A General Measure of Effect Size for Mediation Analysis

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To my family, whose love and support made all this possible

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1 INTRODUCTION

1.1 Effect Size in Psychological Research

Methodological and applied researchers are becoming increasingly aware of the importance of effect size in psychological research. Several definitions of effect size have been offered in the methodological literature (Cohen, 1988; Kelley & Preacher 2012; Kirk, 1996), but, fundamentally, an effect size conceptually is a quantification of some phenomenon of interest (Kelley & Preacher, 2012). In applied research settings, effect sizes have also been defined as communicating of the practical importance of an effect beyond statistical significance (Cohen, 1988), the translation of the effect to a meaningful scale interpretable by other researchers (Kirk, 1996), and interpretation of the effect of interest in different contexts (Greenland, Schlessman, & Criqui, 1986). These definitions highlight the importance of effect sizes in the effective communication of scientific findings.

Many methodologists have advocated for applied researchers to report effect sizes and confidence intervals (CIs) for their results instead of relying on statistical significance. Weaknesses of the null hypothesis significance test (NHST) framework for conveying scientific results have been well-documented (Cohen, 1994; Greenland & Poole, 2013; Wilkinson, 1999). Effect sizes and CIs not only convey the same information as a NHST, but also provide information regarding the magnitude of the effect and precision of the estimate.

In addition to conveying important information regarding the findings of a given study, effect sizes are necessary for sample size planning. The classic approach to sample size planning is power analysis. Although fundamentally based on the NHST framework, power analysis requires the specification of the population effect size(s) to compute the minimum sample size necessary to achieve a desired level of power. More modern approaches to sample size planning (e.g., Monte Carlo simulation, accuracy in parameter estimation) also rely on the specification of effect sizes.

Effect sizes also have a prominent role in meta-analysis. Meta-analysis is an approach to scientific inquiry that collects and synthesizes the results of multiple studies that fundamentally examine the same phenomena of interest. There are many factors that may differ among studies of the same phenomena (e.g., sample, measures, experimental design). Across study differences in these factors are translated into discrepancies among the metrics and magnitudes of point and interval estimates of structural parameters, variances, covariances, and residual variances. Effect sizes translate estimates from discrepant studies into a common metric, facilitating comparisons and synthesis of findings.

Much methodological work has been devoted to accurately defining and clarifying the distinctions among various effect size measures that can be used to quantify the same phenomena. For example, the effect of a binary variable representing treatment and control groups on a continuous outcome can be quantified as either a standardized mean difference between groups, or the strength of association between the treatment variable and the outcome. Which effect size is chosen has implications for how the effect is interpreted and can result in substantially different conclusions regarding the practical importance of the finding. In addition, much work has been devoted to evaluating the properties of sample estimators of effect sizes and CIs to a) elucidate the study design conditions (e.g., sample size, effect magnitude) under which inferences from an estimated effect size would be questionable, and b) develop new estimators that yield valid inferences under broader ranges of design conditions (e.g., small sample size, small effects). Despite the long history of effect sizes in the methodological literature for experimental and observational studies, disagreements regarding the appropriate interpretation and use of effect sizes, as well as improvements to sample estimators and CIs for these traditional designs continue to this day (Algina, Keselman, & Penfield, 2005; Wilcox & Tian, 2011).

Theoretical and computational advancements allow researchers to test models that are substantially more complex than what could be modeled in traditional designs. For example, structural equation modeling (SEM) is a framework where a complex structure of relationships among multiple variables can be modeled simultaneously. With these advances in modeling come new methods to conceptualize and quantify phenomena of interest (i.e., model discrepancy in the population quantified by the root mean square error of approximation [RMSEA]; Steiger & Lind, 1980). Given the importance of effect sizes and CIs in the interpretation of scientific results and increasingly common requirements that they be reported for primary outcomes, methodologists have attempted to keep pace in several ways: a) identifying those existing parameters and estimators in complex models that can appropriately be used as measures of effect size (e.g., Preacher & Kelley, 2011), b) proposing improved estimators for established quantities of interest (e.g., Wilcox & Tian, 2011), and c) proposing effect sizes that represent qualitatively different approaches to quantifying the phenomena of interest (e.g., Fairchild, Mackinnon, Taborga, & Taylor, 2009). Although effect sizes have been established for quantifying some aspects of complex models, there are some phenomena of interest for which consensus has yet to be reached. Perhaps the most important of these phenomena is the indirect effect in mediation analysis.

1.2 Effect Size in Mediation Analysis

Mediation analysis is an increasingly popular analytic method for social sciences researchers. The goal of mediation analysis is to examine the mechanisms through which a predictor variable has its effects on an outcome variable through intervening variables called *mediators* (Baron & Kenny, 1986; MacKinnon, 2008). Mediation analysis can be used to examine the effects of these intervening variables in a system of equations that simultaneously model the effects of the predictors on mediators, and the effects of predictors and mediators on outcomes. For a model where all variables are continuous and effects are linear, the most basic mediation model consists of a system of two equations: a) the regression of the mediator on the predictor, and b) the regression of the outcome on the predictor and mediator. The effect of the predictor on the outcome (i.e., total effect) can be decomposed into a component transmitted directly to the outcome controlling for the mediating variable (i.e., direct effect), or indirectly through the mediating variable (i.e., indirect effect). The indirect effect is often of primary interest for researchers using mediation analysis.

Any effect can be conceived of as a process of indirect effects through intervening variables, whether it be scratching an itch or electing a president. Mediation analysis facilitates a more complete understanding by not only examining *if* the phenomena occur, but also *how* they occur. For clinicians or public policy researchers, these processes can represent points of intervention that could substantially improve the effectiveness of a treatment or policy initiative. It is important, therefore, that researchers using mediation analysis have the tools to effectively communicate their findings to those who may use them in practice.

Several effect size measures have been proposed for mediation analysis to address this gap (Alwin & Hauser, 1975; Fairchild et al., 2009; Kraemer, 2008, 2014; MacKinnon, 2008; Preacher & Kelley, 2011; Sobel, 1982). However, many of these measures have limitations and consensus has not been reached as to which, if any, of the existing measures is recommended. These limitations include logical inconsistencies, and poor or unclear statistical properties of sample estimators.

Recently, Lachowicz, Preacher, and Kelley (2018) proposed a novel measure of effect size for mediation analysis (υ) that addressed many of the limitations of the existing measures. υ translates the indirect effect into the variance in an outcome explained by the predictor through the mediator. In addition to this standardized and interpretable scale, υ has several other desirable properties as effect size measures for the indirect effect in three-variable mediation models. These include independence from sample size, the provision of CIs, and sample estimators with good statistical properties.

Although a select few of the preceding mediation effect sizes are appropriate for indirect effects in three-variable mediation models, this model is often too simplistic to appropriately represent complex behavioral, psychological, or societal processes. The purpose of this research is to propose a generalizable effect size for mediation analysis that can be applied to indirect effects in complex mediation models. This effect size will represent a framework consisting of several extensions of the υ effect size proposed in Lachowicz et al. (2018) for three-variable mediation models. Although there are many factors that can introduce complexity into a mediation model (i.e., non-normality of variables, etc.), for the purposes of this research complexity will be defined as models with covariates, multiple predictors, multiple parallel and/or serial mediators, and moderators. This definition of complexity addresses the vast majority of mediation models currently being employed in social science research.

1.3 Developing a Generalizable Effect Size for Mediation Analysis

A strength of υ as an effect size measure is that it fits into the existing explained variance framework such that effect sizes for indirect effects in mediation analysis have clear analogs to common effect sizes in multiple linear regression (MLR) and analysis of variance (ANOVA), a property that facilitates the interpretation of υ. However, the analogous measures of explained variance in MLR and ANOVA for conditional indirect effects have received little attention in the methodological literature. Because the existence of the MLR and ANOVA analogs are important for the interpretation of υ, the proposed research will develop the concept of explained variance for conditional effects in MLR and ANOVA as a foundation for the extension of υ to conditional indirect effects.

Although it is important for the effect size parameter to have a strong conceptual rationale and meaningful interpretation, it is equally important for the parameter to have an accurate sample estimator. It is well known in the statistical literature that many sample analog estimators of population standardized effect size measures are biased (Ezekiel, 1930; Fisher, 1915; Hedges, 1981). This is also true for υ. Lachowicz et al. (2018) derived the bias of the sample analog estimator of υ ($\hat{\upsilon}$), showing that, like common explained variance estimators such as R^2, the estimator had a consistent positive bias. Lachowicz et al. proposed an adjusted version of the estimator ($\tilde{\upsilon}$) to correct for the sample bias of $\hat{\upsilon}$. It is expected that the estimators of the effect sizes proposed in this research will also be positively biased. The proposed research will include derivations of the bias of sample analog estimators of the population effect sizes, and develop adjusted versions of the estimators to correct for sample bias. The finite sample properties of the unadjusted and adjusted estimators as well as the associated CIs will be investigated with Monte Carlo simulation studies.

Empirical examples will be included in each section to demonstrate the use and interpretation of the υ extensions. In addition, because standardized indirect effects are also a viable effect size measure for the complex mediation models considered in this research, each section will include comparisons of the relevant standardized indirect effects to the υ extensions for the empirical example.

The dissertation will consist of eight chapters. Chapter 2 will provide the necessary background for effect size, mediation analysis, existing effect size measures in mediation analysis, and introduction of the υ effect size measure. Chapter 3 will consist of a review of the explained variance framework of which υ will be a part, including matrix expressions of existing bias-adjusted estimators of explained variance in MLR and ANOVA, and introduction of a general SEM approach for estimating indirect effects. Chapter 4 will develop a matrix-based framework for extending υ to complex mediation models with observed variables. Chapter 5 will extend the υ framework to mediation models with latent variables. Chapter 6 will develop the concept of explained variance for conditional effects in MLR. Chapter 7 will extend the υ framework to mediation models with conditional effects. Chapter 8 will summarize the research, including limitations and future directions.

2 EFFECT SIZE AND MEDIATION ANALYSIS

2.1 Effect size

Despite calls for reporting effect sizes and CIs to supplement NHSTs, conflicting definitions of effect size have made it unclear precisely what qualified as an appropriate effect size measure to report. Particularly problematic was that effect size was defined by several sources in terms of NHSTs (e.g., Barry & Mielke, 2002; Cohen, 1988), the practice which advocates of effect size wanted researchers to avoid. Others sources defined effect size in terms of practical/clinical/scientific significance (e.g., Cohen, 1988). However, this definition is also lacking because effect size and practical significance are not synonymous. Rather, effect size is used to make judgments about practical significance (Kelley & Preacher, 2012).

Kelley and Preacher (2012) defined an effect size as the "quantitative reflection of the magnitude of some phenomenon that is used for the purpose of addressing a question of interest." Because a phenomenon is quantified by population parameters or sample statistics, Kelley and Preacher (2012) also defined an effect size as "a statistic or parameter with a purpose, which is to quantify some phenomenon that addresses a question of interest." This definition of effect size is intentionally broad as any parameter and statistic can be used appropriately as an effect size under the right circumstances.

Several of the benefits of reporting effect sizes can be illustrated by reference to the standardized mean difference (Cohen's *d*). Consider two experimental studies of an arbitrary construct that are equivalent in all respects except in the scale of the outcome measure. Assuming the experimental effect of the manipulation (i.e., difference in means between experimental groups) in both studies is exactly equivalent, the unstandardized effects of the manipulation would differ solely due to the difference in the scales of the outcomes. The difference in scale could be due to different but equally valid measures of the construct, or simply different within-group variances due to sampling variability (Hedges & Olkin, 1985). Importantly, these factors would be considered irrelevant to the research question at hand, namely "does the experimental manipulation result in a difference between the means of the experimental groups?'

Converting the unstandardized mean difference into Cohen's *d* removes the scales from the effect of the manipulation, allowing for an appropriate comparison of the effects in the studies. Whether or not to use a standardized or unstandardized effect size measure should depend on whether the differences in scales are meaningful aspects of the study design. For example, a difference in sample variances may not be simply an artifact of sampling variability, but could reflect meaningful differences in the sampling methods (e.g., inclusion/exclusion criteria, seasonality). In that case, standardizing would confound the effect with frequencies of the predictor and outcome, such that two variables with equivalent unstandardized regression coefficients would have differing standardized coefficients if the variances of the predictor and/or outcome differ (Greenland et al., 1986). This reason is prominent among those proposed by Greenland et al. (1986) in advocating *against* the use of standardized effect size measures. In addition, it can also be shown that the magnitude of a standardized measure of partial association (i.e., partial correlation) can be non-zero when the unstandardized partial association is zero. Although it is important to understand the sources of variance used in standardization, it appears unnecessarily restrictive to eschew all standardized effect sizes in favor of unstandardized measures. First, nothing prevents the reporting of both unstandardized and standardized measures. For example, an effect can appear substantial in a raw metric, but quite small when considered in the context of the variability in the predictor and outcome. Second, this precludes the comparison variables that measure the same phenomenon but have different metrics. Third, this would limit effect sizes to bivariate relationships, and effect size measures such as multiple R^2 cannot be considered.

Kelley and Preacher (2012) proposed properties of good effect size measures. The first property is that the effect size should have an interpretable scale, the importance of which has been the focus of the preceding section. The second property is that effect size estimates should be reported with CIs. CIs convey the precision of point estimates, or the uncertainty with which a point estimate can be used to make inferences about the population. CIs are constructed based on the sampling distribution of the estimator. If the sampling distribution of the estimator is known, CIs can be constructed analytically. If the sampling distribution is unknown, several methods can be used to construct CIs, including semiparametric approaches that use simulation from the known distributions of parameters that comprise an effect (i.e., Monte Carlo), and nonparametric approaches based on resampling to construct an empirical sampling distribution (i.e., bootstrapping). The third property is that the effect size should be independent of sample size. Specifically, this means that the population effect size does not change for different sample sizes. The fourth property is that effect size estimators should have good statistical properties of unbiasedness, consistency, and efficiency. For an estimator to be unbiased means that the expected value of the estimator is the population value it estimates. In some circumstances, however, a biased estimator may have smaller variance than an unbiased estimator (e.g., empirical Bayes predictions of random effects in multilevel modeling). A consistent estimator is one such that, as sample size increases, estimates converge to the population value. Finally, an efficient estimator is one with minimum variance compared to other estimators of the population value.

Effect sizes can be generally classified along a spectrum of ranging from fully parametric to fully nonparametric. Parametric effect sizes rely on specific assumptions about the distributions of parameters in the population (e.g., equality of variances, normality of residuals), whereas nonparametric effect sizes make no such distributional assumptions. The focus of this review of effect sizes will be on parametric effect sizes.

One of the primary purposes of the following review is to identify commonalities in the development of the effect sizes that will aid in the evaluation of existing effect size measures for mediation analysis, and inform the extensions of υ that follow. Methodological work on effect sizes for traditional research designs has been ongoing for over a century (e.g., Fisher, 1915), whereas effect size measures for mediation analysis have been developed relatively recently. Although indirect effects are more complex than effects in traditional designs, there are also common statistical and interpretative elements shared by the effects, such that developments in one effect size may inform the developments of another.

2.2 Types of Effect Sizes

2.2.1 Standardized mean differences

For study designs where the predictor variable is binary and the outcome is continuous, the most frequently used effect size measure by social and educational researchers is the standardized mean difference δ (Cohen, 1988; Hedges & Olkin, 1985; Lipsey & Wilson, 2001). In the population, the standardized mean difference between two groups δ is defined as

$$\delta = \frac{\mu_1 - \mu_2}{\sigma}, \tag{2.1}$$

where μ_1 and μ_2 are the group-specific population means, and σ is the population standard deviation, assumed equal across levels. The effect size δ is interpreted as the difference between population means in terms of standard deviation units (e.g., $\delta = 0.5$ means that the group means differ by on half standard deviation with dummy coding). Several sample estimators have been proposed for δ and typically vary in how the population standard deviation is estimated. Glass (1976) proposed an estimator Δ that estimated the population standard deviation as the within-group standard deviation for the control group ($S_{control}$)

$$\Delta = \frac{\bar{Y}_1 - \bar{Y}_2}{\hat{\sigma}_{control}}, \tag{2.2}$$

where $\bar{Y}_1$ and $\bar{Y}_2$ are the estimated group-specific sample means. Although the choice of $\hat{\sigma}_{control}$ as the estimator is reasonable given the assumption of equal within-group population variances, a more accurate estimator is the pooled within-group standard deviation ($\hat{\sigma}_{pooled}$), which is the estimator used in Cohen's *d*

$$d = \frac{\bar{Y}_1 - \bar{Y}_2}{\hat{\sigma}_{pooled}}, \tag{2.3}$$

where

$$\hat{\sigma}_{pooled} = \sqrt{\frac{(n_1 - 1)\hat{\sigma}_1^2 + (n_2 - 1)\hat{\sigma}_2^2}{n_1 + n_2}}, \tag{2.4}$$

where n_1 and n_2 are the group-specific sample sizes, and $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are estimates of within-level variances. Cohen's *d* is equivalent to the maximum likelihood estimate (MLE) of δ. However, Hedges (1981) showed Cohen's *d* is a biased estimator of δ, particularly for small samples. In practice, an adjusted estimator (Hedges' *g*) is used

$$g = \frac{\bar{Y}_1 - \bar{Y}_2}{s_{pooled}} = d\sqrt{\frac{n_1 + n_2}{n_1 + n_2 - 2}} \tag{2.5}$$

where the denominator $n_1 + n_2 - 2$ corrects for bias in the estimate of $\hat{\sigma}_{pooled}$ (Hedges & Olkin, 1985).

The success of δ can be attributed to several factors. Primary among these factors is that δ has an intuitive interpretation. The fundamental unit of δ is the standard deviation, which is often one of the first statistics taught in introductory methods courses, and also serves as the primary unit for normed measures (e.g., intelligence tests, achievement tests). In addition, Cohen (1988) provided benchmarks for what effect size values of δ should be considered small, medium, and large. Although the mechanistic adherence to benchmarks has been rightfully criticized in the methodological literature (Snyder & Lawson, 1993), it is difficult to understate the importance of providing some criteria for evaluating an effect size, especially for novel measures. As stated in Algina et al. (2005) “By itself, Cohen’s δ, or any other [effect size] for that matter, has little value. What is required is experience in applying the [effect size].” A researcher using a novel measure of effect size by definition does not have experience in applying the effect size, and, therefore, must rely on some external criteria by which to judge the effect size obtained for their effect of interest. As use of an effect size increases, researchers can and should rely less on these rough guidelines and instead on the norms for the measure developed within their specific area of study, but benchmarks can be very important for facilitating the adoption of a new measure.

Although δ is a widely used effect size in the social and behavioral sciences, particularly in meta-analysis (Lipsey & Wilson, 2001; Hedges & Olkin, 1985), there are many other ways to quantify the standardized mean difference in the sample and in the population. For example, Hedges and Olkin (1985) showed the bias in the expected value of Hedges' g as

$$E[g] = \delta\left(1 + \frac{3}{4N-9}\right), \tag{2.6}$$

where $N = n_1 + n_2$. Equation 2.6 means g is positively biased by the factor $3\delta/(4N-9)$. Correcting for this bias yields a new unbiased estimator of the population standardized mean difference

$$\tilde{g} = g\left(1 - \frac{3}{4N-9}\right). \tag{2.7}$$

Methods for constructing CIs for the unadjusted and adjusted estimators are available from several sources (Bird, 2002; Hedges & Olkin, 1985; Kelley, 2007; Steiger & Fouladi, 1997).

2.2.2 Strength of association

Effect size for the effect of a binary predictor on a continuous outcome can be conceptualized not only as a standardized difference in means, but more generally as the strength of association between the two variables. The most commonly used standardized measure of strength of association is the correlation coefficient, which in the special case of the relationship between binary and continuous variables is the point-biserial correlation

$$r = \frac{\sigma_{YX}}{\sqrt{\sigma_Y^2}\sqrt{\sigma_X^2}}, \tag{2.8}$$

where σ_{YX} is the population covariance between the outcome and the predictor, and σ_Y^2 and σ_X^2 are the population variances of the outcome and predictor, respectively. The corresponding sample estimator is

$$\hat{r} = \frac{\hat{\sigma}_{YX}}{\sqrt{\hat{\sigma}_Y^2}\sqrt{\hat{\sigma}_X^2}}, \tag{2.9}$$

The sample estimator expression can be re-expressed in terms similar to that of δ (Cohen, 1988)

$$r = \frac{\delta}{\sqrt{\delta^2 + (p_1 p_2)^{-1}}}, \tag{2.10}$$

where p_1 and p_2 are the proportion of the population within each group. This can be further re-expressed as

$$r = \frac{\mu_1 - \mu_2}{\sigma_Y}\sqrt{p_1 p_2}. \tag{2.11}$$

Equation 2.11 reveals an important distinction between Cohen's *d* and *r*. It can be seen that for a fixed difference in group means, the population point-biserial correlation can vary depending on the relative proportions of subjects within each group, which is not true for Cohen's *d*. In this sense, the point-biserial correlation coefficient is considered sensitive to base rate, whereas Cohen's *d* is insensitive to base rate (McGrath & Meyer, 2006). When groups have similar base rates in the population, the point-biserial correlation has a perhaps more familiar translation into δ (Cohen, 1988; Hedges & Olkin, 1985)

$$r = \frac{\delta}{\sqrt{\delta^2 + 4}}. \tag{2.12}$$

It is also important to note that the benchmarks for small, medium, and large correlations suggested by Cohen (1988) make this assumption of equal base rates for each group.

Although not as commonly used as a measure of effect size for binary predictors as are standardized mean differences, the point-biserial correlation is still a useful effect size. Like the standard deviation metric for Cohen’s *d*, the correlation coefficient as an indicator of the strength of relationship between two variables is a core component of introductory statistics courses. Also like Cohen’s *d*, benchmarks for small (.1), medium (.24), and large (.37) values of the point-biserial correlation were proposed by Cohen (1988).

The correlation coefficient has several advantages over the standardized mean difference as an effect size measure. One primary advantage is flexibility. The point-biserial correlation is a special case of the more general Pearson product-moment correlation, a common effect size for bivariate relationships between continuous variables. Standardized mean differences are not generalizable to designs with continuous predictors because it would be necessary to dichotomize the predictor in some fashion, a practice that has received much criticism in the methodological literature (MacCallum, Zhang, Preacher, & Rucker, 2002). Although a correlation between continuous variables and a correlation between a binary and continuous variable are not strictly comparable (e.g., small [.1], medium [.24], and large [.37] effect sizes for the point-biserial correlation are generally less than small [.1], medium [.3], and large [.5] effect sizes for the correlation), the difference reflects more the difference in the nature of the predictor/outcome association (i.e., correlation at specific levels of the population vs. across the range of possible population levels) than a fundamental discrepancy in the effect size when applied to different study designs. McGrath and Meyer (2006) outlined several additional advantages of the point-biserial correlation over Cohen’s *d* as an effect size measure. These include a) a more direct relationship to statistical power, b) strength of association being a more general concept than group mean differences, and c) being an integral component of general linear models. In addition, although effect size values tend to be of smaller magnitude, base rate dependence can be meaningful in applied settings. For example, Meehl and Rosen (1955) suggest that the correlation is a more ecologically valid indicator of treatment effectiveness. Finally, for circumstances where sensitivity to base rate is not desirable, adjustments to the point-biserial correlation estimates (and effect size benchmarks) can be made to translate the effect into one that is base rate independent (McGrath & Meyer, 2006). Similarly, base rate estimates from prior research could also be used to conduct sensitivity analyses that provide bounds for the expected effect size in the population.

Like Cohen's *d*, the sample correlation coefficient is a biased estimator of the population correlation. Fisher (1915) first derived the bias in the expected value of $\hat{r}$ as

$$E[\hat{r}] = r - \frac{r(1-r^2)}{2N}, \tag{2.13}$$

and proposed as an approximately unbiased estimator $\tilde{r}$

$$\tilde{r} = \hat{r}\left(1 + \frac{1-\hat{r}^2}{2N}\right). \tag{2.14}$$

It is interesting to note that, unlike Hedges' *g*, the bias of $\hat{r}$ is negative. In addition, the relationship between bias and the standard error of $\hat{r}$ (i.e., $SE_{\hat{r}} = [1-\hat{r}^2]/\sqrt{N}$). For example, re-expressing $\tilde{r}$ in terms of $SE_{\hat{r}}$ yields

$$\tilde{r} = \hat{r}\left(1 + \frac{SE_{\hat{r}}}{2\sqrt{N}}\right). \tag{2.15}$$

This suggests that the bias in $\tilde{r}$ can also be understood as a function of imprecision of the estimate. Methods for constructing CIs for unadjusted and adjusted estimators can be found in several sources (Efron, 1987; Hedges & Olkin, 1985).

2.2.3 Explained variance

An alternative class of effect sizes includes measures of explained variance, which are commonly referred to as R^2 in MLR and η^2 in ANOVA. For study designs where the predictor and outcome are both continuous, the proportion of variance explained is equivalent to the squared Pearson correlation r^2. For designs that include a binary predictor and continuous outcome, much of the research and language of explained variance comes from the ANOVA literature, where η^2 is commonly interpreted as the proportion reduction in error variance (PRE) due to an experimental manipulation (Maxwell & Delaney, 2003).

Cohen (1988) proposed η^2 as an effect size measure that quantified the proportion of variance accounted for by group membership in the population, which can be expressed as

$$\eta^2 = \frac{\sigma_y^2 - \sigma^2}{\sigma_y^2}. \tag{2.16}$$

The within-group variance σ^2 can be obtained from a regression model with any number of predictors, meaning η^2 can be used as an effect size for a wider class of effects than the effect sizes previously considered. In the case of a single binary predictor and continuous outcome, η^2 is equivalent to the squared point-biserial correlation (r^2). For multiple continuous predictors, the equivalent of Equation 2.16 in multiple regression is

$$R^2 = \frac{\sigma_{\hat{y}}^2}{\sigma_{\hat{y}}^2 + \sigma_e^2}, \tag{2.17}$$

where $\sigma_{\hat{y}}^2$ is the variance of predicted values of the outcome *y* from the population regression, or the model implied variance, and σ_e^2 is the variance of the residuals in the population. Because η^2 is the proportion of variance due to group membership, it can be expressed in terms of δ

$$\eta^2 = \frac{\delta^2}{\delta^2 + (p_1 p_2)^{-1}}. \tag{2.18}$$

It is clear from Equation 2.18 that, like the point-biserial correlation, η^2 is sensitive to the population base rate. In sample data, the population analog estimator of η^2 ($\hat{\eta}^2$) is quantified as (Maxwell & Delaney, 2003)

$$\hat{\eta}^2 = \frac{(SST)N^{-1} - (SSW)N^{-1}}{(SST)N^{-1}} = \frac{SSB}{SST}, \tag{2.19}$$

where *SST* is total sum of squared errors (i.e., $\sum_i \sum_j [y_{ij} - \bar{y}_{\bullet\bullet}]^2$), *SSW* is the within-group sum of squared errors (i.e., $\sum_i \sum_j [y_{ij} - \bar{y}_{\bullet j}]^2$), and *SSB* is the between-group sum of squared errors (i.e., $\sum_i \sum_j [\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet}]^2$). The analogous estimator of R^2 ($\hat{R}^2$) in multiple regression is

$$\hat{R}^2 = \frac{SSR}{SSR + SSE}, \tag{2.20}$$

where *SSR* is the sum of squares due to regression (i.e., $\sum_i [\hat{y}_i - \bar{y}]^2$), and *SSE* is the sum of squared errors (i.e., $\sum_i [y_i - \hat{y}_i]^2$).

η^2 for binary predictors and R^2 for multiple regression have the advantage of intuitive interpretations as proportions. Cohen (1988) proposed benchmarks for both η^2 and R^2, but, as with correlations, these differ due to the binary vs continuous nature of the predictor. For a binary predictor, the proposed benchmarks were small = .01, medium = .06, and large = .14 effects for η^2 (pp. 285-287). For a single continuous predictor, the proposed benchmarks were small = .01, medium = .09, and large = .25 for r^2 (pp. 79-81). For multiple continuous predictors, the proposed benchmarks were small = .02, medium = .13, and large = .26 for the multiple R^2 (pp. 412-414). Because of the close relationship between proportion of variance and correlation, the advantages of correlation effect sizes over standardized mean differences also apply to R^2 measures. In addition, standardized mean differences and correlations are limited to bivariate relationships, whereas the proportion of variance measures can be used as an effect size for relationships between an outcome and multiple variables.

Like sample estimators of the standardized effect size measures previously considered, the sample estimators $\hat{\eta}^2$ and $\hat{R}^2$ are biased. Although derivations of the bias and adjustments for $\hat{\eta}^2$ and $\hat{R}^2$ developed in parallel in the ANOVA and MLR literature, the findings and proposed corrections shared several common features. The most widely used adjustments to $\hat{\eta}^2$ are $\hat{\varepsilon}^2$ (Kelley, 1935), defined as

$$\hat{\varepsilon}^2 = \frac{SSB-(k-1)MSW}{SST}, \tag{2.21}$$

and $\hat{\omega}^2$ (Hayes, 1973), defined as

$$\hat{\omega}^2 = \frac{SSB-(k-1)MSW}{SST+MSW}, \tag{2.22}$$

where *MSW* is mean square within-group error (i.e., $\sum_i \sum_j [y_{ij} - \bar{y}._j]^2 / [N-k]$). Notably more adjustments have been proposed to correct for the sample bias of $\hat{R}^2$ in multiple linear regression analysis (see Yin & Fan, 2001, for a comprehensive review). However, the most commonly used adjustment was proposed by Ezekiel (1930), which is the default adjusted estimator in several software packages such as SPSS and Stata

$$\tilde{R}^2_{Ezekiel} = 1 - \frac{N-1}{N-p-1}(1-\hat{R}^2). \tag{2.23}$$

A noteworthy advantage of $\tilde{R}^2_{Ezekiel}$ over other $\hat{R}^2$ adjustments is that it can be applied to both $\hat{R}^2$ and the squared Pearson correlation $\hat{r}^2$ (Wang & Thompson, 2007). In addition, Maxwell (1981) demonstrated that $\tilde{R}^2_{Ezekiel}$ is nearly equivalent to $\hat{\varepsilon}^2$, differing by one degree of freedom

$$\hat{\varepsilon}^2 = 1 - \frac{N-1}{N-p}(1-\hat{R}^2). \tag{2.24}$$

Large sample and exact CIs can be constructed for the unadjusted and adjusted R^2 and η^2 estimators (Algina, 1999; Kelley, 2007; Smithson, 2001).

2.2.4 Summary of effect sizes

Several aspects of standardized effect size measures are made salient when reviewing the methodological research on effect sizes for traditional research designs. The first is that although interpretations may substantially differ, correlations and R^2 measures are more generalizable effect size measures than standardized mean differences. Further, because R^2 can quantify effects with more than two variables, R^2 effect size measures are more generalizable than correlations. It is also noteworthy that for correlations and R^2, the effect size benchmarks for small, medium, and large effects differ depending on whether the predictor is binary or continuous, and whether the effect is between an outcome and a single variable or multiple variables. Another aspect common to all of the traditional standardized effect size measures is that estimators that are sample analogues of the population effect size parameters (e.g., Cohen's

d, $\hat{r}$, $\hat{R}^2$) are biased. This bias can be attributed to error variance (Box, 1971), and, therefore, bias is expected to be worse in study conditions where sampling error is relatively high (e.g., small samples, small population effects). In addition, bias tends to be larger for R^2 measures than for correlations and standardized mean differences, suggesting that, in general, sampling error has a greater impact on R^2 estimators. Corrections for sample bias have been proposed for all of the estimators of standardized effect sizes. However, likely because of the heightened sensitivity of R^2 measures to sampling error, bias-adjusted estimators for R^2 are more routinely applied in practice than, for example, bias-adjusted estimators for $\hat{r}$. As methodological advancements spur the creation of novel effect size measures, awareness of the common themes in the effect size literature for traditional designs can prove highly informative for developments in more complex models.

2.3 Mediation Analysis

Mediation analysis provides a method to examine the mechanisms through which variables have their effects, allowing researchers to build a more complete understanding of the causal processes underlying phenomena. More complete knowledge of the relationships among variables can, for example, identify points of intervention in a causal chain that can be expected to have the maximum downstream effects. In addition, mediation analysis can be used to quantify the relative importance of competing causal processes, such as the relative downstream effects of several treatment components.

The fundamental logic and assumptions of mediation analysis can be illustrated with a three-variable mediation model. For this model (illustrated in a path diagram in Figure 1), mediation analysis decomposes the *total effect* of a predictor x on an outcome y into a *direct effect* of x on y controlling for the mediator m, and an *indirect effect* of x on y through m. The relationship between x and y for a given observation (i subscript is left off for convenience) is expressed as

$$y = B_{y \bullet x} + B_{yx} x + \varepsilon_{y \bullet x}, \tag{2.25}$$

where B_{yx} is the total effect of x on y, $B_{y \bullet x}$ is the intercept, and $\varepsilon_{y \bullet x}$ is the error term, where $\varepsilon_{y \bullet x} \sim N(0, \sigma^2_{y \bullet x})$. To examine the direct and indirect effects, it is first necessary to estimate the effect of x on m, expressed as

$$m = B_{m \bullet x} + B_{mx} x + \varepsilon_{m \bullet x}, \tag{2.26}$$

where B_{mx} is the effect of x on m, $B_{m \bullet x}$ is the intercept, and $\varepsilon_{m \bullet x}$ is the error term, where $\varepsilon_{m \bullet x} \sim N(0, \sigma^2_{m \bullet x})$. Next, it is necessary to estimate the effects of x and m on y, expressed as

$$y = B_{y \bullet \mathbf{x}} + B_{yx \bullet m} x + B_{ym \bullet x} m + \varepsilon_{y \bullet \mathbf{x}}, \tag{2.27}$$

where $B_{ym \bullet x}$ is the effect of m on y controlling for x, $B_{yx \bullet m}$ is the effect of x on y controlling for m (i.e., direct effect), $B_{y \bullet \mathbf{x}}$ is the intercept, and $\varepsilon_{y \bullet \mathbf{x}}$ is the error term, where $\varepsilon_{y \bullet \mathbf{x}} \sim N(0, \sigma^2_{y \bullet \mathbf{x}})$ (**x** in subscripts indicates multiple variables). In addition, $\varepsilon_{m \bullet x}$ and $\varepsilon_{y \bullet \mathbf{x}}$ are independent across equations, which implies that the sampling distributions of estimators of regression coefficients are also independent for different outcomes.

The indirect effect can be obtained as either the difference between the total effect and direct effect ($B_{yx} - B_{yx \bullet m}$) or as the product of the effect of x on m and the effect of m on y controlling for x ($B_{mx} B_{ym \bullet x}$). These methods yield equivalent values of the indirect effect when the models are linear in their effects. However, the product of coefficients approach is often preferred because it allows for the calculation of many more types of indirect effects in complex mediation models with multiple indirect paths and/or nonlinear effects.

The population indirect effect is estimated by the product of regression coefficients estimated in the sample. Although the distribution of the product $B_{mx}B_{ym \bullet x}$ is a complex function of the normal distributions, the expected value of the indirect effect distribution is the population parameter. That is, the sample indirect effect is an unbiased estimator of the population indirect effect. Monte Carlo simulation studies have confirmed the unbiasedness of the sample indirect effect (MacKinnon, Warsi, & Dwyer, 1995).

2.3.1 Confidence intervals

Methods of constructing CIs for indirect effects have received much attention in the methodological literature. It is worthwhile to consider the advances in CIs for the indirect effect because the effect size υ is a function of the indirect effect, and it is likely that properties of the indirect effect sampling distribution that pose challenges for constructing accurate CIs will also be challenges for constructing CIs for υ.

For many common estimators with sampling distributions that are asymptotically normal (e.g., means, unstandardized regression coefficients), approximate large sample CIs are typically constructed by inverting the *z* test (or *t* test for small sample sizes). For example, inverting a two-sided *z* test yields the familiar CI for the point estimate $\hat{\theta}$

$$\hat{\theta} \pm z_{1-\alpha/2}[SE(\hat{\theta})], \tag{2.28}$$

where $z_{1-\alpha/2}$ is the *z* value corresponding to the $1-\alpha/2$ quantile of the standard normal distribution. CIs for the indirect effect were constructed first using this parametric approach based on the assumption of asymptotic normality. Specifically, Sobel (1982) used the first-order multivariate delta method to derive the asymptotic variance of the indirect effect ($B_{mx}^2\sigma_{ym\bullet x}^2 + B_{ym\bullet x}^2\sigma_{mx}^2$), which, when substituting sample estimates for parameters, yields a CI for the product of point estimates, $\hat{B}_{mx}\hat{B}_{ym\bullet x}$

$$\hat{B}_{mx}\hat{B}_{ym\bullet x} \pm z_{1-\alpha/2}\sqrt{\hat{B}_{mx}^2\hat{\sigma}_{ym\bullet x}^2 + \hat{B}_{ym\bullet x}^2\hat{\sigma}_{mx}^2}, \qquad (2.29)$$

where $\hat{\sigma}_{mx}^2$ and $\hat{\sigma}_{ym\bullet x}^2$ are the estimated sampling variances of $\hat{B}_{mx}$ and $\hat{B}_{ym\bullet x}$, respectively. However, several simulation studies showed conditions where normal theory CIs for the indirect effect failed to achieve the nominal coverage rate of 95% (MacKinnon et al., 1995; Stone & Sobel, 1990). In addition, even for conditions where nominal coverage was achieved, the proportion of cases where the true value was greater than the upper 95% CI limit or less than the lower 95% CI limit were imbalanced. MacKinnon et al. (1995) found a similar pattern of results for CIs based on a second-order multivariate delta method approximation

$$\hat{B}_{mx}\hat{B}_{ym\bullet x} \pm z_{1-\alpha/2}\sqrt{\hat{B}_{mx}^2\hat{\sigma}_{ym\bullet x}^2 + \hat{B}_{ym\bullet x}^2\hat{\sigma}_{mx}^2 + \hat{\sigma}_{mx}^2\hat{\sigma}_{ym\bullet x}^2}. \qquad (2.30)$$

The putative cause of the poor performance of normal theory CIs is the non-normality of the sampling distribution of the indirect effect (MacKinnon, Lockwood, & Williams, 2004). Although the sampling distributions of the regressions coefficients $\hat{B}_{mx}$ and $\hat{B}_{ym\bullet x}$ are normal, the sampling distribution of the product of normally distributed variables is substantially more complex (Aroian, 1947; Craig, 1936). This is because the mean, variance, skew, and kurtosis of the product distribution are functions of the distributions of $\hat{B}_{mx}$ and $\hat{B}_{ym\bullet x}$. Specifically, the product distribution is narrower than the normal distribution (i.e., leptokurtic), has heavier tails, and when the product of coefficients is non-zero, is skewed left or right depending on the sign of the product (Craig, 1936). In other words, the sampling distribution of a non-zero indirect effect requires an asymmetric CI.

MacKinnon and colleagues (2004, 2007) proposed a method of constructing asymmetric 95% CIs for the indirect effect. The form of the proposed CIs is similar to the normal theory-based CIs (Sobel, 1982) in Equation 2.29, but replaces the *z* values with the critical values based on the distribution of the product ($m_{1-\alpha/2}$; Meeker, Cornwell, & Aroian, 1981)

$$\hat{B}_{mx}\hat{B}_{ym\bullet x} \pm m_{1-\alpha/2}\sqrt{\hat{B}_{mx}^{2}\hat{\sigma}_{ym\bullet x}^{2} + \hat{B}_{ym\bullet x}^{2}\hat{\sigma}_{mx}^{2}}. \tag{2.31}$$

Although this method of constructing CIs for the indirect effect improves the coverage balance, overall coverage does not achieve the nominal level for small sample sizes and small effects (Mackinnon et al., 2004). A possible reason for the poor coverage in these conditions is that, whereas $z_{1-\alpha/2}$ is a fixed value, $m_{1-\alpha/2}$ varies with the magnitudes and variances of the coefficients. Because these values of the asymptotic distribution are not known a priori, they are replaced with their unbiased sample estimates (MacKinnon et al., 2004, 2007). Exactly what effect using sample estimates to obtain critical values has on the performance of the CIs is unclear, but it is reasonable to assume that using sample estimates instead of fixed parameters adds uncertainty to the interval estimation procedure, particularly for those conditions in which the CIs performed poorly.

Nonparametric and semiparametric CIs are viable and increasingly popular alternatives to fully parametric CIs for the indirect effect. Nonparametric approaches do not assume a functional form of the sampling distribution of either the estimator or the variables that compose the estimator, but rather are used to generate an empirical sampling distribution via resampling. The most commonly used resampling technique is called bootstrapping. In its simplest application, bootstrapping is used to create an empirical distribution of the estimator by resampling observations with replacement from the original data, obtaining estimates from the resampled data, and repeating the procedure enough times to obtain a sufficiently close approximation of the true sampling distribution.

Several methods can be used to construct CIs from the empirical sampling distribution. The simplest approach is the percentile method where the CI is constructed using as the lower and upper limits the values of the empirical distribution that correspond to the 100*$\alpha/2$th and 100*$(1-\alpha/2)$th percentiles. However, the performance of percentile CIs is expected to degrade as the empirical sampling distribution deviates from either the normal or a transformation of the normal distribution (e.g., excessive skew, kurtosis; Davison & Hinkley, 1997). Adjustments that improve the performance of CIs in these circumstances include the bias-corrected (BC) and the bias-corrected and accelerated bootstrap (BC_a; Efron, 1987).

The BC bootstrap CI improves accuracy by adjusting the $\alpha/2$ and $1-\alpha/2$ percentiles of the bootstrap distribution. Specifically, the adjusted upper percentile is $\Phi(2\hat{z}_0 + z_{1-\alpha/2})$ and adjusted lower percentile is $\Phi(2\hat{z}_0 + z_{\alpha/2})$, where $\hat{z}_0$ is the *z* score corresponding to the percentile of the observed indirect effect in the empirical distribution, and $\Phi(\bullet)$ represents the cumulative normal distribution. The accuracy of the CI can be further improved by adjusting the $\alpha/2$ and $1-\alpha/2$ percentiles for the skew of the empirical distribution. BC_a CIs adjust for skew by adding an acceleration constant $\hat{a}$ to the BC bootstrap CI

$$\frac{\alpha^*}{2} = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{\alpha/2}}{1-\hat{a}(\hat{z}_0 + z_{\alpha/2})}\right) \quad \text{and} \quad 1-\frac{\alpha^*}{2} = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{1-\alpha/2}}{1-\hat{a}(\hat{z}_0 + z_{1-\alpha/2})}\right). \qquad (2.32)$$

The acceleration constant $\hat{a}$ is an estimate of skew, which can be obtained using an approximation (Efron, 1987) or estimated nonparametrically using bootstrapping or the jackknife (Frangos, 1990).

Semiparametric approaches represent a middle ground between the fully parametric and fully nonparametric approaches to constructing CIs. The term "semiparametric" is quite broad, but for constructing CIs, the semiparametric approach typically involves specifying the sampling distributions of specific variables (e.g., regression coefficients, variances) and using those distributions to approximate the sampling distribution of a more complex estimator that is a function of those variables with known distributions. Monte Carlo CIs are constructed by treating the variables with known distributions as population parameters, simulating a sufficiency large number of values from the parameters, computing estimates of the complex estimator for each set of simulated values, and constructing CIs using as the lower and upper limits the values of the simulated distribution that correspond to the $\alpha / 2$ and $1-\alpha / 2$ percentiles. Monte Carlo CIs have been shown to have good statistical properties for indirect effect estimates (Preacher & Selig, 2012).

The three-variable mediation model represents a relatively simple model of the relationships among constructs. It is more often the case in the social and behavioral sciences that theories consist of complex webs of relationships among multiple constructs, where the effects of one construct are competing with the effects of another, or the effects of a construct vary across levels of another. SEM has become the dominant framework for modeling and testing these complex relationships.

2.4 Effect Size in Mediation Analysis

The concept of effect size in mediation analysis has received comparatively little attention in the methodological literature relative to advancements in point and interval estimation. One possible reason is that, although the indirect effect is composed of regression coefficients with established effect sizes, the indirect effect does not fall neatly within the categories of effect size for traditional study designs previously discussed. For example, one could consider the coefficients composing the indirect effect B_{mx} and $B_{ym \bullet x}$ in either their raw metrics or as standardized effect sizes. However, if the effect sizes of the coefficients differ (e.g., B_{mx} is large and $B_{ym \bullet x}$ is small), it would be unclear how to interpret the magnitude of the indirect effect. Therefore, it would appear to be more practically useful to consider effect size for the total indirect effect rather than for its constituent parts.

The intention of the following review is not to provide a complete evaluation of all existing effect size measures for mediation analysis, but to focus on effect sizes that can be interpreted within the effect size categories previously described for traditional research designs (i.e., standardized mean differences, strength of association, proportion of explained variance). The one exception will be a discussion of ratio measures of effect size because these measures are among the most reported effect sizes for mediation analysis. Because of the similarities between these specific effect sizes for mediation and their corresponding traditional measures, it is useful to consider the common themes that have arisen in the methodological literature for traditional effect sizes (i.e., generalizability, sample bias of standardized effect size estimators, bias corrections related to sampling error).

2.4.1 *Ratio measures*

The earliest effect size measures for mediation analysis were the proportion mediated (*PM*; Alwin & Hauser, 1975) and the ratio mediated (*RM*; Sobel, 1982) measures. For the three-variable mediation model in Figure 1, *PM* is defined as

$$PM = \frac{B_{mx}B_{ym \bullet x}}{B_{yx}}. \tag{2.33}$$

This effect size is interpreted as the proportion of the total effect of *x* on *y* (B_{yx}) mediated by *m* ($B_{mx}B_{ym \bullet x}$). *RM* evaluates the indirect effect relative to the direct effect and is defined as

$$RM = \frac{B_{mx}B_{ym \bullet x}}{B_{yx \bullet m}}. \tag{2.34}$$

This effect size is interpreted as the ratio of the indirect effect of *x* on *y* through *m* ($B_{mx}B_{ym \bullet x}$) to the direct effect of *x* on *y* ($B_{yx \bullet m}$).

The ratio measures of effect size for indirect effects have several significant limitations. One important limitation regards interpretation. For example, it is not clear how one would interpret a large value of *PM* or *RM* if the magnitude of indirect effect is significant but close to zero. For a large value of *PM* or *RM* to be practically important, it must be assumed the total effect is also practically important. Although this may still be a useful interpretation for the indirect effect for some researchers, the interpretation is highly context-dependent, and would not be recommended if the magnitude of the indirect effect is of interest. Another important limitation is the performance of sample estimators of *PM* and *RM*. The estimators of *PM* and *RM* have large variances across repeated samples and require very large samples for the estimators to stabilize (N = 500 for *PM*, and N = 5000 for *RM*; MacKinnon, 1995; Tofighi, MacKinnon, & Yoon, 2009).

2.4.2 Standardized mean differences

Hansen and McNeal (1996) first proposed a standardized effect size for the indirect effect of a binary predictor (e.g., group membership for intervention), and continuous mediators and outcomes. The effect size is expressed as

$$ES = \beta_{mx}\beta_{ym\bullet x} = \frac{\mu_{m1} - \mu_{m2}}{\sigma}\beta_{ym\bullet x}, \tag{2.35}$$

where the effect of the predictor β_{mx} is analogous to δ in Equation 2.1, and $\beta_{ym\bullet x}$ is the standardized effect of m on y from Equation 2.27. This means that the total effect size of an intervention can be decomposed into a direct effect on an outcome, and an indirect effect where the intervention causes a change on the mediator (the effect of the mediator is assumed equal across groups). The effect size is estimated by substituting sample quantities

$$ES_d = \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\frac{\hat{b}_{mx}\hat{b}_{ym}}{\hat{\sigma}_{IE}}, \tag{2.36}$$

where the effect of the predictor is Cohen's *d*, and $\hat{\sigma}_{IE}$ is the asymptotic standard error of the indirect effect in Equation 2.29. In addition, because Cohen's *d* is a biased estimator of δ, the authors also proposed an adjusted estimator analogous to Hedge's *g*

$$ES_g = ES_d\left(1 - \frac{3}{4n_1 + 4n_2 + 9}\right). \tag{2.37}$$

More recently, Kraemer (2008) proposed a similar standardized mean difference measure for mediation models, but more work is necessary to determine its viability as an effect size measure.

2.4.3 Strength of association

Like unstandardized regression coefficients in MLR, indirect effects can be standardized to remove the metrics of the predictor and the outcome from the effect (the scale of the mediator does not factor into the standardization). The interpretation of the standardized indirect effect also changes in a way analogous to standardized regression coefficients where the metric of the standardized variable(s) is the standard deviation. For example, the interpretation of the indirect effect when standardized by the standard deviations of both the predictor and outcome is that a one standard deviation increase in the predictor is associated with some standard deviation change in the outcome through the mediator.

When *x* and *y* are both continuous, the most common form of standardization is complete standardization ($\beta_{mx}\beta_{ym\bullet x}$), which removes the scale from both *x* and *y*

$$\beta_{mx}\beta_{ym\bullet x} = B_{mx}B_{ym\bullet x}(\sigma_x / \sigma_y). \tag{2.38}$$

Complete standardization is appropriate when the scales of both *x* and *y* are arbitrary. However, if the scale of *x* or *y* is meaningful, the indirect effect can be partially standardized by the standard deviation of only the variable with arbitrary scaling. For example, if the scale of *x* is meaningful but not that of *y*, the partially standardized indirect effect is

$$B_{mx}\beta_{ym\bullet x} = B_{mx}B_{ym\bullet x}(1/\sigma_y). \tag{2.39}$$

Partial standardization has been recommended for indirect effects where *x* is binary and *y* is continuous (Hayes, 2013). Specifically, an unstandardized binary variable has an intuitive metric where effects are interpreted as mean differences between groups. Standardizing a binary variable changes the interpretation of the effect into standard deviation units, which makes interpretation more difficult in terms of group mean differences. However, as described in Section 2, the completely standardized effect with binary *x* can be interpreted in terms of strength of association (point-biserial correlation) or explained variance (η^2), which as previously described have several advantages over standardized mean differences as effect size measures.

2.4.4 Explained variance

Several methodologists have proposed explained variance effect size measures for indirect effects (de Heus, 2012; Fairchild et al., 2009; MacKinnon, 2008). These measures ostensibly quantify the variance in *y* that is accounted for jointly by both *m* and *x*. To more clearly understand why this component of explained variance corresponds to the indirect effect, it is helpful to consider the three potential sources of variance in *y* that can be explained by *m* and *x*: a) the variance in *y* explained uniquely by *x*, b) the variance in *y* uniquely explained by *m*, and c) the aforementioned variance in *y* explained jointly by *m* and *x*. The indirect effect is a component of the total effect of *x* on *y*. It stands to reason then that variance not attributable in some way to *x* is irrelevant to the variance explained by the indirect effect, which would rule out the variance in *y* explained by *m* independent of *x* as irrelevant to the indirect effect variance. In addition, the variance in *y* attributable to *x* independent of *m* is more consistent with the definition of the direct effect, meaning the variance in *y* explained jointly by *m* and *x* is the variance attributable to the indirect effect. Fairchild et al. (2009) proposed an effect size to quantify this component of explained variance as

$$R^2_{med} = r^2_{ym} - (R^2_{y \bullet mx} - r^2_{yx}), \tag{2.40}$$

where r^2_{ym} is the squared unconditional correlation between *y* and *m*, r^2_{yx} is the squared unconditional correlation between *y* and *x*, and $R^2_{y \bullet mx}$ is the squared multiple correlation of *y* on both *m* and *x*. Essentially, r^2_{ym} consists of variance in *y* attributable to *m* independent of *x*, and variance in *y* explained jointly by *m* and *x*, and $R^2_{y \bullet mx} - r^2_{yx}$ isolates the variance in *y* uniquely attributable to *m*. Subtracting the unique component from the total r^2_{ym} leaves the joint variance component. MacKinnon (2008) proposed two alternative measures to quantify the joint explained variance component, one based on the partial correlation of *y* and *m* given *x*

$$R^2_{partial1} = r^2_{yx} r^2_{ym \bullet x}, \tag{2.41}$$

and a partial correlation version scaled by $R^2_{y \bullet mx}$

$$R^2_{partial2} = r^2_{yx} r^2_{ym \bullet x} / R^2_{y \bullet mx}. \tag{2.42}$$

de Heus (2012) proposed a measure of the joint variance explained component based on the semipartial correlation of *y* and *m* given *x*

$$R^2_{semipartial} = r^2_{yx} r^2_{y(m \bullet x)}. \tag{2.43}$$

Lachowicz et al. (2018) outlined several limitations of these variance explained measures for the indirect effect. The most notable limitations concern interpretability. For R^2_{med}, it is possible for the measure to return nonzero values of effect size when the indirect effect is in fact zero. Specifically, Lachowicz et al. (2018) show that when $\beta_{ym \bullet x} = 0$ (i.e., no indirect effect), $R^2_{med} = \beta^2_{mx} \beta^2_{yx \bullet m}$. In other words, when the indirect effect is zero, R^2_{med} quantifies what is known in the path analysis literature as “spurious” correlation (Simon, 1957). Although the variance quantified by R^2_{med} may be of interest to some researchers, the variance does not correspond to that uniquely attributable to the indirect effect, and, therefore, is not an appropriate measure for this purpose. For $R^2_{partial2}$, no justification is provided for scaling $R^2_{partial1}$ by $R^2_{y\bullet mx}$, so it is not clear what variance this effect size is quantifying. The partial and semi-partial R^2 measures ($R^2_{partial1}$ and $R^2_{semipartial}$) appear to have more desirable interpretations. Both measures have forms similar to the indirect effect as products of coefficients, and both are bounded by zero and one. However, further examinations of these measures (Lachowicz et al., 2018; Wen & Fan, 2015) demonstrated that these measures lack an important property known as monotonicity. For a measure to lack monotonicity means that with all else held equal in the population, these measures are not one-to-one functions of the indirect effect in either raw or absolute value. The implication for measures that lack of monotonicity is that equivalent indirect effects from two studies could yield different effect sizes. Until the conditions that cause these measures to lack monotonicity are made explicit, their utility as effect size measures is limited.

2.4.5 Effect size υ

Lachowicz et al. (2018) proposed υ as a measure of effect size for mediation analysis. υ is a measure of explained variance, interpretable as the variance in an outcome explained by a predictor through a mediator that appropriately adjusts for variance due to spurious correlation unaccounted for in the Fairchild et al. (2009) R^2_{med} formulation in Equation 2.40, defined as

$$\upsilon = \beta^2_{ym\bullet x} - (R^2_{y\bullet mx} - r^2_{yx}). \tag{2.44}$$

Lachowicz et al. (2018) also showed that for a three-variable mediation model, υ is equivalent to the squared standardized indirect effect ($\beta^2_{mx}\beta^2_{ym\bullet x}$). Because υ is a measure of explained variance, Cohen's (1988) benchmarks for interpreting small, medium, and large effect size are applicable[1]. For an indirect effect with a binary predictor, the appropriate benchmarks are for η^2 (Cohen, 1988, pp. 285-287). For an indirect effect with a continuous predictor, the appropriate benchmarks are those for R^2 (Cohen, 1988, pp. 412-414)[2].

Because sample analog estimators of population standardized effect sizes typically are biased (particularly variance estimators), it was expected that the sample analog estimator of υ ($\hat{\upsilon}$) was also biased. This was confirmed by a Monte Carlo simulation study, showing $\hat{\upsilon}$ was upwardly biased particularly for small sample sizes and for small indirect effects. Although the complete sampling distribution of $\hat{\upsilon}$ is not known or easily derivable, Lachowicz et al. (2018) derived the bias in the expected value of $\hat{\upsilon}$, and proposed an adjusted estimator that adjusted for this bias. Because $\hat{B}_{mx}$ and $\hat{B}_{ym\bullet x}$ are independent and normally distributed,[3] the expected value of $\hat{\upsilon}$ is

[1] If an effect is completely transmitted to an outcome through a mediator, the standardized indirect effect is equivalent to the total effect, which for a single predictor is a correlation coefficient. The squared correlation coefficient is equivalent then to υ, and could theoretically be judged against Cohen's benchmarks for explained variance.

[2] It is important to note that, although Cohen's benchmarks may be applied, υ is not bounded by 0 and 1, and is not considered a proportion. υ can be greater than 1 when suppression is evident (i.e., direct and indirect effects have opposite signs).

[3] As is common practice when standardizing regression coefficients, the variable variances σ^2_x and σ^2_y are assumed fixed, known quantities, and do not vary across samples. Therefore, it is assumed the variance ratio σ^2_x / σ^2_y does not affect the sampling distribution of $\hat{B}^2_{mx}\hat{B}^2_{ym\bullet x}$.

$$
\begin{aligned}
E[\hat{\upsilon}] &= E[\hat{B}_{mx}^2 \hat{B}_{ym\bullet x}^2](\sigma_x^2 / \sigma_y^2) \\
&= E[\hat{B}_{mx}^2]E[\hat{B}_{ym\bullet x}^2](\sigma_x^2 / \sigma_y^2) \\
&= (B_{mx}^2 + \sigma_{mx}^2)(B_{ym\bullet x}^2 + \sigma_{ym\bullet x}^2)(\sigma_x^2 / \sigma_y^2) \\
&= \beta_{mx}^2 \beta_{ym\bullet x}^2 + \beta_{mx}^2 \sigma_{mx}^2 + \beta_{ym\bullet x}^2 \sigma_{ym\bullet x}^2 + \sigma_{mx}^2 \sigma_{ym\bullet x}^2 .
\end{aligned}
\tag{2.45}
$$

This means that the expected value of $\hat{\upsilon}$ yields the parameter of interest $\beta_{mx}^2 \beta_{ym\bullet x}^2$ plus bias $\beta_{mx}^2 \sigma_{ym\bullet x}^2 + \beta_{ym\bullet x}^2 \sigma_{mx}^2 + \sigma_{mx}^2 \sigma_{ym\bullet x}^2$. The bias results from the fact that the effect size is comprised of products of normally distributed coefficients, the properties of which have been detailed in several sources (Arnold, 1982; Bohrnstedt & Goldberger, 1969; Goodman, 1960). Because each term is a function of the sampling variances of the regression coefficients, the magnitude of bias is therefore a function of sample size. In addition, the finding that bias in the expected value of $\hat{\upsilon}$ is equivalent to the asymptotic variance approximation for indirect effects (Equation 2.39) is consistent with prior methodological work on bias reduction showing bias is generally proportional to error variance (Box, 1971).

It follows that the bias of the expected value of $\hat{\upsilon}$ can be adjusted by subtracting a bias term from the sample estimates. However, it is important to note that the bias in Equation 2.45 is the *asymptotic* bias, and must be estimated from the sample. This is addressed by substituting an unbiased estimator of the asymptotic bias ($\hat{B}_{mx}^2 \hat{\sigma}_{ym\bullet x}^2 + \hat{B}_{ym\bullet x}^2 \hat{\sigma}_{mx}^2 - \hat{\sigma}_{mx}^2 \hat{\sigma}_{ym\bullet x}^2$; Goodman, 1960). This approach to adjusting for this bias of $\hat{\upsilon}$ is similar to Ezekiel's (1930) adjustment for R^2 in simple linear regression ($\hat{\varepsilon}^2$ when the predictor is binary). A Monte Carlo simulation study showed that bias of $\tilde{\upsilon}$ was negligible for the vast majority of simulation conditions. For conditions where bias was non-negligible (> 5% relative bias; Boomsma, 2013), relative bias was still relatively small (< 20%). Whereas the information conveyed by $\hat{\upsilon}$ is redundant with that of the standardized indirect effect for three-variable mediation models, the adjusted $\tilde{\upsilon}$ conveys unique information that incorporates imprecision in effect estimates.

As previously discussed with the unstandardized indirect effect, parametric CIs rely on knowledge or a reasonable approximation of the sampling distribution of the estimator. The sampling distribution of the unstandardized indirect effect is a complex function of normally distributed variables, and it is unknown how the distribution changes when the indirect effect is standardized. Therefore, it is difficult to propose even an approximation to the distribution of squared standardized indirect effect. Lachowicz et al. (2018) used nonparametric bootstrapping to construct 95% CIs based on the percentiles of the empirical sampling distribution. Although in many conditions satisfactory CI coverage was achieved according Bradley's criteria (92.5% - 97.5%; Bradley, 1978), coverage tended to be too high (> 97.5%) for conditions with small sample sizes and small effects, suggesting the 95% CIs constructed using the percentile method were overly wide. In addition, even when satisfactory coverage was achieved, the proportion of true values below the lower CI limit and above the upper CI limit were imbalanced, suggesting a small but systematic bias in the interval estimation procedure.

υ has many desirable properties as an effect size measure. It is interpretable as a measure of explained variance and can be compared to existing benchmarks for small, medium, and large effects. It is standardized, so it is invariant under linear transformations of *x*, *m*, and *y*. It is not dependent on sample size in the population. It is a monotonic function in absolute value of the standardized indirect effect. Although more research is needed to develop an accurate interval estimator across a wider range of study conditions, CIs for $\hat{\upsilon}$ and $\tilde{\upsilon}$ can be constructed using a nonparametric bootstrap procedure. Finally, although the sample analog estimator $\hat{\upsilon}$ is biased, the adjusted estimator $\tilde{\upsilon}$ has good statistical properties (i.e., negligible bias, consistent) in many conditions common in applied research. In other words, the development of υ was consistent with the development of effect sizes for traditional research designs.

2.5 Summary

In Chapter 2, the topics of effect size, mediation analysis, and effect size in mediation analysis have been introduced. Several types of effect sizes for various common research designs were described. It was shown how sample analog estimators of these effect sizes are biased, and how these estimators can be adjusted, yielding new estimators with often more desirable statistical properties. Although a review of effect sizes for indirect effects in mediation analysis showed parallels among mediation effect sizes and traditional effect sizes, only the mediation effect size υ was developed considering the properties of the sample estimators. The sampling properties of $\hat{\upsilon}$ will be further explored in later sections in the development of a general bias adjustment procedure. In Chapter 3, a general explained variance framework for MLR and SEM will be introduced, and a general matrix-based formula for the adjusted R^2 will be proposed. In Chapter 4, the explained variance framework will be used to generalize the effect size υ to complex mediation models.

3 GENERAL EXPLAINED VARIANCE FRAMEWORK FOR MLR AND SEM

3.1 Matrix Representation of MLR and Explained Variance

3.1.1 Matrix Representation of MLR

The effect sizes presented in Chapter 2 were applicable to bivariate relationships in traditional research designs, and relationships among three variables for the mediation designs. As designs become more complex, it is more convenient to express models in matrix notation. This section will review the matrix representation of MLR and the computation of explained variance (R^2).

Although the MLR representation can be considered a special case of the SEM framework that will be reviewed in later sections, there are notable advantages to considering the MLR framework first. One is that the MLR representation will provide clearer links between the simpler effect sizes presented in Chapter 2 and the extensions to more complex models presented in later sections, particularly when considering the sampling properties of the estimators for deriving bias of sample estimators. Another notable advantage is that, whereas the adjusted R^2 is a well-established quantity in MLR, an analogous bias-adjusted statistic has not been studied for R^2 in SEM. Expressing the MLR R^2 bias adjustment in matrix form will facilitate the development of a generalizable bias-adjustment for estimators of υ extensions in later sections.

For ease of presentation and without loss of generality, I assume all variables are in standardized form. The regression equation is expressed in matrix form as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{3.1}$$

where $\mathbf{y}$ is a $N \times 1$ vector of responses, **X** is a $N \times p$ matrix of *p* covariates, $\hat{\boldsymbol{\beta}}$ is a $p \times 1$ vector of regression coefficients, and $\boldsymbol{\varepsilon}$ is a $N \times 1$ vector of errors. The model is assumed to be correctly specified, and that the errors are homoscedastic and independent. It is well known that when these assumptions hold, the ordinary least squares (OLS) estimator of $\boldsymbol{\beta}$ ($\hat{\boldsymbol{\beta}}$) that minimizes the sum of squared errors is $(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ (Rencher, 2008; Searle, 1971). In addition, $\hat{\boldsymbol{\beta}}$ is an unbiased estimator of $\boldsymbol{\beta}$ (i.e., $E[\hat{\boldsymbol{\beta}}] = \boldsymbol{\beta}$), and the variance of $\hat{\boldsymbol{\beta}}$ is $\sigma_\varepsilon^2(\mathbf{X}'\mathbf{X})^{-1}$, where σ_ε^2 is the residual variance.

Obtaining variance estimates is most conveniently performed in matrix-based MLR using quadratic forms. The general quadratic form for computing the sums of squares of a vector $n \times 1$ **y** and a $n \times n$ symmetric matrix **A** is $\mathbf{y}'\mathbf{A}\mathbf{y}$, where **A** determines the type of squared quantity. For example, if **A** is a $N \times N$ identity matrix **I**, the quadratic form $\mathbf{y}'\mathbf{I}\mathbf{y}$ yields the *SST*, whereas substituting **I** with the matrix $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ yields the SSR, and $\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ yields the *SSE*. Variance estimates are obtained by dividing these quantities by the sample size.

As described in Section 2.2.3, the above variances are biased estimators of their respective population variances. To determine the degree of bias, it is helpful to know some of the distributional properties of the quadratic form. Assuming $\mathbf{y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and **A** is a matrix of constants, the expected value of $\mathbf{y}'\mathbf{A}\mathbf{y}$ is (Rencher, 2008)

$$E[\mathbf{y}'\mathbf{A}\mathbf{y}] = tr(\mathbf{A}\boldsymbol{\Sigma}) + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu}. \tag{3.2}$$

Providing the assumptions hold, bias in the expected value of a quadratic estimator can be determined from Equation 3.2 as deviations of the expected value from the population parameter. For example, assuming $\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Sigma})$, the expected value of the above biased σ_ε^2 estimator is

$$
\begin{aligned}
E[N^{-1}SSE] &= N^{-1}E[\mathbf{y}'(\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')\mathbf{y}] \\
&= N^{-1}[tr\{(\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')\boldsymbol{\Sigma}\}+\boldsymbol{\beta}'\mathbf{X}'(\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')\mathbf{X}\boldsymbol{\beta}] \\
&= \sigma_\varepsilon^2 N^{-1}[tr\{(\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')\}+\boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}-\boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}] \\
&= \sigma_\varepsilon^2 \frac{N-p-1}{N}.
\end{aligned}
\tag{3.3}
$$

It is clear then the unbiased estimator of σ_ε^2 is $(N-p-1)^{-1}SSE$ (s_ε^2). This approach can also be used to derive the unbiased estimators of the total variance $(N-1)^{-1}SST$ (s_y^2) and the variance due to regression $p^{-1}SSR$.

3.1.2 Explained Variance

Variance explained is defined as the ratio of the variance explained in the outcome by the covariates to the total variance of the outcome, or the proportion reduction in error variance due to the covariates in the model (Cohen, Cohen, West, & Aiken, 2003; Maxwell & Delaney, 2003), commonly expressed as

$$
R^2 = \frac{\sigma_{\hat{y}}^2}{\sigma_y^2} = 1 - \frac{\sigma_\varepsilon^2}{\sigma_y^2}. \tag{3.4}
$$

Other sources define R^2 as the ratio of sums of squares regression to the sums of squares total (Rencher, 2008), expressed as

$$
R^2 = \frac{SSR}{SST} = \frac{\mathbf{y}'[\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\mathbf{y}}{\mathbf{y}'\mathbf{I}\mathbf{y}} = 1 - \frac{\mathbf{y}'[\mathbf{I}-\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\mathbf{y}}{\mathbf{y}'\mathbf{I}\mathbf{y}}. \tag{3.5}
$$

If MLEs are used to estimate variances in Equation 3.4, these definitions are equivalent. Both Equations 3.4 and 3.5 quantify the variance explained in the sample, and are biased estimators of the population R^2. Equation 3.4 can be adjusted for this bias by using the unbiased estimators $s_{\hat{y}}^2$, s_{ε}^2, and s_y^2 instead of the MLEs $\sigma_{\hat{y}}^2$, σ_{ε}^2, and σ_y^2

$$\tilde{R}^2 = \frac{s_{\hat{y}}^2}{s_y^2} = 1 - \frac{s_{\varepsilon}^2}{s_y^2}. \tag{3.6}$$

This is equivalent to the Ezekiel (1930) adjustment that is provided by default in many statistical software programs

$$\begin{aligned} \tilde{R}^2 &= 1 - \frac{s_{\varepsilon}^2}{s_y^2} \\ &= 1 - \frac{N-1}{N-p-1} \frac{\mathbf{y}'[\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\mathbf{y}}{\mathbf{y}'\mathbf{I}\mathbf{y}} \\ &= 1 - \frac{N-1}{N-p-1}\left(1 - R^2\right). \end{aligned} \tag{3.7}$$

This R^2 adjustment formula makes salient the importance of sample size and number of parameters in R^2 bias. Specifically, increasing the number of parameters holding sample size constant will increase bias, and, alternatively, increasing the sample size holding the number of parameters constant will decrease bias.

Although a popular choice for obtaining more precise estimates of R^2, $\tilde{R}^2$ is by no means the only statistic designed for this purpose. Yin and Fan (2001) conducted a thorough review of R^2 estimators, finding at least six different methods for R^2 bias adjustment. Comparing the performance of the estimators, a method proposed by Olkin and Pratt (1958) consistently returned the least biased estimates across a range of conditions. This is unsurprising as the adjustment was derived from the known probability density function of R^2, and therefore accounts for bias in higher order moments (e.g., skew, kurtosis). Although $\tilde{R}^2$ adjusts for bias only in the expected value of R^2, the measure returned largely accurate estimates across many conditions, with bias becoming problematic in conditions with small ratios of small size to number of parameters.

The approach to deriving and adjusting for bias in the expected value considered thus far has proven particularly useful for creating estimators with improved statistical properties (i.e., $\tilde{R}^2$, $\tilde{\upsilon}^2$). Importantly, it demonstrates that in some cases the complete distributional properties of an estimator, although desirable, may not be necessary to obtain accurate estimates. It should be noted, however, that models for which the estimator biases were derived are not particularly complex and contain relatively few parameters. Of interest is a more general approach to reduction in expected value bias, of which the adjusted estimators considered thus far are special cases, and which can be extended to more general modeling frameworks (i.e., SEM).

3.1.3 General Bias Reduction

Many bias-adjusted estimators described in the previous section and in Chapter 2 can be expressed in terms of sample bias subtracted from the biased MLE estimator, which are special cases of a more general procedure for bias reduction (Box, 1971; Cox & Snell, 1968; Cox & Hinkley, 1974). The asymptotic bias of a MLE $\hat{\theta}$ can be expressed generally as

$$b(\theta) = \frac{b_1(\theta)}{N} + \frac{b_2(\theta)}{N^2} ..., \tag{3.8}$$

where N is the sample size, $b_1(\theta)$ is first-order bias, $b_2(\theta)$ is second-order bias, etc. It is often of interest to correct for $b_1(\theta)$ because higher order biases tend to be of negligible magnitude in most applications (Box, 1971; Cox & Hinkley, 1974; Firth, 1993). A bias-adjusted estimator $\hat{\theta}_{BC}$ can then be created by substituting sample estimates for the MLEs

$$\hat{\theta}_{BC} = \hat{\theta} - \frac{b_1(\hat{\theta})}{N}, \tag{3.9}$$

Several methods have been proposed for estimating $b_1(\theta)$, including methods that implicitly estimate $b_1(\theta)$ as part of the iterative estimation algorithm (Firth, 1993; Kosmidis & Firth, 2010), and methods that explicitly estimate $b_1(\theta)$ so it may be subtracted from the MLE. Explicit methods include the jackknife (Quenouille, 1956; Schucany, Gray, & Owen, 1971), bootstrapping (Davison & Hinkley, 1997; Efron, 1975; Hall & Martin, 1988), and asymptotic expansions (Box, 1971; Cox & Snell, 1968; Cox & Hinkley, 1974).

The bias reduction approaches described in previous sections are special cases of a more general method of asymptotic expansion. If $\hat{\theta}$ is a function of a parameter μ that has an estimator *T* with good statistical properties (i.e., unbiasedness, minimum variance), the asymptotic bias of the function $\hat{\theta} = f(T)$, assuming the function is continuous at μ, can be approximated with a Taylor expansion (Cox & Snell, 1968; Cox & Hinkley, 1974)

$$f(T) \approx f(\mu) + (T-\mu) f'(\mu) + \frac{1}{2}(T-\mu)^2 f''(\mu). \tag{3.10}$$

where $f'(\bullet)$ and $f''(\bullet)$ are the first and second derivatives of $f(\bullet)$. It follows that functions with no second derivative (e.g., linear functions) are asymptotically unbiased. Assuming the second derivative exists, the asymptotic bias of the expected value of $\hat{\theta}$ is

$$E[f(T)] \approx f(\mu) + \frac{1}{2}\text{var}(T) f''(\mu). \tag{3.11}$$

An adjusted estimator can then be constructed by rearranging terms and substituting T for μ

$$\hat{\theta}_{BC} = f(T) - \frac{1}{2}\operatorname{var}(T)\, f''(T). \tag{3.12}$$

For estimators consisting of multiple parameters such as R^2, the multiparameter extension of Equation 3.12 is (Box, 1971)

$$\hat{\theta}_{BC} = f(\mathbf{T}) - \frac{1}{2} tr\left\{\mathbf{\Sigma}(\mathbf{T})\mathbf{H}\right\}, \tag{3.13}$$

where $\mathbf{\Sigma}(\mathbf{T})$ is the variance-covariance matrix of parameter estimates, and **H** is a matrix of second derivatives of $f(\mathbf{T})$ (i.e., Hessian matrix). It is possible that for some functions (i.e., $\hat{\upsilon}$) the asymptotic bias term on the right-hand side of equation 3.13 contains nonlinear parameters, which, when replaced with sample estimates, will yield biased estimates of the asymptotic bias. The solution is to apply the asymptotic expansion to the nonlinear functions in the bias term

$$\hat{\theta}_{BC} = f(\mathbf{T}) - \frac{1}{2}\left(tr\left\{\mathbf{\Sigma}(\mathbf{T})\mathbf{H}\right\} - \frac{1}{2} tr\left\{\mathbf{\Sigma}(\mathbf{T})\mathbf{H}_1\right\} \right), \tag{3.14}$$

Where $\mathbf{H}_1$ is the matrix of second derivatives of $tr\left\{\mathbf{\Sigma}(\mathbf{T})\mathbf{H}\right\}$.

Although unbiasedness is a desirable property of estimators, bias-adjusted estimators should be carefully evaluated prior to use in applied research settings. Because bias adjustments typically use sample estimates in place of asymptotic parameters, additional sources of error variance can be introduced into an otherwise unbiased estimator. This bias-variance tradeoff can be evaluated analytically or through simulation if sampling distributions are complex.

3.1.4 General Bias Reduction for R^2

The adjusted $\tilde{R}^2$ (Equation 2.23) can also be re-expressed in a generalizable matrix form. Although to this point the R^2 estimators have been expressed in terms of variance and sums of squares ratios, re-expressing explained variance as the ratio of standardized regression coefficients more clearly shows that $\tilde{R}^2$ follows the same form as the general bias reduction described in the previous section. The biased MLE estimator of R^2 is

$$\hat{R}^2 = \hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}}, \tag{3.15}$$

where $\hat{\boldsymbol{\beta}}$ is a vector of standardized regression coefficients and $\mathbf{R}$ is the correlation matrix among variables (the total variance of the outcome y is one and so it is omitted from the denominator). The expected value of the unadjusted $\hat{R}^2$ is

$$\begin{aligned} E[\hat{R}^2] &= E[\hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}}] \\ &= tr\left\{\mathbf{R}\sigma_\varepsilon^2(\mathbf{X}'\mathbf{X})^{-1}\right\} + \boldsymbol{\beta}'\mathbf{R}\boldsymbol{\beta} \\ &= \frac{\sigma_\varepsilon^2}{N-1} tr\left\{\mathbf{X}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\right\} + \boldsymbol{\beta}'\mathbf{R}\boldsymbol{\beta} \\ &= \frac{p}{N-1}\sigma_\varepsilon^2 + \boldsymbol{\beta}'\mathbf{R}\boldsymbol{\beta} \end{aligned} \tag{3.16}$$

This shows that $\hat{R}^2$ is upwardly biased by the factor $\sigma_\varepsilon^2 p/(N-1)$. In addition, when $\boldsymbol{\beta} = 0$, estimates of $\hat{R}^2$ are upwardly biased by $p/(N-1)$ (Rencher, 2003). The Ezekiel (1930) adjusted $\tilde{R}^2$ can be expressed in matrix form by subtracting the bias factor from the MLE estimator. However, it is again important to note that $\sigma_\varepsilon^2 p/(N-1)$ is an asymptotic parameter, so the MLE residual variance estimator is upwardly biased. An unbiased estimator of the bias factor uses the unbiased estimator s_ε^2

$$\begin{aligned}\tilde{R}^2 &= \hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}} - \frac{p}{N-1}s_\varepsilon^2 \\ &= \hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}} - \frac{p}{N-1}\frac{1-\hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}}}{N-p-1} \\ &= \hat{R}^2 - \frac{p}{N-p-1}(1-\hat{R}^2),\end{aligned} \tag{3.17}$$

which is equivalent to the Ezekiel's $\tilde{R}^2$ in Equation 2.23.

The bias term estimated in Equation 3.17 can also be obtained using the more general bias reduction approach in the previous section. The bias-adjusted $\tilde{R}^2$ is expressed as

$$\tilde{R}^2 = \hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}} - \frac{1}{2}tr\{\mathrm{var}(\hat{\boldsymbol{\beta}})\mathbf{H}\}, \tag{3.18}$$

where **H** is the Hessian matrix of second derivatives of the function $\hat{\boldsymbol{\beta}}'\mathbf{R}\hat{\boldsymbol{\beta}}$ with respect to the elements of $\hat{\boldsymbol{\beta}}$. To show that this expression is equivalent to Equation 3.17, the right-hand side of Equation 3.18 is expanded

$$\begin{aligned}\frac{1}{2}tr\{\mathrm{var}(\hat{\boldsymbol{\beta}})\mathbf{H}\} &= \frac{1}{2}tr\left\{\begin{bmatrix} s_{\beta 1}^2 & \cdots & s_{\beta 1,\beta p} \\ \vdots & \ddots & \vdots \\ s_{\beta p,\beta 1} & \cdots & s_{\beta p}^2 \end{bmatrix}\begin{bmatrix} 2 & \cdots & 2r_{x1,xp} \\ \vdots & \ddots & \vdots \\ 2r_{xp,x1} & \cdots & 2 \end{bmatrix}\right\} \\ &= \sigma_\varepsilon^2 tr\left\{(\mathbf{X}'\mathbf{X})^{-1}\begin{bmatrix} 1 & \cdots & r_{x1,xp} \\ \vdots & \ddots & \vdots \\ r_{xp,x1} & \cdots & 1 \end{bmatrix}\right\} \\ &= \sigma_\varepsilon^2 tr\left\{(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}\right\} \\ &= \frac{\sigma_\varepsilon^2}{N-1} tr\left\{(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\right\} \\ &= \sigma_\varepsilon^2 \frac{p}{N-1}.\end{aligned} \tag{3.19}$$

Substituting this result into Equation 3.18 and replacing the residual variance parameter σ_{ε}^{2} with the sample unbiased estimator s_{ε}^{2} yields an equivalent formula for $\tilde{R}^{2}$.

The two bias-correction approaches considered in this section (i.e., approximation via expectations and asymptotic expansion) yielded identical results in these special cases, but there are situations where using the more general asymptotic expansion approach presented in Section 3.1.3 is likely to be more advantageous. The regression models considered thus far have been linear in their coefficients, omitting nonlinear effects such as polynomials and conditional, or moderated, effects. Although both methods would return the same or similar approximations in many circumstances, differentiation in asymptotic expansion is a much more expedient approach. In addition, methods based on asymptotic expansions can incorporate additional higher-order approximations to account for different sources of bias.

3.1.5 Limitations of MLR

The primary limitation of MLR as a framework for representing mediation effect sizes is the restriction of the models to a single outcome. Although mediation parameters can be estimated using several MLR models as in Equations 2.26 and 2.27, this approach quickly becomes intractable as models become more complex, in particular when accounting for the residual relationships among multiple outcomes. In addition, this approach does not lend itself to a generalizable solution that can be applied to mediation models of various complexities.

3.2 Structural Equation Modeling Framework

SEM is a statistical modeling framework that can be used to simultaneously investigate complex interrelationships among variables, and estimate many substantively meaningful parameters that are not accessible by simpler methods. The SEM framework proposed by Jöreskog (1977), and implemented in M*plus* using a modified LISREL framework (LISCOMP;

Muthén & Muthén, 1998-2018) is a particularly useful and intuitive approach for representing the indirect effects and effect sizes for complex mediation models. The flexibility of the LISCOMP framework also allows for effect size extensions using cutting edge techniques available in M*plus* such as mediation using multilevel SEM (MSEM), time series analysis, and latent class analysis.

The LISCOMP model can be considered to consist of two primary components; a measurement model and a structural model. The measurement model specifies the relationships among the manifest, or indicator, variables and the unobserved latent variables. The measurement model is expressed as

$$\mathbf{y} = \mathbf{\Lambda}\boldsymbol{\eta} + \boldsymbol{\varepsilon}, \tag{3.20}$$

where $\mathbf{\Lambda}$ is a $p \times m$ matrix of factor loadings, $\boldsymbol{\eta}$ is a $m \times 1$ vector of latent variables, and $\boldsymbol{\varepsilon}$ is a $p \times 1$ vector of measurement errors, where $\boldsymbol{\varepsilon} \sim N(0, \mathbf{\Theta})$ (because variables are standardized, the intercept is omitted). This specification assumes that all variables in the model are outcomes (i.e., "all y" specification), considerably simplifying presentation by omitting vectors and matrices specific to exogenous variables. The structural model component of the LISCOMP SEM specifies the relationships among the latent variables. The structural model is expressed as

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\zeta}, \tag{3.21}$$

where $\mathbf{B}$ is a $m \times m$ matrix of slopes for regressions of latent variables on other latent variables, and $\boldsymbol{\zeta}$ is a $m \times 1$ vector of residuals, where $\boldsymbol{\zeta} \sim N(0, \mathbf{\Psi})$.

Although MLR is a special case of SEM where there is a single outcome variable and no latent variables, the methods of parameter estimation markedly differ. The OLS method is most common for MLR, in which parameters are estimated by minimizing the sum of squared errors, and making no distributional assumptions about the variables (though assumptions are needed for valid inferences). By contrast, parameters are estimated in SEM by selecting the values that minimize the discrepancy between the sample covariance matrix **S** and the covariance matrix implied by the model $\hat{\boldsymbol{\Sigma}}$. In other words, the covariance structure of the observations is modelled in SEM, not the individual responses. Parameters are commonly estimated via an iterative maximum likelihood algorithm assuming multivariate normality of the data (i.e., no closed form solution). However, in the special case of a manifest variable regression model, MLEs returned by SEM will be identical to MLEs estimated via OLS under multivariate normality.

3.2.1 Explained variance in SEM

The general definition of R^2 in SEM is analogous to R^2 in MLR (i.e., the relative amount of variance explained in an outcome by a set of predictors; Bollen, 1989; Jöreskog, 2015). However, because the SEM framework can simultaneously model multiple outcome variables, there are more ways to define R^2 in SEM than in MLR. For example, Bollen (1989) outlined two types of explained variance that can be examined in latent variable models. The first type is variance in the manifest variables explained by the model (R_y^2)

$$R_y^2 = 1 - \frac{\left|\hat{\boldsymbol{\Theta}}\right|}{\left|\hat{\boldsymbol{\Sigma}}\right|}, \tag{3.22}$$

where $|\bullet|$ is the determinant, or the *generalized* variance shared by multiple outcomes. The second type of explained variance is the variance in the latent variables explained by other latent variables, or the structural model R_η^2

$$R_\eta^2 = 1 - \frac{|\mathbf{\Psi}|}{|\mathbf{\Psi} + \mathbf{\Phi}|}, \tag{3.23}$$

where $\mathbf{\Phi}$ is a $m \times m$ matrix of latent variable variances and covariances. Although R_η^2 is equivalent to the MLR R^2 in the special case of a path model with a single outcome variable, generalized variance is not commonly thought of as a substantively meaningful quantity. This is likely because, outside of very simple models, shared variance is explained by the pooled effects of other variables in the model, making it exceedingly difficult to isolate the effects of specific variables (Bagozzi & Yi, 1988).

Given the difficulty in generating appropriate research hypotheses regarding generalized variance, a more useful measure of R^2 should quantify the variance explained in each outcome separately, such that variance explained can be attributed to specific causes. An alternative formulation for R_η^2 ($\mathbf{R}_\eta^2$) is a $m \times m$ matrix of explained variances (and covariances), expressed as

$$\begin{aligned} \mathbf{R}_\eta^2 &= \mathbf{D}_\eta^{-1/2}[(\mathbf{I}-\mathbf{B})^{-1} - \mathbf{I}]\mathbf{\Psi}[(\mathbf{I}-\mathbf{B})^{-1} - \mathbf{I}]'\mathbf{D}_\eta^{-1/2} \\ &= [(\mathbf{I}-\mathbf{B}^{st})^{-1} - \mathbf{I}]\mathbf{\Psi}^{st}[(\mathbf{I}-\mathbf{B}^{st})^{-1} - \mathbf{I}]', \end{aligned} \tag{3.24}$$

where $(\mathbf{I}-\mathbf{B})^{-1} - \mathbf{I}$ is referred to as the total effect (Bollen, 1987), $\mathbf{B}^{st}$ is the matrix of standardized effects, $\mathbf{\Psi}^{st}$ is the standardized residual covariance matrix, and $\mathbf{D}_\eta$ is a diagonal matrix of latent variable variances, estimated as (Jöreskog & Sörbom, 2015)

$$\mathbf{D}_\eta = \left\{ diag[((\mathbf{I}-\mathbf{B})^{-1})\mathbf{\Psi}(\mathbf{I}-\mathbf{B})'^{-1}] \right\}. \tag{3.25}$$

$\mathbf{D}_\eta$ can be omitted if latent or manifest variables variances are standardized. When the model is specified to include latent variables, each diagonal element of $\mathbf{R}^2_\eta$ is the variance in that latent outcome explained by the latent predictors. When a path model is specified (i.e., $\mathbf{\Lambda} = \mathbf{I}$ and $\mathbf{\Theta} = \mathbf{0}$), each diagonal is the variance in that manifest variable explained by the manifest predictors, where a model with a single outcome is equivalent to the MLR R^2.

It should be noted that the $\mathbf{\Psi}$ matrix in the center of the quadratic form in Equations 3.24 and 3.25 is a matrix of residual variances and covariances, and not covariances or correlations as is the case for quadratic forms in MLR (e.g., Equation 3.15). This is partially due to the adoption of the "all y" notation for LISCOMP models made throughout, as exogenous variables can be considered outcome variables with no predictors. The original LISREL notation separates exogenous and endogenous variables (Jöreskog, 1977), such that the variance/covariance matrix of the exogenous predictors would be equivalent to the variance/covariance matrix of predictors in MLR. However, the partitions of $\mathbf{\Psi}$ that correspond to outcome variables have no counterpart in MLR, but are essential for simultaneously modeling variables as both predictors and outcomes. The effects of variables that are modeled simultaneously with their causes to predict other downstream variables are residual effects, so it is reasonable to assume that the residual variance (and residual covariances) should be considered when quantifying the contributions of those variables to the overall variance explained in a particular outcome.

3.2.2 Sample Estimator of $\mathbf{R}^2_\eta$

Given that $\mathbf{R}^2_\eta$ is a nonlinear function and parameters are estimated via maximum likelihood, it is reasonable to assume that $\hat{\mathbf{R}}^2_\eta$ is a biased estimator of the population $\mathbf{R}^2_\eta$. Assuming variables are standardized, the bias-adjusted estimator $\tilde{\mathbf{R}}^2_\eta$ is expressed as

$$\tilde{\mathbf{R}}_{\eta}^{2} = [(\mathbf{I}-\hat{\mathbf{B}}^{st})^{-1}-\mathbf{I}]\hat{\mathbf{\Psi}}^{st}[(\mathbf{I}-\hat{\mathbf{B}}^{st})^{-1}-\mathbf{I}]' - \hat{\mathbf{\Delta}}_{bc}, \tag{3.26}$$

where $\hat{\mathbf{\Delta}}_{bc}$ is a $m \times m$ matrix of bias adjustment terms $(1/2)tr\{\mathbf{V}(\mathbf{B}^{st})\mathbf{H}^{st}\}$ corresponding to the *j*th outcome variables in $\hat{\mathbf{R}}_{\eta}^{2}$. Further expansions for nonlinear coefficients in the bias term can be included as well if $\hat{\mathbf{\Delta}}_{bc}$ contains nonlinear functions of $\hat{\mathbf{B}}^{st}$ (Equation 3.14).

Because the bias estimate in Equation 3.26 is asymptotic, sample estimates would replace population parameters. In particular, because bias is proportional to error variance (Box, 1971), many parameters, if not all in some cases, will be variances. In MLR, variances have well-known unbiased sample estimators (e.g., s_{ε}^{2}). However, because variances and covariances of parameter estimates in SEM (i.e., asymptotic covariance, or ACOV, matrix) are estimated via maximum likelihood, these estimates are biased. However, given the assumption multivariate normality is satisfied, unbiased estimates can be obtained by substituting the denominator *N* with *N-1* (Kaplan, 2008).

3.3 Summary

In Chapter 3, a general modeling framework was presented that will provide a foundation for the υ extensions in the following chapters. Although a special case of the general framework, MLR was discussed first to introduce matrix notation and the properties of estimators, particularly for variance and R^2 estimators. It was shown how these properties affect bias in estimation, and also how the properties can be used to construct improved estimators. A general method for bias-adjustment was proposed, and, in the special case of R^2, it was demonstrated to return the common adjusted $\tilde{R}^2$. The LISCOMP SEM framework was introduced as a more powerful modeling framework. SEM analogs of R^2 ($\mathbf{R}_{\eta}^{2}$) were also introduced, and an adjusted version of $\hat{\mathbf{R}}_{\eta}^{2}$ ($\tilde{\mathbf{R}}_{\eta}^{2}$) was proposed using the general bias reduction method. In Chapter 4, the general modeling framework is used to extend υ to complex mediation models, and the general bias reduction method is used to construct an improved estimator. The properties of the adjusted and unadjusted estimators are evaluated and compared via Monte Carlo simulation.

4 EXTENDING υ TO COMPLEX MEDIATION MODELS

This section will provide an extension of υ that allows for computation of effect sizes for a wide variety complex mediation models, most notably models with multiple mediators (parallel and serial), covariates, and multiple predictors (e.g., multicategorical *x* variable). Interpretations of the resulting effect sizes will be provided. Because the effect size sample estimators are expected to be positively biased, a general formula for the expected bias will be derived and used to propose a general form for an adjusted estimator that corrects for this bias. Like bias corrections for traditional standardized effect size measures and υ, the bias will be derived for the expected value of the estimator. This bias-adjusted υ for three-variable mediation models will be compared to the matrix version's bias corrected effect size, demonstrating that the adjusted estimator proposed in Lachowicz et al. (2018) is a special case of a more general bias-adjustment formula. Finite sample properties of point and interval estimators of the unadjusted and adjusted estimators will be evaluated in a Monte Carlo simulation study for a complex mediation model. For the point estimators, the simulation study has three primary goals: a) determine if the general bias adjustment formula yields adjusted estimators that have negligible bias across a range of conditions (e.g., sample size, effect magnitudes) common in applied research settings, b) demonstrate that accuracy of the estimators in estimating the population parameter increases with increasing sample size (i.e., consistency), and c) determine if there are certain conditions where the unadjusted estimator, although biased, may be more accurate than the adjusted estimator (i.e., relative efficiency). For the interval estimator, the purpose of the simulation study is to evaluate the performance of 95% CIs in terms of overall coverage and coverage balance.

4.1 Mediation in the LISCOMP framework

The mediation model represented in Equations 2.26 and 2.27 can be conveniently specified as a single model in the LISCOMP framework. For further convenience, I assume variables are standardized (i.e., $\boldsymbol{\alpha} = 0$ and $\boldsymbol{\nu} = 0$), only manifest variables (i.e., $\boldsymbol{\Lambda} = \mathbf{I}$), and no residuals in the measurement model ($\boldsymbol{\Theta} = 0$). These assumptions mean that Equation 3.20 reduces to

$$\mathbf{y} = \mathbf{I}\boldsymbol{\eta}, \tag{4.1}$$

Solving Equation 3.21 for $\boldsymbol{\eta}$ and substituting into Equation 4.1 yields

$$\mathbf{y} = (\mathbf{I} - \mathbf{B}^{st})^{-1}\boldsymbol{\zeta}. \tag{4.2}$$

Bollen (1987) showed how the total, direct, and indirect effects can be computed using the components of Equation 4.2. The matrix $\mathbf{T}^{st} = (\mathbf{I} - \mathbf{B}^{st})^{-1} - \mathbf{I}$ consists of the total effects of *x* on *m*, and *x* and *m* on *y*

$$\mathbf{T}^{st} = \begin{bmatrix} 0 & 0 & 0 \\ \beta_{mx} & 0 & 0 \\ \beta_{mx}\beta_{ym \bullet x} + \beta_{yx \bullet m} & \beta_{ym \bullet x} & 0 \end{bmatrix}. \tag{4.3}$$

Further, the elements of $\mathbf{M}^{st} = \mathbf{T}^{st} - \mathbf{B}^{st}$ consists of the indirect effects

$$\mathbf{M}^{st} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{mx}\beta_{ym \bullet x} & 0 & 0 \end{bmatrix}. \tag{4.4}$$

Complex mediation models can be conveniently expressed using the LISCOMP SEM matrix notation. For recursive models (i.e., no feedback loops), the $\mathbf{B}^{st}$ matrix can be expressed in general terms as

$$\mathbf{B}^{st} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{mx}^{st} & \mathbf{B}_{m}^{st} & \mathbf{0} \\ \mathbf{B}_{yx}^{st} & \mathbf{B}_{ym}^{st} & \mathbf{0} \end{bmatrix}, \tag{4.5}$$

where the $m \times m$ coefficient matrix $\mathbf{B}^{st}$ consists of four partitions: (a) a $q \times p$ partition $\mathbf{B}_{mx}^{st}$ of the effects of *p* predictors on *q* mediators, (b) a $r \times p$ partition $\mathbf{B}_{yx}^{st}$ of the effects of p predictors on *r* outcomes, controlling for mediators, (c) a $r \times q$ partition $\mathbf{B}_{ym}^{st}$ of the effects of *q* mediators on *r* outcomes, controlling for the predictors, and (d) a $q \times q$ partition $\mathbf{B}_{m}^{st}$ of the effects of mediators on other mediators. Because there are no regressions of predictors on other predictors or outcomes on other outcomes (certain predictors or outcomes would then be considered mediators), the upper left $p \times p$ and lower right $r \times r$ submatrices are $\mathbf{0}$. The $\mathbf{B}_{m}^{st}$ submatrix has zeros on its diagonal, and, whereas all elements of $\mathbf{B}_{mx}^{st}$, $\mathbf{B}_{yx}^{st}$, and $\mathbf{B}_{ym}^{st}$ may appear only in the lower or upper triangle, elements of $\mathbf{B}_{m}^{st}$ may be above and below the diagonal depending on the direction of relationships among mediators. However, for the model to remain recursive, it must be possible to arrange the rows and columns of $\mathbf{B}_{m}^{st}$ to yield a lower triangular matrix with zeros on the diagonal.

A matrix of total effects $\mathbf{T}^{st}$ is computed from $\mathbf{B}^{st}$ as in the previous special case of three-variable mediation (i.e., $\mathbf{T}^{st} = (\mathbf{I} - \mathbf{B}^{st})^{-1} - \mathbf{I}$), which in terms of partitions of $\mathbf{B}^{st}$ in Equation 4.5 is

$$\mathbf{T}^{st} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{mx}^{st} + ((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I})\mathbf{B}_{mx}^{st} & (\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I}) & \mathbf{0} \\ \mathbf{B}_{yx}^{st} + \mathbf{B}_{mx}^{st}\mathbf{B}_{ym}^{st} + \mathbf{B}_{ym}^{st}((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I})\mathbf{B}_{mx}^{st} & \mathbf{B}_{ym}^{st} + \mathbf{B}_{ym}^{st}((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I}) & \mathbf{0} \end{bmatrix}, \tag{4.6}$$

where $\mathbf{I}$ is a $q \times q$ identity matrix. The matrix of indirect effects $\mathbf{M}^{st}$ is then computed (i.e., $\mathbf{M}^{st} = (\mathbf{I}-\mathbf{B}^{st})^{-1} - \mathbf{I} - \mathbf{B}^{st}$), which in terms of partitions of $\mathbf{B}^{st}$ is

$$\mathbf{M}^{st} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ ((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I})\mathbf{B}_{mx}^{st} & (\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I}) - \mathbf{B}_{m}^{st} & \mathbf{0} \\ \mathbf{B}_{ym}^{st}((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I})\mathbf{B}_{mx}^{st} & \mathbf{B}_{ym}^{st}((\mathbf{I}-\mathbf{B}_{m}^{st})^{-1} - \mathbf{I}) & \mathbf{0} \end{bmatrix}. \tag{4.7}$$

The four nonzero partitions in $\mathbf{M}^{st}$ consist of generalizable expressions for computing indirect effects. The partition in the first column, second row of $\mathbf{M}^{st}$ contains the indirect effects of a set of predictors on a set of mediators through another set of mediators. The partition in the second row, second column of $\mathbf{M}^{st}$ contains the indirect effects of a set of mediators on another set of mediators through another set of mediators. The partition in the third row, second column contains the indirect effects of a set of mediators on the outcomes through another set of mediators. Finally, the partition in the third row, first column represents the indirect effects of a set of predictors on a set of outcomes through a set of mediators.

The indirect effects contained in $\mathbf{M}^{st}$ in Equation 4.7 are *total* indirect effects, which are composed of *specific* indirect effects. To illustrate these types of indirect effects, consider a mediation model with a single predictor x, a single outcome y, and two parallel mediators m_1 and m_2 (see Figure 2). There are three indirect effects that could be examined in this model: (a) the specific indirect effect of x on y through m_1, and (b) the specific indirect effect of x on y through m_2, and (c) the total indirect effect of x on y through both m_1 and m_2. The three MLR equations that specify this model represent the effect of x on m_1

$$m_1 = \beta_{m1x} x + \varepsilon_{m1}, \tag{4.8}$$

the effect of x on m_2

$$m_2 = \beta_{m2x} x + \varepsilon_{m2}, \tag{4.9}$$

and the effects of x, m_1, and m_2 on y

$$y = \beta_{yx\bullet\mathbf{x}} x + \beta_{ym1\bullet\mathbf{x}} m_1 + \beta_{ym2\bullet\mathbf{x}} m_2 + \varepsilon_y. \tag{4.10}$$

The specific indirect effects are the pathways through which x affects y through m_1 ($\beta_{m1x}\beta_{ym1\bullet\mathbf{x}}$) and m_2 ($\beta_{m2x}\beta_{ym2\bullet\mathbf{x}}$), and the total indirect effect ($\beta_{m1x}\beta_{ym1\bullet\mathbf{x}} + \beta_{m2x}\beta_{ym2\bullet\mathbf{x}}$) is the sum of the specific indirect effects.

The model coefficients are more conveniently expressed in terms of the partitioned $\mathbf{B}^{st}$ matrix as

$$\mathbf{B}^{st} = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta_{m1x} & 0 & 0 & 0 \\ \beta_{m2x} & 0 & 0 & 0 \\ \hdashline \beta_{yx\bullet\mathbf{x}} & \beta_{ym1\bullet\mathbf{x}} & \beta_{ym2\bullet\mathbf{x}} & 0 \end{array}\right]. \tag{4.11}$$

The matrix of total indirect effects $\mathbf{M}^{st}$ is computed from $\mathbf{B}^{st}$

$$\mathbf{M}^{st} = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline \beta_{m1x}\beta_{ym1\bullet\mathbf{x}} + \beta_{m2x}\beta_{ym2\bullet\mathbf{x}} & 0 & 0 & 0 \end{array}\right], \tag{4.12}$$

Bollen (1987) proposed a method for obtaining these specific indirect effects by recomputing $\mathbf{M}^{st}$ from a modified coefficient matrix $\mathbf{B}^{st*}$. This is accomplished by omitting the rows and columns of $\mathbf{B}^{st}$ that correspond to the mediators that are part of the specific pathway of interest by setting these entries to zero. Recalculating $\mathbf{M}^{st}$ from this modified coefficient matrix yields a matrix of specific indirect effects ($\mathbf{M}^{st*}$).

To replace specific entries of $\mathbf{B}^{st}$ with zeros, $\mathbf{B}^{st}$ is pre- and post-multiplied by an elementary matrix $\mathbf{O}$:

$$\mathbf{B}^{st*} = \mathbf{O}\mathbf{B}^{st}\mathbf{O}, \tag{4.13}$$

where $\mathbf{O}$ is a $m \times m$ diagonal matrix where the elements associated with the variables to be omitted from $\mathbf{B}^{st}$ are set to zero. For example, in order to obtain the specific indirect effect $\beta_{m1x}\beta_{ym1\bullet\mathbf{x}}$ from the previous multiple mediator model, in the mediation model illustrated in Figure 2, the $\mathbf{O}$ that would set the rows columns and columns of $\mathbf{B}^{st}$ associated with m_2 to zero is

$$\mathbf{O} = \begin{array}{c} x \\ m_1 \\ m_2 \\ y \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4.14}$$

which is then substituted into Equation 4.13 to compute $\mathbf{B}^{st*}$

$$
\begin{aligned}
\mathbf{B}^{st*} &= \mathbf{O}\mathbf{B}^{st}\mathbf{O} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\left[\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline \beta_{m1x} & 0 & 0 & 0 \\ \beta_{m2x} & 0 & 0 & 0 \\ \hline \beta_{yx\bullet\mathbf{x}} & \beta_{ym1\bullet\mathbf{x}} & \beta_{ym2\bullet\mathbf{x}} & 0 \end{array}\right]
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \left[\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline \beta_{m1x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline \beta_{yx\bullet\mathbf{x}} & \beta_{ym1\bullet\mathbf{x}} & 0 & 0 \end{array}\right].
\end{aligned}
\tag{4.15}
$$

The modified indirect effect matrix $\mathbf{M}^{st*}$ calculated from $\mathbf{B}^{st*}$ yields the specific indirect effect

$$
\mathbf{M}^{st*} = \left[\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline \beta_{m1x}\beta_{ym1\bullet\mathbf{x}} & 0 & 0 & 0 \end{array}\right]. \tag{4.16}
$$

This framework offers a general solution for obtaining indirect effects for a wide array of mediation models. These include models with multiple parallel or serial mediators, multiple predictors, covariates, and latent variable mediation. In addition, methods are available for decomposing complex indirect effects.

4.2 Matrix Extension of υ

Effect sizes for total and specific indirect effects can be obtaining by deriving a general matrix-based specification of υ in the LISCOMP framework. Specifically, because υ is a measure of explained variance, the quadratic forms used in previous sections for computing R^2 can be applied to compute the matrix extension of υ. In addition, the sampling properties of quadratic forms will also be applicable, allowing for generalizations of the properties of υ discussed in Chapter 2.

The quadratic form for the LISCOMP structural model in Equation 3.24 can be applied to obtain the quadratic form of υ. This is accomplished by substituting the matrix of total effects **T** with the matrix of indirect effects $\mathbf{M}$, yielding the υ matrix $\boldsymbol{\Upsilon}$

$$\boldsymbol{\Upsilon} = \mathbf{D}^{-1/2}\mathbf{M}\boldsymbol{\Psi}\mathbf{M}'\mathbf{D}^{-1/2}, \tag{4.17}$$

An equivalent method of computing $\boldsymbol{\Upsilon}$ is to use the standardized indirect effect matrix $\mathbf{M}^{st}$ and the standardized residual variance matrix $\boldsymbol{\Psi}^{st}$

$$\boldsymbol{\Upsilon} = \mathbf{M}^{st}\boldsymbol{\Psi}^{st}\mathbf{M}'^{st}. \tag{4.18}$$

If the unmodified **B** matrix is used to compute $\mathbf{M}$, $\boldsymbol{\Upsilon}$ yields a matrix of υ effect sizes for the total indirect effects. If the modified matrix $\mathbf{B}^*$ (i.e., the rows and columns associated with certain variables are set to 0), $\boldsymbol{\Upsilon}$ yields a matrix of υ for specific indirect effects.

4.2.1 Special case of a single predictor and outcome, and two parallel mediators

To demonstrate the proposed matrix method, $\hat{\boldsymbol{\Upsilon}}$ is obtained from a mediation model with a single predictor *x*, a single outcome *y*, and two parallel mediators m_1 and m_2 (see Figure 2). There are three effect sizes to consider in this model: (a) the effect size for the total indirect effect of *x* on *y* through both m_1 and m_2, (b) the effect size for the specific indirect effect of *x* on *y* through m_1, and (c) the specific indirect effect of *x* on *y* through m_2. The *unstandardized* regression coefficients for this model can be expressed in the unstandardized partitioned **B** matrix from Equation 4.5:

$$\mathbf{B} = \left[\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline B_{m1x} & 0 & 0 & 0 \\ B_{m2x} & 0 & 0 & 0 \\ \hline B_{yx\bullet\mathbf{x}} & B_{ym1\bullet\mathbf{x}} & B_{ym2\bullet\mathbf{x}} & 0 \end{array}\right]. \tag{4.19}$$

Calculating $\mathbf{M}$ from **B** yields

$$\mathbf{M} = \left[\begin{array}{c|cc|c} 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline B_{m1x}B_{ym1\bullet\mathbf{x}} + B_{m2x}B_{ym2\bullet\mathbf{x}} & 0 & 0 & 0 \end{array}\right], \tag{4.20}$$

where the fourth row, first column element represents to total indirect effect of *x* on *y*, which is the sum of the specific indirect effects through m_1 ($B_{m1x}B_{ym1\bullet\mathbf{x}}$) and m_2 ($B_{m2x}B_{ym2\bullet\mathbf{x}}$). Also, for this example, $\mathbf{\Psi}$ consists of the variance *x*, and residual variances and covariances of m_1, m_2, and *y*

$$\mathbf{\Psi} = \begin{bmatrix} \sigma_x^2 & & & \\ 0 & \zeta_{m1}^2 & & \\ 0 & \zeta_{m1m2} & \zeta_{m2}^2 & \\ 0 & 0 & 0 & \zeta_y^2 \end{bmatrix}, \tag{4.21}$$

and **D** is a diagonal matrix of standard deviations (Jöreskog & Sörbom, 2015)

$$\mathbf{D} = \begin{bmatrix} \sigma_x & & & \\ 0 & \sigma_{m1} & & \\ 0 & 0 & \sigma_{m2} & \\ 0 & 0 & 0 & \sigma_y \end{bmatrix}. \tag{4.22}$$

Pre- and post-multiplying $\mathbf{\Psi}$ by $\mathbf{D}^{-1/2}\mathbf{M}$ and $\mathbf{M}'\mathbf{D}^{-1/2}$ yields

$$
\Upsilon = \begin{bmatrix} \sigma_x^{-1/2} & & & \\ 0 & \sigma_{m1}^{-1/2} & & \\ 0 & 0 & \sigma_{m2}^{-1/2} & \\ 0 & 0 & 0 & \sigma_y^{-1/2} \end{bmatrix} \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline B_{m1x}B_{ym1\bullet\mathbf{x}} + B_{m2x}B_{ym2\bullet\mathbf{x}} & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \zeta_x^2 & & & \\ 0 & \zeta_{m1}^2 & & \\ 0 & \zeta_{m1m2} & \zeta_{m2}^2 & \\ 0 & 0 & 0 & \zeta_y^2 \end{bmatrix} \times
$$

$$
\left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline B_{m1x}B_{ym1\bullet\mathbf{x}} + B_{m2x}B_{ym2\bullet\mathbf{x}} & 0 & 0 & 0 \end{array}\right]' \begin{bmatrix} \sigma_x^{-1/2} & & & \\ 0 & \sigma_{m1}^{-1/2} & & \\ 0 & 0 & \sigma_{m2}^{-1/2} & \\ 0 & 0 & 0 & \sigma_y^{-1/2} \end{bmatrix}
$$

$$
= \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & (\beta_{m1x}\beta_{ym1\bullet\mathbf{x}} + \beta_{m2x}\beta_{ym2\bullet\mathbf{x}})^2 \end{array}\right].
$$

(4.23)

Υ quantifies the expected variance by *x* explained in *y indirectly* through m_1 and m_2. It is also possible to follow the matrix modification method in 4.13 – 4.17 to compute Υ from $\mathbf{M}^*$ for either m_1 ($\beta_{m1x}^2\beta_{ym1\bullet\mathbf{x}}^2$) or m_2 ($\beta_{m2x}^2\beta_{ym2\bullet\mathbf{x}}^2$). It should be noted that when obtaining an effect size for a specific indirect effect, for example, through m_1, it is not equivalent to the effect size of *x* on *y* through m_1 estimated without m_2 in the model. Omitting m_2 from **B** eliminates elements corresponding to m_2, but coefficients are estimated as from the full model. The implication is that the interpretation of υ for the specific indirect effect through m_1 is the variance explained indirectly by *x*, controlling for m_2, or the expected variance in *y* explained by *x* through m_1 that is constant across levels or subpopulations defined by m_2.

4.2.2 Properties of Υ

Like υ for three-variable models, Υ has several desirable properties as an effect size measure for complex mediation models. First, Υ is interpretable as the variance explained in the outcome by a set of predictors through a set of mediators. In addition, as described in Lachowicz et al. (2018), Cohen's benchmarks for R^2 measures (i.e., .02 = small, .15 = medium, .25 = large; Cohen, 1988) can be applied to υ, providing researchers with researchers some general heuristics to help communicate the magnitude of their effects of interest. Second, Υ is standardized, so it is not dependent on linear transformations of the variables. Third, the population Υ is not dependent on sample size, meaning it can be estimated consistently from sample data. Third, although the sampling distribution of Υ is not known, CIs can be constructed using, for example, a bootstrap or Monte Carlo based approach. Fourth, Υ is a monotonic function of the standardized indirect effect, so there is a one-to-one mapping of effect magnitude to effect size (in absolute value). Taken together, the properties of Υ make it a promising method for meaningfully quantifying the magnitude of indirect effects in a wide array of mediation models.

4.3 Sample Estimator of Υ

To this point I have considered the properties of Υ in the population. In order for Υ to be useful as an effect size measure, it will need a sample estimator with good statistical properties. Lachowicz et al. (2018) showed that, like many other estimators of variance parameters, the sample analog estimator $\hat{\upsilon}$ of the population υ for three-variable mediation models is positively biased, where the magnitude of bias is larger for small samples and small effect magnitudes. Because $\hat{\upsilon}$ is a special case of the more general $\hat{\Upsilon}$ and can be expressed as a quadratic function of MLEs, it is expected that $\hat{\Upsilon}$ is also a biased estimator of the population Υ.

4.3.1 Bias adjusted estimator $\tilde{\Upsilon}$

The preceding derivation shows a general form for the bias of Υ estimates. It is important to note, however, that this is the asymptotic bias in the expected value. In order to incorporate this bias into an adjusted estimator, appropriate sample estimates must be substituted for the above parameters.

The bias-reduction approach in Chapter 3 is a general solution for obtaining the appropriate sample estimates for the bias adjusted estimator. Although adjustment for first-order bias is sufficient for many of the effect size estimators considered in previous chapters, the complexity of indirect effects introduces additional sources of bias that are not appropriately accounted for using first-order methods. To illustrate, recall the bias in the expected value of $\hat{\upsilon}$ derived for a three-variable mediation model ($\beta_{mx}^2\sigma_{ym\bullet x}^2+\beta_{ym\bullet x}^2\sigma_{mx}^2+\sigma_{mx}^2\sigma_{ym\bullet x}^2$; Section 2.4.5) is equivalent to the second-order normal-theory approximation of the asymptotic variance for the standardized indirect effect $\hat{\beta}_{mx}\hat{\beta}_{ym}$ (Equation 2.30; MacKinnon et al., 1995). A first-order approximation of the bias term is $\beta_{mx}^2\sigma_{ym\bullet x}^2+\beta_{ym\bullet x}^2\sigma_{mx}^2$ (Equation 2.29; Bollen, 1987; Sobel, 1982), which omits the variance product $\sigma_{mx}^2\sigma_{ym\bullet x}^2$ and, therefore, consistently underestimates bias and overestimate effect sizes.

Also recall that simple substitution of sample estimates for the parameters in the bias term also introduces bias. This is clear by deriving the expected value of a hypothetical adjusted estimator $\tilde{\upsilon}*$ where the bias term is a simple substitution of parameters for estimates

$$\begin{aligned}E[\tilde{\upsilon}*]&=E[\hat{\beta}_{mx}^2\hat{\beta}_{ym\bullet x}^2]-E[\hat{\beta}_{mx}^2\hat{\sigma}_{ym\bullet x}^2]-E[\hat{\beta}_{ym\bullet x}^2\hat{\sigma}_{mx}^2]-E[\hat{\sigma}_{mx}^2\hat{\sigma}_{ym\bullet x}^2]\\&=(\beta_{mx}^2+\sigma_{mx}^2)(\beta_{ym\bullet x}^2+\sigma_{ym\bullet x}^2)-(\beta_{mx}^2+\sigma_{mx}^2)\sigma_{ym\bullet x}^2-(\beta_{ym\bullet x}^2+\sigma_{ym\bullet x}^2)\sigma_{mx}^2-\sigma_{mx}^2\sigma_{ym\bullet x}^2\\&=\beta_{mx}^2\beta_{ym\bullet x}^2+\beta_{mx}^2\sigma_{ym\bullet x}^2+\beta_{ym\bullet x}^2\sigma_{mx}^2+\sigma_{mx}^2\sigma_{ym\bullet x}^2-\beta_{mx}^2\sigma_{ym\bullet x}^2-\sigma_{mx}^2\sigma_{ym\bullet x}^2-\beta_{ym\bullet x}^2\sigma_{mx}^2-\sigma_{ym\bullet x}^2\sigma_{mx}^2-\sigma_{mx}^2\sigma_{ym\bullet x}^2\\&=\beta_{mx}^2\beta_{ym\bullet x}^2-2\sigma_{mx}^2\sigma_{ym\bullet x}^2.\end{aligned}$$

The additional variance product term $-2\sigma^2_{mx}\sigma^2_{ym\bullet x}$ means that simple substitution yields an estimator that overestimates bias and underestimates effect size.

These issues can be addressed simultaneously by modifying the sample estimated bias approximation. Whereas in the previous example the entire sample estimated second-order variance approximation was subtracted from the biased estimator, the modified estimate subtracts the first-order component of the variance approximation and adds the second-order component.

$$\tilde{\boldsymbol{\Upsilon}} = \hat{\mathbf{M}}^{st}\hat{\boldsymbol{\Psi}}^{st}\hat{\mathbf{M}}'^{st} - \hat{\boldsymbol{\Delta}}'_{bias}, \tag{4.24}$$

Where $\hat{\boldsymbol{\Delta}}'_{bias}$ is a $m \times m$ matrix with elements $(1/2)tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}\right\} - (1/4)tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}_1\right\}$ corresponding to outcome variables. To illustrate, I will derive the asymptotic bias for the three-variable mediation model above, showing that it yields equivalent results. Solving for the asymptotic bias yields the matrices

$$\begin{aligned}
\boldsymbol{\Sigma}(\boldsymbol{\beta}) &= \begin{bmatrix} \sigma^2_{mx} & 0 \\ 0 & \sigma^2_{ym\bullet x} \end{bmatrix}, \\
\mathbf{H} &= \begin{bmatrix} 2\beta^2_{ym\bullet x} & 4\beta_{mx}\beta_{ym\bullet x} \\ 4\beta_{mx}\beta_{ym\bullet x} & 2\beta^2_{mx} \end{bmatrix}, \\
\mathbf{H}_1 &= \begin{bmatrix} 2\sigma^2_{mx} & 0 \\ 0 & 2\sigma^2_{ym\bullet x} \end{bmatrix},
\end{aligned} \tag{4.25}$$

where the first bias approximation is

$$\frac{1}{2}tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}\right\} = \beta^2_{ym\bullet x}\sigma^2_{mx} + \beta^2_{mx}\sigma^2_{ym\bullet x}, \tag{4.26}$$

and the approximation to this component is

$$\frac{1}{2} tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}_1\right\} = 2\sigma_{mx}^2 \sigma_{ym\bullet x}^2. \tag{4.27}$$

Substituting these results into $\boldsymbol{\Delta}'_{bias}$ yields the appropriate asymptotic bias approximation

$$\begin{aligned} \Delta'_{yy} &= \frac{1}{2} tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}\right\} - \frac{1}{4} tr\left\{\boldsymbol{\Sigma}(\hat{\boldsymbol{\beta}})\mathbf{H}_1\right\} \\ &= \beta_{ym\bullet x}^2 \sigma_{mx}^2 + \beta_{mx}^2 \sigma_{ym\bullet x}^2 - \sigma_{mx}^2 \sigma_{ym\bullet x}^2. \end{aligned} \tag{4.28}$$

The second bias approximation in Equation 4.28 is multiplied by ½ because the first approximation does not include approximations of second-order terms (i.e., $\sigma_{mx}^2 \sigma_{ym\bullet x}^2$), so adjusting by Equation 4.27 yields an over-correction.

Although unbiasedness is a desirable property for estimators, it is possible for other important properties of these estimators to be deficient, such that a biased estimator of the same parameter is more useful in practice. Particularly notable for estimators are consistency (i.e., the estimator converges to the parameter as sample size increases), and variability. For example, an unbiased estimator with high sampling variability can be substantially less useful than a biased estimator that is more efficient.

4.4 Simulation Study

The purpose of the present simulation study is to examine the sampling properties of the unadjusted and bias-adjusted effect size estimators $\hat{\upsilon}$ and $\tilde{\upsilon}$ in complex mediation models. Of interest is determining whether the matrix generalization and bias-adjustment yield estimators with good statistical properties in terms of bias, variance, and overall accuracy, and whether common interval estimation methods return proper CIs for the estimators. It is of particular importance for researchers to know if there are certain conditions under which the estimators would not be expected to yield accurate estimates.

4.4.1 Simulation Design

The generating model for this simulation was a parallel mediation model with a single predictor *x*, a single outcome *y*, and two mediators m_1 and m_2 (Figure 2). Variables in this simulation were considered standard normal in the population. As described in Section 4.2.1, effect sizes can be estimated for three indirect effects: 1) the specific indirect effect of *x* on *y* through m_1 ($\beta_{m1x}\beta_{ym1 \bullet \mathbf{x}}$), 2) the specific indirect effect of *x* on *y* through m_2 ($\beta_{m2x}\beta_{ym2 \bullet \mathbf{x}}$), and 3) and the total indirect effect of *x* on *y* through m_1 and m_2 ($\beta_{m1x}\beta_{ym1 \bullet \mathbf{x}} + \beta_{m2x}\beta_{ym2 \bullet \mathbf{x}}$). Because of the symmetry in the magnitudes of the specific effects, effect sizes for this simulation will be evaluated for the total indirect and one specific indirect effect, $\beta_{m1x}\beta_{ym1 \bullet \mathbf{x}}$.

4.4.2 Simulation Conditions

Parameter values for the paths were varied among .15, .39, and .59, magnitudes for small, medium, and large standardized coefficients common in applied research. Because of the large number of parameter combinations possible in this model, some generating parameters were constrained to be equal ($\beta_{m2x} = \beta_{ym2 \bullet \mathbf{x}}$) and some were fixed to zero, including the direct effect and the residual correlation of the mediators. Parameter values consistent with the null hypothesis of no indirect effect were not considered in this simulation because zero is on the boundary of the parameter space for υ. In addition, the properties of estimators typically evaluated under the null hypothesis (i.e., Type I error rate, power) are not of interest for υ estimators because $\hat{\upsilon}$ and $\tilde{\upsilon}$ were not intended to be used for null hypothesis significance testing. Sample size was varied among 50, 100, 250, and 500. This yields a total of 3x3x3x4 = 108 total conditions.

Because the sampling distributions of $\hat{\upsilon}$ and $\tilde{\upsilon}$ are unknown and assumed to be non-normally distributed, nonparametric bootstrapping was used to construct 95% CIs. Two bootstrap CI methods were evaluated in this simulation: a) percentile and b) BCa (Section 2.3). Although percentile bootstrap CIs performed satisfactorily in terms of coverage and balance for υ estimates from a three-variable mediation model (Lachowicz et al., 2018), BCa CIs are often recommended for estimators with non-normal, heavy-tailed sampling distributions, which is characteristic of the distributions of both $\hat{\upsilon}$ and $\tilde{\upsilon}$. It is expected that BCa CIs will outperform percentile CIs in terms of coverage and balance, particularly in conditions with small effect magnitudes.

The simulation was conducted in R (version 3.4.1; R Core Team, 2017). 1,000 replications per condition is sufficient to obtain accurate estimates of bias for point estimates and coverage for CIs. For each replication, 1,000 bootstrap resamples are used to construct 95% CIs using the `boot` package (Canty & Ripley, 2017). The point estimators will be evaluated in terms of bias, overall accuracy, and relative efficiency, and CIs will be evaluated in terms of coverage, coverage balance, and CI width.

4.4.3 Evaluation criteria

Bias was evaluated using percent relative bias, defined as the difference between the expected value of the estimator and the population value, divided by the population value

$$bias_{rel}(\hat{\theta}) = \frac{E[\hat{\theta}] - \theta}{\theta}. \tag{4.29}$$

The metric of percent relative bias is often more interpretable than is raw bias, although it is possible for trivially small raw bias to appear quite large in terms of percent relative bias when the population parameter is close to zero. Researchers often use a criterion of 5% for acceptable relative bias (Boomsma, 2013), which I will also use in this study. It was hypothesized that the unadjusted estimator will exhibit positive bias, with the largest biases in conditions with smallest sample sizes and smallest effects. It was hypothesized that the adjusted estimator will exhibit acceptable bias (< 5%) in all conditions.

Accuracy was evaluated in terms of mean square error (MSE). MSE is defined as

$$\text{MSE} = E[bias(\hat{\theta})]^2 + \text{var}(\hat{\theta}). \tag{4.30}$$

It follows that for an unbiased estimator, MSE is equivalent to the estimator variance, and MSE will favor estimators with less variability. However, it possible there are circumstances where a biased estimator can have less variance than an unbiased estimator, such that the biased estimator returns more accurate estimates. To further examine the variability of the estimators, relative efficiency (*RE*) was evaluated by the ratio of the empirical sampling variances

$$RE = \frac{\text{var}(\hat{\theta}_1)}{\text{var}(\hat{\theta}_2)}. \tag{4.31}$$

$RE < 1$ corresponds to the sampling variance of $\hat{\theta}_2 > \hat{\theta}_1$, $RE > 1$ to the sampling variance of $\hat{\theta}_2 < \hat{\theta}_1$, and $RE = 1$ to equal sampling variances. *RE* was defined as the ratio of the sampling variance of $\tilde{\upsilon}$ to $\hat{\upsilon}$ for this simulation. It is not clear if there are conditions where the unadjusted $\hat{\upsilon}$ would be more accurate or have greater sampling variability than the bias-adjusted $\tilde{\upsilon}$, so these questions will be addressed empirically.

CIs were evaluated using average CI width, coverage, and balance of coverage. Average CI width was used to evaluate the precision of the estimates, where smaller widths represent estimates with greater precision. It was unclear which estimator will have smaller average CI widths; this will be evaluated empirically. Coverage is defined as the proportion of CIs that contain the true population value. It is expected that the coverage probabilities of proper CIs are equal to one minus the nominal alpha level. The nominal alpha level for CIs in this study is .05, and coverage will be evaluated according to Bradley's criteria (.925 - .975; Bradley, 1978). It is expected that 95% CIs will have coverage probabilities acceptably close to the nominal .95 level. In addition to achieving nominal coverage, it is assumed for proper CIs that the proportion of times the population value is greater than the upper CI limit and less than the lower CI limit are equal (i.e., 2.5% for 95% CIs). However, it is possible for CIs to achieve the nominal alpha and be imbalanced in the proportion of misses above and below the confidence limits, which results in biased estimates of Type 1 error rates and power. It is expected that the proportion of misses above and below the 95% CI limits are equal.

Finally, it is unclear how or if the sampling properties of the estimators of the effect size for the total indirect effect would differ as compared to the specific indirect effect, so this question will be addressed empirically.

4.5 Simulation Results

4.5.1 Bias

Simulation results for the relative bias of $\hat{\upsilon}$ for the total indirect effect can be found in Table 1, where shaded cells highlight relative bias > 5%. The hypothesis that $\hat{\upsilon}$ would be positively biased, with larger magnitudes of bias at smaller N and for smaller effect sizes, and decreasing magnitudes as N and effect magnitude increased, was supported by the simulation results. The direction of bias for $\hat{\upsilon}$ was positive in all conditions, consistent with analytic results. The largest values of percent relative bias (138.61%, 93.57%, and 97.63%) occurred at the smallest *N* considered in the simulation (*N* = 50), and also for the smallest effects ($\beta = .15$). In addition, for smaller effect sizes, percent relative bias > 5% was evident even at the largest *N* considered (*N* = 500) for the smallest effects. Increasing *N* was associated with decreasing bias, supporting the hypothesis that $\hat{\upsilon}$ is a consistent estimator. Finally, bias of $\hat{\upsilon}$ was negligible for large effect magnitudes of the total indirect effect for all sample sizes.

Simulation results for percent relative bias of $\tilde{\upsilon}$ for the total indirect effect can also be found in Table 1. The hypothesis that bias of $\tilde{\upsilon}$ would be negligible across simulation conditions was largely supported by simulation results. Overall, percent relative biases for $\tilde{\upsilon}$ were of much smaller magnitude than for $\hat{\upsilon}$. For the conditions in which bias was greatest for the $\hat{\upsilon}$, the relative biases of $\tilde{\upsilon}$ were –0.23%, 2.67%, and –1.48%. In total, the largest relative bias across all *N* and effect sizes considered for $\tilde{\upsilon}$ was –7.29% at *N* = 50, and only two other simulation conditions had relative bias > 5% (–5.03%, and –5.95%,). No parameter combination had relative bias > 5% at *N* = 250 and *N* = 500. Although bias was mostly of negligible magnitude at smaller sample sizes, bias tended to be in the negative direction, a tendency that also decreases with increasing sample size. Finally, as with $\hat{\upsilon}$, bias decreased as *N* increased, supporting the hypothesis that $\tilde{\upsilon}$ is a consistent estimator.

Results for relative bias of effect size estimators of the specific indirect effect $\beta_{m1x}\beta_{ym1\bullet\mathbf{x}}$ can be found in Table 2. Findings were generally similar to those for the total indirect effect. For both estimators, hypotheses regarding the magnitude and direction for the specific indirect effect were supported. For $\hat{\upsilon}$, the largest values of relative bias (265.84%, 223.31%, and 234.11%) occurred at the smallest *N* and for the smallest effects. In addition, bias was non-negligible for the smallest effect magnitude even at the largest sample size. For $\tilde{\upsilon}$, relative bias was also non-negligible for the smallest sample size and smallest effects, although of substantially smaller magnitudes than $\hat{\upsilon}$ (-9.14%, -5.35%, and 18.61%). Similarly, the remaining relative biases of $\tilde{\upsilon}$ considered non-negligible were also small magnitude (–10.59%, –5.42%, 5.48%, –6.65%, and 5.51%). Finally, increasing *N* was associated with decreasing bias for both estimators.

4.5.2 Accuracy and Relative Efficiency

Simulation results regarding MSE and *RE* for effect size estimators of the total indirect effect can be found in Table 3, and for the specific indirect effect in Table 4. Shaded cells highlight conditions where MSE of $\tilde{\upsilon}$ was greater than $\hat{\upsilon}$, and where $RE > 1$ (i.e., variance of $\tilde{\upsilon} > \hat{\upsilon}$). Increasing *N* was associated with decreasing MSE for both estimators of total and specific indirect effects, supporting the hypothesis that overall accuracy of the measures would increase with increasing *N*. It was also clear, for both effects, that outside of a few conditions, $\tilde{\upsilon}$ was a more accurate estimator of υ than $\hat{\upsilon}$. In addition, it was clear that across the vast majority of conditions $\tilde{\upsilon}$ was a more efficient estimator. Finally, the magnitudes of the accuracy and efficiency discrepancies between the estimators were dependent on sample size and effect magnitudes, such that differences were largest for the smallest sample sizes and smallest effects.

4.5.3 Confidence Intervals

Results for 95% percentile bootstrap CIs of $\hat{\upsilon}$ and $\tilde{\upsilon}$ for the total indirect effect can be found in Tables 5 and 6, respectively, and for the specific indirect effect in Tables 7 and 8, respectively. Shaded cells highlight conditions where satisfactory coverage (92.5% – 97.5%) was not achieved. The hypotheses that coverage would reach the nominal 95% level as *N* increased, and that the proportions of misses to the left and right of the 95% CI would be balanced, were supported by simulation results. As with bias, satisfactory coverage was achieved with larger effect sizes and at larger *N* for both estimators and effects, such that satisfactory coverage was achieved for all parameter combinations at $N = 250$. In addition, misses to the left and right of the 95% CI were approximately balanced at $N = 250$ for both estimators and effects. When satisfactory coverage was not achieved, it was predominantly due to coverage > 97.5%, or CIs being too wide.

Results for 95% BCa bootstrap CIs of estimators for the total indirect effect can be found in Tables 9 and 10, respectively, and for the specific effect in Tables 11 and 12, respectively. The hypotheses that that nominal coverage would converge to a satisfactory level, and that the proportions of misses to the left and right would achieve balance, were generally supported by the results as well. Similar to the percentile CIs, satisfactory coverage was generally achieved for larger *N* and larger effect magnitudes, and achieved for all parameter combinations at $N = 500$ both estimators and effects. However, there are noteworthy differences between the results for the CI methods. In contrast to the percentile CIs, when satisfactory coverage was not achieved, it was predominantly due to coverage < 92.5%, or *CIs being too narrow*. In addition, for small sample sizes and small effect magnitudes, there tended to be larger deviations from nominal coverage (~80-90%) for total indirect effects, and even greater deviations for specific indirect effects (~70-80%). Another noteworthy difference between the methods is that, whereas percentile CIs for the total and specific indirect effects achieved satisfactory coverage at the same sample size ($N = 250$), a larger sample size was required for BCa CIs to achieve satisfactory coverage for the specific indirect ($N = 500$) than for the total indirect effect ($N = 250$).

4.6 Empirical Example

I now present an empirical example to facilitate interpretation and implementation of υ in a complex mediation model. I use the results from a study conducted by Li, Starr, and Wake (2018) that investigated the pathways through which anxiety can have downstream effects on depression. Specifically, the authors used the National Longitudinal Study of Adolescent to Adult Health (ADD Health; Harris et al., 2009) dataset to examine the indirect effects of anxiety symptoms assessed at Wave 1 (1994-1995) on depressive symptoms at Wave 4 (2008) through insomnia and unrestful sleep assessed at Wave 2 (1996). Analyses were conducted with those participants that had observations on these variables at each study wave (N = 3910). The predictor variable of anxiety symptoms at Wave 1 were assessed using a composite of six items measuring anxious physiological arousal, where higher values indicate greater symptoms of anxiety. The mediator variables of insomnia and unrestful sleep at Wave 2 were assessed using single items asking about difficulty falling asleep and staying asleep and feeling tired upon waking, respectively, where higher values indicate greater sleep problems. The outcome variable of depressive symptoms at Wave 4 was assessed using the Center for Epidemiologic Studies Depression Scale (CES-D), where higher values indicated greater depressive symptomology.

Results presented here differ slightly form the original study results because relevant control variables were excluded to simplify analyses. The total standardized indirect effect of anxiety symptoms on depressive symptoms through both insomnia and unrestful sleep was 0.038 (95% percentile CI = .027, .052), the standardized specific indirect effect through insomnia was 0.021 (95% percentile CI = .009, .033), and the standardized specific indirect effect through insomnia was 0.018 (95% percentile CI = .01, .026).

For the υ estimators, the $\hat{\upsilon}$ effect size estimate for the total indirect effect was 0.0015 (95% percentile bootstrap CI = .0007, .0027), meaning that the variance explained indirectly in depressive symptoms by anxiety symptoms through insomnia and unrestful sleep in this sample was 0.0015. The $\tilde{\upsilon}$ effect size estimate for the total indirect effect was 0.014 (95% percentile bootstrap CI = 0.0007, 0.0027). $\tilde{\upsilon}$ is interpreted as the estimated variance in depressive symptoms by anxiety symptoms through insomnia and unrestful sleep in the population. The $\hat{\upsilon}$ effect size estimate for the specific indirect effects through insomnia and unrestful sleep were 0.0004 (95% percentile bootstrap CI = .00008, .000106) and 0.0003 (95% percentile bootstrap CI = .0001, .0007). This means that the variance explained indirectly in depressive symptoms by anxiety symptoms in this sample through insomnia was 0.0004, and through unrestful sleep was 0.0003. The $\tilde{\upsilon}$ effect size estimate for the specific indirect effects through insomnia and unrestful sleep were 0.0004 (95% percentile bootstrap CI = 0.00006, 0. 000103) and 0.0003 (95% percentile bootstrap CI = 0.00009, 0.0006), respectively. Like the bias-adjusted effect size for the total indirect effect, $\tilde{\upsilon}$ for the specific indirect effects is interpreted as the estimated variance in depressive symptoms explained by anxiety symptoms in the population separately by via insomnia and unrestful sleep.

4.7 Summary

In Chapter 4, the SEM framework described in Chapter 3 was used to generalize the effect size measure υ. A general form of the bias was also derived using the general bias reduction strategy from Chapter 3, and a bias-adjusted estimator of Υ ($\tilde{\Upsilon}$) was proposed. The sampling properties of the unadjusted estimator $\hat{\Upsilon}$ and $\tilde{\Upsilon}$ were evaluated via Monte Carlo simulation. Chapter 5 will review moderated MLR, and apply the results of Chapter 2 to investigate explained variance for moderated regression models. Chapter 6 will combine the findings of Chapters 4 and 5 to extend Υ to moderated mediation models.

5 EXPLAINED VARIANCE IN MODERATED MLR

5.1 Moderated MLR

Moderated, or conditional, indirect effects are an increasingly popular type of complex mediation model in the social sciences. However, measures of explained variance for moderated effects in general have received little attention in the methodological literature. This section will review the literature for these effects in ANOVA and MLR, noting gaps in terms of the themes of standardized effect sizes previously described (e.g., generalizability, biasedness of estimators). The goal of this section is to establish a coherent framework for explained variance for moderated effects in MLR that will serve as a foundation for extending υ to moderated mediation models. An empirical demonstration of the conditional effect size will be provided using the running empirical example, and R software code will be provided.

It is a common in the social sciences to hypothesize that the effect of one variable on another varies across identifiable populations. For example, it is possible that the effect of an early childhood intervention designed to improve reading is different for boys than it is for girls, or for children from lower SES neighborhoods than from higher SES neighborhoods. A more complete understanding of how effects vary in direction and magnitude can have important consequences for the reporting of study results.

In traditional MLR, the partial effect of a variable on an outcome is assumed to be constant across levels of all other variables in the regression model, which precludes investigating conditional effects. However, moderation hypotheses can be investigated in MLR by incorporating additional variables that are products of other variables in the model, where effects of such product terms are often referred to as *interactions*. The unstandardized effect of a variable x_1 on y conditional on levels of a moderating variable x_2 is expressed as

$$y = B_0 + B_1 x_1 + B_2 x_2 + B_3 x_1 x_2 + \varepsilon, \tag{5.1}$$

where B_3 is the partial effect of the interaction of x_1 and x_2 on *y* controlling for x_1 and x_2. Because the effect is nonlinear (the expected value of *y* changes for different values of the predictors), it is assumed the assumptions required for additive regression models (i.e., normality of errors, linearity, homoscedasticity, existence, independence of errors) apply across all combinations of predictor values. The significance of the interaction is determined by testing the significance of B_3, or testing the significance of the increment in R^2 due to including the interaction term in the model (Cohen et al., 2003).

If the effect of x_1 on *y* is of particular interest, x_1 would be considered the focal predictor, and x_2 the moderator variable. Equation 5.1 can be rearranged to more closely resemble this distinction as

$$y = B_0 + (B_1 + B_3 x_2) x_1 + B_2 x_2 + \varepsilon. \tag{5.2}$$

The effect of the focal variable x_1 can now be said to vary across levels x_2. The magnitude of the interaction term B_3 is then the difference in the effect of x_1 on *y* corresponding to a one unit increase in x_2. For example, if x_2 is binary, B_3 is the difference in the effect of x_1 on *y* in one group relative to the effect in a reference group.

If the interaction coefficient is significantly different from zero, the moderation can be further examined by probing and plotting the effect of the focal predictor conditional on values of the moderator (Aiken & West, 1991). The effect of a predictor at a given level of a moderator (typically at the moderator mean, and ± 1 SD) is referred to as a simple slope. The simple slope also may be tested for significance using a conditional standard error. Interpretation of the moderator effects can be facilitated by plotting these simple slopes at various levels of the moderator. An alternative to testing simple slopes at fixed values of the moderator is to construct simultaneous CIs for the effect of the predictor across the range of moderator values (Johnson & Neyman, 1936). Regions where the CIs do not include zero are values of the moderator where the simple slope is significant.

5.1.1 Moderated MLR in LISCOMP

Although the moderated MLR in Equations 5.1 and 5.2 are instructive, it is desirable to express a moderated MLR model in matrix form as in Section 3.1. Despite the long history of methodological research on moderated MLR, the appropriate matrix representation of an interaction in MLR has not been addressed. At issue is how to specify the interaction term not only for notational convenience, but also to make use of the results derived in Section 3.1 regarding the properties of estimators.

An obvious approach would be to model the product term as a new predictor in the matrix specification, as is commonly done when estimating interaction effects in MLR. This specification would yield the correct parameter estimates for the models in Equations 5.1 and 5.2, and including the product variable in the variance/covariance matrix allows for computations of variances, covariances, and R^2. However, the specification also presents some issues. Whereas it is typical to standardize coefficients by scaling the coefficients by the ratio of the standard deviation of the predictor to the standard deviation of the outcome, this is not an appropriate standardization for the product term (Champoux & Peters, 1987; Muthén & Asparouhov, 2015; Wen, Marsh, & Hau, 2010). This could be avoided by computing the product term from standardized variables, a straightforward solution but requiring extra data management. A more problematic issue is that this specification includes a non-linear effect, which is not directly estimable in SEM software. (Kenny & Judd, 1984).

These issues can (in theory) be avoided by specifying the moderated MLR model in reduced form (Equation 5.2) as a SEM. The LISCOMP specification for latent interactions (Klein & Moosbrugger, 2000; Klein & Muthén, 2007), assuming variables are standardized, is expressed as

$$\boldsymbol{\eta}^{st} = \mathbf{B}^{st}\boldsymbol{\eta}^{st} + \mathbf{j}\boldsymbol{\eta}^{st\prime}\boldsymbol{\Omega}^{st}\boldsymbol{\eta}^{st} + \zeta, \tag{5.3}$$

where $\mathbf{j}$ is a $m \times 1$ vector designating the interaction outcome variable, $\boldsymbol{\Omega}^{st}$ is a square matrix of interaction coefficients

$$\boldsymbol{\Omega}^{st} = \begin{bmatrix} 0 & \omega_{1,2} & \cdots & \omega_{1,p} \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \omega_{p-1,p} \\ 0 & \cdots & \cdots & 0 \end{bmatrix}. \tag{5.4}$$

Solving for $\boldsymbol{\eta}^{st}$ and substituting into the measurement model yields interaction model for manifest variables

$$\mathbf{y} = (\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}^{st\prime}\boldsymbol{\Omega}^{st})^{-1}\zeta. \tag{5.5}$$

This means that the outcome is conditional on values of $\boldsymbol{\eta}^{st}$. This avoids the problem of standardization *when centered variables are used to create the product term* because the interaction term will be scaled by a product of standard deviations rather than the variance of the product (Champoux & Peters, 1987). However, if the product term is created from uncentered variables, the variance of the product is a complex function of variable means, variances, and covariances. In addition, it also follows that if the elements of $\boldsymbol{\eta}^{st}$ are centered at 0 and uncorrelated, Equation (5.5) reduces to $\mathbf{y} = (\mathbf{I} - \mathbf{B}^{st})^{-1}\boldsymbol{\zeta}$ (Chapter 3). The equivalent MLR expression is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{X}'\boldsymbol{\Omega}\mathbf{X} + \boldsymbol{\varepsilon}. \tag{5.6}$$

For a MLR model with two predictors and an interaction, the coefficient matrix $\mathbf{B}^{st}$ is

$$\mathbf{B}^{st} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_1 & \beta_2 & 0 \end{bmatrix}, \tag{5.7}$$

and $\boldsymbol{\Omega}^{st}$ is

$$\boldsymbol{\Omega}^{st} = \begin{bmatrix} 0 & \beta_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{5.8}$$

The placement of the β_3 coefficient in $\boldsymbol{\Omega}^{st}$ designates which variable is the focal predictor and which is the moderator. In Equation 5.8 the placement of β_3 designates x_2 as the focal predictor and x_1 as the moderator (i.e., the effect of x_2 varies across levels of x_1). To designate x_1 the focal predictor, the [2,1] element of $\boldsymbol{\Omega}^{st}$ would contain the interaction coefficient, with element [1,2] being 0.

In summary, the LISCOMP framework provides a flexible framework for representing moderated regression models in MLR. Like the alternative specification method where the interaction is modeled as a separate variable, LISCOMP returns the desired parameter estimates and is suitable for expression in quadratic form (Section 3.24). However, a clear advantage is that the specification exists within a more general modeling framework. Although uncentered product terms cannot be easily centered in this specification, it should be noted that it is often recommended by methodologists to center variables when conducting moderated regression analysis as centering can be used to aid in the interpretation of effects and remove non-essential multicollinearity among variables (Aiken & West., 1991).

5.2 Explained Variance in Moderated MLR

Explained variance for a moderated MLR model can be obtained using the SEM formula for $\mathbf{R}_{\eta}^{2}$ in Equation 3.21. Assuming variables are standardized, $\mathbf{R}_{\eta}^{2}$ is expressed in quadratic form as

$$\mathbf{R}_{\eta}^{2} = [(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]\boldsymbol{\Psi}^{st}[(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]'. \tag{5.9}$$

For the moderated MLR model with two variables and their interaction, Equation 5.9 yields

$$\begin{aligned}\mathbf{R}_{\eta}^{2} &= [(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]\boldsymbol{\Psi}^{st}[(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]' \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_1 & \beta_2 + \beta_3\eta_{x1} & 0 \end{bmatrix} \begin{bmatrix} 1 & & \\ \zeta_{x1,x2} & 1 & \\ 0 & 0 & \zeta_{y}^{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 + \beta_3\eta_{x1} \\ 0 & 0 & 0 \end{bmatrix}.\end{aligned} \tag{5.10}$$

Before continuing with the example, it is important to note a complication that arises due to the presence of η_{x1} in $\mathbf{B}^{st}$. Because it was not assumed that η_{x1} was fixed to a specific value as in the analysis of simple slopes, the computation of $\mathbf{R}_{\eta}^{2}$ requires taking the product of the random variables η_{x1} with ζ_{x1} and ζ_{x2} (Muthén & Asparouhov, 2015). For clarity, I will continue the example substituting the SEM estimates with the OLS counterparts

$$\mathbf{R}_\eta^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_1 & \beta_2 + \beta_3\mathbf{x}_1' & 0 \end{bmatrix} \begin{bmatrix} (N-1)^{-1}\mathbf{x}_1'\mathbf{x}_1 & & \\ (N-1)^{-1}\mathbf{x}_1'\mathbf{x}_2 & (N-1)^{-1}\mathbf{x}_2'\mathbf{x}_2 & \\ 0 & 0 & \sigma_\varepsilon^2 \end{bmatrix} \begin{bmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 + \beta_3\mathbf{x}_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= (N-1)^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_1\mathbf{x}_1'\mathbf{x}_1 + (\beta_2 + \beta_3\mathbf{x}_1')\mathbf{x}_2'\mathbf{x}_1 & \beta_1\mathbf{x}_1'\mathbf{x}_2 + (\beta_2 + \beta_3\mathbf{x}_1')\mathbf{x}_2'\mathbf{x}_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 + \beta_3\mathbf{x}_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= (N-1)^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \begin{matrix} \beta_1^2\mathbf{x}_1'\mathbf{x}_1 + \beta_1(\beta_2 + \beta_3\mathbf{x}_1')\mathbf{x}_2'\mathbf{x}_1 + \\ (\beta_1\mathbf{x}_1'\mathbf{x}_2 + \beta_2\mathbf{x}_2'\mathbf{x}_2 + \beta_3\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_2)(\beta_2 + \beta_3\mathbf{x}_1) \end{matrix} \end{bmatrix}.$$

$$= (N-1)^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \begin{matrix} \beta_1^2\mathbf{x}_1'\mathbf{x}_1 + \beta_2^2\mathbf{x}_2'\mathbf{x}_2 + 2\beta_1\beta_2\mathbf{x}_2'\mathbf{x}_1 + \\ 2\beta_1\beta_3\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_1 + 2\beta_2\beta_3\mathbf{x}_2'\mathbf{x}_2'\mathbf{x}_1 + \beta_3^2\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_2\mathbf{x}_1 \end{matrix} \end{bmatrix}.$$

(5.11)

Distributing $(N-1)^{-1}$ shows the first three terms are the familiar coefficients β_1^2, β_2^2, and $2\beta_1\beta_2 r_{x1,x2}$. However, the second three terms contain element-wise, or *Hadamard*, products of the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ (Gentle, 2017; Searle, 1982). The result is that product variables are created in order to obtain the variance of the interaction variable ($\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_2\mathbf{x}_1 = (N-1)^{-1}\sigma_{x1x2}^2$), and covariances between the interaction variable and x_1 ($\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_1 = (N-1)^{-1}\sigma_{x1x2,x1}$) and x_2 ($\mathbf{x}_2'\mathbf{x}_2'\mathbf{x}_1 = (N-1)^{-1}\sigma_{x2x2,x1}$).[4] This yields the R^2 formula

$$R_\eta^2 = \beta_1^2 + \beta_2^2 + \beta_3^2\sigma_{x1x2}^2 + 2\beta_1\beta_2 r_{x1,x2} + 2\beta_1\beta_3\sigma_{x1x2,x1} + 2\beta_2\beta_3\sigma_{x1x2,x2}, \tag{5.12}$$

[4] Bohrnstedt and Goldberger (1969) showed that variables are mean-centered $\sigma_{x2x2,x1} = \sigma_{x1x2,x2}$.

which is the expected R^2 if the regression had been conducted with an interaction variable created from standardized x_1 and x_2. The variances and covariances of the interaction term are estimated in the LISCOMP framework not by explicitly creating product variables, but modeling the non-normal distribution of the product variable as a mixture of normal distributions and estimating variances and covariances via maximum likelihood (Klein & Moosbrugger, 2000).

5.2.1 Explained variance for conditional effects

The R_η^2 formulation in Equation 5.12 quantities the total variance explained by a model with an interaction term. However, as described in the previous section, the moderator was not fixed to specific values as is often done in the analysis of simple slopes (Aiken & West, 1991). Given the importance of the plotting and probing in the interpretation of conditional effects, it would be of interest for R_η^2 to also be applicable to these conditional effects.

Returning to the moderated MLR in Equation 5.2 where x_1 is considered the focal predictor and x_2 moderator, the expected value of *y* can be expressed in terms of x_1 where x_2 is fixed at a specific level *c* as

$$E[y \mid x_1, x_2 = c] = B_0 + (B_1 + B_3 c)x_1 + B_2 c. \tag{5.13}$$

If variables are standardized as in Section 5.2, the commonly used conditional levels of the moderator (i.e., mean, one standard deviation above and below the mean) reduce Equation 5.13 to three simple equations

$$\begin{aligned} &E[y^* \mid x_1^*, x_2^* = 0] = \beta_1 x_1^*, \\ &E[y^* \mid x_1^*, x_2^* = -1] = (\beta_1 - \beta_3)x_1^* - \beta_2, \\ &E[y^* \mid x_1^*, x_2^* = 1] = (\beta_1 + \beta_3)x_1^* + \beta_2, \end{aligned} \tag{5.14}$$

where y^*, x_1^*, and x_2^* denote standardized y, x_1, and x_2. The variance of the predicted values of the standardized simple regressions in Equation 5.14 (i.e., mean *SSR*) can be obtained by application of the quadratic form in Section 3.1

$$
\begin{aligned}
&\mathrm{var}[y^* \mid x_1^*, x_2^* = 0] = \beta_1^2, \\
&\mathrm{var}[y^* \mid x_1^*, x_2^* = -1] = \beta_1^2 + \beta_3^2 \sigma^2_{x_1^* x_2^*} - 2\beta_1 \beta_3 \sigma_{x_1^* x_2^*, x_1^*}, \\
&\mathrm{var}[y^* \mid x_1^*, x_2^* = 1] = \beta_1^2 + \beta_3^2 \sigma^2_{x_1^* x_2^*} + 2\beta_1 \beta_3 \sigma_{x_1^* x_2^*, x_1^*},
\end{aligned}
\tag{5.15}
$$

where $\sigma^2_{x_1^* x_2^*}$ is the variance of the product of standardized x_1^*, and x_2^*, and $\sigma_{x_1^* x_2^*, x_1^*}$ the covariance of x_1^* and the standardized product term (Section 5.2). β_2 drops out of the variances in Equation 5.15 because x_2^* is constant and, therefore, has a variance of zero. The variance of the product term $\sigma^2_{x_1^* x_2^*}$ is included in Equation 5.15 because the variance of the product is not the product of the variances of the variables that comprise the interaction, meaning the variance of the product of standardized variables is not one (unless the covariance of the variables that comprise the interaction is exactly zero). This result suggests that, from the definition of R^2 in Section 3.1.2, the ratio of the conditional variance due to regression to the total conditional variance of the outcome (assumed to be one when *y* is standardized at the specific level of the moderator) yields a conditional version of R_η^2.

It should be noted that the conditional variances in Equation 5.15 represent a subset of the variance components of R_η^2 in Equation 5.12. This suggests a conditional form of R_η^2 in Equation 5.12 could be expressed as

$$
R_\eta^2 = \beta_1^2 + \beta_2^2 c_{x2}^2 + \beta_3^2 c_{x2}^2 \sigma^2_{x1x2} + 2\beta_1\beta_2 c_{x2} r_{x1,x2} + 2\beta_1\beta_3 c_{x2} \sigma_{x1x2,x1} + 2\beta_2\beta_3 c_{x2}^2 \sigma_{x1x2,x2}, \tag{5.16}
$$

where c_{x2} corresponds to the level at which the moderator variable x_2 is fixed. For Equation 5.15, c_{x1} is omitted and c_{x2} takes the values 0, 1, and -1. Because x_2 is fixed, it variable has zero variance and covariances, so the terms $\beta_2^2 c_{x2}^2$, $2\beta_1\beta_2 c_{x1} c_{x2} r_{x1,x2}$, and $2\beta_2\beta_3 c_{x1} c_{x2}^2 \sigma_{x1x2,x2}$ drop out, yielding the results in Equation 5.15.

5.2.2 Bias-adjusted R^2 estimator

The bias-correction approach implemented thus far is also appropriate for approximating the bias of the expected value of the moderated MLR $\hat{R}^2$, and constructing an improved estimator. The bias can be approximated using Equation 3.26, and the resulting adjusted estimator is

$$\mathbf{R}_\eta^2 = [(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]\boldsymbol{\Psi}^{st}[(\mathbf{I} - \mathbf{B}^{st} - \mathbf{j}\boldsymbol{\eta}'^{st}\boldsymbol{\Omega}^{st})'^{-1} - \mathbf{I}]' - \boldsymbol{\Delta}_{bias}, \tag{5.17}$$

where the elements of $\boldsymbol{\Delta}_{bias}$ corresponding to outcome variables contain $(1/2)\,tr\{\text{var}(\mathbf{B})\mathbf{H}\}$ and zero otherwise. For a moderated MLR model with two predictors x_1 and x_2 and their interaction, the estimated bias is

$$\begin{aligned}\frac{1}{2}tr\{\text{var}(\mathbf{B}^{st})\mathbf{H}\} &= \frac{1}{2}tr\left\{\begin{bmatrix}\sigma_{\beta1}^2 & & \\ \sigma_{\beta2,\beta1} & \sigma_{\beta2}^2 & \\ \sigma_{\beta3,\beta1} & \sigma_{\beta3,\beta2} & \sigma_{b3}^2\end{bmatrix}\begin{bmatrix}2 & 2r_{x1,x2} & 2\sigma_{x1x2,x1} \\ 2r_{x1,x2} & 2 & 2\sigma_{x1x2,x2} \\ 2\sigma_{x1x2,x1} & 2\sigma_{x1x2,x2} & 2\sigma_{x1x2}^2\end{bmatrix}\right\} \\ &= \sigma_{\beta1}^2 + \sigma_{\beta2}^2 + \sigma_{\beta3}^2\sigma_{x1x2}^2 + 2\sigma_{\beta2,\beta1}r_{x1,x2} + 2\sigma_{\beta3,\beta1}\sigma_{x1x2,x1} + 2\sigma_{\beta3,\beta2}\sigma_{x1x2,x2}.\end{aligned}$$

(5.18)

A second order correction is not necessary as the parameters in the bias term are linear. Equation 5.17 may also be applied to estimates of conditional R^2 in Section 5.2.1.

5.3 Summary

Chapter 5 reviewed the concept of moderation in MLR, and proposed a general method for expressed explained variance and bias adjustment for moderated MLR models. In Chapter 6, the general solutions for explained variance and bias adjustment are applied to extend the effect size to moderated mediation models.

6 EXTENDING υ TO MODERATED MEDIATION MODELS

6.1 Moderated Mediation

Researchers often theorize that the effect of a construct varies across populations with differing characteristics. In multiple regression analysis, this is commonly modeled by including an interaction term in the regression equation so the effect of variable, called a focal predictor, on the outcome can vary across levels of another variable, called a moderator. This modeling approach can be extended to mediation analysis where indirect effects can vary across levels of a moderator. This is referred to as moderated mediation analysis, or conditional process modeling (Edwards & Lambert, 2007; Hayes, 2013; Preacher, Rucker, & Hayes, 2007). Although several conflicting definitions of moderated mediation exist in the statistical literature (Preacher et al., 2007), moderated mediation is defined as occurring when indirect effects vary across levels of at least one moderator, regardless of which constituent path of the indirect effect is moderated. Indirect effects in a moderated mediation model are termed *conditional indirect effects*.

Preacher et al. (2007) illustrated five prototypical examples in which mediation effects can be moderated by a single covariate or by multiple covariates. In subsequent sections, effect sizes will be proposed for each of the cases of moderated mediation as examples of the general effect size framework. In the first case x is also a moderator of the relationship between m and y. The equation representing the effect of x on m is the same as in the simple three-variable model (Equation 2.27), and the equation for the effect of m and x on y where the effect of m is moderated by x is

$$y = B_0 + B_{yx\bullet\mathbf{x}}x + B_{ym\bullet\mathbf{x}}m + B_{yxm\bullet\mathbf{x}}xm + \varepsilon_y, \tag{6.1}$$

where $B_{yxm\bullet\mathbf{x}}$ is the effect of the interaction effect of *x* and *m* on *y*. The effect of the moderator on the indirect effect can be seen more clearly by re-expressing Equation 6.1 in reduced form as

$$y = B_0 + B_{yx\bullet\mathbf{x}}x + (B_{ym\bullet\mathbf{x}} + B_{yxm\bullet\mathbf{x}}x)m + \varepsilon_y, \tag{6.2}$$

Therefore, the conditional indirect effect for Case 1 is $B_{mx}(B_{ym\bullet\mathbf{x}} + B_{yxm\bullet\mathbf{x}}x)$.

Case 2 considered in Preacher et al. (2007) is where a covariate *w* is a moderator of the effect of *x* on *m*. Equation 2.26 is now expanded to include the covariate and the interaction term

$$m = B_0 + B_{mx\bullet\mathbf{x}}x + B_{mw\bullet\mathbf{x}}w + B_{mxw\bullet\mathbf{x}}xw + \varepsilon_m, \tag{6.3}$$

where $B_{mw\bullet\mathbf{x}}$ is the effect of the covariate *w* on *m* controlling for *x* and the interaction *xw*, and $B_{mxw\bullet\mathbf{x}}$ is the effect of the interaction of *x* and *w* on *y*. The model for the effects of *x* and *m* on *y* remain the same as in the simple three-variable model (Equation 2.27), so the conditional indirect effect for Case 2 is $(B_{mx\bullet\mathbf{x}} + B_{mxw\bullet\mathbf{x}}w)B_{ym\bullet x}$.

Case 3 is where *z* is a moderator of the effect of *m* on *y*. The equation relating *x* and *m* is the same as in Equation 2.26, and the equation for the effect of *m* on *y* as moderated by *z* controlling for *x* is

$$y = B_0 + B_{yx\bullet\mathbf{x}}x + B_{ym\bullet\mathbf{x}}m + B_{yz\bullet\mathbf{x}}z + B_{ymz\bullet\mathbf{x}}mz + \varepsilon_y, \tag{6.4}$$

where $B_{ymz\bullet\mathbf{x}}$ is the interaction effect of *m* and *z* on *y* controlling for *x*, *m*, and *z*. The effect of *m* on *y* controlling for *x* in Equation 6.4 is conditional on levels of *z*, so the conditional indirect effect is $B_{mx}(B_{ym\bullet\mathbf{x}} + B_{ymz\bullet\mathbf{x}}z)$.

Case 4 is where a covariate *w* moderates the effect of *x* on *m* and a covariate *z* moderates the effect of *m* on *y*. The equation relating *m* to *x* and *w* is the same is in Equation 6.3, and the equation relating y to m, x, and z is the same as in Equation 6.4. The conditional indirect effect is $(B_{mx\bullet\mathbf{x}} + B_{mxw\bullet\mathbf{x}}w)(B_{ym\bullet\mathbf{x}} + B_{ymz\bullet\mathbf{x}}z)$.

Case 5 considered is where w moderates the effect of x on m, as well as the effect m on y. The effect of x and w on m is the same as in Equation 6.4, and the effects of x, m, and w on y are

$$y = B_y + B_{yx\bullet\mathbf{x}}x + B_{ym\bullet\mathbf{x}}m + B_{yw\bullet\mathbf{x}}w + B_{ymw\bullet\mathbf{x}}mw + \varepsilon_y. \tag{6.5}$$

The conditional indirect effect for this model is $(B_{mx\bullet\mathbf{x}} + B_{mxw\bullet\mathbf{x}}w)(B_{ym\bullet\mathbf{x}} + B_{ymw\bullet\mathbf{x}}w)$.

Overall, the five cases presented in this section represent a small subset of the possible conditional indirect effects that may be examined in moderated mediation (Hayes, 2013). However, these cases are instructive for conveying the complexities introduced when applying the effect size υ.

6.2 υ for Moderated Mediation

υ can be obtained for each of the moderated mediation models previously presented by combining the moderated MLR findings of Chapter 5 with the general matrix framework in Chapter 4. Assuming variables are standardized, the LISCOMP expression for the total effects for a moderated mediation model (Chapter 5) from a matrix of conditional effects $\mathbf{B}^{st}_{MOD}$ ($\mathbf{B}^{st}_{MOD} = \mathbf{B}^{st} + \mathbf{j}\boldsymbol{\eta}^{st\prime}\boldsymbol{\Omega}^{st}$) is

$$\mathbf{T}^{st}_{MOD} = (\mathbf{I} - \mathbf{B}^{st}_{MOD})^{-1} - \mathbf{I}, \tag{6.6}$$

which is modified as in Chapter 4 to obtain the matrix of conditional indirect effects $\mathbf{M}^{st}_{MOD}$

$$\mathbf{M}^{st}_{MOD} = \mathbf{T}^{st}_{MOD} - \mathbf{B}^{st}_{MOD}. \tag{6.7}$$

The matrix of conditional indirect effects can then be substituted into Equation 4.17 to obtain a matrix of conditional effect sizes $\boldsymbol{\Upsilon}_{MOD}$

$$\begin{aligned}\boldsymbol{\Upsilon}_{MOD} &= \mathbf{D}^{-1/2}\mathbf{M}_{MOD}\boldsymbol{\Psi}\mathbf{M}'_{MOD}\mathbf{D}^{-1/2} \\ &= \mathbf{M}^{st}_{MOD}\boldsymbol{\Psi}^{st}\mathbf{M}^{st}_{MOD}{}',\end{aligned} \tag{6.8}$$

where $\mathbf{D}$ consists of the appropriate variances for standardizing interaction terms (Muthén & Asparouhov, 2015; Wen et al., 2010). It should be noted that, whereas the moderated R^2 formulas of Chapter 5 were presented for the total model R^2, the indirect effects for moderated mediation are conditional on specific values of the moderator, so the resulting effect sizes are also conditional on moderator values.

6.2.1 *Effect sizes for prototypical moderated mediation cases*

For clarity of presentation, models for the five prototypical cases of moderated mediation are considered to be for manifest rather than latent variables. For Case 1 (the predictor moderates the effect of the mediator on the outcome), the matrix of conditional effects $\mathbf{B}^{st}_{MOD}$ is expressed as

$$\begin{aligned}\mathbf{B}^{st}_{MOD} &= \mathbf{B}^{st} + \mathbf{j}\boldsymbol{\eta}^{st\prime}\boldsymbol{\Omega}^{st} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ \beta_{mx} & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & \beta_{ym\bullet\mathbf{x}} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} \eta^{st}_x \\ \eta^{st}_m \\ \eta^{st}_y \end{bmatrix}' \begin{bmatrix} 0 & \beta_{ymx\bullet\mathbf{x}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ \beta_{mx} & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & \beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta^{st}_x & 0 \end{bmatrix}.\end{aligned} \tag{6.9}$$

The matrix of indirect effects $\mathbf{M}^{st}_{MOD}$ from $\mathbf{B}^{st}_{MOD}$ is

$$\mathbf{M}_{MOD}^{st} = (\mathbf{I} - \mathbf{B}_{MOD}^{st})^{-1} - \mathbf{I} - \mathbf{B}_{MOD}^{st}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{mx}(\beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta_x^{st}) & 0 & 0 \end{bmatrix}. \tag{6.10}$$

Υ_{MOD} is then calculated from $\mathbf{M}_{MOD}^{st}$

$$\Upsilon_{MOD} = \mathbf{M}_{MOD}^{st}\mathbf{\Psi}^{st}\mathbf{M}_{MOD}^{st}{}'$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{mx}(\beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta_x^{st}) & 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta_x^{st2} & & \\ 0 & \zeta_m^{st2} & \\ 0 & 0 & \zeta_y^{st2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta_{mx}(\beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta_x^{st}) & 0 & 0 \end{bmatrix}' \tag{6.11}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\beta_{mx}(\beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta_x^{st}))^2\,\zeta_x^{st2} \end{bmatrix}.$$

As described in Chapter 5, the effect size $(\beta_{mx}(\beta_{ym\bullet\mathbf{x}} + \beta_{ymx\bullet\mathbf{x}}\eta_x^{st}))^2\,\zeta_x^{st2}$ consists of products of η_x^{st}, which are the covariance between η_x^{st} and η_x^{st2} (σ_{x,x^2}), and the variance of η_x^{st2} ($\sigma_{x^2}^2$). Substituting these findings into Equation 6.11 yields the υ for Case 1 as

$$\upsilon_{MOD} = \beta_{mx}^2\beta_{ym\bullet\mathbf{x}}^2 + \beta_{mx}^2\beta_{ymx\bullet\mathbf{x}}^2\eta_x^{st2}\sigma_{x^2}^2 + 2\beta_{mx}^2\beta_{ym\bullet\mathbf{x}}\beta_{ymx\bullet\mathbf{x}}\eta_x^{st}\sigma_{x^2,x}, \tag{6.12}$$

where *x* is the conditional value of the predictor at which υ_{MOD} is evaluated. This means that, because *x* is standardized, υ_{MOD} at the mean of *x* is equivalent *in form* to υ for a simple three-variable mediation model. However, when $\beta_{mx}^2 \neq 0$, $\upsilon_{MOD} = \upsilon$ only if $\beta_{ym\bullet\mathbf{x}}^2 = \beta_{ym\bullet x}^2$, or when there is no effect of the interaction.

υ has several desirable properties as an effect size measure for conditional indirect effects in moderated mediation models illustrated in Case 1. As demonstrated in Lachowicz et al.

(2018), υ is interpretable as the variance explained indirectly in an outcome by a predictor (or set of predictors) through a mediator (or set of mediators). In a moderated mediation model, the variance explained indirectly is conditional on the values of the moderator. υ is standardized, so it is invariant to changes in the scales of the predictor, mediator, and outcome. υ is also monotonically related to its respective conditional indirect effect. In addition, 95% CIs can be obtained for υ using a bootstrapping procedure. Finally, the matrix-based approach allows for υ to be obtained for moderated mediation models more complex than the five cases previously illustrated (e.g., moderated mediation with covariates, multiple mediators, multiple moderators).

The derivation of υ for Case 2 (covariate *w* moderates the effect of *x* on *m*) is not as straightforward as for Case 1. Specifically, following the same steps as in the derivation for υ for Case 1 results in an effect size for a quantity that is not the squared standardized conditional indirect effect. Repeating the procedure followed in Case 1, the reduced form matrix of standardized conditional regression coefficients $\mathbf{B}_{MOD}^{st}$ (*x* is designated the focal predictor) is

$$\begin{aligned}
\mathbf{B}_{MOD}^{st} &= \begin{array}{c} x \\ w \\ m \\ y \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{mx\bullet\mathbf{x}} & \beta_{mw\bullet\mathbf{x}} & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & 0 & \beta_{ym\bullet\mathbf{x}} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \eta_x^{st} \\ \eta_w^{st} \\ \eta_m^{st} \\ \eta_y^{st} \end{bmatrix}' \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_{mxw\bullet\mathbf{x}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
&= \begin{array}{c} x \\ w \\ m \\ y \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta_w^{st} & \beta_{mw\bullet\mathbf{x}} & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & 0 & \beta_{ym\bullet\mathbf{x}} & 0 \end{bmatrix}.
\end{aligned} \tag{6.13}$$

The matrix of standardized conditional indirect effects $\mathbf{M}_{MOD}^{st}$ is calculated from $\mathbf{B}_{MOD}^{st}$ as

$$\mathbf{M}^{st}_{MOD} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})\beta_{ym\bullet\mathbf{x}} & \beta_{mw\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}} & 0 & 0 \end{bmatrix}, \tag{6.14}$$

and Υ_{MOD} is calculated from $\mathbf{M}^{st}_{MOD}$ is

$$\Upsilon_{MOD} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \begin{matrix} ((\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})\beta_{ym\bullet\mathbf{x}})^2 \zeta^{st\,2}_{x} + (\beta_{mw\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}})^2 \zeta^{st\,2}_{w} + \\ 2(\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})\beta_{ym\bullet\mathbf{x}}\beta_{mw\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\zeta^{st}_{xw} \end{matrix} \end{bmatrix}. \tag{6.15}$$

υ in this case consists of variance explained *y* indirectly by *x* through *m* conditional on *w* ($(\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})^2 \beta^2_{ym\bullet\mathbf{x}} \zeta^{st\,2}_{x}$), variance explained in *y* indirectly by *w* through *m* ($(\beta_{mw\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}})^2$), and the covariance of the indirect effects of *x* and *w* on *y* through *m* ($2(\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})\beta_{ym\bullet\mathbf{x}}\beta_{mw\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\zeta^{st}_{xw}$). This shows that straightforward application of the matrix method results in υ that quantifies the total variance explained indirectly from all of the conditional indirect effects on a specific outcome. However, in many cases it is of interest to report an effect size for a specific conditional indirect effect, and υ obtained here consists of variance explained indirectly from several sources.

υ is derived for a specific conditional indirect effect using a modification of the matrix method described in Chapter 4. To obtain the specific υ for the conditional indirect effect $(\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta^{st}_{w})^2 \beta^2_{ym\bullet\mathbf{x}} \zeta^{st\,2}_{x}$, $\mathbf{B}^{st}_{MOD}$ is modified by pre- and post- multiplication by an elementary matrix **O** that replaces the regression coefficient $\beta_{mx\bullet\mathbf{x}}$ with zero, resulting in a modified matrix of coefficients $\mathbf{B}^{st*}_{MOD}$

$$\mathbf{B}_{MOD}^{st*} = \mathbf{O}\mathbf{B}_{MOD}^{st}\mathbf{O}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta_w^{st} & \beta_{mw\bullet\mathbf{x}} & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & 0 & \beta_{ym\bullet\mathbf{x}} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.16}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta_w^{st} & 0 & 0 & 0 \\ \beta_{yx\bullet\mathbf{x}} & 0 & \beta_{ym\bullet\mathbf{x}} & 0 \end{bmatrix}.$$

The matrix of indirect effects $\mathbf{M}_{MOD}^{st*}$ is calculated from $\mathbf{B}_{MOD}^{st*}$ as

$$\mathbf{M}_{MOD}^{st*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta_w^{st})\beta_{ym\bullet\mathbf{x}} & 0 & 0 & 0 \end{bmatrix}, \tag{6.17}$$

and $\boldsymbol{\Upsilon}_{MOD}^{*}$ is calculated from $\mathbf{M}_{MOD}^{st*}$ as

$$\boldsymbol{\Upsilon}_{MOD}^{*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ((\beta_{mx\bullet\mathbf{x}} + \beta_{mxw\bullet\mathbf{x}}\eta_w^{st})\beta_{ym\bullet\mathbf{x}})^2 \end{bmatrix}. \tag{6.18}$$

As in Case 1, υ_{MOD} incorporates products of variances and covariances of product terms, expressed as

$$\upsilon_{MOD} = \beta_{mx}^2\beta_{ym\bullet\mathbf{x}}^2 + \beta_{mx}^2\beta_{ymx\bullet\mathbf{x}}^2\eta_w^{st2}\sigma_{xw}^2 + 2\beta_{mx}^2\beta_{ym\bullet\mathbf{x}}\beta_{ymx\bullet\mathbf{x}}\eta_w^{st}\sigma_{xw,x}, \tag{6.19}$$

where σ_{xw}^2 is the variance of the product term *xw*, and $\sigma_{xw,x}$ is the covariance of *x* with the product term. In addition, as in Case 1, the conditional variance in *y* explained indirectly by *x* through *w* at the mean of *w* is equivalent to υ from the simple three-variable mediation model.

υ for Cases 1 – 5 can be found in Table 13. For models where the specific and total υ differ as in Case 2, the specific and total υ are provided assuming that the indirect effect of *x* is of primary interest. Derivations for υ in the remaining Cases 3 – 5 follow the same procedures as in Cases 1 and 2.

6.2.2 *Bias-adjusted estimator of conditional* υ

The bias-correction approach used thus far is again implemented to adjust for bias in the expected value of $\hat{\boldsymbol{\Upsilon}}_{MOD}$ to yield an improved estimator $\tilde{\boldsymbol{\Upsilon}}_{MOD}$. The general form of the bias-adjusted estimator is

$$\tilde{\boldsymbol{\Upsilon}}_{MOD} = \hat{\mathbf{M}}_{MOD}^{st}\hat{\boldsymbol{\Psi}}^{st}\hat{\mathbf{M}}_{MOD}^{\prime st} - \hat{\boldsymbol{\Delta}}_{bias}^{\prime}. \tag{6.20}$$

where the elements of $\hat{\boldsymbol{\Delta}}_{bias}^{\prime}$ corresponding to outcome variables consist of $(1/2)tr\{\mathrm{var}(\mathbf{B})\mathbf{H}\} - (1/4)tr\{\mathrm{var}(\mathbf{B})\mathbf{H}_1\}$, and zero otherwise (Equation 3.26). For the Case 1 moderated mediation model, the estimated bias correction is

$$\begin{aligned}\hat{\Delta}_{bias,y}^{\prime} = {} & \hat{\beta}_{ym\bullet\mathbf{x}}^2\hat{\sigma}_{mx}^2 + \hat{\beta}_{mx}^2\hat{\sigma}_{ym\bullet\mathbf{x}}^2 + 2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymx\bullet\mathbf{x}}\eta_x^{st}\hat{\sigma}_{mx}^2 + 2\hat{\beta}_{mx}^2\eta_x^{st}\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}} + \hat{\beta}_{ymx\bullet\mathbf{x}}^2\eta_x^2\hat{\sigma}_{mx}^2 + {} \\ & \hat{\beta}_{mx}^2\eta_x^{st2}\hat{\sigma}_{ymx\bullet\mathbf{x}}^2 - \hat{\sigma}_{ym\bullet\mathbf{x}}^2\hat{\sigma}_{mx}^2 - 2\eta_x^{st}\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}}\hat{\sigma}_{mx}^2 - \eta_x^{st2}\hat{\sigma}_{ymx\bullet\mathbf{x}}^2\hat{\sigma}_{mx}^2.\end{aligned} \tag{6.21}$$

At the mean of the moderator (i.e., mean of standardized *x*), the bias-adjustment is equivalent to $\tilde{\upsilon}$ for the three-variable mediation model ($\hat{\beta}_{ym\bullet\mathbf{x}}^2\hat{\sigma}_{mx}^2 + \hat{\beta}_{mx}^2\hat{\sigma}_{ym\bullet\mathbf{x}}^2 - \hat{\sigma}_{ym\bullet\mathbf{x}}^2\hat{\sigma}_{mx}^2$), and at +1 standard deviation above the mean of *x* is

$$\hat{\beta}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}+\hat{\beta}^2_{mx}\hat{\sigma}^2_{ym\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}+2\hat{\beta}^2_{mx}\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}}+\hat{\beta}^2_{ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}+$$
$$\hat{\beta}^2_{mx}\hat{\sigma}^2_{ymx\bullet\mathbf{x}}-\hat{\sigma}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-2\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-\hat{\sigma}^2_{ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}.$$

Bias adjustments for the five prototypical cases of moderated mediation previously described can be found in Table 13.

6.3 Simulation Study

The purpose of the present simulation study is to examine the sampling properties of the unadjusted and bias-adjusted effect size estimators $\hat{\upsilon}$ and $\tilde{\upsilon}$ in a moderated mediation model. As in the simulation in Chapter 4, of interest is determining whether the matrix generalization and bias-adjustment yield estimators with good statistical properties in terms of bias, variance, and overall accuracy, and whether common interval estimation methods return proper CIs for the estimators.

6.3.1 Simulation Design

The generating model for this simulation was a moderated mediation model with a single predictor *x*, outcome *y*, mediator *m*, and interaction between *x* and *m* (Figure 3). Variables in this simulation were considered standard normal in the population. Because effect sizes can be estimated for the indirect effect conditional on levels of the moderator, two levels were chosen for the simulation: a) the conditional indirect effect of *x* on *y* through m at the mean of standardized *x* ($\beta_{mx}\beta_{ym\bullet\mathbf{x}}$), and b) at +1 SD above the mean of *x* ($\beta_{mx}(\beta_{ym\bullet\mathbf{x}}+\beta_{ymx\bullet\mathbf{x}})$).

6.3.2 Simulation Conditions

Parameter values for the paths were varied among .15, .39, and .59, magnitudes for small, medium, and large standardized coefficients common in applied research. As in the previous simulation, values consistent with the null hypothesis of no indirect effect were not considered in this simulation because zero is on the boundary of the parameter space. Sample size was varied among 50, 100, 250, and 500. This yields a total of 3x3x3x4 = 108 total conditions. Percentile and BCa 95% CIs (Section 2.3) were evaluated for $\hat{\upsilon}$ and $\tilde{\upsilon}$. As in the previous simulation, it was expected that BCa CIs will outperform percentile CIs in terms of coverage and balance, particularly in conditions with small effect magnitudes.

1,000 replications per condition is sufficient to obtain accurate estimates of bias for point estimates and coverage for CIs, and 1,000 bootstrap resamples are used to construct 95% CIs. The point estimators will be evaluated in terms of bias, overall accuracy, and relative efficiency, and CIs will be evaluated in terms of coverage and coverage balance.

6.3.3 Evaluation criteria

Evaluation criteria were those detailed in Section 4.4.3. It is unclear how or if the sampling properties of the estimators of υ would differ at the mean of *x* as compared to +1 SD above the mean of *x*, so this question will be addressed empirically.

6.4 Simulation Results

6.4.1 Bias

Simulation results for the relative bias of $\hat{\upsilon}$ for the conditional indirect effect at the mean of standardized *x* can be found in Table 14, where shaded cells highlight relative bias > 5%. Results were generally consistent with results from the previous simulation. The hypothesis that $\hat{\upsilon}$ would be positively biased, with larger magnitudes of bias at smaller *N* and for smaller effect sizes, and decreasing magnitudes as *N* and effect magnitude increased, was supported by the simulation results. The direction of bias for $\hat{\upsilon}$ was positive in all conditions, consistent with analytic results. For the smallest effect magnitudes ($\beta = .15$), bias was non-negligible at all sample sizes, and in those conditions with the smallest *N* (*N* = 50), percent relative bias

(290.84%, 238.9%, and 206.6%) was of the largest magnitude across all conditions. Increasing *N* was associated with decreasing bias, supporting the hypothesis that $\hat{\upsilon}$ is a consistent estimator. Finally, bias of $\hat{\upsilon}$ was negligible for large effect magnitudes of the conditional indirect effect for all sample sizes.

Simulation results for percent relative bias of $\tilde{\upsilon}$ also can be found in Table 14. The hypothesis that bias of $\tilde{\upsilon}$ would be negligible across simulation conditions was largely supported by simulation results. As in the previous simulation, percent relative biases for $\tilde{\upsilon}$ were of much smaller magnitude than for $\hat{\upsilon}$. For the conditions in which bias was greatest for the $\hat{\upsilon}$, the relative biases of $\tilde{\upsilon}$ were –3.98%, -4.44%, and –9.88%. Overall, the largest magnitude of relative bias across conditions for $\tilde{\upsilon}$ was –24.39% at $N = 50$. 12 of the 27 conditions at $N = 50$ demonstrated non-negligible bias, 6 at $N = 100$, 2 at $N = 250$, and none at $N = 500$. Finally, as with $\hat{\upsilon}$, bias decreased as *N* increased, supporting the hypothesis that $\tilde{\upsilon}$ is a consistent estimator.

Results for relative bias of effect size estimators of the conditional indirect effect at +1 standard deviation above the mean of standardized *x* can be found in Table 15. Findings were generally similar to those for the conditional indirect effect at the mean of *x*. For both estimators, hypotheses regarding the magnitude and direction for the specific indirect effect were supported. For $\hat{\upsilon}$, the largest values of relative bias (272.15%, 105.65%, and 103.89%) occurred at the smallest *N*, and relative bias was non-negligible for these effects even at the largest sample size. For $\tilde{\upsilon}$, relative bias was non-negligible in 18 of the 27 effect magnitude conditions at $N = 50$, in 3 conditions at $N = 100$, and in no conditions at $N = 250$ and $N = 500$. Although bias tended to be negative at smaller sample sizes, the magnitude of bias showed no clear relationship with effect magnitude. Finally, increasing *N* was associated with decreasing bias for both estimators.

6.4.2 *Accuracy and Relative Efficiency*

Simulation results of MSE and *RE* for effect size estimators of the conditional indirect effect at the mean of *x* can be found in Table 16, and for the conditional indirect effect at +1 standard deviation above the mean of *x* in Table 17. Shaded cells highlight conditions where the MSE of $\tilde{\upsilon}$ was greater than that of $\hat{\upsilon}$, and where $RE > 1$ (i.e., variance of $\tilde{\upsilon} > \hat{\upsilon}$). Increasing *N* was associated with decreasing MSE for both estimators of both conditional effects, supporting the hypothesis that overall accuracy of the measures would increase with increasing *N*. It was also clear that, for both effects, outside of a few conditions, $\tilde{\upsilon}$ was a more accurate estimator of υ than $\hat{\upsilon}$. In addition, it was clear that across the vast majority of conditions $\tilde{\upsilon}$ was a more efficient estimator. Finally, the magnitudes of the accuracy and efficiency discrepancies between the estimators were dependent on sample size and effect magnitudes, such that differences were largest for the smallest sample sizes and smallest effects.

6.4.3 *Confidence Intervals*

Results for 95% percentile bootstrap CIs of $\hat{\upsilon}$ and $\tilde{\upsilon}$ for the conditional indirect effect at the mean of *x* can be found in Tables 18 and 19, respectively, and for the conditional indirect effect +1 standard deviation above the mean of *x* in Tables 20 and 21, respectively. Shaded cells highlight conditions where satisfactory coverage (92.5% – 97.5%) was not achieved. The hypotheses that coverage would reach the nominal 95% level as *N* increased, and that the proportions of misses to the left and right of the 95% CI would be balanced, were supported by simulation results. As with bias, satisfactory coverage was achieved with larger effect sizes and at larger *N* for both estimators and effects, such that satisfactory coverage was achieved for all parameter combinations at *N* = 250. In addition, misses to the left and right of the 95% CI were approximately balanced at $N = 250$ for both estimators and effects. When satisfactory coverage was not achieved, it was predominantly due to coverage > 97.5%, or CIs being too wide.

Results for 95% BCa bootstrap CIs of estimators for the conditional indirect effect at the mean of x can be found in Tables 22 and 23, respectively, and for the conditional indirect effect at +1 standard deviation above the mean of x in Tables 24 and 25, respectively. The hypotheses that that nominal coverage would converge to a satisfactory level, and that the proportions of misses to the left and right would achieve balance, were generally supported by the results as well. Similar to the percentile CIs, satisfactory coverage was generally achieved for larger N and larger effect magnitudes, and achieved for all parameter combinations at $N = 500$ both estimators and effects. As with the CI methods for the multiple mediator model, there were noteworthy differences between the results. In contrast to the percentile CIs, when satisfactory coverage was not achieved, it was predominantly due to coverage < 92.5%, or *CIs being too narrow*. In addition, whereas percentile CIs for both conditional effects achieved satisfactory coverage at the same sample size ($N = 250$), BCa CIs required sample sizes of 500 for satisfactory coverage in the majority of conditions.

6.5 Empirical Example

I now present an empirical example to facilitate interpretation and implementation of υ for a moderated mediation model. I use the results from a study conducted by Alexopolous and Cho (in press) that investigated the conditional pathway through which risk taking has a downstream effect on sexual behavior. Like the example in Chapter 4, the authors used ADD Health (Harris, 2009). The authors examined the indirect effect of risk taking on sexual behavior through alcohol use, conditional on level of parent child communication. Analyses were conducted with those female participants that had observations on these variables at Wave 4 (N =

1233). The predictor variable of risk taking was assessed using a single item asking whether the participant “liked to take risks”, where higher values indicated greater propensity for risk taking. The mediator variable of alcohol use was assessed using a single item of the number of drinking days in the past 30 days. The outcome variable of sexual behaviors was assessed using a single item of the participant’s total number of sexual partners. The moderating variable of parental communication was assessed using a composite of six items, where three items measure the frequency and quality of communication between the participant and their mother, and the remaining three items measure the frequency and quality of communication between the participant and their father.

Results presented here differ slightly form the original study results because relevant control variables were excluded to simplify analyses. The standardized indirect effect of risk taking on sexual behavior through alcohol use at the mean of parental communication (i.e., 0) was 0.0097 (95% percentile CI = .0032, .0181), at one standard deviation below the mean of parental communication was 0.0029 (95% percentile CI = –.0106, .0162), and at one standard deviation above the mean of parental communication was 0.0164 (95% percentile CI = .0051, .0332).

For the υ estimators, the $\hat{\upsilon}$ effect size estimate for the indirect effect at the mean of parental communication was 0.00009 (95% percentile bootstrap CI = .00002, .00056), meaning that the variance explained indirectly in sexual behaviors by risk taking through alcohol use at the mean of parental communication was 0.00009. The $\tilde{\upsilon}$ effect size estimate for the indirect effect at the mean of parental communication was 0.00008 (95% percentile bootstrap CI = 0.00001, 0.00056). $\tilde{\upsilon}$ is interpreted as the estimated variance in sexual behaviors explained by risk taking through alcohol use at the mean of parental communication in the population. The $\hat{\upsilon}$

effect size estimate for the indirect effect at one standard deviation below and above the mean of parental communication were 0.00013 (95% percentile bootstrap CI = .00001, .00033) and 0.00015 (95% percentile bootstrap CI = .00002, .00059), respectively. This means that the variance explained indirectly in sexual behaviors by risk taking through alcohol use at one standard deviation below the mean of parental communication was 0.00013, and at one standard deviation above the mean of parental communication was 0.00015. The $\tilde{\upsilon}$ effect size estimate for the indirect effect at one standard deviation below and above the mean of parental communication were 0.00011 (95% percentile bootstrap CI = .000003, .00059) and 0.00012 (95% percentile bootstrap CI = .000009, .00053), respectively.

6.6 Summary

In Chapter 6, the effect size υ was extended to moderated mediation models, or models with conditional indirect effects. Unadjusted and bias-adjusted estimators were derived for five prototypical examples of moderated mediation effects. A Monte Carlo simulation was conducted to investigate the sampling properties of the effect size estimators. Chapter 7 will summarize the findings of the dissertation, and discuss limitations and future directions.

7 CONCLUSION AND DISCUSSION

7.1 Summary

The goal of the dissertation was to develop a generalizable effect size measure for mediation analysis. Chapter 2 reviewed effect sizes for common study designs, mediation analysis, and effect size in mediation analysis. Introduced in Chapter 2 was the effect size measure upsilon, which was shown to have many desirable properties for a general effect size measure for indirect effects in mediation analysis. Chapter 3 reviewed a matrix-based framework for MLR models and SEM that would be used as a general framework for extending the effect size υ to more complex models. Chapter 4 proposed a generalizable effect size measure and sample estimators for complex mediation models including models with multiple mediators, predictors, and covariates. Chapter 5 proposed a general matrix framework for explained variance in moderated MLR. Chapter 6 further extended the mediation effect size to conditional indirect effects.

7.2 Primary Contributions

7.2.1 General effect size measure for mediation

The primary contribution of this work is a generalizable measure of effect size for indirect effects in mediation analysis. The effect size Υ was shown to be applicable to a wide variety of mediation models used in applied psychological research, including models with multiple parallel and serial mediators, multiple predictors, and covariates in Chapter 4, and conditional indirect effects in Chapter 6. The effect size υ proposed in Lachowicz et al. (2018) for simple three-variable mediation models served as basis for these extensions. In addition, the concept of explained variance in moderated MLR was developed to extend Υ to conditional indirect effects.

The effect size Υ has several desirable properties as an effect size for mediation analysis. First, Υ is a standardized measure of effect size, which yields effect sizes for indirect effects that are comparable within and across studies. Second, Υ is interpretable as a measure of explained variance. Although benchmarks are commonly applied for small, medium, and large proportions of variance in traditional research designs (Cohen, 1988), it is unclear whether the effects from traditional designs are comparable to indirect effects from complex mediation models (e.g., total indirect effect of several mediators, conditional indirect effect). Therefore, it would desirable to develop benchmarks for these effects from the accumulation of findings within specific research domains. It is also important to note that Υ can be greater than 1 when suppression is evident (i.e., direct and indirect effect have opposite signs), so this measure is not strictly proportion. Third, the matrix-based framework (i.e., LISCOMP) for Υ makes the effect readily generalizable to many types of mediation models. Fourth, two sample estimators were proposed for Υ ($\hat{\Upsilon}$ and $\tilde{\Upsilon}$). Although bias of both estimators was negligible for the largest total, specific, and conditional indirect effect magnitudes, bias for $\tilde{\Upsilon}$ was negligible in many more conditions than $\hat{\Upsilon}$. In addition, $\tilde{\Upsilon}$ was demonstrated to be a more accurate estimator in terms of MSE than $\hat{\Upsilon}$ in the vast majority of study conditions. Fifth, 95% CIs can be constructed for Υ using a bootstrap procedure. Although percentile and BCa methods perform satisfactorily in terms of coverage and balance at large sample sizes for the indirect effects considered in the simulations (i.e., $N > 500$), results show that the percentile method outperforms the BCa method for smaller sample sizes.

The review of effect sizes in Chapter 2 showed that many sample estimators of standardized effect sizes are biased, and bias-adjusted estimators are often recommended. With the exception of the effect size based on the standardized mean difference (Hansen & McNeal, 1996), Υ is the only standardized effect size for mediation analysis with a bias-adjusted estimator. However, unlike the Hansen and McNeal (1996) measure, the sampling properties of the unadjusted and bias-adjusted Υ estimators and CIs were evaluated with Monte Carlo simulation studies.

As a measure of explained variance, υ is generalizable to a larger class of mediation models than other standardized measures. More specifically, as shown in Chapters 2 and 3, measures of explained variance can be used to quantify not only bivariate relationships, but also relationships between an outcome and several predictors. This means that, in addition to indirect effects with a single predictor and outcome, Υ can be used to quantify the variance explained in an outcome indirectly by several predictors, including the overall variance explained via the indirect effect of a multi-categorical predictor.

7.3 Limitations

7.3.1 Variable standardization

The effect size derivations in this dissertation assumed the variances of the variables used to standardize indirect effects were fixed, known quantities. Although this is common practice in coefficient standardization, variable variances can also be considered to vary randomly across samples. Assuming these variances are stochastic rather than fixed means the sampling distribution of $\hat{\Upsilon}$ would also incorporate the distributions of the variables used for standardization, which would add further complexity to the derivation of the expected value of $\hat{\Upsilon}$ for bias adjustment. However, simulation results show that bias for $\tilde{\Upsilon}$ was negligible in the majority of simulation conditions for both the multiple mediator and conditional mediator models, and of relatively small magnitude in those conditions where bias was non-negligible. It is possible that the sampling variability of the variances has a systematic effect on the bias of $\tilde{\Upsilon}$, perhaps responsible for the negative bias trend observed in $\tilde{\Upsilon}$ for small sample sizes and small effect magnitudes.

7.3.2 υ for latent variable models

Because the Υ extensions use the LISCOMP SEM framework, it is possible to obtain effect sizes for indirect effects among latent variables. However, the results of the dissertation are limited to mediation models with manifest variables, and caution should be taken if Υ is to be applied to models with latent variables. The primary reason for this caution is that, because SEM separates construct relevant variance from error variance, it is questionable if R^2 estimates obtained from a latent structural model are comparable to estimates from models with manifest variables. More specifically, it would be expected that R^2 estimates would be larger for latent variable models as compared to manifest variable models because the total variance of manifest outcomes includes measurement error, a source a variance that would not be explainable by predictors.

Also relevant is the issue of standardization in latent variable models. The standardization in 3.24 is of the same form used when standardizing coefficients in MLR (i.e., dividing by variable standard deviations), but in SEM there is no estimated parameter that corresponds to the total variance for endogenous variables, only parameters for residual variances ($\mathbf{\Psi}$). This complicates the process of standardization, where the improper standardization (e.g., analyzing a correlation matrix as a covariance matrix) can result in misleading measures of model fit and standard error estimates (Cudeck, 1989; McDonald & Ho, 2002).

7.3.3 *Explained variance in non-recursive models*

The interpretation of R^2 in SEM becomes more complicated when models are non-recursive (Bentler & Raykov, 2000; Teel, Bearden, & Sharma, 1984). When a model contains a feedback loop, it is unclear how to attribute variance explained to specific sources. For example, if two variables x_1 and x_2 are considered predictors and outcomes of one another, the variance explained in x_1 can partially be attributed to not only x_2, but also x_1 because x_1 is considered a cause of x_2. Using traditional measures of R^2 in these circumstances results in inflated estimates of explained variance. Although methodologists have proposed more general computational forms for R^2 in SEM to account for this issue (Bentler & Raykov, 2000; Teel et al., 1986), consensus has not yet been reached regarding how to most appropriately define and compute R^2 in non-recursive models. Because of this uncertainty, and the more general issues of interpretation for non-recursive models, the effect size extensions described in this dissertation are limited to recursive models.

7.4 Future Directions

The results of this dissertation offer several promising directions for future research. One is investigation of Υ for indirect effects in latent variable models. Given the ease with which latent variables are modeled with the LISCOMP framework, it is important that the desirable properties of Υ estimators translate to indirect effects in latent variable models, including models with conditional effects among latent variables (i.e., latent interactions; Klein & Moosbrugger, 2000). Another important extension of Υ is to mediation models in clustered data, which can be modeled in the LISCOMP framework using multilevel structural equation modeling (MSEM; Preacher, Zyphur, & Zhang, 2010). This extension poses additional challenges, particularly for the derivation of bias-adjusted sample estimators, because the assumption of independence for regression coefficients across equations can be violated independence is a key assumption when deriving the bias in the expected value of $\hat{\Upsilon}$ in Chapters 4 and 6.

Another promising direction for future research is in evaluation and comparison of additional CI estimators. Although the percentile CIs performed well in many of the simulation conditions, both percentile and BCa CIs were unstable for small sample sizes and for small indirect effect magnitudes. Additional interval estimators that could be evaluated for performance in these conditions include bias-corrected bootstrap CIs without an acceleration constant (MacKinnon et al., 2004), Monte Carlo CIs (Preacher & Selig, 2012), and Bayesian credibility intervals (Yuan & MacKinnon, 2009).

7.5 Conclusion

In conclusion, the results of this dissertation show that Υ is a theoretically meaningful and useful measure of effect size for indirect effects of many types of mediation models. Furthermore, the bias-adjusted sample estimator $\tilde{\Upsilon}$ has been demonstrated to have good statistical properties in many study design conditions common in applied research.

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Figure 1. *Path diagram for a three-variable mediation model*

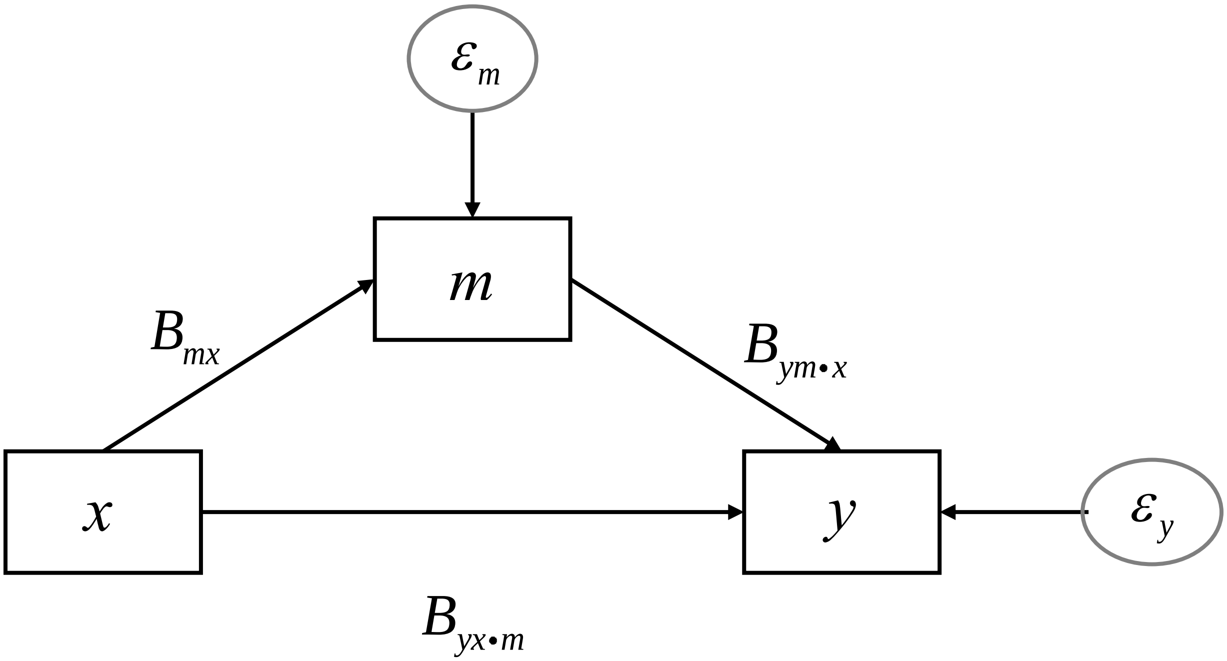

Figure 2. *Path diagram for a multiple parallel mediation model*

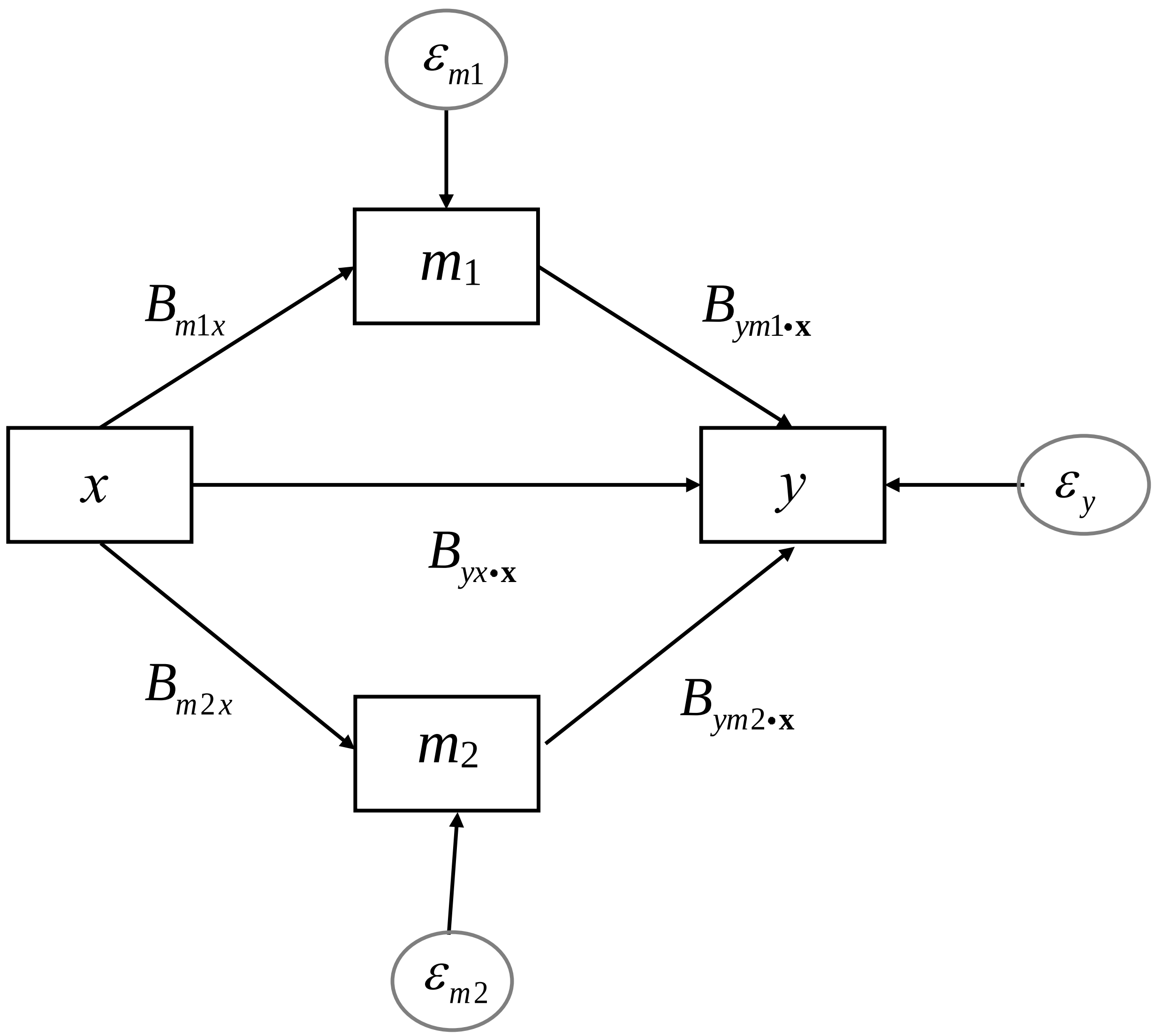

Figure 3. *Path diagram for a moderated mediation model with predictor X mediator interaction*

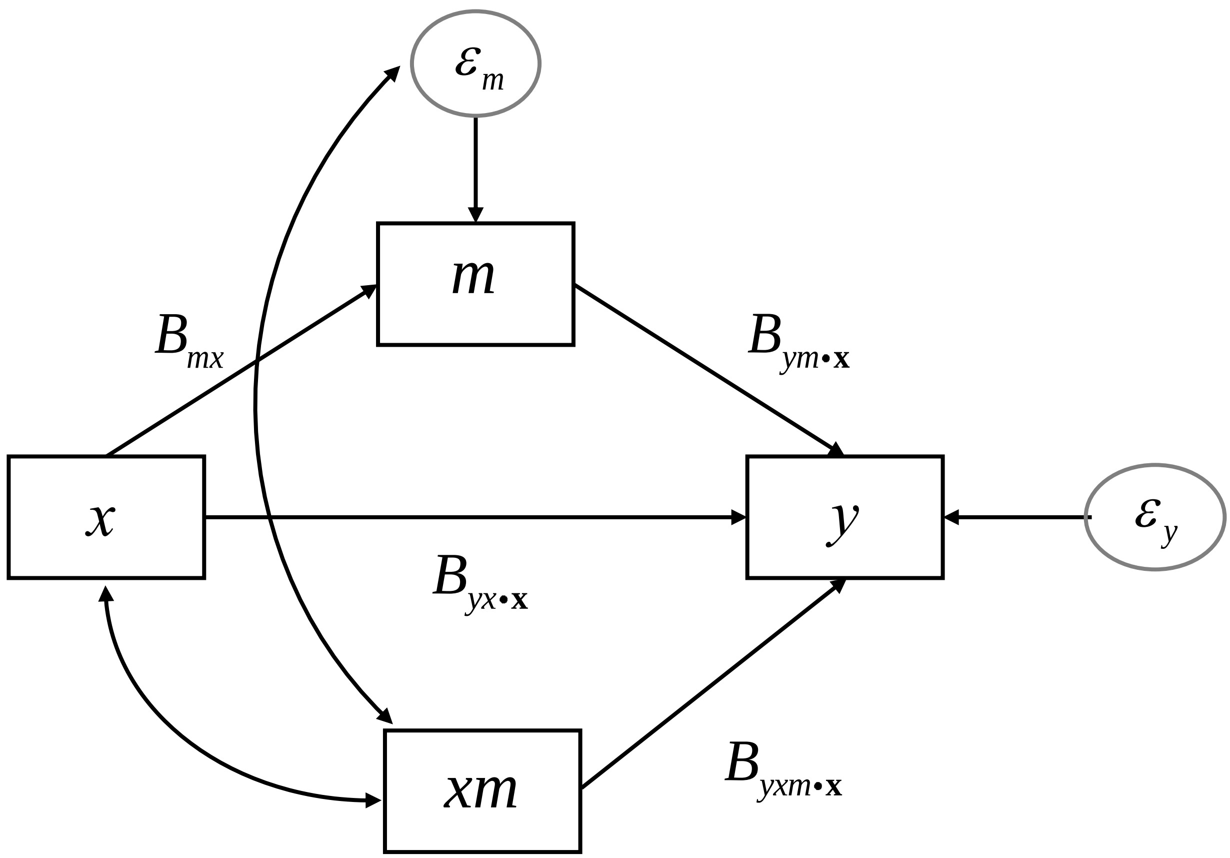

Table 1. *Percent relative bias of the total indirect effect of two mediators*

β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	$\hat{\upsilon}$ N=50	$\hat{\upsilon}$ N=100	$\hat{\upsilon}$ N=250	$\hat{\upsilon}$ N=500	$\tilde{\upsilon}$ N=50	$\tilde{\upsilon}$ N=100	$\tilde{\upsilon}$ N=250	$\tilde{\upsilon}$ N=500
.15	.15	.15	138.606	50.251	20.885	12.724	0.234	-2.845	1.616	3.465
		.39	19.391	9.954	3.563	0.706	-4.777	-1.292	-0.725	-1.391
		.59	6.889	2.596	0.665	0.171	-2.754	-1.911	-1.089	-0.698
	.39	.15	60.960	36.110	14.047	6.756	-7.293	1.056	0.784	0.327
		.39	25.013	6.976	3.328	1.838	3.842	-2.862	-0.479	-0.053
		.59	5.711	2.693	0.192	1.055	-2.604	-1.277	-1.355	0.284
	.59	.15	69.406	34.151	13.882	2.120	1.514	1.125	0.993	-4.237
		.39	17.037	8.092	3.789	1.992	-3.672	-1.917	-0.166	0.034
		.59	6.756	2.432	1.108	0.513	-0.991	-1.339	-0.375	-0.227
.39	.15	.15	93.566	32.291	14.668	7.484	2.665	-5.949	-0.274	0.240
		.39	17.917	7.431	1.558	1.484	-4.869	-3.091	-2.487	-0.513
		.59	4.723	3.651	1.331	0.686	-4.196	-0.622	-0.309	-0.130
	.39	.15	17.856	8.991	3.961	0.667	-1.007	-2.247	-0.328	-1.433
		.39	10.283	2.732	2.404	1.798	-1.624	-2.830	0.228	0.723
		.59	1.576	0.281	0.056	0.862	-3.928	-2.930	-1.089	0.348
	.59	.15	10.011	6.793	1.134	0.272	-5.029	-0.476	-1.678	-1.124
		.39	3.529	2.564	0.198	0.427	-4.917	-1.526	-1.403	-0.367
		.59	2.966	0.886	0.492	0.147	-2.967	-1.023	-0.264	-0.523
.59	.15	.15	97.628	46.858	19.923	13.093	-1.447	-2.894	0.883	3.678
		.39	25.150	8.185	6.160	4.120	-2.812	-4.806	1.103	1.606
		.59	9.034	1.142	-0.038	0.097	-0.868	-3.489	-1.858	-0.802
	.39	.15	16.083	9.160	4.248	2.164	-4.379	-0.500	0.547	0.335
		.39	7.997	1.643	1.080	0.157	-2.393	-3.182	-0.806	-0.775
		.59	0.080	-0.116	0.599	0.564	-4.371	-2.265	-0.240	0.148
	.59	.15	7.664	5.001	1.586	0.928	-1.966	0.425	-0.173	0.059
		.39	0.765	1.238	0.530	0.223	-4.707	-1.410	-0.506	-0.736
		.59	0.736	0.170	0.105	0.095	-3.019	-1.271	-0.538	-0.120

Note: Shaded cells indicate relative bias > 5%; $\hat{\upsilon}$ is the unadjusted effect size estimator; $\tilde{\upsilon}$ is the bias-adjusted effect size estimator; β_{m1x} and β_{m2x} are the effects of x on m_1 and m_2; β_{ym1} and β_{ym2} are the effects of m_1 and m_2 on y controlling for x, respectively.

Table 2. *Percent relative bias for a specific effect of a mediator model with two parallel mediators*

β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x} = \beta_{ym2\bullet\mathbf{x}}$	$\hat{\upsilon}$ N=50	$\hat{\upsilon}$ N=100	$\hat{\upsilon}$ N=250	$\hat{\upsilon}$ N=500	$\tilde{\upsilon}$ N=50	$\tilde{\upsilon}$ N=100	$\tilde{\upsilon}$ N=250	$\tilde{\upsilon}$ N=500
.15	.15	.15	265.837	107.800	40.819	26.050	-9.142	-0.092	2.194	3.290
		.39	223.305	107.955	42.167	12.215	-5.352	5.484	5.505	-4.314
		.59	234.106	84.231	25.468	15.950	18.606	0.216	-4.475	1.049
	.39	.15	98.228	53.592	20.186	11.355	-10.587	1.151	0.113	1.524
		.39	110.225	43.319	17.090	8.357	3.373	-6.648	-2.362	-1.293
		.59	100.006	53.747	16.566	7.725	-2.052	4.891	-2.348	-1.668
	.59	.15	90.469	46.002	20.088	4.300	-3.236	-0.212	1.805	-4.743
		.39	80.844	47.242	16.813	9.117	-2.191	1.542	-1.265	0.140
		.59	82.988	42.880	17.885	7.279	-2.411	-2.420	0.170	-1.582
.39	.15	.15	135.606	54.912	25.950	12.020	3.470	-4.410	2.769	0.822
		.39	104.253	49.085	15.902	8.535	-3.536	-3.602	-4.035	-1.169
		.59	82.194	41.097	17.500	3.814	-5.423	0.846	2.310	-3.544
	.39	.15	17.762	10.322	3.777	1.039	-3.151	-2.299	-1.085	-1.361
		.39	25.106	8.754	4.777	2.739	1.064	-2.399	0.417	0.590
		.59	16.910	8.390	3.735	2.562	-2.123	-0.721	0.196	0.812
	.59	.15	11.266	7.583	2.095	4.128	-4.868	-0.342	-1.010	-1.666
		.39	9.124	5.414	1.533	1.969	-4.656	-1.813	-1.306	0.555
		.59	6.708	5.283	2.883	2.153	-1.956	-0.926	0.425	-1.068
.59	.15	.15	142.380	68.212	27.463	18.545	-1.895	-2.893	0.040	0.902
		.39	131.656	56.393	26.015	17.071	0.951	-3.127	2.939	1.556
		.59	94.401	40.291	15.701	6.989	2.718	-2.494	-1.235	-1.266
	.39	.15	17.123	10.002	5.102	2.771	-3.233	-0.688	0.969	0.726
		.39	15.580	5.773	3.162	2.871	-2.815	-2.819	-0.185	-0.783
		.59	9.720	5.884	1.996	1.561	-1.974	0.262	-0.165	0.488
	.59	.15	7.846	5.014	1.450	1.797	-2.205	0.203	-0.414	-0.126
		.39	1.655	2.772	0.829	1.121	-1.029	-0.979	-0.633	-0.848
		.59	1.711	0.117	0.849	0.583	-2.896	-2.327	-0.021	0.152

Note: Shaded cells indicate relative bias > 5%; $\hat{\upsilon}$ is the unadjusted effect size estimator; $\tilde{\upsilon}$ is the bias-adjusted effect size estimator; β_{m1x} and β_{m2x} are the effects of x on m_1 and m_2; β_{ym1} and β_{ym2} are the effects of m_1 and m_2 on y controlling for x, respectively.

Table 3. *MSE and relative efficiency total effect*

			N=50			N=100			N=250			N=500		
			MSE*			MSE*			MSE*			MSE*		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*
.15	.15	.15	0.071	0.044	0.701	0.017	0.013	0.777	0.004	0.004	0.874	0.002	0.002	0.928
		.39	1.065	0.895	0.868	0.433	0.390	0.919	0.154	0.148	0.966	0.075	0.073	0.982
		.59	6.167	5.739	0.944	3.201	3.088	0.968	1.150	1.134	0.987	0.545	0.541	0.993
	.39	.15	0.233	0.178	0.820	0.090	0.075	0.896	0.026	0.024	0.951	0.012	0.011	0.974
		.39	2.133	1.843	0.917	0.734	0.689	0.952	0.292	0.284	0.980	0.130	0.128	0.990
		.59	7.540	7.210	0.968	3.420	3.344	0.984	1.360	1.351	0.993	0.657	0.652	0.997
	.59	.15	0.642	0.526	0.925	0.259	0.232	0.962	0.090	0.086	0.984	0.037	0.036	0.992
		.39	2.664	2.496	0.972	1.336	1.293	0.984	0.490	0.483	0.995	0.233	0.231	0.997
		.59	9.056	8.904	1.001	4.320	4.307	1.002	1.700	1.698	1.001	0.822	0.822	1.001
.39	.15	.15	0.383	0.286	0.828	0.097	0.082	0.886	0.033	0.030	0.944	0.013	0.012	0.971
		.39	1.955	1.697	0.897	0.812	0.754	0.941	0.312	0.304	0.975	0.139	0.137	0.988
		.59	8.783	8.289	0.950	3.761	3.628	0.974	1.437	1.417	0.990	0.740	0.735	0.994
	.39	.15	1.065	0.894	0.860	0.455	0.413	0.921	0.154	0.147	0.966	0.069	0.068	0.982
		.39	3.776	3.421	0.928	1.716	1.644	0.961	0.681	0.665	0.983	0.297	0.292	0.992
		.59	10.212	9.892	0.970	4.499	4.428	0.984	1.953	1.941	0.994	0.956	0.949	0.997
	.59	.15	2.435	2.229	0.928	1.126	1.063	0.960	0.392	0.386	0.984	0.192	0.191	0.992
		.39	5.605	5.411	0.968	3.035	2.974	0.984	1.160	1.154	0.994	0.557	0.555	0.997
		.59	11.598	11.718	1.010	5.584	5.609	1.006	2.130	2.133	1.003	1.099	1.101	1.001
.59	.15	.15	1.237	0.978	0.895	0.427	0.374	0.951	0.146	0.137	0.979	0.060	0.057	0.988
		.39	4.017	3.627	0.953	1.783	1.704	0.968	0.616	0.595	0.986	0.328	0.320	0.993
		.59	13.276	12.521	0.965	5.541	5.440	0.983	2.347	2.329	0.992	1.118	1.114	0.996
	.39	.15	3.341	2.940	0.906	1.602	1.502	0.958	0.561	0.545	0.983	0.291	0.286	0.991
		.39	7.983	7.459	0.950	3.534	3.432	0.972	1.337	1.320	0.989	0.673	0.669	0.994
		.59	15.097	14.710	0.974	6.413	6.336	0.988	2.506	2.489	0.995	1.281	1.274	0.997
	.59	.15	6.450	5.981	0.942	3.106	2.966	0.970	1.018	1.001	0.987	0.532	0.527	0.994
		.39	10.077	9.839	0.971	4.451	4.375	0.985	1.943	1.930	0.994	0.972	0.970	0.997
		.59	11.437	11.671	1.021	5.467	5.533	1.011	2.014	2.022	1.004	1.043	1.045	1.002

Note: Shaded cells indicate MSE and variance $\tilde{\upsilon} > \hat{\upsilon}$; $\hat{\upsilon}$ is the unadjusted effect size estimator; $\tilde{\upsilon}$ is the bias-adjusted effect size estimator; β_{m1x} and β_{m2x} are the effects of x on m_1 and m_2; β_{ym1} and β_{ym2} are the effects of m_1 and m_2 on y controlling for x, respectively. *MSE scaled by 1,000 for presentation.

Table 4. *MSE and relative efficiency specific effect*

			N=50			N=100			N=250			N=500		
			MSE*			MSE*			MSE*			MSE*		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x} = \beta_{ym2\bullet\mathbf{x}}$	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*
.15	.15	.15	0.015	0.009	0.570	0.004	0.003	0.674	0.001	0.001	0.793	< 0.001	0.001	0.868
		.39	0.012	0.006	0.529	0.004	0.003	0.693	0.001	0.001	0.801	< 0.001	< 0.001	0.873
		.59	0.030	0.010	0.654	0.008	0.001	0.679	0.002	0.001	0.813	0.001	< 0.001	0.893
	.39	.15	0.116	0.114	0.786	0.039	0.034	0.869	0.011	0.011	0.940	0.005	0.005	0.966
		.39	0.120	0.098	0.809	0.031	0.027	0.885	0.010	0.010	0.944	0.005	0.005	0.972
		.59	0.087	0.074	0.867	0.035	0.031	0.920	0.012	0.010	0.968	0.004	0.004	0.983
	.59	.15	0.328	0.298	0.910	0.141	0.135	0.955	0.054	0.054	0.980	0.022	0.025	0.991
		.39	0.328	0.311	0.947	0.145	0.143	0.971	0.048	0.047	0.991	0.020	0.021	0.993
		.59	0.278	0.279	0.992	0.113	0.118	1.001	0.042	0.044	1.004	0.020	0.020	1.002
.39	.15	.15	0.194	0.154	0.792	0.049	0.044	0.856	0.016	0.015	0.933	0.006	0.005	0.965
		.39	0.109	0.084	0.773	0.033	0.028	0.857	0.011	0.012	0.930	0.005	0.004	0.964
		.59	0.108	0.062	0.764	0.028	0.023	0.856	0.009	0.008	0.933	0.004	0.005	0.967
	.39	.15	0.733	0.641	0.838	0.285	0.258	0.906	0.100	0.096	0.959	0.047	0.046	0.979
		.39	0.683	0.605	0.855	0.248	0.226	0.915	0.094	0.092	0.964	0.042	0.043	0.981
		.59	0.478	0.394	0.879	0.226	0.193	0.939	0.072	0.066	0.973	0.036	0.036	0.986
	.59	.15	1.962	1.802	0.917	0.890	0.853	0.955	0.301	0.295	0.982	0.152	0.152	0.991
		.39	1.457	1.366	0.938	0.774	0.750	0.969	0.276	0.273	0.987	0.142	0.142	0.994
		.59	1.143	1.145	1.001	0.584	0.583	0.999	0.242	0.242	0.999	0.113	0.114	1.000
.59	.15	.15	0.746	0.674	0.879	0.247	0.233	0.945	0.085	0.083	0.977	0.035	0.040	0.986
		.39	0.534	0.511	0.910	0.199	0.194	0.939	0.060	0.063	0.977	0.033	0.037	0.986
		.59	0.395	0.347	0.885	0.154	0.131	0.942	0.052	0.043	0.969	0.022	0.021	0.985
	.39	.15	2.569	2.299	0.894	1.195	1.140	0.952	0.442	0.439	0.981	0.222	0.221	0.990
		.39	2.349	2.153	0.916	0.935	0.891	0.952	0.340	0.334	0.981	0.166	0.165	0.990
		.59	1.394	1.230	0.919	0.645	0.598	0.958	0.235	0.231	0.983	0.111	0.110	0.991
	.59	.15	5.158	4.824	0.935	2.584	2.501	0.966	0.870	0.861	0.986	0.450	0.447	0.993
		.39	3.855	3.683	0.946	1.820	1.768	0.972	0.708	0.701	0.989	0.354	0.352	0.995
		.59	2.053	2.038	0.997	0.971	0.979	0.999	0.417	0.415	0.999	0.176	0.176	1.000

Table 5. *Percentile bootstrap CI coverage total effect* $\hat{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	99.5	5	0	98.8	8	4	95.9	17	24	95.4	18	28
		.39	96.3	12	25	94.9	11	40	95.7	23	20	94.6	26	28
		.59	96.0	19	21	93.7	26	37	94.5	28	27	94.8	22	30
	.39	.15	98.4	15	1	96.0	28	12	95.8	20	22	94.7	22	31
		.39	95.9	25	16	95.1	13	36	95.4	23	23	95.0	20	30
		.59	94.2	16	42	95.3	19	28	94.3	28	29	94.9	25	26
	.59	.15	97.9	20	1	97.2	21	7	94.8	26	26	95.0	23	27
		.39	96.1	19	20	93.3	29	38	95.0	25	25	95.9	25	16
		.59	94.3	25	32	94.0	22	38	95.2	24	24	95.5	20	25
.39	.15	.15	98.4	15	1	98.3	10	7	96.0	19	21	95.3	23	24
		.39	96.1	10	29	94.7	18	35	93.0	31	39	95.3	18	29
		.59	96.0	17	23	95.6	21	23	95.4	24	22	94.2	32	26
	.39	.15	94.8	11	41	94.1	19	40	94.8	26	26	95.6	18	26
		.39	95.9	18	23	94.5	20	35	94.2	30	28	96.1	19	20
		.59	94.6	15	39	95.8	14	28	94.6	20	34	94.2	33	25
	.59	.15	95.8	18	24	93.9	20	41	95.0	22	28	95.6	13	31
		.39	95.1	14	35	94.5	24	31	95.0	28	22	95.0	17	33
		.59	94.3	28	29	95.6	18	26	94.4	28	28	94.7	31	22
.59	.15	.15	97.9	20	1	97.3	27	0	95.3	33	14	95.5	29	16
		.39	97.1	19	10	93.8	26	36	95.3	23	24	95.0	34	16
		.59	95.0	19	31	95.2	20	28	93.6	26	38	94.7	28	25
	.39	.15	96.1	13	26	95.5	27	18	95.1	29	20	94.9	29	22
		.39	95.9	14	27	94.3	25	32	94.7	28	25	94.8	25	27
		.59	94.3	22	35	95.7	14	29	96.0	22	18	94.1	30	29
	.59	.15	95.0	21	29	94.3	29	28	96.1	15	24	94.9	27	24
		.39	93.8	20	42	96.0	15	25	94.8	31	21	94.0	26	34
		.59	93.4	23	43	96.8	18	1 4	95.3	24	23	96.1	19	20

Table 6. *Percentile bootstrap CI coverage total effect* $\tilde{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	100.0	0	0	97.6	4	20	95.7	6	37	94.7	15	38
		.39	93.7	6	57	93.9	6	55	94.8	19	33	94.0	21	39
		.59	94.4	8	48	93.6	12	52	94.4	19	37	94.6	18	36
	.39	.15	99.5	2	3	96.3	9	28	95.0	15	35	93.8	22	40
		.39	94.3	11	46	93.8	10	52	94.7	22	31	94.8	18	34
		.59	93.3	9	58	94.5	14	41	95.1	18	31	94.7	22	31
	.59	.15	99.5	3	2	96.5	9	26	93.6	21	43	94.8	18	34
		.39	94.7	13	40	93.3	16	51	94.6	18	36	95.8	21	21
		.59	94.1	16	43	93.4	16	50	94.9	20	31	95.9	16	25
.39	.15	.15	99.2	4	4	97.7	8	15	94.8	16	36	95.1	16	33
		.39	94.7	5	48	94.2	11	47	93.3	20	47	95.3	14	33
		.59	94.4	12	44	95.1	12	37	95.1	21	28	94.3	25	32
	.39	.15	93.3	3	64	92.8	11	61	95.1	16	33	95.0	16	34
		.39	95.4	11	35	94.6	11	43	94.2	23	35	95.9	16	25
		.59	94.1	8	51	94.8	10	42	94.1	17	42	93.9	32	29
	.59	.15	94.6	11	43	94.1	11	48	95.4	14	32	95.0	11	39
		.39	94.5	8	47	94.1	18	41	94.0	26	34	94.8	15	37
		.59	93.8	21	41	95.4	14	32	94.8	21	31	94.5	28	27
.59	.15	.15	99.5	4	1	98.2	14	4	94.4	24	32	95.8	19	23
		.39	96.0	13	27	93.2	15	53	95.3	14	33	95.6	26	18
		.59	93.4	11	55	94.5	18	37	93.4	23	43	94.6	26	28
	.39	.15	94.6	5	49	95.2	17	31	95.0	22	28	94.7	26	27
		.39	94.9	8	43	93.8	22	40	94.6	23	31	94.8	21	31
		.59	93.5	13	52	95.2	13	35	95.3	20	27	93.9	29	32
	.59	.15	93.4	13	53	94.3	21	36	95.5	12	33	94.4	26	30
		.39	93.4	13	53	95.7	12	31	95.0	26	24	94.0	24	36
		.59	93.8	10	52	94.8	22	30	95.3	20	27	95.7	17	26

Table 7. *Percentile bootstrap CI coverage specific effect* $\hat{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	99.2	7	1	98.6	13	1	96.7	14	19	93.3	25	42
		.39	99.1	8	1	98.8	11	1	96.2	15	23	96.1	10	29
		.59	99.0	10	0	98.1	10	9	95.3	13	34	94.4	17	39
	.39	.15	98.4	15	1	97.2	19	9	95.1	27	22	94.2	28	30
		.39	97.4	22	4	97.9	18	3	95.3	27	20	95.0	18	32
		.59	96.9	27	4	96.9	28	3	95.0	24	26	95.6	23	21
	.59	.15	97.4	25	1	96.8	28	4	94.9	25	26	95.7	22	21
		.39	97.2	28	0	97.1	28	1	95.2	26	22	96.6	21	13
		.59	97.3	27	0	98.0	20	0	95.0	28	22	94.8	25	27
.39	.15	.15	97.7	18	5	98.3	15	2	95.2	30	18	94.7	24	29
		.39	98.7	12	1	97.6	15	9	94.8	19	33	95.5	21	24
		.59	97.6	16	8	97.1	19	10	95.7	18	25	94.3	22	35
	.39	.15	93.6	17	47	94.0	13	47	95.3	21	26	95.3	16	31
		.39	93.8	21	41	94.5	16	39	93.7	25	38	96.3	19	18
		.59	95.4	13	33	94.3	23	34	96.6	13	21	93.7	33	30
	.59	.15	94.0	25	35	94.8	20	32	95.2	23	25	94.4	17	39
		.39	93.9	16	45	94.5	24	31	94.6	22	32	94.9	31	20
		.59	94.2	21	37	93.2	31	37	93.1	40	29	94.7	25	28
.59	.15	.15	97.1	27	2	98.0	20	0	96.6	31	3	95.5	29	16
		.39	97.5	25	0	96.9	31	0	96.5	24	11	94.2	36	22
		.59	97.4	26	0	97.1	24	5	95.3	20	27	94.1	23	36
	.39	.15	96.3	14	23	94.7	29	24	95.1	30	19	94.4	32	24
		.39	94.7	21	32	94.2	25	33	95.7	22	21	95.3	23	24
		.59	94.3	20	37	94.7	25	28	94.3	20	37	94.0	29	31
	.59	.15	95.3	14	33	94.0	31	29	96.2	16	22	94.4	28	28
		.39	94.0	15	45	94.8	22	30	94.1	28	31	94.5	29	26
		.59	94.7	15	38	94.9	19	32	95.3	28	19	96.1	22	17

Table 8. *Percentile bootstrap CI coverage specific effect* $\tilde{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	99.8	0	2	95.7	3	40	94.4	3	53	92.8	10	69
		.39	99.5	0	5	95.7	2	41	94.4	3	53	94.1	7	52
		.59	98.6	2	12	95.2	0	48	93.0	4	66	93.3	10	57
	.39	.15	97.6	7	17	96.0	9	31	94.7	13	40	94.4	17	39
		.39	98.5	5	10	97.9	8	13	95.2	14	34	93.9	10	51
		.59	98.3	8	9	97.8	10	12	94.2	12	46	95.2	15	33
	.59	.15	99.4	5	1	98.0	9	11	93.6	18	46	95.0	18	32
		.39	98.9	9	2	97.8	11	11	94.2	14	44	96.6	15	19
		.59	98.8	11	1	99.1	8	1	94.0	20	40	94.7	17	36
.39	.15	.15	98.2	4	14	97.6	6	18	94.1	19	40	94.6	17	37
		.39	98.4	3	13	98.3	2	15	93.2	15	53	94.6	14	40
		.59	97.1	1	28	95.3	8	39	95.4	6	40	93.5	16	49
	.39	.15	90.9	9	82	92.9	10	61	95.5	15	30	94.9	11	40
		.39	92.2	10	68	93.6	10	54	93.2	21	47	96.5	14	21
		.59	92.9	4	67	93.9	17	44	95.6	11	33	93.7	31	32
	.59	.15	93.4	14	52	94.0	11	49	95.3	18	29	93.7	16	47
		.39	93.2	7	61	94.0	14	46	94.5	19	36	95.7	21	22
		.59	93.7	11	52	93.3	23	44	93.3	35	32	94.4	22	34
.59	.15	.15	99.2	6	2	98.5	14	1	96.9	18	13	95.6	18	26
		.39	98.8	10	2	98.2	13	5	95.7	12	31	94.1	26	33
		.59	98.5	13	2	97.7	10	13	94.3	14	43	94.2	16	42
	.39	.15	95.1	7	42	94.3	19	38	94.4	25	31	94.1	29	30
		.39	93.1	11	58	93.7	20	43	95.4	17	29	95.3	17	30
		.59	93.0	12	58	94.7	18	35	94.1	19	40	94.0	26	34
	.59	.15	94.7	8	45	94.0	23	37	96.2	13	25	94.6	25	29
		.39	93.3	10	57	95.3	13	34	94.4	22	34	94.8	25	27
		.59	93.4	14	52	94.6	16	38	95.0	24	26	95.9	18	23

Table 9. *BCa bootstrap CI coverage total effect $\hat{\upsilon}$*

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\cdot\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\cdot\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	81.6	15	169	84.6	16	138	95.0	23	27	96.2	21	17
		.39	93.5	17	48	94.6	19	35	96.0	25	15	94.9	31	20
		.59	95.2	23	25	93.6	34	30	94.4	32	24	94.8	23	29
	.39	.15	84.3	19	138	88.0	31	89	95.1	26	23	94.6	24	30
		.39	93.1	31	38	95.8	18	24	95.4	26	20	95.2	22	26
		.59	94.1	21	38	95.1	26	23	94.7	28	25	94.8	27	25
	.59	.15	85.9	28	113	87.1	31	98	93.9	27	34	95.0	24	26
		.39	92.7	22	51	93.0	33	37	94.9	25	26	95.3	30	17
		.59	94.0	29	31	93.7	25	38	94.9	26	25	95.9	19	22
.39	.15	.15	83.7	21	142	88.8	12	100	94.6	24	30	95.2	26	22
		.39	92.9	16	55	95.4	18	28	93.5	33	32	95.3	21	26
		.59	95.6	25	19	96.0	21	19	95.4	27	19	94.7	28	25
	.39	.15	92.4	16	60	94.6	25	29	95.2	30	18	95.4	23	23
		.39	96.1	25	14	94.8	21	31	93.9	36	25	95.9	21	20
		.59	94.4	21	35	95.8	16	26	94.6	24	30	94.2	34	24
	.59	.15	95.4	23	23	94.5	26	29	95.3	23	24	95.2	19	29
		.39	95.1	18	31	94.9	26	25	94.9	30	21	94.5	22	33
		.59	93.8	31	31	95.3	21	26	94.1	29	30	94.9	29	22
.59	.15	.15	81.0	33	157	84.4	33	123	91.4	35	51	95.1	31	18
		.39	89.0	30	80	92.6	30	44	95.5	23	22	94.9	35	16
		.59	94.5	27	28	94.7	25	28	94.0	26	34	94.4	31	25
	.39	.15	94.0	17	43	95.2	32	16	94.9	29	22	94.6	32	22
		.39	95.8	20	22	94.7	28	25	94.5	31	24	94.8	25	27
		.59	94.8	22	30	95.5	17	28	95.6	26	18	93.9	32	29
	.59	.15	95.2	23	25	94.8	34	18	96.1	22	17	94.5	30	25
		.39	94.2	25	33	95.9	17	24	95.0	29	21	94.2	25	33
		.59	93.1	25	44	94.7	29	24	95.1	27	22	96.1	17	22

Table 10. *BCa bootstrap CI coverage total effect* $\tilde{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\cdot\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\cdot\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	97.2	1	27	85.9	4	137	95.8	12	30	95.5	19	26
		.39	93.0	11	59	95.1	11	38	95.9	22	19	94.6	28	26
		.59	95.3	19	28	93.3	28	39	94.5	29	26	94.5	23	32
	.39	.15	83.6	6	158	86.2	17	121	95.1	18	31	94.0	22	38
		.39	94.1	17	42	95.2	13	35	95.3	24	23	94.9	19	32
		.59	94.1	20	39	94.8	23	29	94.8	23	29	94.6	26	28
	.59	.15	82.0	10	170	86.7	10	123	92.8	22	50	94.9	17	34
		.39	92.1	15	64	92.9	24	47	94.7	21	32	95.5	23	22
		.59	94.3	25	32	93.5	22	43	94.6	26	28	95.9	17	24
.39	.15	.15	82.0	9	171	85.1	9	140	94.3	16	41	95.0	19	31
		.39	93.0	8	62	95.1	14	35	93.3	29	38	95.4	16	30
		.59	95.7	20	23	96.0	18	22	95.5	24	21	94.5	27	28
	.39	.15	92.2	8	70	94.7	17	36	95.6	20	24	94.9	21	30
		.39	96.8	16	16	94.7	17	36	94.1	28	31	95.8	16	26
		.59	94.0	20	40	95.3	15	32	94.1	24	35	93.8	34	28
	.59	.15	95.0	15	35	94.4	18	38	95.3	16	31	95.0	16	34
		.39	95.1	10	39	94.5	22	33	94.4	28	28	94.7	18	35
		.59	93.3	28	39	95.2	20	28	94.1	29	30	94.9	28	23
.59	.15	.15	77.4	6	220	80.6	15	179	91.0	24	66	95.6	21	23
		.39	88.5	18	97	93.0	20	50	95.5	17	28	95.4	30	16
		.59	94.9	21	30	94.9	20	31	94.0	24	36	94.5	28	27
	.39	.15	93.4	9	57	95.4	20	26	95.1	23	26	94.5	28	27
		.39	95.8	15	27	94.5	22	33	94.5	27	28	94.9	20	31
		.59	94.6	18	36	95.6	16	28	95.6	24	20	93.9	30	31
	.59	.15	94.5	18	37	94.7	27	26	95.9	16	25	94.8	26	26
		.39	94.0	18	42	95.6	16	28	95.0	28	22	94.1	24	35
		.59	92.8	23	49	94.3	29	28	95.3	25	22	95.9	16	25

Table 11. *BCa bootstrap CI coverage specific effect* $\hat{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\cdot\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\cdot\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	82.0	14	166	83.2	23	145	92.0	23	57	94.0	30	30
		.39	80.5	13	182	85.5	17	128	92.2	23	55	96.0	20	20
		.59	80.2	19	179	84.9	17	134	93.6	21	43	94.8	26	26
	.39	.15	82.7	24	149	84.6	34	120	91.7	29	54	93.5	36	29
		.39	83.8	32	130	86.1	28	111	93.0	30	40	94.2	24	34
		.59	84.1	41	118	86.5	38	97	90.8	31	61	95.7	23	20
	.59	.15	84.4	30	126	84.6	32	122	91.5	30	55	95.4	25	21
		.39	83.4	32	134	83.9	34	127	91.4	27	59	96.9	21	10
		.59	82.8	30	142	87.7	24	99	91.8	31	51	94.7	26	27
.39	.15	.15	84.5	22	133	85.7	27	116	90.5	34	61	94.5	25	30
		.39	85.2	22	126	86.3	24	113	91.6	25	59	95.5	22	23
		.59	86.1	21	118	86.8	29	103	94.2	26	32	94.7	27	26
	.39	.15	92.1	21	58	94.7	19	34	95.6	27	17	95.5	19	26
		.39	94.2	25	33	94.9	21	30	94.1	32	27	96.3	23	14
		.59	94.6	23	31	94.7	29	24	96.4	19	17	94.3	35	22
	.59	.15	94.5	28	27	94.6	26	28	95.3	26	21	94.5	19	36
		.39	93.4	23	43	94.7	28	25	95.2	25	23	94.7	33	20
		.59	92.7	29	44	92.8	37	35	93.1	40	29	94.6	28	26
.59	.15	.15	83.1	31	138	82.9	23	148	88.4	35	81	94.5	30	25
		.39	83.4	30	136	84.4	32	124	90.6	28	66	93.3	39	28
		.59	85.8	31	111	86.3	26	111	92.9	24	47	94.3	23	34
	.39	.15	93.2	21	47	93.9	38	23	94.5	32	23	94.9	34	17
		.39	93.4	26	40	94.3	26	31	95.7	24	19	95.7	24	19
		.59	94.4	27	29	94.9	30	21	94.1	25	34	94.0	31	29
	.59	.15	95.3	20	27	95.2	30	18	96.1	19	20	94.3	31	26
		.39	94.9	18	33	95.3	24	23	95.1	27	22	94.3	32	25
		.59	95.0	18	32	94.5	23	32	95.3	30	17	96.0	23	17

Table 12. *BCa bootstrap CI coverage specific effect* $\tilde{\upsilon}$

			N=50			N=100			N=250			N=500		
β_{m1x}	$\beta_{ym1\bullet\mathbf{x}}$	$\beta_{m2x}=\beta_{ym2\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	84.0	0	160	76.1	4	235	87.0	8	122	93.8	19	43
		.39	85.2	0	148	77.1	8	221	87.4	8	118	96.5	9	26
		.59	82.5	2	173	77.7	1	222	90.8	6	86	94.2	18	40
	.39	.15	69.3	10	297	77.7	13	210	91.2	20	68	93.9	18	43
		.39	72.0	10	270	80.4	12	184	92.0	19	61	93.4	14	52
		.59	72.9	12	259	82.1	17	162	90.5	13	82	95.0	16	34
	.59	.15	75.4	9	237	81.4	9	177	91.1	18	71	94.8	17	35
		.39	76.0	14	226	82.7	12	161	91.3	16	71	96.7	16	17
		.59	76.1	14	225	85.3	10	137	92.6	17	57	94.7	18	35
.39	.15	.15	70.5	9	286	76.6	8	226	88.9	22	89	94.7	16	37
		.39	72.9	4	267	79.9	3	198	90.7	17	76	94.8	18	34
		.59	73.5	9	256	82.3	13	164	94.3	12	45	93.5	19	46
	.39	.15	90.2	13	85	94.5	15	40	95.4	19	27	95.2	14	34
		.39	93.6	14	50	94.8	14	38	94.5	22	33	96.5	18	17
		.59	94.2	8	50	94.9	24	27	95.8	12	30	94.0	33	27
	.59	.15	93.7	22	41	94.3	19	38	95.4	21	25	93.8	17	45
		.39	93.2	9	59	93.9	23	38	94.6	21	33	95.1	27	22
		.59	92.6	18	56	93.8	22	40	92.9	37	34	93.9	25	36
.59	.15	.15	70.0	9	291	75.9	15	226	88.1	17	102	94.1	21	38
		.39	72.8	12	260	79.3	17	190	89.9	15	86	93.2	30	38
		.59	74.7	14	239	82.6	11	163	92.8	15	57	93.8	18	44
	.39	.15	92.1	10	69	94.8	22	30	94.7	25	28	94.8	30	22
		.39	92.5	17	58	94.3	21	36	95.5	22	23	95.5	20	25
		.59	95.3	15	32	95.1	20	29	94.1	21	38	93.8	30	32
	.59	.15	95.4	10	36	94.7	26	27	95.9	16	25	94.7	26	27
		.39	94.7	11	42	95.2	21	27	94.7	25	28	94.3	31	26
		.59	94.3	16	41	94.7	18	35	95.5	25	20	96.0	22	18

Table 13. *Prototypical conditional indirect effects with unadjusted effect size and expected bias*

Case	Indirect effect	υ	$\hat{\Delta}'_{bias}$
1	$B_{mx\bullet\mathbf{x}}(B_{ym\bullet\mathbf{x}}+B_{ymx\bullet\mathbf{x}}\eta_x)$	$\beta^2_{mx}\beta^2_{ym\bullet\mathbf{x}}+\beta^2_{mx}\beta^2_{ymx\bullet\mathbf{x}}\eta^2_x\sigma^2_{x^2}+2\beta^2_{mx}\beta_{ym\bullet\mathbf{x}}\beta_{ymx\bullet\mathbf{x}}\eta_x\sigma_{x^2,x}$	$\hat{\beta}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}+\hat{\beta}^2_{mx}\hat{\sigma}^2_{ym\bullet\mathbf{x}}+\hat{\beta}^2_{ymx\bullet\mathbf{x}}\eta^2_x\hat{\sigma}^2_{mx}+\hat{\beta}^2_{mx}\eta^2_x\hat{\sigma}^2_{ymx\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymx\bullet\mathbf{x}}\eta_x\hat{\sigma}^2_{mx}+$ $2\hat{\beta}^2_{mx}\eta_x\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}}-\hat{\sigma}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-2\eta_x\hat{\sigma}_{ym\bullet\mathbf{x},ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-\eta^2_x\hat{\sigma}^2_{ymx\bullet\mathbf{x}}\hat{\sigma}^2_{mx}$
2	$(B_{mx\bullet\mathbf{x}}+B_{mxw\bullet\mathbf{x}}\eta_w)B_{ym\bullet\mathbf{x}}$	$\beta^2_{mx\bullet\mathbf{x}}\beta^2_{ym\bullet x}+\beta^2_{ym\bullet x}\beta^2_{mxw\bullet\mathbf{x}}\eta^2_w\sigma^2_{xw^2}+2\beta_{mx\bullet\mathbf{x}}\beta_{mxw\bullet\mathbf{x}}\beta^2_{ym\bullet x}\eta_w\sigma_{x^2,w}$	$\hat{\beta}^2_{ym\bullet x}\hat{\sigma}^2_{mx\bullet\mathbf{x}}+\hat{\beta}^2_{mx\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet x}+\hat{\beta}^2_{mxw\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{ym\bullet x}+\hat{\beta}^2_{ym\bullet x}\eta^2_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}+2\hat{\beta}^2_{ym\bullet x}\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+$ $2\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mxw\bullet\mathbf{x}}\eta_w\hat{\sigma}^2_{ym\bullet x}-\hat{\sigma}^2_{ym\bullet x}\hat{\sigma}^2_{mx\bullet\mathbf{x}}-2\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet x}-\eta^2_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet x}$
3	$B_{mx\bullet\mathbf{x}}(B_{ym\bullet\mathbf{x}}+B_{ymz\bullet\mathbf{x}}\eta_z)$	$\beta^2_{mx}\beta^2_{ym\bullet\mathbf{x}}+\beta^2_{mx}\beta^2_{ymz\bullet\mathbf{x}}\eta^2_z\sigma^2_{xz^2}+2\beta^2_{mx}\beta_{ym\bullet\mathbf{x}}\beta_{ymz\bullet\mathbf{x}}\eta_z\sigma_{x^2,z}$	$\hat{\beta}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}+\hat{\beta}^2_{mx}\hat{\sigma}^2_{ym\bullet\mathbf{x}}+\hat{\beta}^2_{ymz\bullet\mathbf{x}}\eta^2_z\hat{\sigma}^2_{mx}+\hat{\beta}^2_{mx}\eta^2_z\hat{\sigma}^2_{ymz\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymz\bullet\mathbf{x}}\eta_z\hat{\sigma}^2_{mx}$ $+2\hat{\beta}^2_{mx}\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}-\hat{\sigma}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-2\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}\hat{\sigma}^2_{mx}-\eta^2_z\hat{\sigma}^2_{ymz\bullet\mathbf{x}}\hat{\sigma}^2_{mx}$
4	$(B_{mx\bullet\mathbf{x}}+B_{mxw\bullet\mathbf{x}}\eta_w)(B_{ym\bullet\mathbf{x}}+B_{ymz\bullet\mathbf{x}}\eta_z)$	$\beta^2_{mx\bullet\mathbf{x}}\beta^2_{ym\bullet\mathbf{x}}+2\beta_{mx\bullet\mathbf{x}}\beta_{mxw\bullet\mathbf{x}}\beta^2_{ym\bullet x}\eta_w\sigma_{x^2,w}+$ $\beta^2_{ym\bullet\mathbf{x}}\beta^2_{mxw\bullet\mathbf{x}}\eta^2_w\sigma^2_{xw^2}+\beta^2_{mx\bullet\mathbf{x}}\beta^2_{ymz\bullet\mathbf{x}}\eta^2_z\sigma^2_{xz^2}+$ $2\beta^2_{mx\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\beta_{ymz\bullet\mathbf{x}}\eta_z\sigma_{x^2,z}+4\beta_{mx\bullet\mathbf{x}}\beta_{mwx\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\beta_{ymz\bullet\mathbf{x}}\eta_w\eta_z\sigma_{xw,xz}+$ $2\beta_{ym\bullet\mathbf{x}}\beta^2_{mwx\bullet\mathbf{x}}\beta_{ymz\bullet\mathbf{x}}\eta^2_w\eta_z\sigma_{xw^2,xz}+2\beta_{mx\bullet\mathbf{x}}\beta_{mwx\bullet\mathbf{x}}\beta^2_{ymz\bullet\mathbf{x}}\eta_w\eta^2_z\sigma_{xw,xz^2}+$ $\beta_{mwx\bullet\mathbf{x}}\beta_{ymz\bullet\mathbf{x}}\eta^2_w\eta^2_z\sigma_{xw^2,xz^2}$	$\hat{\beta}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx\bullet\mathbf{x}}+\hat{\beta}^2_{mx\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet\mathbf{x}}-\hat{\sigma}^2_{ym\bullet x}\hat{\sigma}^2_{mx\bullet\mathbf{x}}+2\hat{\beta}^2_{ym\bullet x}\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+$ $2\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mxw\bullet\mathbf{x}}\eta_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}-2\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet\mathbf{x}}+\hat{\beta}^2_{mxw\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}+\hat{\beta}^2_{ym\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}-$ $\eta^2_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mxw\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymz\bullet\mathbf{x}}\eta_z\hat{\sigma}^2_{mx\bullet\mathbf{x}}+2\hat{\beta}^2_{mx\bullet\mathbf{x}}\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}-2\eta_z\hat{\sigma}^2_{mx\bullet\mathbf{x}}\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}+$ $4\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymz\bullet\mathbf{x}}\eta_w\eta_z\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+4\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mwx\bullet\mathbf{x}}\eta_w\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}-$ $4\eta_w\eta_z\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}+2\hat{\beta}^2_{mwx\bullet\mathbf{x}}\eta^2_w\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymz\bullet\mathbf{x}}\eta^2_w\eta_z\hat{\sigma}^2_{mwx\bullet\mathbf{x}}-$ $2\eta^2_w\eta_z\hat{\sigma}_{ym\bullet\mathbf{x},ymz\bullet\mathbf{x}}\hat{\sigma}^2_{mwx\bullet\mathbf{x}}+\hat{\beta}^2_{ymz\bullet\mathbf{x}}\eta^2_z\hat{\sigma}^2_{mx\bullet\mathbf{x}}+\hat{\beta}^2_{mx\bullet\mathbf{x}}\eta^2_z\hat{\sigma}^2_{ymz\bullet\mathbf{x}}-\eta^2_z\hat{\sigma}^2_{mx\bullet\mathbf{x}}\hat{\sigma}^2_{ymz\bullet\mathbf{x}}+$ $2\hat{\beta}^2_{ymz\bullet\mathbf{x}}\eta_w\eta^2_z\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+2\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mwx\bullet\mathbf{x}}\eta_w\eta^2_z\hat{\sigma}^2_{ymz\bullet\mathbf{x}}-2\eta_w\eta^2_z\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ymz\bullet\mathbf{x}}+$ $\hat{\beta}^2_{ymz\bullet\mathbf{x}}\eta^2_w\eta^2_z\hat{\sigma}^2_{mxw\bullet\mathbf{x}}+\hat{\beta}^2_{mxw\bullet\mathbf{x}}\eta^2_w\eta^2_z\hat{\sigma}^2_{ymz\bullet\mathbf{x}}-\eta^2_w\eta^2_z\hat{\sigma}^2_{mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ymz\bullet\mathbf{x}}$
5	$(B_{mx\bullet\mathbf{x}}+B_{mxw\bullet\mathbf{x}}\eta_w)(B_{ym\bullet\mathbf{x}}+B_{ymz\bullet\mathbf{x}}\eta_w)$	$\beta^2_{mx\bullet\mathbf{x}}\beta^2_{ym\bullet\mathbf{x}}+2\beta_{mx\bullet\mathbf{x}}\beta_{mxw\bullet\mathbf{x}}\beta^2_{ym\bullet x}\eta_w\sigma_{x^2,w}+\beta^2_{ym\bullet\mathbf{x}}\beta^2_{mxw\bullet\mathbf{x}}\eta^2_w\sigma^2_{xw^2}+$ $\beta^2_{mx\bullet\mathbf{x}}\beta^2_{ymw\bullet\mathbf{x}}\eta^2_w\sigma^2_{xw^2}+2\beta^2_{mx\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\beta_{ymw\bullet\mathbf{x}}\eta_w\sigma_{x^2,w}+$ $4\beta_{mx\bullet\mathbf{x}}\beta_{mwx\bullet\mathbf{x}}\beta_{ym\bullet\mathbf{x}}\beta_{ymw\bullet\mathbf{x}}\eta^2_w\sigma^2_{xw}+2\beta_{ym\bullet\mathbf{x}}\beta^2_{mwx\bullet\mathbf{x}}\beta_{ymw\bullet\mathbf{x}}\eta^3_w\sigma_{xw^2,xw}+$ $2\beta_{mx\bullet\mathbf{x}}\beta_{mwx\bullet\mathbf{x}}\beta^2_{ymw\bullet\mathbf{x}}\eta^3_w\sigma_{xw^2,xw}+\beta_{mwx\bullet\mathbf{x}}\beta_{ymw\bullet\mathbf{x}}\eta^4_w\sigma^2_{xw^2}$	$\hat{\beta}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mx\bullet\mathbf{x}}+\hat{\beta}^2_{mx\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet\mathbf{x}}-\hat{\sigma}^2_{ym\bullet x}\hat{\sigma}^2_{mx\bullet\mathbf{x}}+2\hat{\beta}^2_{ym\bullet x}\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+$ $2\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mxw\bullet\mathbf{x}}\eta_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}-2\eta_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ym\bullet\mathbf{x}}+\hat{\beta}^2_{mxw\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}+$ $\hat{\beta}^2_{ym\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}-\eta^2_w\hat{\sigma}^2_{ym\bullet\mathbf{x}}\hat{\sigma}^2_{mxw\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymw\bullet\mathbf{x}}\eta_w\hat{\sigma}^2_{mx\bullet\mathbf{x}}+2\hat{\beta}^2_{mx\bullet\mathbf{x}}\eta_w\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}-$ $2\eta_w\hat{\sigma}^2_{mx\bullet\mathbf{x}}\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}+4\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymw\bullet\mathbf{x}}\eta^2_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+4\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mwx\bullet\mathbf{x}}\eta^2_w\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}-$ $4\eta^2_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}+2\hat{\beta}^2_{mwx\bullet\mathbf{x}}\eta^3_w\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}+2\hat{\beta}_{ym\bullet\mathbf{x}}\hat{\beta}_{ymw\bullet\mathbf{x}}\eta^3_w\hat{\sigma}^2_{mwx\bullet\mathbf{x}}-$ $2\eta^3_w\hat{\sigma}_{ym\bullet\mathbf{x},ymw\bullet\mathbf{x}}\hat{\sigma}^2_{mwx\bullet\mathbf{x}}+\hat{\beta}^2_{ymw\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{mx\bullet\mathbf{x}}+\hat{\beta}^2_{mx\bullet\mathbf{x}}\eta^2_w\hat{\sigma}^2_{ymw\bullet\mathbf{x}}-\eta^2_w\hat{\sigma}^2_{mx\bullet\mathbf{x}}\hat{\sigma}^2_{ymw\bullet\mathbf{x}}+$ $2\hat{\beta}^2_{ymw\bullet\mathbf{x}}\eta^3_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}+2\hat{\beta}_{mx\bullet\mathbf{x}}\hat{\beta}_{mwx\bullet\mathbf{x}}\eta^3_w\hat{\sigma}^2_{ymw\bullet\mathbf{x}}-2\eta^3_w\hat{\sigma}_{mx\bullet\mathbf{x},mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ymw\bullet\mathbf{x}}+$ $\hat{\beta}^2_{ymw\bullet\mathbf{x}}\eta^4_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}+\hat{\beta}^2_{mxw\bullet\mathbf{x}}\eta^4_w\hat{\sigma}^2_{ymw\bullet\mathbf{x}}-\eta^4_w\hat{\sigma}^2_{mxw\bullet\mathbf{x}}\hat{\sigma}^2_{ymw\bullet\mathbf{x}}$

Table 14. *Percent relative bias for moderated mediation model at the mean of x*

β_{mx}	$\beta_{ym \cdot \mathbf{x}}$	$\beta_{ymx \cdot \mathbf{x}}$	$\hat{\upsilon}$ N=50	$\hat{\upsilon}$ N=100	$\hat{\upsilon}$ N=250	$\hat{\upsilon}$ N=500	$\tilde{\upsilon}$ N=50	$\tilde{\upsilon}$ N=100	$\tilde{\upsilon}$ N=250	$\tilde{\upsilon}$ N=500
.15	.15	.15	290.835	104.918	35.019	22.106	-3.977	-7.553	-3.868	3.388
		.39	206.595	103.834	40.712	16.492	-4.438	-1.116	4.431	-0.488
		.59	238.189	95.903	32.693	16.474	-9.883	-0.145	0.279	1.281
	.39	.15	89.855	50.198	18.018	11.840	-24.387	-4.092	-1.936	1.796
		.39	112.770	48.281	17.848	8.790	-3.158	-5.210	-2.101	-0.950
		.59	114.832	57.859	16.687	7.617	0.980	6.682	-2.718	-1.833
	.59	.15	91.134	45.689	19.311	4.795	-9.055	-1.399	0.803	-4.323
		.39	85.549	49.766	17.503	9.539	-15.052	1.608	-0.839	0.429
		.59	86.832	44.202	20.047	8.512	-13.345	-3.436	2.060	-0.485
.39	.15	.15	133.223	60.926	26.377	6.869	-5.211	-0.534	2.948	-4.361
		.39	130.009	56.188	20.757	9.124	7.477	3.041	0.733	-0.591
		.59	113.888	53.806	27.190	6.379	22.241	12.931	11.882	-0.907
	.39	.15	17.177	9.329	3.649	2.363	-10.492	-3.579	-1.250	-0.069
		.39	22.700	8.933	2.288	2.341	-2.939	-2.735	-2.124	0.141
		.59	17.670	8.373	3.919	1.256	-3.832	-1.517	0.165	-0.577
	.59	.15	12.433	6.941	1.731	0.051	-4.822	-1.175	-1.418	-1.603
		.39	13.461	4.726	1.373	2.021	-2.942	-2.833	-1.576	0.566
		.59	10.680	5.078	2.557	0.403	-4.265	-1.745	-0.076	-0.899
.59	.15	.15	150.394	62.749	21.214	14.581	-9.495	-9.462	-6.218	1.052
		.39	132.700	56.855	20.755	8.297	2.639	-1.846	-1.742	-2.862
		.59	101.967	55.838	19.823	10.886	17.721	16.836	4.535	3.406
	.39	.15	20.415	8.402	2.921	3.070	-2.820	-2.506	-1.223	1.013
		.39	16.217	8.770	0.918	1.966	-3.189	-0.170	-2.497	0.269
		.59	16.417	9.385	1.468	2.616	3.146	3.167	-0.868	1.454
	.59	.15	6.512	4.215	1.579	0.867	-3.825	-0.718	-0.310	-0.067
		.39	2.388	4.198	2.110	0.253	-6.163	0.076	0.530	-0.524
		.59	8.712	2.382	1.488	1.929	2.449	-0.484	0.381	1.382

Table 15. *Percent relative bias for moderated mediation model at +1 standard deviation above the mean of x*

			$\hat{\upsilon}$				$\tilde{\upsilon}$			
β_{mx}	$\beta_{ym\cdot\mathbf{x}}$	$\beta_{ymx\cdot\mathbf{x}}$	N=50	N=100	N=250	N=500	N=50	N=100	N=250	N=500
.15	.15	.15	272.146	108.189	34.205	24.001	0.865	0.097	-3.659	4.510
		.39	98.977	52.556	25.485	10.554	-18.906	-3.869	3.663	-0.209
		.59	95.840	50.650	17.250	7.809	-4.284	2.715	-1.738	-1.597
	.39	.15	103.887	57.916	21.373	13.258	-22.184	-1.807	-0.522	2.319
		.39	105.654	46.401	17.350	8.136	-2.740	-4.645	-2.118	-1.497
		.59	95.489	51.802	16.763	8.429	0.689	5.703	-1.699	-0.694
	.59	.15	95.962	48.031	20.074	5.302	-9.235	-1.186	0.845	-4.153
		.39	85.642	51.078	17.948	9.299	-11.943	3.128	-0.483	0.115
		.59	83.751	43.933	19.905	7.745	-8.893	-1.758	2.128	-1.147
.39	.15	.15	102.961	49.137	21.206	6.432	-9.511	-2.894	1.083	-3.319
		.39	26.779	12.915	6.730	1.428	-5.491	-2.137	0.810	-1.472
		.59	10.338	8.038	2.392	0.944	-7.465	-0.718	-1.048	-0.767
	.39	.15	25.004	12.707	5.403	2.238	-10.063	-3.986	-0.993	-0.925
		.39	19.130	7.354	1.830	2.361	-2.849	-3.156	-2.215	0.327
		.59	5.430	4.259	1.525	0.733	-9.776	-3.252	-1.447	-0.738
	.59	.15	15.539	8.614	2.752	0.223	-4.880	-1.139	-1.028	-1.633
		.39	10.111	4.874	2.266	2.298	-6.178	-2.907	-0.824	0.772
		.59	6.667	5.202	2.476	-0.207	-6.654	-1.249	-0.064	-1.471
.59	.15	.15	99.414	45.345	15.264	8.206	-10.046	-5.234	-4.106	-1.368
		.39	19.098	5.395	0.434	2.923	-4.376	-5.550	-3.827	0.800
		.59	1.585	-0.202	0.737	0.220	-7.783	-4.802	-1.092	-0.685
	.39	.15	25.221	10.261	4.166	3.607	-4.352	-3.841	-1.229	0.929
		.39	8.333	3.659	0.857	1.163	-5.786	-3.035	-1.756	-0.138
		.59	-1.238	0.527	-0.023	0.403	-8.394	-2.989	-1.400	-0.284
	.59	.15	8.539	5.382	2.116	1.210	-4.677	-1.032	-0.350	-0.011
		.39	-0.682	2.211	0.797	-0.230	-9.005	-1.884	-0.793	-1.019
		.59	-1.785	-1.053	-0.077	0.308	-6.861	-3.535	-1.063	-0.182

Table 16. *MSE and relative efficiency for conditional indirect effect at mean x*

			N=50			N=100			N=250			N=500		
			MSE*			MSE*			MSE*			MSE*		
β_{mx}	$\beta_{ym\bullet\mathbf{x}}$	$\beta_{ymx\bullet\mathbf{x}}$	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*	$\hat{\upsilon}$	$\tilde{\upsilon}$	*RE*
.15	.15	.15	0.021	0.012	0.630	0.003	0.002	0.645	0.001	<0.001	0.778	<0.001	<0.001	0.872
		.39	0.011	0.006	0.587	0.003	0.002	0.683	0.001	0.001	0.805	<0.001	<0.001	0.873
		.59	0.015	0.009	0.637	0.003	0.002	0.702	0.001	0.000	0.814	<0.001	<0.001	0.889
	.39	.15	0.112	0.081	0.784	0.042	0.034	0.866	0.011	0.010	0.938	0.005	0.005	0.966
		.39	0.149	0.109	0.812	0.038	0.031	0.880	0.011	0.010	0.946	0.005	0.005	0.969
		.59	0.134	0.101	0.850	0.041	0.034	0.902	0.011	0.010	0.949	0.005	0.004	0.972
	.59	.15	0.392	0.317	0.929	0.161	0.140	0.948	0.054	0.050	0.980	0.022	0.022	0.989
		.39	0.391	0.321	0.924	0.171	0.149	0.957	0.050	0.048	0.985	0.021	0.020	0.989
		.59	0.341	0.285	0.963	0.132	0.119	0.990	0.048	0.045	0.988	0.023	0.022	0.994
.39	.15	.15	0.181	0.127	0.795	0.048	0.037	0.858	0.015	0.013	0.942	0.005	0.005	0.968
		.39	0.165	0.112	0.771	0.045	0.036	0.872	0.013	0.012	0.937	0.005	0.005	0.967
		.59	0.129	0.090	0.789	0.040	0.032	0.879	0.014	0.012	0.941	0.005	0.004	0.971
	.39	.15	0.683	0.569	0.844	0.297	0.267	0.908	0.114	0.109	0.958	0.052	0.051	0.979
		.39	0.661	0.534	0.843	0.281	0.253	0.910	0.100	0.096	0.963	0.049	0.047	0.981
		.59	0.646	0.548	0.870	0.235	0.213	0.922	0.083	0.080	0.968	0.040	0.040	0.983
	.59	.15	1.990	1.782	0.912	0.881	0.827	0.953	0.289	0.283	0.981	0.153	0.152	0.990
		.39	1.793	1.615	0.926	0.782	0.748	0.962	0.267	0.263	0.985	0.150	0.147	0.992
		.59	1.487	1.396	0.956	0.682	0.658	0.974	0.270	0.265	0.989	0.129	0.129	0.995
.59	.15	.15	1.024	0.809	0.913	0.274	0.239	0.954	0.077	0.073	0.978	0.040	0.038	0.988
		.39	0.796	0.642	0.934	0.206	0.177	0.951	0.066	0.062	0.978	0.032	0.032	0.991
		.59	0.518	0.408	0.894	0.198	0.171	0.948	0.064	0.060	0.980	0.028	0.027	0.989
	.39	.15	3.025	2.677	0.920	1.253	1.179	0.955	0.442	0.431	0.981	0.229	0.225	0.990
		.39	2.438	2.170	0.917	1.033	0.968	0.957	0.366	0.361	0.982	0.185	0.183	0.991
		.59	1.972	1.760	0.926	0.887	0.833	0.963	0.300	0.295	0.983	0.139	0.137	0.992
	.59	.15	5.368	4.965	0.932	2.592	2.479	0.966	0.950	0.932	0.985	0.448	0.444	0.993
		.39	4.105	3.910	0.941	2.183	2.090	0.969	0.785	0.769	0.988	0.381	0.379	0.993
		.59	3.755	3.506	0.960	1.650	1.605	0.977	0.630	0.620	0.989	0.314	0.310	0.995

Table 17. *MSE and relative efficiency for conditional effect at +1 standard deviation above the mean of x*

			50			100			250			500		
			MSE			MSE			MSE			MSE		
β_{mx}	$\beta_{ym\bullet\mathbf{x}}$	$\beta_{ymx\bullet\mathbf{x}}$	$\hat{\upsilon}$	$\tilde{\upsilon}$	RE	$\hat{\upsilon}$	$\tilde{\upsilon}$	RE	$\hat{\upsilon}$	$\tilde{\upsilon}$	RE	$\hat{\upsilon}$	$\tilde{\upsilon}$	RE
.15	.15	.15	0.048	0.022	0.543	0.011	0.006	0.634	0.002	0.001	0.762	0.001	0.001	0.871
		.39	0.164	0.109	0.732	0.055	0.042	0.828	0.019	0.016	0.913	0.007	0.006	0.952
		.59	0.576	0.446	0.874	0.226	0.191	0.917	0.067	0.063	0.967	0.030	0.029	0.982
	.39	.15	0.155	0.100	0.716	0.057	0.043	0.827	0.016	0.014	0.919	0.007	0.007	0.955
		.39	0.426	0.313	0.842	0.136	0.114	0.903	0.041	0.038	0.955	0.019	0.018	0.978
		.59	0.948	0.771	0.929	0.343	0.295	0.957	0.113	0.107	0.978	0.047	0.046	0.989
	.59	.15	0.455	0.350	0.892	0.190	0.160	0.922	0.062	0.057	0.969	0.025	0.025	0.984
		.39	0.800	0.650	0.918	0.358	0.308	0.948	0.107	0.101	0.983	0.044	0.042	0.990
		.59	1.267	1.089	0.996	0.518	0.469	0.998	0.194	0.183	0.996	0.089	0.087	0.998
.39	.15	.15	0.349	0.223	0.763	0.109	0.079	0.829	0.034	0.029	0.926	0.013	0.012	0.958
		.39	1.214	0.948	0.822	0.521	0.454	0.897	0.189	0.177	0.955	0.083	0.081	0.977
		.59	2.912	2.610	0.902	1.429	1.331	0.949	0.548	0.534	0.979	0.244	0.241	0.989
	.39	.15	0.930	0.733	0.821	0.383	0.332	0.893	0.151	0.141	0.951	0.067	0.065	0.975
		.39	2.174	1.833	0.879	0.934	0.859	0.931	0.372	0.362	0.971	0.176	0.173	0.986
		.59	3.346	3.183	0.937	1.869	1.806	0.969	0.715	0.706	0.988	0.342	0.340	0.994
	.59	.15	2.366	2.056	0.895	1.043	0.960	0.942	0.345	0.334	0.976	0.178	0.177	0.987
		.39	3.298	3.058	0.938	1.728	1.666	0.969	0.626	0.616	0.987	0.341	0.336	0.993
		.59	5.698	5.664	0.994	2.892	2.843	0.995	1.223	1.213	0.998	0.565	0.568	0.999
.59	.15	.15	1.680	1.248	0.924	0.536	0.441	0.945	0.155	0.144	0.978	0.080	0.076	0.987
		.39	3.904	3.391	0.919	1.894	1.818	0.959	0.653	0.650	0.982	0.354	0.346	0.992
		.59	7.568	7.374	0.952	3.630	3.607	0.975	1.591	1.577	0.990	0.667	0.665	0.995
	.39	.15	3.725	3.214	0.924	1.534	1.437	0.959	0.547	0.531	0.982	0.282	0.274	0.990
		.39	5.769	5.344	0.935	2.846	2.751	0.969	1.026	1.014	0.985	0.550	0.544	0.993
		.59	8.481	8.507	0.965	4.220	4.194	0.984	1.893	1.891	0.994	0.893	0.890	0.997
	.59	.15	6.201	5.706	0.933	2.899	2.760	0.968	1.074	1.051	0.985	0.502	0.496	0.993
		.39	7.807	7.782	0.959	4.108	4.010	0.977	1.578	1.565	0.992	0.830	0.830	0.996
		.59	10.268	10.640	1.002	5.530	5.625	1.001	2.167	2.175	1.000	1.022	1.021	1.000

Table 18. *Percentile bootstrap CI coverage for* $\hat{\upsilon}$ *conditional effect at the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym\bullet\mathbf{x}}$	$\beta_{ymx\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	98.9	10	1	99.1	9	0	96.1	9	30	95.1	20	29
		.39	99.7	3	0	99.2	8	0	95.8	14	28	95.6	14	30
		.59	99.4	5	1	99.1	9	0	96.3	8	29	95.2	13	35
	.39	.15	98.5	13	2	97.2	21	7	95.8	20	22	94.9	24	27
		.39	98.2	18	0	97.8	18	4	96.2	24	14	94.3	22	35
		.59	97.2	27	1	97.5	23	2	95.7	19	24	96.0	18	22
	.59	.15	97.5	23	2	97.7	23	0	94.7	24	29	94.7	25	28
		.39	97.3	27	0	96.0	39	1	95.8	25	17	96.6	23	11
		.59	97.6	24	0	98.0	20	0	95.1	26	23	94.6	28	26
.39	.15	.15	98.1	17	2	97.3	22	5	96.4	21	15	95.8	17	25
		.39	98.1	18	1	97.2	21	7	95.8	20	22	94.6	19	35
		.59	98.2	15	3	97.9	19	2	95.2	28	20	96.0	13	27
	.39	.15	94.7	11	42	94.3	14	43	94.2	22	36	94.0	26	34
		.39	95.6	14	30	93.7	22	41	93.9	21	40	94.7	29	24
		.59	93.1	30	39	94.9	17	34	95.8	19	23	94.5	25	30
	.59	.15	94.3	22	35	94.3	21	36	95.6	17	27	94.6	22	32
		.39	93.9	21	40	94.4	20	36	96.1	19	20	94.9	27	24
		.59	94.1	24	35	94.3	29	28	94.5	30	25	94.7	22	31
.59	.15	.15	97.7	22	1	97.1	27	2	97.4	24	2	94.3	27	30
		.39	97.1	29	0	98.5	14	1	97.1	23	6	94.1	25	34
		.59	97.5	25	0	97.1	29	0	96.2	26	12	95.5	31	14
	.39	.15	95.7	16	27	94.6	28	26	94.8	30	22	94.4	29	27
		.39	95.1	23	26	94.2	24	34	95.1	22	27	93.9	24	37
		.59	94.3	23	34	93.9	32	29	95.1	18	31	96.8	19	13
	.59	.15	94.7	17	36	94.8	26	26	94.8	21	31	95.0	22	28
		.39	94.8	14	38	94.6	27	27	95.2	21	27	95.7	15	28
		.59	94.5	34	21	94.7	25	28	94.6	32	22	94.3	37	20

Table 19. *Percentile bootstrap CI coverage for $\tilde{\upsilon}$ conditional effect at the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym\bullet\mathbf{x}}$	$\beta_{ymx\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	99.9	1	0	95.5	1	44	93.4	4	62	94.3	12	45
		.39	100.0	0	0	96.7	2	31	93.4	6	60	94.0	8	52
		.59	99.9	1	0	97.5	2	23	94.2	2	56	93.5	7	58
	.39	.15	97.5	4	21	97.1	7	22	94.1	7	52	94.5	18	37
		.39	98.3	5	12	97.7	7	16	96.0	12	28	94.0	11	49
		.59	98.4	5	11	97.4	9	17	93.7	11	52	95.5	13	32
	.59	.15	99.1	5	4	98.2	11	7	94.0	16	44	94.6	16	38
		.39	99.0	7	3	98.2	15	3	94.7	13	40	96.9	16	15
		.59	99.4	5	1	98.9	10	1	94.8	14	38	94.8	17	35
.39	.15	.15	97.8	4	18	97.9	5	16	94.9	15	36	94.5	11	44
		.39	99.0	1	9	97.8	4	18	94.6	14	40	94.6	12	42
		.59	98.2	2	16	97.9	9	12	94.8	17	35	95.4	8	38
	.39	.15	91.2	7	81	93.5	9	56	93.3	20	47	93.8	22	40
		.39	93.3	10	57	91.9	11	70	93.5	16	49	94.4	26	30
		.59	92.0	18	62	95.0	7	43	95.5	15	30	94.9	19	32
	.59	.15	92.8	12	60	93.3	16	51	95.6	14	30	94.4	19	37
		.39	93.0	14	56	94.1	12	47	95.9	15	26	95.1	22	27
		.59	93.0	14	56	93.8	22	40	95.4	17	29	94.4	20	36
.59	.15	.15	99.2	6	2	98.9	11	0	96.6	16	18	93.2	17	51
		.39	99.1	8	1	98.7	9	4	97.2	11	17	94.0	13	47
		.59	99.0	10	0	98.4	15	1	95.0	21	29	95.5	23	22
	.39	.15	94.4	5	51	93.6	20	44	94.9	23	28	94.2	25	33
		.39	93.9	12	49	94.2	18	40	95.3	16	31	93.8	20	42
		.59	93.9	14	47	94.1	26	33	94.4	16	40	96.7	15	18
	.59	.15	93.0	11	59	94.0	22	38	94.5	20	35	94.8	19	33
		.39	93.5	9	56	94.9	18	33	94.9	19	32	95.3	14	33
		.59	94.4	24	32	94.4	20	36	94.7	28	25	94.5	34	21

Table 20. *Percentile bootstrap CI coverage for $\hat{\upsilon}$ conditional effect at +1 standard deviation above the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym\bullet\mathbf{x}}$	$\beta_{ymx\bullet\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	97.5	25	0	97.3	27	0	96.4	11	25	95.7	25	18
		.39	98.3	16	1	97.4	19	7	94.5	29	26	95.6	16	28
		.59	97.3	25	2	97.2	25	3	96.4	21	15	94.1	22	37
	.39	.15	97.3	25	2	97.1	28	1	95.8	23	19	95.1	24	25
		.39	97.4	26	0	97.3	26	1	95.9	25	16	94.1	21	38
		.59	95.6	42	2	97.5	23	2	94.3	26	31	95.6	20	24
	.59	.15	97.3	26	1	97.1	29	0	94.7	25	28	95.5	24	21
		.39	97.2	28	0	96.4	36	0	95.6	22	22	96.7	21	12
		.59	97.1	29	0	97.8	22	0	95.1	29	20	95.0	24	26
.39	.15	.15	96.9	28	3	96.6	26	8	94.8	34	18	95.8	22	20
		.39	93.8	19	43	93.3	19	48	94.4	19	37	95.6	19	25
		.59	93.4	11	55	92.4	20	56	94.0	22	38	95.4	13	33
	.39	.15	95.0	21	29	95.0	18	32	93.8	33	29	94.2	28	30
		.39	94.2	17	41	93.8	19	43	92.8	26	46	95.0	23	27
		.59	94.2	14	44	94.1	22	37	94.9	18	33	95.6	16	28
	.59	.15	94.0	31	29	94.1	27	32	96.0	18	22	94.2	24	34
		.39	94.0	21	39	93.9	26	35	95.0	21	29	94.3	31	26
		.59	93.5	22	43	93.9	30	31	92.9	35	36	94.9	21	30
.59	.15	.15	94.8	52	0	95.9	40	1	95.5	33	12	94.5	34	21
		.39	94.5	22	33	93.1	23	46	94.1	13	46	94.6	21	33
		.59	92.7	14	59	92.6	8	66	94.1	19	40	96.1	12	27
	.39	.15	96.0	27	13	94.3	33	24	95.5	25	20	94.3	34	23
		.39	94.7	15	38	94.1	20	39	95.0	21	29	94.0	23	37
		.59	92.2	15	63	93.6	19	45	94.6	16	38	95.0	18	32
	.59	.15	95.1	18	31	94.9	28	23	94.5	25	30	94.6	27	27
		.39	94.1	15	44	94.0	21	39	94.1	27	32	93.6	19	45
		.59	94.0	16	44	93.4	24	42	94.7	22	31	94.8	25	27

Table 21. *Percentile bootstrap CI coverage for* $\tilde{\upsilon}$ *conditional effect at +1 standard deviation above the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym \cdot \mathbf{x}}$	$\beta_{ymx \cdot \mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	99.9	1	0	96.5	2	33	93.6	3	61	95.3	15	32
		.39	97.9	4	17	97.5	4	21	94.3	11	46	95.2	10	38
		.59	97.8	12	10	96.8	12	20	94.7	13	40	93.6	15	49
	.39	.15	98.2	5	13	97.7	7	16	94.0	10	50	94.4	21	35
		.39	99.0	5	5	97.6	12	12	95.1	10	39	94.2	12	46
		.59	98.2	13	5	98.2	9	9	93.6	10	54	95.2	14	34
	.59	.15	99.5	4	1	98.3	11	6	94.1	15	44	94.9	17	34
		.39	99.1	8	1	98.4	14	2	94.2	16	42	97.2	14	14
		.59	99.3	6	1	98.8	6	6	94.5	17	38	94.7	17	36
.39	.15	.15	97.3	6	21	95.5	9	36	95.0	22	28	95.0	14	36
		.39	91.2	6	82	91.7	10	73	93.2	13	55	95.0	13	37
		.59	90.4	5	91	92.0	11	69	93.8	15	47	95.5	9	36
	.39	.15	92.3	7	70	93.3	12	55	94.3	19	38	93.9	22	39
		.39	92.3	10	67	91.8	12	70	91.8	19	63	94.5	20	35
		.59	91.9	6	75	93.9	14	47	95.0	15	35	95.3	15	32
	.59	.15	92.4	13	63	93.4	14	52	95.1	13	36	94.3	18	39
		.39	93.0	11	59	93.2	20	48	94.9	18	33	94.8	26	26
		.59	92.3	14	63	94.2	19	39	92.5	32	43	95.2	16	32
.59	.15	.15	98.4	10	6	97.5	11	14	96.0	13	27	94.5	21	34
		.39	94.1	7	52	91.7	10	73	93.5	9	56	94.3	16	41
		.59	91.4	4	82	92.0	4	76	93.8	15	47	95.8	10	32
	.39	.15	95.2	11	37	94.1	19	40	94.5	18	37	94.2	27	31
		.39	92.5	12	63	92.7	15	58	94.8	17	35	94.0	20	40
		.59	90.5	12	83	93.3	16	51	93.9	15	46	94.6	16	38
	.59	.15	93.3	13	54	94.1	19	40	94.5	20	35	94.8	23	29
		.39	92.5	7	68	93.4	14	52	94.2	22	36	93.3	16	51
		.59	92.4	14	62	93.1	19	50	94.5	18	37	94.9	21	30

Table 22. *BCa bootstrap CI coverage for $\hat{\upsilon}$ conditional effect at the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym\cdot\mathbf{x}}$	$\beta_{ymx\cdot\mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	81.5	18	167	81.9	16	165	92.5	14	61	94.9	26	25
		.39	83.4	12	154	83.2	12	156	90.5	24	71	96.3	16	21
		.59	81.3	12	175	81.7	20	163	92.4	14	62	95.9	17	24
	.39	.15	82.7	24	149	85.2	28	120	91.5	29	56	94.6	29	25
		.39	83.8	27	135	86.3	26	111	93.6	28	36	94.2	25	33
		.59	85.0	35	115	87.1	30	99	91.3	25	62	95.5	23	22
	.59	.15	84.5	33	122	84.5	29	126	91.7	25	58	94.5	25	30
		.39	83.2	35	133	83.5	42	123	91.7	25	58	96.4	24	12
		.59	83.2	26	142	87.9	25	96	92.8	26	46	94.7	27	26
.39	.15	.15	83.8	27	135	84.9	31	120	92.9	21	50	95.5	19	26
		.39	84.1	25	134	86.4	28	108	91.1	25	64	94.3	22	35
		.59	85.0	20	130	83.9	29	132	91.6	30	54	95.5	17	28
	.39	.15	92.8	16	56	95.4	20	26	94.7	25	28	94.3	27	30
		.39	94.8	19	33	94.2	26	32	93.5	27	38	94.7	31	22
		.59	92.3	31	46	95.9	20	21	96.2	20	18	94.6	27	27
	.59	.15	94.5	24	31	94.8	25	27	95.6	22	22	94.5	24	31
		.39	93.2	29	39	94.9	23	28	96.1	20	19	94.8	31	21
		.59	93.7	27	36	95.1	26	23	94.7	30	23	94.2	24	34
.59	.15	.15	82.4	30	146	81.1	35	154	89.7	23	80	93.2	27	41
		.39	81.2	35	153	85.7	17	126	89.4	26	80	93.3	22	45
		.59	85.5	26	119	84.2	35	123	91.5	27	58	95.1	32	17
	.39	.15	92.0	26	54	94.5	29	26	94.9	32	19	94.3	32	25
		.39	93.0	25	45	94.0	28	32	95.2	23	25	94.1	24	35
		.59	93.2	25	43	93.9	31	30	94.7	21	32	96.3	21	16
	.59	.15	94.3	23	34	94.8	27	25	94.5	27	28	95.1	25	24
		.39	95.5	17	28	95.1	27	22	95.0	23	27	95.9	16	25
		.59	94.9	30	21	93.9	29	32	94.9	29	22	94.1	37	22

Table 23. *BCa bootstrap CI coverage for $\tilde{\upsilon}$ conditional effect at the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym \cdot \mathbf{x}}$	$\beta_{ymx \cdot \mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	88.8	3	109	77.7	3	220	85.9	8	133	95.1	18	31
		.39	87.5	3	122	77.6	3	221	85.6	10	134	95.5	13	32
		.59	85.6	2	142	76.1	5	234	87.5	6	119	95.5	11	34
	.39	.15	71.7	6	277	76.9	16	215	90.3	17	80	94.2	22	36
		.39	72.6	9	265	79.8	14	188	92.6	16	58	93.8	18	44
		.59	73.2	9	259	80.6	12	182	90.8	12	80	95.6	15	29
	.59	.15	76.2	8	230	80.4	13	183	91.2	19	69	94.2	18	40
		.39	75.4	13	233	81.2	19	169	91.7	15	68	96.8	15	17
		.59	75.5	7	238	84.2	10	148	92.6	15	59	94.7	16	37
.39	.15	.15	72.9	5	266	78.1	10	209	91.4	16	70	94.4	11	45
		.39	71.8	5	277	79.0	8	202	89.9	17	84	94.4	15	41
		.59	74.7	4	249	79.6	12	192	90.4	22	74	95.5	12	33
	.39	.15	90.5	9	86	94.9	15	36	94.3	19	38	94.2	25	33
		.39	92.4	10	66	94.2	16	42	93.6	22	42	94.9	25	26
		.59	91.6	21	63	95.8	13	29	95.8	17	25	95.0	23	27
	.59	.15	93.7	16	47	94.4	19	37	95.9	14	27	94.5	20	35
		.39	93.1	16	53	95.2	12	36	95.6	19	25	95.1	24	25
		.59	92.9	17	54	94.7	21	32	94.9	23	28	93.9	19	42
.59	.15	.15	71.4	8	278	74.1	13	246	87.9	15	106	92.6	19	55
		.39	72.0	11	269	78.8	9	203	88.5	14	101	92.5	15	60
		.59	75.5	13	232	81.5	17	168	91.0	20	70	95.1	23	26
	.39	.15	91.4	11	75	94.1	21	38	94.9	26	25	93.8	28	34
		.39	91.7	16	67	93.5	23	42	95.3	17	30	93.8	21	41
		.59	93.3	17	50	94.1	27	32	94.2	18	40	96.5	16	19
	.59	.15	94.1	14	45	94.8	22	30	94.4	22	34	95.0	23	27
		.39	95.6	12	32	95.2	20	28	95.0	21	29	95.6	14	30
		.59	94.4	25	31	94.3	22	35	95.0	26	24	94.4	32	24

Table 23. *BCa bootstrap CI coverage for $\hat{\upsilon}$ conditional effect at +1 standard deviation above the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym \bullet \mathbf{x}}$	$\beta_{ymx \bullet \mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	86.7	20	113	84.4	29	127	93.2	14	54	95.4	27	19
		.39	84.7	22	131	87.2	29	99	90.9	34	57	95.4	21	25
		.59	83.6	31	133	84.3	33	124	92.5	27	48	93.8	27	35
	.39	.15	82.3	26	151	84.7	31	122	91.9	30	51	94.7	29	24
		.39	82.7	35	138	85.1	31	118	92.6	33	41	94.4	23	33
		.59	82.2	51	127	85.9	35	106	90.9	29	62	95.4	25	21
	.59	.15	84.4	33	123	84.2	30	128	91.7	27	56	95.3	23	24
		.39	83.6	32	132	83.6	38	126	91.8	24	58	96.7	21	12
		.59	82.1	31	148	87.6	25	99	91.9	32	49	95.2	24	24
.39	.15	.15	89.1	31	78	89.9	25	76	93.2	35	33	94.6	20	34
		.39	90.9	20	71	93.2	23	45	93.9	27	34	95.2	23	25
		.59	92.4	24	52	92.4	30	46	94.3	26	31	94.8	22	30
	.39	.15	92.1	19	60	94.4	25	31	94.0	31	29	94.2	32	26
		.39	93.2	26	42	93.2	26	42	92.7	32	41	94.4	32	24
		.59	94.2	19	39	94.0	28	32	95.2	20	28	95.4	19	27
	.59	.15	93.0	32	38	94.6	27	27	96.0	20	20	93.9	27	34
		.39	92.4	28	48	93.6	35	29	94.4	29	27	94.1	34	25
		.59	92.4	28	48	93.9	31	30	92.2	41	37	94.6	25	29
.59	.15	.15	87.6	37	87	88.0	37	83	92.6	25	49	93.3	26	41
		.39	92.1	23	56	90.7	30	63	93.1	20	49	94.2	30	28
		.59	91.6	21	63	92.5	14	61	93.7	29	34	96.0	19	21
	.39	.15	90.8	31	61	93.4	30	36	94.6	26	28	94.6	32	22
		.39	93.5	21	44	93.8	26	36	94.6	28	26	93.4	29	37
		.59	91.8	22	60	93.7	29	34	94.3	23	34	94.3	26	31
	.59	.15	94.5	18	37	94.6	28	26	94.8	26	26	95.1	26	23
		.39	93.8	22	40	93.7	26	37	93.6	35	29	92.9	28	43
		.59	93.7	21	42	93.4	29	37	94.6	26	28	94.7	29	24

Table 25. *BCa bootstrap CI coverage for $\tilde{\upsilon}$ conditional effect at +1 standard deviation above the mean of x*

			N=50			N=100			N=250			N=500		
β_{mx}	$\beta_{ym \cdot \mathbf{x}}$	$\beta_{ymx \cdot \mathbf{x}}$	Cov	High	Low	Cov	High	Low	Cov	High	Low	Cov	High	Low
.15	.15	.15	90.6	4	90	80.6	6	188	91.3	4	83	95.3	17	30
		.39	74.2	6	252	79.3	9	198	90.5	19	76	95.0	14	36
		.59	72.4	15	261	79.5	18	187	91.4	16	70	92.5	21	54
	.39	.15	75.2	7	241	79.0	9	201	91.0	14	76	94.2	21	37
		.39	75.5	8	237	81.3	15	172	91.9	21	60	93.8	14	48
		.59	75.2	16	232	82.8	15	157	90.1	17	82	94.9	17	34
	.59	.15	78.0	7	213	80.7	12	181	91.5	17	68	94.8	18	34
		.39	76.3	12	225	81.3	20	167	91.2	16	72	96.8	16	16
		.59	75.6	6	238	85.3	7	140	92.2	19	59	95.2	15	33
.39	.15	.15	75.1	6	243	82.6	8	166	91.8	20	62	93.5	14	51
		.39	89.5	4	101	92.4	12	64	92.9	22	49	95.0	20	30
		.59	90.9	12	79	92.0	21	59	93.9	21	40	95.3	13	34
	.39	.15	89.3	13	94	93.7	10	53	94.4	21	35	94.0	25	35
		.39	92.6	12	62	92.2	19	59	92.3	26	51	94.0	28	32
		.59	92.9	7	64	93.7	23	40	95.0	18	32	95.4	17	29
	.59	.15	93.1	15	54	94.0	20	40	95.5	14	31	93.7	22	41
		.39	92.6	14	60	93.3	23	44	94.2	23	35	93.9	34	27
		.59	91.7	19	64	94.1	24	35	92.5	34	41	94.9	19	32
.59	.15	.15	77.5	8	217	85.1	10	139	90.7	12	81	92.7	19	54
		.39	90.4	10	86	89.8	20	82	92.9	15	56	93.6	24	40
		.59	91.0	12	78	91.9	12	69	93.8	24	38	95.9	16	25
	.39	.15	89.3	12	95	92.9	20	51	93.7	20	43	93.9	27	34
		.39	92.3	15	62	92.3	20	57	94.7	21	32	93.6	25	39
		.59	91.3	16	71	93.2	23	45	94.2	20	38	94.3	22	35
	.59	.15	93.5	11	54	94.1	21	38	94.4	23	33	94.8	23	29
		.39	92.5	15	60	93.6	19	45	93.9	29	32	93.5	21	44
		.59	92.9	17	54	93.5	22	43	94.2	24	34	94.6	26	28