# GAME THEORETIC MULTIDISCIPLINARY OPTIMIZATION FOR SYSTEM-OF-SYSTEMS ANALYSIS

By

Quentin C. Noreiga

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**APPROVED** 

Professor Sankaran Mahadevan

Professor Mark P. McDonald

Professor Mark Ellingham

Professor John Conley

To my loving wife, Cheryl, my children, and my parents...

#### **ABSTRACT**

High-speed rail (HSR) planning models have not considered the response of airline operations to introducing high-speed rail to their commercial transportation networks. While considering the decision processes of travelers in predicting transportation system demand, HSR planning models have assumed airline response to be static; therefore, the overall objective of this research is to model and analyze travel demand in an intercity transportation system consisting of highway, conventional rail, air, and (possibly) high speed rail, for the purposes of anticipating system-wide shifts in travel demand resulting from the introduction of high-speed rail projects. In this dissertation, the approach to formulate, decompose, and solve this problem consists of the following tasks: (1) development of a computationally inexpensive model to estimate the interregional travel demand, performing model verification, uncertainty propagation, and sensitivity analysis. (2) Integration of the simplified surface transportation systems planning models with airline fleet optimization models to capture the optimal cooperative response of the aviation sector. (3) Apply the simplified models from objective 1 and the optimization methods from objective 2 to determine equilibrium resourcing and pricing conditions for competitive airlines given levels of service for HSR and airlines to determine the validity of pricing assumptions. These tasks are performed using the Cambridge Systematics travel demand model of the California Corridor.

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Mark McDonald, Ph.D., Assistant Professor of Civil and Environmental Engineering, Lipscomb University, & Adjunct Professor of Civil and Environmental Engineering, Vanderbilt University School of Engineering

Sankaran Mahadevan, Ph.D., Professor of Civil and Environmental Engineering, Vanderbilt University School of Engineering

Mark Ellingham, Ph.D., Professor of Mathematics, Vanderbilt College of Arts and Sciences

John Conley, Ph.D., Professor of Economics, Vanderbilt College of Arts and Sciences

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#### **CHAPTER I**

#### INTRODUCTION

"Nothing in this world can take the place of persistence. Talent will not; nothing is more common than unsuccessful people with talent. Genius will not; unrewarded genius is almost a proverb. Education will not; the world is full of educated failures. Persistence and determination alone are omnipotent."

- Calvin Coolidge.

#### 1.1 Overview

Our nation's commercial air and highway transportation networks are overly congested. The year 2000 produced record delays with more than one quarter of flights arriving at least 15 minutes behind schedule (Mayer and Sinai 2003). With some 75 million licensed drivers in heavily populated areas, each averaging roughly 16,000 kilometers per year within those areas, there are approximately 1,200 billion kilometers driven annually in metropolitan areas, bringing the total delay to 6 billion vehicle-hours each year (Arnott and Small 1994). Both statistics are indications of the transportation congestion facing U.S. regions. In response to both highway and air congestion, all states have established State Transportation Improvement Programs for the purposes of addressing and solving their highway, and air transportation issues. These decision makers and transportation planners need models to support decisions involving the numerous solution strategies which include expanding the capacity of existing networks, creating new networks, and determining optimal methods to manage existing resources (Daganzo 1976). Of the new network possibilities, some regions are considering the introduction of high-speed rail to their commercial transportation network.

The California Corridor is one example of an overly congested, large-scale, intercity multi-modal commercial network in which transportation planners are faced with decisions involving resource acquisition, and resource allocation. As a result, California is currently planning to introduce high-speed rail (HSR) network to the California Corridor. The planned network would connect the San Francisco Bay area, Sacramento, Fresno, Bakersfield, Los Angeles, San Diego, and Las Vegas via high speed rail as shown in Figure 1.1.

High-speed rail is a common form of interregional transportation in Europe and Asia (Potter 1989). In the past few years, transportation planners have been conducting the analysis to establish high-speed rail in various sectors of the United States. Other projects include the Midwest Corridor HSR project which plans to link Chicago, Detroit, and St. Louis (Mathur and Srinivasan 2009), and the Northeast Corridor HSR project which plans to link Washington D.C., New York, and Boston (Chen 2010). In recent years, high-speed rail has been considered as a potential competitor to commercial air. For this body of work, it is assumed that high-speed rail is a strong competitor with regards to commercial air and that price is a critical factor for user mode choice decisions between high-speed rail and commercial air.

The addition of HSR in the California Corridor will have obvious impacts on the total transportation system (Cambridge Systematics 2008). The Cambridge Systematics HSR planning models have considered the decision processes of travelers in predicting network demand; however, the model does not consider competitor responses such as airline operations to the introduction of high-speed rail and have assumed them to be static. In addition, one iteration of the Cambridge Systematics model takes approximately four days to run. The computational expense of this model along with the static response assumption are examples of common critiques of systems analysis techniques.

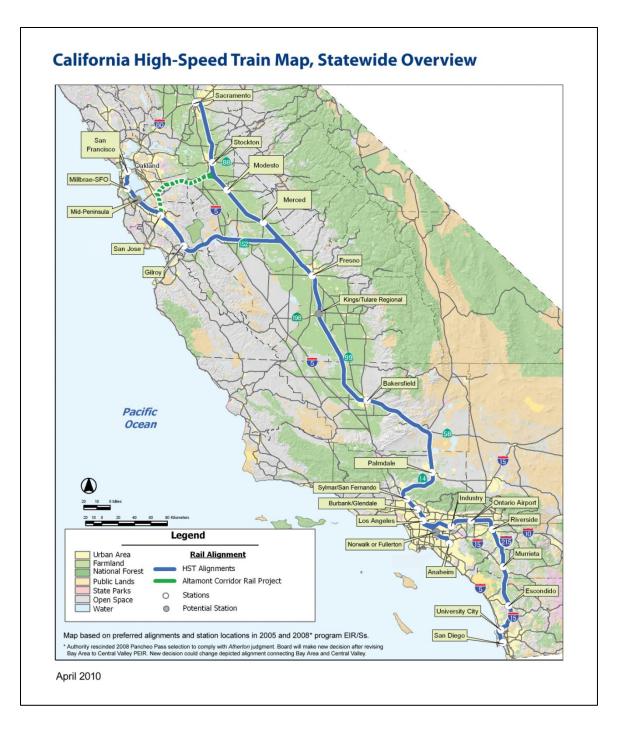


Figure 1.1: California High-Speed Train Map

To address the issues of computational expense, the proposed research seeks to develop a smaller model that looks at the problem from a higher level of resolution. To address the issue

of assuming a static competitor response, the proposed research seeks to define a cause and effect relationship between the CS main mode choice modeling and the resource allocation and pricing decisions of commercial air. The CS main mode choice decision model utilizes several input factors. Among these input factors are pricing and resourcing. As a result, any subsequent changes to the resource capacity and/or pricing by commercial air in response to the introduction of high-speed rail would affect the main mode choice decision modeling results used by Cambridge Systematic. Using the California high-speed rail problem as a case study, the overall objective of this research is to provide a framework to model and analyze a system and its effects on other systems. This type of analysis is considered system-of-systems analysis. Solving the overall problem of decision support for the total transportation system in Southern California requires modeling the total system as a system of systems.

This research provides a framework to conduct system-of-systems analysis. While several definitions exist for a system-of-systems, in this research, a system-of-systems is defined as a network of systems. This collective view of systems analysis seeks to provide a method for quantifying the effects that systems have on each other. System-of systems analysis is often utilized where decisions directly affecting one system also affect the conditions in another system. Two examples of systems often included in system-of-systems analysis are transportation and business systems which are directly affected by customer or user decision choice.

Systems analysis models are often used to estimate and predict system conditions and response. Due to the large amount of data required to model a system, systems analysis models can become very large and complex. As a result, systems analysis models are often criticized for their computational expense and failure to consider the impacts of conditions related to, but

outside the immediate scope of their model. This research seeks to provide a framework to account for the computational expense and narrow scope of systems analysis by decomposing systems analysis into manageable steps. To solve the problem of computational expense, decomposition is necessary (Papatheodorou, Magirou, and Kiountouzis 1993)(Ostertag et al. 2009). The decomposition steps suggested by this research are to develop a feasible model, integrate that model in a systems network, conduct a short run pricing analysis, and determine both system optimal and user equilibrium points.

The first step accounts for the model computational expense. The remaining steps consider both the response from and the effects on other systems. Parsimonious modeling will be used for feasible model development. A short run analysis will be conducted by observing model output across the likely range of critical input parameters. Multidisciplinary optimization will be used for the system optimal model integration. Game theoretic optimization will be used for determining a competitive network user equilibrium. This analysis is designed to show how the network of systems operates under certain specific conditions. To illustrate the proposed method, this research will consider a transportation system-of-systems consisting of multiple commercial service providers competing within a specified region. The inherent difficulties of system-of-systems analysis and transportation systems analysis in particular are in accurately capturing the interdependencies of related systems given multiple decision makers utilizing various operational strategies.

The primary challenge of this type of work is in dealing with the large amount of variables and data. This research proposes that utilizing a reduced or parsimonious modeling approach will mitigate the data requirement and provide a framework for developing a model feasible for repetitive analysis. The challenge with using a reduced or parsimonious model is its

accuracy as compared to the original or parent model. This research will utilize model calibration to ensure model accuracy. The second challenge centers around the concept of induced demand which simply implies that the total network demand increases as capacity increases (Cervero 2002). Induced demand typically refers to an increase in the total user demand in a given network; however, in this analysis induced demand is defined in terms of demand shifts from one mode of transportation to another based on changes in the model inputs.

#### 1.2 Research Questions & Objectives

The overall objective of this dissertation is to apply a multidisciplinary optimization method to model and analyze the travel demand of an intercity transportation system consisting of highway, conventional rail, air, and (possibly) high speed rail, for the purposes of anticipating system-wide shifts in travel demand resulting from the introduction of high-speed rail to an existing commercial transportation network. In particular, the proposed dissertation research will focus on the California Corridor. Utilizing the Cambridge Systematics model as a case study and accounting for the anticipated research challenges, listed below are the primary questions that this research seeks to answer.

- 1. Since the CS model is proprietary (only underlying equations available in CAHSRA reports), and computationally expensive (4+ days to evaluate once), can a useful simplified model, suitable for sensitivity analysis, uncertainty quantification, and optimization studies be specified, estimated, and validated?
- 2. To which model parameters is the CS model most sensitive?
- 3. What is the contribution of the uncertainty in key parameters to the uncertainty in model predictions?

- 4. Can the response of the total air transportation system to the presence of HSR be predicted? If so, how?
- 5. Can competition between airlines be modeled in a system-of-systems context?
- 6. Will the presence of HSR shift this balance? If so, how?
- 7. Given that pricing decisions are made on a shorter time scale than airline resource acquisition and schedule design, can pricing strategies be identified using the simplified planning model?
- 8. What can be said about the ridership and revenue projections for air and HSR as a result of equilibria and pricing games?
- 9. What are the implications for the viability of HSR in California?

This research will develop and analyze a feasible travel demand model for the purpose of performing repetitive analysis to demonstrate the effects of and on outside systems. This research will also address the issue of induced demand by accounting for changes in user demand through network equilibrium analysis and short run pricing analysis. Based on these overall questions, the research objectives of this research are listed below.

Objective 1 is the development, verification, and exploitation of a computationally inexpensive model to estimate the interregional travel demand in California. This task consists of simplifying the Cambridge Systematics model and performing model verification, uncertainty propagation, and sensitivity analysis. This task will address the problem that the Cambridge Systematics model is too computationally expensive for use in analysis requiring repetitive model evaluations. Utilizing parsimonious travel demand modeling, the proposed research will contribute to the field of demand modeling by providing a methodology to simplify, verify, and conduct uncertainty quantification and sensitivity analysis of a travel demand model in the

California Corridor. This will be accomplished by changing the model from a small town level analysis to a county level analysis. The proposed research will contribute to the analysis of system-of-systems by developing a model computationally feasible for optimization and detailed analysis.

Objective 2 is the integration of transportation systems planning model with a fleet assignment model using multidisciplinary optimization to determine the system optimal resource acquisition and resource allocation aircraft requirements. Using game theory, a short run analysis will be conducted to identify the optimal price input parameters for the model integration. The short run analysis compares variations in critical input parameters to illustrate the potential effects of competition on travel demand and profit. This objective will address the unrealistic assumption that commercial air resourcing will remain static upon the introduction of high-speed rail to the California commercial transportation network. The contribution of this objective will be the development of a framework to integrate demand modeling and fleet assignment modeling by defining the inputs and outputs of both models are how they interact assuming cooperative decision making.

Objective 3 is the formulation and solution of a game theoretic optimization problem to determine the user equilibrium airline level-of-service conditions for a multi-modal intercity transportation network. The major accomplishment of this objective will be the utilization of game theory to determine the optimal resourcing and pricing for a commercial transportation network. This will be accomplished by applying the simplified models from objective 1 and the optimization methods from objective 2 to determine equilibrium conditions for competitive airlines.

Completing these tasks will provide decision makers with a methodology to solve a complex large scale multi-modal intercity commercial transportation network analysis and design problem by fulfilling the requirement for a feasible model for use in repetitive analysis, integrating that model with an outside but related system to illustrate the cause and effects on and from outside systems, establishing system and user equilibrium conditions for a long run analysis, and conducting a short run pricing analysis.

#### **CHAPTER II**

#### LITERATURE REVIEW

"There is nothing impossible to him who will try." – Alexander the Great

A summary of the concepts and literature relevant to this research include the Cambridge Systematics Integrated Transportation Management System (ITMS) model final report, system-of-systems, transportation systems planning, airline schedule planning, Mean Value First Order Second Moment (MVFOSM) methods, multidisciplinary optimization, and game theory.

## 2.1 Transportation Systems Planning

The Cambridge Systematics parent model used in this research is a traditional transportation systems planning model. Classical urban transportation planning model consists of four stages: trip generation, trip distribution, mode choice, and route assignment similar to the transportation study conducted for the Chicago area (Chicago Area Transportation Study 1959). Trip generation determines the frequency or number of trips for an origin pair based on socioeconomic data. Trip distribution efficiently matches origin-destination (OD) pairs and provides the basis for trip paths made up of three or more OD pairs. Mode choice allocates the proportion of OD trips that will utilize one mode of transportation over another. Route assignment assigns trips to each OD pair via a particular mode of transportation (Meyer and Miller 2001). Assignment can be based on user equilibrium, travel demand, and travel time. Link Performance must be assessed in the form of delays, and passenger queues.

Researchers such as Nagurney, Dafermos, Scheffi, and McFadden (Machovec 1995)(Machovec 1995) have all contributed to the study of transportation research. Nagurney's work defines supernetworks as various combinations of systems which can include transportation (A. Nagurney 2006). Her work helped to provide the framework for a synthesized study of systems which were traditionally only considered as separate entities. The multiple modes of transportation in the CS model qualify as supernetwork. Sheffi's research of urban transportation networks considered systems analysis which used optimization to solve for deterministic user equilibrium and system optimal conditions (Sheffi 1984). Dafermos and Sparrow contributed to the field of transportation analysis through their work on traffic assignment and traffic equilibrium studies(Dafermos 1980). McFadden conducted work in the field of econometrics on travel demand models and behavior (Domencich 1975).

Like other systems analysis, transportation systems planning analysis is often conducted in isolation of systems which impact it and vice versa. This research explores transportation systems planning and shows its integration with another transportation system analysis, namely airline schedule planning. The models which comprise transportation systems planning follow.

## 2.1.1 Trip generation

Trip generation modeling uses socioeconomic measures to estimate and predict aggregate numbers of travelers. This type of model typically uses logit regression to approximate the probability of 0, 1, and 2+ trips (Sheffi 1984). The output of the trip generation model is an origin matrix which quantifies the initiation of expected movements from a given region. This matrix becomes a critical input to the destination choice model.

#### 2.1.2 Destination Choice

Destination choice modeling is a means of approximating the attraction between two entities. In this work, the attraction is measured in travel demand. Destination choice modeling is conducting using the traditional gravity model shown below (de Grange, Fernández, and de Cea 2010). The use of a gravity model allows for a reasonable estimate of the travel demand.

$$T_{ij} = P_i \left[ \frac{F(t_{ij})A_j}{\sum_z F(t_{ij})A_j} \right]$$

where

 $T_{ij}$  = number of trips produced in zone i and attracted to zone j

 $P_i$  = total number of trips produced in zone i

F() = the decay function; the rate at which a zone's attraction declines with increasing travel time:  $(1/distance_{ii})^2$ 

 $t_{ii}$  = the minimum zone-to-zone travel time.

 $A_i$  = number of trips attracted to zone j based of the number of households in zone j.

z = the total number of zones.

The results of the gravity model take the form of an origin-destination matrix and become the primary input to the main choice model.

#### 2.1.3 Mode choice

Mode choice modeling, like trip generation modeling, uses socioeconomic measures to estimate and predict user decisions regarding mode of travel given a set of transportation mode choices.

This type of model also uses logit regression to approximate the probability of individual or group travel. The output of the mode choice model is a probability matrix which provides the likelihood that a particular mode of transportation will be utilized for a given origin-destination pair. The output of the mode choice model becomes the input demand for the route assignment model.

### 2.1.4 Route assignment

Route assignment modeling seeks to establish equilibrium conditions based on one of two primary strategies. The two strategies are system optimality (Koike 1970) and user equilibrium (Konishi 2004). Transportation providers most benefit from a network operating under system optimal conditions where the system is at its most efficient. User equilibrium conditions most benefit the transportation user such that no benefit is achieved from unilaterally making a transportation route choice change. The aggregate analysis of user-level route choices in transportation systems planning model is the study of network equilibrium.

## 2.1.5. Network Equilibrium

This research explores network equilibrium conditions from the point of system optimality in the model integration chapter as well as user equilibrium in the game theoretic optimization chapter to explain the cause and effect relationships that related systems have with each other. System optimality is often the goal of transportation service providers. When operating at system optimal conditions, a network is at its most efficient and cost effective state. User equilibrium implies that a network or system is balanced such that an individual user gains no advantage by

making an alternative transportation decision. User equilibrium can be described in many practical terms to include traffic flow, and user decision choice. Network equilibrium can be described in terms of Wardrop principles of route choice (Wardrop 1953). The first deals with user equilibrium and states that "the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route". The second principle deals with system optimality and states that "at equilibrium the average journey time is minimum". Both principles address the conditions surrounding users traveling to and from the same origin and destination given multiple travel routes. Using the first principle, users seeks to minimize their own cost of travel where cost is judged in travel time. Equilibrium is reached when a single user cannot further minimize their individual cost by choosing an alternative route. Using the second principle, users behave cooperatively for the benefit of minimizing the overall system cost without regard for their individual cost.

## 2.2 Cambridge Systematics Travel Study

The Cambridge Systematic Travel Study is used as the parent model for the model simplification analysis. A study of travel behaviors in the California Corridor was performed by Cambridge Systematics (CS) under commission from the California High Speed Rail Authority in 2008 (Cambridge Systematics 2008). The Cambridge Systematics Study was conducted to assess the interregional commercial traffic in the state of California and assess the ridership and revenue of the California High-Speed Rail project to justify the building of high-speed rail as a means of alleviating the commercial air demand and congestion in California.

The overall model design included urban travel, interregional travel, external travel and trip assignment. Urban travel included areas beginning and ending in the San Francisco Bay

area, Greater Los Angeles, or San Diego regions. Interregional trips included those with both ends in California but in different regions. External trips consisted of trips with one end in California and the other end outside of California. The study consisted of the development of numerous demand models to estimate the trip frequency, destination choice, access/egress, and main mode choice of California travelers. The access/egress models were nested by main mode choice into drive/park, drop off, rental car, and those that didn't drive. Those that didn't drive were further segmented by those who traveled by taxi, transit, and walkers/bikers. The main mode choice modes were segmented by auto and non-auto. The non-drivers were further segmented by air, conventional rail, and high-speed rail travelers.

Model analysis was conducted using traffic analysis zones focusing on the small town and city level. The interregional travel models included travel survey data sources, highway and transit networks, and socioeconomic data. The survey data consisted of revealed preference and stated-preference mode choice data from air, rail, and auto passengers. A total of 3,172 surveys were conducted: 1,234 airline, 430 rail, and 1,508 auto. The socioeconomic data consisted of household, and employment data. The household data consisted of household size, income group, number of workers and car ownership. The employment data consisted of retail, service, and other.

The Cambridge Systematic study was conducted based on traffic analysis zones across 14 regions of California for a total of 4,667 zones. The California Corridor problem contains four modes of transportation: commercial air, high-speed rail, conventional rail, and privately owned vehicle. The travel market was segmented by purpose: business, commute, recreation and other, and by trip length: long trips (>100 miles) and short trips (<100 miles). The travel market was also segmented by those who traveled alone and those who traveled in a group.

The Institute of Transportation Studies (ITS) at Berkeley conducted a review of the California High-Speed Rail Ridership and Revenue Forecasting Study (Brownstone, Hansen, and Madanat 2010). Below are excerpts from the Berkeley ITS Review.

The review found the demand forecasting models unreliable for policy analysis...The mode choices of the individuals surveyed were not representative of California interregional travelers...The mode shares actually used by the travelers were not representative of traveler population...Since it is likely that travelers on different modes attach different degrees of importance to different service attributes (e.g. air travelers care more about travel time than auto travelers), it is likely that the resulting model gives a distorted view of the tastes of the average California traveler... Unfortunately, the methodology employed by CS for adjusting the model parameters has been shown to be incorrect for the type of model they employed. The parameters are therefore invalid and the forecasts based on them, in particular of high speed rail mode shares, are unreliable. (It should be noted that at the time CS performed the study the incorrectness of their adjustment method was not known.)...CS changed key parameter values after the model development phase because the resulting estimates did not accord with the modelers' a priori expectations...Specifically, the modelers increased the parameter for headway (the time between successive aircraft or train departures) and set it to a value typically found in intra-regional travel demand models. This adjustment made the predicted shares of the travel modes very sensitive to changes in frequency...The CS model employed a model structure that does not allow for travelers to choose between high speed rail stations...In the model validation phase, several parameters of the mathematical model were adjusted...As a result of this process, many of the model parameters were assigned values that were considerably different from those obtained in the model development phase. In some instances changes to the model parameters were informed by professional judgments of the consulting team as well as the goal of replicating observed behavior. The resulting "validated" model, which is used to generate subsequent high speed rail ridership forecasts, provides reasonably accurate "backcasts" for the year 2000, reflects certain patterns of behavior observed in the traveler surveys, and accords with professional judgments of the consultant. However, the combination of problems in the development phase and subsequent changes made to model parameters in the validation phase implies that the forecasts of high speed rail demand—and hence of the profitability of the proposed high speed rail system—have very large error bounds. These bounds, which were not

quantified by CS, may be large enough to include the possibility that the California HSR may achieve healthy profits and the possibility that it may incur significant revenue shortfalls.

This research addresses two critiques of the Cambridge Systematics model: its computational expense and its assumption of no response from other transportation service providers such as commercial airlines. The model simplification of the first research objective is designed to address the computational expense critique. The multidisciplinary optimization model integration and game theoretic competitive equilibrium analysis of objectives two and three are designed to address the assumption of not considering the competitive response from other transportation service providers. Objective two utilizes two system-of-systems practical applications: transportation systems planning and airline schedule planning. These systems consist of individual models with interdependent inputs and outputs. Using the interactions of these inputs and outputs, this research seeks to address the modeling, resourcing, pricing, and viability issues surrounding the California high-speed rail project.

## 2.3 Mean Value First Order Second Moment

The mean value first order second moment (MVFOSM) method is used to determine the percent contribution to variance of the input parameter coefficients for the model simplification. Other analytical reliability techniques include Monte Carlo Simulation (MCS)(Mooney 1997), First-Order Reliability Method (FORM) (Chiralaksanakul and Mahadevan 2005) and Second-Order Reliability Method (SORM) (Hohenbichler et al. 1987). MCS is the most accurate and computationally expensive of the four methods. Both FORM and SORM are less accurate and computationally expensive than MCS, but more accurate and computationally expensive than

MVFOSM. Given the low level accuracy of the parent model, the MVFOSM is the best choice taking both accuracy and computational expense into account. MVFOSM is based on a first – order Taylor series approximation linearized at the mean values of the random variables. The probability of failure is based on a safety index defined as the ratio of the mean to the standard deviation where Z is the performance function, R is the resistance, and S is the load.

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

The probability of failure in terms of the safety index follows.

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta)$$

The generalization of the performance function for multiple random variables is shown below.

$$Z = g(X) = g(X_1, X_2, ..., X_n)$$

Next is a Taylor series expansion of the performance function about the means values followed by the first-order approximate mean and variance of Z.

$$Z = g(\mu_X) + \sum_{i=1}^n \frac{\partial g}{\partial X_i} \left( X_i + \mu_{X_i} \right) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g}{\partial X_i \partial X_j} \left( X_i - \mu_{X_i} \right) \left( X_j - \mu_{X_j} \right) + \cdots$$

$$\mu_Z \approx g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n})$$

$$\sigma_Z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \frac{\partial g}{\partial X_i} \frac{\partial g}{\partial X_j} Cov(X_i, X_j)$$

Where  $Cov(X_i, X_j)$  is the covariance of  $X_i$  and  $X_j$ . Assuming uncorrelated variables, the variance is shown below. This is the basis for determining the percent contribution to variance of the input parameters utilized for the model simplification.

$$\sigma_Z^2 \approx \sum_{i=1}^n \left(\frac{\partial g}{\partial X_i}\right)^2 Var(X_i)$$

## 2.4 System-of-Systems

Each mode of the commercial transportation network in this analysis can be categorized as its own system, so the combined multimodal network by definition is a system-of-systems. A system-of-systems, in this research, is defined as a network of systems. Figure 2.1 displays a general systems interaction using optimization in system-of-systems modeling (Smith 2007).

Systems-of-systems have also been described as supernetworks (Anna Nagurney and Toyasaki 2003). A multi-modal transportation network such as the California high-speed rail problem is one example of a supernetwork. According to Nagurney, "Supernetworks may be comprised of such networks as transportation, telecommunication, logistical and financial networks, among others." Nagurney's studies link human choice and network performance within a complex network (A. Nagurney 2006). This is applicable as the proposed research will link human choice to the performance of a complex network of multiple modes. Keating and his co-authors describe systems of systems as meta-systems that "are themselves comprised of multiple autonomous embedded complex systems that can be diverse in technology, context,

operation, geography and conceptual frame (Keating et al. 2009)." The modes of transportation in this research are typically analyzed as separate entities. This research treats the California network as a system of systems. System-of-systems analysis requires the analysis of multiple stakeholders. In this research, stakeholders include the airport, airline, high-speed rail, and highway transportation mangers. The last stakeholder, and potentially the most difficult to predict, is the interregional passenger of this multi-modal network. Accurate modeling of interregional passenger requires the use of multidisciplinary optimization, while accurate modeling of the competitive airline requires the use the game theory. Formulating this problem includes aspects of mode choice, link performance, and user equilibrium. While previous works considered the modes as separate entities, the proposed research considers them under one analysis.

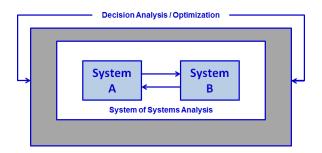


Figure 2.1: Generic System-of-Systems Framework

Systems analysis is typically application and problem domain specific. As a result, domain and application specific systems analysis model output does not lend well to collaboration with other systems. The models lack the cohesive structure required to synthesize analysis across multiple domains and applications. The system-of-systems problem domains

and applications used in this research include transportation systems planning, airline schedule planning, and network equilibrium through multidisciplinary optimization and game theory.

## 2.5 Multidisciplinary Optimization

Multidisciplinary optimization problems solve problems spanning multiple disciplines while conventional methods solve problems with a single discipline (Arora 2007). Multidisciplinary optimization is used to perform the model integration of transportation systems planning and airline schedule planning. The general form of the multidisciplinary optimization problem is shown in Figure 2.2. Two particular MDO methods are utilized during this analysis, multidisciplinary feasible (MDF) and simultaneous analysis and design (SAND). MDF requires convergence of the analysis codes at every iteration of the optimizer (and at every finite difference point if numerical approximation of the gradients is to be used). SAND does NOT require interdisciplinary compatibility (convergence of analysis codes) until the end of the analysis. The model integration utilizes the multidisciplinary feasible method as the fleet acquisition and resourcing design variables are utilized to maximize profit subject to the price condition state variables. The game theoretic optimization utilizes the simultaneous analysis and design method are the both the resource acquisition and allocation variables along with the pricing states variables are utilized to maximize profit.

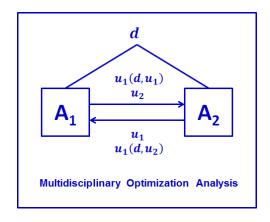


Figure 2.2: Multidisciplinary Optimization Analysis

## 2.5.1 Multidisciplinary Feasible

The multidisciplinary feasible (MDF) formulation is an 'all-in-one' method where the interaction of two systems or analysis methods is the basis of a single optimization (Arora 2007). The MDF formulation follows.

Analysis 1: 
$$A_1(d, u_1, u_2) = 0$$

Analysis 2:  $A_2(d, u_1, u_2) = 0$ 
 $Min_d c(d)$ 

Subject to:

$$A_1\left(d,u_1,u_2
ight)=0 
ightarrow e.g.$$
 Fluid Dynamics 
$$A_2\left(d,u_1,u_2
ight)=0 
ightarrow e.g.$$
 Structural Engineering 
$$g\left(d,u_1,u_2
ight)\geq 0$$

In this method, the cost associated with the design variable, d, is minimized subject to the interaction of two analyses or systems with interdependent states variables such as boundary constraints and capacities,  $u_i$ .

## 2.5.2 Simultaneous Analysis and Design (SAND)

In Simultaneous Analysis and Design (SAND) problems, both the design variables and state variables are included as optimization variables (Arora 2007). The SAND formulation follows.

$$Min_{d,u_{1}^{*},u_{2}^{*}} c(d)$$

Subject to:

$$u_1(d, u_2^*) - u_1^* = 0$$

$$u_2(d, u_1^*) - u_2^* = 0$$

$$g(d, u_1^*, u_2^*) \ge 0$$

In the SAND formulation, the cost of supporting the design variable, d, along with the state variables,  $u_i$ , are minimized.

## 2.6 Airline Systems Planning

Airline schedule planning and transportation systems planning are related systems with inputs and outputs that affect each other. Unfortunately, these two systems are commonly studied in isolation. This research bridges the gap of research conducted in isolation by showing the integration of these two transportation systems through the use of multidisciplinary optimization.

Airline schedule planning consists of four airline planning problems which include schedule design, fleet assignment, crew scheduling, and aircraft maintenance scheduling (Barnhart and Cohn 2004). This work focuses primarily on the fleet assignment and schedule design aspects of airline schedule planning. Airline schedule planning problems are traditionally formulated as optimization problems designed to minimize some cost, or maximize some benefit given the requirement to provide transportation service to a transportation demand distributed across multiple origin-destination pairs.

Airline schedule planning and its four components have been developed by the work of researchers such as Barnhart and Cohn (Barnhart and Cohn 2004). The schedule design problem identifies the origins and destination pairs serviced by an airline and at what frequency (Jiang and Barnhart). Fleet assignment models assign aircraft to support customer demand. The goal of the fleet assignment problem is to minimize the cost of providing aircraft to meet demand (Dumas, Aithnard, and Soumis 2009). The aircraft maintenance routing problem determines how best to allocate aircraft to support passenger demand while adhering to maintenance requirements (Gopalan and Talluri 1998). The crew scheduling problem assigns flight crew to serviced flights with the goal of minimizing cost (Cohn and Barnhart Jun2003). This work focuses on the schedule design problem and the fleet assignment problem. A more detailed review of schedule design and fleet assignment follow.

#### 2.6.1 Schedule Design

The schedule design problem determines the frequency that flights are scheduled in support of customer demand (Barnhart and Cohn 2004) The goal of the schedule design problem is to minimize the operational cost of supporting customer demand by assigning aircraft flights across

multiple origin-destination pairs. The schedule design problem can have various objectives to include minimize cost, maximize profit, maximize revenue, or maximizing expected market share. The schedule design formulation is shown below

$$Min_{x_{od}}C(x_{od}^f)$$

where

$$\sum_{o} \sum_{d} x_{od}^{f} < MOPD_{f} * N_{f} \forall f \in F$$

(Resource Constraint)

The decision variables become  $x_{od}^f$ 

$$C(x_{od}^f) = Total Aircraft Operating Expenses$$

C = Cost

*Fleets:*  $f \in F$ 

*Origins:*  $o \in O$ 

*Destinations:*  $d \in D$ 

 $N_f$  = Number of aircraft of fleet f

 $MOPD_f = Max \ Ops \ per \ day \ for \ an \ aircraft \ of \ fleet \ f$ 

## 2.6.2 Fleet Assignment

The resource allocation optimization portion of this problem can best be described as a fleet assignment problem. Abara defines "the goal of the fleet assignment problem is to assign as many flight segments as possible in a schedule pattern to one or more aircraft types while optimizing some objective and meeting various operational constraints." (Abara 1989) These constraint equations ensure that each flight is flown by only one fleet and maintain the conservation of flow of aircraft (Subramanian et al. 1994)(Cordeau et al. 2001)(Ioachim et al. 1999). Current fleet assignment models for passenger transportation are primarily unimodal

(Shan Lan, Clarke, and Barnhart 2006). Davendralingam and Crossley formulate a dynamic programming formulation for aircraft design using passenger demand models (Davendralingam and Crossley 2010). This problem contains many complexities. Resource acquisition and resource allocation decisions are made over time. The problem can be multi-objective. The goals of maximizing profit and minimizing total travel time or travel delay will most likely have differing optimal solution sets. Conflicting optimal solution sets means that no single solution is likely to solve all problems. The multi-modal aspect of this problem requires the analysis of multiple vehicle types. Without some method to evaluate the total network as a whole, alternatives are hard to evaluate on their own. Lastly, this problem must model user mode choice given network conditions at various stages in time. Based on these complexities, this problem is too big to manage without a disciplined approach.

The fleet assignment problem, in this chapter, assigns aircraft to support passenger demand while minimizing the cost of operating expenses. The objective of the fleet assignment problem is to minimize operating costs. The decision variables are the number of aircraft from a given list of fleets which are assigned to each origin-destination pair based on the travel demand. Typically, a fleet assignment problem has several different constraint types: fleet size, flight coverage, flow balance, continuity and schedule balance (Barnhart, Belobaba, and Odoni 2003). Fleet size restricts the analysis to the number of available vehicles. Flight coverage (cover rows) ensures only one fleet covers a leg. Flow balance requires the number of aircraft departing from and arriving at a given airport is equal. Continuity ensures low volume leg operation in a multicity route. Schedule balance ensures same fleet services a multi-city route. The fleet assignment formulation is shown below.

$$Min_{x_{od},p_{od}}C(x_{od}^f)$$

where

$$\sum_{o} \sum_{d} x_{od}^{f} < MOPD_{f} * N_{f} \forall f \in F$$

(Resource Constraint)

(Aircraft Conservation/Flow Balance)

The decision variables become  $x_{od}^f$ 

$$C(x_{od}^f) = Total Aircraft Operating Expenses$$

C = Cost

*Fleets:*  $f \in F$ 

Origins:  $o \in O$ 

*Destinations:*  $d \in D$ 

 $N_f = Number of aircraft of fleet f$ 

 $MOPD_f = Max \ Ops \ per \ day \ for \ an \ aircraft \ of \ fleet \ f$ 

where

$$f = \{1, 2, 3, 4\}$$

$$x_{od}^f \geq 0$$
, integer

# 2.7 Game Theory

Game theory is a theoretical framework for analyzing decision scenarios encountered by multiple decision-makers in a common scenario. In this research, game theory is used to determine the

equilibrium pricing conditions for the model integration problem and to determine the optimal resourcing and pricing for the multiple airline optimization analysis. Game theory is used in this research to address the following potential situation:

"Suppose HSR creates a dominating presence in a given travel market and forces an airline out of heavily servicing that market; therefore, it has these planes available. What will the airline do with those planes?"

In this research, optimization is combined with game theory to determine equilibrium conditions of a system with a non-linear relationship between player decisions and their payoffs. This equilibrium relationship is defined as Nash Equilibrium. A Nash equilibrium exists when no player benefits from unilaterally changing their strategy (Giocoli 2004). A Nash Equilibrium is a solution of a two or more player game where the equilibrium strategies of all players are commonly known by each player (Osborne 1994). The Nash equilibrium can be found by determining simultaneous best responses using best response functions. The best response function derives the most beneficial reaction to the conditions presented by the remaining players in a non-cooperative game (Rey-Biel 2009).

There are two categories of games to include cooperative or non-cooperative games. In cooperative games, the players operate under agreed upon conditions often defined by a contract (Driessen 1988). In non-cooperative games, the players operate to satisfy self-serving goals (Nash 1951). This work involves non-cooperative games where the players compete by choosing the optimal resourcing and pricing with the goal of maximizing profit or ridership. A competitive game consists of several components to include players, strategies, and payoffs (Rasmusen 1989). The players are the decision-makers whose decisions are based on strategies which dictate their actions and result in payoffs and penalties. Strategies are actions that define

player decisions such as cost minimization, and revenue, profit, or market share maximization (Stahl 1988). Payoffs are the benefits gained by making a particular decision given other stated actions. Payoffs are often defined in terms of functions based on player decisions. In this research, the competing airlines and high-speed rail decision-makers are the players. Optimizing resourcing decisions and prices to maximize profit and ridership becomes the player strategies. The resulting profit and ridership are the payoffs. This research contains several modeling assumptions. (1) Multiple players produce a homogenous product. (2) Players do not cooperate. (3) The number of players is held constant. (4) Players choose product or service quantities simultaneously. (5) Players are assumed to know each other's potential decision options and payoffs. (6) Players are also expected to choose the option that is the most beneficial to them.

#### **CHAPTER III**

# PARSIMONIOUS TRAVEL DEMAND MODELING FOR MULTIMODAL TRANSPORTATION SYSTEM OF SYSTEMS

"Two roads diverged in a wood and I, I took the one less traveled by and that has made all the difference." – Robert Frost

#### 3.1 Introduction

Transportation system planning requires the use of demand models for decision support. Transportation demand models are widely used to forecast interregional travel demand for the purpose of providing decision support in choosing potential transportation projects such as high-speed rail (HSR) projects in various corridors in the United States, to include the Midwest Corridor HSR project which plans to link Chicago, Detroit, and St. Louis, and the Northeast Corridor HSR project which plans to link Washington D.C., New York, and Boston.

Transportation demand models are often large and computationally expensive to evaluate due to the amount of required data and calculations performed with these large data sets. Furthermore, due to the sparseness of the data, model input parameters are uncertain. It is important to account for the uncertainty in model predictions when using these models for decision support to assume the reliability of predictions of future system wide conditions. Using these models for iterative model analyses is required for decision support activities such as sensitivity analysis, uncertainty quantification, and optimization. In this chapter, the required computational efficiency will be achieved by developing simplified models for demand

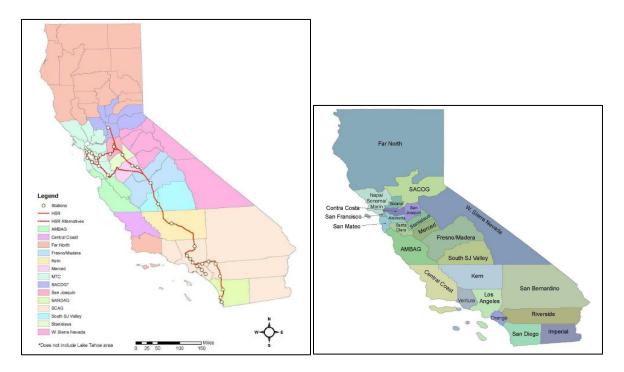
estimation. Also the simplified travel demand model will be exploited using computationally efficient methods for sensitivity analysis and uncertainty quantification.

To address the issue of computational expense in travel demand modeling, one solution is proposed: a *parsimonious travel demand model* (PTDM) to estimate multimodal travel demand. A parsimonious model refers to a model which utilizes a reduced number of data parameters or input variables (Ho and Chong 2003). The PTDM proposed in this chapter is derived from a proprietary parent model; in this case, the Cambridge Systematics (CS) Interregional Travel Model System (ITMS) trip frequency and main-mode choice models for the California HSR Ridership and Revenue Study (Cambridge Systematics 2008). This research develops the PTDM, a derived and simplified model of travel demand in the California corridor, and uses it to perform uncertainty quantification and sensitivity analysis for parameters listed in the Cambridge Systematics ITMS trip frequency and main-mode choice models for the California High-Speed Rail (HSR) Ridership and Revenue Study (Cambridge Systematics 2008).

Neither this chapter, nor the PTDM with uncertainty quantification, addresses the accuracy or reliability of the CS model forecasts. The PTDM simplifies a complex travel demand model for the purpose of reducing computational expense, while the uncertainty quantification illustrates a method to identify the key input parameters for use in repetitive optimization and sensitivity analysis. This chapter does not attempt to assess the reliability of the parent model forecasts, and the results in this chapter should not be used to infer the level of uncertainty in the California High-Speed Rail Authority's Ridership & Revenue forecasts. The analysis of the planned California High-Speed Rail system is only used in this chapter to illustrate how the proposed methodology can be applied.

# 3.2 Parent Model Description

The California Corridor is an example of a large-scale multimodal interregional transportation network. It consists of automobile, commercial air, and conventional rail networks. To address its interregional transportation congestion, California is planning to add HSR to its commercial passenger transportation network. The planned HSR network shown in Figure 3.1 will connect the cities of the San Francisco Bay Area, Sacramento, Fresno, Bakersfield, Los Angeles, and San Diego.



(AMBAG – Association of Monterey Bay Area Governments, MTC - Metropolitan Transportation Commission, SACOG – Sacramento Area Council of Governments, SANDAG – San Diego Association of Governments, SCAG – Southern California Association of Governments)

Figure 3.1: California Regions & Proposed HSR Station Locations

Cambridge Systematics (CS) was tasked to perform an interregional travel demand forecasting study in California for the California High-Speed Rail Authority in 2008 (Cambridge Systematics 2008). The CS study consisted of numerous logit demand models to estimate the

trip frequency, destination choice, access/egress mode choice, and main-mode choice of California travelers utilizing four modes of transportation: car, commercial air, conventional rail, and HSR. The trip frequency model segments the model output into zero, one, or two trips per household per day. The destination choice model contains variables reflecting the influence of different area types and destination districts, as well as other factors listed in Table 3.1, where the destination districts are shown in Figure 3.1. The main-mode choice model alternative set consists of car, air, conventional rail, and high-speed rail. All model components were estimated using data from stated preference surveys and revealed preference surveys collected at the household level for intraregional trips and intercept surveys at airports and other locations. Based on these surveys, models of California interregional travel were developed that were segmented by travel purpose, and distance traveled, with variables reflecting household size, year 2000 household income range, household automobile ownership, number of workers in a household, and travel party size. The access/egress mode choice model alternative set consists of drive/park, drop off, rental car, and those that didn't drive. Those that didn't drive are further categorized into taxi, transit, and walk/bike. The access and egress mode choice models are based on reported mode use from survey data and include variables for trip cost, in-vehicle travel time, out-of-vehicle travel time, and household demographics.

The CS ITMS follows the traditional four-step travel demand modeling approach and consists of sequential logit demand models to estimate the trip frequency, destination choice, access/egress mode choice, and main-mode choice of California travelers utilizing the four modes of transportation noted above (Cambridge Systematics 2008). The purpose of the CS model system study was to forecast future ridership on the proposed California HSR system and to provide decision support for planning the HSR system as a means of reducing the demand on

existing modes of transportation in order to alleviate future transportation system congestion in California.

The CS model is complex and computationally expensive to run when conducting repetitive applications to explore the sensitivity of the outcomes. For example, one execution of the CS model can take several days due to the large number of travel analysis zones used in the model. The CS model adopted a travel analysis zone system based on a statewide model developed by the California Department of Transportation, with greater resolution in selected urban areas using the travel analysis zone system developed for those regions by the respective metropolitan planning organizations for their regional travel demand models, and in consequence contains 4,667 zones (Cambridge Systematics 2008). Adjusting the travel analysis zone resolution is a primary feature of the PTDM implementation.

Trip Frequency	<b>Destination Choice</b>	Main-Mode Choice
Level of Service	Level of Service	Main-Mode Characteristics
Intraregion accessibility	Mode choice logsum	Constants
Mode/destination choice logsum	Distance (miles)	Car (base)
	Distance squared/100	
Household Characteristics	Distance cubed/10,000	Air
Medium Income	Anna Trma	Conventional Rail
High Income Fewer cars than workers in Household	Area Type Urban destination	_ High-Speed Rail
rewer cars than workers in Household No cars in Household	Rural destination	Level of Service
Fraction of household who are workers	Urban to Urban	Cost (\$)
No workers in household	Suburban to Suburban	In-vehicle time (min)
Household Size	Rural to Rural	Service Headway (min)
1 person household		Reliability (% on time)
3+ person household	<b>Destination District</b>	
	Alameda	Trip Characteristics
Location Variables	AMBAG	Travel in a Group
SACOG resident	Central Coast	Car
SANDAG resident	Contra Costa	Air
SCAG resident	Far North	
MTC resident	Fresno	Household Characteristics
	Kern	Household Size
Constants	Los Angeles	Car
1 trip	Marin/Sonoma/Napa	
2+ trips	Merced	Income
	Orange Riverside	High - car
	S. San Joaquin	High - air High - conventional rail
	San Bernardino	High - high-speed rail
	San Francisco	nigh - high-speed ran
	San Joaquin	Fewer Cars than Workers
	San Mateo	Car
	SANDAG	
	Santa Clara	Nesting & Scaling
	Solano	Nest - air, rail, high-speed rail
	Stanislaus	Access mode choice logsum
	Ventura	Egress mode choice logsum
	W. Sierra Nevada	
	Regional Interactions	_
	MTC to SCAG	
	MTC to SANDAG	
	SACOG to SANDAG	
	SCAG to MTC	
	SCAG to SACOG SANDAG to MTC	
	SANDAG to MTC SANDAG to SACOG	
	Size Variables	_
	Other Employment	
	Households Retail Employment - Low Income	
	Retail Employment - Low Income Retail Employment - Medium Income	
	Retail Employment - High Income	
	Service Employment - Low Income	
	Service Employment - Medium Income	
	Service Employment - High Income	
SACOG - Sacramento Area Council of Gov		
SANDAG - San Diego Association of Gover		
SCAG - Southern California Association o		
MTC - Metropolitan Transportation Comm	issian (Can Francisca)	

Table 3.1: Cambridge Systematics Model Variables

# 3.3 PTDM Implementation

The PTDM, unlike the proprietary parent CS model system, contains only three primary model types: trip frequency, destination choice, and main-mode choice. The PTDM was constructed using input parameters and their coefficients from the parent CS model; however, the data used to populate the input parameters was derived from publically available sources as the CS model data was not available. PTDM data sources include census 2000 data, California Department of Motor Vehicles and Bureau of Transportation Statics. The PTDM uses data and models that are similar to the CS models to illustrate how issues of travel demand uncertainty quantification can be answered. The PTDM described in this chapter adjusts the model resolution by redefining the travel analysis zones to the counties of California, thereby reducing the number of analysis zones from the 4,667 zones used in the ITMS to 58 (Cambridge Systematics 2008). For each travel analysis zone, distances to and from other counties, airports, conventional rail, and HSR stations are based on the most populated city in each county. Interregional travelers originate from a county in one California region and have a destination in another region as shown in Figure 3.1. The model assumes that the county-level household characteristic attributes are uniform throughout each county. The access and egress mode choice models were eliminated from the PTDM since access and egress models would require multiple county-level distance metrics. Based on this elimination, this PTDM assumes little to no impact on the trip frequency and mainmode choice output from the access and egress models.

		Trip Freq	luency Mod	el Varial	ole Coeffic	ients				
	Long Trips				Short Trips				CS Model	PTDM
	Business	Commute	Recreation	Other	Business	Commute	Recreation	Other	* = usec	l in model
Level of Service										
Intraregion accessibility	-0.128	-0.217	-0.4	-0.532	-0.329	-0.176	-0.438	-0.536	*	Omitted
Mode/destination choice logsum	0.466	0.123	0.656	0.159	0.205	0.262	0.262	0.22	*	Optimized
Household Characteristics										
Medium Income	0.527	0.188	-	-	0.331	1.045	0.355	-	*	*
High Income	1.139	0.291	-0.246	0.393	0.835	1.523	0.432	-	*	*
Fewer cars than workers in Household	-0.412	-0.457	-0.922	-0.915	-0.947	-0.225			*	*
No cars in Household	-	-	-	-	-	-	-1.27	-0.736	*	*
Fraction of Household who are workers	0.537	1.274	-	-	1.153	1.57			*	*
No workers in Household	-2.098	-2.668	-	0.372	-0.863	-2.163	0.493	-	*	*
Household Size	-	=	=	-	-	=	-0.136	-		
1 person household	_	-	-	-0.424	-	-	-0.401	-	*	*
3+ person household	-	-	-0.482	-0.379	-	-	-	-	*	*
Location Variables										
SACOG resident	0.976	0.918	1.084	2.527	977	-2.736	-1.241	-1.177	*	*
SANDAG resident	-0.704	-0.419	1.344	0.92	-0.88	-1.446	-1.802	-0.66	*	*
SCAG resident	-1.176	-1.644	-0.031	0.259	-1.969	-1.524	-1.16	-1.265	*	*
MTC resident	-1.372	-0.729	1.011	1.134	-1.275	-1.982	-0.25	-0.524	*	*
Constants										
1 trip	-15.67	-6.48	-3.416	-0.493	-4.946	-8.242	-2.881	-0.845	*	*
2+ trips	-16.3	-7.914	-5.083	-2.823	-5.513	-9.07	-3.787	-1.624	*	*

Table 3.2: Trip Frequency Model Coefficients

SANDAG - San Diego Association of Governments SCAG - Southern California Association of Governments MTC - Metropolitan Transportation Commission (San Francisco)

Cambridge Systematic model coefficients given in the Cambridge Systematics model documentation were used in the PTDM trip frequency models as shown in Table 3.2. These coefficients were applied to socioeconomic data derived at the county level from publically available sources. The PTDM trip frequency models and the Cambridge Systematic trip frequency models utilize a similar structure. These models segment trip frequency into short and long trips by trip purpose to include business, commute, recreation, and other, for a total of eight models. The trip frequency models for both long and short trips, and trips greater than or less than 100 miles, contain continuous and categorical variables. The level-of-service variables are continuous, while the location and number-of-trip variables are categorical. The household characteristic variables are continuous, but in some cases were treated as dummy variables or

categorical variables. Two variables from the CS model were not used in the PTDM model: interregional accessibility, and mode/destination choice logsum. The logsum measures, used in the CS model, are a means to estimate a weighted average of travel time and cost that can be fed from one model to another (Cambridge Systematics 2008). For the initial CS model estimation, a synthesized network zone accessibility measure was used, the details of which are not available from the model documentation. The destination/main-mode choice logsum was also not computed as the estimation of the CS destination choice model used a mode choice logsum calculation from a Caltrans statewide model (Cambridge Systematics 2008) the details of which are also not available from the model documentation; instead, an equivalent logsum value for each county was inferred via optimization as shown below to calibrate the PTDM to the CS model. The optimization minimized the sum of squares error between the CS model and PTDM regional trip frequency output results. The CS model coefficients were not re-estimated for use in the PTDM main-mode choice models.

$$Min \sum (CS_i - P_i)^2$$

with respect to the equivalent logsums

where

 $CS_i = CS Model trip frequency of region i$ 

 $P_i = PTDM trip frequency of region i$ 

The intraregion accessibility variable was not utilized due to the change in model resolution. This variable distinguished between locations with destinations within their home region, outside their home region but within 100 miles, and those outside their region and over 100 miles from

their origin. Changing the model resolution to the county level made the intraregion accessibility variable impractical to use since the county distances are based on the most populated city in each county. The impact of not using this variable was not significant to the PTDM results, since the PTDM was not independently estimated, but calibrated to the CS model.

The trip frequency models contained thirty-six segments per model. One example of a household population segment is the high income, three-person household with two workers and fewer vehicles than workers. Each county household population was segmented in accordance with the location and household characteristic variables. While census data provided county household population data in various income brackets, available household size data only provided household averages. Household size segmentation probabilities were derived using Poisson distribution parameters. The Poisson distribution is used to estimate the probability of a single event or a specific number of events given the average occurrence of the event ' $\nu$ ' (Haldar 2000) where ' $\nu$ ' is the average household demographic as shown below.

$$P(Specific \ Number \ of \ People \ in \ a \ Household) = \frac{(vt)^x}{x!}e^{-vt}$$

As an example, the average household size per county was used to estimate the percentage of households with 1, 2, and 3+ members in a household. These probabilities were then applied to estimate the number households in each county with 1, 2, and 3+ household members. A similar approach was taken to estimate the number of workers in a household, the number of households with specific numbers of vehicles, and those with fewer vehicles than workers. Upon household population segmentation, the model variable coefficients were applied to logit formulas to

establish model segment dis-utilities and probabilities for 0, 1, and 2+ trips by household for each segment. These probabilities were then applied to the segmented populations by county to determine the trip frequency for each county.

To reduce the model expense due to complexity, the PTDM destination choice model was conducted utilizing a traditional gravity model utilizing county populations and distances based the highest populated cities in each county. As the PTDM does not quantify the impacts of uncertainty in the destination choice model, the use of a gravity model allows for a reasonable estimate of the travel demand. The gravity model estimation was conducted for each county origin-destination pair from PTDM results calibrated to the CS model results.

The gravity model based destination choice model results shown in Table 3.3, as expected, indicated that a large concentration of interregional travel stems from the San Joaquin Valley, Los Angeles, and San Francisco regions. Due to the county level model resolution, there is a loss between the trip frequency and destination totals for the short trips output. This occurs since several counties do not have destination pairs outside of their region where their most populated cities are within 100 miles of each other. These counties include San Bernardino, Imperial, Del Norte, Humboldt, Lassen, Modoc, Shasta, Siskiyou, and Trinity.

The PTDM main-mode choice models and the Cambridge Systematic main-mode choice models utilize a similar structure. This series of models segments main-mode choice into short and long trips by trip purpose. These purposes include business and other for long trips, and business, commute, and other for short trips for a total of five models. The CS model coefficients, as shown in Table 3.4, were not re-estimated, but used to populate the PTDM main-mode choice models. The main-mode choice models for both long and short trips contain categorical, continuous, and dummy variables. The main-mode choice constant, nesting, and trip

characteristic coefficients as shown in Table 3.4, are categorical, while the level-of-service variables are continuous. Similar to the trip frequency models, the household characteristic variables in the main-mode choice models are continuous; but in some cases were treated as dummy variables or categorical variables. Two variables from the CS main-mode choice models were not used in the PTDM: the 'access mode choice logsum', and 'egress mode choice logsum' variables. This PTDM assumes little to no impact on the trip frequency and main-mode choice output from the access and egress models.

	Short Trips				Long Trips						
Proposed Model Adjusted Destination Choice Results	Commute	Business	Recreation	Other	Commute	Business	Recreation	Other	PTDM Destination Choice Total	CS Model Destination Choice Input	Absolute Difference
LA to Sacramento	4,081	555	1,437	1,684	1,521	1,609	3,824	1,158	15,869	12,414	3,455
LA to San Diego	63,874	7,953	37,228	22,595	29,010	10,660	66,528	15,489	253,336	262,936	9,600
LA to SF	10	3,190	7,362	11,031	16,681	7,989	27,393	4,874	78,531	54,898	23,633
Sacramento to SF	14	6,376	3,783	8,280	10,558	3,651	13,380	2,667	48,708	139,580	90,872
Sacramento to San Diego	646	120	144	128	138	37	245	64	1,521	3,033	1,512
San Diego to SF	0	85	30	42	136	40	197	40	569	14,939	14,370
LA/SF to SJV	28,926	9,935	15,550	23,847	6,345	4,719	23,979	3,343	116,645	209,536	92,891
Other to SJV	101,623	21,273	29,274	69,303	12,868	5,041	9,344	1,767	250,493	282,337	31,844
To/From CC	125,810	16,228	38,818	45,565	35,188	10,755	27,956	5,434	305,753	280,431	25,322
To/From Far North	66,590	17,464	33,171	56,020	30,090	7,094	9,311	1,482	221,222	187,527	33,695
To/From W. Sierra Nevada	30,467	6,264	12,771	22,586	6,514	2,176	2,749	351	83,878	59,871	24,007
Total	422,041	89,442	179,567	261,080	149,050	53,771	184,906	36,669	1,376,526	1,507,502	130,976

Table 3.3: Calibrated Destination Choice Model Output Comparison

M	ain Mode C	Choice M	odel Varia	ble Coeffi	cients			
	Lo	ng	Short			CS Model PTDM		
	Business	Other	Business	Commute	Other	* = used	in model	
Main Mode Characteristics		-	•					
Constants								
Air	-0.1645	0.6898	-	-	-	*	*	
Conventional Rail	-0.387	0.6149	-0.268	4.232	-0.3847	*	*	
High-speed rail	-0.3503	1.434	-1.557	4.048	0.5041	*	*	
Level of Service								
Cost (\$)	-0.01626	-0.035	-0.109	-0.148	-0.109	*	*	
In-vehicle time (min)	-0.016	-0.011	-0.5	-0.025	-0.014	*	*	
Service Headway (min)	-0.003	-0.003	-0.006	-0.0023	-0.009	*	*	
Reliability (% on time)	0.001	0.005	-0.023	0.006	0.004	*	*	
Trip Characteristics								
Travel in a Group								
Car	0.8492	1.417	-	-	-	*	*	
Air	-0.3375	-0.5061	-	-	-	*	*	
Household Characteristics								
Household Size								
Car	0.0704	0.225	-	0.655	-	*	*	
Income								
High - car	-	-	-1.211	-1.247	-	*	*	
High - air	1.018	-	-	-	-	*	*	
High - conventional rail	0.5237	-	-	-	-	*	*	
High - high-speed rail	0.9807	-	-	-	-	*	*	
Fewer cars than Workers								
Car	-0.7696	-0.4354	-0.7873	-2	-	*	*	
Nesting and scaling								
Nest - air, rail, high-speed rail	0.8514	0.7426	0.5159	0.5892	0.6855	*	*	
Access mode choice logsum	0.115	0.2134	0.4628	0.33	0.3148	*	Omitted	
Egress mode choice logsum	0.1561	0.3974	0.4628	0.33	0.3148	*	Omitted	

Table 3.4: Main-Mode Choice Variable Coefficients

The trip segments developed for the trip frequency models are utilized in the main-mode choice models. The primary effort for the main-mode choice models consists of establishing level-of-service parameters for HSR and assigning conventional rail, commercial air, and HSR stations to county origin-destination pairs containing multiple rail stations, and airports. Using the trip frequency household population segmentation, the model variable coefficients were also

applied to logit models to establish model segment dis-utilities and probabilities for car, air, conventional rail, and HSR travel by segment. These probabilities were then applied to the segmented populations by county to determine main-mode choice distributions for each county. Utilizing year 2000 trip frequency estimates, Table 3.5 shows the main-mode choice splits from regional origins.

Region	Car		Air		CR		HSR		Total	
LA	261,830	20.3%	2,974	19.4%	1,299	13.5%	2,296	1.7%	268,399	
SAC	112,220	8.7%	661	4.3%	175	1.8%	15,719	11.8%	128,775	
SD	85,795	6.7%	4,220	27.5%	5,973	62.2%	20,611	15.4%	116,599	
$\mathbf{SF}$	197,045	15.3%	2,394	15.6%	617	6.4%	36,225	27.1%	236,281	
SJV	344,103	26.7%	2,562	16.7%	533	5.6%	33,290	24.9%	380,488	
CC	179,539	14.0%	2,339	15.2%	917	9.6%	16,499	12.3%	199,294	
FN	82,020	6.4%	161	1.0%	63	0.7%	6,887	5.2%	89,130	
WSN	24,339	1.9%	56	0.4%	23	0.2%	2,177	1.6%	26,594	
Total	1,286,892		15,367		9,600		133,701		1,445,560	
		LA = Los	Angeles			SJV = Sa	n Joaquin '	Valley		
		SAC = Sa	cramento			CC = Central Coast				
		SD=San	Diego							
		SF = San	Francisco			WSN = V	Vest Sierra	Nevada		
		CR = Con	ventional	Rail						

Table 3.5: PTDM Main-Mode Choice Prediction Results using year 2000 data.

This implementation of the PTDM has a primary limitation as compared to the proprietary parent CS Model. The PTDM modeling resolution is at the county level or aggregation of counties. Larger geographic areas are limited to combinations of multiple counties. Model results that include sections of a county are not possible. This research was conducted without the actual CS model and without the input data used in the CS model. Due to the lack of data availability, this analysis was conducted with public record sources, including

the Bureau of Transportation Statistics, California Department of Motor Vehicles, the Bureau of Labor & Statistics, and Census data for the year 2000.

# 3.4 Uncertainty Quantification

After reducing the model, decision makers will need a means of quantifying the uncertainty in the PTDM output data arising from uncertainty in model parameters and input data. Parameter and input uncertainty is quantified using an analytical uncertainty propagation method to provide a mean and standard deviation for the travel demand model output. The model uncertainty quantification is computed using the Mean Value First Order Second Moment (MVFOSM) method which is based on a first-order Taylor series approximation of the output function (Haldar 2000). The MVFOSM method, with evaluations approximately equal to the number of inputs parameters (N+1), has a significant computational advantage over the numerous iterations required for Monte Carlo simulation. The partial derivatives shown below are approximated using finite difference.

$$Var[g(x)] \approx \sum_{i} \left(\frac{\partial g(x)}{\partial x_i}\right)^2 Var[x_i]$$

where

g(x) = travel demand model output function

 $x_i = model \ coefficient$ 

 $Var[x_i]$  = the variance in  $x_i$  derived from the t-statistics given in the CS report.

The CS report provided a mean and t-statistic for each model input parameter which was used as the basis for the assigned probability distributions. This data is reproduced in Table 3.6. Based on the input parameter coefficient analysis, the mean HSR ridership using year 2000 data is 133,701 riders per day with a standard deviation of 6,888, assuming HSR prices at 77% of their comparable commercial air prices for a coefficient of variation of about 5%. This mean and standard deviation provide a notional HSR ridership for the year 2000 if the network existed at that time. Obviously, HSR did not exist in 2000; however, the year 2000 model can be used as a basis to determine HSR ridership in 2030 assuming an identical relationship in demand model input parameters and assuming a ridership annual growth factor from the year 2000 to 2030 to which the PTDM results would be dependent. If the PTDM were to assume a 1.4% annual increase in ridership, the PTDM would estimate 2030 daily HSR ridership to be 202,896, while the CS model estimates 2030 daily ridership to be 202,740 (Brinckerhoff).

### 3.5 Sensitivity Analysis in Uncertainty Quantification

The objective of sensitivity analysis in uncertainty quantification is to determine the relative magnitudes of contribution to the output uncertainty arising from input uncertainty. This is a different concept from parameter 'elasticity' where sensitivity is defined as the percent change in the output divided by the percent change in the input. The sensitivity analysis for the demand input variable parameters is based on each input parameter's contribution to the variance of the total demand model follows.

The PTDM sensitivity analysis uses techniques from the field of structural reliability, which has been developed over the last fifty years by researchers such as (Der Kiureghian 2008), (Ditlevsen 1996), (Rackwitz 2001), and Mahadevan (McDonald and Mahadevan 2008), to name

a few. Particularly, the PTDM utilizes the Mean Value First Order Second Moment (MVFOSM) method (David 1996)(Haldar 2000) to identify the input uncertainties in the parameter coefficients that most strongly contribute to the model output uncertainty. These outcomes are then used to rank order the variables based on their influence on the demand model.

The percent contribution to HSR Ridership Variance is shown below. For an input parameter to be a dominant contributor to the model output variance, the input parameter must contribute greatly to the value of the model output and must have a large variance. When both of these conditions are present, it is possible to have one or two parameters completely dominate the total model variance.

% Contribution to HSR Ridership Variance = 
$$\frac{\left(\frac{\partial g(x)}{\partial x_i}\right)^2}{Var\left[g(x)\right]} \, Var\left[x_i\right]$$

Of the many uncertain parameters in the models, the PTDM sensitivity analysis results indicate that the uncertainty in the output may be predominantly caused by very few uncertain inputs. Table 3.8 indicates the contributors to the PTDM variance. The variables considered for their contribution to the variance in the PTDM are listed in the trip frequency and main-mode choice model variable coefficients tables, Figure 3.2 and Figure 3.3, Table 3.6, and Table 3.7. Both the trip frequency models and main-mode choice models contain model coefficients which contribute to the total model variance, using the CS model parameters and t-statistics to determine the coefficient means and standard deviations assuming a normal distribution.

Categorical variable model coefficients based on the number of trips generated per household dominate the variance in the trip frequency models, while the in-vehicle travel time coefficients dominate the variance in the main-mode choice models. The trip frequency model coefficients used to determine the number of trips generated per household contribute about 98% of the total model variance as shown in Table 3.8. For example, Table 3.6 shows that the average number of household workers variable in the short-commute trip frequency model has a large partial difference (-158,584) and a small variance (.0146), while the no household workers variable in the long-commute trip frequency model has a small partial difference (-5036) and a large variance (0.52). Both variables have a low percent contribution to variance, 0.32% and .0116%. In contrast, the coefficient of 1-Trip constant in the long-recreation trip frequency model has a relatively high contribution to variance of 13.92% resulting from both a large partial difference (-114070) and a large variance (1.2143).

It is not surprising that the dominant input uncertainties are one-trip and two-trips or more trip constants in the trip frequency models. Intercity travel is much less common that intracity travel, and these trips originate with relatively small probabilities. In order to accurately estimate these probabilities, a large amount of data is required. Because these models use revealed preference data based on a limited number of individual travel logs, the uncertainty in the probability that an individual would choose to travel on any given day is quite large. Further, these constants have a large impact on ridership forecasts because the projected HSR ridership will vary in proportion to the total travel demand, on which the constants have a large impact. Figures 3.2 and 3.3 indicate that the predictive capability of the PTDM is primarily based on subjective inputs, in this case, the coefficients assigned to the total number of travelers and number of passengers using a particular mode of travel.

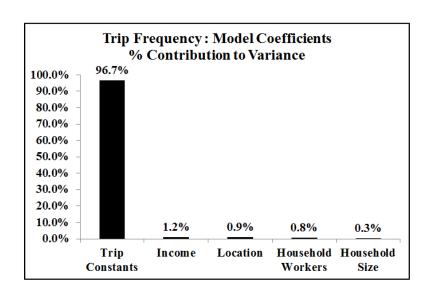


Figure 3.2: PTDM Trip Frequency Coefficient % Contribution to Variance

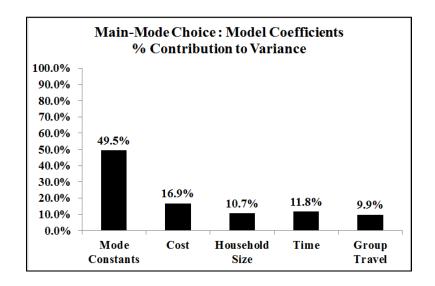


Figure 3.3: PTDM Main-Mode Choice Coefficient % Contribution to Variance

Trip Frequency Coefficient % Contribution to Variance								
(2D/2=\ <sup>2</sup> + \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \								
Trip Purpose	Variable	$\partial \mathbf{R}/\partial \mathbf{x}$	$\mathbf{E}[\mathbf{x}]$	Var(x)	$(\partial \mathbf{R}/\partial \mathbf{x})^2 *$	Contribution to		
					Var(x)	Variance		
Long Business	2 Trips	-25612	-16.3	46.1267	30,257,964,687	26.6510%		
Long Business	1 Trip	-23942	-15.67	46.4176	26,607,445,045	23.4357%		
Long Recreation	1 Trip	-114070	-3.416	1.2143	15,799,932,059	13.9165%		
Short Recreation	2 Trips	-160462	-3.787	0.4414	11,365,415,065	10.0106%		
Short Commute	1 Trip	-166050	-8.242	0.2940	8,106,918,450	7.1405%		
Short Commute	2 Trips	-157660	-9.07	0.2950	7,332,037,479	6.4580% 2.9030%		
Short Recreation Short Other	1 Trip 2 Trips	-85686 -87220	-2.881 -1.624	0.4489 0.3901	3,295,864,469 2,967,956,412	2.6142%		
Long Recreation	2 Trips	-43392	-5.083	1.2210	2,299,026,047	2.0250%		
Short Business	2 Trips	-42938	-5.513	0.5403	996,178,310	0.8774%		
Short Business	1 Trip	-37518	-4.946	0.5450	767,075,261	0.6756%		
Short Commute	Medium Income	-136906	1.045	0.0303	568,558,352	0.5008%		
Short Commute	High Income	-129468	1.523	0.0314	525,687,660	0.4630%		
Long Recreation	MTC	-74118	1.011	0.0884	485,726,570	0.4278%		
Short Commute	HH Workers	-158584	1.57	0.0146	366,801,697	0.3231%		
Long Recreation	HHS 3+	-146810	-0.482	0.0153	329,212,366	0.2900%		
Long Commute	HH Workers	-70980	1.274	0.0482	243,083,152	0.2141%		
Short Business	Logsum	228910	0.205	0.0022	113,744,891	0.1002%		
Long Commute	SCAG	-38096 40550	-1.644	0.0681	98,828,291	0.0870%		
Short Business Short Recreation	HH Workers Medium Income	-40550 -64344	1.153 0.355	0.0532 0.0202	87,438,022 83,481,001	0.0770% 0.0735%		
Long Business	High Income	-22380	1.139	0.0202	83,481,991 72,197,989	0.0735%		
Short Other	SCAG	-31636	-1.265	0.0695	69,512,308	0.0612%		
Short Commute	No HHW	-22712	-2.163	0.1344	69,329,772	0.0611%		
Short Business	High Income	-29622	0.835	0.0726	63,661,713	0.0561%		
Short Commute	SACAG	-32556	-2.736	0.0487	51,600,142	0.0454%		
Short Recreation	No HHW	-67390	0.493	0.0105	47,907,364	0.0422%		
Short Recreation	High Income	-43348	0.432	0.0238	44,729,038	0.0394%		
Long Recreation	SANDAG	-16754	1.344	0.1475	41,390,385	0.0365%		
Short Recreation	SCAG	-28774	-1.16	0.0479	39,661,097	0.0349%		
Short Commute	SCAG	-42866	-1.524	0.0195	35,920,540	0.0316%		
Long Business	SCAG	-18052	-1.176	0.1067	34,774,452	0.0306%		
Short Commute	SANDAG	-22278	-1.446	0.0691	34,305,488	0.0302%		
Short Business	No HHW	-14296	-0.863	0.1192	24,354,020	0.0215%		
Short Business Long Other	SANDAG MTC	-10750 -12766	-0.88 1.134	0.1600 0.1112	18,490,000 18,129,171	0.0163% 0.0160%		
Short Business	MTC	-16172	-1.275	0.0579	15,135,476	0.0133%		
Short Recreation	HHS = 1	-24282	-0.401	0.0238	14,025,261	0.0124%		
Short Other	SACAG	-13902	-1.177	0.0716	13,829,362	0.0122%		
Long Commute	No HHW	-5036	-2.668	0.5200	13,186,807	0.0116%		
Short Commute	MTC	-29684	-1.982	0.0134	11,837,498	0.0104%		
Long Commute	MTC	-12380	-0.729	0.0632	9,685,016	0.0085%		
Long Other	HHS 3+	-22628	-0.379	0.0183	9,381,120	0.0083%		
Long Recreation	SACAG	-12278	1.084	0.0607	9,149,734	0.0081%		
Long Other	2 Trips	-25230	-0.2823	0.0138	8,807,117	0.0078%		
Short Recreation	SANDAG	-6386	-1.802	0.2135	8,706,391	0.0077%		
Short Other	MTC	-12256	-0.524	0.0519	7,796,585	0.0069%		
Long Commute	SACAG SACAG	-14058 -8668	0.918	0.0381	7,539,399 6,858,044	0.0066%		
Short Business Long Other	High Income	-8668 -13962	-0.997 0.393	0.0913 0.0350	6,827,187	0.0060% 0.0060%		
Long Business	MTC	-6562	-1.372	0.1452	6,254,256	0.0055%		
Long Other	No HHW	-13936	0.372	0.0240	4,665,946	0.0041%		
Short Recreation	SACAG	-8686	-1.241	0.0491	3,705,162	0.0033%		
Long Business	No HHW	-2758	-2.098	0.3808	2,896,287	0.0026%		
Short Business	SCAG	-7406	-1.969	0.0524	2,875,160	0.0025%		
Long Recreation	HHV <hhw< td=""><td>-3940</td><td>-0.922</td><td>0.1476</td><td>2,291,035</td><td>0.0020%</td></hhw<>	-3940	-0.922	0.1476	2,291,035	0.0020%		
Long Business	SACAG	-4852	0.976	0.0696	1,638,090	0.0014%		
Long Other	SACAG	-3962	2.527	0.0602	944,855	0.0008%		
Short Business	HHV <hhw< td=""><td>-2382</td><td>-0.947</td><td>0.1557</td><td>883,407</td><td>0.0008%</td></hhw<>	-2382	-0.947	0.1557	883,407	0.0008%		
Long Other	HHS = 1	-3576	-0.424	0.0449	574,734	0.0005%		
Long Other	HHV <hhw household="" td="" vehicles<=""><td>-774</td><td>-0.915</td><td>0.1730</td><td>103,628</td><td>0.0001%</td></hhw>	-774	-0.915	0.1730	103,628	0.0001%		
	Household Venicles Household Workers				113,533,937,315	1		
	Household Size							
	San Francisco Region							
	Los Angeles Region							
	San Diego Region							
	Sacramento Region							

Table 3.6: Contribution to PTDM Trip Frequency Model Variance in Projected Daily Ridership

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Main-Mode Choice Coefficient % Contribution to Variance

Trip Purpose	Variable	∂R/∂x	E[x]	Var(x)	$\frac{(\partial \mathbf{R}/\partial \mathbf{x})^2}{\mathbf{Var}(\mathbf{x})}$	% Contribution to Variance
Long Other	HSR - constant	-21,488	1.434	0.041966	19,377,342	46.68062%
Short Business	Cost	-131,160	-0.109	0.000407	7,009,197	16.88537%
Long Other	HHS - Car	45,086	0.225	0.002108	4,286,041	10.32521%
Long Other	Group Car	11,604	1.417	0.024247	3,264,919	7.86529%
<b>Short Commute</b>	Time	585,180	-0.025	0.000007	2,496,356	6.01380%
Short Other	Time	560,114	-0.014	0.000007	2,274,062	5.47829%
<b>Short Commute</b>	CR - constant	582	4.232	2.649382	897,409	2.16189%
Long Business	Group Car	4,394	0.849	0.040881	789,299	1.90145%
Long Business	FCW - CAR	1,198	-0.770	0.102827	147,578	0.35552%
<b>Short Commute</b>	HHS - Car	1,120	0.665	0.110556	138,682	0.33409%
Long Other	Time	471,608	-0.011	0.000001	133,466	0.32152%
Short Business	HI - Car	658	-1.211	0.277225	120,029	0.28915%
Long Business	HI - HSR	-1,644	0.981	0.041744	112,822	0.27179%
Long Other	FCW - CAR	1,966	-0.435	0.024180	93,460	0.22515%
Long Other	Air - constant	1,058	0.690	0.060692	67,936	0.16366%
Long Other	CR - constant	1,010	0.615	0.055932	57,056	0.13745%
Long Business	Air - constant	658	-1.645	0.122500	53,038	0.12777%
Short Business	HSR - constant	384	-1.557	0.309215	45,596	0.10984%
Long Other	Group Air	1,472	-0.506	0.018710	40,540	0.09766%
Long Business	HI - Air	880	1.018	0.051176	39,631	0.09547%
Short Commute	HSR - constant	108	4.048	2.621809	30,581	0.07367%
Long Business	Group - Air	802	-0.338	0.015625	10,050	0.02421%
Short Business	Time	31,362	-0.500	0.000007	7,170	0.01727%
Short Business	Headway	-35,156	-0.006	0.000006	7,119	0.01715%
Long Other	Headway	-98,324	-0.003	0.000001	7,103	0.01711%
Long Business	Time	37,956	-0.016	0.000002	2,993	0.00721%
Long Business	Cost	-15,428	-0.016	0.000002	384	0.00093%
Long Business	Headway	-23,230	-0.003	0.000001	355	0.00085%
Short Commute	Headway	-8,866	-0.002	0.000001	72	0.00017%
Short Other	Cost	582	-0.109	0.000177	60	0.00014%
<b>Short Commute</b>	Cost	582	-0.148	0.000172	58	0.00014%
Long Other	Cost	-3,630	-0.035	0.000004	47	0.00011%
Short Other	Headway	-2,128	-0.009	0.000003	12	0.00003%

HSR = High-Speed Rail

CR = Conventional Rail

HI = High-Income

Table 3.7: Contribution to PTDM Main-Mode Choice Model Variance in Projected Daily Ridership

PTDM	<b>%</b>	Contribution	to	Variance

41,510,464

1

Contributors to Variance	Variance	%
Trip Frequency Coefficients	368,644,254,254	98.0%
Main-Mode Choice Coefficients	7,563,326,739	2.0%
Total	376,207,580,993	

Table 3.8: Contribution to PTDM Variance in Projected Daily Ridership

### 3.6 PTDM Computational Expense

The PTDM model analysis was conducted on a Dell Studio 1747 Intel(R) Core i7 1.60 GHz, 64-bit operating system with 8.0 GB RAM. The computation time for the PTDM sensitivity analysis described in this chapter is approximately 14 hours. Each of the five main-mode choice models utilized in the PTDM methodology has a run time of approximately 2.5 hours. The eight trip frequency models utilized in the PTDM methodology have a run time of approximately 10 minutes each for a total of 80 minutes. The parent CS model has a computational expense of several days. The difference in computational expense is attributed the complexity of the model output. While the main-mode choice output differentiated between four possible travel possibilities: HSR, commercial air, conventional rail and car, the trip frequency models only produced one output, travel demand.

#### 3.7 Conclusion

In order to overcome the hurdles of computational expense in travel demand models, a PTDM was proposed which utilized a model resolution different from that of its proprietary parent model. This parent model was used to calibrate and verify the results of the PTDM. The PTDM reduced computational expense so that repetitive model simulations, optimization analysis, and sensitivity analysis are more feasible.

To identify key model parameters, MVFOSM methods were used for uncertainty propagation and sensitivity analysis based on percent contributions to the model output variance. These key model parameters were found to be the constant terms in the trip frequency submodel. Knowing the percent contribution of the different parameters to the overall PTDM variance is

critical when conducting repetitive analysis. Only the input parameter coefficients determined to be significant, based on their model coefficient contributions to variance, need to be considered during repetitive decision analysis. All other parameters can be satisfactorily left at their means as the uncertainty in some parameter values contributes little to the uncertainty in the model output.

The PTDM prediction of HSR ridership, which has a coefficient of variation of 5%, assumes no bias, a valid model form, and the coefficient variances given in the documentation of the CS model. If these assumptions are valid, the PTDM results imply that the CS trip frequency and main-mode choice models provide reasonably reliable estimates for ridership and revenue, for any given set of input assumptions. However, these results do not determine the reliability of the parent model forecasts, since this analysis does not consider variance in the input values of the model variables, while the PTDM uses a different level of spatial resolution and does not include several of the model components in the parent model. Therefore, the results in this chapter should not be used to evaluate the reliability of the California High-Speed Rail Authority's Ridership & Revenue forecasts.

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#### **CHAPTER IV**

# MULTIDISCIPLINARY OPTIMIZATION FOR SYSTEM OPTIMAL MODEL INTEGRATION

"There are no secrets to success. It is the result of preparation, hard work, and learning from failure." – Colin Powell

#### 4.1 Introduction

A system-of-systems consists of a network of systems whose inputs and outputs affect the decision parameters of the other systems within the network. These inputs and outputs can act as boundary constraints for the affected network system. System-of-systems decision analysis, as shown in Figure 4.1 is based on the interaction of multiple systems using an optimization approach. The study of system performance or systems analysis spans many educational disciplines and practical applications. The total effects of these systems and applications can rarely be appreciated when analyzed in isolation. When systems interact with each other due to performing similar functions or impacting either the inputs or operational conditions of other systems, a more holistic approach may be warranted. The study of interrelated systems, known as system-of-systems analysis is the overall focus of this objective. In this chapter, system-of-systems analysis will be discussed from a transportation engineering perspective.

The study of transportation engineering is one example of system-of-systems analysis.

Transportation systems perform similar functions and are impacted by network conditions,

operational conditions such as weather, as well as each other. The California high-speed rail study is one example of a system-of-systems as it involves the analysis of multiple modes of transportation. Using the California high-speed rail program as a case study, this research will approach systems analysis from network conditions and related systems and will not consider the impacts of weather or other operational conditions. This type of network systems analysis, due to its complexity, requires analytical methods for decision analysis support. Decision support for system-of-systems engineering involves a synthesis of systems modeling, and optimization which consists of describing the network as a series of models, and optimizing across the network (McInvale, McDonald, and Mahadevan 2011).

Utilizing a multidisciplinary optimization methodology as shown in Figure 4.2, this research defines the interactions between airline fleet acquisition, assignment and travel demand forecasting using the PTDM by showing how aircraft allocation affects user demand and how user demand affects allocation.

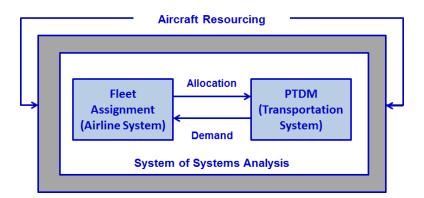


Figure 4.1: Case Study Multidisciplinary Optimization System-of-Systems Analysis

Two examples of transportation system-of-systems problems that can be analyzed through the use of multidisciplinary optimization are transportation systems planning and airline scheduling planning. Both of these model constructs contain individual models designed for specific purposes which both affect and are affected by other systems both within and outside of their respective systems families. The analysis of these two transportation systems typically involves the study of a single mode of transportation. Like most system-of-systems models, transportation system planning and airline schedule planning are traditionally conducted in isolation. Studying the interactions between these two systems-of-systems requires multimodal analysis. Multimodal interaction analysis such as this greatly increases the modeling complexity required for decision analysis. As problem complexity increases, feasible solution methods for system-of-systems analysis become more critical.

Current solution methods to solve multi-modal intercity commercial transportation SoS decision problems are too computationally expensive.. As the problem becomes more complex, the number of required simulations also increases which adds to the computational expense. Depending on the level of complexity and computational expense, systems problems such as these require decomposition into smaller more manageable elements. While some research exists that combine particular elements of transportation systems planning and airline schedule planning (Sherali, Bae, and Haouari 2010), currently no work exists that synthesizes transportation system planning and airline schedule planning given multimodal transportation providers. Unfortunately, current mathematical models which focus solely on one system often lack the synthesis required to interact with other complex mathematical models.

A key complexity to system-of-systems analysis is integrating network models especially when these networks span multiple disciplines of study. Methods for multidisciplinary

synthesized decision support for system-of-systems analysis are critical to systems planners. Due to the growing complexity of essential intricate systems such as commercial transportation systems, methods for synthesized decision support are expected to increase in importance. As a result of their complexity and computational expense, mathematical models have become a common way to conduct decision support analysis methods for systems of systems. The purpose of these models is typically to estimate the results of a specific system. To address these issues, this research utilizes a method of multidisciplinary optimization for solving the integrated transportation system-of-systems problem consisting of the main-mode choice problem of transportation systems planning and the fleet assignment problem of airline schedule planning. When integrating the main-mode choice and fleet assignment optimal decisions, the resource allocation problem becomes the decision which drives the user main-mode decision while the resource allocation problem is a fleet assignment problem where a known capacity is used to support a user demand.

Utilizing current decision support methods which include transportation systems planning modeling, parsimonious travel demand modeling, airline scheduling planning modeling, schedule design modeling, and fleet assignment modeling, this chapter implements a decision support method for multimodal transportation system planning models and airline schedule planning models from the perspective of a market leader or dominant market player. It is assumed that the market leader establishes the initial market conditions to which all other players in the market respond. The research in this chapter provides a methodology for decision makers to establish those initial market conditions. In the following chapter, this research provides a synthesized methodology for determining the competitive response to changes in an established network due to changes in market conditions.

This decision support method combines the user-level multimodal decision choice of transportation system planning with the airline schedule design and fleet assignment decisions of airline schedule planning using multidisciplinary optimization. The contribution of this work is the synthesized decision support method for multimodal transportation system planning models and airline schedule planning models by combining the user-level multimodal decision choice of transportation system planning with the airline schedule design and fleet assignment decisions of airline schedule planning using multidisciplinary optimization and game theory.

The overall scope of the California high-speed rail project to include the connected cities, cost estimates, and project passenger revenue reported early in California's planning process is still in question. Media reports indicate that much planning still remains concerning the scope of the California high-speed rail program (Mckinley 2011). Reports such as these give credence to the fact that decision-makers require measures to effectively model and predict network usage. In this case, decision makers require a means of gauging the viability of high-speed rail as a commercial transportation service provider. Assessing the viability of high-speed rail has centered on two primary questions:

- What average price for commercial air and high-speed rail results in maximum profit?
- At what price is high-speed rail viable?

Both of these questions assume a relatively certain cost estimate for high-speed rail which was recently called into question. Another issue worth researching is the effect of high-speed rail on commercial air demand. It has been wondered whether the existence of high-speed rail could alleviate current commercial air congestion. To address these questions, a short-run economic

analysis is provided comparing commercial air and high-speed rail average prices using game

theory.

This chapter is organized into five main sections. The first section discusses the

parameters of the models integrated in this research followed by the proposed methods and

formulations. Next, a case study is provided to illustrate the synthesized decision support

method followed by results and discussion, and a conclusion which explain how the

multidisciplinary optimization method provides a synthesized method for multimodal

transportation system planning models and airline schedule planning models as illustrated by the

case study.

4.2. Single Airline Optimization Methodology

This analysis assumes a feasible pricing range for the average price of both commercial air and

high-speed rail. This analysis considers only these two factors and holds the model conditions

for conventional rail and transportation via private-owned vehicle constant. Using the PTDM,

this analysis estimates the high-speed rail demand and commercial air profit for the feasible

ranges of commercial air and high-speed rail. The simultaneous analysis and design (SAND)

and multidisciplinary feasible (MDF) methods follow for the system optimal fleet assignment.

SAND Formulation

 $Maximize_{x,y^*,d^*} \pi(x,y^*,d^*)$ 

Subject to:

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$$y(d_1^*x) - y^* = 0$$

$$d(y^*) - y^* = 0$$

$$S(y^*) - d^* \ge 0$$

$$\sum y^* = x_{fod} \, MOPD$$

Where

 $Y(unstarred) = optimal fleet assignment at fixed d^*$ .

D(unstarred) = demand given fixed fleet assignment y\*.

MDF Formulation

$$Maximize_x \pi(x)$$

Subject to:

$$S^*(x) \geq D^*\left(S^*(x)\right)$$

Fleet assignment and user mode choice models directly impact each other through the interaction of their respective input and output elements. The main input to the fleet assignment problem is the customer demand. The customer demand by mode of transportation is the output of the user mode choice model. A main input to the user mode choice model is the resource allocation or capacity which is the output of the fleet assignment model. The other primary input to the user mode choice model is ticket price.

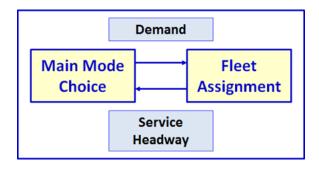


Figure 4.2: User Mode Choice & Fleet Assignment Variable Interactions

Figure 4.2 illustrates the interaction between transportation systems planning and airline schedule planning through fleet assignment and user mode choice decision modeling. The user mode split determines the user travel demand for both high-speed rail and commercial air from ticket price & service headway. The user mode choice input elements consist of the origin-destination ticket prices and the service headways, while the output element is the travel demand by mode. Commercial air demand determines resource allocation (capacity) & the ticket prices required for a profit. One primary assumption in this integration is that capacity is set to meet passenger travel demand for both commercial air and high-speed rail.

### 4.3 Case Study

Transportation demand models have been used to estimate high-speed rail travel demand for the purpose of providing transportation planning decision support for potential high-speed rail projects. As stated in chapter three, the proposed California Corridor High-Speed Rail project (Cambridge Systematics 2008) plans to link Sacramento, San Francisco, Los Angeles, and San Diego. As California and other U.S. regions are considering the addition of high-speed rail as a new mode of transportation to their commercial transportation networks, transportation providers

are now faced with new decision support questions regarding both the acquisition and allocation of transportation resources. These decision support questions require a synthesized approach for conducting multidisciplinary multimodal decision support over time.

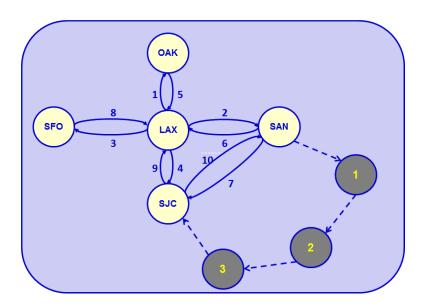


Figure 4.3: Air Transportation Network Diagram

To illustrate the integration of multimodal user decision choice and airline resource acquisition and resource allocation, this case study problem utilizes travel demand results from the parsimonious travel demand model from chapter three and the air transportation network diagram displayed in Figure 4.3 to develop airline schedule and allocation results assuming a single or dominant air service provider. This synthesized decision support method for multimodal transportation system planning models and airline schedule planning models will be conducted by providing and discussing the cause and effect relationships between the input and output parameters of the respective models. The prices for both high-speed rail and commercial air are based on PTDM pricing used to initially estimate California interregional travel demand.

As current pricing for high-speed rail in the United States does not exist, high-speed rail pricing, without any government subsidizing, is set at 77% of comparable commercial air pricing as in the parent ITMS model (Cambridge Systematics 2008).

### **4.3.1 Problem Description**

The problem of resource acquisition and resource allocation occurs over time. The first stage of the problem is to determine the optimal number of vehicles by type to purchase or acquire in order to support the estimated future air travel demand. Upon receiving or acquiring the desired number of vehicles by type, the second stage of the problem determines the optimal resource allocation to maximize profit.

Utilizing the daily interregional travel demand estimates from the PTDM, this chapter describes the effects of transportation policy on commercial air and high-speed rail demand and on commercial air resource allocation. Although a relatively new mode of public transportation, high-speed rail policy analysis has been developed through researchers such as Gunn who studied the methods for scenario based high-speed rail forecast generation (Gunn, Bradley, and Hensher 1992).

### **4.3.2** Case Study Formulation

This analysis assumes a feasible pricing range for the average price of both commercial air and high-speed rail. This analysis considers only these two factors and holds the model conditions for conventional rail and transportation via private-owned vehicle constant. Using the PTDM to estimate the transportation demand splits based on changes in the average ticket prices for

commercial air and high-speed rail, the case study considers the average commercial air prices ranging from \$200 to \$350 and for average high-speed rail prices ranging from \$175 to \$250 and reports the corresponding ridership and revenue. These model outputs are derived over combinations of commercial air and high-speed rail prices over the listed ranges. An analysis of these model outputs over the specified ranges makes up the short run pricing analysis of commercial air and high-speed rail. The pricing analysis is used to determine the viability of high-speed rail in the California by utilizing the main-mode choice decision models of the PTDM to determine how revenue and ridership responds to changes in the average prices commercial air and high-speed rail in California. The short run analysis reported in this chapter reflects the initial scope of California high-speed rail project.

This short run analysis assumes constant conditions for the main mode choice model parameters with the exception of the commercial air and high-speed rail prices. In this analysis the commercial air and high-speed rail prices are averaged across the entire California Corridor commercial transportation network. Other model parameters such as the cost of traveling by car resourcing parameters which define the level of transportation service provided, demographic parameters, and socioeconomic parameters are not adjusted in this analysis. The ridership results from the short run analysis are used as inputs in the fleet assignment optimization.

The fleet assignment formulation optimizes the number of aircraft to purchase maximizes the profit associated with purchasing the aircraft required to support the quantity demand given an available fleet. This analysis assumes aircraft are dedicated to specific origin-destination links. The resource acquisition optimization involves a schedule design optimization to determine the optimal number of flights per day to support the customer demand for each origin-destination pair. The resource allocation optimization assigns aircraft to specific origin

destination pairs in support of customer demand for each origin-destination pair. The aircraft acquisition and allocation master problem follows.

$$\begin{aligned} \mathit{Max}\ \mathit{Profit}_{x,y} \\ \mathit{Profit}_{x,y} &= \mathit{REV}_y - \mathit{CC}(x) + \mathit{OC}(y) \\ \\ \sum_{o \in O} \sum_{d \in D} y_{fod} &\leq x_f(\mathit{MOPD}) \ \forall \ f \in \mathit{F} \\ \\ \\ \sum_{f \in \mathit{F}} y_{fod}(\mathit{PAX}_f) &\geq \mathit{Q}_{od}(y) \ \forall \ o \in \mathit{O}, d \in \mathit{D} \end{aligned}$$

$$x^L \le x \le x^U$$

$$y \ge 0$$

$$y \ge 0$$

Where

REV = revenue

 $CC = capital \ cost$ 

OC = operational cost

PAX = passengers

*MOPD* = maximum number of operations per aircraft

 $x = aircraft \ acquisitions$ 

 $y = aircraft \ allocations$ 

L = lower bound

U = upper bound

O = origin

D = destination

Q = demand

The amount of available aircraft is dependent on the first stage resource acquisition decision. The available fleets for this case study are listed in Table 4.1. The listing of available aircraft is based on the current aircraft fleets of the primary commercial airlines serving California. These airlines include United, American Airlines, and Southwest Airlines. The aircraft fleet listed in Table 4.1 is derived from the websites of the six primary airlines serving California. This problem assumes a mixed aircraft fleet and that the mixed fleet capacity can support traveler demand. Each flight leg is flown by only one fleet. The fleet assignment allocation optimization formulation minimizes the operational costs using an airline cost model. The cost model used to estimate aircraft operating cost was based on an aircraft costing model by Harris in 2005. The basic operating cost equation is shown in Figure 4.4.

Fleet Assignment Problem Aircraft

Aircraft Model	Engine Model	Seating Capacity	Cost in Millons	
ATR 72	PW120	64	\$22.7	
CRJ-100 ER	Allied Signal LF507	50	\$31.0	
B 737 - 3/7	CFM56-7B24	137	\$70.8	
A 320	CFM56-5B4/P	138	\$88.3	
В 767 -3	PW 4060	225	\$175.0	
В 737 - 5	CFM56-3CI	122	\$65.0	

Table 4.1: Case Study Available Aircraft

Flight crew expenses are based on the airline's business approach, whether the flight is regional, domestic or international, the assigned aircraft maximum take-off gross weight (MTOGW) and the number of block hours assigned to the flight. A block hour is defined as the amount of time from push-off from the departure gate to arriving at the arrival gate. Fuel & Oil expenses are obviously based primarily on the cost of fuel per gallon, the number of cruise and non-cruise gallons per departure based on the flight distance, and the number of departures. The number of gallons required for each departure is based on the engine-specific fuel consumption, and the engine take-off thrust for a jet or the brake horsepower for a piston or turbroprop. Insurance expenses are based on the number of aircraft owned by an airline, while the rental expense is based on the number of aircraft leased by an airline. Although a significant aspect of airline costs, the aircraft cost model in this chapter does not consider the age of the aircraft fleet, maintenance expenses, or specific airport related expenses.

 $C(x_{od}^f) = Total \ Aircraft \ Operating \ Expenses + All \ Other \ Expenses$ 

Total Aircraft Operating Expenses = Flying Operation + Flight Equip Maint + Flight Equip. Depr. and Amort.

All Other Operating Expenses =
Passenger Service + Landing Fees + Rest of All Other + Transport Related

Figure 4.4: Airline Total Operating Cost Framework

A fractional allocation of aircraft to 10-link system is assumed; therefore, flow conservation is not required. The number of aircraft available to the 10-link system is limited by a maximum operations per aircraft per day (MOPD). In this research the MOPD is 5.

$$x \; \equiv \; \frac{\textit{Number of Aircraft Available to } 10 - \textit{link system}}{\textit{MOPD}}$$

Supply provided on a link must be greater than PTDM-predicted demand.

$$\sum_{f} y_{fod} \times PAX_{f} \ge Q(y)$$

where

$$y_{fod} = aircraft \ allocation$$

Capital Cost is amortized into daily cost over a 5 year period. The operational cost model is based on "An Economic Model of U.S. Airline Operating Expenses" by Harris in 2005. Lastly, aircraft are dedicated to specific links.

## 4.4 Results

# 4.4.1 Profit and Ridership Results

The following tables show the commercial air and high-speed rail ridership from the PTDM and the commercial air profit over the specified average price ranges for commercial air and high-speed rail given the presence of high-speed rail. These results are the basis of the normal-form game which ultimately compares commercial air to high-speed rail.

High-Speed Rail Ridership										
Air Prices	\$350	51,711	47,594	44,723	43,545					
	\$325	54,060	49,966	46,189	44,296					
	\$300	57,376	52,171	48,788	45,541					
	\$275	62,049	55,164	51,253	48,132					
Trices	\$250	67,428	59,832	54,216	50,921					
	\$225	73,742	65,729	59,066	54,135					
	\$200	82,589	72,619	65,618	59,639					
		\$175	\$200	\$225	\$250					
			ligh-Speed	l Rail Prices						

Table 4.9: Daily High-Speed Rail Ridership

High-speed rail ridership is highest when high-speed rail prices are at their lowest and commercial air prices are at their lowest as listed in Table 4.9. These counterintuitive results are based on the nested structure of the PTDM where air and high-speed rail ridership are nested entities so their combined ridership (air & HSR) increases when their prices are lowest as shown later in Table 4.12. The range of high-speed rail ridership spans from approximately 43K to 82K.

	Commercial Air Ridership											
	\$350	97,701	98,437	98,831	99,177							
	\$325	110,574	113,370	113,997	114,502							
	\$300	118,019	119,675	120,460	121,167							
Air Prices	\$275	120,447	121,627	122,627	123,582							
Tilees	\$250	122,639	123,936	125,212	126,457							
	\$225	125,466	127,064	128,675	130,254							
	\$200	129,397	131,414	133,448	135,448							
		\$175	\$200	\$225	\$250							
	High-Speed Rail Prices											

Table 4.10: Daily Commercial Air Ridership

Commercial air ridership is highest when high-speed rail prices are at their highest and commercial air prices are at their lowest as listed in Table 4.10. The range of commercial air ridership spans from approximately 97K to 135K.

Car Ridership										
	\$350	1,286,589	1,289,938	1,292,399	1,293,221					
	\$325	1,271,393	1,272,650	1,275,776	1,277,152					
Air Prices	\$300	1,260,667	1,264,162	1,266,730	1,269,251					
	\$275	1,253,612	1,259,250	1,262,120	1,264,260					
	\$250	1,246,103	1,252,320	1,256,605	1,258,620					
	\$225	1,237,044	1,243,360	1,248,342	1,251,646					
	\$200	1,224,375	1,232,210	1,237,090	1,241,005					
		\$175	\$200	\$225	\$250					
			High-Speed	l Rail Prices						

Table 4.11: Car Ridership

Car ridership is highest when both high-speed rail prices and commercial prices are at their highest as listed in Table 4.11. The range of car ridership spans from approximately 1.22M to 1.29M.

Co	mbined I	Ridership:	Air & Hi	gh-Speed	Rail
	\$350	149,412	146,031	143,554	142,722
	\$325	164,634	163,335	160,186	158,798
	\$300	175,395	171,846	169,247	166,709
Air Prices	\$275	182,496	176,792	173,880	171,714
Trices	\$250	190,067	183,768	179,428	177,378
	\$225	199,208	192,793	187,742	184,389
	\$200	211,986	204,032	199,066	195,088
		\$175	\$200	\$225	\$250
			High-Speed	l Rail Prices	;

Table 4.12: Combined Expect Daily Ridership

Com	Combined Ridership: Car, Air & High-Speed Rail										
	\$350	1,436,001	1,435,969	1,435,953	1,435,943						
	\$325	1,436,027	1,435,985	1,435,962	1,435,950						
	\$300	1,436,062	1,436,008	1,435,977	1,435,959						
Air Prices	\$275	1,436,108	1,436,041	1,436,000	1,435,974						
	\$250	1,436,171	1,436,088	1,436,033	1,435,998						
	\$225	1,436,252	1,436,153	1,436,083	1,436,035						
	\$200	1,436,361	1,436,242	1,436,156	1,436,093						
	·	\$175	\$200	\$225	\$250						
	High-Speed Rail Prices										

Table 4.13: Combined Car, Air, High-Speed Rail Ridership

Combined commercial air ridership and high-speed rail ridership is highest when both high-speed rail prices and commercial air prices are at their lowest as listed in Table 4.12. The range of combined commercial air ridership and high-speed rail ridership spans from approximately 142K to 211K. This combined ridership indicates that commercial air and high-speed rail are nested modes of transportation.

Combined car, commercial air ridership, and high-speed rail ridership is highest when both high-speed rail prices and commercial air prices are at their lowest as listed in Table 4.13. The range of all three modes of transportation varies little from 1.435M to 1.436M. It is expected that as both high-speed rail and commercial air prices increase, overall travel decreases. The model seems to account for one of the primary influencers of transportation prices, fuel cost. So if conditions exist that result in high air and rail prices, a similar reduction effect is experienced in car travel as well.

Commercial air profit is highest when commercial air prices are highest as shown in Table 4.14. Even though commercial air ridership is lowest at their highest price range, the revenue gained by higher ticket prices offsets the decreased ridership.

Commercial Air Profit											
	\$350	\$30,021,850	\$30,869,292	\$30,801,864	\$30,547,364						
	\$325	\$29,516,331	\$28,926,436	\$28,101,140	\$27,979,998						
	\$300	\$25,185,524	\$25,360,378	\$25,054,836	\$24,650,800						
Air Prices	\$275	\$20,087,341	\$20,066,472	\$18,942,861	\$18,546,525						
	\$250	\$13,512,354	\$15,120,791	\$15,985,885	\$16,615,839						
	\$225	\$14,898,949	\$14,521,663	\$14,779,876	\$15,153,489						
	\$200	\$11,651,751	\$11,596,315	\$11,640,930	\$11,228,014						
		\$175	\$200	\$225	\$250						
	High-Speed Rail Prices										

Table 4.14: Commercial Air Profit

# **4.4.2** Model Integration Results

The fleet assignment results from this chapter demonstrate the expected cause and effect relationships regarding the introduction of high-speed rail to an existing commercial network. Table 4.15 shows the cooperative fleet assignment and profit without high-speed rail for the six aircraft used in the case study. This base scenario assumes commercial air fares based on year 2000 figures where HSR prices are set at 77% of air fares. Table 4.16 shows the aggregate fleet assignment and profit by the six aircraft used in the case study network upon the introduction of high-speed rail. This aggregate fleet assignment is the average of aircraft fleet assignments by origin-destination pair and the associated profit by aircraft across the given ranges of high-speed rail and commercial air prices.

Base Scenario: Cooperative Fleet Assignment without High-Speed Rail

Airports			Available Aircraft						
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	Boeing 737-5	ATR 72	Boeing 767-3	Total	
LAX	OAK	20	17	20	18	19	12	107	
LAX	SAN	29	24	28	27	25	26	158	
LAX	SFO	11	14	11	11	15	6	68	
LAX	SJC	20	19	20	19	21	14	114	
OAK	LAX	19	19	20	18	21	12	109	
SAN	LAX	28	25	28	26	26	26	160	
SAN	SJC	14	16	15	13	18	5	81	
SFO	LAX	13	16	13	12	18	2	74	
SJC	LAX	20	19	20	19	21	13	113	
SJC	SAN	12	15	13	11	17	9	77	
Profit		\$7,719,243	\$2,687,751	\$8,049,321	\$6,544,682	\$4,096,417	\$557,129	\$29,654,543	

Table 4.15: Base Scenario: Cooperative Fleet Assignment Without High-Speed Rail

Aggregate Model Integration Fleet Assignment with High-Speed Rail

Air	ports							
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	Boeing 737-5	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	14	9	28	31	8	8	98
LAX	SAN	28	50	18	19	75	15	205
LAX	SFO	12	22	9	7	17	5	73
LAX	SJC	16	21	23	25	27	8	120
OAK	LAX	30	34	15	16	17	6	118
SAN	LAX	37	<b>2</b> 7	19	21	43	22	169
SAN	SJC	6	7	10	11	17	15	66
SFO	LAX	13	17	10	8	12	7	66
SJC	LAX	14	28	24	25	27	7	124
SJC	SAN	10	7	15	16	18	6	73
Pr	rofit	\$5,687,766	\$2,465,230	\$5,332,741	\$4,895,609	\$4,375,067	-\$1,993,381	\$20,763,031

Table 4.16: Aggregate Model Integration Fleet Assignment With High-Speed Rail

One preconceived notion of HSR is that it would reduce air demand because it is perceived as a similar, cheaper, and possibly superior mode of travel to air. Based on the findings of this research, a reduction in air demand will not necessarily reduce the number of aircraft operations servicing Southern California. Instead, the introduction high-speed rail resulted in a greater use of smaller aircraft. This increase of aircraft provided an increased level-of-service resulting in increased schedule flexibility for commercial air. The most notable result is the ~\$9M reduction in profit for commercial air upon the introduction of high-speed rail to the commercial transportation network. In order to reduce airline operations, additional external incentives may need to be provided to the airlines. Air level-of-service will probably stay relatively close to what it is now, and congestion at California airports will probably be as severe as it was before the addition of HSR. Many environmental impacts (e.g., noise, carbon emissions) are related to the number of operations more strongly than the size of aircraft. It is likely that many of the environmental benefits of HSR are significantly overstated. Demands on

air traffic control will likely remain unchanged or could increase if more flights of smaller aircraft are scheduled as an airline response to reduced demand arising from the presence of HSR. Tables 4.17 to Table 4.23 show the fleet assignment allocations and profits for all the price comparisons in the normal-form game.

Model Integration Fleet Assignment (Air \$350 / HSR \$175)

_									
	Air	ports	Air Price= \$350		HSR Price=	HSR Price= \$175		97,701	
	Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	Boeing 737-5	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	1	20	31	35	6	0	93
	LAX	SAN	32	48	3	5	123	0	211
	LAX	SFO	11	35	0	0	38	0	84
	LAX	SJC	4	23	21	24	47	0	119
	OAK	LAX	42	47	3	7	6	0	105
	SAN	LAX	47	50	5	7	82	0	191
	SAN	SJC	1	13	11	14	44	0	83
	SFO	LAX	11	35	0	1	38	0	85
	SJC	LAX	2	22	22	26	45	0	117
	SJC	SAN	1	13	11	14	45	0	84
	Profit		\$6,498,327	\$4,569,134	\$4,471,983	\$4,901,212	\$9,963,099	-\$381,905	\$30,021,850

Model Integration Fleet Assignment (Air \$350 / HSR \$200)

			0	-			,	
Air	ports	Air Price= \$350		HSR Price=	\$200	Demand= 98,437		
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	2	19	31	35	5	1	93
LAX	SAN	32	47	4	6	123	1	213
LAX	SFO	11	34	1	1	38	0	85
LAX	SJC	5	22	22	25	46	1	121
OAK	LAX	43	<b>4</b> 7	4	7	6	1	108
SAN	LAX	47	50	6	8	81	1	193
SAN	SJC	1	12	12	15	44	0	84
SFO	LAX	11	34	1	1	38	0	85
SJC	LAX	2	21	23	26	45	1	118
SJC	SAN	1	13	11	14	45	0	84
Pı	rofit	\$6,675,941	\$4,496,702	\$4,759,461	\$5,028,757	\$9,897,723	\$10,708	\$30,869,292

Model Integration Fleet Assignment (Air \$350 / HSR \$225)

		-		- \				
	Airports	Air Price= \$350		HSR Price=	\$225	Demand=	98,831	
Orig	gin Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LA	X OAK	3	15	34	36	0	0	88
LA	X SAN	34	44	7	8	114	0	207
LA	X SFO	13	30	4	2	29	0	78
LA	X SJC	6	18	25	26	38	1	114
OA	K LAX	44	42	7	9	0	0	102
SA	N LAX	49	46	9	9	73	0	186
SA	N SJC	3	8	14	16	35	1	77
SF	O LAX	13	30	4	2	29	0	78
SJ	C LAX	4	17	26	28	36	0	111
SJ	C SAN	3	8	14	16	36	1	78
Profit		\$7 388 080	\$3.818.304	\$5 933 941	\$5 548 317	\$8 219 822	-\$106 600	\$30 801 864

Model Integration Fleet Assignment (Air \$350 / HSR \$250)

Air	ports	Air Price=	\$350	HSR Price=	\$250	Demand=	99,177	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	4	8	35	36	0	1	84
LAX	SAN	36	39	9	8	107	0	199
LAX	SFO	15	23	6	3	21	1	69
LAX	SJC	8	12	26	27	30	1	104
OAK	LAX	45	36	8	9	0	0	98
SAN	LAX	52	42	11	10	66	0	181
SAN	SJC	5	1	16	17	27	2	68
SFO	LAX	15	24	6	3	21	1	70
SJC	LAX	6	11	28	28	29	1	103
SJC	SAN	4	1	15	16	31	2	69
Pı	rofit	\$8,120,317	\$2,947,217	\$6,644,762	\$5,777,684	\$7,057,087	\$298	\$30,547,364

Table 4.17: Model Integration Fleet Assignment (Air \$350)

Model Integration Fleet Assignment (Air \$325 / HSR \$175)

_									
	Air	ports	Air Price=	\$325	HSR Price=	\$175	Demand=	110,574	
	Origin	Destination	Airbus 320	CRJ-100 ER	<b>Boeing 737-3</b>	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	6	5	38	38	0	3	90
	LAX	SAN	40	38	13	11	103	4	209
	LAX	SFO	17	20	8	4	15	3	67
	LAX	SJC	11	9	30	29	25	4	108
	OAK	LAX	47	33	11	11	0	2	104
	SAN	LAX	55	41	15	13	62	4	190
	SAN	SJC	6	0	19	18	21	4	68
	SFO	LAX	17	21	8	5	15	3	69
	SJC	LAX	8	9	31	30	24	4	106
	SJC	SAN	6	0	18	18	25	4	71
	Profit		\$8,311,293	\$2,421,555	\$7,280,551	\$5,928,788	\$5,698,263	-\$124,120	\$29,516,331

Model Integration Fleet Assignment (Air \$325 / HSR \$200)

			0					
Air	ports	Air Price=	\$325	HSR Price=	HSR Price= \$200		Demand= 113,370	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	6	5	38	38	0	3	90
LAX	SAN	40	38	14	12	101	5	210
LAX	SFO	17	20	9	4	13	4	67
LAX	SJC	11	9	30	29	23	5	107
OAK	LAX	47	33	11	11	0	3	105
SAN	LAX	56	41	15	14	60	4	190
SAN	SJC	7	0	19	19	19	4	68
SFO	LAX	17	21	9	5	13	4	69
SJC	LAX	9	9	31	31	22	5	107
SJC	SAN	6	0	18	18	23	4	69
Pı	Profit		\$2,316,109	\$7,205,664	\$5,802,762	\$5,382,915	-\$137,236	\$28,926,436

Model Integration Fleet Assignment (Air \$325 / HSR \$225)

Air	rports	Air Price=	\$325	HSR Price=	\$225	Demand= 113,997				
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total		
LAX	OAK	4	6	37	<b>3</b> 7	0	5	89		
LAX	SAN	39	39	13	12	99	7	209		
LAX	SFO	15	21	8	4	11	6	65		
LAX	SJC	10	10	29	29	20	8	106		
OAK	LAX	45	34	10	10	0	5	104		
SAN	LAX	54	42	14	13	57	7	187		
SAN	SJC	5	1	18	18	17	6	65		
SFO	LAX	16	21	8	4	11	6	66		
SJC	LAX	7	10	30	30	20	7	104		
SJC	SAN	5	1	17	17	21	7	68		
Pı	rofit	\$7,833,177	\$2,578,466	\$7,052,118	\$5,921,544	\$5,074,931	-\$359,097	\$28,101,140		

Model Integration Fleet Assignment (Air \$325 / HSR \$250)

Air	ports	Air Price=	\$325	HSR Price=	\$250	Demand=	114,502	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	5	6	38	38	0	5	92
LAX	SAN	39	40	13	12	98	7	209
LAX	SFO	15	21	8	4	10	6	64
LAX	SJC	10	10	30	29	20	8	107
OAK	LAX	46	34	10	11	0	5	106
SAN	LAX	54	42	15	14	57	7	189
SAN	SJC	5	1	19	18	16	7	66
SFO	LAX	16	21	8	5	11	6	67
SJC	LAX	8	10	31	31	19	7	106
SJC	SAN	5	1	18	18	20	7	69
Pı	Profit		\$2,489,675	\$7,114,307	\$5,838,286	\$5,007,899	-\$225,179	\$27,979,998

Table 4.18: Model Integration Fleet Assignment (Air \$325)

Model Integration Fleet Assignment (Air \$300 / HSR \$175)

_									
	Air	ports	Air Price=	\$300	HSR Price=	\$175	Demand=	118,019	
	Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	5	6	38	38	0	6	93
	LAX	SAN	40	40	14	12	98	8	212
	LAX	SFO	16	21	9	5	10	7	68
	LAX	SJC	10	10	30	29	20	8	107
	OAK	LAX	46	34	11	11	0	5	107
	SAN	LAX	55	42	16	14	57	8	192
	SAN	SJC	6	1	19	19	16	7	68
	SFO	LAX	16	21	9	5	10	7	68
	SJC	LAX	8	10	32	31	19	8	108
	SJC	SAN	5	1	18	18	20	7	69
	Profit		\$7,145,530	\$2,230,624	\$6,571,946	\$5,366,799	\$4,503,649	-\$633,025	\$25,185,524

Model Integration Fleet Assignment (Air \$300 / HSR \$200)

Air	ports	Air Price= \$300		HSR Price=	\$200	Demand= 119,675		
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	6	1	39	38	0	7	91
LAX	SAN	41	38	15	13	94	10	211
LAX	SFO	16	16	10	5	4	9	60
LAX	SJC	11	6	31	30	14	11	103
OAK	LAX	46	29	12	11	0	7	105
SAN	LAX	56	40	17	15	52	10	190
SAN	SJC	6	0	20	19	9	9	63
SFO	LAX	17	16	10	5	4	9	61
SJC	LAX	9	5	33	31	13	10	101
SJC	SAN	6	0	19	18	14	9	66
P	rofit	\$7,638,319	\$1,905,037	\$7,169,842	\$5,693,742	\$3,774,073	-\$820,635	\$25,360,378

Model Integration Fleet Assignment (Air \$300 / HSR \$225)

Aiı	rports	Air Price= \$300		HSR Price=	\$225	Demand= 120,460					
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total			
LAX	OAK	6	1	39	38	6	7	97			
LAX	SAN	41	38	16	13	93	10	211			
LAX	SFO	17	16	10	5	4	9	61			
LAX	SJC	11	6	32	30	14	11	104			
OAK	LAX	47	29	12	11	0	7	106			
SAN	LAX	56	40	17	15	52	10	190			
SAN	SJC	6	0	20	19	9	9	63			
SFO	LAX	17	16	10	5	4	9	61			
SJC	LAX	9	5	33	32	13	10	102			
SJC	SAN	6	0	20	18	13	9	66			
P	rofit	\$7,605,230	\$1,775,041	\$7,141,279	\$5,568,311	\$3,786,354	-\$821,379	\$25,054,836			

Model Integration Fleet Assignment (Air \$300 / HSR \$250)

		4: D: 6200		HCD D: 6250		D 1 101 175		
Air	rports	Air Price=	\$300	HSR Price=	\$250	Demand=	121,167	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	5	3	35	36	7	8	94
LAX	SAN	35	40	17	16	90	12	210
LAX	SFO	17	16	7	0	0	13	53
LAX	SJC	14	10	26	27	16	12	105
OAK	LAX	42	31	12	12	1	8	106
SAN	LAX	50	42	18	17	52	12	191
SAN	SJC	17	10	12	15	16	5	75
SFO	LAX	15	18	9	6	4	9	61
SJC	LAX	9	7	32	31	13	11	103
SJC	SAN	6	3	18	18	14	9	68
P	rofit	\$7,337,992	\$2,277,486	\$6,531,249	\$5,535,753	\$3,912,555	-\$944,236	\$24,650,800

Table 4.19: Model Integration Fleet Assignment (Air \$300)

Model Integration Fleet Assignment (Air \$275 / HSR \$175)

_									
	Air	ports	Air Price=	\$275	HSR Price=	\$175	Demand=	120,447	
	Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	19	13	20	23	16	11	102
	LAX	SAN	29	63	25	30	52	8	207
	LAX	SFO	8	24	8	9	20	6	75
	LAX	SJC	20	25	20	22	25	8	120
	OAK	LAX	14	38	14	17	34	10	127
	SAN	LAX	25	13	23	29	29	30	149
	SAN	SJC	1	9	0	3	12	30	55
	SFO	LAX	13	14	13	17	14	0	71
	SJC	LAX	16	43	16	19	33	9	136
	SJC	SAN	14	9	15	17	13	5	73
	Profit		\$5,002,876	\$2,802,393	\$4,994,190	\$5,260,754	\$3,914,038	-\$1,886,910	\$20,087,341

Model Integration Fleet Assignment (Air \$275 / HSR \$200)

Air	ports	Air Price=	\$275	HSR Price=	\$200	Demand=	121,667	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	19	17	19	22	20	10	107
LAX	SAN	29	61	26	30	51	10	207
LAX	SFO	9	23	8	9	18	6	73
LAX	SJC	20	24	21	22	24	9	120
OAK	LAX	15	37	15	17	33	10	127
SAN	LAX	26	13	25	30	28	29	151
SAN	SJC	1	7	0	2	10	31	51
SFO	LAX	13	13	13	16	13	2	70
SJC	LAX	16	42	16	18	33	9	134
SJC	SAN	14	9	14	16	13	6	72
P	rofit	\$5,082,141	\$2,760,333	\$5,115,094	\$5,203,949	\$3,834,934	-\$1,929,978	\$20,066,472

Model Integration Fleet Assignment (Air \$275 / HSR \$225)

			0	-	· ·		,	
Air	rports	Air Price=	\$275	HSR Price=	\$225	Demand= 122,627		
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	19	19	20	22	21	10	111
LAX	SAN	26	62	25	29	50	13	205
LAX	SFO	10	23	10	10	18	5	76
LAX	SJC	21	24	21	22	24	9	121
OAK	LAX	16	37	16	18	32	9	128
SAN	LAX	19	2	16	23	17	49	126
SAN	SJC	1	8	0	3	10	32	54
SFO	LAX	13	12	13	15	12	2	67
SJC	LAX	17	42	17	19	32	9	136
SJC	SAN	14	9	15	16	12	6	72
Pı	Profit		\$2,616,105	\$4,944,152	\$4,982,827	\$3,617,058	-\$2,052,089	\$18,942,861

Model Integration Fleet Assignment (Air \$275 / HSR \$250)

	Air	ports	Air Price=	\$275	HSR Price=	\$250	Demand=	123,582	
0	)rigin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
1	LAX	OAK	20	18	20	22	21	9	110
I	LAX	SAN	12	63	15	21	51	32	194
1	LAX	SFO	10	22	10	10	17	5	74
I	LAX	SJC	19	24	20	22	24	11	120
(	OAK	LAX	17	36	17	18	31	9	128
5	SAN	LAX	21	1	19	25	17	45	128
5	SAN	SJC	3	7	3	4	10	28	55
5	SFO	LAX	14	11	14	16	12	2	69
:	SJC	LAX	18	41	18	19	31	9	136
	SJC	SAN	13	20	13	15	22	4	87
	Profit		\$4,543,531	\$2,717,082	\$4,733,537	\$4,884,340	\$3,727,099	-\$2,059,064	\$18,546,525

Table 4.20: Model Integration Fleet Assignment (Air \$275)

Model Integration Fleet Assignment (Air \$250 / HSR \$175)

_								,	
	Air	ports	Air Price=	\$250	HSR Price=	\$175	Demand=	122,639	
	Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	20	21	20	22	24	7	114
	LAX	SAN	7	61	11	18	48	40	185
	LAX	SFO	10	21	10	11	17	4	73
	LAX	SJC	15	35	15	20	35	12	132
	OAK	LAX	18	35	18	19	30	7	127
	SAN	LAX	19	0	16	22	16	50	123
	SAN	SJC	5	7	5	6	10	25	58
	SFO	LAX	1	15	1	4	15	22	58
	SJC	LAX	19	40	19	20	30	7	135
	SJC	SAN	14	19	14	15	21	2	85
	Profit		\$3,500,610	\$2,482,311	\$3,660,840	\$3,991,860	\$3,449,304	-\$3,572,571	\$13,512,354

Model Integration Fleet Assignment (Air \$250 / HSR \$200)

Air	ports	Air Price= \$250		HSR Price=	\$200	Demand=	123,936	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	21	20	22	23	23	7	116
LAX	SAN	9	60	13	19	48	38	187
LAX	SFO	12	20	12	12	16	2	74
LAX	SJC	16	34	16	21	34	11	132
OAK	LAX	19	34	19	19	29	6	126
SAN	LAX	19	1	16	21	17	51	125
SAN	SJC	13	8	12	12	9	12	66
SFO	LAX	0	16	2	4	16	21	59
SJC	LAX	20	40	22	20	31	4	137
SJC	SAN	16	15	14	15	15	4	79
Pı	Profit		\$2,439,796	\$4,177,104	\$4,159,243	\$3,350,286	-\$2,943,611	\$15,120,791

Model Integration Fleet Assignment (Air \$250 / HSR \$225)

8 8 ( +)									
Air	rports	Air Price=	\$250	HSR Price=	\$225	Demand=	125,212	_	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total	
LAX	OAK	27	18	27	26	21	0	119	
LAX	SAN	10	60	14	20	47	38	189	
LAX	SFO	12	20	12	12	16	2	74	
LAX	SJC	17	31	15	22	32	13	130	
OAK	LAX	20	33	20	20	28	7	128	
SAN	LAX	21	1	18	22	17	49	128	
SAN	SJC	14	7	13	13	9	11	67	
SFO	LAX	2	15	4	5	16	19	61	
SJC	LAX	21	40	23	21	30	4	139	
SJC	SAN	17	15	15	16	14	4	81	
P	Profit		\$2,334,981	\$4,540,069	\$4,406,612	\$3,231,738	-\$2,844,103	\$15,985,885	

Model Integration Fleet Assignment (Air \$250 / HSR \$250)

			0		,			
Air	ports	Air Price=	\$250	HSR Price=	\$250	Demand=	126,457	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	27	17	27	26	20	0	117
LAX	SAN	11	59	15	20	47	38	190
LAX	SFO	13	19	13	13	15	2	75
LAX	SJC	17	30	16	22	31	13	129
OAK	LAX	20	33	20	20	28	6	127
SAN	LAX	22	1	19	22	16	49	129
SAN	SJC	15	7	14	13	8	11	68
SFO	LAX	3	15	5	5	16	19	63
SJC	LAX	22	39	23	21	30	3	138
SJC	SAN	17	14	16	16	14	4	81
Pı	Profit		\$2,295,634	\$4,718,194	\$4,500,018	\$3,182,977	-\$2,577,887	\$16,615,839

Table 4.21: Model Integration Fleet Assignment (Air \$250)

Model Integration Fleet Assignment (Air \$225 / HSR \$175)

_									
	Air	ports	Air Price=	\$225	HSR Price=	\$175	Demand=	125,466	
	Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
	LAX	OAK	16	0	25	27	6	16	90
	LAX	SAN	25	40	25	26	60	19	195
	LAX	SFO	9	17	8	9	13	10	66
	LAX	SJC	26	27	25	26	31	0	135
	OAK	LAX	22	35	22	24	28	2	133
	SAN	LAX	31	40	26	27	41	20	185
	SAN	SJC	9	20	7	10	22	13	81
	SFO	LAX	13	10	12	12	3	8	58
	SJC	LAX	18	44	24	24	23	5	138
	SJC	SAN	8	2	8	6	0	25	49
	Profit		\$4,278,991	\$2,026,806	\$4,610,737	\$4,208,899	\$2,895,197	-\$3,121,681	\$14,898,949

Model Integration Fleet Assignment (Air \$225 / HSR \$200)

Air	ports	Air Price=	\$225	HSR Price=	\$200	Demand=	127,064	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	16	0	25	27	5	16	89
LAX	SAN	25	39	26	26	59	20	195
LAX	SFO	10	17	8	9	13	10	67
LAX	SJC	27	26	25	26	30	1	135
OAK	LAX	21	35	21	23	28	3	131
SAN	LAX	31	40	26	27	41	21	186
SAN	SJC	9	19	8	10	21	14	81
SFO	LAX	13	10	12	12	2	9	58
SJC	LAX	18	43	23	24	23	6	137
SJC	SAN	9	1	9	6	0	25	50
P	rofit	\$4,369,049	\$1,986,813	\$4,641,429	\$4,224,281	\$2,854,427	-\$3,554,337	\$14,521,663

Model Integration Fleet Assignment (Air \$225 / HSR \$225)

			0	-	· ·		,	
Air	rports	Air Price=	\$225	HSR Price=	\$225	Demand=	128,675	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	19	4	25	26	9	14	97
LAX	SAN	27	41	27	28	59	18	200
LAX	SFO	11	17	10	10	14	8	70
LAX	SJC	26	26	25	25	30	3	135
OAK	LAX	19	34	19	22	27	8	129
SAN	LAX	32	40	27	28	41	21	189
SAN	SJC	11	19	10	12	21	10	83
SFO	LAX	13	9	12	12	2	10	58
SJC	LAX	18	42	21	22	23	9	135
SJC	SAN	10	1	9	6	0	25	51
P	Profit		\$2,016,400	\$4,685,394	\$4,265,965	\$2,891,476	-\$3,538,101	\$14,779,876

Model Integration Fleet Assignment (Air \$225 / HSR \$250)

ports	Air Price=	\$225	HSR Price=	\$250	Demand=	130,254	
Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
OAK	23	3	23	25	11	14	99
SAN	28	35	28	27	56	21	195
SFO	10	18	6	8	16	12	70
SJC	19	31	19	21	32	12	134
LAX	19	24	19	20	20	14	116
LAX	31	41	33	35	44	13	197
SJC	7	9	5	6	13	24	64
LAX	12	21	12	12	19	4	80
LAX	19	40	21	22	25	10	137
SAN	16	14	16	18	18	3	85
Profit		\$2,037,538	\$4,598,257	\$4,353,638	\$3,223,540	-\$3,497,028	\$15,153,489
	Destination OAK SAN SFO SJC LAX LAX SJC LAX LAX SJC LAX LAX	Destination         Airbus 320           OAK         23           SAN         28           SFO         10           SJC         19           LAX         19           LAX         31           SJC         7           LAX         12           LAX         19           SAN         16	Destination         Airbus 320         CRJ-100 ER           OAK         23         3           SAN         28         35           SFO         10         18           SJC         19         31           LAX         19         24           LAX         31         41           SJC         7         9           LAX         12         21           LAX         19         40           SAN         16         14	Destination         Airbus 320         CRJ-100 ER         Boeing 737-3           OAK         23         3         23           SAN         28         35         28           SFO         10         18         6           SJC         19         31         19           LAX         19         24         19           LAX         31         41         33           SJC         7         9         5           LAX         12         21         12           LAX         19         40         21           SAN         16         14         16	Destination         Airbus 320         CRJ-100 ER         Boeing 737-3         Boeing 737-5           OAK         23         3         23         25           SAN         28         35         28         27           SFO         10         18         6         8           SJC         19         31         19         21           LAX         19         24         19         20           LAX         31         41         33         35           SJC         7         9         5         6           LAX         12         21         12         12           LAX         19         40         21         22           SAN         16         14         16         18	Destination         Airbus 320         CRJ-100 ER         Boeing 737-3 Boeing 737-5         ATR 72           OAK         23         3         23         25         11           SAN         28         35         28         27         56           SFO         10         18         6         8         16           SJC         19         31         19         21         32           LAX         19         24         19         20         20           LAX         31         41         33         35         44           SJC         7         9         5         6         13           LAX         12         21         12         12         19           LAX         19         40         21         22         25           SAN         16         14         16         18         18	Destination         Airbus 320         CRJ-100 ER         Boeing 737-3         Boeing 737-5         ATR 72         Boeing 767-3           OAK         23         3         23         25         11         14           SAN         28         35         28         27         56         21           SFO         10         18         6         8         16         12           SJC         19         31         19         21         32         12           LAX         19         24         19         20         20         14           LAX         31         41         33         35         44         13           SJC         7         9         5         6         13         24           LAX         12         21         12         12         19         4           LAX         12         21         12         12         19         4           LAX         19         40         21         22         25         10           SAN         16         14         16         18         18         3

Table 4.22: Model Integration Fleet Assignment (Air \$225)

Model Integration Fleet Assignment (Air \$200 / HSR \$175)

			0	-			,	
Air	ports	Air Price=	\$200	HSR Price=	\$175	Demand=	129,397	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	24	1	23	31	2	12	93
LAX	SAN	26	64	26	30	58	14	218
LAX	SFO	0	27	10	13	25	8	83
LAX	SJC	20	24	20	23	25	13	125
OAK	LAX	20	31	20	23	32	5	131
SAN	LAX	28	15	26	34	32	29	164
SAN	SJC	5	7	3	3	12	29	59
SFO	LAX	17	6	18	13	1	5	60
SJC	LAX	19	38	18	20	23	13	131
SJC	SAN	19	8	19	21	13	0	80
Pı	Profit		\$1,630,176	\$3,980,017	\$4,030,949	\$2,467,468	-\$4,134,617	\$11,651,751

Model Integration Fleet Assignment (Air \$200 / HSR \$200)

Air	ports	Air Price=	\$200	HSR Price=	\$200	Demand=	131,414	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	24	0	23	31	1	13	92
LAX	SAN	26	65	26	29	58	16	220
LAX	SFO	14	28	11	12	25	0	90
LAX	SJC	20	25	21	23	26	13	128
OAK	LAX	20	32	21	22	32	6	133
SAN	LAX	27	13	24	32	29	34	159
SAN	SJC	3	6	0	0	10	34	53
SFO	LAX	16	6	18	12	1	6	59
SJC	LAX	20	40	20	22	26	10	138
SJC	SAN	19	7	20	21	13	0	80
P	rofit	\$3,864,278	\$1,646,090	\$3,978,442	\$3,937,343	\$2,451,998	-\$4,281,836	\$11,596,315

Model Integration Fleet Assignment (Air \$200 / HSR \$225)

			0	-			,	
Air	ports	Air Price=	\$200	HSR Price=	\$225	Demand=	133,448	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	Boeing 767-3	Total
LAX	OAK	23	6	22	29	7	14	101
LAX	SAN	26	65	27	29	57	17	221
LAX	SFO	14	27	11	11	24	1	88
LAX	SJC	20	22	22	23	23	14	124
OAK	LAX	21	31	21	22	32	7	134
SAN	LAX	28	14	26	33	30	33	164
SAN	SJC	3	6	0	0	10	35	54
SFO	LAX	17	6	19	12	1	6	61
SJC	LAX	18	43	18	21	32	11	143
SJC	SAN	18	9	19	20	15	2	83
P	rofit	\$3,891,345	\$1,687,675	\$4,035,258	\$3,889,374	\$2,550,722	-\$4,413,445	\$11,640,930

Model Integration Fleet Assignment (Air \$200 / HSR \$250)

Air	ports	Air Price=	\$200	HSR Price=	\$250	Demand=	135,448	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	23	5	22	29	6	15	100
LAX	SAN	26	63	27	29	56	19	220
LAX	SFO	14	27	11	11	23	2	88
LAX	SJC	22	28	22	24	28	11	135
OAK	LAX	20	31	20	22	31	9	133
SAN	LAX	29	14	27	34	30	33	167
SAN	SJC	3	7	1	1	11	35	58
SFO	LAX	17	5	19	12	0	6	59
SJC	LAX	19	44	19	21	33	11	147
SJC	SAN	16	10	17	18	15	5	81
Pı	rofit	\$3,898,865	\$1,720,952	\$4,026,919	\$3,865,031	\$2,581,252	-\$4,865,005	\$11,228,014

Table 4.23: Model Integration Fleet Assignment (Air \$200)

## 4.5 Conclusion

The benefits of transportation policy analysis, more specifically the study of transportation systems planning and airline schedule planning, has far reaching benefits. Demonstrating the links between transportation policy, travel demand modeling, and resource allocation modeling such as fleet assignment, provides transportation planners with a means to view the cost and travel demand implications of their resource allocation and pricing decisions.

The study of transportation systems planning models and airline schedule planning models can take several forms. Integrating both planning models through user-mode choice and fleet assignment is one method of bridging the gap between sectors of transportation theory commonly studied in isolation. Other methods of integration can involve the other transportation forecasting models and airline schedule planning models which include trip generation, trip distribution, route assignment, schedule design, aircraft maintenance routing, and crew scheduling. Integrated analysis can be used to show how policy decisions, especially those which directly impact transportation policy, can affect multimodal public transportation.

The integration of transportation systems planning and fleet assignment resulted in feedback relationship between the main mode choice determinations of transportation systems planning and the resource allocation of the fleet assignment problem. Given high-speed rail & commercial air ridership and profit forecasts, transportation planners need a model to estimate the resources required to support multimodal interregional travel demand. More specifically, planners need to know how many planes are required to support California travel. The synthesized decision support method for multimodal transportation system planning models and airline schedule planning models can provide decision makers with the required synthesized decision making tool.

Without high-speed rail, reducing the price of commercial air resulted in an increase in commercial air ridership and a reduction in the number of car users. Since the majority of the high-speed rail ridership stems from a reduction in the number of car users, one can assume that the addition of high-speed rail to a multimodal transportation network should alleviate highway congestion, but not a have a great impact on commercial air congestion. Conventional rail ridership was not greatly impacted by other model changes. There was little change to the conventional rail ridership as a result of having or not having high-speed rail. Similar to the ridership results without high-speed rail, there was minimal change to the conventional rail ridership results based on changes to both commercial air and high-speed rail.

As stated previously, a Nash Equilibrium is defined where no player in a competitive game benefits from unilaterally changing their strategy. In this case of commercial air and high-speed rail pricing for the purpose of maximizing commercial air profit and high-speed rail ridership, a Nash equilibrium exists where commercial air sets its average price at \$350 and high-speed rail sets its average price at \$175. Commercial air loses profit and high-speed rail loses ridership if either decides to change their average price.

A possible scenario regarding the introduction of high-speed rail is the idea of subsidizing commercial air in order to allow a reduction high-speed rail pricing for the purpose of stimulating high-speed rail ridership and revenue. Reducing commercial air prices an average of ~\$25 results in a \$1M profit loss resulting in a high-speed rail revenue increase of ~\$300K. As a result, subsidizing commercial air for the benefit of high-speed rail is not cost effective.

The pricing strategy most beneficial to the future of commercial air and high-speed rail as alternate modes of transportation to car results in the worst short-term revenue benefit. Decision

makers will have to decide whether to set costs based on short-term or long-term benefit. Listed below are the conclusions which motivated the short-run analysis:

- What average prices for commercial air and high-speed rail maximize air profit and high-speed rail ridership? The maximum profit for commercial air and ridership for high-speed rail are achieved at the Nash equilibrium price of \$350 for commercial air and \$175 for high-speed rail.
- At what price is high-speed rail viable? The viability of high-speed rail depends on the
  cost of building and operating it. Without network cost information for high-speed the
  question of viability cannot be answered.

This chapter demonstrated how policy decisions through pricing subsidies can affect user mode choice decisions which in turn affect fleet assignment.

#### CHAPTER V

# GAME THEORETIC MULTIDISCIPLINARY OPTIMIZATION FOR MULTIPLE MARKET NETWORK USER EQUILIBRIUM DECISION ANALYSIS

"In competitive behavior, someone always loses."

- John Forbes Nash - "A Beautiful Mind"

## 5.1 Introduction

Developing and anticipating system responses in a competitive transportation network is a key task for decision makers managing multiple competitive markets. In the competitive airline industry, multiple airlines can assign multiple fleets to optimize for various objectives to include minimizing cost, maximizing market share, and maximizing profit. Current analysis of the commercial airline industry to conduct such optimization studies typically focuses internally on competing airlines and specific markets. As decision makers are considering and planning for the introduction of high-speed rail to their competitive commercial transportation markets, this work outlines a multidisciplinary methodological approach for analyzing a network of competitive markets which include commercial air and high-speed rail. This work seeks to establish the optimal resourcing conditions amongst competitors in a transportation network comprised of multiple origin-destination pairs such that any unilateral shifts result in either increased operational costs or a loss of market share. Transportation service provider resourcing affects the level-of-service provided to the transportation customer.

The term of level-of-service is used by transportation officials to measure the effectiveness of transportation systems (Papacostas 2001). Level-of-service is typical used to describe vehicle transit flow conditions where the various levels describe traffic flow conditions from free-flow operations to a breakdown in vehicle flow. While levels of service can be quantified in terms of vehicle headway or car length spaces between vehicles, the term level-of-service is often subjective. For the purpose of this synthesized research, airline levels-of-service will be considered in terms of service headway or the amount of distance, measured in time, between transportation service vehicles (Wardman 2004). Assuming a constant operating day, level-of-service will be described by the number of flights conducted per day between an origin-destination pair. Level-of-service conditions become a key constraint when conducting systems analysis, since level-of-service conditions define network capacity.

This chapter is the next step in the multidisciplinary transportation systems analysis framework provided by this research. This research began with the construction of a parsimonious travel demand modeling for California high-speed rail followed by the integration of transportation systems planning and airline schedule planning through the use of multidisciplinary optimization. This chapter expands the concept of determining equilibrium conditions from a network perspective consisting of multiple individual markets using a game theoretic and multidisciplinary optimization method for multidisciplinary analysis.

This remainder of this chapter is organized into five main sections. The first section presents the methodology and formulations. Next, a case study is provided to illustrate the synthesized decision support method followed by results and discussion, and a conclusion which explain how the proposed method provides a synthesized method for user equilibrium decision analysis.

# 5.2 Game Theoretic Multidisciplinary Optimization Methodology

In this chapter, game theoretic multidisciplinary optimization is used to solve a network user equilibrium decision analysis problem. Game theoretic optimization combines the principles of optimization to solve game theoretic problems. A coupling of game theory and optimization was proposed by Palomar et al which explored the theoretic principles and techniques of game theory convex optimization and variational inequality theory and discussed their relationships to Nash Equilibrium problems (Scutari et al. 2010). A study of game theory and transportation system modeling was conducted by Fisk analyzed the problem of operator competition and uses game theory to formulate a solution (Fisk 1984). A game theoretic approach to urban public transport integration policy was proposed by Roumboutos and Kapros to predict the outcomes of various fare and location dependent strategies for public and private transportation operators (Roumboutsos and Kapros 2008). The game theoretic optimization methodology follows.

Master Problem for airline a: Given  $x_{f \propto}$ ,  $+y_{k \propto}$  for all airlines except airline a.

$$\begin{aligned} Max_{x_a,y_a,p_a}\pi \\ \pi &= R_a - CC_a(x_a) - OC_a(y_a) \\ \\ \sum_{o \in O} \sum_{d \in D} y_{fodka} &\leq x_f(MOPD) \ \forall \ f \in F, k \in K \\ \\ \\ \sum_{o \in O} \sum_{d \in D} y_{fodka}(PAX_f) &\geq Q_{oda}(Y_k) \ \forall \ o \in O, k \in K \end{aligned}$$

$$x_a^L \le x \le x_a^U$$
$$y \ge 0$$

The objective function is formulated for each player. The objective function constraints for each player are dependent on decisions made by the other players as these decisions affect the user demand supported by each player. For this optimization formulation, the solution is derived by iterating through the objective functions of each player while holding constant the decisions for the other competitors. In this research, consecutively optimizing the objective functions of each player will be considered one optimization iteration.

Another approach to the optimization iteration approach is to optimize the decisions of all players simultaneously. Based on the number of players and subsequent degrees of freedom, this simultaneous formulation approach can quickly become intractable due to computational expense. After each optimization iteration, re-evaluate user demand. User demand can be re-evaluated through the use of a travel demand model such as the PTDM or by using a demand redistribution decision rule. The process of re-evaluating user demand and conducting an optimization iteration continues until either the demand distribution or its optimal resourcing converges. For this analysis, user demand re-evaluation will be is based on a decision rule. The decision rule assumes equal service provider utilization based on the capacity provided by each service provider.

# 5.3 Case Study

Optimization and game theory models have been used to estimate responses to changes in network conditions. Changes in resourcing and pricing are amongst the primary potential network responses to a market entrant. Given the potential entrance of high-speed rail to US commercial, transportation providers require decision support to be prepared for competitor responses. These decision support questions require a synthesized approach for conducting multidisciplinary multimodal decision support over time. To illustrate the method of anticipating network responses to changes in network conditions, the following case study problem utilizes travel demand results from the parsimonious travel demand model from chapter three to develop a competitive response model which outlines the relationship between three competitive airlines responding to the entrance of high-speed rail and to each other. This analysis assumes aircraft are dedicated to specific origin-destination links.

# **5.3.1** Problem Description

This case study problem involves three competitive airlines allocating aircraft to support travel demand for ten origin-destination pairs assuming four possible demand distribution scenarios. The four scenarios are based on the four HSR price values as shown in Table 5.25. The three airlines in this case study problem are United Airlines, Southwest Airlines, and American Airlines. The data used in this game is generally based on Bureau of Transportation Statistics data from 2007 to 2009. The problem description, constraints, and solutions provided are meant to be used for illustrative purposes only and not directly indicative of previous or anticipated airline behavior. In this problem, each airline has two fleets to choose from as shown in Table. 5.24.

Airlin	Airline Allocation Aircraft									
Airline	Aircraft Model	Seating Capacity								
United	CRJ-100 ER	50								
United	A 320	138								
Southwest	B 737 - 5	122								
Southwest	B 737 - 3/7	137								
American	ATR 72	64								
American	В 767 - 3	225								

Table 5.24: Airline Allocation Aircraft

This airline allocation game involves three competitive commercial airlines seeking to minimize the cost of allocating resources to support customer demand. The airline available aircraft, seating capacities, and allocation initial conditions are illustrated in Tables 5.24 to Table 5.26. The objective function for each player is to maximize the profit of allocating aircraft in support of customer demand. The demand allocation is assumed to be dependent on resource capacity provided by each player. The cost is based on the NASA cost model used in previous chapters (Harris 2005).

In this case study, there are six input parameters corresponding to the six fleet choices of the three competing airlines. The ranges of the six parameters are based on the feasible ranges of the input parameters given the customer demand initial conditions. The problem of resourcing and its relationship to user main-mode choice is assumed to be an instantaneous process although network conditions often take time to reach equilibrium conditions.

Aircraft Demand (Air Price: \$350)

0-1-1-	Destination	Distance		HSR	Prices	
Origin	Destination	(miles)	\$175	\$200	\$225	\$250
LAX	OAK	369.5	9,966	10,041	10,081	10,116
LAX	SAN	125.3	15,730	15,848	15,912	15,967
LAX	SFO	388.1	5,764	5,808	5,831	5,851
LAX	SJC	347.7	10,552	10,631	10,674	10,711
OAK	LAX	369.5	9,966	10,041	10,081	10,116
SAN	LAX	125.3	15,730	15,848	15,912	15,967
SAN	SJC	463.7	6,839	6,891	6,918	6,942
SFO	LAX	388.1	5,862	5,906	5,930	5,951
SJC	LAX	347.7	10,454	10,533	10,575	10,612
SJC SAN		463.7	6,839	6,891	6,918	6,942
	Total Deman	ıd	97,701	98,437	98,831	99,177

Table 5.25: Aircraft Demand By Scenario

The pricing used for this case study is an average of the ticket prices across the 10 origin-destination pairs. As expected, the resulting demands for each scenario vary based on pricing conditions.

**Airline Aircraft Initial Conditions** 

Airline	U	nited	South	west	Ame	rican
OD Pair	A 320	CRJ-100 ER	B 737 - 3/7 B 737 - 5		ATR 72	В 767 - 3
1	10	10	44	44	0	0
2	45	37	0	0	125	20
3	25	25	0	0	40	5
4	10	10	26	26	45	10
5	57	37	13	13	0	0
6	55	37	0	0	80	25
7	0	0	20	20	45	10
8	25	25	0	0	40	5
9	10	10	30	30	45	10
10	0	0	18	18	45	10

Table 5.26: Airline Allocation Initial Conditions

The resource allocation problem is non-linear and as a result is subject to the network initial allocation conditions. These aircraft distribution initial conditions are generally based on aircraft allocations in the California Corridor from 2007 to 2009. Based on the aircraft resource allocation initial conditions, Table 5.27 shows the initial competitive airline demands.

Commercial Airline Initial Demand Split

0	Dantingtion		97,701			98,831			98,437			99,177	
Origin	Destination	United	SWA	AA									
LAX	OAK	1,411	8,554	0	1,422	8,619	0	1,428	8,653	0	1,433	8,684	0
LAX	SAN	6,166	0	9,563	6,213	0	9,635	6,238	0	9,674	6,260	0	9,708
LAX	SFO	3,231	0	2,533	3,255	0	2,552	3,268	0	2,563	3,280	0	2,572
LAX	SJC	1,443	5,170	3,938	1,454	5,209	3,968	1,460	5,230	3,984	1,465	5,248	3,998
OAK	LAX	7,401	2,565	0	7,457	2,584	0	7,486	2,594	0	7,513	2,603	0
SAN	LAX	7,356	0	8,373	7,412	0	8,436	7,442	0	8,470	7,468	0	8,500
SAN	SJC	0	3,436	3,403	0	3,462	3,429	0	3,476	3,442	0	3,488	3,454
SFO	LAX	3,286	0	2,576	3,311	0	2,596	3,324	0	2,606	3,335	0	2,615
SJC	LAX	1,330	5,496	3,628	1,340	5,537	3,656	1,345	5,559	3,670	1,350	5,579	3,683
SJC	SAN	0	3,256	3,583	0	3,281	3,610	0	3,294	3,624	0	3,305	3,637
	Total	31,625	28,477	37,599	31,863	28,691	37,882	31,991	28,806	38,034	32,103	28,907	38,167

Table 5.27: Commercial Aircraft Demand By Airline

### 5.4 Results

Similar to the previous, this chapter models resource allocation associated with four price-related customer demands. Unlike the previous chapter, this chapter provides a method for conducting analysis for a network made up of multiple markets. As expected, the customer demands with the higher demand values have slightly increased resource allocations. This relatively constant allocation is due to varying aircraft percent utilizations. The airlines wish to maximize profitability and are forced to increase their level of service and possibly reconsider pricing strategy. Airlines maximize profitability by flying more small aircraft.

Tables 5.28 and 5.29 list the competitive airline fleet allocations with and without high-speed rail. As expected, there was a significant decrease in air profit (~\$7M) upon the

introduction of high-speed rail. In addition, the resource allocations vary slightly amongst the four customer demand values.

Tables 5.30 to 5.36 list the competitive prices for the three airlines given the four demand values. Surprisingly the optimal prices in general were outside the feasible range of the pricing model.

Base Scenario: Multiple Airline Fleet Assignment - Air Demand: 121,180

Air	ports	Un	ited	Sout	hwest	Am	erican	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	23	9	25	0	26	9	92
LAX	SAN	31	31	25	20	59	10	174
LAX	SFO	0	39	3	11	4	7	64
LAX	SJC	20	16	25	1	54	1	118
OAK	LAX	7	47	15	10	27	7	114
SAN	LAX	30	35	24	22	59	9	178
SAN	SJC	13	5	0	18	9	7	53
SFO	LAX	0	39	14	0	5	7	65
SJC	LAX	22	10	23	3	30	7	96
SJC	SAN	13	5	16	0	11	7	52
Pı	Profit \$		\$5,616,357	\$11,399,286	\$4,977,415	\$8,963,441	\$5,733,717	\$47,430,230

Table 5.28: Base Scenario: Cooperative Fleet Assignment Without High-Speed Rail

Multiple Airline Fleet Assignment - Air Demand: 97,701

Air	ports	Un	ited	South	nwest	Ame	erican	
Origin	Destination	Airbus 320	CRJ-100 ER	<b>Boeing 737-3</b>	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	21	10	23	1	29	6	91
LAX	SAN	27	30	24	17	59	6	163
LAX	SFO	0	38	6	9	7	6	67
LAX	SJC	22	10	25	1	32	6	97
OAK	LAX	8	43	15	11	29	6	113
SAN	LAX	26	35	22	18	59	6	166
SAN	SJC	14	7	3	15	13	6	59
SFO	LAX	0	39	12	2	8	6	68
SJC	LAX	22	10	22	4	32	6	96
SJC	SAN	14	7	14	3	13	6	58
Pı	rofit	\$10,109,854	\$4,743,074	\$10,911,683	\$3,733,637	\$8,899,047	\$3,950,743	\$42,348,038

Table 5.29: GTO Fleet Assignment With High-Speed Rail – Demand 97,701

Multiple Airline Fleet Assignment - Air Demand: 98,437

Air	rports	Un	ited	Sout	hwest	Am	erican	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	21	8	24	0	25	8	87
LAX	SAN	28	30	23	17	58	7	162
LAX	SFO	0	39	3	12	4	8	65
LAX	SJC	20	16	25	1	54	0	116
OAK	LAX	8	46	15	10	27	7	113
SAN	LAX	26	33	22	19	59	7	166
SAN	SJC	15	5	0	19	9	8	55
SFO	LAX	0	39	14	0	5	7	66
SJC	LAX	22	9	23	3	30	7	94
SJC	SAN	15	5	16	1	11	7	55
P	rofit	\$10,360,203	\$5,499,649	\$11,167,958	\$4,769,249	\$8,887,221	\$5,247,150	\$45,931,429

Table 5.30: GTO Fleet Assignment With High-Speed Rail – Demand 98,437

Multiple Airline Fleet Assignment - Air Demand: 98,831

		Munipic	ZIII IIIIC I ICO	ct 21551511111CI	t m Demu	10. 70,001		
Air	rports	Un	ited	Sout	ıwest	Ame	erican	
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	21	10	23	1	29	7	91
LAX	SAN	28	30	24	17	59	7	164
LAX	SFO	0	38	6	9	7	7	67
LAX	SJC	22	10	25	1	32	7	98
OAK	LAX	9	44	15	11	29	7	114
SAN	LAX	26	35	23	18	59	7	167
SAN	SJC	14	8	3	15	13	7	60
SFO	LAX	0	39	12	3	8	7	68
SJC	LAX	22	10	22	4	32	7	97
SJC	SAN	14	8	14	3	13	7	58
P	rofit	\$10,506,212	\$5,648,774	\$11,269,798	\$4,943,130	\$8,881,362	-\$1,068,763	\$40,180,512

Table 5.31: GTO Fleet Assignment With High-Speed Rail – Demand 98,831

Multiple Airline Fleet Assignment - Air Demand: 99,177

Air	ports	Un	ited	Sout	hwest	Am	erican	
Origin	Destination	Airbus 320	CRJ-100 ER	<b>Boeing 737-3</b>	<b>Boeing 737-5</b>	ATR 72	<b>Boeing 767-3</b>	Total
LAX	OAK	21	10	23	1	26	7	89
LAX	SAN	28	29	24	17	57	8	162
LAX	SFO	0	38	6	9	5	7	66
LAX	SJC	20	16	25	2	56	0	118
OAK	LAX	9	44	15	11	27	7	112
SAN	LAX	26	34	23	18	57	7	166
SAN	SJC	15	6	3	15	10	7	57
SFO	LAX	1	38	12	3	5	7	66
SJC	LAX	22	10	22	4	29	7	95
SJC	SAN	14	7	14	3	10	7	56
P	rofit	\$10,432,086	\$5,506,637	\$11,136,247	\$4,731,000	\$8,911,512	\$5,335,594	\$46,053,077

Table 5.32: GTO Fleet Assignment With High-Speed Rail – Demand 99,177

Multiple Airline Optimal Prices - Air Demand: 97,701

A :-			ited	•	4		erican	OD D :
All	rports	UI	iitea	Souti	iwest	Am	erican	OD Pair
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	Boeing 737-5	ATR 72	Boeing 767-3	Average Price
LAX	OAK	\$500	\$313	\$500	\$313	\$500	\$313	\$407
LAX	SAN	\$500	\$357	\$500	\$357	\$500	\$357	\$429
LAX	SFO	\$313	\$500	\$313	\$500	\$500	\$313	\$406
LAX	SJC	\$500	\$319	\$500	\$319	\$500	\$319	\$409
OAK	LAX	\$313	\$500	\$500	\$313	\$500	\$313	\$407
SAN	LAX	\$500	\$357	\$500	\$357	\$500	\$357	\$429
SAN	SJC	\$500	\$300	\$300	\$500	\$500	\$300	\$400
SFO	LAX	\$313	\$500	\$500	\$313	\$500	\$313	\$406
SJC	LAX	\$500	\$319	\$500	\$319	\$500	\$319	\$409
SJC	SAN	\$500	\$313	\$500	\$313	\$500	\$313	\$407
Aircraft A	verage Price	\$444	\$378	\$461	\$360	\$500	\$322	\$411

Table 5.33: GTO Optimal Prices With High-Speed Rail – Demand 97,701

Multiple Airline Optimal Prices - Air Demand: 98,831

Air	ports	Un	iited	Sout	hwest	Am	erican	OD Pair
Origin	Destination	Airbus 320	CRJ-100 ER	Boeing 737-3	Boeing 737-5	ATR 72	<b>Boeing 767-3</b>	Average Price
LAX	OAK	\$500	\$500	\$500	\$500	\$500	-	\$500
LAX	SAN	\$500	\$500	\$500	\$500	\$500	-	\$500
LAX	SFO	\$500	\$500	\$500	\$500	\$500	-	\$500
LAX	SJC	\$500	\$500	\$500	\$500	\$500	-	\$500
OAK	LAX	\$500	\$500	\$500	\$500	\$500	-	\$500
SAN	LAX	\$500	\$500	\$500	\$500	\$500	-	\$500
SAN	SJC	\$500	\$500	\$500	\$500	\$500	-	\$500
SFO	LAX	\$500	\$500	\$500	\$500	\$500	-	\$500
SJC	LAX	\$500	\$500	\$500	\$500	\$500	-	\$500
SJC	SAN	\$500	\$500	\$500	\$500	\$500	-	\$500
Aircraft A	Aircraft Average Price		\$500	\$500	\$500	\$500	-	\$500

Table 5.34: GTO Optimal Prices With High-Speed Rail – Demand 98,831

Multiple Airline Optimal Prices - Air Demand: 98,437

Multiple Airline Optimal Prices - Air Demand: 98,437												
Airports		United		Southwest		American		OD Pair				
Origin	Destination	Airbus 320	CRJ-100 ER	<b>Boeing 737-3</b>	Boeing 737-5	ATR 72	<b>Boeing 767-3</b>	Average Price				
LAX	OAK	\$500	\$430	\$500	\$430	\$500	\$430	\$465				
LAX	SAN	\$500	\$500	\$500	\$500	\$500	\$500	\$500				
LAX	SFO	\$380	\$500	\$380	\$500	\$500	\$380	\$440				
LAX	SJC	\$500	\$477	\$500	\$477	\$500	\$477	\$488				
OAK	LAX	\$430	\$500	\$500	\$430	\$500	\$430	\$465				
SAN	LAX	\$500	\$499	\$500	\$499	\$500	\$499	\$499				
SAN	SJC	\$500	\$393	\$393	\$500	\$500	\$393	\$446				
SFO	LAX	\$381	\$500	\$500	\$381	\$500	\$381	\$440				
SJC	LAX	\$500	\$436	\$500	\$436	\$500	\$436	\$468				
SJC	SAN	\$500	\$393	\$500	\$393	\$500	\$393	\$446				
Aircraft Average Price		\$469	\$463	\$477	\$454	\$500	\$432	\$466				

Table 5.35: GTO Optimal Prices With High-Speed Rail – Demand 98,437

Multiple Airline Optimal Prices - Air Demand: 99,177

Airports		United		Southwest		American		OD Pair
Origin	Destination	Airbus 320	CRJ-100 ER	<b>Boeing 737-3</b>	Boeing 737-5	ATR 72	<b>Boeing 767-3</b>	Average Price
LAX	OAK	\$500	\$430	\$500	\$430	\$500	\$430	\$465
LAX	SAN	\$500	\$500	\$500	\$500	\$500	\$500	\$500
LAX	SFO	\$380	\$500	\$380	\$500	\$500	\$380	\$440
LAX	SJC	\$500	\$477	\$500	\$477	\$500	\$477	\$488
OAK	LAX	\$430	\$500	\$500	\$430	\$500	\$430	\$465
SAN	LAX	\$500	\$499	\$500	\$499	\$500	\$499	\$499
SAN	SJC	\$500	\$393	\$393	\$500	\$500	\$393	\$446
SFO	LAX	\$381	\$500	\$500	\$381	\$500	\$381	\$440
SJC	LAX	\$500	\$436	\$500	\$436	\$500	\$436	\$468
SJC	SAN	\$500	\$393	\$500	\$393	\$500	\$393	\$446
Aircraft Average Price		\$469	\$463	\$477	\$454	\$500	\$432	\$466

Table 5.36: GTO Optimal Prices With High-Speed Rail – Demand 99,177

## 5.5 Conclusions

Game theoretic optimization can serve as an effective means for conducting transportation systems analysis. This work demonstrated how optimization can be utilized within the context of game theory to determine equilibrium points in terms of pricing scenarios for the analysis of multimodal systems to conclude commercial and high-speed rail. This work demonstrated how optimal pricing condition amongst competitors such that any unilateral shifts result in a loss of either profit or market share. Lastly, this work showed how pricing strategies can affect the identification of equilibrium points that determine profit, revenue, and market share. As the Game Theoretic Optimization best response results are not sensitive to initial conditions, it is assumed that each unique combination of best response functions either have a unique equilibrium point or none at all.

## **CHAPTER VI**

## **CONCLUSIONS**

"Let me tell you the secret that has led me to my goal. My strength lies solely in my tenacity." – Louis Pasteur

This research provided a framework to conduct system-of-systems analysis using two specific system-of-systems analysis applications directly affected by customer or user decision choice namely transportation systems planning and airline schedule planning. This framework was designed to aid system-of-systems decision makers in conducting resourcing and pricing analysis. While this research focused on two transportation system-of-systems, the methodologies provided in this research are generalizable to other system-of-systems domains. This research addressed two of primary criticisms of system-of-systems analysis, complexity and computational expense along with not considering the effects of outside systems by successfully developing a feasible transportation demand model for repetitive analysis, the parsimonious travel demand model using the California high-speed rail study conducting by Cambridge Systematics as a parent model for model reduction and calibration. This research considered both the effects of and on outside system-of-systems models by integrating the fleet assignment model from airline schedule planning and showing how both models impact each other through highlighting the input and output variables of both models in the context of multidisciplinary optimization. This research further demonstrated the cause and effect relationships between the

user mode choice models of transportation systems planning and fleet assignment models of airline schedule planning. Using game theoretic optimization, this research demonstrated a methodology to model and estimate system-of-systems resource requirements.

This research provided a normal-form game methodology to conduct near-term pricing analysis for a transportation system-of-systems network. This analysis methodology provided a means to quantitatively identify network conditions and interactions critical for network analysis. In the case of the California high-speed rail study, this research identified the nested relationship between commercial air and high-speed rail passengers as compared to car travelers. This type of analysis in meant to address large-scale transportation network concerns such as the goal of U.S. transportation decision-makers to address their highway, and air transportation congestion issues. In response to this specific concern in the California Corridor, this research concludes that congestion will not be considerably mitigated by the introduction of high-speed rail in California regardless of the scope of the California high-speed rail project. Due to the effect of induced demand, it is concluded that aggregate travel demand will increase as more transportation capacity is created.

Given that pricing decisions are made on a shorter time scale than airline resource acquisition and schedule design, pricing strategies can be identified using the simplified planning model as shown by the formulation of HSR/Air pricing as a normal form game, the prediction of HSR/Air ridership and profit as a function of ticket pricing, and the identification of equilibrium pricing strategies. Based on the PTDM model, the ridership and profit equilibrium pricing is \$350 for air & \$175 for HSR. The assessment of the viability of HSR in California is dependent on highly subjective modeling constants and accurate cost assumptions.

Based on the results of the model simplification chapter, this research concludes that a useful simplified model, suitable for sensitivity analysis, uncertainty quantification, and optimization studies can be specified, estimated, and validated. The PTDM is most sensitive to the trip constant and mode constant model parameters followed by travel cost and income parameters. Other parameters are insignificant in the model prediction uncertainty. The contribution of the key parameters to the uncertainty in model predictions is 96.7% for the trip constants and 49.5% for the mode constants. The response of the total air transportation system to the presence of HSR can be predicted provided that outside influencers are considered in the model analysis. The competition between multiple airlines can be modeled in a system-of-systems context. The presence of HSR will shift the balance of competitive airlines but not a great deal. As airlines need to become more competitive, smaller aircraft will become more heavily utilized to allow airlines to remain competitive.

As shown by the analysis the Cambridge Systematics travel demand model, transportation systems do not operate in a vacuum and as a result cannot be analyzed without taking into consideration its effects on and from outside sources such as other transportation models and environmental conditions. Future travel demand analysis should consider the resourcing, pricing, and infrastructure effects on a transportation network. The demand modeling, uncertainty quantification, and sensitivity analysis methodologies utilized applied here to mitigate computational expense can be used in other transportation modeling applications. I recommend conducting similar travel demand, resourcing, and pricing analysis for other regions considering the introduction of high-speed rail to their commercial transportation network such as the Northeast, and Midwest Corridors. As high-speed rail does not currently exist in the U.S., cost models do not exist for U.S. based high-speed rail, I recommend further analysis be

conducted to develop high-speed rail cost models. Lastly, the parsimonious travel demand model developed in this research was conducted at the county-level, as a result, I recommend exploring different degrees of model resolution to find the best balance between model accuracy and computational expense and complexity.

### APPENDIX: U.S. AIRLINE COST MODEL

(Harris 2005)

# SUMMARY OF EXPENSE ESTIMATING EQUATIONS

This appendix provides a concise summary of the several expense-estimating equations associated with this economic model. Explanatory notes are provided as appropriate or required. Each equation yields the *yearly expenses* of one aircraft at the flight equipment level, not expenses per block hour or per trip or per airborne hour. The expenses are in 1999 dollars. No estimate of inflation or other major changes within the industry is considered.\*

Several equations require an assessment of the airline's approach to business, quantified by an airline factor. The airline factor attempts to account for such things as a start-up situation, a charter airline approach, a "lean and mean" philosophy, the average airline, a mature but low-fare airline, or a mature major airline. In some equations, the range of this airline factor is large. However, this reflects the industry as it existed in 1999.

Appendix 3 tabulates representative values for all aircraft parameters required by this economic model.

Any number of comparisons driven by the variables—not by the airline factor—can be made using this economic model. One need only set any given airline factor to average and then proceed.

## Flight Crew Expenses (page 17)

## Fuel & Oil Expenses (page 21)

$$Fuel \, Expense = \frac{Fuel \, Cost}{Gallon} \Bigg( \frac{Non\text{-}cruise \, gallons}{Departure} + \frac{Cruise \, gallons}{Departure} \Bigg) Departure$$

<sup>\*</sup>This model does not, for example, attempt to reflect the disruption of September 11, 2001. The model's basis is industry data of 1999 and the model was developed during the period January 2000 through July 2002. The first draft of this report was completed in early September of 2002.

Takeoff gross weight (TOGW) for passenger aircraft assumes one passenger equals 225 pounds. Fuel weight is 6.5 lbs/gal. Fuel load is increased by 50 percent to provide a reserve.

$$TOGW = Operating WE + 225 (Available Seats) (Load Factor) + 1.5 \left(\frac{6.5 \text{ lbs}}{\text{gal}}\right) \left(\frac{\text{Fuel in lbs}}{\text{Departure}}\right)$$

Cargo aircraft TOGW assumes one ton of cargo equals 2,000 pounds. Fuel weight is 6.5 lbs/gal. Fuel load is increased by 50 percent to provide a reserve.

TOGW = Operating WE + 2000 (Available Tons) (Load Factor) + 1.5 
$$\left(\frac{6.5 \text{ lbs}}{\text{gal}}\right) \left(\frac{\text{Fuel in lbs}}{\text{Departure}}\right)$$

$$\frac{\text{Non-cruise } \text{Gal.}}{\text{Departure}} = \frac{0.001713 \left(\text{SFC}_{jet} \times \text{Thrust}\right)_{Takeoff}}{\left(\text{Thrust/TOGW}\right)_{Takeoff}^{2}} \left(\text{turbojet/turbofan - driven airplane}\right)$$

Non-cruise Gal. 0.01113 (SFC ... × BHP)

$$\frac{\text{Non-cruise } \text{Gal.}}{\text{Departure}} = \frac{0.01113 \left( \text{SFC}_{\text{piston}} \times \text{BHP} \right)_{\text{Takeoff}}}{\left( \text{Thrust/TOGW} \right)_{\text{Takeoff}}^{2}} \quad \left( \text{turboprop-driven airplane} \right)$$

Start Cruise at Winitial = Takeoff Gross Weight - 6.5 lb/gal (Non-cruise Gallons)

$$\begin{split} & \frac{\text{Cruise Gallons}}{\text{Departure}} = \frac{W_{\text{initial}} \left(1 - e^{-K}\right)}{6.5 \text{ lbs/gal}} \\ & \text{where } K_{\text{Jet}} = \frac{\text{Range} \times \text{SFC}_{\text{Cruise}}}{\left(V \times L/D\right)_{\text{Average}}} \text{ and } K_{\text{Prop}} = \frac{\text{Range} \times \text{SFC}_{\text{Cruise}}}{\left(375 \times \eta_p \times L/D\right)_{\text{Average}}} \end{split}$$

#### Nomenclature:

- Thrust refers to the sum of thrusts from all engines or propellers. Units are pounds.
- BHP is the sum of brake horsepower from all engines driving propellers. Units are hp.
- SFC is specific fuel consumption in fuel pounds/hour per pounds of thrust for jets or fuel pounds/hour per BHP for engines driving propellers.
- d. V is average cruise speed in statute miles per hour. (See T-2, z410.0/z650.0)
- e. Range is statute miles per departure (See T-2, z410.0/z510.0)
- The lift to drag ratio (L/D) has no units.
- g. Propeller efficiency (η<sub>p</sub>) has no units.
- h. Operating Weight Empty. Units are pounds.
- Fuel cost per gallon in 1999 was \$0.51.

## Calculation Notes:

The fuel calculations require iteration because the TOGW depends on the pounds of fuel required by the departure (i.e., trip); but the fuel required depends on the TOGW. Initiate the iteration with the takeoff gross weight at maximum. Then run through the equations and recalculate the TOGW. If the second TOGW is higher than the maximum TOGW, stop the calculation at one iteration and use the calculated fuel. (This result means the 50 percent fuel reserve is too high.) If the second TOGW is lower than the maximum TOGW, proceed to iterate until the calculation converges.

### Insurance Expenses (page 25)

Insurance Expense = 0.0056 (Capital Invested)

This insurance covers what is called "hull" insurance for aircraft owned by the airline. Lacking a more appropriate insurance company policy contract, use the aircraft purchase price in the year the aircraft was bought by the airline. The constant, 0.0056, is associated with the industry in 1999.

## Rental Expenses (page 26)

Rental Expense = 0.0835 (Capital Invested By Leasing Company)

A leasing company buys an aircraft and then leases or rents the aircraft to an airline. Use the aircraft purchase price in the year the aircraft was bought by the leasing company. The constant, 0.0835, is associated with the industry in 1999. This rental expenses assumes a "dry" lease where the airline pays for the fuel and oil.

## Other Flying Operation Expenses (page 27)

Other FO Expenses = 0.04 (Flight Crew + Fuel & Oil + Insurance + Rental )

## Flying Operation Expenses (page 27)

Flying Operation Expenses = Flight Crew

- + Fuel & Oil
- + Insurance
- + Rental
- + Other FO

#### Flight Equipment Maintenance Expenses (page 30)

Flight Equipment Maintenance Expenses = Airframe Maint. + Engine Maint.

$$\begin{split} & \text{Airframe = K} \left\{ \left( \text{Ref.W} \right)^{0.72118} \left( \text{FH} \right)^{0.46050} \left( \text{DP} \right)^{0.32062} \left( \text{NAC} \right)^{0.20700} \left( 1 + \frac{\text{Inhouse AF}}{\text{Total AF}} \right)^{-0.43177} \right\} \\ & \text{Engine = K} \left\{ \left( \text{Thrust} \right)^{0.89650} \left( \text{N}_{\text{E}} \right)^{0.92340} \left( \text{FH} \right)^{0.15344} \left( \text{DP} \right)^{0.37535} \left( \text{NAC} \right)^{0.4429} \left( 1 + \frac{\text{Outside Eng.}}{\text{Total Eng.}} \right)^{-0.34704} \right\} \end{split}$$

The constant K depends on 4 considerations as

$$K = ST[1.73(CF)(MF)(ET)]$$

#### where

```
ST = Service Type (Passenger = 1.0, Cargo = 1.3252)

ET = Engine Type (Turbofan = 1.0, Turboprop = 1.2644)

MF = Aircraft Model Factor (Earliest = 1.0, Early = 0.7104, Recent = 0.514,

Latest = 0.4260, Very Latest = 0.35)

CF = Airline Cost Factor (Very Low = 0.4470, Low = 0.8339, Average = 1.0, High = 1.3019)
```

The constant K introduces an aircraft Model Factor to reflect the aircraft generation and quantify aircraft age. The logic here is that the airlines operated, in 1999, a wide range of aircraft models. However, in the jet engine propelling group, for example, all the aircraft have swept wings. The fundamental type begins with the earliest Boeing 707 class, passes through smaller and larger variations, and ends with the very latest Boeing 777 class. While the takeoff gross weight varies a great deal between classes, the fundamental technology remains. Improvements over the 4 decades have occurred, however, which lowered maintenance expenses. In this light, the earliest swept-wing, jet-propelled model in a given class has been assigned a Model Factor of one. More recent versions have a reduced value Model Factor. The table at the end of this appendix should help in conveying the author's logic.\*

The table at the end of this appendix lists, qualitatively, the classification of all aircraft in the industry's fleet in terms like earliest, recent, latest, etc. The numerical values assigned to the qualitative classifications were found by iterations so that the predicted flight equipment expenses correlated with DOT, Form 41, reported data.

Finally, the definitions of variables used in the airframe and engine maintenance equations are

```
Thrust ≡ Propulsion Unit's Thrust at Sea Level Standard Day, in pounds

N<sub>p</sub> ≡ Number of Propulsion Units per Aircraft
```

Ref.  $W \equiv Reference Weight of Aircraft$ 

= Minimum Operational Weight Empty LESS Engine Dry Weight, in pounds

 $FH \equiv Flight Hours Flown by the Fleet in One Year, in hours$ 

DP 

■ Departures Performed by the Fleet in One Year

NAC 

Number of Aircraft in Fleet for the Year

In following this logic, the author would assign the SST, the first in its technology and class, with a Model Factor = 1. Similarly, should a commercial airliner evolve from the military tiltrotor program, it would be "the first" and receive MF = 1. Should models evolve (i.e., introducing a SST-200 after the now flying SST-100) from either of these two unique technologies, that aircraft would advance from MF = 1.0 to early and MF = 0.7104. The assumption is, of course, that improvements, which reduce maintenance expenses, are incorporated in an ongoing process. Thus, maintenance experiences from all preceding aircraft will be addressed in the next aircraft to be produced.

It should be noted that the equations were developed from entities having many more than 1 aircraft in the fleet. The author believes, however, that the two equations are valid for NAC = 1. The reason for this statement is that there is only the slightest evidence of economy of scale. For example, the airframe maintenance could be rewritten as

$$Airframe = K \left\{ \left( \text{Ref.W} \right)^{0.72118} \left( \frac{FH}{NAC} \right)^{0.46050} \left( \frac{DP}{NAC} \right)^{0.32062} \left( \text{NAC} \right)^{0.98812} \left( 1 + \frac{Inhouse\,AF}{T\,otal\,AF} \right)^{-0.43177} \right\}$$

and the engine maintenance as

$$Engine = K \left\{ \left(Thrust\right)^{0.89650} \left(N_{E}\right)^{0.92340} \left(\frac{FH}{NAC}\right)^{0.15344} \left(\frac{DP}{NAC}\right)^{0.37535} \left(NAC\right)^{0.97169} \left(1 + \frac{Outside\,Eng.}{T\,otal\,Eng.}\right)^{-0.34704} \right\}$$

Written in this form shows that the exponent of NAC in both equations is, for practical purposes, 1.0. This result says that flight equipment maintenance expenses are directly proportional to number of aircraft.

### Flight Equipment Depreciation & Amortization Expenses (page 34)

Depr. & Amort. Expense = 
$$APP \frac{(1-RV)}{DP}$$
  
 $APP = Aircraft Purchase Price$   
 $RV = Residual Value$   
 $DP = Depreciation Period$ 

This expense applies to the aircraft owned by the airline. The purchase price is in then year dollars.

### Total Aircraft Operating Expenses (page 38)

### Passenger Service Expenses (page 42)

Passenger Service Expenses = 1.6(55,500) (Number of Flight Attendants)

where

$$No. of Flt. Attendants = \begin{bmatrix} \left(\frac{Aircraft Block Hours per Year}{Attendant Hours per Year}\right) \\ \times \left(\frac{FAA Req. Attendants}{No. of Seats}\right) \\ \times \left(\frac{No. of Seats}{Aircraft}\right) \\ \times \left(Number of Aircraft\right) \end{bmatrix}$$

$$(1.3647 + .02351 \frac{Block Hours}{Departure})$$

The factors 1.6 and \$55,500 per attendant are representative of the industry in 1999.

## Landing Fees (page 46)

Landing Fees = 0.00147(ST)(RF)(MLW)(Departures)

ST ≡ Service Type Factor (Passenger = 1.0, Cargo = 0.89)

RF ≡ Route Factor (Domestic = 1.0, Atlantic = 2.36, Latin America = 1.64, Pacific = 4.28)

MLW ≡ Maximum Landing Weight, in pounds

## Rest of All Other Operating Expenses (page 47)

Rest of AOOE in 1999 = AF 
$$\begin{cases} 11,604 \text{ (No. of Overhead Employees)} \\ +71,186 \text{ (Passengers)} +161,768 \text{ (Cargo)} \end{cases}$$

$$AF = Airline \ Factor \begin{pmatrix} Low = 0.8, \ Very \ Low = 0.6 \\ Average = 1.0 \\ High = 1.2, \ Very \ High = 1.5 \end{pmatrix}$$

$$Passengers = \sum_{n=1}^{N} \left( \frac{Available Seats}{Aircraft} (Passenger Load Factor) (No. of AC) \right)_{n}$$

$$Cargo = \sum_{n=1}^{N} \left( \frac{Available Tons}{Aircraft} (Cargo Load Factor) (No. of AC) \right)_{s}$$

# Transport Related Expenses (page 51)

Transport Related Expenses = 1.035 (Rest of All Other Operating Expenses)

The factor 1.035 is a reasonable allocation for passenger airlines in 1999. For cargo airlines, such as FedEx and some others, a more representative value would be 1.5 to 2.0, as Figure 45 suggests.

## All Other Operating Expenses (page 53)

All Other Operating Expenses = Passenger Service

- + Landing Fees
- + Rest of All Other
- + Transport Related

# Total Operating Expenses (page 56)

Total Operating Expenses = Total Aircraft Operating Expenses + All Other Operating Expenses

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