms again [226]), and tearing the tape in the correct place <13>. Lauryn comments that "That's a good idea" [227].

- [220] Natalie: Wait. *((Walks back toward her vertex.))* Let's like make sure it's like exact like put that back at the °corner?° *((Tahir picks the tape up and holds it back at his vertex. As he kneels, Lauryn bends down and places her end down toward the ground.))*, and like, okay. *((<5> Natalie waits until Tahir is set, then stands up with her end and starts walking toward the middle.))* And then *((Lauryn bends down and places her end down toward the ground.))* like hold it there <6>? Just wait there. And then,
- [221] Lauryn: Okay and then I'll *((<7> places hand down on the tape Natalie is folding over))* hold this one here,



- [222] Natalie: *((<8> Stops walking as the tape pulls taut against Lauryn's pressure.))*  
Wait- no.
- [223] Lauryn: Oh *((<9> Lets up a little on the tape))*.=
- [224] Natalie: =Just, put it- *((places her end at Tahir's vertex))* there <10>, and like hold it THERE. And then *((<11> pointing at where Lauryn is holding the fold))* pull it- tight *((Lauryn pulls at the fold to make the folded over tape taut.))* so you find the middle. ExACTly <12>. Yeah.
- [225] Lauryn: So the middle's here. *((Lets go of the roll of tape and rips the tape where her left hand is holding the fold.))*
- [226] Natalie: Yeah=
- [227] Lauryn: =<13> Okay. That's a good idea.

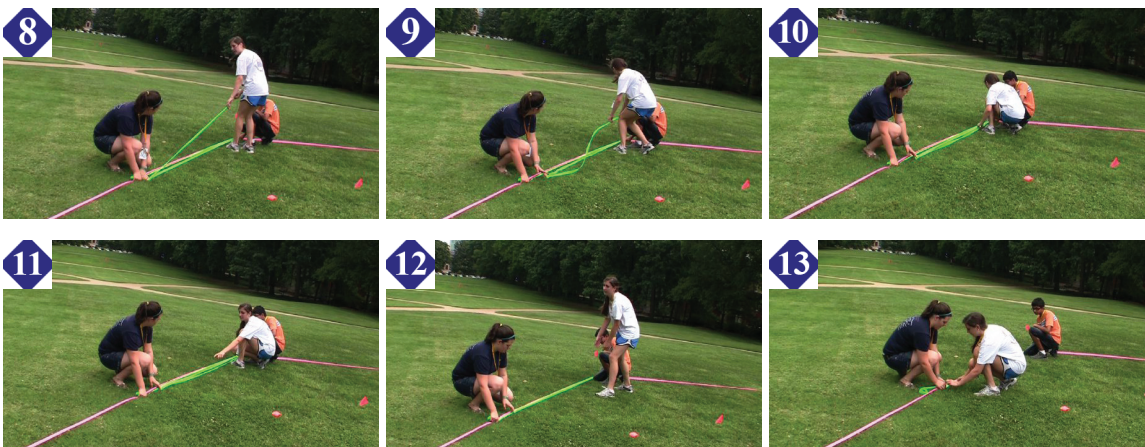


Figure 4-21. Excerpt 2, "So the Middle's Here"

**Excerpt 2 discussion.** When Natalie was trying to halve the original “one” in order to find the midpoint, Lauryn’s action of putting her hand down on the tape provided physical feedback to Natalie that indicated her strategy for creating 1.5 was not shared by Lauryn. In general the WSG tasks made agreements and misunderstandings available through individuals’ visible actions with the materials and also the physical feedback they received as a result of others’ actions. The feedback was often indirect, a form of visual and haptic “overhearing” (Jordan & Putz, 2004) afforded by the disruptions in tools and division of labor in WSG tasks. This feedback served as formative assessment for students as they endeavored to negotiate strategies and accomplish tasks together.

This is a phenomenon that is apparent in all episodes of WSG at both sites. Trouble and disagreement (and sometimes innovation—see “What We Needed the Yardsticks For,” previous episode) stemmed from this much larger, richer visual and physical problem solving space. For KCMS students, indications of misunderstandings in typical classroom activity came from student or teacher talk, and the inscriptions available on paper or whiteboard. During WSG activities, students’ whole bodies, their manipulations of everyday materials as mathematical tools-in-the-making, and the connections between bodies and materials are resources that provide rich visual and physical feedback during problem solving.

Once Natalie detected that Lauryn and Tahir had different ideas about how to scale the side, she took over directing the action. Explaining her strategy to them amounted to a demonstration achieved by giving them specific directions about where to move their bodies and what to do with the tape. This mode of explanation has the demands of constantly monitoring others’ understandings to the extent that they are

executing the demonstration correctly. In the next chapter I will argue that this mode of explanation, in combination with the use of everyday materials, allows for students with less sophisticated understandings of the problem solving strategy to participate in meaningful, consequential ways.

### **Walking Scale Geometries Produce Trouble**

As I mentioned in the discussion of “How You Wanna Do It?”, the fragility, interconnectedness, and other material properties of the inscriptional system resulting from the disruptions of WSG often produced in trouble for the pragmatics of drawing and maintaining mathematical representations. At the same time, “What We Needed the Yardsticks For” provided an example of how these properties supported the invention of a inscriptional innovation. Additionally, these properties can highlight particular mathematical concepts that are normally tacit or hidden in typical classroom mathematics activity.

Future iterations of the WSG design study will attend more explicitly to and try to anticipate this kind of trouble, so that it has more potential to be productive than detrimental for learning. Tasks can be designed to either invite and leverage this kind of trouble, or avoid it. Classroom instruction following the tasks can do the same.

In this section, I describe two episodes where the representational and inscriptional system of WSG produced trouble which was then resolved among group members. In the first episode, four KCMS students encounter trouble when they realize that they have prepared six pieces of rope to make a triangle. In the second episode, students in SEC reuse a piece of green flagging tape previously measured and cut for a different pur-

pose. Conceptual residue from this mathematical use and its physical properties conflict with the group's ongoing subgoals.

### Assembling Materials and Bodies for an Argument: “Each Person Holds Two Ropes” (KCMS, Day 7)

a)

1. Watch another group's instructional video. Take notes on the back of yesterday's worksheet. You will have to follow these instructions when we go outside.
2. OUTSIDE:
  - a. Make an isosceles triangle by following the instructions in the video you watched. Tell us when you are finished, and we will take a picture from above.

b)



*Figure 4-22.* Task and students in “Each Person Holds Two Ropes” a) The task. b) Felicia, Kaitlin, Kimberly, and Marla discuss how each person can hold two ropes.

In this episode, trouble arises as the students negotiate how they will organize the materials and their bodies in order to draw an isosceles triangle. The resources for their negotiation come from talk and gesture, referring to future time material and embodied connections. The trouble these students encounter is a result of the WSG representational property that side lengths can be determined and produced before they are arranged and placed.

The episode took place toward the middle of the WSG lesson sequence at KCMS, and involves Felicia, Kaitlin, Kimberly, and Marla. On the previous two (rainy) days, the students were told that they would be creating an instructional video for making an isosceles triangle, and that the next day another group would watch their video and try to follow their steps to make the triangle. We asked them to figure out how their group would make an isosceles triangle at large scale, outside, encouraging them to use the materials in the WSG toolkit to try things out. Then we asked them to write down their plan, and write a script for what they would say and show in their video. We then video recorded their presentation, and, after a whole class viewing and discussion, we gave them the opportunity to revise it.

This activity was designed in part because the weather prevented us from going outside, but also because Ms. H and I wanted to give students more practice making and articulating their plans for making WSG drawings. We also felt that it would be important for the students' work to have an authentic purpose, so we had them use each other's videos the next day to make isosceles triangles outside.

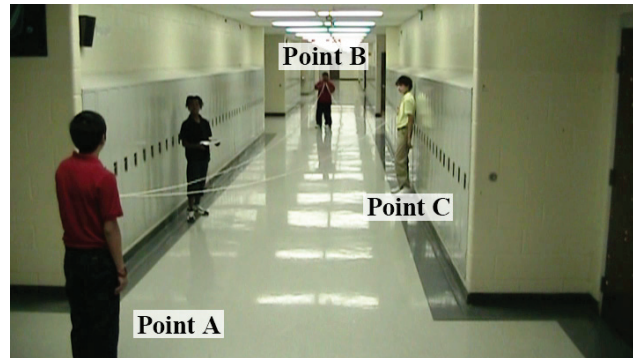
In this episode the group has watched the instructional video made by another group of students. The instructional video consisted of one boy reading from his list of instructions, and three other boys holding a pre-made isosceles triangle (Figure 4-23).



a)

First, you need to get measuring tape. Second, measure the base 20 feet. Two people holding it at each end of the rope. Third, measure the rope 30 feet. One person hold it with rope. Fourth, measure it again 30 feet. Five, have one person hold the rope at point A. At point A ((points at A)). Second, have one person hold two ropes at point B ((points to B)). Third, have one person hold two ropes at point C. And that's it. And that's our triangle.

b)



c)

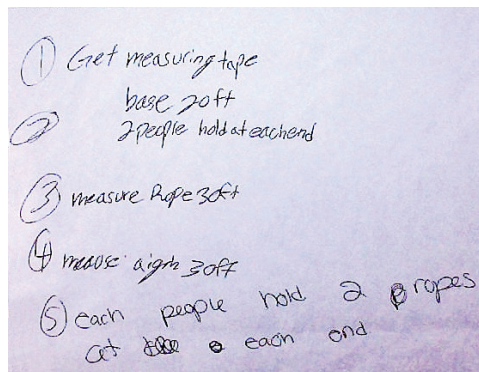


Figure 4-23. Instructional video for “Each Person Holds Two Ropes” a) The instructions. b) A screenshot from the instructional video, with points A, B, and C labeled. c) The group’s notes.

The group watched the video, took notes on what to do (Figure 4-23c), and cut lengths of pink rope according to the instructions before heading outside to make the triangle. (I do not have video record of this indoor activity). As the episode begins, the group has just gotten outside and started to untangle their rope pieces. They have a disagreement about how many pieces they need, and what the instructions meant. Kaitlin

and Marla remember very clearly that each person is to hold two ropes, and they each have different understandings of this. The notes, which Marla took and only Marla looks at while they are outside, say “each people hold two ropes at each end.” The disruptions in being able to see and manage (perspective and division of labor) the whole figure engaged these students in a discussion about the numbers of sides and vertices in the triangle, and how many sides meet at each vertex. As the episode progresses, Felicia and Kimberly convince Kaitlin and Marla that they only need three pieces of ropes. I argue that this is achieved through a progression of increasingly complex demonstrations and explanations by Felicia and Kimberly that recruits multiple perspectives, scales, and bodies.

**Excerpt 1 description.** Kaitlin begins by taking a verbal inventory of their materials: “So what do we need, we need, do we need, hold on, four thirties and two twenties?” There is silence for 2 seconds, and then she adds, “Cause the base is twenty” [2]. Kaitlin asks rather than states this, repeats “we need” three times, and interrupts herself with “hold on,” all of which indicates that this is not exactly what she expects. Then, after a silence where Kimberly and Felicia look at her and Marla looks at her sheet of notes, she justifies her reasoning, saying “Cause the base is twenty.” Her utterance is followed by several overlapping turns by all four group members debating the issue.

Kimberly asks, “How many triangles are we MAKing?” [3]. For Kimberly, the number of lengths of string that they have (six) does not correspond with the number of triangles she thinks they are making (one). If walking scale triangles are made with one rope per side, and they are making one triangle, they should only need three lengths of rope.

Marla agrees with Kaitlin, saying “Yeah, two twenties” [4]. Kaitlin gains a little momentum now, talking over Marla, “Cause two- each person has to hold two pieces of rope” [5]. Kaitlin is making a direct reference to the instructions they were given for making their triangle. Near the end of the video, they are told “Second, have one person hold two ropes at point B. Third, have one person hold two ropes at point C.”

Felicia says, “Yeah. Cause, like look” [6] while she walks toward the ends of the thirty foot lengths and bends down. It is unclear who she is agreeing with, however. When she stands up she has not picked anything up, and does not demonstrate anything as her utterance indicated she would. Meanwhile, Kaitlin has picked up the end of one of the remaining unmeasured lengths of rope, and has started to walk back to the far end of the measuring tape. She is ready to end the discussion and continue the activity of measuring.

Kimberly, however, wants to know more about the two pieces of rope. She turns toward Kaitlin and asks, “Like this <1>, or like- like <2> that?” [7, 10]. Her gestures are difficult to see, since Marla and Felicia stand between her and the camera. However, from what is visible of her body as she moves and from her shadow, it is clear that she is demonstrating different ways of holding two ropes. For the first, she holds her hands together in front of her body. For the second, she holds them out to either side. For Kaitlin, the important information is that there are two pieces of rope. Kimberly wants to know more; specifically, she wants to know how they will hold them. Kaitlin responds to Kimberly’s question, starting to say “Like thi-” but then correcting herself, saying “Like that” [11] and mirroring Kimberly’s second gesture, which she is still holding <3>. At the same

time, Marla says, “Wait. Hold on” [12] and points in the air as if counting <3>. She and Felicia join Kaitlin and Kimberly’s f-formation (Kendon, 1990).

- [1] *((All walking toward the center where their things are on the ground.))*
- [2] Kaitlin: So what do we need, we need, do we need, (1s) *((Marla picks up the piece of paper she was taking notes on and looks at it))* hold on, four thirties and two twenties? (2s) Cause the [base is twenty.
- [3] Kimber: [How many triangles are we  
[MAking?
- [4] Marla: [Yeah, two [twenties,
- [5] Kaitlin: [Cause two- each person has to hold two pieces of [rope.  
*((Kaitlin bends down to pick up another piece of rope, and starts walking back toward the far end of the measuring tape.))*
- [6] Felicia: [Yeah.  
Cause, like look *((walks over to the ends of the thirty foot ropes, and bends down, but doesn't pick anything up.))*
- [7] Kimber: Like this *((<1> Kimberly holds her two hands in fists in front of her, as if holding ropes in each.))*, or like=
- [8] Kaitlin: =Like two [pieces=
- [9] Marla: [We need six thirties. *((Stops walking, turns to face the others.))* =
- [10] Kimber: =like *((<2> Kimberly holds her two hands in fists out to either side, as if holding ropes in each.))* that? *((Turns to face Kaitlin))*
- [11] Kaitlin: *((Stops walking to look at Kimberly.))* Like [thi- Like that <3>.
- [12] Marla: [Wait. Hold on. *((Marla points in the air as if counting <3>. Marla and Felicia go stand with Kimberly and Kaitlin.))*

Figure 4-24. Excerpt 1a, “Each Person Holds Two Ropes”

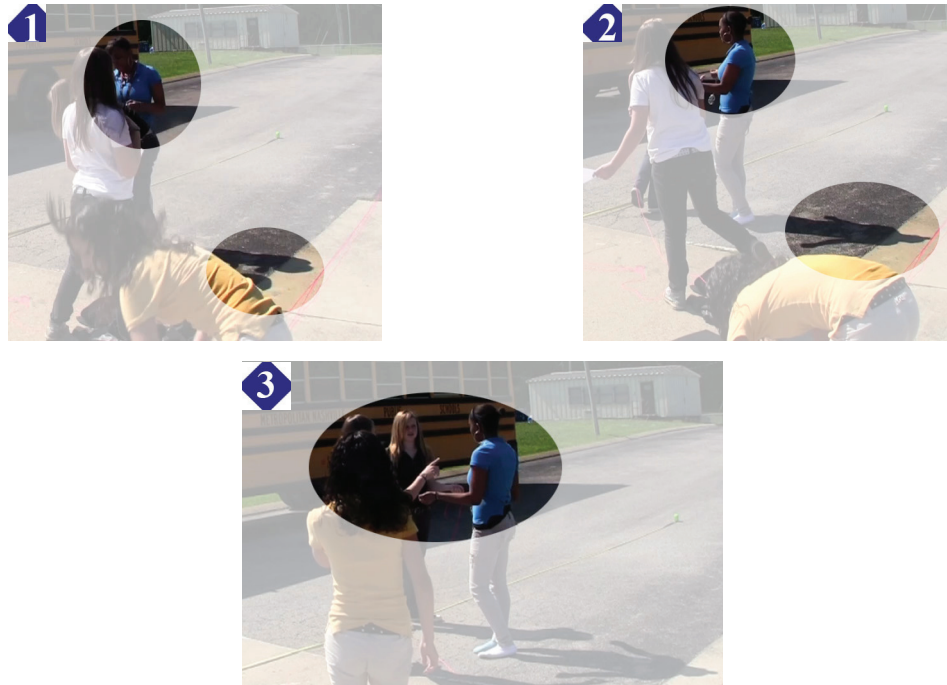


Figure 4-24, continued. Excerpt 1a, “Each Person Holds Two Ropes”

In response to Kaitlin’s “Like that” and gesture <3>, Kimberly responds, “Yeah” [13]. Kaitlin continues, saying “Cause it’s gonna go::o” [14] and traces a triangle with her fingers, from bottom up, with a vertex at the bottom <4-7>. She then reverses her trace, and Kimberly echoes this reverse trace in her own gestural stage <7-10>.

From this exchange it seems as though Kaitlin and Kimberly are in agreement. Their understandings of how they will hold the ropes seem to match, and what they expect their triangle to look like seems to match. However, their agreement does not extend to how many lengths of rope they’ll need, and Kaitlin does not want to discuss it anymore. She immediately turns away, saying “I don’t know, I’m just” [16] and continues to walk toward the far end of the measuring tape to measure the length of rope she picked up in [5], the first time she tried to dismiss the controversy.

At the same time, Marla confirms, “We need four thirties. We need four thirties and one twenty- two twenties” [17]. Kaitlin responds quickly, saying “Ok, so we got three over there I think this is our last thirty” [18]. The students walk back to measure, and Kimberly says, “I don’t understand this at all” [20]. However, she still goes to stand by Kaitlin while she measures the rope she is holding. Kaitlin and Marla both respond. Marla starts to explain by saying “They said you hold two ropes” [22], but is interrupted by Kaitlin, who is recruiting Marla as someone else who understands: “Marla do you get this? I do” [23]. Kaitlin does not try to explain though, she just says, “I dunno I just get it” [23].

[13] Kimber: Yeah.

[14] Kaitlin: Cause <4> it’s <5> gonna <6> go<7>::o <8>, ((Traces a triangle with her fingers, then traces the reverse. As Kaitlin traces the reverse, Kimberly also traces a triangle with her fingers.))

[15] Felicia: (Dyall) <9> just have <10>.=

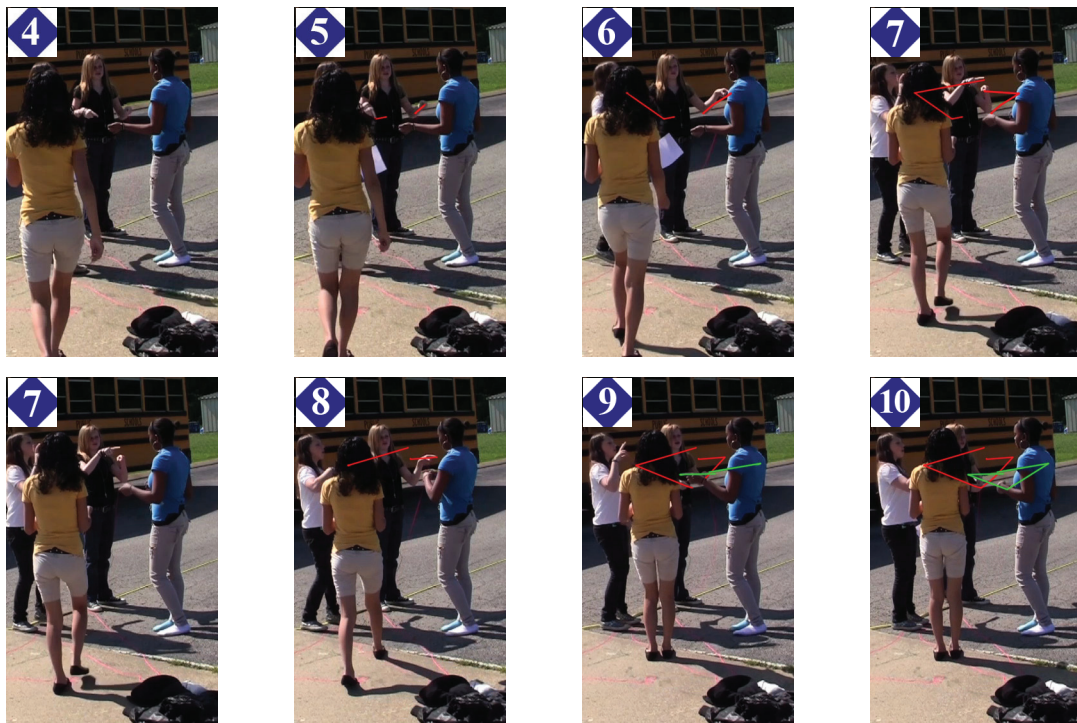


Figure 4-25. Excerpt 1b, “Each Person Holds Two Ropes”

- [16] Kaitlin: =[I don't know I'm just, (*walks away toward far end of measuring tape.*)]
- [17] Marla: [We need four thirties. We need four thirties and one twenty- two twenties=]
- [18] Kaitlin: =Ok, so we got three over there I think this is our last thirty.
- [19] (2s) (*They all walk back to their places where they were measuring*)
- [20] Kimber: I don't understand this at all.
- [21] Kaitlin: I do.
- [22] Marla: They said you [hold two ropes
- [23] Kaitlin: [Marla do you get this? I do. The- I dunno I just get it.
- [24] Marla: This was thirty.
- [25] Kaitlin: Ok so we got all our thirties this is four of em.

*Figure 4-25, continued. Excerpt 1b, "Each Person Holds Two Ropes"*

For the next minute and a half the group finishes measuring the last two pieces of rope, the two twenty foot lengths. They do these two at the same time. They discover that the two lengths are slightly different, so Marla asks Felicia for a pair of scissors and trims them while they are still stretched out together next to the measuring tape. After Felicia hands Marla the scissors, she walks toward Kaitlin and asks her, "Wha? We have to grab two for each?" <11>[28]. Felicia is not answered, and Kaitlin asks "who wants to hold the base piece?" [30]. Kimberly says she does not, but Marla says she will.

- [26] (*1 minute and 23 seconds pass while they measure the two twenty foot lengths.*)
- [27] (*Marla is at the close end of the measuring tape with one end of the two pink ropes, trimming them to be the same length with a pair of scissors. Kaitlin is at the other end. Kimberly is standing over Marla, and Felicia is walking toward the middle, facing Kaitlin.*)
- [28] Felicia: Wha? We have to grab two for each? (( <11> Stops walking, looking at Kaitlin.))

*Figure 4-26. Excerpt 1c, "Each Person Holds Two Ropes"*



[29] Marla: Ok now they're both twenty.

[30] Kaitlin: Ok who wants to hold the base piece? ((Marla stands up.))

[31] Kimber: I don't.

[32] Marla: I will.

*Figure 4-26, continued.* Excerpt 1c, "Each Person Holds Two Ropes"

Kaitlin says, "Hold on. We gotta picture their thing again. Because-" [33]. She interrupts herself, however, when Marla starts to take the two twenty foot pieces of rope in the direction of the thirty foot pieces. Kaitlin does not want them to get mixed up again, and tells Marla, "Keep the twenty over here! So we don't mix it up again" [33]. Marla changes direction, and Felicia picks up a different section of the two ropes to help. She asks if they are the legs, meaning the two congruent sides of the triangle, and both Kaitlin and Marla respond at the same time, "No." Kaitlin says that "the legs are thirty" [35] and Marla says that they "are over there" [36], pointing to the four thirty foot pieces of rope. Then, at the same time again, they address the base. Kaitlin says that "the base is twenty"[37] and Marla says that "This is the base. You hold them together" [38]. As she says this, Felicia is backing up with the two twenty foot pieces of rope sliding through her hands so that it remains taut between herself and Marla. As she gets to the end, they are holding the ropes as they would if they were making the base of the triangle, with both



ropes as the same line segment. She asks, “<12> Why did we cut the two?” [39]. Marla responds, “Because that’s what they said. Two ropes- for each side” [40].

Turns [38] and [40] are the first times that a group member has specified, in talk or gesture, this version of why they need six pieces of rope. According to Marla, the instructions said that they need two ropes for each side, and “you hold them together.” In fact, the instructions said that each person holds two ropes, not that each side is two ropes. It is possible to Marla realized her mistake, since her talk halts and hesitates right after she says “two ropes-” and before she finishes with “for each side.” No one responds, and in the next 1.5 seconds Marla flips her hair to the side with one hand and shifts her feet. She then says, “This is gonna be a big triangle,” [42] and Felicia and Kimberly agree.

- [33] Kaitlin: Hold on. We gotta picture their thing again. *((Puts down her end of the ropes, and picks up the measuring tape. Marla starts to pull on the ropes, and turns her body to the right.))* Because- *((To Marla))* Keep the twenty over here! So we don’t mix it up again.
- [34] Felicia: *((Picking up the pink rope toward the middle as Marla walks left with the ends. Felicia walks backward until she is holding the other ends.))* Aren’t these the legs? (Or just)
- [35] Kaitlin: [No the legs are thirty
- [36] Marla: [No the legs are over there.
- [37] Kaitlin: [The base is twenty.
- [38] Marla: [This is the base. You hold them together.

*Figure 4-27. Excerpt 1d, “Each Person Holds Two Ropes”*

[39] Felicia: *((Holding the ends now.))* <12> Why did we cut the two?



[40] Marla: Because that's what they said. Two ropes- for each side.

[41] (1.5s) *((Marla flips some of her hair to the side with her right hand and shifts her feet.))*

[42] Marla: This is gonna be a big triangle.

[43] Felicia: Yeah.

[44] Kimber: Yeah it is. ( )

*Figure 4-27, continued.* Excerpt 1d, "Each Person Holds Two Ropes"

Kaitlin jumps in, saying, "No, lo- they're like, each person holds two ropes <13>" [45] as she walks over and stands to face Marla. She is objecting to something about Marla's claim, but it is unclear what. Felicia responds by saying "I'mma hold fo::our" [46] and shows them that she will have two in her right hand <14> and two in her left <15> [48]. Marla agrees, saying "Yeah" [49], but Kaitlin does not, saying "No" at the same time and sighing [50]. Felicia repeats that she's going to hold four [51], Kimberly renews her disagreement [52, 54] and Kaitlin rethinks her stance, and decides, "I think the base is only ONE rope" [56]. Kimberly agrees, demonstrating how she thinks Marla and Felicia would hold one rope as a base [57, 59]. Marla also agrees, talking over Kimberly, saying "let's make the base one" [58] and "Do the base is one rope" [60]. After Marla makes her declaration the other three students all agree, saying "Yeah" at the same time [61-63].

- [45] Kaitlin: No- lo- they're like, each person holds two ropes. ((<13> *Walking over and facing Marla.*))



- [46] Felicia: I'mma hold fo::our.=  
 [47] Marla: =Ok [ya-  
 [48] Felicia: [Cause these are two ((<14> *moving her right hand, holding the ropes, up and down*)) and this ((<15> *holding her left hand out as if holding ropes in it*)) is gonna be two.



- [49] Marla: [yeah.  
 [50] Kaitlin: [No, ghhhh,  
 [51] Felicia: I'mma hold four  
 [52] Kimber: Y'all got a lot of [( )]  
 [53] Kaitlin: [No, this is gonna be confusing.  
 [54] Kimber: Cause I thought-  
 [55] Felicia: [I think we're just s-  
 [56] Kaitlin: [I think the base is only ONE rope.  
 [57] Kimber: Yeah cause li- I thought we're s- ((*takes the ropes from Marla and separates them*)) ok like [she holds like that ((*hands the end of one rope to Marla*))  
 [58] Marla: [Yeah let's make the base one.  
 [59] Kimber: and then this one like= ((*points toward Felicia*))  
 [60] Marla: =Do the base is one rope.  
 [61] Felicia: [Yeah.  
 [62] Kaitlin: [Yeah.  
 [63] Kimber: [Yeah.

Figure 4-28. Excerpt 1e, "Each Person Holds Two Ropes"

**Excerpt 1 discussion.** Just as in any other setting when a strategy or procedure is just given to students, we should not be surprised that the group ran into trouble executing the isosceles triangle instructions. They did not make mathematical sense of the instructions until they tried to implement them, and even in their discussion, agreement about the base came only after many bids for attention from Kimberly and Felicia. The disruptions in space, tools, perspective, and division of labor both caused a problem for the group and, eventually, provided resources for its resolution. Inside, while watching the video, they could only anticipate what to attend to in the video and in their notes. Their access to the resources for engaging in the task was limited or altered in the classroom space. They could use and manipulate the ropes and their bodies, but actually trying to make WSG triangles indoors was not a legitimized activity at that time.

Leaving the classroom space where the video, viewed on a laptop situated on a desk, was accessible, forced the group to rely on their memories and notes of the instructions. Once they got outside and began to actually make the triangle they discovered trouble. Because they had yet to make sense of how the ropes and bodies would become an isosceles triangle based on the instructions, Kaitlin and Marla were still treating the ropes as ropes that needed to be measured and cut. They remembered that “each person holds two ropes,” and since there were three people, they concluded that there should be six ropes. With their intrinsic perspectives of standing at a vertex holding two ropes, their reasoning did not extend to the entire triangle and how each person could hold two ropes while using only three ropes.

The inscriptional peculiarity of the materials of WSG and the method of using pre-measured lengths of rope that are later arranged and placed to create a geometric

figure also contributed to this trouble. The question of how many sides in relation to the number of vertices in a triangle is moot in paper and pencil versions of drawing triangles. In WSG, the relation between the number of edges and vertices in a polygon, and how they can be arranged, is salient and, in this case, problematic. As described above, four days later when “How You Wanna Do It?” occurred, this relationship was still salient, although less problematic for almost the same group of students (with the addition of Bianca). Kaitlin did originally claim that all students needed to hold a side, but there is no evidence of whether this is because she thought they needed five bodies or if she did not realize that there were five, rather than four students.

Because of the division of labor in WSG tasks, all the group members had to agree before they could continue. Granted, Kimberly and Felicia could have gone along with Marla’s idea of holding two ropes together, and just silently disagreed. There are also some clear issues regarding status (Cohen, 1994) here in how the girls agree or disagree with each other, and how they manage to come to a consensus by the end of the episode. There is evidence here and in our other observations of her (as in “How You Wanna Do It?”) that Kaitlin had both social and academic status, and the other students, in particular the girls, tended to defer to her. She generally took charge of and directed the action in groupwork. She was also a frequent contributor to whole class discussions.

However, the students each wanted the activity to make sense, and there is evidence of all four group members trying to make sense of the instructions, the materials, and their plan in this episode. The students tried to justify the six rope plan (Marla and Kaitlin) or make objections to it (Kimberly and Felicia) through talk, gesture, and demonstration with the ropes. This is not to say that moments of confusion or non-sense were

not tolerated. After making her initial objection, Kimberly remained quiet and continued helping the group measure and sort the ropes until Felicia renewed the concern. This was common in the way students engaged in WSG tasks. Questions, confusions, and disagreements might be voiced, but not always pressed immediately. Students would engage in the plan, waiting for it to play out, and then (re)raise the question when it was directly relevant to the current action, and trouble was either imminent or in progress (there is another example of this in the next chapter, in “I Get it Now.”)

Argumentation and the resolution of what to do with the base in this excerpt depended entirely on talk and demonstration. The WSG resources for developing and negotiating shared understandings, as seen in “What We Needed the Yardsticks For” or “So the Middle’s Here?”, were not yet relevant for this group, as they had not started to build the figure yet. Materials and bodies were not yet connected or in coordination, and the visual environment contained bodies and ropes not yet assembled as line segments or triangle sides. As the episode progresses, the students begin to introduce, develop, and build on different kinds of resources.

Early on, Kimberly introduced into this exchange a first person gesture of holding two ropes (without actual ropes) [10] (which will be repeatedly used by the group throughout this excerpt and the rest of the episode). Next, Kaitlin traced an imagined triangle in the air [14], mirrored by Kimberly (this will be reused but adapted by Kimberly later [72]). When Marla suggested that each side is made up of two ropes [38-40], she and Felicia had created a WSG line segment between them, providing a new resource in the material environment for the students’ reasoning. Felicia countered that then she would be holding four ropes [46], and demonstrated by breaking Kimberly’s gesture up,

first imagining two ropes in her right hand, then gesturing with the two in her left [48], which were already part of a WSG line segment (with a future as a triangle side). It is at this point that the group decided Felicia must be correct about the base, and they all agree that they should “do the base as one rope.”

**Excerpt 2 description.** Now that this decision is made, Kaitlin gets back to business, directing the others to put the extra rope away, in the bag. Marla readily takes on this task, separating the two ropes and repeating, “Put...” [67,71, 73]. Felicia, however, is not satisfied. After the group has all agreed to make the base one rope, she begins, “I think” [64] but is cut off by Kaitlin. She tries again at [66], saying “I think all of them is one.” She wants them to agree that every side should be made of just one rope, not just the base. Kimberly agrees [69], and Felicia says it again [70].

Kimberly then narrates and demonstrates the triangle. She puts her arms out to her sides with her hands positioned as if she is holding a rope in each, and says, “like you hold it like that” [72]. Then she draws the three sides of the triangle with her fingers in the space in front of her, each side drawn separately and unconnected, as a collection of three line segments (as opposed to the two continuous traces that Kaitlin drew in the air earlier). This collection of line segments more closely resembles the WSG strategy of bringing three precut ropes together to draw a triangle. Felicia watches intently and agrees, faintly echoing her gestures.

- [64] Felicia: I think=  
 [65] Kaitlin: =Ok so you c- put the base in the bag, [the other extra  
 [66] Felicia: [I think all of them is[ one.  
 [67] Marla: [Put the-  
           Put tha-  
 [68] *((Marla and Felicia separate the two twenty foot pieces of rope))*  
 [69] Kimberly: Yeah.  
 [70] Felicia: I think [all of them is one. They mean (     )  
 [71] Marla: [Put-  
 [72] Kimber: [Like you- like (*<16> putting her hands up like she is holding ropes in each*)) you [hold it like that. [Like (*<17> traces a line diagonally, top left to bottom middle, with her left hand*)) one a-[(*<18> traces a a line diagonally, top right to bottom middle, with her right hand*)) one like (*<19> traces a horizontal line at the top, right to left, with her right hand.*))and then like that.  
 [73] Marla: [No I sa- [Put- [Put that one- (*(pointing at the rope Felicia is holding, then pointing over to where Kaitlin is standing, where the bag of supplies is)*)  
 [74] Kaitlin: [No but look!  
 [75] Felicia: [Yeah [(two)



Figure 4-29. Excerpt 2a, “Each Person Holds Two Ropes”





Figure 4-29, continued. Excerpt 2a, “Each Person Holds Two Ropes”

Kaitlin responds to the group by making an appeal to the instructional video, saying “you gotta watch it make it how THE::EIRS waszah” [76]. Felicia’s “Yeah” seems to indicate that what she and Kimberly just demonstrated is in line with the instructions. She says, “I’mma hold two ROpes” [78], holding her hands out to demonstrate. Just like the instructions from the video, Felicia will hold two ropes. Kimberly looks at Kaitlin and says, “I thought they had just the other rope and that it was:s,” [80] and trails off, swinging her foot in front of her then back. The two ropes meant just the other side, not two

ropes per side. Kaitlin does not understand, and protests again, “No they said each person holds two ropes” [81]. Felicia says again, “I AM gonna hold two ropes” [82], and Kaitlin finishes her thought, “and it’s three people” [83].

Kimberly repairs “two ropes” by saying “two ENDS like,” [84] and Felicia takes up the language of ends, narrating and demonstrating ends on the rope she is holding <20> and how she will hold two ends <21>, and Kimberly agrees, saying “Like that” and pointing at Felicia. Felicia continues her explanation, that Kimberly will hold two ends, and one of the ends Kimberly holds will be the same rope as Felicia’s. And Marla will hold two also [88]. Kaitlin and Marla both respond emphatically, with “O::oh ye::eah!” [89] and “Oh my god!” [90].

- [76] Kaitlin: °Oh my bad.° But y[ou gotta watch it we gotta [make it how THE:EIRS waszah.
- [77] Felicia: [So, [it’s just,
- [78] Felicia: Yeah, I’mma hold [two ROpes. ((*holding one in her left hand*)) This one was be the other then.
- [79] Kaitlin: [They had,
- [80] Kimber: I thought they had just the other rope and that it was:s, ((*swings left foot and looks at Kaitlin.*))
- [81] Kaitlin: No they said each person holds two ropes.
- [82] Felicia: I AM gonna hold two ropes.
- [83] Kaitlin: and it’s three people.
- [84] Kimber: Like- ((*puts hands out*)) [two E:NDS, like, like=
- [85] Marla: [No- that’s how it IS=
- [86] Felicia: =Like the two ((<20> *backing away from them and holding out the rope in front of her with her right hand*)) ENDS?=
- [87] Kimber: =Like that.= ((*Pointing at Felicia*))
- [88] Felicia: =I’mma ((<21> *holds her hands out in front of her*)) hold like that and then ((*pointing at Kimberly*)) she’s gonna hold those two and ((*pointing at Marla*)) she’s gonna hold those two=

Figure 4-30. Excerpt 2b, “Each Person Holds Two Ropes”



[89] Kaitlin: =O::oh ye::ah!=

[90] Marla: =Oh my god!

*Figure 4-30, continued. Excerpt 2b, “Each Person Holds Two Ropes”*

**Excerpt 2 discussion.** While Kaitlin and Marla behaved as if the issue was settled, busying themselves with getting rid of the extra rope, Felicia continued making bids to argue that every side must be just one rope [64, 66, 70]. Kimberly was in agreement, and the two coordinated their bodies and ropes, real and imaginary, with their talk to show Kaitlin and Marla how each person would hold two ropes, yet they would only need three total ropes.

Kimberly began by merging her rope holding gesture with the third person bird’s-eye-view trace of the triangle in the air [71]. After Kaitlin objected again that “each person holds two ropes,” Kimberly repaired “ropes” to be “E:NDS,” and Felicia animated with her hands and a part of the rope that she was holding the rope “ends” [86]. Felicia then demonstrated Kimberly’s rope holding gesture, acting as a vertex, and recruited the bodies of Kimberly and Marla to animate the other two vertices. It is at this point, after building multi-scaled gestural representations of the WSG triangle, that Marla and Kaitlin were convinced by Kimberly and Felicia’s argument.

The disruptions in space, tools, perspective, and division of labor all contributed to the trouble encountered by this group. The inside space and outdoor space held different meaning for the students and had different affordances for activity. We asked them

to anticipate future activity while inside, and to remember past activity while outside. The properties of WSG representations and the method of constructing polygons out of pre-measured lengths of rope bring forward mathematical properties that are otherwise hidden and unproblematized (as discussed in “How You Wanna Do It?”). The intrinsic perspectives and necessary division of labor of students as vertices makes it difficult for students to see the whole figure at once, or what other students are experiencing.

However, these disruptions also provided the resources for Felicia and Kimberly’s increasingly sophisticated explanations and demonstrations. Because of the group’s shared histories with a third person extrinsic perspective on triangles in the classroom and with a first person intrinsic perspective on embodying rope-holding vertices in past WSG tasks, these experiences could be coordinated together in argumentation. In the end, Felicia built (in talk and gesture) a full WSG triangle by recruiting the bodies-as-vertices, each holding two ropes, of Kimberly and Marla. Felicia and Kimberly added and laminated different perspectives and bodies into their arguments, building a complex, multi-scaled gestural representation of the triangle made of three people, each holding two ropes, but only three ropes.

In addition to the complex multi-bodied and multi-scaled demonstrations assembled and orchestrated by Felicia and Kimberly the juxtaposition of the materials and material manipulations with mathematically meaningful uses and features in talk and gesture served as a resource for Kaitlin and Marla to come to the same mathematical conclusion as the other two students. Treating the ropes not as measured lengths of twenty and thirty feet, but rather as “base” and “legs,” and calling the function of the person-as-vertex as not just holding ropes but holding “ends” supported Felicia and Kimberly’s

argument and Kaitlin and Marla's developing understandings. These increasingly rich, multi-modal, multiply-scaled, multiple-bodied explanations and arguments developed not just to resolve disputes, but also to explain strategies to others and to revise students' own thinking (another example of this can be found in "Four Points Over There," in the next chapter).

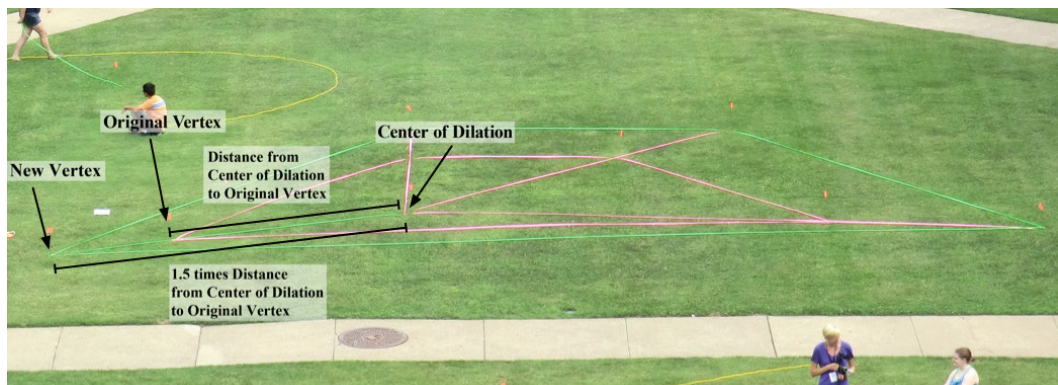
In the next episode, a piece of green tape that had already been measured and cut for a particular mathematical use gets repurposed for a new strategy. Not only does the old meaning of the piece of green tape make it difficult for students to recalibrate their interpretations and pragmatic manipulations, but it also causes trouble for the students' understandings and implementations of the new solution strategy.

### **Shifting Mathematical Meanings : "How Did You Decide on this Green Thing?" (SEC).**

This episode is constituted of excerpts from "I Get It Now," which follows "So the Middle's Here," and is described fully in the next chapter. As a reminder, the students are Lauryn, Natalie, and Tahir, and the task was to draw a quadrilateral 1.5 times the size the quadrilateral that they had just drawn (Figure 4-19a). For the purposes of discussing trouble produced by the disruptions of WSG, I will provide abbreviated episode descriptions along with associated transcript lines and images that highlight how the shifting and developing mathematical meanings of the green piece of tape as a geometric representation and tool caused trouble for the group.

In "So the Middle's Here," the group had begun to implement Lauryn and Tahir's strategy of scaling each side of the original quadrilateral 1.5 times, then spreading them out. They had used a green piece of tape to find 1.5 times one side of the quadrilateral.

After the episode ended, Natalie convinced them (with the help of Nate, the researcher operating the camera) to try her strategy of dilating from a point in the middle of the original of the quadrilateral to find corresponding vertices for the new similar quadrilateral (Figure 4-31). While the other two agreed to try this idea, they did not fully understand what Natalie was proposing. As this episode opens, the group is getting ready to start implementing Natalie's strategy.



*Figure 4-31.* Natalie's dilation strategy, completed. They picked an arbitrary point in the middle of the original quadrilateral, calling it the "center of dilation." They then used a piece of tape to measure from the center of dilation to each original vertex, then extended that piece of tape by half its original measure. The new vertex was placed at the end of that tape, now 1.5 times the distance from the center of dilation to the original vertex. This process was repeated for all four vertices, then a long piece of tape was wrapped around the four new vertices to draw the new, similar, 1.5 times quadrilateral.

**Excerpt 1 description.** Lauryn brings the green piece of tape over to Natalie, asking, "Are we gonna use the same length?" [255], then justifies using it "Cause this is the one and a half" [257]. For Lauryn, the green tape is still 1.5 times the side of the quadrilateral, and this measure is relevant for its next use. At the same time, Natalie says, "Guess we can use this to measure" [256]. She wants to use it simply as a piece of tape to measure where the new vertex will be. The tape itself will not be a part of the new quad-

ilateral. When she hears Natalie’s reason that it is “the one and a half,” she disagrees mildly, saying, “Well” [258], but she does not pursue the thought out loud.

[255] Lauryn: [Are we gonna use the same length? (*<1> Holding the green tape that was just measured to be 1.5 times the length of the side, and handing one end to Natalie.*)]

[256] Natalie: [Guess we can use this to measure.

[257] Lauryn: Cause this is [the one and a half,=

[258] Natalie: [Well,



Figure 4-32. Excerpt 1a, “How Did You Decide on this Green Thing?”

The group proceeds, under Natalie’s direction to stretch the green piece of tape from the center of dilation through one of the original vertices. They pause their activity while Tahir untangles some rope, and the three of them discuss what they are doing. At [305], standing at the end of the green tape exterior to the original quadrilateral, Lauryn looks at their work and asks if they are going to make the new, scaled quadrilateral “this way.” With her arm, she traces a quadrilateral rotated so that the green tape is one of its sides. In contrast, Natalie tells her that they are “going around it,” and traces a quadrilateral around her own body. Natalie’s gesture does not include the green tape at all.

[305] Lauryn: So, we're gonna go <2>, this way <3-4>? ((Standing at the end of the green tape, Lauryn looks toward the middle of the quadrilateral. She uses her forearm to trace a quadrilateral rotated from the original, treating the green length as a side.))

[306] Natalie: Like, we're going a<5>round <6> it <7>. Like the <8> same growth factor. ((Kneeling at the center of dilation, Natalie traces a quadrilateral around her own body.))

[307] Lauryn: O::::OH!

[308] Natalie: Yeah.

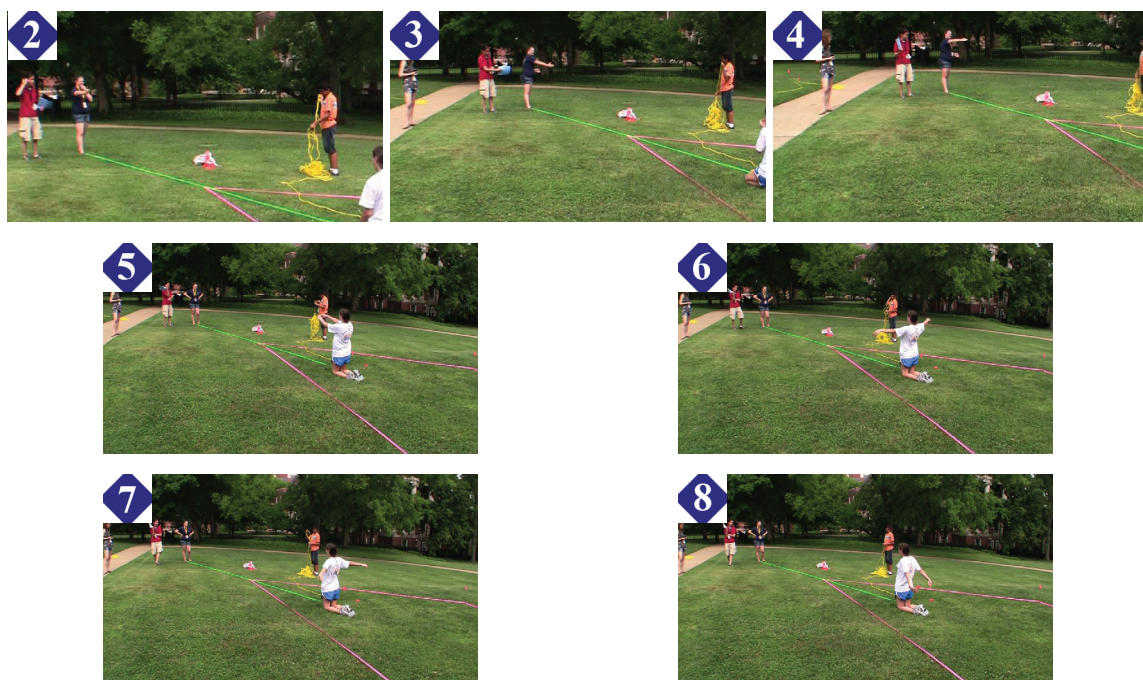


Figure 4-33. Excerpt 1b, “How Did You Decide on this Green Thing?”

**Excerpt 1 discussion.** For Lauryn, the length of the piece of green tape, measured to be 1.5 times the length of one of the sides of the original quadrilateral, is still relevant. Having given it this meaning, she continues to interpret their ongoing activity in relation to the idea that this piece of tape is a side of the new quadrilateral. At [255], she says that they can just use the “same length,” since they’ve already measured it. At [305], she imagines the new quadrilateral in relation to the green tape, tracing out the figure based on where it would be if the green tape were one of its sides.



**Excerpt 2 description.** Soon after, I walk over to check on the group. I see the green piece of tape lying on the ground, and I ask, “how did you decide on this green thing?” [323]. Natalie begins to say “Well, we’re just waiting-” [324], indicating some future identity for the green tape once Tahir has finished untangling the yellow rope. However, Tahir interrupts and says, “Well the green thing is actually one point five times the length of that” [325], and points to the side of the quadrilateral. His use of the present tense “is” indicates that it is still a scaled version of that side, and his use of “actually” implies that this fact might not be apparent in its current state, or that they are misusing it. Both Lauryn and Natalie agree with Tahir, but Natalie follows up by telling me that “we were gonna do something else but then we realized like the angles would be hard to like, copy, so, now we’re just getting this untangled, so we can l-” [329]. For Natalie, the green tape happens to be 1.5 times the length of that side because of a past plan. She hints at a new plan for it again, but, unfortunately, I interrupt her. We know, however, that her intention is to use it to measure 1.5 times the distance from the center of dilation to the original vertex in order to find the new vertex, a use that has no relevance to its prior measurement.

[323] Jasmine: So- how did you decide on this green thing?

[324] Natalie: Well, we’re just waiting-

[325] Tahir: Well the green thing is actually one point five times the length of that.

[326] Lauryn: Yeah.

[327] Natalie: Yeah.

[328] Jasmine: Ok.

[329] Natalie: So we we- we were gonna do something else but then we realized like the angles would be hard to like, copy, so, now we’re just getting this untangled, so we can l-

[330] Jasmine: Oh, so you’re avoiding copying angles. I see.

*Figure 4-34. Excerpt 2a, “How Did You Decide on this Green Thing?”*

Finally, as Natalie is trying to orchestrate the procedure to find 1.5 times the distance between the center of dilation and the vertex [338-349], this past mathematical identity of the tape again interferes with the group's progress. This time, it is not the meaning that the students gave it as 1.5 times the length of the original side, the corresponding physical properties (endpoints). Natalie halves the part of the tape lying between the center of dilation and vertex by folding the end that was at the center to the vertex. She then asks Lauryn to "fold...the other green back over" [338]. Lauryn brings the end of the tape over to the vertex, and leans down to stop there [340]. In its previous role as a side of the new quadrilateral, both of the ends of the green tape had meaning as marking the endpoints of that measured line segment. However, for the Natalie's dilation strategy, the procedure that they are implementing will determine the new endpoint of that green tape, which will also tell them where the new vertex should be. As a length of tape with as yet undetermined length, the end of the tape that Lauryn was holding does not have any mathematical meaning. Lauryn attributes some (probably unarticulated) meaning to it by bringing it to the vertex.

Natalie stops her, and Lauryn hands over her end of the tape. As Natalie begins to complete the measurement of 1.5 times, she makes a similar action, placing the end down at the fold that demarcated the halfway point between the vertex and the center of dilation [342]. She then hesitates, stumbles in her talk, then begins to pull the tape back so that it is folded over at the vertex. As soon as she tears the green length at the point it overlaps with her halfway fold, Tahir exclaims that he understands how the newly ripped tape is 1.5. Lauryn agrees that she understands, and there is evidence as they continue on with the rest of the vertices that she does. Tearing the excess destroys the previous math-

ematical measure (and corresponding physical properties) of the green tape, allowing it to take on the properties of the scaled measure of the distance between vertex and center of dilation.

[338] Natalie: ... And then just, could you hold that there? With that. Okay. And then, fold? (*<9> points down the length of the green tape.*) hnn, the green, the other green back over?

[339] Lauryn: [This one?

[340] Natalie: [Just so it's the same length at this (*Points to the folded segment she and Tahir are holding.*)? But keep- holding that there (*points to vertex*). So we like measure off, (*<10> Lauryn brings her end of the tape over to the vertex.*) NO like,=

[341] Lauryn: =Okay now I'm confused.



[342] Natalie: Like bring this (hh) over (hh)here. (*<11>Lauryn hands Natalie her tape end.*) [And then like, (*<12>Puts the end at the end of her folded half.*)

[343] Lauryn: [So it's the same length as that.

[344] Natalie: Yah. And so, wait. We need to, (*<13>Pulls the green end behind her until the tape is folded at the vertex.*) pull it until, it's right, <14> there, and hold it there. (Like,)



Figure 4-35. Excerpt 2b, “How Did You Decide on this Green Thing?”

- [345] Jasmine: So you're re-measuring now? What's happening?  
 [346] Tahir: [(I don't know.)  
 [347] Natalie: [Now we're=  
 [348] Lauryn: =I'm not sure.=  
 [349] Natalie: =going- out the- point- ((*Tears the excess off the green tape.*)) five. So  
 we [already have the one that's within?  
 [350] Tahir: [I get it!  
 [351] Natalie: [And then if we, continue, so,  
 [352] Lauryn: [Oh I get it. [Yeah.  
 [353] Tahir: [It's, yeah I get it I get how that's, [one point five

*Figure 4-35, continued.* Excerpt 2b, "How Did You Decide on this Green Thing?"

**Excerpt 2 discussion.** As illustrated in all the other episodes, but in particular "How You Wanna Do It?" and "We Can't Lose Dean's Height," students' shared histories of using and mathematizing the WSG materials can contribute to future productive negotiations of shared understanding and making mathematical sense. Here, the group's shared history of making this particular green piece of tape a 1.5 times scaled transformation of the side of the quadrilateral caused trouble both for Lauryn and Tahir's developing understanding of Natalie's strategy and for Natalie's own implementation of her 1.5 times scaling routine.

The inscriptional properties of WSG materials, unlike typical paper and pencil drawing, allow objects to be reused and repurposed, with new mathematical meanings and referents. In "We Can't Lose Dean's Height," the group repurposed their line segment to jump rope and play other games. At the same time, mathematical properties of the line segment were preserved, constraining their play. Similarly, in "What We Needed the Yardsticks For," the mathematical properties of the triangle and the physical demands of keeping it intact constrained the group's play. At the same time, the group's play supported the development of a mathematical innovation. However, in these episodes and

others described thus far, materials were not repurposed mathematically in the midst of a task. In “We Can’t Lose Dean’s Height” and “What We Needed the Yardsticks For” the materials were repurposed for play, but maintained their mathematical characteristics.

Here, the green piece of tape was an object with newly developing mathematical meanings, but students still attributed it with meanings from its most recent mathematical identity. The difficulty for the students to avoid this is apparent in this episode. Its measure, its physically available and visible endpoints, and its mathematical relationship with the original quadrilateral all played a role in how students made sense of it, their current measurement task, and Natalie’s dilation strategy as a whole.

### **Summary**

These six episodes illustrate examples of the resources that the WSG activity context made available, and ways that students engaged together in solving WSG problems. The disruptions in space, tools, perspective, and division of labor in WSG tasks made aspects of participation problematic, but also supported the recruitment of rich resources not typical of classroom teaching and learning, for problem solving. The resources recruited in the WSG setting can be leveraged for classroom instruction that supports productive hybridity.

The shift in the space of problem solving from contained indoor spaces to open outdoor spaces gave rise to whole body activity, visual access others’ actions, and playfulness not typical of classroom engagements.

The everyday materials of WSG, including students’ whole bodies, were mathematical tools-in-the-making, with familiar physical properties, constantly (re)negotiated

by students for the pragmatic purposes of the mathematical task at hand. They became visible and physical resources for argumentation, explanation, and formative assessment as groups negotiated strategies and implemented them. Students' shared local histories in using the materials similarly became resources. At the same time, uses of the materials could shift focus to typically unexplored mathematical topics or cause other trouble for problem solving.

Distributed, first person, intrinsic WSG perspectives sometimes caused trouble for students in representations and manipulations of geometric figures. However, this could have the effect of raising interesting new mathematical questions. We also saw that students coordinated third person, extrinsic perspectives on geometric representations and concepts with large scale WSG descriptions of them. WSG versions could include differently scaled versions of a mathematical object or the assembly of multiple intrinsic perspectives.

The division of labor associated with WSG tasks promoted negotiation of ideas and shared understandings, but did not require it. It provided multiple viewpoints on the representational space, as well as the distributed handling of materials that allowed for the rich visual and physical resources that students recruited for making arguments, giving explanations, assessing each other's understandings, and, in the case of Dean and Eddy, developing innovations.

These disruptions, in combination, also provided more access to problem solving activities for students with differing understandings of the task and the proposed solutions. The division of labor made it more difficult for students to be left out, and the everyday material manipulations gave them opportunities to participate in problem solv-

ing in more peripheral, but not trivial ways. I will discuss this in more detail in the next chapter.

## CHAPTER V

### RESOURCES FOR ACCESS TO PARTICIPATION AND SENSE-MAKING IN WALKING SCALE GEOMETRY

The disruptions of WSG were designed not just to provide new and different available resources and modes of engagement in geometry problem solving, but also to support more opportunities to learn for diverse populations of students. In other words, students should have a variety of ways of participating, at different levels of mathematical sophistication (Greeno & Gresalfi, 2008). In order for WSG to support productive hybridity, we should consider opportunities to learn in the WSG context as well as the ongoing classroom context.

In this chapter I use three episodes to ground descriptions of some of the ways I have found that WSG design supports participation in problem solving. The first episode, “Four Points Over There,” illustrates how a student develops a strategy and an explanation. I argue that, for her, the contributions of the other two students, as well as material and spatial concerns, are critical resources. In this way, the other two students have consequential roles to play in the WSG activity, and this entry point for participation, which includes mathematical reasoning and making arguments, is a legitimate form of engagement.

The next episode, “I Get It Now,” shows how the everyday qualities of the materials of WSG are a resource for students to participate in executing a solution strategy, regardless of their levels of mathematical sophistication in relation to that strategy. The



last episode is a retelling of “Each Person Holds Two Ropes,” this time with an eye toward how bodies can be mathematized as representations or conceptual tools. This practice of giving mathematical meanings to students’ bodies supports students’ participation, both in terms of negotiating shared understandings, and also for making sense of a problem or strategy.

### Access to Participation and Sense-Making

#### Material, Spatial, and Social Resources for Developing a Strategy: “Four Points Over There” (SEC).

a)

**As a group, draw each geometric object with the given materials, using the lawn as your paper. Each object should have sides at least the length of 2 of your bodies. When you are finished, let us know so that we can take a picture from ground level and from the top of Wyatt.**

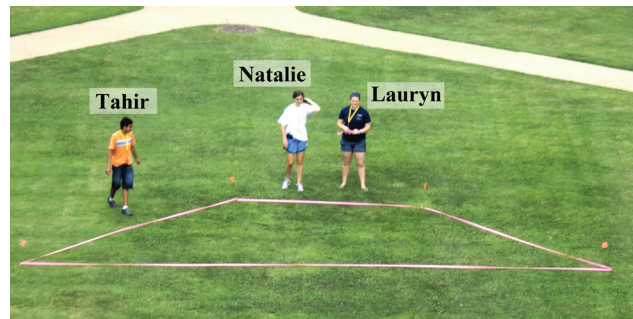
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5. A **quadrilateral 1.5 times** the size of the one you just drew

To think about:

- What stays the same?
- What changes?
- How can you be sure that the new quadrilateral is 1.5 times the size of the original? What could you do to check?

b)



*Figure 5-1.* Task and students in “Four Points Over There” a) The task. b) Lauryn, Natalie, and Tahir stand by their WSG quadrilateral.

This episode takes place at the beginning of the WSG task described in “So the Middle’s Here” and “How Did You Decide on this Green Thing?” (Figure 5-1a). The group contains Lauryn, Natalie, and Tahir (Figure 5-1b). In this sequence Lauryn, Natalie, and Tahir have made a quadrilateral, and are now developing a strategy to make a new quadrilateral 1.5 times the size of the original.

Natalie wants to try a dilation with an arbitrary center of dilation somewhere inside the original quadrilateral. In other words, a flag representing a point, the center of dilation, would go inside the quadrilateral somewhere. Then, they would measure the distance from the center of dilation to one of the vertices, then measure 1.5 times that distance. The new corresponding vertex is that distance from the center of dilation on the straight line passing through the original vertex (Figure 4-31).

Natalie is having trouble getting the other two to understand her idea. At the same time, she has not exactly formulated it yet. Specifically, she is not sure what the scale factor for the distance from the center of dilation to the new vertex should be. As Lauryn and Tahir respond to her explanations and engage her by either objecting to or trying to help plan the idea, Natalie has opportunities to revise her explanation and her thinking. The group’s shared history with the materials and building the original quadrilateral, Lauryn and Tahir’s responses, as well as the existing representation and its spatial location all provide resources for these revisions.

**Excerpt 1 description.** This is the first task in the sequence where the group has stopped to talk about what they will do. Lauryn asks, “Ok, so how are we gonna do this?” [147]. Tahir suggests using a ruler, but before he says what they’d use it for he says it’s a terrible idea [149-151]. Natalie then says, somewhat quietly, “So the angles are gonna

stay the same it's just gonna be like-" then interrupts herself and follows up, in a louder voice, "Do you wanna um, pick a point in the <1> center" [152]. As she says this, she points into the center of their quadrilateral.

Lauryn interrupts Natalie at this point, and their talk overlaps. Natalie and Lauryn say "and" at the same time, but then Lauryn pauses. Natalie continues, saying "go out," and Lauryn picks up on these words and takes over as Natalie says "a certain-." Lauryn says, "go all the way out and then come back in?" [152-153]. Although Natalie and Lauryn's talk overlaps here, their gestures do not. As Natalie says "go out [<2> a certain-," she points out at one of the far vertices of the quadrilateral. Meanwhile, Lauryn is pointing with both hands at the middle of the far side, and as she says "[<2> go all the way out <3>" she sweeps her arms apart so that she is pointing at each of the far vertices of the quadrilateral. As she completes her suggestion, "and then [come back in <4>?" she leaves her left arm pointing at that vertex, and brings her right arm back toward the middle of the side. She is animating pulling tape along the length of the far side, then halving it by doubling it back over to find the side's halfway point. This will become a part of the group's routine for finding 1.5 times a length, but this is not what Natalie is referring to. Natalie's talk and gestures refer to her idea for how to scale the entire quadrilateral, not just one side of it. Her strategy will not require scaling any of the sides, but instead she wants to "go out a certain" scaled distance from some "point in the center," through each vertex in order to find new, corresponding vertices.

Neither Natalie nor Lauryn detect this misunderstanding, however, and Natalie says "Yeah" as Lauryn says "come back in" [153-154], and begins walking and saying, "So let's, um," before Lauryn has completed her sentence.

- [147] Lauryn: Ok, so how are we gonna do this?  
 [148] Natalie: WEll.  
 [149] Tahir: Let's get a ruler!  
 [150] Lauryn: A ruler  
 [151] Tahir: That's a terrible idea.  
 [152] Natalie: °So the angles are gonna stay the same it's just gonna be like-° Do you wanna um, pick a point in the ((<1> Points into the middle of the quadrilateral)) center, [and then go out [((<2> Points toward the far vertex of the quadrilateral.))a certain-  
 [153] Lauryn: [And- [((<2> Points with both hands to the middle of the far side of the quadrilateral.)) go all the way out ((<3> Sweeps arms apart so that each hand is pointing to one of the far vertices of the quadrilateral)) and then [come back in ((<4> Leaving her left hand pointing at the quadrilateral, Lauryn brings her right arm back in, and points at the middle of the side))]?  
 [154] Natalie: [Yeah. So let's, um.

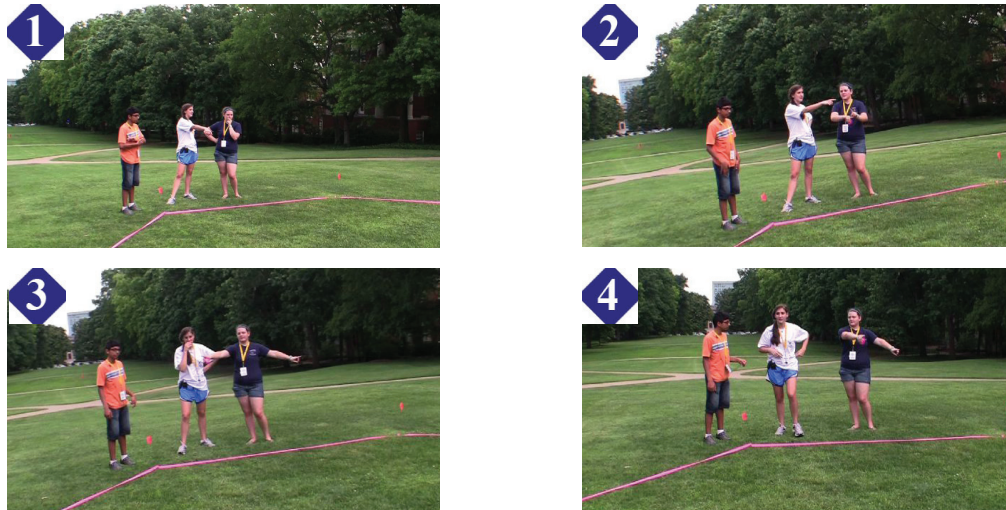


Figure 5-2. Excerpt 1, “Four Points Over There”

**Excerpt 1 discussion.** In this episode a student with more formal geometry learning experiences than the rest of her group, Natalie, advanced a relatively sophisticated strategy for constructing a similar quadrilateral. At this stage of the task, she did not yet have a coherent way of articulating her idea. Natalie’s first attempt, “Do you wanna um, pick a point in the center, and then go out ((pointing to the far vertex of the quadrilateral)) a certain-” [152], was incomplete because she was interrupted by Lauryn, but

it contained the beginning of the procedure she imagined. As the episode continues, it becomes clear that it is not just the explanation that is under development, but the strategy itself is as well.

Natalie's talk and gestures here focus only on a center of dilation ("a point in the center"), and the dilation to produce one vertex. Her talk also indicates a mathematical constraint that is salient for her, that the angles of the new quadrilateral will have to stay the same [152]. Her interactions with Lauryn and Tahir, as well as representational materials in the environment, and her experiences in her high school geometry class serve as resources to help her to build on this initial idea.

**Excerpt 2 description.** Lauryn takes a step forward to follow Natalie, and holds out the roll of pink tape in her hands, saying "This is gonna have to be cut, cause it- there's not gonna be a way we can do it" [155]. For Lauryn, whose plan is to scale each side of the quadrilateral and then make sure the endpoints of the new side connect to form vertices, it only makes sense to break the tape up into separate segments. For Natalie, whose strategy involves finding four new corresponding vertices, they should still be able to wrap the pink tape around, keeping it intact.

Natalie tells her, "I'm sure we can" [156], and Lauryn responds saying "It's not a big deal" to just cut it [157]. She laughs, uncomfortable or unsure about the exchange. She persists, though, saying "Cause there's gonna have to be four points over here," pointing to the space behind them, and we're gonna have to keep coming back with," and she holds out a short length of the tape [159]. Lauryn envisions that they will be putting four vertices in a different space, and they will have to "keep coming back" to the original to find the new side lengths.

Natalie quietly repeats “Four points over there-” and interrupts herself, saying “Or do we wanna do it like bigger out of it. Like if we dilate it? Outward?” [160]. When Natalie understands that Lauryn means that they will produce a new quadrilateral in a separate space, she offers an alternative, using the word “Or.” Lauryn’s idea involves all four vertices in the space behind the original quadrilateral; Natalie’s next explanation is oriented toward the spatial relationship between the original and new quadrilaterals. The new quadrilateral will be “bigger out of it,” or “Outward.” As she says “bigger out of it,” she holds both hands out in front of her, projecting to a space out past the side of the quadrilateral that she is facing <5>. As she says “dilate,” she pushes her hands outward again in a smaller version of the same gesture <6>. Then, when she says “Outward,” she points each index finger outward, in a yet smaller version of that same gesture <7>. Lauryn responds, “Mm hmm” [161].

- [155] Lauryn: This is gonna have to be cut, cause it- there’s not gonna be a way we can do it. ( )
- [156] Natalie: I’m sure we can.
- [157] Lauryn: We can just cut it. It’s not a big deal. ((laughs))
- [158] Natalie: Like, if we, like, °I don’t think it’s°, Ok so [do we wanna just like,
- [159] Lauryn: [Cause there’s gonna have to be four points over here, and we’re gonna have to keep comin back with,
- [160] Natalie: °Four points over there-° Or do we wanna do it like ((<5> holds both arms out in front of her, pointing hands forward)) bigger out of it. Like if we like ((<6> points hands outward)) dilate it? Outward ((<7> points each index finger out))?
- [161] Lauryn: Mm hmm.



Figure 5-3. Excerpt 2a, “Four Points Over There”

Natalie continues her explanation, “We could do that. So we could have like, <8> this in the center, and like, <9> double, or, maybe not double, but like,” [162]. She steps forward to animate the center of dilation <8> and points again to the vertex to show the length that needs to be dilated to find the new corresponding vertex <9>.

Tahir interrupts Natalie with his suggestion of how this will work, that they will have to “break” their quadrilateral, perhaps at the vertex Natalie was pointing to, and “scoot all these others out” [163]. Tahir speaks as if he is adding on to Natalie’s idea. What he is saying is in line with Natalie’s strategy in that he is thinking about an expansion outward of the original quadrilateral. However, he imagines taking the original quadrilateral itself and reusing the materials that constitute it in order to create the new one.

[162] Natalie: We could do that. So we could have like, ((<8> takes a step forward, looking down at her feet)) this in the center, and like, ((<9> points to the vertex in front of her)) double, or, maybe not double, but like,

[163] Tahir: We’re gonna have to break it at one part, that’s, I think, we can break it right there, scoot all these others, out.

[164] Natalie: Or we co-=-

[165] Tahir: =Yeah.



Figure 5-4. Excerpt 2b, “Four Points Over There”

Natalie treats Tahir’s contribution as a different idea, and describes her idea as another alternative: “Or we could put like a flag in here and we could like go out to each like point and maybe double that or like, halve THAT and like, so it’s like, °going, out.°

Or like you know like dilating [on a grid?]” [166]. Natalie tries her explanation for the third time, adding details like putting a flag where she had earlier indicated a “center” and that they will “go out” each point. As she says this, she points down at her feet to indicate where the flag will go <10>, points to the vertex and traces the distance between it and the “center” with her finger <11-13>, and points to each of the other vertices <14-16>. Then she invokes more literally the paper and pencil version of the procedure, “dilating on a grid,” and uses her hands to form a grid-like gesture <17>. Lauryn is quick to respond that she knows the procedure Natalie is referring to. When she does, Natalie says, hopefully, “We could do THAT. Do you wanna do that?” [168].

It is not clear if Lauryn really does understand Natalie’s strategy or if she knows about dilating on a grid, but later events and talk in this sequence indicate that she does not. Regardless, she does not see why they would do something so complicated when they could just make another trapezoid, and Tahir seems to agree [169-170]. Natalie makes the argument that getting the angles right would be hard to do, and with her way “you have the angles already formed” [171]. Lauryn agrees, and the group tries to figure out what is next. Tahir and Lauryn make suggestions about the color of tape they should use [176-178], and Natalie suggests they use the rope to measure. At this point, Nate interrupts them to tell them they might not have time to start something right now, but after their study hall they will come back and they can finish.



[166] Natalie: Or we could put like a ((<10> points down to her feet)) flag in here and we could like go ((<11> points out to the vertex again)) out to each like point and maybe double that or like, ((<12><13> traces in from and out to vertex)) halve THAT and like, ((<14> points to vertex behind Lauryn)) so it's like, ((<15> points to vertex to her left)) °going, ((<16>points to vertex behind her)) out. ° Or like you know like ((<17> holding hands in front of her in a rectangular shape)) dilating [on a grid?

[167] Lauryn:

[Yeah.

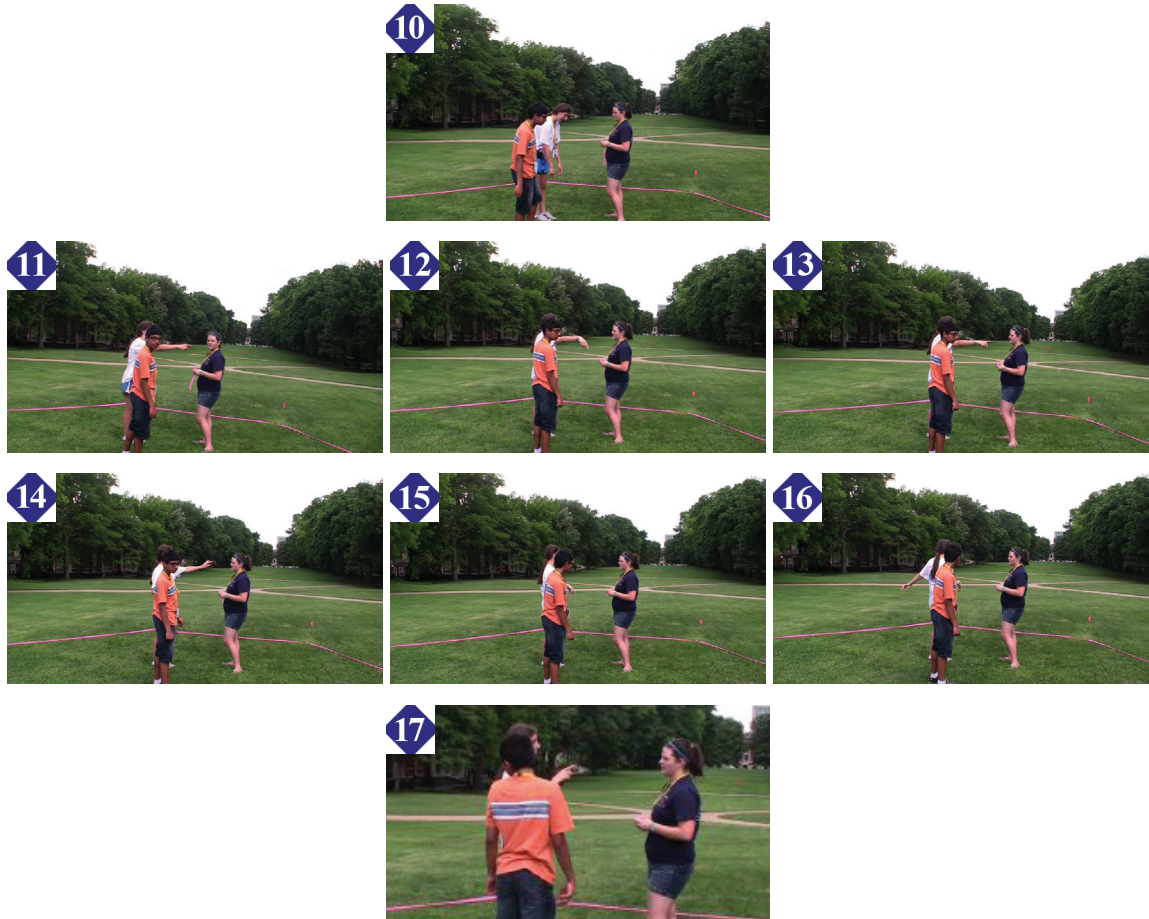


Figure 5-5. Excerpt 2c, “Four Points Over There”

- [168] Natalie: We could do THAT. Do you wanna do that?  
 [169] Lauryn: We could do that or we could just do another, tr(hh)apezoid. Like, [( )]  
 [170] Tahir: [I  
 think it would be easier to-  
 [171] Natalie: [It  
 would be hard to make the angles exact so it might be easier to just like  
 make it around this, and you have the angles already formed.  
 [172] Lauryn: Ok.  
 [173] Natalie: Do you wanna- Is that what y-  
 [174] Tahir: So, what-  
 [175] Natalie: Ok, so do you all wanna get like,  
 [176] Tahir: We should probably make it out of a different color of tape.  
 [177] Natalie: Yeah. Let's um,  
 [178] Lauryn: Yeah like the green one.  
 [179] Natalie: Yah and then we can just use rope to like measure.  
 [180] Nate: We're not gonna have time- well? We've got like 5 minutes.  
 [181] Tahir: What time is it?  
 [182] Natalie: Do youall think we can do it?

*Figure 5-5, continued. Excerpt 2c, "Four Points Over There"*

**Excerpt 2 discussion.** As Natalie explained what she was thinking, and as her groupmates responded or moved to take action, she had opportunities to repair her explanation in the making (Schegloff, 1997). At the same time, Lauryn and Tahir played crucial roles in providing these opportunities and were co-constructors of the explanation.

When Lauryn said that they would have to cut the tape and that the four new vertices would be in the space behind the original quadrilateral [155-159], Natalie tried a second time to explain her idea: "Or do we wanna do it like *((holding both arms out in front of her, pointing hands forward))* bigger out of it. Like if we like *((pointing hands outward))* dilate it? Outward *((pointing each index finger out))*? ... We could do that. So we could have like, *((taking a step forward, looking down at her feet))* this in the center, and like, *((pointing to the vertex in front of her))* double, or, maybe not double, but like," [160, 162]. This second explanation had a few new features. Ahead of the procedural

steps she offered an overview, that they do it “bigger out of it” or “dilate it.” This overview attended to the spatial relationship between the original and new quadrilaterals. The new explanation also included new mathematical language (dilate) that she continued to use after this explanation, as well as a try at the scaling factor that they might use (double), which previously she had not specified (“a certain-” [152]).

Tahir tried to add on by suggesting they break the quadrilateral, at which point Natalie made her third attempt: “Or we could put like a *((pointing down to her feet))* flag in here and we could like go *((pointing out to the vertex again))* out to each like point and maybe double that or like, *((tracing in from and out to vertex))* halve THAT and like, *((pointing to vertex behind Lauryn))* so it’s like, *((pointing to vertex to her left))* °going, *((pointing to vertex behind her))* out. ° Or like you know like *((holding hands in front of her in a rectangular shape))* dilating [on a grid?” [166]. This time, Natalie completed her procedural account, recasting the “center” as “a flag in here,” using “flag” as an alternative reference for the point. She also revised “double” to “halve,” refining her own idea of what the dilation factor might be. She finished with another overview, but this time she used a paper and pencil scaled school geometry concept, “dilating on a grid.” For this third explanation, Natalie grounded her strategy in the materials (flag) and the existing quadrilateral (pointing and tracing), and at the same time referenced the classroom version of her idea.

After this explanation, the other two did not argue that they did not understand her idea, but that they thought it was too complicated. This gave Natalie the opportunity to justify her idea. She told them that her strategy simplifies a problem with their idea: “It would be hard to make the angles exact so it might be easier to just like make it

around this, and you have the angles already formed” [171]. At this point, the other two conceded.

Natalie’s thinking and explanation was repeatedly revised as the other students responded to her. Lauryn and Tahir’s utterances signaled to Natalie that they did not understand her strategy the way she meant for them to. These responses included a concern about their material manipulations (Lauryn wanted to cut the tape), spatial locations (Lauryn indicated where the four new points will be), an attempt at engaging in the planning process (Tahir suggested they break the quadrilateral and scoot the vertices out), and a challenge (Lauryn and Tahir both protested that there was an easier way). Natalie was positioned to answer them four times, and she took each opportunity to revise her explanation. Her responses were grounded in the existing quadrilateral that they had just built and the material steps that the group would have to execute in order to accomplish the task. Through each exchange, Natalie’s own thinking also continued to develop, at least in terms of the dilation factor.

In part due to the division of labor requirements of WSG tasks in general, and Natalie’s solution strategy in particular, Natalie was invested in providing a sensible explanation to the other two students, so that they would and could help her. On the flip side, Lauryn and Tahir had reason to push Natalie. If they wanted to try their own strategy she had to be in agreement, and if they were to help her, they had to understand hers. This aspect of WSG supported a sociomathematical norm (Yackel & Cobb, 1996) of explanation that included needing to convince all members of the group to try and strategy and coordinating understandings enough so that they could carry it out together.

Talk, gesture, and the shared representation, unsurprisingly, played a large role in Natalie's developing explanation. Her final explanations were also multi-scaled, positioning herself and her groupmates both within the walking scale quadrilateral and as potential consumers of the paper and pencil version of "dilating on a grid," similar to Felicia and Kimberly's demonstrations in "Each Person Holds Two Ropes." Additionally, the material and spatial concerns of the new construction were shared considerations for next steps in task completion. Finally, problems or suggestions raised by Lauryn and Tahir concerning material manipulations and spatial configurations became resources for Natalie to respond to her groupmates' understandings and to revise her own thinking and explanation. In this episode the three students, as a group, co-constructed (with Natalie leading) the explanation and the strategy.

### **Material and Physical Manipulations as Access to Participation: "I Get It Now" (SEC)**

This episode occurs soon after "So the Middle's Here." Lauryn, Natalie, and Tahir continue to work on producing the quadrilateral that is 1.5 times their original quadrilateral. After the group makes the first 1.5 times scaled side of the original quadrilateral, Lauryn tells them they have to spread the new side out next to the original. Natalie worries about getting angles right again, and with the help of Nate, the researcher holding the video camera, she convinces the group to try her dilation strategy.

In this episode, the group repurposes the green tape that was previously cut to be 1.5 times the side of the original quadrilateral. They use the tape now to measure the distance from the center of dilation to the vertex, then find 1.5 times that distance, and locate the new corresponding vertex. As they work, Lauryn and Tahir follow the directions of

Natalie, mainly given as physical manipulations of the materials (tape, rope, flags, and bodies). They sometimes ask questions, sometimes challenge her, and sometimes try to anticipate the next move. However, although they do not agree with or understand her solution strategy, they have opportunities to participate in meaningful and eventually more central ways.

**Excerpt 1 description.** As Lauryn hands Natalie the end of the green tape that they have just measured to be 1.5 times the length of the side, the two speak at the same time. Lauryn asks, “Are we gonna use the same length? Cause this is the one and a half” [255, 257] while she hands Natalie one end of the green tape. At the same time, Natalie says, “I guess we can use this to measure” [256]. To Lauryn, the green tape is still one of the sides of the new quadrilateral, measured to be 1.5 times the length of the corresponding side on the original. She asks if they will use the same “length, Cause this is the one and a half” [255, 257]. Meanwhile, Natalie no longer thinks of it as a part of their new quadrilateral, but just a long piece of tape that they can “use ... to measure.”

Tahir is still unsure that this plan will work, and points out that the flag is not at the center of the quadrilateral. Natalie tells him that it does not have to be. She then kneels down with her end of the green tape at the flag, and tells Lauryn to “make this line go out from <1> that point?” [260]. Lauryn does not understand, and Tahir tells her to “Cross THIS <2> over here <3> (on) the flag- [(front) flag” [263], pointing to the vertex of the quadrilateral that is between Lauryn and Natalie. Lauryn does this, and Natalie asks her to put the tape to the ground and hold it there, and then asks Tahir to hold the tape “there,” pointing at the front flag.

- [255] Lauryn: [Are we gonna use the same length? ((Holding the green tape that was just measured to be 1.5 times the length of the side, and handing one end to Natalie.))
- [256] Natalie: [Guess we can use this to measure.
- [257] Lauryn: Cause this is [the one and a half,=
- [258] Natalie: [Well,
- [259] Tahir: =That's not the center point though.
- [260] Natalie: It doesn't need to be. You can dilate from any point. Just as long as you go out the same distance from ea- ((Kneels down at the center of dilation flag with her end of the green tape in her hand, and points to the front left vertex.)) So like make this line go out from <1> that point?
- [261] Lauryn: What?
- [262] Natalie: Like(h),
- [263] Tahir: Cross THIS <2> over here <3> (on) the flag- [(front) flag
- [264] Natalie: [Yeah. Okay.=
- [265] Lauryn: =Like that?=  
 [266] Natalie: =Put it to the ground? <4> And hold it there? And then- could you like, <5> hold it- [°there?
- [267] Tahir: [(Yeah. Like here?) <6>

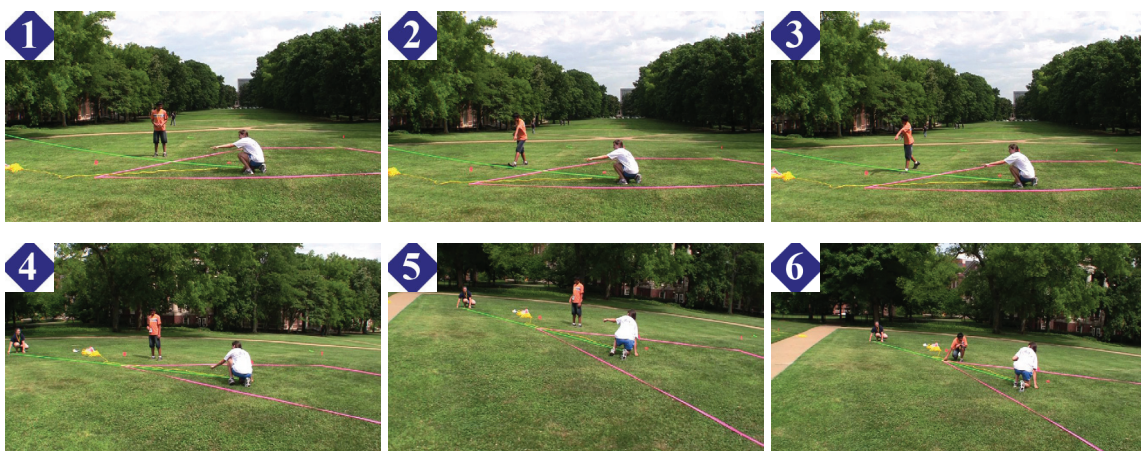


Figure 5-6. Excerpt 1a, “I Get It Now”

Now that Lauryn and Tahir are in place, Natalie starts to bring her end of the green tape over to Tahir, but hesitates and goes back to make small adjustments to the placement of the green tape. She brings her end to Tahir again, then decides that they should use the yellow rope “just to make sure we’re on the same line” [268]. While Natalie has a general plan both about how they will dilate the quadrilateral and how they will

find the new corresponding vertices, the exact actions they will have to execute are still under development for her.

They discover that the yellow rope is tangled, and as Tahir untangles it the group again discusses Natalie's dilation idea [273-288]. Natalie, knowing that the others are not in agreement with her about the strategy, looks at Nate and says that she feels like it is "really stupid" [270]. Tahir responds, "I don't think either of us really understand what you're doing" [271] and Lauryn agrees. Natalie tries to explain again, this time appealing directly to the paper and pencil version of the procedure: "You know, like dilating on a, grid? Like when you get like a triangle and you pick a point in the center and you go out with your ruler? To like make it bigger?" [273]. Lauryn just shrugs in response. She and Tahir do not know about this, because they have not yet learned it in school.

- [268] Natalie: Yes. *((Stands up with tape, walks toward vertex.))* Okay. Okay *((Backs up, kneels))* sorry. *((Mic transmitter falls))* Yikes! Okay. And then like, here. Loosen up just a tad? Just so it can like, pull out here? Okay. Yah. And then like, *((Walks with end of the tape to vertex.))* find whatever half of THIS *((looks at and reaches back to the doubled tape, then turns to Tahir))*, is? Like hold that tight there too? *((walks back, pulling doubled tape through her hand until taut.))* Okay. So we're probably gonna have to keep *((picks up yellow rope, and takes the end to the center flag))*, like THIS just to make sure we're on the same line. So put- *((points at yellow rope))* the- yellow rope like *((points at vertex))* over there too? But like make sure it can still reach- *((pulls on yellow rope))* eheh(hh) it's mostly,
- [269] Tahir: Yeah, the rope is kinda tied [(together)
- [270] Natalie: [It's okay, jus::s- I think it's better now, cause- *((faces Nate))* I feel like I'm like, doing something really stupid.
- [271] Tahir: I don't think either of us really understand what you're doing.
- [272] Lauryn: Yeah I'm not really,
- [273] Natalie: You know like dilating on a, grid? Like when you get like a triangle and you pick a point in the center and you go out with your ruler? To like make it bigger? Like *((Lauryn shrugs.))*

Figure 5-7. Excerpt 1b, "I Get It Now"



They continue to talk ([277-288], not shown), until suddenly, Lauryn notices that, “from this angle, our sides are really off” [293]. Lauryn, standing in her position from outside the quadrilateral, is attending to their representation and assessing it. Nate reminds her that they are not off since they just made a quadrilateral, not a trapezoid [298]. After Jasmine walks over and asks Lauryn what she means that their sides are off [303], two seconds pass, and Lauryn asks, “So, we’re gonna go <7>, this way<8-9>?” [305]. Natalie responds, “Like, we’re going a<10>round <11> it <12>. Like the <13> same growth factor” [306]. In response to this, Lauryn exclaims, “O:::::OH! Okay I get it now” [307, 309].

(...)

[289] Lauryn: So how is this gonna, like,

[290] Natalie: *((Sighs, and looks back at Nate))*. Well I don’t know yet.

[291] Lauryn: Yeah.

[292] Natalie: Well I mean like, I don’t know what like the, [growth factor is supposed to be.

[293] Lauryn: [Wow, from this angle, our sides are really off.

[294] Tahir: (yeah)

[295] Natalie: Yeah they are.

[296] Lauryn: I didn’t notice that before.

(...)

[305] Lauryn: So, we’re gonna go<7>, this way<8><9>? *((She uses her forearm to trace a quadrilateral rotated from the original, treating the green length as a side.))*

[306] Natalie: Like, we’re going a<10>round<11> it<12>. Like the <13> same growth factor.*((Kneeling at the center of dilation, Natalie traces a quadrilateral around her own body.))*

[307] Lauryn: O:::::OH!

[308] Natalie: Yeah.

[309] Lauryn: Okay I get it now.

Figure 5-8. Excerpt 1c, “I Get It Now”

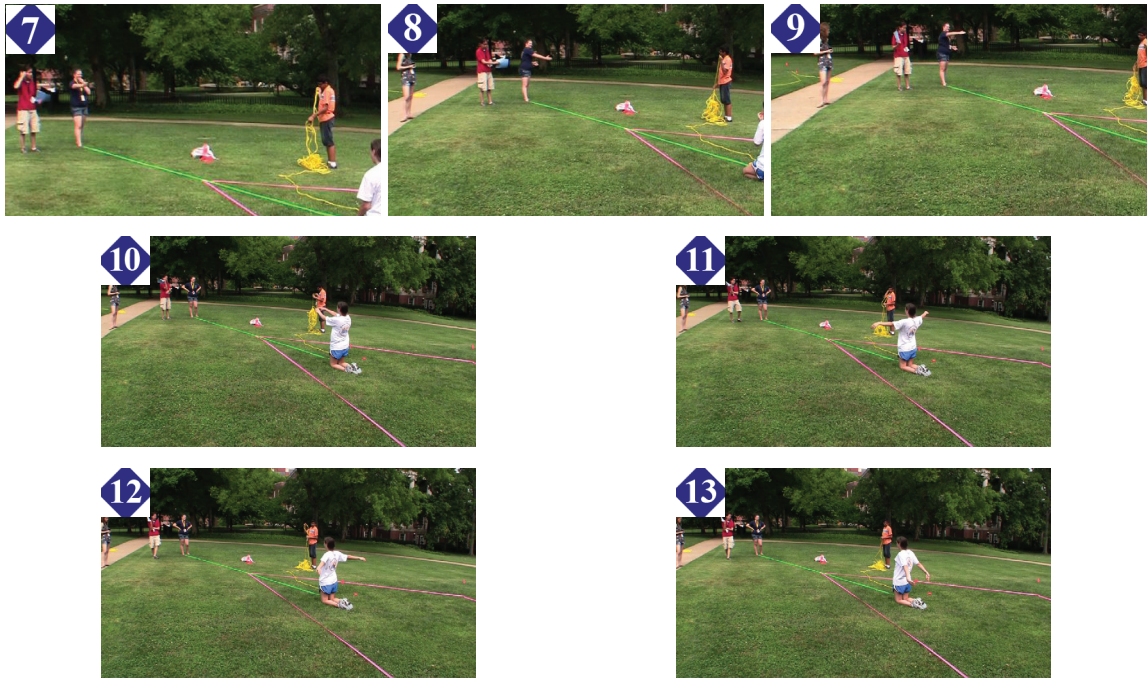


Figure 5-8, continued. Excerpt 1c, "I Get It Now"

**Excerpt 1 discussion.** Before they even began to execute Natalie's strategy, Lauryn tried to anticipate what to do, bringing the green piece of tape over and asking if they were going to use it. While Lauryn was mistaken about why they were going to use it, Natalie suggested using it at the same time, legitimizing Lauryn's idea. As they began, Natalie gave Lauryn instructions using mathematical language to refer to parts of the materials they are using ("So like make this line go out from that point?" [260]). Lauryn did not know what Natalie wanted her to do, and Tahir revoiced the instruction for her. Tahir's version of the instructions substituted gestures and language referring to the materials ("Cross THIS over here (on) the flag" [263]) for the mathematical entities that Natalie referred to ("this line," and "that point"). Natalie approves of Tahir's revision of her direction, and Lauryn does as she is asked.

This is not to say that Lauryn does not know what a point or line is, or that she is not familiar with the group's now established practice of representing points with flags and lines with tape. However, it is not clear to Lauryn which tape-as-line and which flag-as-point Natalie is talking about. Additionally, with her still-developing understanding of Natalie's strategy, and her lingering interpretation of the green tape as one of the sides of the new quadrilateral, it would not make sense to place it where Natalie is asking her to place it. However, the ability for Lauryn to engage with everyday language and objects allows her to execute the direction, even if she has not yet made mathematical sense of it. In this way, both Lauryn and Tahir have access to participating in the task even as they are still making sense of the solution strategy.

Lauryn and Tahir also make bids for further explanation or feedback. They do this by anticipating next moves ("Are we gonna use the same length?" [255]; "So, we're gonna go, this way?" [305]) or by challenging Natalie ("That's not the center point though" [259]; "I don't think either of us really understand what you're doing" [273]). As the episode progresses, Lauryn and Tahir's physical actions and engagements with the materials serve as openings for, if not requests for, further explanation or feedback from Natalie.

**Excerpt 2 description.** Jillian and Jasmine ask the group some clarifying questions about the center of dilation and the green length of tape [310-330], not shown. At this point, Tahir finishes untangling the yellow rope, and Natalie asks him to bring it over. Lauryn says that she gets it, that "I just didn't really understand what you were saying for a minute, 'til we actually started doin' it" [335]. But then when she tries to anticipate the

next move, “And is he going to this one?” [337] pointing to the vertex to her left, Natalie says no.

Natalie then walks her end of the green tape over to the vertex, and asks Tahir to hold it down. She backs up to the folded halfway point, and tells Lauryn to “fold? <14> hnn, the green, the other green back over? Just so it’s the same length at this? But keep-holding that there ((points to vertex)). So we like measure off,” [338, 340]. She wants Lauryn to bring her end of the green tape over and fold it at the vertex, and pull it over the folded “half” between Natalie and Tahir. The spot where Natalie is holding it will be the spot that they tear it to produce a length 1.5 times the distance from the center of dilation to the vertex. Lauryn brings her end to the vertex <15>. Natalie tells her “NO like,” and Lauryn quickly responds, “Okay now I’m confused” [341]. Lauryn’s actions make it clear to Natalie that they do not share an understanding of the plan.

(...)

- [331] Tahir: Okay I think this is [probably (about )]  
[332] Natalie: [Okay. Okay. Do you wanna just like, pull it over to that corner? Just so we have like a straight [line as comparison  
[333] Lauryn: [O:I get it NOW=  
[334] Natalie: =Yeah. I felt really stupid like- I- like=  
[335] Lauryn: =I- I just didn’t really understand what you were saying for a minute, ‘til we actually started doin’ it.  
[336] Natalie: Okay. Yah. And just like leave it there?  
[337] Lauryn: And is he going to this one? ((Points at vertex to her left.))

*Figure 5-9. Excerpt 2a, “I Get It Now”*

- [338] Natalie: Uh, no. Not yet. And then just, could you hold that there? With that. Okay. And then, fold? ((<14> *pointing down the length of the green tape*)) hnn, the green, the other green back over?
- [339] Lauryn: [This one?
- [340] Natalie: [Just so it's the same length at this ((*Points to the folded segment she and Tahir are holding.*))]? But keep- holding that there ((*points to vertex*)). So we like measure off, ((<15> *Lauryn brings her end of the tape over to the vertex.*)) NO like,=
- [341] Lauryn: =Okay now I'm confused.



Figure 5-9, continued. Excerpt 2a, “I Get It Now”

Since Lauryn did not understand her directions, Natalie gestures for Lauryn to bring the green tape end over, and Lauryn hands it to her <16>. Natalie then struggles to implement her plan herself (closer description of this event can be found in the previous chapter, “How did you decide on this green thing”). She puts the end of the tape at the fold that she is holding <17>, then hesitates, before pulling it back past her until it is folded at the vertex that Tahir is holding <18-19>.

At this point I come over to see what they are doing. Both Tahir and Lauryn are quick to say that they do not know. Natalie tells me, “Now we’re going- out the- point- five. So we already have the one that’s within?” [347, 349]. As she says this, Tahir exclaims, “I get it!” [350], then “It’s, yeah I get it I get how that’s, one point five” [353]. Lauryn agrees that she gets it too. I state my understanding of the 1.5, “[So that’s one ((Points with her hands at the center of dilation and vertex)), and then that’s point five ((Points with her hands at vertex and other end of green tape))]? [And so that’s how you

know your dilation?” [357]. Natalie agrees, and then narrates the end of the procedure, “and then we need, this is our vertex corresponding to THAT one” [358].

[342] Natalie: Like bring this (hh) over (hh)here. ((<16> Lauryn hands Natalie her tape end.)) [And then like, ((<17> Puts the end at the end of her folded half.))

[343] Lauryn: [So it’s the same length as that.

[344] Natalie: Yah. And so, wait. We need to, ((<18> Pulls the green end behind her until the tape is folded at the vertex.)) pull it until, it’s right, <19> there, and hold it there. (Like,).



[345] Jasmine: So you’re re-measuring now? What’s happening?

[346] Tahir: [(I don’t know.)

[347] Natalie: [Now we’re=

[348] Lauryn: =I’m not sure.=

[349] Natalie: =going- out the- point- ((Tears the excess off the green tape.)) five. So we [already have the one that’s within?

[350] Tahir: [I get it!

[351] Natalie: [And then if we, continue, so,

[352] Lauryn: [Oh I get it. [Yeah.

[353] Tahir: [It’s, yeah I get it I get how that’s, [one point five

[354] Natalie: [If someone wants to- [carry THAT back, out?

[355] Lauryn: This way?

[356] Natalie: Yeah. To there. Yeah.

Figure 5-10. Excerpt 2b, “I Get It Now”

**Excerpt 2 discussion.** In this excerpt, Lauryn’s manipulation of the green tape is not what Natalie expects or wants. Natalie does it herself. Unlike at the beginning of excerpt 1, when Tahir revoiced Natalie’s instruction for Lauryn [263], the spatial configuration of this task is such that it is not difficult for Natalie to take it over. Although, in this case, Natalie took away one way for Lauryn to engage in the task, by doing so she provided feedback to Lauryn about her current understandings. She was able to do so because of the visual availability of Lauryn’s actions. Like in the “So the Middle’s Here” episode, the large scale physical space, whole body engagements, and distributed perspective of WSG give students additional resources for assessing each other in the midst of problem solving, and the division of labor often makes it necessary to give each other feedback in order for the activity to progress. Even in situations when there is not a more knowledgeable or leading group member, these properties of WSG could allow students to assess their own progress. For example, in “How you wanna do it?” Kaitlin realized something must be wrong, since her tape ends did not connect.

This episode provides a good example of how students with differing understandings of the mathematics can still participate in the task. As they do so they have more opportunities to participate more centrally in mathematically significant ways. Lauryn and Tahir attended intently to what Natalie said and did, and followed Natalie’s instructions carefully. As they progressed, the group continued to use gestures and the language of the materials rather than the mathematical referents while negotiating and coordinating their actions.

However, as the procedure played out, as Lauryn and Tahir continued to engage physically with the materials, and as the group continued to talk about the strategy and

carrying it out [350, 352], the two finally caught on. There is evidence for this as the group proceeds with the task. For the next vertex, all three of them co-narrated the process as they implemented it together, and as they completed the dilation, there was no more disagreement or trouble. The three group members, at differing levels of understanding of the plan, all had access to participating in the task, which led to increased and more mathematically engaged levels of participation as the activity progressed. This is not to say that Lauryn, Tahir, or even Natalie understood why the dilation strategy worked. This could be a question for future design of WSG tasks or of parallel classroom instruction that leverages this kind of WSG solution.

Lauryn and Tahir's trajectory of engagement in this task could be described as intent peripheral participation (Lave & Wenger, 1991; Rogoff, Paradise, Arauz, Correa-Chavez, & Angelillo, 2003). They were not yet central participants in the problem solution because they could not anticipate or direct the whole process. However, they participated to the extent that they could, intently observing and attending to the proceedings, with the intention of later being able to be more central participants. The WSG tasks supported this kind of intent, peripheral participation because of the disruptions in tools and division of labor. The everyday materials of WSG were constantly mathematical tools-in-the-making, as determined by the activity of the group. As the students shared a local history of engaging in the tasks together, the materials' mathematical meanings or uses may have become more stabilized and tacit, which may have served as an impediment to developing new understandings (see "What's This Green Thing For?" episode in the next chapter). However, as students negotiated meanings and uses for the materials, even when just one student was leading the problem solution (like Natalie, in this case),



the others could, and because of the division of labor, had to, still manipulate their bodies and the materials in the context of the mathematical activity, and thus had some access to participation.

Additionally, the disruptions in space, tools, and perspectives also provided affordances for ongoing assessment, while the division of labor makes giving feedback to each other worthwhile. The large scale spatial configurations, interconnected whole body and material manipulations, and distributed perspective of WSG could allow students visual and physical access to each other's actions. When these actions were counter to a student's idea of the solution strategy, or prevented a student from completing his or her own action, there was a clear indicator of divergent understandings. In this episode and in "So the Middle's Here," Natalie could see and feel Lauryn's actions, easily identifying when they were misaligned with her dilation plan. In "What We Needed the Yardsticks For," the visual availability of the students' actions and the physical actions and reactions in the service of keeping the triangle intact supported the convergence of an idea. Because of the need for multiple bodies to complete the WSG tasks, it was in the best interest of students trying to complete the tasks to give each other feedback and continue to negotiate their understandings.

In this episode, Lauryn and Tahir tried to understand what Natalie wanted to do, and they followed along with her instructions, actively making sense of their own actions and the developing scaling routine. In addition to moving their bodies and manipulating materials according to Natalie's directions, Lauryn and Tahir also tried to anticipate, restate, or ask about the unfolding strategy. These modes of engagement all supported

Lauryn and Tahir's eventual shared understanding of Natalie's strategy for scaling the distance from center of dilation to vertex, and for her dilation strategy.

However, not all students on the periphery might choose to participate in this way, and task and group structures may not always allow for it. Kimberly, in the "How You Wanna Do This?" episode, was content to stand aside and watch the other four students make the rectangle. When she was instructed by a demanding Kaitlin to come help, she did, but as soon as Kaitlin realized they did not need a fifth person Kimberly was summarily dismissed. She spent the rest of the task standing off to the side. It is possible that, given just four students in the group, this may not have happened. It is also possible that with just four students in the group Kaitlin might have found another way to ostracize Kimberly and impede her participation. Yet another possibility is that Kimberly's (or anyone else's) participation was productive for accomplishing the task, but she only executed the physical movements without making mathematical meaning of what she and her group are doing. These are issues to be addressed further in task implementation and instruction that develops norms for participation, or affordances or constraints that supports participation. What this episode demonstrates is that the disruptions of the WSG context can support this kind of engagement, and it is not uncommon for it to occur.

For the next episode, I revisit "Each Person Holds Two Ropes" from the previous chapter. For the purposes of discussing opportunities to learn, in this rendition I describe how the use of bodies as mathematical representations provides access for the students to make sense of the instructions they were given.

## **Coordinating Bodies and Mathematics: “Each Person Holds Two Ropes” (KCMS, Day 7)**

This episode was previously described in Chapter Four, starting on page 111. In this episode, Felicia, Kaitlin, Kimberly, and Marla are drawing an isosceles triangle based on an instructional video made by another group of students. Felicia and Kimberly argue that they only need three ropes in order to make their isosceles triangle, while Marla and Kaitlin seem convinced that they need six, since the instructions told them that “each person holds two.”

**Abbreviated episode description.** Kimberly, then Kimberly and Felicia challenge Kaitlin and others in the group. In the beginning, Kimberly raises the challenge (“How many triangles are we MAKing?” [3], but when Kaitlin dismisses the question after a few exchanges, Kimberly goes along with the group. Later, after the group finishes measuring all six lengths of rope, Felicia revives the challenge (“Wha? We have to grab two for each?” [27]). Kaitlin and Marla ignore her for a few turns at talk, but Kaitlin herself hesitates, saying that “We gotta picture their thing again” [32]. When Marla articulates her vision of how they would use the six pieces of rope, that “You hold them together” [37] because the instructions said “Two ropes- for each side” [39], Kaitlin remembers that the instructions actually said “each person holds two ropes” [44]. Felicia takes up this language, showing how, if they follow Marla’s idea, she would be holding four ropes, not two [45, 47, 50]. Kaitlin then decides that the base should only be one rope, and the others agree [52-60].

Kaitlin behaves as though the issue was settled and directs the others to put the extra base back in the bag. Felicia continues to question their plan though, making repeated bids for the floor, so that she can tell them, “I think all of them is one” [61, 63,

67]. Kimberly agrees. The two of them assemble increasingly complex demonstrations of people standing at vertices holding two ropes each, one on each side, to form a triangle. They do this by holding real and imaginary ropes in their hands out to their sides and tracing the sides of triangles in the air. Finally, they communicate the idea of rope ends [81, 83] and the fact that two people would hold the same rope [83, 85]. Felicia then simulates the WSG triangle by animating one person-as-vertex, and recruiting Kimberly and Marla as the other two. It is at this point that Kaitlin and Marla agree, and the conflict is resolved.

**Episode discussion.** The group developed agreement on the mathematization of bodies and ropes over the course of these interactions. The salient point in the instructions that Kaitlin and Marla remembered was that each person would hold two ropes. They also remembered that the “legs” of the isosceles triangle would be thirty feet each, and the base would be twenty feet. However, if each length of rope is understood to be one side, then six ropes were clearly too many to make a triangle. Marla’s interpretation of needing six ropes was that each side would be made of two ropes, held overlapping. For Kaitlin, however, the lengths of ropes were not yet parts of a triangle, just line segments with specific measurements. As she began to treat them as triangle sides (“Who wants to hold the base piece?” [29]), she hesitated (“Hold on. We gotta picture their thing again” [32]), and recognized the problem. For Kaitlin, the rope progressed from tangled materials to measured line segment to triangle side, and as she dealt with each meaning for the rope new goals and problems emerged.

In the same way that the division of labor requirements of WSG supported the revisions of explanation in “Four Points Over There,” the same requirements supported

the sustained exchange in this episode. The students' divergent understandings were made available by their need to coordinate their actions and manipulations of the materials, and resolution was sought so that they could complete the task.

Aside from the constraints of the division of labor that supported the negotiation of shared understandings among group members in WSG, the openness of possibilities for students to mathematize materials and bodies and to communicate about these mathematical meanings served as a resource. In "I Get It Now" I described how the everyday understandings of materials, in coordination with their mathematical relevancies for the solution, provided access for Lauryn and Tahir to participate in the dilation strategy. Because Lauryn and Tahir could respond to Natalie's directives in terms of everyday language and materials, they were able to intently participate in the problem solving process, and had the opportunity to become more central participants as they progressed.

Here, I argue, in addition to uses of everyday materials, it was the flexibility with which students recruited and interpreted their own and others' bodies mathematically that provided access to participation. For example, as Kimberly's holding two ropes gesture was adopted and repeatedly deployed by members of the group, they placed themselves in the imagined triangle as vertices holding ropes. Even without a full picture of the figure and its interconnected components, this mathematization of the body allowed all students to be active participants in the (future) solution, in a mathematically consequential way. Even when Marla was still in disagreement with Felicia and Kimberly, she could still be recruited as the third vertex of Felicia's imagined triangle, standing and holding two ropes, one in each hand. At the same time it gave students a familiar material resource with which to reason mathematically.

Students' bodies did not need to be recruited as representatives of mathematical objects to have access to participation in a WSG task. In "We Can't Lose Dean's Height," the students used Dean's height as the unit of measure for their line segment. Harry made repeated bids to become a new, longer unit of measure, since he was taller and having a longer piece of rope would enhance their play. Even though the group did not agree to change the unit of measure to Harry's height, his body was a resource for their reasoning about the length of their line segment, and positioned him as a participant in their drawing activity in a mathematically significant way.

### **Summary**

These three episodes illustrate some of the ways that the WSG context supports opportunities to learn for a heterogeneous group of students. The disruptions associated with WSG created affordances for multiple entry points and different forms of participation (Cohen, 1994, Lotan, 2003). The disruptions of space, tools, and division of labor provide a configuration of engagement that supports students' participation in developing and making sense of solutions strategies. In "Four Points Over There," Lauryn and Tahir's contributions to the development of the dilation idea and explanation, and Natalie's responses to them, depended heavily on existing material uses, projected future manipulations, spatial layouts, multiple scales, and classroom versions of the procedure.

In the execution of a strategy, material and physical manipulations, in combination with the interconnected multi-party property of the inscriptional system allowed all students in the group to participate mathematically significant (but more or less peripheral) ways, even if they were not yet all in agreement about the solution. The visual and

physical availability of the representational space provided a resource for students to engage intently, receive different forms of feedback (visual and haptic), and eventually participate in more central ways, taking the lead on parts of the solution.

Finally, that bodies were used as tools for mathematics positioned students to be participants in problem solving not just from an extrinsic, third person point of view, but as a mathematical entity. Students could participate as part of the representational system. In “Each Person Holds Two Ropes” this was a resource for Kaitlin and Marla to imagine future body and material configurations. In “How You Wanna Do It?” and “What We Needed the Yardsticks For” students’ positions inside the figures gave them access to visual and haptic feedback otherwise unavailable. Incorporating the body as mathematically meaningful also provided students an additional resource, of which they have had a lifetime of knowledge and experiences, with which to reason.

## CHAPTER VI

### DISCUSSION

The change in scale of WSG tasks introduces disruptions that expand the set of available resources and opportunities to participate for learners. I use “available” here in the sense that they are recognizable as relevant to students within their problem solving context. Available resources may include shared local histories of mathematical sense-making, visible or tangible material artifacts, and norms for negotiating shared understanding. They may also include individual histories of sense-making, experiences with and understandings of material artifacts, and ways of negotiating understanding. The disruptions in space, tools, perspective, and division of labor promote and support the recruitment and coordination of shared histories of learning as well as individual trajectories of participation within and across learning contexts.

In this chapter I summarize the findings from the previous two chapters with respect to the four disruptions of WSG. I then provide a brief, informal comparison between the KCMS and SEC contexts, raising some questions for future design and study. Finally, I discuss more thoroughly some design considerations both for the WSG setting and the parallel classroom setting, based on findings presented in this dissertation. I begin by acknowledging some practical complications related to implementing WSG in schools, then make three proposals for future design. Finally, I also make a proposal for an additional principle for designing for PDE (Engle, 2011; Engle & Conant, 2002).



## **Disruptions, Engagements, and Opportunities to Learn**

In this section, I will summarize findings from the previous two chapters organized according to the disruptions of WSG. I will address both of the research questions, and discuss how the disruptions support problem solving and access to participation in WSG. While the four disruptions of WSG seldom stand alone in affecting participation and engagement, they are often significant enough that it is not unreasonable to consider them individually. When it is clear that it is a combination of disruptions that results in a particular property of engagement or aspect of participation, I say so explicitly.

### **Space**

In shifting the spaces of mathematical activity from inside classrooms to large, open spaces, students' uses of their bodies, their interactions with each other, and their interpretation of materials become situated in a different place where they have histories of a broad variety of activities and experiences. The outdoors is a space where bodies are free and unconstrained (Nespor, 1997); where there is space to move whole bodies; where material manipulations extend beyond peripersonal space (in grasping distance); and where students have participated in largely non-classroom-related, non-mathematical experiences. In both of the KCMS and SEC sites students used the outdoor space in ways atypical of mathematics class activities, running, jumping, dancing, throwing, and yelling. Almost always, these engagements were constrained (and afforded) by the emerging goals of students' mathematical problem solving. Ben, Dean, Eddy, and Harry's play in "We Can't Lose Dean's Height" depended on maintaining the length of the line segment that they had measured to be twice Dean's height, then later on producing and maintain-

ing straightness for their photograph. At times, these engagements were recruited as resources for mathematical activity and problem solving. The group's play around the idea of breaking their triangle in "What We Needed the Yardsticks For," but the need to keep the triangle intact for Jasmine's on the ground photograph led to Eddy's breaking the triangle by stabbing it, which transformed into the discovery of a new mathematical use for yardsticks.

These moments of play or exploiting the outdoor space did not usually lead to such direct instances of mathematical reasoning, but they did provide students experiences with their bodies, the materials, and each other in the context of (and therefore with the constraints and affordances of the goals of) mathematics problem solving that could be recruited as resources. Students were often observed running, dancing, laying down, and twirling, whipping, and throwing materials, individually and together. In typical classroom settings, like that of KCMS, students' mathematical engagements were typically bounded as activity at desks, in seated positions, using their eyes and upper bodies (arms and hands), with representations and materials within reach. The whole body, non-classroom engagements of WSG, infused with and by students' mathematical activities, position students to make new and different meanings of their current mathematical engagements. For example, later during the task involving Ben, Dean, Dwayne, Eddy, and Harry that was described in the introduction, Dean introduced (and Dwayne eventually co-narrated) a baseball game in which a player (Dean standing at home plate), runs around their large scale rope quadrilateral, from home to first ("There's Harry") to second ("There's Eddy") to third ("where Ben is") and back to home, after which Dwayne agreed that their quadrilateral qualified as a diamond.

These engagements can also be viewed as impediments to learning, or “off task” activities. The analysis here shows that the students may not have been directly engaged in problem solving, but their play was certainly *within* task, if not qualified as *on* task. The group’s mathematical activity was not temporarily suspended or set aside while they waited and played. Instead play incorporated the group’s shared history of mathematizing the materials. That they could repurpose the ropes and their bodies to maintain relevant mathematical properties while pursuing other goals could be viewed as evidence of (or opportunities to engage in) mathematical flexibility. In the case of the yardstick innovation, students took up an opportunity to continue their mathematical reasoning even after the task had been completed.

It is interesting to note that this kind of play occurred more often in the KCMS setting than during SEC. It is not surprising, given the SEC students’ positive school and school mathematics identities. A question to be further investigated is whether this may provide fewer learning opportunities for SEC students, or whether different kinds of tasks might promote play. However, the shift in the space should still be considered to be a productive disruption for these students. The whole bodied, large scale affordances of this disruption supported engagements with the materials not possible in the classroom (including changes in distributions of materials and labor, discussed subsequently).

## **Tools**

The WSG disruption in tools from typical mathematics classroom tools like pencil and paper, rulers, and protractors reconfigures the representational infrastructure of geometry and provides students opportunities to invent new representational tools and prac-

tices. Drawing with everyday materials like ropes, tape, flags, and bodies has different properties than pen-like inscriptional tools. Instead of moving an object to leave a trace behind, WSG materials have pre-existing qualities like length, straight- or curved-ness, locations in space, and connective relations to each other. Students need to make mathematical sense of these together in their groups in order to accomplish their tasks. In doing so, they invent ways of using and manipulating the materials. Inventing and exploring representational possibilities provides both conceptual agency and rich opportunities for mathematical meaning making for students (Enyedy, 2005; Greeno & Hall, 1997; Hall & Greeno, 2008; Nemirovsky, Tierney, & Wright, 1998). Additionally, the everyday materials of WSG provide opportunities for students to make mathematical sense of objects with which they have previous, non-schooled experiences.

This disruption was problematic for students, but in productive ways. In “Each Person Holds Two Ropes” Kaitlin and Marla struggled to make connections between how each of three people could hold two ropes and how the resulting triangle could be composed of only three ropes. The group’s initial activities of preparing the materials had assumed that there would be six ropes, and they had measured and cut that many. It is unlikely, given instructions to draw an isosceles triangle with specific side measurements using paper, pencil, and a ruler, that they would have produced six sides. At the same time, the disruption of using ropes to draw edges provided an opportunity for students to explore the geometric property of figures relating the number of vertices and edges. In the WSG context of drawing an isosceles triangle, the group came to consensus on this relationship.

The possibility for the materials to be reused and re-mathematized, even in forms that maintained previously established mathematical properties, also became problematic for students. In “How Did You Decide on this Green Thing,” the previously measured and cut green tape representing 1.5 times the side of the original quadrilateral provided resources for students to use it in ways incongruent with the new strategy of scaling the distance from the center of dilation to the vertex. While this did not turn out to be fatal for the problem solving efforts of Lauryn, Natalie, and Tahir, it is an important consideration for WSG design efforts. In this episode it was the re-mathematization of an everyday object that provoked trouble. Tacit assumptions about, and the shared local history of, that object was an obstacle for Lauryn and Tahir’s understandings to converge with Natalie’s, and tripped Natalie up in the implementation of her own plan. At the same time, tacit assumptions about (and experiences with) everyday materials, as well as developing shared local histories with them is a major aspect of my design conjectures and findings for how students engage in the WSG context. These assumptions and histories of use can, and often do, become resources for problem solving and access to participation. It is still an open question as to how WSG tasks and accompanying instruction can be designed to both leverage this property but prevent trouble that cannot be overcome.

The disruption to tools also provided opportunities for students to participate in problem solving, both at the individual and group levels. The possibility for Dean and Harry to use their bodies as units of length measure allowed them to play active roles as mathematical objects, and for the group to have familiar entities with which to reason. Similarly, bodies as mathematical representation and inscription devices (bodies as vertices) provided students with new experiences within geometric figures, supporting under-

standings of relations between parts of those figures. Lastly, the properties of the everyday materials and students' experiences with them provided resources for manipulating them for the purposes of problem solving. Students folded ropes to divide and multiply lengths, stretched them taut to produce straightness, and bent or rotated them to produce angles.

### **Perspective**

The WSG context disrupts perspective in two related ways. Students' views are from within the geometric figures, so that what they see is significantly different from the typical bird's-eye-view of mathematics, and common geometry objects are more difficult to identify (e.g., acute and obtuse angles). Students' views are also different from each other, depending on where they are standing in relation to the figure.

The difficulty in coordinating these unfamiliar and disparate views both presents students with problems for negotiating their understandings (individually and with others), but also supports the need to do so. For example, in the episode described in the introduction, Dwayne insisted that, from where he was standing, the rectangle they had made already looked like a diamond (parallelogram with no right angles). In the end, Dwayne had Dean take over his vertex so that he could walk around their quadrilateral and look at it from different perspectives. During tasks, it was not unusual for students to report to the group what they saw, or ask each other what kinds of angles they were holding, in addition to walking around to get different views of the figure. In "How You Wanna Do It?", when Kaitlin announced that the sides of the quadrilateral at her vertex

“don’t connect,” the other three students promptly responded with the status of their respective vertices.

The unfamiliar and disparate views not only make it difficult to identify normally familiar geometric objects, and difficult to negotiate understandings, but the resulting unavailability of the whole figure hides geometric relationships between parts. In “Each Person Holds Two Ropes,” Kaitlin and Marla’s developing plans to use six ropes to draw their isosceles triangle persisted in part because of their (anticipated) positions as vertices, and the resulting partial view. While this can become an unsurmountable obstacle for student learning, it also makes salient some tacitly understood geometric relationships that are usually hidden by bird’s-eye-view perspectives.

Not only does the relationship between the number of sides and vertices in a polygon become important in WSG activity but also how those sides and vertices can fit together. Additionally, students’ intrinsic perspectives on vertices and sides provide new opportunities for understanding straightness and angles. While making a line segment, looking down the length of the segment from the endpoint highlights what Henderson and Taimina (2005) call rigid-motion-along-itself symmetry (“Any portion of a line may be moved along the line without leaving the line,” p. 18) and 3-dimensional rotation symmetry (“In a 3-dimensional space, rotate the line around itself through *any* angle using itself as an axis,” p. 19).

Standing at a vertex holding a polygon side in each hand highlights the rotational and directional properties of angles. When drawing on pencil and paper, angle measures can be adjusted by erasing and re-drawing at least one of the sides of the angle. For a stu-

dent with an angle side in each hand, opening and closing one or both arms changes the angle measure.

The highlighting of these normally tacit geometric relationships and properties may be beneficial for supporting the learning of typically taught geometry content, or may be important in and of themselves. I will address more of this question later in this chapter.

### **Division of Labor**

The disruption to division of labor in WSG supports students' negotiations of shared understanding. Because WSG tasks generally require more than one person to solve the problems, all students in a group must coordinate their ideas so that, together, they can complete the tasks. In many of the episodes presented in this dissertation students engaged in sustained exchanges about what they should do, and why (e.g., "We Can't Lose Dean's Height," "Four Points Over There," and "Each Person Holds Two Ropes"). I do not mean to imply that students understand the task and solution in the same way once they are finished; rather, they share an understanding of the task and solution to the extent that they can jointly coordinate to achieve their problem solving goals.

This has significant consequences for access to participation. It is more difficult for students to be left behind by others who arrive at answers faster, since everyone needs to be included. In "Four Points Over There" and "I Get it Now," even though Lauryn and Tahir had very partial understandings of Natalie's dilation strategy, she was compelled to continue revising her explanation and to give versions of instructions that Lauryn and



Tahir could follow. This gave them an opportunity to participate in nontrivial ways, and they eventually were able to anticipate and take the lead on other parts of the plan.

This division of labor, in combination with the large spaces and the tools of WSG, distributes the activity across bodies and materials while at the same time connecting them physically. This physical connection provides an important resource for students to develop their own thinking. In “What We Needed the Yardsticks For,” the interconnectedness of the materials made other students’ actions consequential for Eddy’s own activity. As he adjusted his own body and the way he held the rope, in response to others’ movements, and in the service of maintaining the triangle with his yardstick holding up his vertex, he significantly contributed to the yardstick innovation.

The physical connection also allows students to have access to each other’s thinking. In “So the Middle’s Here,” Lauryn, Natalie, and Tahir were all alerted to their differing plans for producing 1.5 times the side of the quadrilateral when Natalie and Tahir both stood up, allowing their line segment to blow away in the wind.

The interconnectedness was a resource not just for problem solving, but also for students’ opportunities to engage in WSG tasks in a variety of ways, and supported learning. For example, in “So the Middle’s Here” the connected manipulations of the green tape not only made the three students’ divergent strategies for scaling the line segment available to each other, but also supported a need for renegotiation of their plan.

### **Comparisons Between KCMS and SEC**

Although different tasks were implemented in KCMS and SEC, and there was little classroom instruction in SEC, having both of the contexts provides valuable com-

parisons to be made between them. My primary motivation for conceptualizing the WSG setting as a designed disruption for productive hybridity was to contribute to work on supporting *all* students' mathematics learning. The data from KCMS and SEC allows me to compare, informally, how these two different groups of students engaged in WSG tasks. In this section I also comment on some further questions that arise as a result of this comparison.

In general, SEC students recruited more classroom geometry knowledge, and more often, than did KCMS students. This is not surprising, since the older SEC students had all had more geometry coursework than the KCMS students, and had more positive mathematical identities. This difference, however, did not hinder the participation of either group, in the sense that both groups still engaged in the tasks in mathematical ways, and the tasks were not trivial for either group. However, in the SEC context, it remains a question as to whether or how design and instruction can better support students' learning about mathematical relationships when they apply procedures remembered from the classroom context to WSG tasks. For example, in the case of Lauryn, Natalie, and Tahir's dilation, I have argued that all three students had access to participation in the task, and they all engaged in mathematically significant ways. However, we do not have evidence that they have considered the mathematical relationships that allow this procedure to work. How can design be revised to support attention to these relationships, either in the WSG setting or in the classroom? It is also worth future systematic investigation to see if the recruitment of "too much" classroom mathematical knowledge in the WSG setting can be detrimental to engagement or access to participation, or can somehow override the disruptions.

Another unsurprising difference in engagement was that it was much less likely for SEC students to disengage, even when a student did not have a specific role in ongoing activity. Students remained nearby, listened and watched, asked questions and gave suggestions, and made bids for additional responsibilities. This was not always true in KCMS. In “How You Wanna Do It?” Kimberly allowed herself to be brushed aside. Although she remained close by and appeared to be monitoring the group’s activity, she did not make bids for increased participation. Had I been able to, I would have kept the group sizes to four students for that task, but given the practical realities of schooling, choosing group sizes is not always possible. How else can WSG tasks be designed to support the engagement of all students within a group? Especially given the nature of the setting, where students are expected to invent strategies, it is often difficult to predict if someone might be left out.

In a related difference, students in SEC were much less likely to engage in play, either for extended periods of time (like Ben, Dean, Eddy, and Harry did) or even in fleeting moments. While they waited for their photographs, for example, Lauryn, Natalie, and Tahir familiarized themselves with the next task, prepared their materials, or occasionally just stood quietly. While this is not necessarily a concern with respect to these students’ opportunities to learn, the informal, carefree experience of the outdoor space and everyday materials in combination with doing mathematics may be a resource for learning that these students are missing out on.

## **Design Considerations**

In this chapter I have summarized different ways in which students engaged in WSG geometry problem solving in both the KCMS and SEC settings, and different forms of access to participation. The disruptions of space, tools, perspective, and division of labor denied students access to typical ways of doing and learning geometry. At the same time, they invited and supported unique ways of engaging in problem solving, made available or relevant particular kinds of resources, and sometimes highlighted typically hidden mathematical concepts. In this section I consider some of the pragmatic obstacles of implementing the WSG setting in schools, then make some suggestions for future WSG design and classroom instructional design that could leverage students' WSG experiences. I do this by describing one attempt at an approximation of WSG that I implemented in KCMS, proposing a design of WSG as "ensemble mathematics," and discussing how mathematics concepts might be made general across the WSG setting and the classroom.

### **The Pragmatics of Walking Scale Geometry**

WSG as a learning context has a lot of potential for providing more resources for mathematical sense-making for more students. However, the disruptions associated with WSG are disruptions to aspects of typical classroom mathematics activity that are, both pragmatically, and disciplinarily, difficult to disrupt. The space, tools, perspective, and division of labor properties of typical classroom mathematics have been developed and sedimented over time for various reasons. There are many reasons for resistance to making changes (Tyack & Tobin, 1994). While WSG is not attempting to reform classroom

mathematics in any way, it is creating a setting that is part of classroom mathematics that disrupts those aspects in major ways. In this section, I discuss some of the practical difficulties of implementing WSG. I include this section both to acknowledge pragmatic considerations of the design, but also to raise questions about the commitments of typical classroom mathematics instruction.

One of the major requirements of WSG is a large amount of flat space, outside the classroom, preferably outside the building. In plenty of places (New York City, for example), this is difficult to come by, especially an area close enough to the school so that students can transition between it and the classroom without spending too much valuable instructional time. If such a space is available and close by, the logistics of being able to go there and use it during planned times must align. Considerations include access to the space, weather, having enough time, and having permission to leave the classroom. Students not only need access to the space, but also that it be clear enough of other activity that they can engage in the tasks without their work being disturbed. The weather is consequential not just because it may prevent students from being outside (e.g., rain and tornadoes), but students must also be able to engage in the physical activity of WSG tasks. For example, the heat and humidity encountered during the KCMS and SEC studies was a challenge for students and adults, alike. As for time, the effort of gathering materials, walking to the outdoor space, and setting up to begin a task, then cleaning up materials and walking back to the classroom, was significant in relation to the typical 70 minute class period at KCMS. Finally, as mentioned in Chapter Three, KCMS regularly experienced lock-downs, where students, teachers, and staff could neither enter nor exit the building. These factors were significant enough for me at KCMS that my plans for

10 days of WSG were reduced to only six outdoor days. In SEC, where I had much more freedom, having access to a large university campus and the luxury of the summer program's rules, it was much easier to go outside for WSG (although we barely just avoided a major thunderstorm).

Once outside, because of the large scale representational space of WSG, talk, gesture, and material resources (e.g., inscriptions on paper or a whiteboard) are no longer available as easily shared. Students are often far away from each other within a group, and there is almost no talk between groups. The teacher can generally only attend to one group at a time, and rarely overhears what other groups are doing or can detect inappropriate behavior within other groups. The distance between groups makes it difficult to rotate between them during a single session and keep up with what each group is doing, the problems they encounter, and the discussions they have. The ability to stop everyone during a task and bring them all together to address any common, developing questions or issues is also much more difficult.

That there are no small scale shareable (visually and gesturally) mathematical inscriptions available that students and teacher can talk and reason over is significant both for student engagement and instructional strategies. As discussed in the previous section, the division of labor and disparate perspectives of WSG can be a resource for learning and for opportunities to participate. At the same time, inscriptions are a foundational aspect to mathematics and scientific practices, with which practitioners (re)present and communicate information, make arguments, and convince others (Latour, 1990). Not only are they useful for these purposes, but learning to create and use them is an important aspect of mathematics learning.

These obstacles highlight two aspects of schooling and classroom instruction, other than the four discussed throughout this dissertation, that WSG disrupts. First, the heavily disciplined version of students' bodies in schools and in classrooms that makes it difficult to leave the classroom and that constrains disciplinary engagements to small scale activities largely confined to the modalities of peripersonal space denies the wide range of experiences and resources that could be available to students (Hall, 1996; Hall & Nemirovsky, 2008) and what often constitutes their preferred modes of interacting in the world (Nespor, 1997). The WSG tasks, as implemented in KCMS and in SEC, allowed students to participate in geometry problem solving with their whole bodies, intermixed with engagements that can be classified as play or other "off-task" behavior, yet still productive for learning.

Second, it is worth questioning the assumptions behind what is made problematic by the distribution of WSG across large scale space. Keeping students all together, in view of the teacher, with all of their activity available to teachers (mathematically oriented and otherwise), assumes responsibilities and motivations for both teachers and students. It also assumes students need to be managed and policed, or they will likely behave inappropriately and avoid any form of disciplinarily productive participation. If somehow they are compelled to engage in mathematical activity, teachers had better be there to catch it, or it will be lost for any possibilities of future instruction. I do not mean to belittle teachers' roles in the learning of students, or many forms of instruction that I believe to be highly effective (and awe-inspiring). Making student thinking visible, making sense of student thinking, being able to coordinate and orchestrate the developing thinking of a class full of students, posing timely questions, and producing carefully crafted goals and

subgoals (in the midst of problem solving activity) are all important, and critical aspects to high quality instruction. It remains an open question, and one for continued design and research efforts, how students can be supported in participating more actively in (and contributing to) these forms of instruction.

All of these obstacles to implementing WSG are also important for making design decisions about when and how to use WSG tasks as a part of regular classroom instruction. Given the obstacles to implementation, when are WSG tasks the most worthwhile, and how can they provide the most leverage for classroom teaching and learning? In my conceptualization of designing disruptions for productive hybridity, the WSG tasks constitute a mediating setting that supports students' recruitment of novel resources for geometry problem solving and makes available new forms of engagement and access to participation in problem solving activity. Next, I discuss some design implications based on these pragmatic concerns and the findings from the previous two chapters and summarized above.

### **Materials and Mathematics to Support Productive Hybridity**

One major requirement of WSG problem solving is for students to develop mathematical meanings for everyday materials and bodies in the context of the task at hand. These meanings sometimes carry over across tasks (e.g., ropes as line segments), but may also shift deliberately (e.g., the tape that was a scaled version of a quadrilateral side is now a measuring device) or tacitly (e.g., rope ends also become points of intersection that become triangle vertices). These shifts in meanings can become problematic in the WSG task setting, especially when not all students are aware of them. In the classroom setting,



removed in space and time from the WSG context, they may be even more difficult to capture and leverage for hybridity and learning. For example, how can Natalie's method of scaling a quadrilateral side 1.5 times be shared in a classroom setting in a way that is meaningful for all members of the class, when it was difficult enough to explain it in the WSG context?

At the same time, it is exactly this property of WSG that provides students with opportunities to take up conceptual agency. That students are invited to animate everyday objects as mathematical entities and invent representational tools and strategies supports them in making their own sense of mathematical relationships and properties. So then it is important to design ways that these meanings can be made durable enough so that they are still available and robust in classroom activity.

One possibility is to develop tools that approximate the WSG context but that are importable into the classroom. My solution to this problem at KCMS and SEC was to have photographs taken from different intrinsic perspectives and from above, to have multiple views of in progress and completed task solutions. However, the two dimensional quality of these photographs, the distortions resulting from being taken not-quite-directly-above, and the difficulty recovering what students were doing in in-progress images made them poor representations of students' engagement with the materials and related mathematical reasoning.

On the day after the final day of WSG in KCMS, I tried a miniature version of WSG, which I called "Desktop Geometry." With string, eyelet screws, tape, and any other materials they wanted to use, along with sheets of thick cardboard as a drawing surface, I asked students to engage in WSG-like tasks in pairs (Figure 6-1). This activity main-

tained the disruption of tools of WSG, but did not incorporate those of space, perspective, or division of labor. Occasionally students' strategies required more than two hands, but more often than not a second set of hands got in the way.

a)

Use your desktop geometry kit to do the following. You may use any of the outdoor geometry materials, and anything else in the classroom (except protractors and rulers).

- A. Make an isosceles triangle. Make a line segment, in a different color, from the top angle to the middle of the base.
- B. Make another isosceles triangle. This triangle should have the same angle measures as A, but with sides twice as long. Make a line segment, in a different color, from the top angle to the middle of the base.
- C. Make another isosceles triangle. This triangle should have the same angle measures as A, but with sides half as long. Make a line segment, in a different color, from the top angle to the middle of the base.
- D. Check another group's triangles. Ms. H will assign the groups. Are the angles all the same size? Are the sides the lengths they should be?

b)



Figure 6-1. Desktop Geometry a) The Desktop Geometry task. b) The Desktop Geometry solutions of Bianca, Felicia, and Kaitlin (left), and Dean and Eddy (right).

These tasks did bridge between the classroom and WSG, but also highlighted the importance of the interplay between all the disruptions. It also became clear that this desktop version of WSG had practical properties of its own that could be problematic (or, possibly, leveraged) for learning (e.g., tying knots was difficult, and Dean and Eddy struggled to get their line segments straight and tie the knots tightly).

While Desktop Geometry, at least in the form that I implemented it, was not an ideal solution for leveraging students' experiences of mathematizing everyday materials in WSG, I believe that using an intermediate scale and maintaining many of the everyday materials has potential to be a good replacement or supplement for the photographs, either in classroom discussion following WSG experiences or for classroom tasks.

In general, the design of other forms of approximating the WSG setting to capture the resources made available through mathematizing everyday materials within the classroom setting could support bridging the WSG experience with classroom learning to support hybridity in the classroom. This may include abbreviated demonstrations or performances of WSG drawing, which I will discuss in the next section.

### **Ensemble Mathematics for Future Iterations of Walking Scale Geometry**

There has been a steadily growing body of literature concerning embodiment and mathematical learning (Abrahamson, 2009; Enyedy, 2005; Nemirovsky & Ferrara, 2009; Núñez, Edwards, & Filipe Matos, 1999; Perry, Church, & Goldin-Meadow, 1988; Rasmussen, Stephan, & Allen, 2004). This research tends to focus on individual student bodies, and is often limited to gesture. Studies have not yet been conducted investigating how whole bodies, in coordination with each other and other material resources, can be lever-

aged to support mathematical learning. This analysis of the WSG context contributes to the embodied mathematical cognition literature by providing accounts of students solving geometry tasks that involve multi-party, whole-bodied engagements, often interconnected through material manipulations. In this section, I make proposals for how this form of mathematical problem solving can support learning.

In an ethnographic study of competitive high school marching bands, Rogers Hall and I have documented structures of learning that are highly productive for a large group (over 100) of students who have varying levels of marching experience and abilities (Hall & Ma, 2011; Ma & Hall, 2011). Over the course of a marching season (late summer through the end of the fall semester), the students, along with band directors and instructors, worked together to learn a seven minute show, performed on football fields, that would be performed at and ranked favorably in competitive regional and national venues. Three aspects of this rich learning context are relevant to WSG. First, the intergenerational and heterogeneous nature of learning was highly consequential for the success of the group and the identities of marchers. Second, marchers progress from individual concerns of finding their positions (or “dots”) on the field in relation to the timing of the musical score to attending to their coordinations with other marchers to create aesthetically striking forms designed by instructors. Finally, marching successfully was driven by the goal of successfully performing the show repeatedly in a variety of similarly structured contexts.

These three aspects of learning to march point to the ways that WSG can support classroom instruction in a type of mathematics that we have come to think of as “ensemble mathematics.” In ensemble mathematics, a group of students engage together

to accomplish a mathematical task, with the design and performance of a mathematical routine as a subgoal of the activity. In WSG, this routine might be something like scaling a line segment in the context of finding the distance to a new vertex in a polygon dilation strategy, or drawing a quadrilateral. In the analysis presented in this dissertation, I have shown how students with different levels or forms of mathematical sophistication and different mathematical dispositions can still have access to participating in WSG tasks. Additionally, these students can successfully participate together, with each student likely to support the developing thinking of the others. For example, Lauryn, Natalie, and Tahir were all active participants in constructing and producing the dilation strategy that Natalie introduced.

The second two aspects of ensemble mathematics, moving from individual to relational, coordinated engagements with the task, and performance as a subgoal, need to be further explored in the WSG setting. It is clear from my analysis that the relational and coordinated engagements of WSG problem solving has properties that support both problem solving activity and access to participation through the availability of novel sources of feedback. These engagements also have the benefit of highlighting mathematical relationships between the parts of geometric wholes in ways not typically made explicit in classroom geometry activity. In order to successfully complete tasks, students must coordinate their distinct, individual perspectives. In future iterations of this design study, where classroom instruction that leverages students' experiences in the WSG setting is made a more explicit concern of design activities, these coordinations will be further investigated as resources for students' mathematical activity at paper and pencil scale.

Finally, the goal of performance at the end of learning is not new for educational settings. Summative assessments are often called performances, although in mathematics classrooms they rarely involve whole bodies or multiple parties. What are the advantages, for learning geometry, of ensemble performance? What difference does it make for performance to be for the purposes of explanation and argumentation rather than assessment? Admittedly, in the marching band context, performances were assessed in highly public, highly technical arenas, and design and rehearsals of the show were explicitly conducted with the anticipated judgments of competition officials in mind. At the same time, performances were also produced with the goals of assembling a narrative and creating a dramatic experience and response for audiences. In the domain of the performing arts, these are forms of explanation and argument. Learning to perform, then, for the marchers, involved participating in a collective performance, where individual engagements were only successful if they were conducted in coordination with the rest of the band. Students did not just have to find their dots, but to adjust appropriately if the rest of the band was half a step off.

If solutions to (or routines related to) WSG tasks are thought of as performances, students would be asked to be able to solve the “same” task more than once. I qualify “same,” since multi-party whole-bodied engagements are always subject to contingencies. If they were not, then performing a marching band show perfectly time after time would be trivial. For WSG tasks this is mathematically consequential, since embodied movements and manipulations of materials constitute the inscriptional results.

In the design of WSG tasks for this dissertation, students were only asked to perform their problem solutions once, and the results were fixed by the photographs that we

took. In the case of Lauryn, Natalie, and Tahir, where they performed (though not for an external audience) their 1.5 times scaling routine of a distance (from center of dilation to a vertex) four different times, the repeated rehearsals of this routine not only contributed to the development of a stable procedure for accomplishing the task, but also contributed to the learning of all students in the group. In each iteration of the process, students took on increased and different responsibilities in executing the routine, and responded to contingencies arising from each other's changing engagements. Future WSG designs could more explicitly exploit the possibilities of rehearsals and performance as a resource for learning.

From the point of view of scaling 1.5 times as a concept, Lauryn, Natalie, and Tahir actually performed the scaling routine five times. Before they began the dilation plan, the group scaled the side of the original quadrilateral 1.5 times. As documented in "How Did You Decide on this Green Thing?" and "I Get It Now," that first experience with scaling, which ended with Lauryn declaring "That's a good idea," did not mean that the first scaling for the purposes of dilation proceeded smoothly. Certainly, the second iteration of scaling for dilation was far less problematic in comparison to either of those other two. This could be attributed to the confusion of Lauryn and Tahir about the dilation strategy as a whole. However, Natalie, who did have an image of the whole dilation plan, also encountered trouble with the routine. That the scaling was being executed in the context of two different strategies is significant for these students' understanding and execution of the 1.5 times concept. In the next section, I discuss the idea of generality as a mathematics learning goal, and how the WSG context as a disruption for productive hybridity can support it.

## **Generalizing Across Walking Scale Geometry and Classroom Instruction**

Generalizing from more specific cases is a major goal of mathematics (and geometry) education (Driscoll, 2007; National Council of Teachers of Mathematics, 2000). From a situative perspective, generalizing is not about acquiring and being able to apply existing, externally available, abstract concepts to a variety of problems. Instead, concepts are socially constructed in the context of activity, developed over time by groups with shared goals, tools, and forms of communication and interpretation (Hall & Greeno, 2008). Generalizing, then, is also a socially constructed accomplishment that involves making concepts relevant, meaningful, and useful across contexts (from scaling a quadrilateral side to scaling the distance from an arbitrary center of dilation to a vertex), and across settings (from WSG tasks to pencil and paper geometry tasks; Hall & Greeno, 2008). In a mathematics learning setting, we want to support students in making sense of and making use of concepts across tasks and situations. In this sense, the WSG setting provides more than a mediating experience through which productive hybridity is supported in the classroom. It is an additional activity in which students grapple with concepts.

In a study of an eighth grade class engaged in a population modeling project, Jurow (2004) identified linking and making conjectures as practices that support generalization. Classroom instruction can be designed to support linking between WSG experiences and more classroom appropriate tasks, while inviting conjectures about future, possibly imagined WSG problem solving. This can be done not just by designing approximations of the WSG setting, but also with tasks structurally similar enough so as to make WSG experiences relevant (Greeno, Moore, & Smith, 1993; Hall, Wieckert,



& Wright, 2010; Jurow, 2004). We have already seen that classroom understandings of some mathematical concepts, like congruence for line segments (in KCMS and SEC) and dilation (for Natalie in KCMS) are readily recruited in WSG tasks. The question remains about what concepts are developed in WSG that can and should be adapted and made meaningful in the classroom context?

One that has arisen from this dissertation study is the relationship between sides and vertices in a polygon. How can we help students develop more general ways of thinking about how it is that a triangle has three sides and three vertices, and each vertex is adjacent to two sides in a triangle? Or that a polygon has  $n$  sides and  $n$  vertices, and each vertex is adjacent to two sides? How can we help students develop more general ways of thinking about the possibilities for translating and rotating line segments of fixed length so that they form a triangle? Or a quadrilateral? The WSG tasks I designed for this dissertation highlight these mathematical relationships within the WSG setting. Future classroom design can address these concepts in tasks appropriate for the constraints of that setting.

That the WSG setting can support generalization in addition to providing a shared experience and resources that support productive classroom hybridity raises an important question for Productive Disciplinary Engagement (PDE) design principles (Engle, 2011; Engle & Conant, 2002). In addition to problematizing content, supporting students in taking on intellectual authority, holding students accountable to each other and disciplinary ideas, and assuring that students have access to appropriate resources, a design for PDE should provide a variety of settings in which students can make sense of and adapt concepts. By this I mean more than a variety of problems that address the same concept

in varying degrees of similar or different ways. While this is an important resource for promoting generality, I believe that the analysis of the WSG tasks demonstrate that they provide more than just an alternative wrapper for the same problem.

WSG, in large part because of its four constituent disruptions, is a brand new setting with its own goals; spatial, material, and physical concerns; division of labor; and conceptual and representational tools. Concepts, like “triangles,” are the “same” in that one could identify three sides and three angles in a WSG triangle and on paper. However, triangles in WSG have brand new forms of coming into existence, with a new collection of problems for representation and inscription, producing distinct practical and mathematical concerns for students. In the WSG setting, Felicia, Kaitlin, Kimberly, and Marla came to treat triangles not just as a figure with three straight sides (properties given in whole class discussion before the task began), but as a figure with three ropes (straight sides) where each person (as a vertex) holds two rope ends. This is a new and mathematically rich description for a triangle, useful in the classroom setting, and relevant not only to the demands of the WSG setting. In other words, adapting the concept of “triangle” in the WSG setting provided an opportunity for Felicia, Kaitlin, Kimberly, and Marla to make it more general.

## APPENDIX A

### KCMS Lesson Plans

#### Day 1: Tuesday, April 26, 2011 (RAIN)

Class time: 70 min

Summary: Talk about norms of discussion, including a trial discussion about what the best school lunch is. Then, push the desks to the perimeter of the room and use the center of the floor as “paper.” Do WS straight line and triangle tasks with classroom tools here, as a whole class.

Objectives: Begin developing norms for discussion; Problematize drawing at large scale

Materials: Poster paper, anything that’s in the classroom

1. DO NOW: Draw a straight line segment.

Draw a triangle.

Discuss: Share as many strategies as we can get for drawing them.

2. DISCUSSION on discussion (30 min)

- We’re going to try a new way of talking about our math ideas. We want everybody to have a chance to share their ideas, even if they are new ideas. And we want to be able to compare our ideas, and help each other build on them.
- Let’s start by agreeing on ways to make sure everyone can feel comfortable sharing their ideas, even if they’re new and still developing.
  - What are some ways we might tell someone that we agree with their idea?
  - What are some ways we might tell someone that we disagree with their idea, without hurting their feelings?
  - What are some ways we can ask someone a question about their idea, without hurting their feelings?
- Try out our discussion techniques: “What is the best lunch they serve in the cafeteria?”
- Point out ways students justified their answers—“because” statements—and anything else relevant.

3. Large Scale Geometry (20 min)

Push all the desks to the perimeter of the room, facing in. Students stay in desks. The floor in the center of the room is now our “paper” for drawing geometric objects.

- How would you draw a large line segment, using the floor as your paper?
  - Write how you would do it on your paper (Use the “WSG Activity 1” worksheet to write individually.)
  - Share strategies with the class, trying them out together
    - There is a good chance they’ll use the floor tiles. Still, we’ll have to problematize how to “draw” something on the floor (since we can’t draw on the floor with marker without getting in trouble).
    - How can we tell it’s straight?

- How would you draw a large triangle, using the floor as your paper?
  - Same as above
- What tools are useful? What do you wish you had? What would you need on the field?

4. EXIT SLIP: Describe to someone who was absent today the difference between doing geometry at your desk on paper and doing it at walking scale, on the floor. List as many differences as you can. You may use words and pictures in your answer.

### **Walking Scale Geometry Activity 1**

1. Using the floor as your “paper,” draw a straight line segment. It should be AT LEAST as long as two of you put together are tall. You may use any of the tools that we have brought.

Describe what each member of your group did. You may use words and pictures.

2. Using the floor as your “paper,” draw a LARGE triangle. Each side of the triangle should be AT LEAST as long as two of you put together are tall. You may use any of the tools that we have brought.

Describe what each member of your group did. You may use words and pictures.

## **Day 2: Thursday, April 28, 2011**

Class time: 70 min

Summary: Today we go outside and construct our geometric objects at walking scale with the tools that students asked for yesterday. In the first half, we go outside and make the objects. The project team will be taking photographs of students' work from above and from the perspective of students. In the second half, we discuss as a class what each group did. Then they will get a photograph of their group's triangles work from above and one from their perspective (ground level) and explain what's happening in it.

Objectives: Develop strategies for using materials for drawing straight lines and triangles at WS; Support students in describing their strategies in writing, pictures, and talk

Materials: Items students asked for yesterday; digital cameras; printer; poster

1. **DO NOW:** Explain, in words and pictures, how you plan to create your triangle outside, at walking scale, using the tools that we brought you.

Discuss: Ask for students to share their strategies, especially different ones.

2. **WSG with their tools, outside.** (This should only take max 10 minutes each task)

Students work in groups of 3, and fill out the worksheet individually.

See first page of "Walking Scale Geometry Activity 2"

Research team members take photographs from above and from ground level, to be printed out for second session.

3. **DISCUSSION** (while JYM prints photos): 1) what materials did they use, 2) how did they use them, 3) how did they know their line/triangle sides were straight 4) are there tools you wish you had?

4. **LABELLING PHOTOS:** Students get two photographs of their triangles work—1 from above, 1 from ground level. We will give them the one from above. As they work on this one, we will go around and have each student choose the one from ground level, and print it out for them.

See pages 2 and 3 of "Walking Scale Geometry Activity 2"

## Walking Scale Geometry Activity 2

### Straight Line, Triangle

1. Using the soccer field as your “paper,” draw a straight line segment. It should be AT LEAST as long as two of you put together are tall. You may use any of the tools that we have brought out. **Tell a research team member when you are finished so we can take a picture.**

**Describe what each member of your group did. You may use words and pictures.**

2. Using the soccer field as your “paper,” draw a LARGE triangle. Each side of the triangle should be AT LEAST as long as two of you put together are tall. You may use any of the tools that we have brought out. **Tell a research team member when you are finished so we can take a picture.**

**Describe what each member of your group did. You may use words and pictures.**

3. Tape your photograph from above here:

Label on your photo:

1. Each member of your group
2. Your triangle
3. The materials you used
4. Anything else you think is important

What kind of triangle is this? \_\_\_\_\_

What did your group do to make it? You may use words and pictures to explain.

Does it look how you expected it to look? Why or why not?

4. Tape your photograph from ground level here:

Label on your photo all the important parts.

What are you doing in this photograph? How is it helping your group make the triangle? You may use words and pictures to explain.



### Day 3: Friday, April 29, 2011

Class time: 70 min

Summary: Students, in groups, will go outside and create a large obtuse angle. They will then look at it through their windows from lots of different viewpoints. They'll trace what they see through their window to get the idea of how the angle looks different as they walk around it.

Objectives: Problematize perspective at walking scale – things might look different on the ground than they do from above, and they look different on the ground depending on your perspective. Our ways of characterizing “acute” and “obtuse” and “right” angles, and even of “eyeballing” the measures of angles are based on perspectives looking down on them.

Materials: WSG materials, Plexiglass windows, Dry Erase markers

#### 1. DO NOW (10 min)

Today you will go outside and make an obtuse angle with our tools. How will you make sure that it is really obtuse? Describe two ways you could check if it is obtuse.

*This is “Walking Scale Geometry Activity 3: Viewing angles on the ground” #1*

#### 2. WSG Angles and Windows activity with their tools, outside. (20 min)

Students will work in groups of three. Each group will make an obtuse angle of their choosing. They will then stand at different predetermined (by us) spots around the angle, and look at it through their windows. They will use an Expo marker to trace the angle onto the window in the right spot.

*See “Supplement to Walking Scale Geometry Activity 3: Viewing angles on the ground”*

#### 3. WORKSHEET (10)

Individually, they trace their angles from the window onto their worksheet. Group members will have to take turns, but they can answer the questions on the worksheet while they wait.

#### 4. DISCUSSION, if there is time:

- When did your angle look obtuse?
- When did it look acute?
- When did it look right?
- How did you know you made an obtuse angle when you made it?
- If it looked acute from where you were standing, did that make it acute?
- If an obtuse angle might look acute depending on where you are standing, what makes it an obtuse angle? (Push to the idea that “obtuse” and “acute” are meant for looking down on an object from above, as we normally do on paper. We aren't looking down on the field in these activities, since we're standing on the ground practically at the same level as the angles. We can't even get really “above” pictures from where Kevin/Nate stands. How will we know the actual measures of our angles? I think we can get some ideas, but we might not answer this question immediately. If we don't, it'll be a question that we keep thinking about the next few weeks. This may be a terrible way to approach this?)

#### 5. EXIT SLIP, if no time for discussion:

If an obtuse angle *looks* acute when you're standing in a certain location, do you think that makes it an acute angle? Why or why not?

### Walking Scale Geometry Activity 3:

#### Viewing angles on the ground

#### DO NOW

Today you will go outside and make an obtuse angle with our tools. How will you make sure that it is really obtuse? Describe two ways you could check if it is obtuse. You may use words and pictures in your answer.

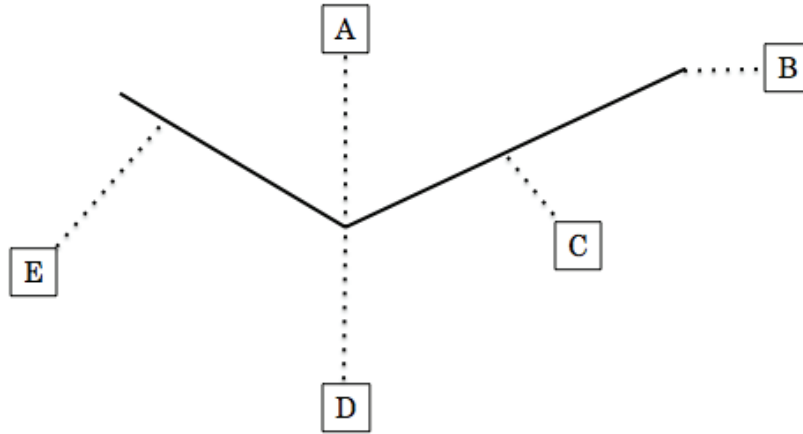
1.	2.
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#### Supplement to WSG Activity 3: Viewing angles on the ground

1. Make a large obtuse angle. You may use any tools that we have brought out.

**Tell us when you are ready, and we will take a photograph of your angle at "A."**

2. Stand at the locations indicated below around your angle, and trace what you see on your window. The solid line is an example of an obtuse angle. The dotted lines are to help you determine where each location is.



3. Go to a place where your angle looks like a right angle. Make sure everyone in your group agrees. Trace the angle in your window in the space marked “F.” Also label where you are standing in relation to the angle.

**Trace your angle from the window at each location in the table below.**

A	B	C
D	E	F (remember to label where F is)

**QUESTIONS:**

How did what the angle looks like change as you walk around it? When did it get smaller? When did it get bigger?

Is there a time when an angle on the field might look like a straight line, even though it's not? You may use words and pictures in your answer.

**EXIT SLIP QUESTION:**

If an obtuse angle *looks* acute when you're standing in a certain location, do you think that makes it an acute angle? Why or why not?

**Day 4: Monday, May 2, 2011 (RAIN)**

Class time: 70 min

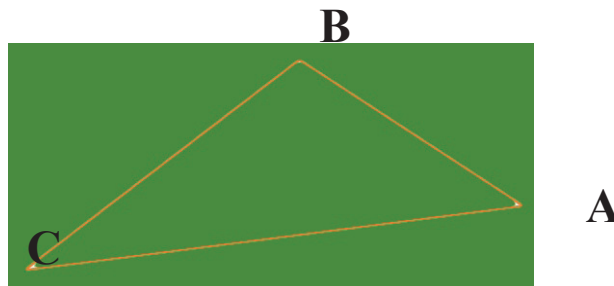
Summary: Students begin by doing an activity similar to the Windows and Angles perspective activity, but at paper scale. We then move to making isosceles and congruent triangles at WS outside, and discuss how we did that.

Objectives: Ground level vs. overhead perspectives, Congruent line segments/ isosceles triangles, describing/explaining strategies

Materials: WSG materials

1. DO NOW (10 min)

Jasmine made a triangle on her lawn this weekend that looked like this from above:



Sketch what you think it might look like if you are standing at positions A, B, and C.  
[PHOTOS are on the next page.]

2. DISCUSSION for each position:

- What did you change? Why?
- Show photo of what each looked like, and discuss why they look this way
- Does it matter what height Jasmine takes the picture from?

3. PHOTOS discussion:

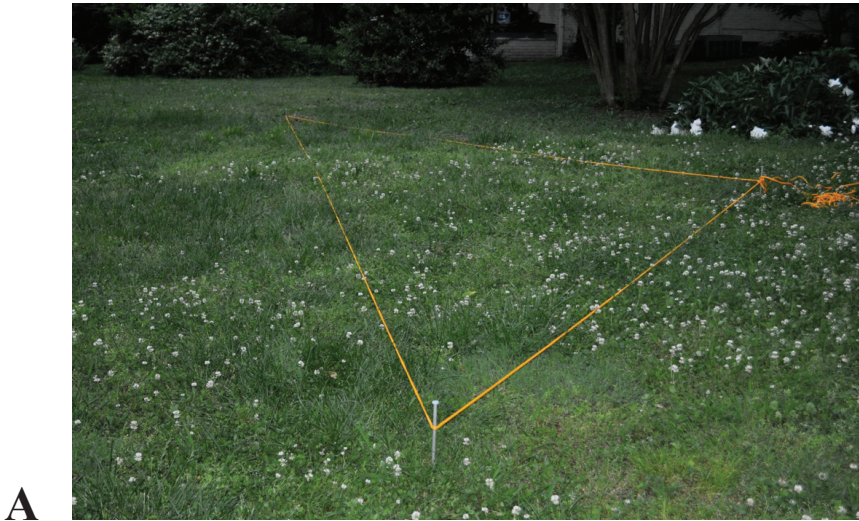
- On the projector, look at the photos of the activities from last week. Point out instances where:
  - The photo makes it look different than we might expect (due to viewing angle or scale)
  - Different strategies and tools (including how bodies are used) are being used to measure lengths; make straight lines; hold rope/tape still and in place.

4. VOCABULARY TASK: MRH made a list of vocab that was coming up in discussion (Ground level; Air level; Straight; Dipped line; Dimension) and had the kids, in groups, explain what they meant and why they were important on easel paper.

5. DISCUSSION: Compare different groups' definitions and descriptions.

6. EXIT SLIP: Give explicit plans to a group of 3 students for making a large ISOSCELES TRIANGLE on the field. Assume that they have never done geometry on

the field before, but are very good at following instructions. You can have them use any of the tools that we have had. You can use words and pictures in your instructions. Tomorrow, another group will use your instructions to try and make an isosceles triangle. We'll see which group has the instructions that produce the best triangle.



## Day 5: Tuesday, May 3, 2011

Class time: 70 min

*Summary:* Students make an instructional video for creating a large scale isosceles triangle.

*Objectives:*

- Students generate strategies for creating large scale isosceles triangles, which helps them understand what an isosceles triangle is (at least 2 congruent sides) and how to make 2 congruent line segments at large scale.
- Students learn to describe their strategies in detail through giving instructions to others

Materials: WSG materials, video cameras.

### 1. DO NOW (10 min)

Jasmine has done her best to give you instructions for drawing her secret shape. Follow her instructions closely to draw the shape on paper with your pencil.

1. Draw a straight line, and label the endpoints A and B.
2. Draw a straight line upward and to the left from A. Label the new endpoint C.
3. Draw a straight line to the right, the same length as AB. Label the new endpoint D.
4. Connect D and B.

### 2. DISCUSSION

- As students work, walk around and see what they've come up with. Ask them to share their drawings on the board, getting as much variety as possible.
- Why did the drawings come out differently? (Go through what they did in each step that made a difference- direction of line in #1; angle of line in #2; length of line in #2; direction of line in #3).
- I will draw my actual secret shape on the board, following the instructions.
- Why didn't everyone get Jasmine's secret shape? What could she have said in her instructions so that they are better?

### 3. VIDEO ACTIVITY (read through all these steps before they start):

- a) You are going to create an instructional video for another group to make an isosceles triangle. First, you will have to figure out how your group would make an isosceles triangle at LARGE scale. You may use the materials in our large scale geometry toolkit to try things out. Write down your plan. (done by 9:25)
- b) Then, write a script for what you will SAY and SHOW in your video. (done by 9:45)
- c) When you are finished, practice your instructional presentation to make sure it makes sense. Then tell a teacher that you are ready, and practice it one more time for her. (done by 10:00)
- d) When your teacher approves, we will video record your presentation. (done by 10:20)
- e) TOMORROW, another group will watch your instructional video and try to follow your steps to make the triangle.

### 4. EXIT SLIP (5 min)

Explain how you can check to see if a LARGE triangle using our outside tools is ISOS-CELES. What will you DO to make sure two sides are the same length? Write down specific instructions for doing this.



## Day 6: Wednesday, May 4, 2011

Class time: 35 + 35 min

*Summary:* Students watch and revise their videos

*Objectives:*

- Students generate strategies for creating large scale isosceles triangles, which helps them understand what an isosceles triangle is (at least 2 congruent sides) and how to make 2 congruent line segments at large scale.
- Students learn to describe their strategies in detail through giving instructions to others
- [Students understand that a triangle can be determined by 3 sides (SSS). We can also get into other combinations of information given: SAS, AAA, ASA. ]

*Materials:* WSG materials; videos and scripts from yesterday, laptops, video camera.

### SESSION 1:

#### 1. DO NOW (10 min)

List three important things that you talked about in your instructional video?

In discussion, we want to highlight:

- Materials
  - How to make sides (straight line segments)
  - How to hold the sides still (people? Stakes? Flags?)
  - How to make two sides congruent
  - How to make the third side
  - How to join the sides (at vertices)
  - What are the angles going to be?
2. REVISE INSTRUCTIONAL VIDEO:
- Each group watches their own instructional video
  - Groups plan for what they want to change, revise their scripts, re-record video

### SESSION 2:

3. If some groups didn't finish recording their videos, do that now.

4. Watch videos as a class, and compare how the groups tackled each element:

- Materials
- How to make sides (straight line segments)
- How to hold the sides still (people? Stakes? Flags?)
- How to make two sides congruent
- How to make the third side
- How to determine side lengths
- How to join the sides at vertices (bringing rope ends together?)
- What are the angles going to be?

*[It'll probably come up that some groups didn't address some elements. e.g., side lengths or angle measures might not be relevant to some groups. We want to talk about what this matters for. If you know three side lengths (SSS), does that determine the angles? What if you don't know all 3 side lengths (just 2 side lengths? SAS?) How many different triangles could you make? We could actually have them get up and demonstrate with rope in front of the class, trying to make different triangles given different pieces of information]*

**Day 7: Thursday, May 5, 2011**

Class time: 70 min

*Summary:* Students use each other's instructional videos, to create large scale isosceles triangles. They'll then "check to see if other groups followed their instructions correctly. Finally, they will convert their isosceles triangles into equilateral triangles.

*Objectives:*

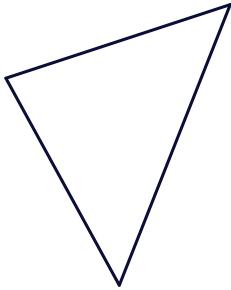
- Students create isosceles triangles at large scale
- Students consider the relationship between isosceles and equilateral triangles.
- Students begin to notice patterns in changes in parts of a triangle
- Students continue to develop the idea of what constitutes "good instructions"

*Materials:* WSG materials; videos, scripts, and notes from yesterday, still cameras.

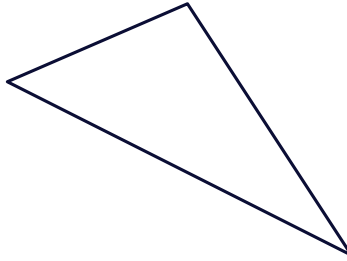
1. DO NOW (10 min)

Circle all the isosceles triangles below.

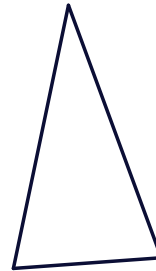
a.



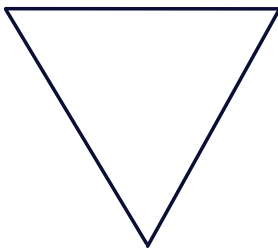
b.



c.



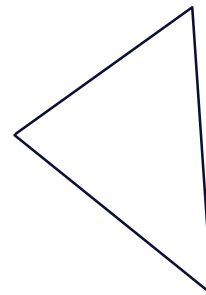
d.



e.



f.



2. VIDEO VIEWING (7 min)

Each group watches the instructional video of another group and takes notes on what to do when they get outside. They can use the back of yesterday's chart. When they go outside, they'll have the notes and the info on the chart to help them make their triangle.

3. WGS: (30 min)

- Each group makes an isosceles triangle according to the instructions in the video they watched. We will have one laptop outside that they can use to review the videos.
- We will take a picture from above of each group's triangle once they are finished.
- The group who made the video will go check to see if the group who made the tri-

angle did it according to the instructions.

- Make your isosceles triangle into an equilateral triangle with the fewest number of steps. Tell us when you're done so we can take a picture from above.

#### 4. DISCUSSION (15 min)

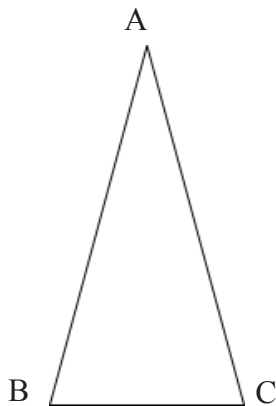
- How well did the instructions work?
  - Were there other things that the instructional videos could have done to make it easier? (This might be missing steps, or not enough showing, etc. We want them to see how specific you have to get, with words and actions, if you want to explain how to do something.)
- What did you do to make your isosceles triangle into an equilateral triangle? What changed and what stayed the same?

*[We want them to notice that:*

- 1. The angle opposite the "base" of the isosceles triangle and the length of the base changed, and they both changed in the same way (bigger or smaller)*
- 2. The two base angles change the opposite way of the other parts*
- 3. The two "legs" stayed the same length (unless they used a different strategy to change their isosceles triangle into an equilateral one.)*

#### 5. EXIT SLIP (5 min)

The triangle below, DABC, is an isosceles triangle with  $AB \cong AC$ . Draw a triangle that is an equilateral triangle with the sides congruent to AB and AC.



1. What happened to  $\angle A$ ?
2. What happened to side BC?
3. What happened to  $\angle B$  and  $\angle C$ ?

## Walking Scale Geometry Activity 6

1. **Watch another group's instructional video. Take notes on the back of yesterday's worksheet. You will have to follow these instructions when we go outside.**
  
2. **OUTSIDE:**
  - a. **Make an isosceles triangle by following the instructions in the video you watched. Tell us when you are finished, and we will take a picture from above.**
  - b. **Check the triangle of the group who watched your video. Did they follow your instructions? DO NOT disturb their triangle when you check it.**
  - c. **Go back to the triangle you made. Make it into an equilateral triangle using as few steps as you can. Tell us when you are finished, and we will take a picture from above.**

**Day 8: Friday, May 6, 2011**

Class time: 70 min

*Summary:* Students will make WSG triangles with some constraints provided by us (2 side lengths given; angle twice or half as big; vertex moved in)

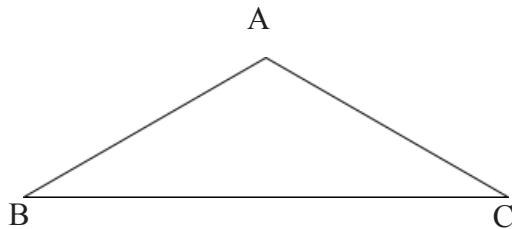
*Objectives:*

- Students understand that an infinite number of triangles can be constructed given 2 side lengths.
- Students understand that the side opposite an angle in a triangle grows and shrinks in correspondence with that angle.
- Students begin to think about how “growing” and “shrinking” is relational. (If the angle grows but the opposite side does not, the other two sides must shrink.)
- Students begin to think about how to halve/double angles at WS.

*Materials:* WSG materials; still cameras

1. DO NOW (10 min)

The triangle below,  $\triangle ABC$ , is an isosceles triangle with  $AB \cong AC$ . Draw a triangle that is an equilateral triangle with the sides the same length as  $AB$  and  $AC$ .



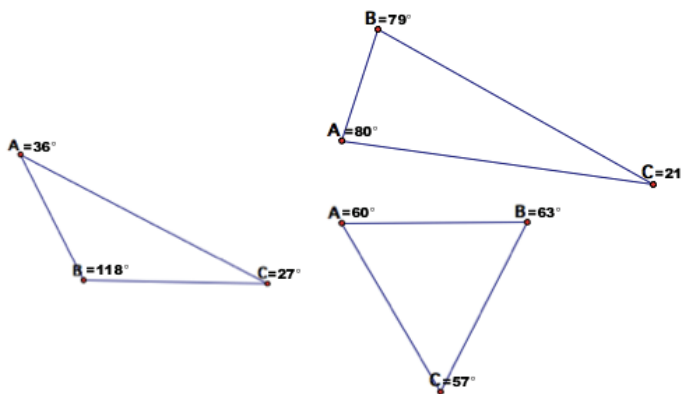
4. What happened to  $\angle A$ ?
  5. What happened to side  $BC$ ?
  6. What happened to  $\angle B$  and  $\angle C$ ?
2. WGS: (35 min)
- a) Each group will receive 2 congruent lengths of rope from us. We will ask them to make an isosceles triangle using these pieces of rope, and any other materials in the box. We'll take a picture of what they made.
  - b) If the group made an acute isosceles, we will ask them to make the angle at the vertex that joins the two congruent sides twice as big, and adjust the triangle. If they made an obtuse isosceles, we'll ask them to make that angle half as big. We'll take a picture of what they made.
  - c) Then we will ask them to move the vertex that joins the two congruent sides 5 giant paces in, and adjust the triangle *without moving the base*. We'll take a picture of what they made.

3. PHOTO DISCUSSION (15 min) [Use photos from outside to discuss what they did]

- Look at every group's 2a. [*Hopefully they will all be different triangles.*]
  - Every group had the same two lengths of rope, why are the triangles all different?
- Look at every group's 2b.
  - For each, ask: How did the angle change? How did the base of the triangle change?
  - Once we've looked at all of them: What do you notice about what happens to the side opposite the angle that changes? [*Get them to generalize that the side grows if the opposite angle grows, and vice versa*]
  - Would this happen for all triangles? Scalene triangles?
- Look at every group's 2c. [*Every group should have had to shorten the two "legs" of their isosceles triangle*]
  - What happened to the angle? [*It got bigger*]
  - What happened to the base? [*It stayed the same*]
  - How could this be? We just said that when an angle grows, the opposite side grows. How come the opposite side didn't grow here? [*It grew in relation to the other sides*]

4. EXIT SLIP (5 min)

Circle the longest side of the triangles.



**WSG Day 9: Monday, May 9, 2011**

Class time: 70 min

*Summary:* Students make a rectangle at WS, then change it to a parallelogram with no right angles. They then make a parallelogram congruent to this one.

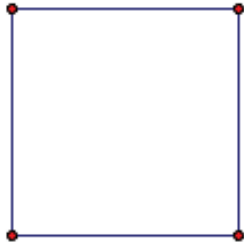
*Objectives:*

- Students think about making right angles and parallel lines at WS
- Students see how sides can move but still remain parallel in a parallelogram
- Students develop strategies for making congruent angles at WS

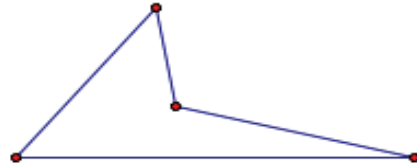
*Materials:* WSG tools

1. DO NOW (10 min): Categorizing quadrilaterals

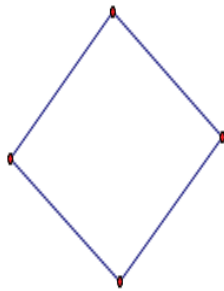
Cross out the terms that DO NOT APPLY to the figures below



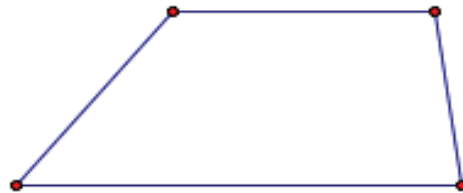
Square  
Rectangle  
Parallelogram  
Rhombus  
Trapezoid  
Quadrilateral



Square  
Rectangle  
Parallelogram  
Rhombus  
Trapezoid  
Quadrilateral



Square  
Rectangle  
Parallelogram  
Rhombus  
Trapezoid  
Quadrilateral



Square  
Rectangle  
Parallelogram  
Rhombus  
Trapezoid  
Quadrilateral

2. WSG: (30 min) Groups should be 4 or more. They can bring the worksheet out with them to fill out as they go.

- a) Each group will make a rectangle.
- b) Change the rectangle into a parallelogram with NO RIGHT ANGLES.
- c) Make a congruent parallelogram next to the original.

3. DISCUSSION (25 min) Compare strategies for each WGS task *[If I can get it together in time, we'll use the pictures to help us with these]*

- How did you make your rectangle *[Highlight parallel lines and right angles]*
- How did you make your parallelogram *[Get kids to stand up and demonstrate]*
  - Did the angles of the rectangle change? *[Yes, two got bigger and two got smaller]*
  - o Did the lengths of the sides change? *[No. We want them to notice that the rule that we just discussed with triangles does not apply to quadrilaterals. If we have time, we can have them speculate on why this is]*
  - o Why did the two pairs of parallel sides stay parallel, even though you moved them and the angles changed? *[The complicated case is the two sides that change direction. As the "base" and "top" sides slide apart, the corresponding angles stay congruent, which maintains the parallelness of the other set of sides. This is foreshadowing for when they do properties of parallel lines with a transversal later in geometry]*
- How did you make your congruent parallelogram?
  - o What had to be the same? *[BOTH sides and angles]*
  - o How did you make sure your sides were the same? *[This should be old hat by now]*
  - o How did you make sure your angles were the same? *[Really push for precision here. "Looking" is not good enough. We might want to point out that making sure the side lengths were the same did not guarantee that the angles stayed the same, like in making triangles.]*

4. EXIT SLIP (5 min) Three new things that you learned today in class. *[They can do it on the back of the Do Now]*



## Walking Scale Geometry Activity 8

### Quadrilaterals

**1.** *Make a large rectangle. When you are finished, ask for a picture from above and at ground level.*

**Explain what you did to make your rectangle. You may use words and pictures.**

**2.** *Change your rectangle into a parallelogram with no right angles. When you are finished, ask for a picture from above and at ground level.*

**What did each person in your group do to change your rectangle?**

**3.** *Make a new parallelogram next to the original. The new parallelogram should be congruent to the original. When you are finished, ask for a picture from above and at ground level.*

**How did your group make the new parallelogram? How did you make sure that the sides and angles were the same?**

## WSG Day 10: Tue, May 10, 2011

Class time: 70 min

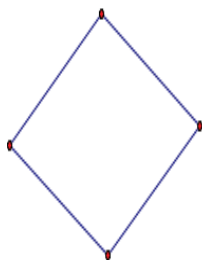
*Summary:* Students will make a quadrilateral at WS, then one congruent to it.

*Objectives:*

- Students develop strategies for making congruent angles at WS

*Materials:* WSG tools

### 1. DO NOW (10 min):



Draw a quadrilateral congruent to the one drawn below. Make it as precise as possible. Explain what you did. You may use words and pictures, but be sure to label your pictures.

*[possible strategies include looking and drawing, tracing, using a ruler and protractor. Push how they can get angles congruent—could be by estimating past a known angle (right/straight), or tracing/comparing. The latter is more precise than the first. One way to do this is to draw an analogy to how we make congruent line segments (comparing)]*

### 2. WSG: (30 min) Groups should be 4 or more. There's no worksheet for this, but we can project the directions from page 2 of "JB-WSG-9.docx"

- a) Make a LARGE SCALE quadrilateral that is NOT a square or a rectangle. Let us know when you're done so we can take pictures from above and on the ground. *[On the field, we should push to ask them how they know it's not a square or rectangle. They should be able to answer both at the level of properties (it doesn't have 4 right angles) and at the level of WSG (if I compare the angle to the corner of the plexi window, the angle is clearly bigger)]*
- b) Make a quadrilateral congruent to the one you chose. Let us know when you are done so we can take pictures from above and on the ground. *[on the field, we should try and get them to make congruent angles more precisely than they have been. They can "fix" an angle by tracing it on a plexi window, using two stakes/flags to replicate the angle, ...]*

### 3. DISCUSSION (25 min) Compare strategies for each WGS task

- How did you make your congruent quadrilateral?
  - o What had to be the same? *[BOTH sides and angles]*
  - o How did you make sure your sides were the same? *[This should be old hat by now]*
  - o How did you make sure your angles were the same? *[Really push for pre-*

*cision here. “Looking” is not good enough. We might want to point out that making sure the side lengths were the same did not guarantee that the angles stayed the same, like in making triangles.]*

- Look at pictures and see if their quadrilaterals look congruent. I’ll try to get it so that we can look at the quadrilaterals and the angles side by side.
- If there’s time, we should talk about the importance of the straightness of the sides of an angle. Could be from looking at pictures.

4. EXIT SLIP (5 min) *[They can do this on the back of the Do Now]*

Describe to a student who was absent today how you make a congruent angle on the field. What do you have to do to make sure it is the right size?

### **INSTRUCTIONS FOR OUTSIDE**

- a) **Make a LARGE SCALE quadrilateral that is NOT a square or a rectangle. Let us know when you’re done so we can take pictures from above and on the ground.**
- a) **Make a new quadrilateral congruent to the first one you made. Let us know when you are done so we can take pictures from above and on the ground.**

## APPENDIX B

### SEC Lesson Plans

#### **PART 1**

**As a group, draw each geometric object with the given materials, using the lawn as your paper. Each object should have sides *at least* the length of 2 of your bodies. When you are finished, let us know so that we can take a picture from ground level and from the top of Wyatt.**

**1. A line segment**

To think about:

- How can you be sure that your line segment is really straight?

**2. A triangle**

To think about:

- What happens to the other angles and the sides of the triangle if one of the angles gets bigger?
- What happens to the other angles and the sides of the triangle if one vertex of the triangle moves closer to the opposite side?

**3. A triangle congruent to one that another group has drawn**

To think about:

- What has to stay the same for the triangles to be congruent? What can change?
- How can you be *sure* that they really are congruent? What could you do to check?

**4. A quadrilateral**

To think about:

- How many different quadrilaterals could you draw with these side lengths?
- How many different quadrilaterals could you draw with these angles?

**5. A quadrilateral 1.5 times the size of the one you just drew**

To think about:

- What stays the same?
  - What changes?
  - How can you be *sure* that the new quadrilateral is 1.5 times the size of the original? What could you do to check?
6. (If there is time in part 1) A **polygon of your choice** with more than 4 sides

## **PART 2**

**If you did not finish Part 1, finish it up to #5.**

**As a group, draw each geometric object with the given materials, using the lawn as your paper. Make each object BIG. When you are finished, let us know so that we can take a picture from ground level and from the top of Wyatt.**

1. A **circle**

To think about:

- How is drawing a curve at walking scale different than drawing a line segment at walking scale?
- What kinds of symmetries does your circle have?

2. A **parabola**

To think about:

- What makes this a parabola?
- What kinds of symmetries does your parabola have?

3. A **spiral**

To think about:

- What kinds of symmetries does your spiral have?

## APPENDIX C

### KCMS Interview Questions

1. What did you learn about in math class over the last six weeks?
2. What do you remember the most about math class over the last six weeks?  
*(prompt for specific experiences or events)*
3. What did you like about math class over the last six weeks? *(prompt for specific experiences or events)*
4. What did you not like about math class over the last six weeks? *(prompt for specific experiences or events)*
5. What were you really good at in math class during the last six weeks? *(prompt for specific experiences or events)*
6. What was really hard in math class during the last six weeks? *(prompt for specific experiences or events)*
7. How was math class during the last six weeks different than math class usually?  
How was it the same? *(prompt for specific experiences or events)*
8. Is there something we have not asked but you want us to know about math class the last six weeks?

## APPENDIX D

### SEC Interview Questions

1. When you look across the different activities of the course, which were the most memorable for you? Why? What did you learn from them?
2. How would you describe this course to a friend who has not been here? What is the “big picture” of the course?
3. What, if anything, will you use or think about from this course when you return to your life away from VSA? Has this course already made you think about your life at home differently? If so, in what ways?
4. What ideas do you think are most important for learning to think spatially?
5. Select 3 items from your thumb drive that you think are most meaningful for showing different aspects of your spatial thinking. Show each of these and describe
  - a. what the different aspects are, and
  - b. what kinds of spatial thinking are involved in each of these items
6. Of the different major themes to the course—math, media, and mobility, which of them do you identify with the most? (Which feels most like “you”?) Why?
7. Of the major themes from the course, which was the most difficult for you to think about or understand? Why?
8. Have you used your body to think spatially in this course? How? How is using your body to think spatially different from just using your brain?
9. Have the activities and thinking you’ve been doing in this course similar or different to the activities and thinking you use in school? In what ways are they the same or different?
10. What was your favorite and least favorite reading and why?
11. Some people would argue that the main idea of learning to think spatially is to learn to read and interpret maps. Do you agree or disagree with this? Why?
12. What plans do you have (if any) to continue to explore spatial thinking in your life beyond this course?

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