Design and Predicted Effects of a Carbon Tax with Border Carbon Adjustment in the United States

Jacob Vest

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Advised by Mario J. Crucini

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Table of Contents:

I. Introduction

With Climate Change at the center of many global political and policy debates, a Pigouvian tax on carbon dioxide emissions remains a favorite solution among economists and other policy experts. However, asymmetric implementation of a carbon tax across the globe gives rise to several problems. A country which implements a carbon tax while others do not faces relatively higher energy and manufacturing costs than its taxless peers. As a result, its energy-intensive industries are made less competitive, and there is potential for significant carbon leakage. Border carbon adjustments (BCAs) are one measure designed to protect domestic firms and prevent carbon leakage, but there is not yet consensus on what form they might take. The goal of this thesis is to make several key suggestions for the design of a BCA, and to predict the effect of a carbon tax combined with BCA on US production and carbon emissions.

The relationship between energy sources and production technology is inherently complex, and to simplify the development of a model to predict the effects a carbon tax, several strong assumptions are made. First, we assume that output is produced according to a constant elasticity of supply (CES) function of energy and other goods. This top layer of the model reflects the ability of producers/consumers to substitute to less energy-intense inputs/products. Second, we assume that energy is produced according to a CES combination of energy sources: petroleum, natural gas, coal, and renewables. Though the energy produced by a unit of each of these sources is in reality a coefficient determined scientifically, a CES model captures the imperfect substitution seen in the market's response to price shocks in these energy sources. This imperfect substitution arises because not all forms of energy are equally useful in different economic contexts. For example, while three cubic meters of natural gas and gallon of gasoline might contain a similar amount of energy, only one can fit inside any reasonably-sized gas tank.

While the assumptions used in the model are strong, they help isolate the two main avenues of substitution that reduce carbon emissions: from dirty energy to clean, and from energy-intense consumption to energy-efficient consumption. One assumption which we wish to relax is that each source of energy is equally sensitive to changes in price, which is a consequence of the CES function. Since the literature suggests there is considerable difference in the own-price elasticities of different energy sources, we compare the CES model of energy to an alternative log-linear model of energy demand which allows the elasticities of energy sources to vary.

One additional topic we investigate is how different sectors of the economy, the residential, commercial, industrial, and transportation sectors, would respond to a carbon tax. We find that the residential and commercial sector shift their energy composition the most in response to a carbon tax, while the industrial sector shifts less and the transportation sector very little at all. This last finding is relevant to policy-makers who wish to eventually achieve net-zero emissions. While a carbon tax may be a powerful tool to decarbonize most of the economy, the transportation sector's emissions will be especially resistant to carbon pricing.

Lastly, this thesis makes three major recommendations on the design of BCAs which balance the goals of the policy and administrative complexity. First, a BCA should not include export rebates due to their costs and associated carbon leakage. Second, a BCA may be constructed only using US production data for significant administrative savings without endangering the key objectives of the policy. Third, a BCA is a complement of a Nordhaus-style “climate club” policy, and an ideal climate policy would include measures beyond a BCA to encourage international adoption of carbon pricing.

II. Literature

An estimate of the social cost of carbon is necessary for any policy involving a carbon price. This thesis uses the estimate developed by the Interagency Working Group on Social Cost of Greenhouse Gases, a group of economists and other experts spanning several government agencies that the Obama administration assembled to estimate the social cost of carbon. Their central estimate, assuming a three percent discount rate, is a cost of $42 per metric ton of carbon dioxide emitted in 2020 (measured in 2007 dollars)[1]. In today’s dollars, this is approximately $50, which is the figure used by this thesis. Estimates of the social cost of carbon vary widely based on various assumptions of the predicted severity of climate change and appropriate discount rates, and $50/ton is near the middle of the pack of social cost of carbon estimates.

How responsive consumption of energy sources is in response to price changes is an important determinant in the effect of a carbon tax. Several authors have examined the elasticities of different energy sources in different times and countries, and the findings of the papers I read are displayed in Table 1. I compare these findings to the results of my own log-linear model, which is introduced in section VI. The most noticeable trend is that coal is especially sensitive to price while other sources are less so. Pindyck analyzes several countries’ elasticities, and he finds that the US and Canada are outliers for their high own-price elasticity of oil. He speculates that the high availability of alternative fuels such as natural gas in North America explains some of this difference. Hang’s finding of a positive own-price elasticity for electricity is very surprising, and

[1] “Interagency Working Group on Social Cost of Greenhouse Gases, United States Government (2016)

he explains that it is likely due to steadily increasing electricity demand in China over his estimation period. Since the higher demand resulted in concurrent increases in price and quantity of electricity, this led to a positive rather than the expected negative relationship between price and quantity.

Hang's finding reveals a significant limitation in attempts to estimate elasticities: it is extremely difficult to disentangle simultaneous changes in supply and demand. This thesis is concerned with how changes in price affect the quantity of energy demanded, since that is the route through which a carbon tax decreases emissions. In estimating elasticities, we hope to capture supply shocks which lead to higher or lower prices, altering quantity demanded in the way a carbon tax would. Because the data may also reflect price changes that are demand-driven and display the opposite relationship between price and quantity, our calculation could underestimate elasticities. In addition, there are other factors, such as increasing regulation over our estimation period, that might lead to changes in quantity simultaneous to changes in quantity due to price, further biasing our estimation, though perhaps towards overestimation.

Table 1: Own-price elasticities by energy source

Author	Oil	Natural Gas		Coal	Electricity
Pindyck -US 1970s	-1.1	-0.52		-2.17	-0.08
Cho - Korea 1980-90s	-0.092	Not estimated		-0.924	-0.708
Hang -China late 1990s	-0.059	Not estimated		-1.591	0.427
Log-linear model	-0.38	-0.24		-0.93	-0.27

There is extensive literature on the design and potential pitfalls of BCAs. Some authors are skeptical that BCAs would fulfill their intended functions. Kortum and Weisbach note that a

BCA with an export rebate fails to prevent carbon leakage[2]. While such a policy would increase the price of energy domestically and reduce domestic consumption, the global price of energy would fall as energy or energy-intensive goods previously consumed domestically could be exported duty-free. As a result, the energy consumption of all other countries would increase along with their carbon emissions, creating carbon leakage.

Another intended function of BCAs is to pressure foreign countries to adopt a carbon tax, and thereby escape carbon duties on their exports. Nordhaus finds that the punitive effect of BCAs are relatively small compared to the carbon externalities of foreign countries.[3] Since the vast majority of a country's emissions serve domestic production, a tax on the small fraction of carbon-intense goods that leave the country provides little incentive to institute major environmental reforms. Instead, Nordhaus proposes a uniform 2% tariff on all countries that fail to adopt a carbon price. This produces a cost roughly proportional to most countries' carbon externalities, which are calculated by multiplying each country's carbon emissions by a $25/ton social cost of carbon.

Other authors are more ambitious in their vision of the potential functions of a BCA. Flannery et. al. describe a BCA scheme which requires each company in energy-intense, trade-exposed industries to provide reporting of its direct and indirect greenhouse gas emissions in order to calculate both import duties and export rebates.[4] Flannery takes care that his approach would not run afoul of WTO guidelines, which require that duties not exceed domestic indirect taxes and not discriminate by country of origin. However, as shown by Nordhaus, any duty sufficient to sway behavior of foreign countries will necessary be larger than the indirect tax on carbon and be contingent on each country's carbon price. Moreover, it is unclear that the administration of such

[2] Kortum and Weisbach (2016)
[3] Nordhaus (2015)
[4] Flannery et. al (2016)

a scrupulous accounting of carbon would be practical. Flannery argues that the necessary data are already voluntarily reported by many companies. But when a company's tax burden becomes directly proportional to its carbon emissions, there emerges an incentive to underreport. A process for verifying reported emissions is not described in his paper. To avoid such administrative headaches, this thesis will propose a BCA scheme that prioritizes simplicity over accuracy.

III. CES model of energy

Our benchmark model focuses on the carbon-content of the basic energy mix used to produce a unit of final aggregate U.S. consumption. The fundamental goal of the model is to predict how the composition of energy use and level of energy consumption in the United States would shift as the result of a carbon tax. Specifically, we are interested in the following four sources: Oil (O), Natural Gas (NG), Coal (C), and Renewables (R) (which includes nuclear), and we track the British thermal units (analogous to kilowatt hours) consumed of each source divided by the total British thermal units (BTU) consumed of all sources. The model seeks to understand how differential taxation of the intermediate inputs into U.S. energy generation can be used to tilt the inputs toward less carbon intensive inputs, such as natural gas and renewables, and away from more carbon intensive inputs, such as coal. In our model, energy (E) is produced from a combination of the four major sources:

[1] $$E = z[b_1O^{1-w} + b_2NG^{1-w} + b_3C^{1-w} + b_4R^{1-w}]^{1/(1-w)}$$

Each source combines according to a CES production function with parameter w, where $\sigma_E = 1/w$ corresponds to the elasticity of substitution between the energy sources. Energy in equation [1] is not measured in raw BTU. If it were, the equation for energy as a function of energy sources would be a linear combination of each source modified by each one's scientifically determined

BTU content per unit. This would not generate economically interesting results, as it would result in a corner solution. Instead, E represents effective energy: the mix of energy sources necessary to produce aggregate output (Y). While the model results in a somewhat muddled interpretation of E, it captures the economy's imperfect substitution between energy sources. To predict the relative use of each energy source, we assume the economy minimizes the cost [2] of producing effective energy.

$$[2] \qquad Cost_E = P_O*O + P_{NG}*NG + P_C*C + P_R*R$$

where P_j are the market prices of the primary energy inputs, j = oil, natural gas, coal and renewable.[5] Formally, the cost minimizing combination of energy sources is solved using the Lagrangian

$$[3] \qquad Lagrangian = \min\{ P_O*O + P_{NG}*NG + P_C*C + P_R*R$$

$$+ P_E(E - z[b_1O^{1-w} + b_2NG^{1-w} + b_3C^{1-w} + b_4R^{1-w}]^{1/1-w}\}$$

Where P_E is the unit price of effective energy with cost minimizing choices of the primary inputs given their prices. This minimization gives rise to four first-order conditions, the first of which describes Oil and is given in equation [4].

$$[4] \qquad O = b_1^{1/w} * (P_O/P_E)^{-1/w} *E$$

[5] For oil, I use retail prices, which are inclusive of direct and indirect taxes. For natural gas, I use citygate prices, since different retail prices are faced by different sectors. This price does not include distribution costs to final consumers. For Coal, I use the average market price, and for renewables, I estimate prices by dividing total expenditures on renewables by its quantity in million BTU. Expenditure data is supplied by the EIA: https://www.eia.gov/state/seds/data.php?incfile=/state/seds/sep_prices/total/pr_tot_US.html&sid=US

The remaining three first order conditions take an identical form to [4] and can be found in the appendix. The price index for energy (P_E) is defined by equating the total cost of energy from equation [2] to the quantity of energy times its price index:

[5] $$P_E * E = P_O*O + P_{NG}*NG + P_C*C + P_R*R$$

Dividing both sides by E, the quantity of energy, the price index of energy is given by the prices of each energy source weighted by its proportion of the total energy.

[6] $$P_E = P_O*O/E + P_{NG}*NG/E + P_C*C/E + P_R*R/E$$

To solve the model, we calculate the expenditure shares of each energy source using EIA data from 2018 and use this information to estimate each b_i in the model. The necessary equations and calculations are found in the appendix. An estimate for σ_E is given by the volatility ratio between energy sources: the standard deviation of the log ratio of their relative quantities divided by the standard deviation of the log ratio of their relative prices.

[7] $$\sigma_E = 1/w = sd[\ln(O/NG)] / sd[\ln(P_O/P_{NG})]$$

where sd represents the standard deviation operator. Equation [7] gives just one of the six pairwise estimates possible between four sources. The calculated estimates of σ_E are given for all six pairs in Table 1 (see appendix), averaging to 0.75, which is the estimate we use.

The next layer of our model relates energy to aggregate output. We assume that energy and value-added (VA) combine according to a CES function to produce aggregate output. We further assume that output equals consumption and investment demanded and that consumption and investment goods have an identical energy profile. Value-added is itself a function of labor and capital.

[8] $$Y = C^D + I^D = [v_1(VA)^{1-\beta} + v_2(E)^{1-\beta}]^{1/(1-\beta)}$$

[9] $$VA = N^{\alpha}K^{1-\alpha}$$

The substitution between energy and value-added is assumed to be a constant $\sigma_Y = 1/\beta$. We estimate σ_Y by comparing the relationship between the relative prices of energy and value-added and their relative quantities. Prices for energy and value-added are measured by the EIA's average price of energy and core CPI respectively, and quantities are derived from energy expenditures and total expenditures minus energy expenditures. Like σ_E, σ_Y is estimated by calculating the volatility ratio (see [10]). I calculate σ_Y to be 0.51 using yearly data from 2001-2018.

[10] $$\sigma_Y = sd[\ln(E/VA)] / sd[\ln(P_E/P_{OG})]$$

The cost function for Y is given by:

[11] $$Cost_Y = P_Y*Y = P_v*VA + P_E*E$$

where P_v is the price index of value-added and P_E is the price index of energy. The cost per unit of consumption is solved using the Lagrangian.

[12] $$Lagrangian = \min\{P_v*VA + P_E*E$$

$$+ P_Y(Y - z_1[v_1(VA)^{1-\beta} + v_2(E)^{1-\beta}]^{1/(1-\beta)}\}$$

Where P_Y is the unit price of aggregate output with cost minimizing choices of the primary inputs given their prices. This minimization gives rise to two first-order conditions:

[13] $$VA = v_1^{1/\beta} * (P_v/P_Y)^{-1/\beta} *Y$$

[14] $$E = v_2^{1/\beta} * (P_E/P_Y)^{-1/\beta} * Y$$

Substituting [13] and [14] into [11] and rearranging, the price index P_Y is given by:

[15] $$P_Y = [v_1^{1/\beta} * P_v^{(\beta-1)/\beta} + v_2^{1/\beta} * P_E^{(\beta-1)/\beta}]^{\beta/(\beta-1)}$$

As in the model of energy consumption, v_1 and v_2 are estimated using the expenditure shares from 2018, and relative energy consumption is predicted by the following equation:

[16] $$E^{t+1}/E^t = (P_E^t / P_E^{t+1} * P_Y^{t+1} / P_Y^t)^{1/\beta} * Y^{t+1}/Y^t$$

E^t represents the quantity of energy consumed pre-carbon tax, and E^{t+1} represents energy consumed post-carbon tax. Relative energy consumption between periods is proportional to the relative prices modified by the elasticity ($1/\beta$) and directly proportional to relative consumption. In order to predict consumption, we use conditions from the solution to [9], which is a simple Cobb-Douglas case, to obtain the price index for value added (P_v) and cost minimizing first order conditions.

[17] $$P_v = w^{\alpha} r^{1-\alpha}$$

[18] $$K/N = w/r * (1-\alpha)/\alpha$$

We can find an expression for the level of output by substituting [9] and [17] into [13].

[19] $$N^{\alpha} K^{1-\alpha} = v_1^{1/\beta} * (w^{\alpha} r^{1-\alpha}/P_Y)^{-1/\beta} * Y$$

Then, rearranging [15] and dividing by N on both sides results in an expression of output per unit of labor:

[20] $$Y/N = v_1^{-1/\beta} * (K/N)^{1-\alpha} * (w^{\alpha} r^{1-\alpha}/P_Y)^{1/\beta}$$

We assume that N=1 is fixed in the long run and substitute [14] into [16].

[21] $$Y = v_1^{-1/\beta} * (w/r * (1-\alpha)/\alpha)^{1-\alpha} * (w^{\alpha} r^{1-\alpha})^{1/\beta} * P_Y^{-1/\beta}$$

We assume the impact of energy prices on wages and rental rates are sufficiently small that they may be ignored, and we obtain the following expression for post-carbon tax output relative to pre-carbon tax output:

[22] $$Y^{t+1}/Y^{t} = (P_Y^{t}/P_Y^{t+1})^{1/\beta}$$

IV. Results from CES model of energy

To generate results from the model, we introduce a $50/ton carbon tax which creates a price shock relative to 2018 prices of the three polluting sources. The price changes for each source are shown in Table 3.

Table 2

Unit	Gallon of Gasoline	1000 ft^3 of Natural Gas	Short ton of coal
2018 Price[6]	$2.81	$4.23	$32.69
CO2 per unit (in kgs)[7]	9.80	54.98	1803.16
Carbon tax per unit	$0.49	$2.75	$90.16
Post-tax price	$3.30	$6.98	$122.85
Percent change in price	17.4%	65.0%	275.8%

Applying the post-tax prices to the model, we first convert each price to dollars per million BTU to facilitate comparison. An alternative Table 2 with each source denominated in million BTU is available in the appendix. The 2018 price, along with data on 2018 expenditures allow calculation of each b_i in the model (see appendix). With estimates for all parameters in the

[6] 2018 prices are obtained from excel spreadsheets downloaded from the EIA website.
[7] Data on CO2 coefficients from "Carbon Dioxide Emissions Coefficients," Energy Information Administration.

model, we can calculate how the post-tax change in energy prices will affect how much of each energy source is consumed as a proportion of energy consumption. We assume that in the long run, supply is infinitely elastic and do not consider the transition costs of moving from one energy mix to another. Table 4 displays each energy source's BTU as a percentage of total BTU in the economy before and after a hypothetical $50/ton carbon tax. The equations and calculations needed to generate the predictions in Table 4 are available in the appendix.

Table 3: Changes in shares of inputs into energy production in response to price shock

Source	Oil	Natural Gas	Coal	Renewables
Percent change in price	17.4%	65.0%	275.8%	0%
Share pre-tax	36.5%	30.8%	13.1%	19.6%
Share post-tax	41.5%	27.1%	6.2%	25.2%

The model predicts significant changes in the composition of energy. Coal declines by more than 50% as a proportion of total energy consumption; it is especially sensitive to a carbon tax due to its low cost and high emissions. Natural gas sees a small decline, less than coal as it is naturally cleaner and more expensive. Surprisingly, despite being a carbon-intense form of energy, petroleum increases as a proportion of BTU consumption. This effect is driven by petroleum's high cost per BTU relative to other sources, which restricts its increase in price to about 35%, compared to a 65% and 276% increase for natural gas and coal respectively. While petroleum's increase in price exerts some downward pressure on its proportion, the fact that proportions must add up to one means that losses in shares of coal and natural gas must be accompanied by increases in the shares of other sources. Renewables absorb most of this increase, rising almost six percentage points, but petroleum's share also increases by five percentage points. This does not mean, however, that the economy will consume more petroleum in absolute terms. Since the carbon tax increases the average price of energy, we expect the consumption of energy to decrease overall. The next section reports how large we expect this decrease to be.

V. Results of full CES model

To find the full effect of the carbon tax on emissions, we must look beyond the composition of energy sources and consider that a higher price of energy reduces energy consumption overall. Let t denote the current period and t+1 denote the period in which the carbon tax has taken effect. Normalizing P_E^t to 1, we obtain $P_E^{t+1} = 1.34$ by plugging in post-tax prices and proportions to equation [6]. This suggests that the average cost of energy will rise by approximately 34%. Returning to the function for output, the increased price of energy leads to a substitution away from energy and towards labor/capital. The energy consumption post carbon tax (t+1) relative to pre carbon tax (t) is given by the following:

$$E^{t+1}/E^t = (P_E^t / P_E^{t+1} * P_Y^{t+1} / P_Y^t)^{1/\beta} * Y^{t+1}/Y^t \qquad [23]$$

Relative energy consumption between periods is proportional to the relative prices modified by the elasticity (1/ β) and directly proportional to relative consumption. The change in energy price leads to a 13% decline in energy per unit of output. The price index of consumption increases by 1.8%, and this spurs a 3.4% decrease in total consumption according to equation [22], leading to a proportional drop in energy consumption.

The combination of these three factors: substitution towards less carbon-intensive energy sources, decrease in energy consumption per unit of output, and a decline in overall output lead to emissions reductions. The carbon dioxide emitted per million BTU of each energy source is obtained from carbon coefficient data from the EIA, summarized in table 2.

Table 4

Source	Kilograms CO2/ million BTU
Coal	95.35
Natural Gas	53.07
Gasoline (Oil)	71.3[8]

From these data, the total emitted carbon dioxide is given by equation 27:

[24] $$CO2 = O*71.3 + NG*53.1 + C*95.4 + R * 0$$

where CO2 is measured in kilograms and each source is measured in million BTU.

The total environmental effect of the $50/ton carbon tax is displayed in Table 5. The first row is obtained from EIA data from 2018, and the second row reports the predictions of our model, incorporating the change in energy proportions, energy-intensity of output, and decline in GDP. The last two rows are obtained by applying equation [24] to the two previous rows and making appropriate conversions.

Table 5

Source	Oil	Natural Gas	Coal	Renewables	Total
Quadrillion BTU pre-tax	36.9	31.1	13.2	19.8	101.0
Quadrillion BTU post-tax	35.2	23.0	5.3	21.4	84.7
Million Metric Tons CO2 pre-tax	2,628	1,651	1,262	0	5,541
Million Metric Tons CO2 post-tax	2,508	1,219	501	0	4,228

[8] Data in table from "Carbon Dioxide Emissions Coefficients," Energy Information Administration.

The full effect of the carbon tax leads to an approximately 23.7% reduction in CO_2 emissions in this model, driven mostly by a decline in energy consumption and a shift away from coal towards renewable energy.

VI. Log-linear model

The CES model of energy requires that each energy share the same elasticity of substitution. To allow for heterogeneity of substitution between energy sources, we compare the CES results to those of a log-linear of energy demand, where the relationship between logged price and quantity is given by the following equation:

$$\log(Q_i) = \beta_{0i} + \sigma_i * \log(P_i) \qquad [25]$$

where Q_i represents the quantity in BTU of the i^{th} energy source and P_i its price in dollars. The same price and quantity data used in the CES model, ranging from 2001-2018, are used to regress log quantity on log price for each energy source according to equation [25]. Since small changes in log approximate percent changes, the interpretation of σ_i is the i^{th} energy source's own-price elasticity. The results of the regression analysis for each energy source are displayed in table 6.

Table 6

Source	Oil	Natural Gas	Coal	Renewable
Estimate of σ	-0.38	-0.24	-0.93	-0.27
Standard error	0.217	0.0354	0.293	0.148
p-value	0.099	0.00001	0.006	0.085

Consistent with the literature, we find that coal is especially sensitive to changes in price, while the other energy sources are less so. As mentioned before, the above estimates do not control for potential simultaneous changes in quantity due to demand or regulation, so we are not confident they reflect the true own-price elasticity of demand. However, since our results seem reasonably in line with earlier work, we assume they can still offer informative predictions about the effect of a price shock in the form of a carbon tax.

Applying the estimated elasticities to our model of log-linear demand, we find that the results (see Table 7) are similar to the CES model, but with a more dramatic decrease in the proportion of energy from coal, reflecting its greater own-price elasticity compared to the other energy sources.

Table 7

Source	Oil	Natural Gas	Coal	Renewables	Total
Pre-proportion	36.5%	30.8%	13.1%	19.6%	100.0%
Post-proportion	40.3%	32.1%	4.5%	23.1%	100.0%
BTU pre-tax	36.9	31.1	13.2	19.8	101.0
BTU post-tax	34.6	27.6	3.9	19.8	85.9
Percent change	-6.1%	-11.4%	-70.7%	0.2%	-15.0%

To predict the decline in energy consumption, we return to the same CES model of output in our benchmark model (equation [8]). Because the log-linear model results in different proportions of each energy source, we find its associated price index is slightly lower than the CES model of energy (1.31 compared to 1.34). The 31% increase in the average price of energy leads to a 13.7% decline in energy consumption and a corresponding according to our model, which along with the cleaner composition of energy sources leads to a predicted 22.4% decline in total emissions.

VII. Sector-specific results

The log-linear model allowed us to examine how each energy source responds differently to price changes, and now we turn to how each sector of the economy might differ in its response to a carbon tax. To analyze the residential, commercial, industrial, and transportation as defined by the EIA, we use the same procedure as for the economy as a whole. We investigate how the quantity of each energy source consumed by each sector covaries with its price from 2001-2018 to generate estimates of elasticities of substitution between energy sources. However, since there is not available data on the prices and quantities of other goods consumed by each sector, we do not repeat the analysis of the elasticity of substitution between energy and value-added. We report results as if each sector mirrored the economy as a whole in substitution between energy and value-added, but this is unlikely to be true. Thus, the most informative results in this section are how each sector shifts its proportional energy composition between different sources.

For brevity, we only report the results from the log-linear model, as they reveal interesting interactions between sector and the own-price elasticity of each energy source and do not differ too much from those of the CES model. The log-linear model for j^{th} sector is given by the following equation:

[26] $$log(Q_{ij}) = \beta_{0ij} + \sigma_{ij} * log(P_{ij})$$

The parameter σ_{ij} is estimated by the same procedure described in the previous section, and the regression results are displayed in the appendix. We summarize the results for each sector in a table and corresponding graph of the sector's energy composition from 2001-2018.

Table 8: Residential Sector

Source	Oil	Natural Gas	Coal	Renewables	Total
Estimated σ	-2.58	-0.19	-0.81	-0.23	NA
Pre-proportion	4.9%	43.5%	21.4%	30.2%	100.0%
Post-proportion	4.0%	49.3%	9.1%	37.6%	100.0%
BTU pre-tax	1.0	9.3	4.6	6.5	21.4
BTU post-tax	0.7	9.0	1.7	6.8	18.2
Percent change	-30.3%	-3.7%	-63.9%	5.8%	-15.0%

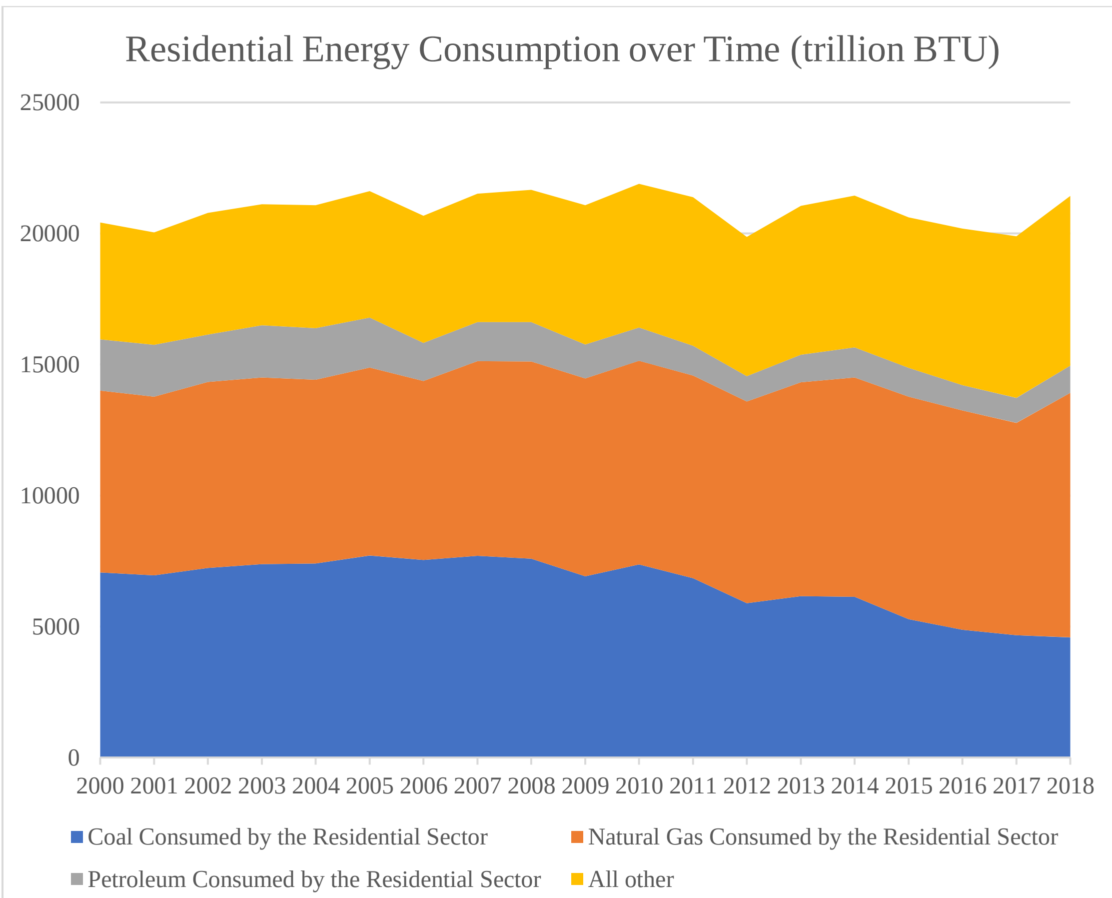
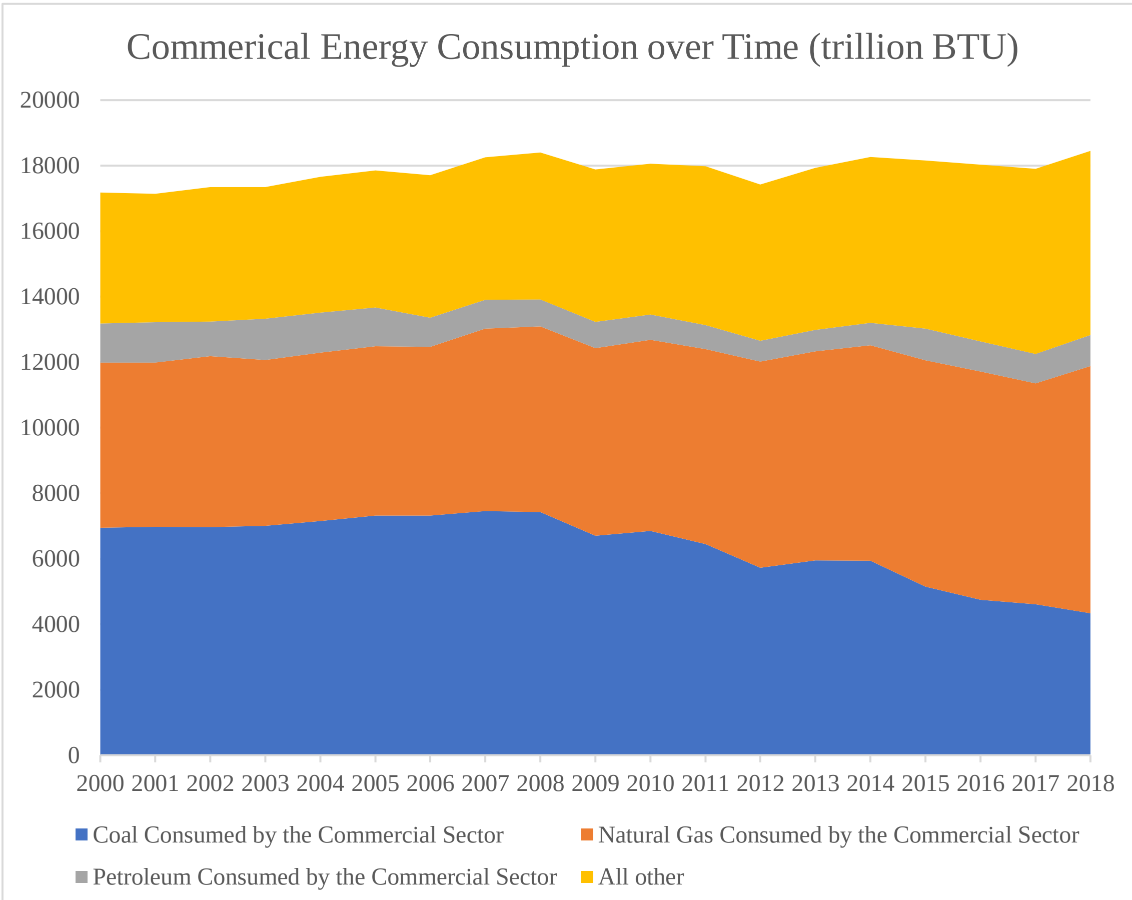


Table 9: Commercial Sector

Source	Oil	Natural Gas	Coal	Renewables	Total
Estimated σ	-2.45	-0.31	-0.83	-0.2	NA

Pre-proportion	5.1%	40.9%	23.5%	30.5%	100.0%
Post-proportion	4.5%	46.0%	10.2%	39.7%	100.0%
BTU pre-tax	0.9	7.5	4.3	5.6	18.5
BTU post-tax	0.7	7.2	1.6	6.2	15.7
Percent change	-11.8%	12.5%	-56.6%	30.2%	-15.0%



The residential and commercial sectors behave quite similarly since their energy needs revolve mostly around heating and lighting buildings, and almost 70% of each sector's energy consumption is from electricity. The most surprising result is an estimated own-price elasticity of oil of -2 or greater (in magnitude), which diverges sharply from the elasticity estimated for the economy as a whole. According to the EIA, oil use in the residential sector is primarily for the purposes of heating water and buildings[9]. Since fuel oils can be easily substituted for natural gas or electricity for heating in the residential or commercial context, it is not surprising that oil consumption in both sectors is more sensitive to changes in price than it is in other parts of the economy where oil is less substitutable. The graph "Residential Energy Consumption Over Time" displays the large reduction in the residential sector's oil consumption over the 2001-2018 time period, with more than a 50% decrease, which was concurrent with an increase in the price of oil. Assuming the residential and commercial sector follow the economy as a whole with a 12.1% reduction in energy consumption, they see a 29% and 31% decline in emissions respectively, indicating these sectors are more sensitive to a carbon tax.

Table 10: Industrial Sector

Source	Oil	Natural Gas	Coal	Renewables	Total
Estimated σ	-0.57	-0.23	-1.14	-0.17	NA
Pre-proportion	27.0%	40.3%	13.1%	19.6%	100.0%
Post-proportion	29.6%	43.3%	3.5%	23.6%	100.0%
BTU pre-tax	8.9	13.2	4.3	6.4	32.8
BTU post-tax	8.3	12.1	1.0	6.6	27.9
Percent change	-6.8%	-8.7%	-77.3%	2.4%	-15.0%

[9] Energy Information Administration, "Use of Energy Explained: Energy Use in Homes," April 8, 2019, https://www.eia.gov/energyexplained/use-of-energy/homes.php.

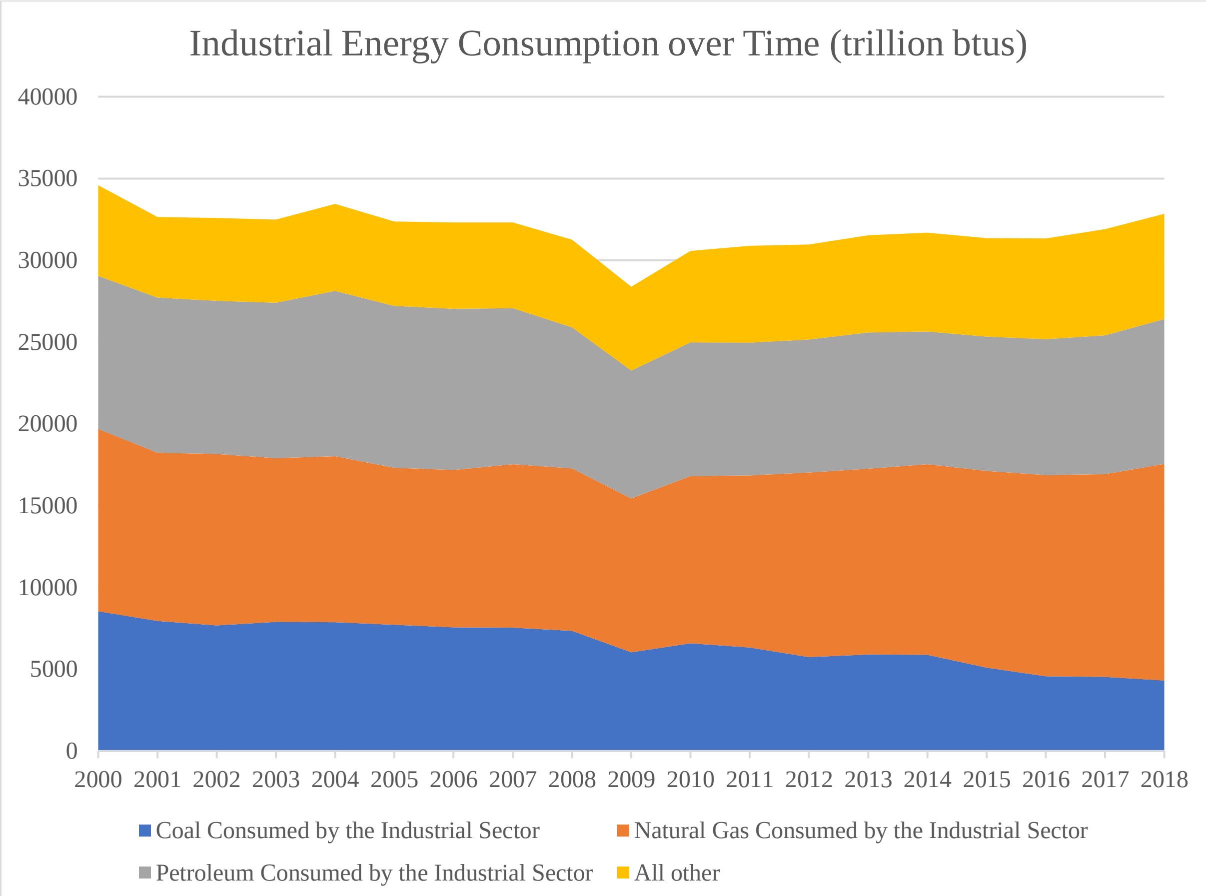
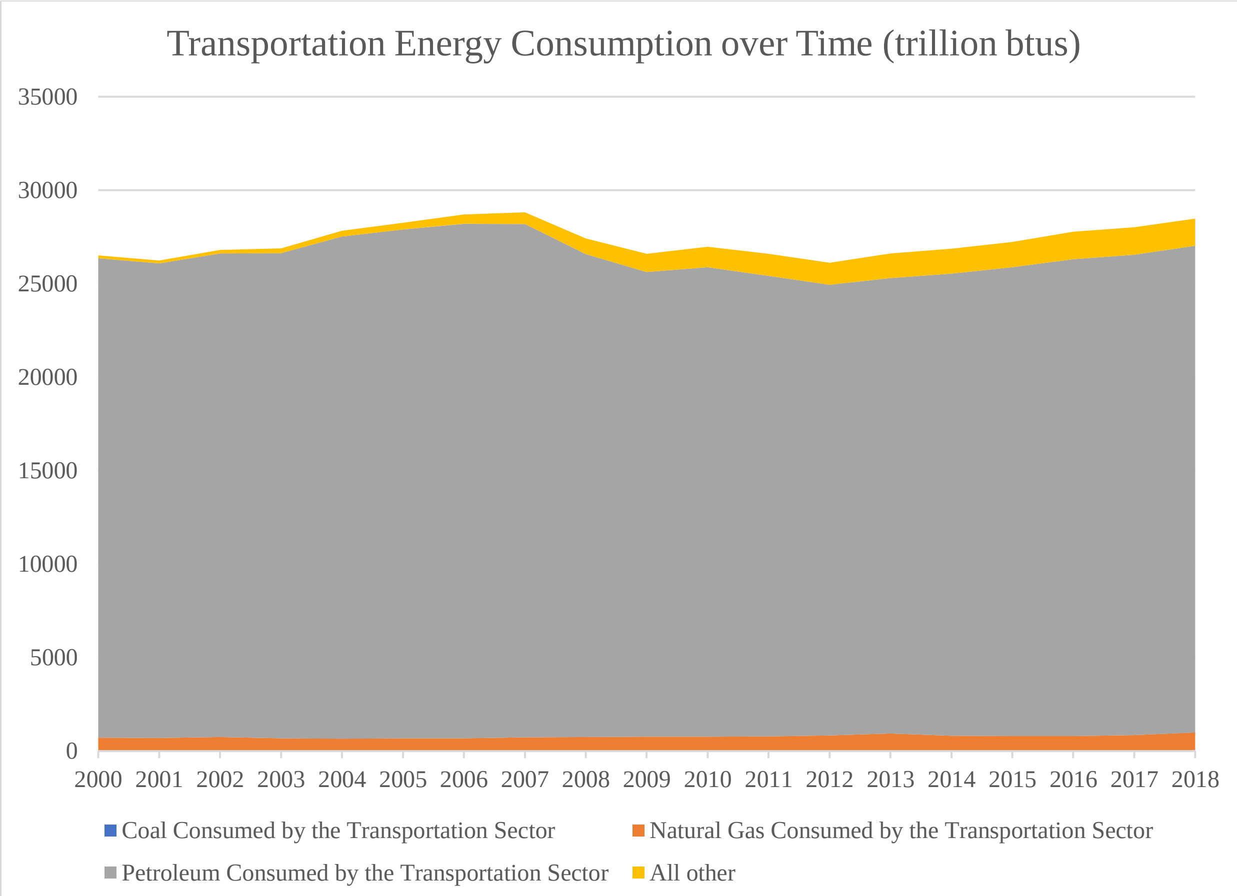


Table 11: Transportation Sector

Source	Oil	Natural Gas	Coal	Renewables	Total
Estimated σ	-0.11	-0.3	-0.86	-2.46	NA
Pre-proportion	91.4%	3.4%	0.1%	5.1%	100.0%
Post-proportion	91.8%	3.0%	0.0%	5.2%	100.0%
BTU pre-tax	26.0	1.0	0.0	1.5	28.5
BTU post-tax	22.2	0.7	0.0	1.3	24.2
Percent change	-14.6%	-25.0%	-70.2%	-13.7%	-15.0%



While the industrial sector mostly mirrors the economy as a whole, the transportation sector is starkly different. More than 90% of its energy consumption is from petroleum, and this figure is very constant over time despite significant price fluctuations. Because the energy used in the transportation sector must be in most cases easily transportable, fuels like oil have few substitutes. The gradual introduction of renewables into the transportation sector, as visualized by the yellow bar, is accounted for almost entirely by the rapid growth of biofuel consumption. Since the biofuel component was almost negligible at the beginning of the period, it increased by more than 1000% by 2018. This dramatic increase in quantity, combined with a moderate decrease in the price of renewables over the period, resulted in a very high estimate for the own-price elasticity of renewable energy. But since our definition of renewable is an amalgamation of all non-fossil energy sources, the renewable price has very little relation to the price of biofuels, and this estimate is not meaningful. Because the own-price elasticity of oil is so low in the transportation sector and oil comprises 90% of the sector's energy consumption, the transportation does little to shift its energy composition in response to a carbon tax. A stronger presence of substitutes, either in the form of biofuels or electric cars, is necessary for a significant emissions reduction in the transportation sector.

VIII. Design of Border Carbon Adjustment

A major quandary in the design of a BCA is the trade-off between administrative simplicity and accuracy. This thesis falls heavily towards the former. To find the exact carbon footprint of a final good entering the U.S. is a task of enormous complexity. Not only must one track the emissions of the firm directly producing the good, one must also track the emissions of every intermediate good and those of the precursors to the intermediates. Applying this process to every good imported by every company from every country in the world would require an army of accountants. Additionally, there would be strong incentives to circumvent the taxes without strong enforcement, and oversight is difficult on such a large scale and across international borders. The potential value of determining carbon footprints exactly is that exporting firms would internalize the costs of emissions for goods destined to be consumed in the U.S, leading them to make economically efficient choices about their level of pollution. However, it seems unlikely that this efficiency gain would be worth the administrative costs in all but the largest firms and most simply-produced products. While there may be a place for a Flannery-style scheme for imports that fall under that subset, this thesis proposes a simpler solution.

In order to prevent domestic products from becoming less competitive compared to imports, one need only ensure that imports are taxed at least as much domestic products under a carbon tax. Thus, only domestic production account data are required for calculating the minimum BCA necessary to prevent a competitive disadvantage for domestic firms. While foreign production technologies may be significantly dirtier or cleaner than domestic, this information is not relevant to the protective function of a BCA. As long as imports' cost rises equivalently to domestic, there is no incentive for consumers to substitute towards foreign-made goods. I propose a BCA derived from the estimated costs U.S. industries would face as the result of an upstream carbon tax. Let $CO2_j$ be the tons of CO2 released per dollar of output in domestic industry j, then the equivalent ad valorum tariff necessary to tax imports equivalently to domestic products, τ_j, is given by equation 16. (Recall the proposed carbon tax of $50/ton of CO2.)

[27] $$\tau_j = 50 * CO2_j$$

A U.S-derived BCA schedule has the advantage of relying on easily-accessible and verifiable data. The Bureau of Economic Analysis (BEA) provides detailed information on the relationship between goods in production, and the industry-by-industry total requirements table details how much each industry consumes of all other industries to produce one dollar of output, both directly and indirectly.[10] For instance, if the total requirements table reports the grain industry uses $0.10 of the petroleum industry in producing one dollar of output, this includes the petroleum used by tractors, but also the petroleum involved in manufacturing the tractors and other intermediate goods. The Energy Information Administration (EIA) reports data on the carbon-intensity of energy sources, measured in terms of CO2 emitted per million BTU of

[10] BEA, "Total Requirements Table," 2012

energy. In order to estimate each industry's estimated carbon tax burden, I identify three major categories of emission-producing fossil fuels: coal, natural gas, and petroleum. While not all subtypes of coal and petroleum produce the same emissions, the data in the BEA's economic accounts do not include detail beyond these broad categorizations, the finer distinctions between these subtypes cannot be applied to generate a more accurate emissions profile for goods. The EIA's emission intensity calculations for the three major fossil fuel sources are presented in Table 2.

Since the BEA's accounts report dollar value of inputs and not physical quantities, it is necessary to estimate the quantity of physical goods represented by each dollar of output in order to calculate embodied emissions. Instead of relying on variable price data, I find the total emissions produced by each energy source sector and divide it by the total value of its output to obtain an estimation of tons of CO2 per dollar. The results of this calculation are displayed in Table 9.

Table 9

Industry	NAICS Name	Output (in Quad btus)	Million tons CO2/quad btu	Million tons CO2 emitted	Output in M$	Tons CO2/$
Coal	Coal mining	20.8	95.4	1,983	52,000	0.0378
Petroleum	Petroleum refineries	36	71.3	2,567	784,000	0.0033
Natural Gas	Natural gas distribution	17.2	53.1	914	79,000	0.0116
Natural Gas[11]	Electric power generation[12]	7.4	53.1	392	413,000	0.0009

[11] Accounts for natural gas consumed by electric utilities
[12] Output represents the sum of the three electric generation industries: Electric power generation, transmission, and distribution, Federal electric utilities, State and local government electric utilities

The choice of industry for coal was straightforward, as all coal must originate from the coal mining industry. However, some challenges are posed by the BEA's aggregation of the oil and gas extraction industries since these industries have drastically different carbon footprints per dollar but are lumped together in the same industry definition, "Oil and Gas Extraction". To disaggregate these two industries, I rely on the petroleum refineries industry as a proxy for oil and a combination of the electric industry and natural gas distribution industry as a proxy for natural gas. Since almost all crude petroleum is refined before use, it is not difficult to justify that downstream industries' use of the Petroleum refineries should be roughly proportional to their oil consumption.

Natural gas is not as easily traced. About 36% of natural gas energy is used to generate electricity and the remaining 64% is used directly by end-use sectors[13]. For the electricity generation sector, I assume the 36% percent of natural gas is distributed evenly between all dollars of industry output and thereby calculate the natural gas carbon footprint of electricity. For the remaining 64%, I assume that end-use sectors must utilize the natural gas distribution industry to receive natural gas to burn and that use of this sector is proportional to direct consumption of natural gas.

To arrive at the carbon tax each industry pays to produce a dollar of output, I add up the output's embodied CO2 emissions from petroleum, coal, and natural gas. The embodied CO2 from each source is calculated by multiplying the value of energy consumed by the CO2 per dollar. For example, if one dollar of grain industry output required \$0.10 of petroleum industry output to produce, multiplying this by 0.0031 tons CO2/dollar (from Table 6) results in 0.00031

[13] Energy Information Administration, "U.S. energy consumption by source and sector, 2018"

tons of CO_2. Finally, the embodied CO_2 is multiplied by the carbon tax rate of $50/ton. In our example for grain, if petroleum is the only source of carbon, the tax per dollar would be 0.00031 tons multiplied by $50/ton which equals $0.015. Thus, the ad valorum equivalent is 1.5%. Six of the most carbon-intense industries is given in Table 7. Figure 1 provides a summary of the carbon tax burden by industry, sorted from greatest to least. While several energy-intense industries see cost increases in the range of 5-20%, the large majority of industries have cost increases of less than 2%.

Under this proposal for a BCA, the value-added duty levied on imports would be equivalent to these estimated cost increases for countries without a carbon tax. Imports from countries with an equal or greater carbon price would not need to be taxed, as they would likely face equal or greater carbon price costs than U.S. firms. As a result, there would be no competitive disadvantage or carbon leakage for a BCA to correct. For countries with a carbon price but one that is lower than the U.S. rate, a partial BCA adjustment is necessary. Such a country's BCA is given by equation [28].

$$[28] \qquad \tau^{i}_{j} = \tau_{j} * (CP^{h} - CP^{i})/CP^{h}$$

where τ^{i}_{j} is the partial BCA for the i^{th} country on the j^{th} industry, τ_{j} is the domestic ad valorum equivalent calculated in equation 16, CP^{h} is the home carbon price, and CP^{c} is the foreign carbon price. Equation [28] acts to fill the gap between the lower carbon price of the foreign country and the higher domestic price, equalizing the carbon costs firms in the two countries bear.

The conditionality of import duties on foreign countries' carbon prices addresses one last objective considered when crafting a BCA: encouraging foreign countries to adopt carbon prices of their own. A Border Carbon Adjustment is unwieldy, inaccurate, and inefficient compared to an ideal world in which every country agrees to set a carbon price, rendering a BCA unnecessary. Further, broad international coordination is necessary to make significant reductions in global emissions necessary to mitigate the vast economic and environmental damage climate change will cause. BCAs are best seen as an intermediate policy bridge between a world with very few effective carbon prices and one in which they are ubiquitous. BCAs allow first movers to spare domestic firms from adverse competitive affects and prevent carbon leakage while the rest of the international community hopefully follows suit.

While this proposal's conditional duties provide some small incentive for foreign countries to adopt carbon prices of their own, as seen in the work of Nordhaus, this incentive pales in comparison with their carbon externalities. Any BCA proposal should be paired with a more significant effort to encourage adoption of carbon prices in other countries, such as Nordhaus's climate club policy.

IX. Conclusions/Discussion

We find that a $50/ton carbon tax prompts a significant shift in composition of U.S. energy sources. However, the magnitude of the shift differs substantially between energy sources and sectors of the economy. Coal and the residential and commercial sectors are most sensitive to a carbon tax, while petroleum and the transportation sector respond the least. According to both our CES and log-linear model of energy, carbon dioxide emissions would decline between 20-25% with slightly more of the decline attributed to reduced energy consumption than a shift in the composition of energy sources.

It is clear that strong policies to combat climate change are long overdue. A Pigouvian carbon tax is an obvious first step, yet many countries fear unilateral action would result in little environmental gain at the cost of economic hardship and reduced competitiveness of domestic industry. A border carbon adjustment smooths the uneven adoption of carbon prices by alleviating at least the latter of these concerns. This proposal reveals that protection of domestic industry and prevention of carbon leakage need not be excessively administratively complex, a frequent criticism of BCAs.

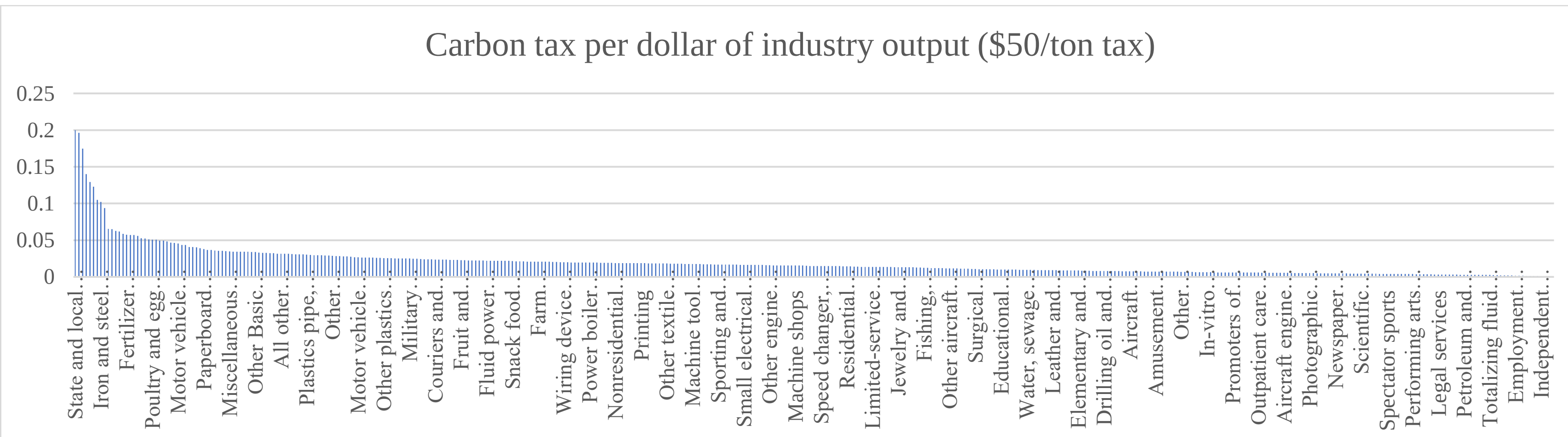
However, it is important to note that unilateral adoption of a carbon tax and BCA are not sufficient to spur sufficient abatement of carbon emissions, as the U.S. accounts for only about 15% of emissions. As the largest economy in the world, the U.S. is in a unique position to pressure other countries to adopt carbon prices, and a BCA does not produce this pressure on its own. Beyond a BCA, the U.S. must create significant incentives for other countries to adopt a carbon price in order to ensure that the BCA is a temporary policy.

X. Appendix a-Tables and Figures

Table 1

Pair	Estimate of sigma
Natural Gas and Oil	0.239
Natural Gas and Coal	0.337
Natural Gas and Renewables	0.171
Petroleum and Coal	0.404
Petoleum and Renewables	0.203
Coal and Renewables	0.575
Average	0.321

Figure 1



Note: all industries sorted by carbon tax exposure, excluding coal and natural gas for the sake of the y-axis scale.

Table 7

Industry	State and local government electric utilities	Petroleum refineries	Cement manu-facturing	Electric power generation, transmission, and distribution	Alumina refining and primary aluminum production	Coffee and tea manu-facturing	Museum, historical sites, zoos, and parks
Coal mining	0.12089	0.00424	0.06374	0.03356	0.02295	0.00304	0.00149
Electric power generation, transmission, and distribution	0.01380	0.01827	0.11011	1.04083	0.19691	0.02000	0.02255
Natural gas distribution	0.00264	0.00624	0.03622	0.00173	0.00986	0.00539	0.00287
Petroleum refineries	0.04440	1.06133	0.07130	0.08394	0.06058	0.03636	0.01617
Federal electric utilities	0.00051	0.00066	0.00401	0.00153	0.00736	0.00073	0.00084
State and local government electric utilities	1.00227	0.00297	0.01790	0.00682	0.03288	0.00327	0.00374
Carbon tax per dollar ($50/ton tax)	0.19992	0.17474	0.13995	0.12949	0.06527	0.01487	0.00808

Note: Selected industries' direct and indirect use of polluting industries and corresponding carbon tax per dollar. All quantities in dollars.

Alternative Table 2

Unit	Oil	Natural Gas	Coal
2018 $/million BTU	$20.48	$4.08	$1.73
CO2 per unit (in kgs)	71.3	53.07	95.35
Carbon tax per unit	$3.57	$2.65	$4.77
Post-tax price	$24.04	$6.74	$6.50
Percent change in price	17.4%	65.0%	275.8%

Appendix b- Model Equations and Calculations

Model of Energy

Remaining First Order Conditions

$NG = b_2^{1/w} * (P_{NG}/P_E)^{-1/w} *E$

$C = b_3^{1/w} * (P_C/P_E)^{-1/w} *E$

$R = b_4^{1/w} * (P_R/P_E)^{-1/w} *E$

Expenditure Shares

$P_O*O/P_E*E = b_1^{1/w} * (P_O/ P_E)^{(w-1)/w}$

$P_{NG}*NG/P_E*E = b_2^{1/w} * (P_{NG}/ P_E)^{(w-1)/w}$

$P_C*C/P_E*E = b_3^{1/w} * (P_C/ P_E)^{(w-1)/w}$

$P_R*R/P_E*E = b_4^{1/w} * (P_R/ P_E)^{(w-1)/w}$

Calculation of Proportions

The proportion of each relative to total BTU pre-tax is given by:

$$O^t/(O^t + NG^t + C^t + R^t) = [b_1^{1/w}(P_O^t)^{-1/w}] / [b_1^{1/w}(P_O^t)^{-1/w} + b_2^{1/w}(P_{NG}^t)^{-1/w} + b_3^{1/w}(P_C^t)^{-1/w} + b_4^{1/w}(P_R^t)^{-1/w}] = 0.365$$

$$NG^t/(O^t + NG^t + C^t + R^t) = [b_2^{1/w}(P_{NG}^t)^{-1/w}] / [b_1^{1/w}(P_O^t)^{-1/w} + b_2^{1/w}(P_{NG}^t)^{-1/w} + b_3^{1/w}(P_C^t)^{-1/w} + b_4^{1/w}(P_R^t)^{-1/w}] = 0.308$$

$$C^t/(O^t + NG^t + C^t + R^t) = [b_3^{1/w}(P_C^t)^{-1/w}] / [b_1^{1/w}(P_O^t)^{-1/w} + b_2^{1/w}(P_{NG}^t)^{-1/w} + b_3^{1/w}(P_C^t)^{-1/w} + b_4^{1/w}(P_R^t)^{-1/w}] = 0.131$$

$$R^t/(O^t + NG^t + C^t + R^t) = [b_4^{1/w}(P_R^t)^{-1/w}] / [b_1^{1/w}(P_O^t)^{-1/w} + b_2^{1/w}(P_{NG}^t)^{-1/w} + b_3^{1/w}(P_C^t)^{-1/w} + b_4^{1/w}(P_R^t)^{-1/w}] = 0.196$$

where prices are converted to price per BTU.

The proportion of each relative to total BTU post-tax is given by:

$$O^{t+1}/(O^{t+1} + NG^{t+1} + C^{t+1} + R^{t+1}) = [b_1^{1/w}(P_O^{t+1})^{-1/w}] / [b_1^{1/w}(P_O^{t+1})^{-1/w} + b_2^{1/w}(P_{NG}^{t+1})^{-1/w} + b_3^{1/w}(P_C^{t+1})^{-1/w} + b_4^{1/w}(P_R^{t+1})^{-1/w}] = 0.415$$

$$NG^{t+1}/(O^{t+1} + NG^{t+1} + C^{t+1} + R^{t+1}) = [b_2^{1/w}(P_{NG}^{t+1})^{-1/w}] / [b_1^{1/w}(P_O^{t+1})^{-1/w} + b_2^{1/w}(P_{NG}^{t+1})^{-1/w} + b_3^{1/w}(P_C^{t+1})^{-1/w} + b_4^{1/w}(P_R^{t+1})^{-1/w}] = 0.271$$

$$C^{t+1}/(O^{t+1} + NG^{t+1} + C^{t+1} + R^{t+1}) = [b_3^{1/w}(P_C^{t+1})^{-1/w}] / [b_1^{1/w}(P_O^{t+1})^{-1/w} + b_2^{1/w}(P_{NG}^{t+1})^{-1/w} + b_3^{1/w}(P_C^{t+1})^{-1/w} + b_4^{1/w}(P_R^{t+1})^{-1/w}] = 0.062$$

$$R^{t+1}/(O^{t+1} + NG^{t+1} + C^{t+1} + R^{t+1}) = [b_4^{1/w}(P_R^{t+1})^{-1/w}] / [b_1^{1/w}(P_O^{t+1})^{-1/w} + b_2^{1/w}(P_{NG}^{t+1})^{-1/w} + b_3^{1/w}(P_C^{t+1})^{-1/w} + b_4^{1/w}(P_R^{t+1})^{-1/w}] = 0.252$$

Numerical values obtained by R-script (see Appendix d– Code).

Derivation of b_i's:

From before, the expenditure shares (ES) are given as:

$ES_O = P_O * O / P_E * E = b_1^{1/w} * (P_O / P_E)^{(w-1)/w}$

$ES_{NG} = P_{NG} * NG / P_E * E = b_2^{1/w} * (P_{NG} / P_E)^{(w-1)/w}$

$ES_C = P_C * C / P_E * E = b_3^{1/w} * (P_C / P_E)^{(w-1)/w}$

$ES_R = P_R * R / P_E * E = b_4^{1/w} * (P_R / P_E)^{(w-1)/w}$

Rearranging to solve for each $b_i^{1/w}$ and defining $P_E=1$ in the initial period, we obtain:

$b_1^{1/w} = ES_O * (P_O / P_E)^{(1-w)/w} = 0.293$

$b_2^{1/w} = ES_{NG} * (P_{NG} / P_E)^{(1-w)/w} = 0.073$

$b_3^{1/w} = ES_C * (P_C / P_E)^{(1-w)/w} = 0.016$

$b_4^{1/w} = ES_R * (P_R / P_E)^{(1-w)/w} = 0.131$

Numerical values obtained by R-script (see Appendix d– Code).

Expenditure Shares for Consumption Model

$(P_{OG} * OG)/(P_Y * Y) = v_1^{1/\beta} * (P_{OG} / P_Y)^{(\beta -1)/\beta}$

$(P_E * E)/(P_Y * Y) = v_2^{1/\beta} * (P_E / P_Y)^{(\beta -1)/\beta}$

Derivation of v_1 and v_2, normalizing P_Y to 1:

$v_1^{1/\beta} = ES_{OG} * (P_{OG}/P_Y)^{(1-\beta)/\beta} = 0.942$

$v_2^{1/\beta} = ES_E * (P_E/P_Y)^{(1-\beta)/\beta} = 0.015$

Numerical values obtained by R-script (see Appendix d– Code).

Appendix c– Regression Tables

Each regression estimates the relationship between log price and quantity, with each coefficient corresponding to an elasticity parameter from the log-linear model. Regression tables are given as direct output from R and report estimates, standard error, t-values, and p-values.

Economy-wide Estimates

Oil

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              3.67283    0.04124  89.053   <2e-16 ***
data$logPetroleumPriceN -0.37972    0.21744  -1.746   0.0999 .  
```

Natural Gas

```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)               2.97827    0.03809  78.197  < 2e-16 ***
data$logNaturalGasPriceN -0.23907    0.03543  -6.748 4.68e-06 ***
```

Coal

```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)   
(Intercept)          0.7988     0.6793   1.176  0.25680   
data$logCoalPriceN  -0.9288     0.2928  -3.172  0.00592 **
```

Renewables

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              2.78166    0.02817  98.752   <2e-16 ***
data$logRenewablePriceN -0.27194    0.14835  -1.833   0.0855 .  
```

Residential Sector Estimates

Oil

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)               7.6678     0.1682  45.593   <2e-16 ***
data$logPetroleumPriceN  -2.5797     0.8867  -2.909   0.0102 *  
```

Natural Gas

```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)               8.75018    0.03767 232.277  < 2e-16 ***
data$logNaturalGasPriceN -0.19198    0.03504  -5.478 5.06e-05 ***
```

Coal

```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)          6.8942     0.6867  10.040  2.6e-08 ***
data$logCoalPriceN  -0.8122     0.2960  -2.744   0.0144 *  
```

Renewables

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              8.56609    0.02561 334.424   <2e-16 ***
```

```
data$logRenewablePriceN -0.23050    0.13490  -1.709    0.107
```

Commercial Sector Estimates

Oil
```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)               7.2429     0.1219  59.394  < 2e-16 ***
data$logPetroleumPriceN  -2.4467     0.6429  -3.806  0.00155 ** 
```

Natural Gas
```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)               8.36320    0.04618 181.090  < 2e-16 ***
data$logNaturalGasPriceN -0.31331    0.04296  -7.293  1.8e-06 ***
```

Coal
```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)          6.8087     0.6639  10.256 1.93e-08 ***
data$logCoalPriceN  -0.8337     0.2862  -2.913   0.0102 *  
```

Renewables
```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              8.44137    0.02616 322.677   <2e-16 ***
data$logRenewablePriceN -0.19909    0.13778  -1.445    0.168    
```

Industrial Sector Estimates

Oil
```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              9.18481    0.05925 155.027   <2e-16 ***
data$logPetroleumPriceN -0.56791    0.31236  -1.818   0.0878 .  
```

Natural Gas
```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)               9.05035    0.04534 199.626  < 2e-16 ***
data$logNaturalGasPriceN -0.22839    0.04217  -5.415 5.72e-05 ***
```

Coal
```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)          6.1047     0.7094   8.605 2.13e-07 ***
data$logCoalPriceN  -1.1443     0.3058  -3.742  0.00178 ** 
```

Renewables
```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              8.62574    0.01995 432.433   <2e-16 ***
data$logRenewablePriceN -0.16901    0.10505  -1.609    0.127    
```

Transportation Sector Estimates

Oil

```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)              10.17187    0.03061 332.323   <2e-16 ***
data$logPetroleumPriceN -0.10663    0.16137  -0.661    0.518    
```

Natural Gas

```
Coefficients:
                          Estimate Std. Error t value Pr(>|t|)    
(Intercept)                6.29595    0.04664 135.000  < 2e-16 ***
data$logNaturalGasPriceN -0.29744    0.04338  -6.856 3.86e-06 ***
```

Coal

```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)  
(Intercept)          1.5576     0.7464   2.087   0.0533 .
data$logCoalPriceN  -0.8581     0.3217  -2.667   0.0169 *
```

Renewables

```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)                6.5337     0.1452  44.999  < 2e-16 ***
data$logRenewablePriceN  -2.4570     0.7647  -3.213  0.00543 ** 
```

Appendix d– Code

The following excerpts do not include the code used to generate estimates for the specific sectors, since the process is almost identical. The output calculations (second excerpt) were performed for both the CES model and log-linear model using the same script, manually typing in the appropriate price index of energy. The many excel sheets used in this analysis are available upon request.

Estimation of CES model of energy

```
> #12/12/19
> #Derivation of elasticity parameter using price and quantity data for energ
y sources
```

```
> setwd("C:/Users/jacob/OneDrive/Desktop/Senior Thesis Project/Backup/EIA Dat
a/Price Data")
> library(readxl)
> library(xlsx)
> data <- read_excel("Master Annual Price Dataset.xlsx")
> totalBtu <- data$CoalBtus + data$NaturalGasBtus + data$PetroleumBtus + data
$RenewableBtus
> data$totalBtus = totalBtu
>
> #Compute log price ratios
> data$logPriceNG_P <- log(data$NaturalGasPrice/data$PetroleumPrice)
> data$logPriceNG_C <- log(data$NaturalGasPrice/data$CoalPrice)
> data$logPriceNG_R <- log(data$NaturalGasPrice/data$RenewablePrice)
> data$logPriceP_C <- log(data$PetroleumPrice/data$CoalPrice)
> data$logPriceP_R <- log(data$PetroleumPrice/data$RenewablePrice)
> data$logPriceC_R <- log(data$CoalPrice/data$RenewablePrice)
>
> #Compute log quantity ratios
> data$logBtusNG_P <- log(data$NaturalGasBtus/data$PetroleumBtus)
> data$logBtusNG_C <- log(data$NaturalGasBtus/data$CoalBtus)
> data$logBtusNG_R <- log(data$NaturalGasBtus/data$RenewableBtus)
> data$logBtusP_C <- log(data$PetroleumBtus/data$CoalBtus)
> data$logBtusP_R <- log(data$PetroleumBtus/data$RenewableBtus)
> data$logBtusC_R <- log(data$CoalBtus/data$RenewableBtus)
>
>
> estimate_labels <- c("NG/P", "NG/C", "NG/R", "P/C", "P/R", "C/R")
> estimate_values <- c(1:6)
>
> #compute ratio of standard deviation of log quantity ratio to sd of log pri
ce ratio
> for(i in 1:6) {
+   #(logprice ratios start in column 13 of the dataframe, log Btu ratios sta
rt in column 19)
+   estimate_values[i]<- sd(data[[i+18]])/sd(data[[i+12]])
+ }
>
> mean <- 0
> for (i in 1:6) {
+   print(paste("Estimate for", estimate_labels[i], "is", estimate_values[i])
)
+   mean <- mean + estimate_values[i]
+ }
[1] "Estimate for NG/P is 0.365270765360965"
[1] "Estimate for NG/C is 0.654963735417636"
[1] "Estimate for NG/R is 0.160477290151696"
[1] "Estimate for P/C is 1.19772238352537"
[1] "Estimate for P/R is 0.693871520766984"
[1] "Estimate for C/R is 1.44258377118494"
>
> mean <- mean/6
>
> w <- 1/mean
>
> #Calculation of bi's (to the 1/w power)
> preprices <- c(2.81, 4.23, 32.69, 16)
> postprices <- c(3.30, 6.98, 122.85, 16)
```

```
> expshare <- c(0.618, 0.104, 0.0187, 0.26)
> btuconverstion <- c(137381, 1036000,      18911000, 1000000)
> #convert to per million btu prices
> for (i in 1:4) {
+   preprices[i] <- preprices[i]*1000000/btuconverstion[i]
+   postprices[i] <- postprices[i]*1000000/btuconverstion[i]
+ }
>
> b <- c(1:4)
> for (i in 1:4) {
+   b[i] <- expshare[i]*(preprices[i])^((1-w)/w)
+ }
> print(b)
[1] 0.29278334 0.07341831 0.01633076 0.13089754
>
> #Calculate Proportions
> preproportions <- c(1,4)
> #generate demoninator of proportion equation
> sum <- 0
> for (i in 1:4) {
+   sum <- sum + b[i]*(preprices[i])^(-1/w)
+ }
> for (i in 1:4) {
+   preproportions[i] <- b[i]*(preprices[i])^(-1/w)/sum
+ }
>
> postproportions <- c(1,4)
> #generate demoninator of proportion equation
> sum <- 0
> for (i in 1:4) {
+   sum <- sum + b[i]*(postprices[i])^(-1/w)
+ }
> for (i in 1:4) {
+   postproportions[i] <- b[i]*(postprices[i])^(-1/w)/sum
+ }
> print(preproportions)
[1] 0.3651098 0.3077992 0.1307242 0.1963668
> print(postproportions)
[1] 0.41513257 0.27094652 0.06194427 0.25197665
>
> #Generate price index
> prePI <- 0
> for (i in 1:4) {
+   prePI <- preproportions[i]*preprices[i] + prePI
+ }
>
> postPI <- 0
> for (i in 1:4) {
+   postPI <- postproportions[i]*postprices[i] + postPI
+ }
>
> print(postPI/prePI)
[1] 1.342256
```

Calculation of Output as a function of Energy and Value-added

```
> #2/5/20
> #Estimation of elasticity of substitution between energy and nonenergy spending
> #12/12/19
> #Derivation of elasticity parameter using price and quantity data for energy sources
> setwd("C:/Users/jacob/OneDrive/Desktop/Senior Thesis Project/Backup/EIA Data/Price Data
> library(readxl)
> data <- read_excel("Energy vs nonenergy consumption analysis.xlsx")
>
> #Calculate "quantities" using expenditures average btu price and Core CPI to find quant
of consumption baskets
> data$energy_quant <- data$energy_expend/data$energy_price
> data$nonenergy_quant <- data$nonenergy_expend/data$coreCPI
>
>
> #Compute log price ratios
> data$logPriceE_NE <- log(data$energy_price/data$coreCPI)
>
> #Compute log quantity ratios
> data$logExpendE_NE <- log(data$energy_quant/data$nonenergy_quant)
>
>
>
> #compute ratio of standard deviation of log quantity ratio to sd of log price ratio
>
> estimate<- sd(data$logExpendE_NE)/sd(data$logPriceE_NE)
>
>
> B <- 1/estimate
>
> #Change in logs
> pricediff <- c(1:16)
> quantdiff <- c(1:16)
> for (i in 1:16) {
+   pricediff[i] <- data$logPriceE_NE[i+1] - data$logPriceE_NE[i]
+   quantdiff[i] <- data$logExpendE_NE[i+1] - data$logExpendE_NE[i]
+ }
>
> estimate2<- sd(quantdiff)/sd(pricediff)
>
> #Calculation of bi's
> # 15.47 is the weighted average price of a million BTUs in 2018
> Pe1 <- 1 * 15.47715091
> Pe2 <- 1.31 * 15.47715091
> #Price of labor/capital is assumed to be constant
> Pnk <- 1
> preprices <- c(Pnk, Pe1)
> postprices <- c(Pnk, Pe2)
>
> expshare <- c(0.942, 0.0582)
>
> v <- c(1:2)
> for (i in 1:2) {
+   v[i] <- expshare[i]*(preprices[i])^((1-B)/B)
+ }
> print(v)
[1] 0.94200000 0.01512939
>
>
> #Calculation of Price Index
> Py <- 0
> for (i in 1:2) {
```

```
+   Py <- Py + v[i]*postprices[i]^((B-1)/B)
+ }
> Py <- Py^(B/(B-1))
> print(Py)
[1] 1.017289
>
> #GDP in 2018
> Y <- 19519.4235 * 10^12
>
>
> #Calculation of energy relative change
> RelativeE <- (Pe1/Pe2 * Py/1)^(1/B)
> print(RelativeE)
[1] 0.8794011
```

Log-linear model calculations

```
> #Jacob Vest
> #2/18/20
> #Linearized Model
> #All data is to be log differenced and price data normalized by CPI
> setwd("C:/Users/jacob/OneDrive/Desktop/Senior Thesis Project/Backup/EIA Dat
a/Price Data")
> library(readxl)
> data <- read_excel("Prices normalized to million btu.xlsx")
>
> #Normalize Prices by Price Index
> data$PetroleumPriceN <- data$PetroleumPrice/data$PriceIndex
> data$NaturalGasPriceN <- data$NaturalGasPrice/data$PriceIndex
> data$CoalPriceN <- data$CoalPrice/data$PriceIndex
> data$RenewablePriceN <- data$RenewablePrice/data$PriceIndex
>
> #Generate Log Prices and Quantities
>
> data$logPetroleumPriceN <- log(data$PetroleumPriceN)
> data$logNaturalGasPriceN <- log(data$NaturalGasPriceN)
> data$logCoalPriceN <- log(data$CoalPriceN)
> data$logRenewablePriceN <- log(data$RenewablePriceN)
>
> data$logPetroleumBtus <- log(data$PetroleumBtus)
> data$logNaturalGasBtus <- log(data$NaturalGasBtus)
> data$logCoalBtus <- log(data$CoalBtus)
> data$logRenewableBtus <- log(data$RenewableBtus)
>
> #Levels
> #Generate Regression Results
> Petroleum <- lm(data$logPetroleumBtus ~ data$logPetroleumPriceN)
> summary(Petroleum) # show results

Call:
lm(formula = data$logPetroleumBtus ~ data$logPetroleumPriceN)

Residuals:
     Min       1Q   Median       3Q      Max
-0.06786 -0.04117 -0.01711  0.03519  0.09105

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)               3.67283    0.04124  89.053   <2e-16 ***
data$logPetroleumPriceN -0.37972    0.21744  -1.746   0.0999 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.05492 on 16 degrees of freedom
Multiple R-squared:  0.1601,   Adjusted R-squared:  0.1076
F-statistic:  3.05 on 1 and 16 DF,  p-value: 0.09993

> PetrolCoeff <- summary(Petroleum)$coefficients[2, 1]
> Petrolpvalue <- summary(Petroleum)$coefficients[2, 4]
> Petrolse <- summary(Petroleum)$coefficients[2, 2]
> NaturalGas <- lm(data$logNaturalGasBtus ~ data$logNaturalGasPriceN)
> summary(NaturalGas) # show results

Call:
lm(formula = data$logNaturalGasBtus ~ data$logNaturalGasPriceN)

Residuals:
      Min        1Q    Median        3Q       Max
-0.087307 -0.026280  0.004563  0.023663  0.139916

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)               2.97827    0.03809  78.197  < 2e-16 ***
data$logNaturalGasPriceN -0.23907    0.03543  -6.748 4.68e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.05235 on 16 degrees of freedom
Multiple R-squared:   0.74,    Adjusted R-squared:  0.7237
F-statistic: 45.53 on 1 and 16 DF,  p-value: 4.683e-06

> NGCoeff <- summary(NaturalGas)$coefficients[2, 1]
> NGpvalue <- summary(NaturalGas)$coefficients[2, 4]
> NGse <- summary(NaturalGas)$coefficients[2, 2]
> Coal <- lm(data$logCoalBtus ~ data$logCoalPriceN)
> summary(Coal) # show results

Call:
lm(formula = data$logCoalBtus ~ data$logCoalPriceN)

Residuals:
     Min       1Q   Median       3Q      Max
-0.25058 -0.08795  0.02680  0.07611  0.25965

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.7988     0.6793   1.176  0.25680
data$logCoalPriceN  -0.9288     0.2928  -3.172  0.00592 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1509 on 16 degrees of freedom
Multiple R-squared:  0.386,    Adjusted R-squared:  0.3477
F-statistic: 10.06 on 1 and 16 DF,  p-value: 0.005919
```

```
> CoalCoeff <- summary(Coal)$coefficients[2, 1]
> Coalpvalue <- summary(Coal)$coefficients[2, 4]
> Coalse <- summary(Coal)$coefficients[2, 2]
> Renewable <- lm(data$logRenewableBtus ~ data$logRenewablePriceN)
> summary(Renewable) # show results

Call:
lm(formula = data$logRenewableBtus ~ data$logRenewablePriceN)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.12712 -0.09213 -0.01486  0.04218  0.21535 

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)              2.78166    0.02817  98.752   <2e-16 ***
data$logRenewablePriceN -0.27194    0.14835  -1.833   0.0855 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.119 on 16 degrees of freedom
Multiple R-squared:  0.1736,	Adjusted R-squared:  0.1219 
F-statistic:  3.36 on 1 and 16 DF,  p-value: 0.08546

> RenewCoeff <- summary(Renewable)$coefficients[2, 1]
> Renewpvalue <- summary(Renewable)$coefficients[2, 4]
> Renewse <- summary(Renewable)$coefficients[2, 2]
> #
> #Generate emissions predictions
> #
> Coeff <- c(PetrolCoeff, NGCoeff, CoalCoeff, RenewCoeff)
> pvalue <- c(Petrolpvalue, NGpvalue, Coalpvalue, Renewpvalue)
> se <- c(Petrolse, NGse, Coalse, Renewse)
> #import prices
> preprices <- c(2.81, 4.23, 32.69, 16)
> postprices <- c(3.30, 6.98, 122.85, 16)
> logchange <- c(1:4)
> for (i in 1:4) {
+   logchange[i] <- log(postprices[i]) - log(preprices[i])
+ }
> 
> 
> preproportions <- c(1:4)
> sum <- 0
> for (i in 1:4) {
+   sum <- sum + data[[18, 2*i+1]]
+ }
> 
> for (i in 1:4) {
+   preproportions[i] <- (data[[18, 2*i+1]])/sum
+ }
> 
> postproportions <- c(1:4)
> 
> #Using elasticity estimated by coeffs, calculate the change in proportion
> for (i in 1:4) {
```

```
+   postproportions[i] <- exp(log(preproportions[i]) +  Coeff[i]*logchange[i]
)
+   #change negative values to 0
+   if (postproportions[i] < 0) {
+     postproportions[i] <- 0
+   }
+ }
>
> #Renormalize so proportions add up to one
> sum <- 0
> for (i in 1:4) {
+   sum <- sum + postproportions[i]
+ }
> for (i in 1:4) {
+   postproportions[i] <- postproportions[i]/sum
+ }
>
> btuconverstion <- c(137381,  1036000,       18911000, 1000000)
> #convert to per million btu prices
> for (i in 1:4) {
+   preprices[i] <- preprices[i]*1000000/btuconverstion[i]
+   postprices[i] <- postprices[i]*1000000/btuconverstion[i]
+ }
>
> #Generate price index
> prePI <- 0
> for (i in 1:4) {
+   prePI <- preproportions[i]*preprices[i] + prePI
+ }
>
> postPI <- 0
> for (i in 1:4) {
+   postPI <- postproportions[i]*postprices[i] + postPI
+ }
>
> print(Coeff)
[1] -0.3797170 -0.2390747 -0.9287633 -0.2719448
> print(pvalue)
[1] 9.992553e-02 4.683223e-06 5.918687e-03 8.545639e-02
> print(se)
[1] 0.21744073 0.03543062 0.29282334 0.14835012
> print(preproportions)
[1] 0.3649251 0.3075803 0.1311164 0.1963782
> print(postproportions)
[1] 0.40347319 0.32068170 0.04505811 0.23078699
> print(postPI/prePI)
[1] 1.310111
```

Appendix e– References

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5. Kortum and Weisbach, "Border Adjustments for Carbon Emissions: Basic Concepts and Design," *Resources for the Future*, 2016.
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11. Bureau of Economic Analysis, "Use Table," 2012.