MONEY AND PRICES IN A MULTIPLE MATCHING DECENTRALIZED TRADING MODEL

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Money and Prices in a Multiple Matching Decentralized Trading Model

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Abstract

We study price determination and exchange patterns in a monopolistically competitive economy, in which both goods and (fiat) money are perfectly divisible. The decentralized trading environment features 'multiple matches,' in which households consume bundles of commodities and have a preference for consumption variety. Although each household contacts many sellers, the variety of the consumption basket that results under barter mediated exchange is sparser than that obtained with monetary exchange. In the latter setting, households need only to locate a good they want, while in the former the more stringent double coincidence of wants must be satisfied. We examine pricing and consider the effects of monetary policy. We show that a sufficiently rapid expansion of the money supply leads to the gradual emergence of barter. Under these circumstances sellers accept both goods and cash payments and workers receive part of their remuneration in kind.

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I. Introduction

In their celebrated contribution Kiyotaki and Wright (1989) provide an elegant formalization of Jevon's notion of the problem of the double coincidence of wants and, in so doing, offer a parsimonious characterization of the advantages of monetary exchange relative to barter. In their framework all trade takes place at unit prices, as money and goods are assumed indivisible and agents can hold at most one unit of *any* object at any given point in time (i.e., exchange entails one-for-one swaps). Yet, once these latter restrictions are relaxed price determination becomes extremely problematic in this class of model, since the random (Poisson) arrival of trade opportunities at the individual level induces a refractory aggregate distribution of cash holdings.[1] In this paper we present a model of price determination, with endogenous exchange patterns, that attempts to preserve the key insights of the early search approach (viz. the difficulty of *quid pro quo* trading in the presence of the problem of the double coincidence of wants), while simultaneously relaxing the strong storage and divisibility assumptions made elsewhere in the literature.

We consider a dynamic monopolistically competitive environment, wherein: (*i*) a continuum of firms, each produce a differentiated product using labor as the sole factor of production and (*ii*) a continuum of households supply labor inelastically to firms and seek variety in the basket of goods they consume.[2] However, our model departs from the prototypical approach in two critical respects. First, we abandon the coordinating function of the Walrasian auctioneer and instead consider a decentralized trading process, wherein each agent meets pairwise with large numbers (measures) of agents every period (any given meeting is characterized by a single or a double coincidence of wants or neither). Second, we impose neither *a priori* restrictions on the medium of exchange nor on the means of factor (wage) payments. Consequently, the firms

1 Several recent papers have studied price determination in more general environments. Shi (1995) and Trejos and Wright (1995*a*,*b*) assume indivisible money and divisible goods, leading to prices in terms of goods per unit money. Diamond and Yellin (1987,1990), Green and Zhou (1995), and Zhou (1996) admit divisible money, in an a setting in which households desire a fixed quantity of the consumption good per trade (regardless of their current cash holdings). Corbae and Camera (1996) relax the unit upper bound restriction on cash holdings, while Molico (1999) admits both divisible goods and money. However, these latter models are analytically complex and numerical methods are used to characterize the equilibrium. Finally, Casella and Feinstein (1990) study pricing (under conditions of hyperinflation), but impose a cash-in-advance constraint.

2 A similar framework is proposed by Blanchard and Kiyotaki (1987), but in a very different context.

in our economy can pay their workers in cash and/or in kind and, for their part, households can purchase goods using cash and/or goods.

We focus on symmetric steady-state equilibria. The multiple matching structure eliminates (via the law of large numbers) idiosyncratic trading uncertainty, leading to deterministic demand functions at the individual level and a simple steady-state distribution of money holdings. We show there is a *pure-barter equilibrium* (*PBE*) in which money is not valued and workers are paid in kind. However, monetary equilibria exist as well. For sufficiently low monetary growth rates only a *pure monetary equilibrium* (*PME*) exists, in which money is used on one side of every transaction. For sufficiently rapid rates of monetary growth the *PME* is unsustainable and barter begins to emerge, resulting in a *mixed-trading equilibrium* (*MTE*).

The variety of goods that accrue in the consumption basket that pertains in the *PBE* is sparser than in either the *PME* or the *MTE*. In the latter two settings, households need only locate a good they want, while in the former the more stringent double coincidence of wants must be satisfied. Thus, our model points to the drawback of barter relative to monetary exchange as stemming from atemporal trade frictions that stymie consumption variety. This feature differs from related search papers, where, under barter mediated exchange, the problem of the double coincidence of wants leads to costly delays between consumption episodes (temporal frictions). Money is neutral in both the *PME* and the *MTE*. This result, familiar in the Walrasian literature, stands in sharp contrast to many other papers in the money and search literature. It arises because the divisibility of money and the lack of storage restrictions, sever the somewhat artificial link between the nominal money supply and the fraction of money traders in the economy. Although money is neutral it is not superneutral. Perhaps our most striking non-superneutrality result concerns the shift in trade patterns that takes place, for sufficiently rapid rates of monetary growth, in the transition from the *PME* to the *MTE*. We show that in the *MTE* the rate of inflation and the volume of barter transactions are positively related (indeed, in the limiting case the *MTE* converges to the *PBE*), This finding seems to offer a plausible explanation for the frequently observed patterns of exchange in hyperinflation, in which sellers accept both goods and cash and workers often receive part of their remuneration in the form of their employer's output.[3]

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3 Holtfrerich (1986) and Huang (1948) provide evidence pertaining to such exchange patterns during the German and Chinese hyperinflations respectively. Even under moderately high inflation in the confederacy at the time of the civil war, Lerner (1969) remarks: "As early as 1862 some Southern firms stopped selling their

At first glance, our approach resembles that proposed by Shi (1997). Indeed, both our respective models admit divisible goods/money and invoke the law of large numbers to yield a simple degenerate distribution of money *ex post*.[4] However, a key difference is that Shi imposes important *a priori* restrictions on the exchange protocol, resembling a cash-in-advance constraint at the individual level. More precisely, each family assigned to hold money (and trade) forgoes the opportunity of production and barter for the period. This is not so in our model. Households are allowed to simultaneously conduct barter as well as monetary transactions and the patterns of trade that emerge are determined as a feature of the equilibrium.

II. The Model

Time is discrete and is indexed by $t \in \mathbb{N}$. The commodity space, $\Omega_0 = [0,N] \subseteq \mathbb{R}_+$, consists of a continuum of distinct varieties of goods, indexed ω, arranged around a circle with circumference N. The economy is populated by a continuum of infinitely-lived households indexed $h \in H_0 \equiv [0,H]$ and a continuum of infinitely lived owners, indexed $\hat{h} \in \hat{H}_0 = [0,N]$. While they discount the future at the common rate $\beta \in (0,1)$, these agents differ in their endowments and in their preferences. Specifically, each household possesses an indivisible unit of labor that is supplied without disutility to at most one firm at a time, while each owner both owns and controls a firm that has unique access to the technology used to produce one of the differentiated commodities $\omega \in \Omega_0$.[5] We identify each owner by $\omega \in \Omega_0$, since each of them controls a unique

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products for currency alone, and customers were forces to offer commodities as well as notes to buy things." A notable recent illustration of this phenomenon is Russia's 'virtual economy,' where payments in kind are common as barter has increased from 5% of sales in 1992 to 45% in 1997 (see Ericson and Ickes (2000) and the references cited therein).

4 In Shi's framework, each household is populated by a continuum of members (c.f., Lucas (1990)) and, at any given point in time, specializes in producing a single good. Each period the family first assigns a fraction of its members to barter and production and the rest to holding money and trading. Once this is done, random matching then takes place between the members of alternative households. In this setting, each family member is subject to considerable trading uncertainty, but the family unit is not (as it is populated by a continuum of members). In contrast, in our framework each household (populated by a single member) faces uncertainty in the search for a *specific* product, but no aggregate uncertainty concerning the *type* of basket of goods it will ultimately consume (since each household locates many products).

5 As in Diamond and Yellin (1987,1990), this structure allows us to avoid explicitly modelling an equity market or the Arrow-Debreu redistribution of firms' profits. Incorporating this feature into a barter environment is problematic, since dividend payments are in the form of goods. The present ownership structure avoids this problem, puts barter and monetary exchange on the same footing, and allows a precise characterization of the difficulties of the former relative to the latter grounded in tastes (the problem of the double coincidence) and trade frictions.

firm and each firm produces a unique product. We assume that the set of firms in the economy is exogenously given.[6] We denote measures by $\sigma[\cdot]$ and normalize: $\sigma[\hat{H}_0] = \sigma[H_0] = N = H = 1$. This simplifies the subsequent arithmetic, since the ratio of workers per firm is then unity.

Preferences

In order to capture the 'problem of the double coincidence of wants,' we assume that agents possess idiosyncratic preferences. More specifically, each household, h, and each owner, $\hat{h}$, derives utility by consuming goods only in the respective intervals $\Omega(h)^*$ and $\hat{\Omega}(\hat{h})^*$ drawn, independently, and at random from Ω_0 at the beginning of each of the periods. We assume that these intervals are of equal length, but differ uniformly in their location on the commodity circle. We define, $x \equiv \sigma[\hat{\Omega}^*]/N = \sigma[\Omega^*]/N \in [0,1]$. In this context, the term $x \in [0,1]$ is the degree of specialization in tastes. In a given meeting between two agents endowed with distinct goods $\omega, \omega' \in \Omega_0$, the probabilities of a single coincidence of wants and the double coincidence of wants are x and x^2 respectively. If $x = 1$, agents are 'generalists' in consumption and all meetings can result in trade. Assumption 1 describes formally each agent's periodic utility function,

Assumption 1.

(*a*) Each household's periodic utility is given by,

$$U(t) = U[D(t)] \tag{1a}$$

where $U(\cdot)$ is a strictly increasing and strictly concave utility function, satisfying the boundary conditions $U(0) = 0$ and $lim_{D \to \infty} U(D) = \bar{u} << \infty$ and

$$D(t) = \left[\int_{\Omega^*} c(\omega,t)^{(\gamma-1)/\gamma} d\omega \right]^{\gamma/(\gamma-1)} \tag{1b}$$

where $\gamma > 1$ and $c(\omega;t)$ is the date t consumption of good ω.

(*b*) Each owner ω's periodic utility is:

$$\hat{U}(t) = \hat{D}(t) = \hat{c}(\omega,t) + \int_{\hat{\Omega}^*} \hat{c}(\omega',t) d\omega' \tag{2}$$

and $\hat{c}(\cdot,t)$ is consumption by owners.

6 In Laing, Li, Wang, (1999) we admit free entry and study the effects of inflation on welfare. None of the arguments presented in this paper depend upon the fixed-entry assumption.

In equation (1*b*), $U[D(t)]$ is the utility derived by consuming a basket of goods. The concavity assumption is standard and the asymptotic upper bound $\bar{u}$, as explained later, ensures the convergence of welfare under barter and monetary exchange as search frictions vanish. Observe from (1*b*) that the value obtained from any given basket of goods depends upon the variety of commodities contained therein [see Dixit and Stiglitz (1977)]. The parameter γ is the constant elasticity of substitution (CES) between goods. To ensure the existence of a well-defined monopolistically-competitive pricing game we impose $\gamma > 1$, implying that goods are substitutes. Finally, in (2) we assume, without significant loss of generality, that owners do not enjoy consumption variety *per se* and that their periodic utility is linear in $\hat{D}$.[7]

Technology

Assumption 2 describes the technology,

Assumption 2. (*Technology*)

(*a*) The technology of firm $\omega \in \Omega_0$ is,

$$y = y(\omega) = f(l(\omega)) \tag{3}$$

where $y(\omega)$ is output, $l(\omega)$ is employment. The technology $f(\cdot)$ is strictly concave and satisfies the boundary and Inada conditions: $f(0) = 0$ and $lim_{l \to 0} f'(l) = \infty$.

(*b*) Households are equally talented at producing any of the differentiated commodities.

(*c*) Firms and households can store their production good alone, without cost and in unlimited quantity. Goods stored in inventory depreciate at the common rate $\delta \in [0,1]$.

In part (*a*) each firm's production is governed by a standard concave technology, with labor as the sole factor of production. Note that in part (*b*) it is immaterial whether or not a worker accepts employment at a firm that produces a good in his consumption set. By virtue of the integral used to define the household's preferences [equation (1*b*)] the contribution to utility from any such source is precisely zero. The assumptions in part (*c*) that agents can store their production good (in any amount) and only their production

7 Given that households preferences are strictly concave in $D(t)$, this restriction is inconsequential. Owner preferences may be derived from (1*a*) by setting $U(\hat{D}) = \hat{D}$ and from (1*b*) by considering the limiting case, $\gamma \to \infty$.

good are important. The former, by minimizing the significance of money as a store of value, enables us to focus on its role as a medium of exchange. The latter feature precludes the emergence of commodity monies, which would complicate the analysis considerably. In what follows we denote household h's and firm ω's inventory holdings of good ω by $k(\omega,h)_t$ and $\hat{k}(\omega)_t$ respectively.

Owners make all of the hiring, production, and pricing decisions relevant to the firm they control. Thus, in any given period, each owner, $\omega \in \Omega_0$: (*i*) hires $l(\omega)$ workers, by offering a labor contract $\nu(\omega)_t = (W(\omega)_t, s(\omega)_t)$, where $W(\omega)_t \geq 0$ is a monetary payment (the nominal wage) and $s(\omega)_t \geq 0$ is a payment in terms of the firm's output and (*ii*) posts the monetary price $P(\omega)_t$ and the relative goods-for-goods prices, $r(\omega,\omega')_t$, at which it is willing to trade. (I.e., $r(\omega,\omega')$ is the amount of good $\omega' \in \Omega_0$ that firm ω receives per unit of ω traded by it). As we shall see later, one consequence of symmetry is that relative prices take the simple form: $r(\omega,\omega') \in \{0,r\}$ for each firm ω and each good ω'.

Fiat Money

The stock of fiat money is M_t. Fiat money is not intrinsically valued by any agent, cannot be privately produced (think of paper currency for example), and is perfectly divisible. We assume free disposal of cash balances, implying that $M_t \geq \int_{H_0} M(h)_t dh + \int_{\hat{H}_0} \hat{M}(\omega)_t d\omega$, where $M(\cdot)_t$ and $\hat{M}(\cdot)_t$ are, respectively, household h's and owner ω's nominal cash holdings. As in Lucas (1990) and Fuerst (1992) we assume that the money supply grows over time, as a consequence of a lump sum injection, T_t, given to firms each period.[8] The stock of money evolves as: $M_{t+1} = M_t + T_t = (1+\mu)M_t$, where $\mu \geq 0$ is the constant rate of monetary growth. Given the constant rate of monetary growth, μ, let $z_t \equiv (1+\mu)^t$. We use z_t to transform all of the nominal variables. Accordingly, let $m(h)_t \equiv M(h)_t/z_t$, $\hat{m}(h)_t \equiv \hat{M}(h)_t/z_t$, $w_t \equiv W_t/z_t$, and $p_t \equiv P_t/z_t$ and define $q(\omega)_t = (p(\omega)_t, r(\omega)_t)$. In what follows we shall have recourse to consider only these transformed variables.

Time Sequence

Figure I depicts the sequence of events during any given period t. In stage I the idiosyncratic preference shock is realized and both households and owners learn the respective intervals $\Omega(h)^*$ and $\hat{\Omega}(\omega)^*$ over which

8 We could equally well assume that, as in for example Casella and Feinstein (1990), the cash injection is received by households ('buyer's') rather than firms ('sellers'). Laing, Li, and Wang (1999) study the effects of a variety of monetary injection mechanisms. The focus of that paper, though, is quite different from that pursued here.

their preferences are defined for that period. In stage II the owner of each firm $\omega \in \Omega$ (*i*) offers $l(\omega)$ workers the contract $v(\omega) = \{w(\omega), s(\omega)\}$ and (*ii*) posts the prices $q(\omega)$. Once firms make their hiring commitments for the period, production commences and the terms of the contract are executed (stage III). In stage IV matching takes place and all trades occur. At stage V firms receive the monetary transfer, T, from the government. Finally, in stage VI each agent chooses a consumption and savings plan.

Markets

We further assume a competitive labor market, in which firms can hire labor provided their contractual offer, v, provides workers with a lifetime utility of at least V_0 (determined in a market for labor contracts). The competitive labor market is warranted by an assumed free mobility of labor and the assumption that households are equally talented at any firm.[9] We assume throughout that neither firms nor workers have access to capital markets.[10] The product market is characterized by significant search frictions. Assumption 3 describes the matching process in this market,

Assumption 3. (*The Product Market*)

(*a*) Matching takes place only between households and firms.

(*b*) Each household matches randomly and simultaneously with a subset of firms, $Z_0(N) \subseteq \Omega_0$, with measure:

$$\sigma[Z_0(N)] = \alpha(N) \qquad [\leq N] \tag{4}$$

where $\alpha(\cdot)$ is non decreasing in N and $lim_{N\to\infty}\, \alpha(N) = \alpha_0$ (possibly infinite).[11]

(*c*) Anonymity.

9 As is standard in (optimal) contracting environments, only the distribution of utility between workers and firms depends upon the competitive-labor-market assumption and not the (essential) properties of the contract. Thus, if V_0 is determined in a monopsonistic, or even search, labor market, then firms must simply offer contracts, v, that provide at least this reservation utility.

10 This implies that at stage III of the time sequence, firms must use beginning of period cash balances and/or inventory holdings to finance the firm's contractual obligations. Likewise, in stage IV households can procure goods only with their current income and/or any savings carried over from the previous period. The assumption that the firm cannot use *current* output to finance goods' payments to workers is inconsequential.

11 Our decentralized trading environment possesses elements in common with Coles and Muthoo (1998) and with Shi (1997). To avoid the measurability issues, *a lá* Judd (1985), that stem from drawing a continuum of random variables, we assume, as a primitive, that the sets $Z_0(N)$ are measurable.

Part (*a*) of the assumption simplifies admissible steady-state exchange patterns by precluding inter-household and inter-firm trades. In appendix *A* we show, at the cost of additional notation, that this pattern can be derived from more primitive assumptions on preferences and worker skills.

In part (*b*), each firm matches with a continuum of firms of measure $\alpha(N)$.[12] The earlier normalization $N = 1$, implies that $\alpha = \alpha(1)$ [≤ 1] also equals the fraction of firms that households contact each period. The parameter α captures the extent of search frictions in the underlying environment (a frictionless economy is consequently one in which $\alpha = 1$). Whenever a household meets a firm, then (as an identity) a firm must also meet a household. Given our earlier population normalizations α is also the fraction of households that each firm contacts during the period. Under random matching, αx is the measure of contacts that satisfy the single coincidence of wants (from either the perspective of households or firms). This gives αx^2 as the measure of contacts that satisfy the more stringent double coincidence of wants condition. The simultaneity assumption ensures that prices are independent of the *order* in which agents procure goods. In part (*c*) the anonymity assumption rules out the emergence of informal credit arrangements. However, this is inconsequential if $lim_{N\to\infty} \{\alpha(N)/N\} = 0$, since the probability of a subsequent random re-match is zero in any case.[13]

The intuition we intend to capture with our conception of the matching process is as follows. Think of a consumer who does his week's shopping at a local market or bazaar. While at the market we view the

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[12] The assumption that each household matches with a mass of traders is made for technical convenience. More specifically, it eliminates idiosyncratic consumption risk (by virtue of the law of large numbers); ensures that periodic utility is both positive and finite (i.e., utility may be written as an integral over a set with positive measure); provides a natural parameterization of market frictions (in terms of the measure, α, of agents contacted each period) and, finally, ensures that each firm is negligible (in the sense that its pricing and employment strategies have no effect upon aggregate consumer wealth). Alternatively, if households contact a *countable* number of sellers, Z, with number 'n.' The analog of equation (2) in this case (equation (1*b*) follows similarly) is, $\hat{D}(n) = \hat{c} + \Sigma_{j=1}^{j=n} \hat{c}(\omega_j)$. For finite n, $\hat{D}(n)$ is bounded and n itself parameterizes trade frictions. Our arguments remain valid (as an approximation) for 'n sufficiently large that the law of large numbers applies.' The use of a continuum lends itself to exact results and avoids this latter vagueness.

[13] The limiting property of $\alpha(N)$ is important. If $lim_{N\to\infty}\alpha(N) = \alpha_0 << \infty$, we can study the properties of an economy populated by an infinite array of monopolistically competitive firms, implying limitations on consumption variety are not an *intrinsic* feature of the economic environment. Here, as search frictions also vanish (i.e., $\alpha_0 \to \infty$) both monetary and barter equilibria might be expected, *a priori*, to converge in welfare terms. This offers a natural consistency check of the model's basic structure, by ensuring that it is not set up to favor one exchange protocol over another from the outset.

household as, in essence, simultaneously matching with many products (but not every product in the economy) and for realism conceive of him selectively purchasing a basket of commodities (but not every good offered for sale). The 'large numbers' assumption is intended to capture the notion that, although the consumer may be uncertain about the specific group of goods offered for sale that week, he anticipates 'almost surely' the *type* of basket of commodities that he will obtain. The simultaneity assumption implies that the prices the consumer faces at each stall are independent of the order in which he executes his shopping plan. Finally, in order to study both barter and monetary exchange we assume that each market stall posts both monetary and goods for goods prices and allow households to finance their purchases using cash and/or goods.

The Equilibrium Concept

In what follows we focus on stationary-symmetric Nash equilibria, in which (given each household's optimal behavior) each firm's choice of employment, l, contract, ν, and price q is optimal given the perceived behavior of other firms. Each firm is negligible in the continuum and treats as exogenous the worker reservation utility V_0 and the prices posted by other firms. Households optimally supply their labor on the basis of the contractual offers made by firms and take as given the prices set by firms. However, each firm is fully cognizant of the fact that households have met many other sellers and that they will substitute toward other commodities if the price it sets is unfavorable.

Our ultimate goal is to solve for the model's symmetric steady state Nash equilibria. This is accomplished in three steps. We first characterize each consumer's optimal demand functions for the differentiated products for a given price sequence. Once this is done, we determine each firm's best response function around any given (stationary) symmetric price configuration. The third and final step uses these best response functions to derive the model's Nash equilibria.

III. Household Behavior

In this Section we examine the behavior of an arbitrary household $h \in H_0$ endowed with $k_t = k(\omega', h)_t$ of good ω' and with money holdings m_t. We study the household's behavior within a stationary environment, wherein:

(*i*) the household's employer offers the labor contract, $\nu \equiv \{w,s\} = \nu_t \; \forall t$ and (*ii*) other firms post, *a.e.*, prices $\mathbf{q} \equiv (\mathbf{p},\mathbf{r}) = \mathbf{q}_t \; \forall \; t$.[14]

At the beginning of each period the household, in accordance with Assumption 3, matches with a set of firms Z_0 with measure $\sigma[Z_0] = \alpha$. Define the set, $Z \equiv \{\omega \in Z_0 \colon \omega \in \Omega(h)^*\}$, as the set of firms, ω, that match with household h and whose goods also belong to household h's consumption set, $\Omega(h)^*$. We then partition the set Z into two subsets, Z_B and Z_M, representing, respectively, matches that satisfy the double coincidence of wants and the household's (but not the owner's) single coincidence of wants. The set of matches that provide the household with zero utility are denoted Z_N. It is easily checked that the measures of these sets are: $\sigma[Z] = \alpha x$; $\sigma[Z_B] = \alpha x^2$; $\sigma[Z_M] = \alpha x(1\text{-}x)$; and $\sigma[Z_N] = \alpha(1\text{-}x)$.

If $w + m_t > 0$ *and* $k_t + s > 0$, the household h can use money and/or goods to finance purchases in the set Z_B. For the set Z_M, the household is obliged to use money, as these matches do not satisfy the double coincidence of wants. It is helpful to decompose the procurement of each good $\omega \in Z$ according to its financing. Thus,

$$c(\omega) \equiv c(\omega)_b + c(\omega)_m \qquad\qquad \textit{for all } \omega \in Z \tag{5}$$

where $c(\omega)_b$ is that part of $c(\omega)$ *financed* using goods' payments and $c(\omega)_m$ is that part *financed* with money. Note that with this convention, $c(\omega)_b = 0$ for all $\omega \in Z_M$ as households must use cash for meetings that do not satisfy the double coincidence of wants. The household's problem is,

$$V(k,m) = max_{\{c \in Z_B\}} \Big[U(D) + \beta V(k',m')\Big] \tag{6a}$$

$$s.t., \qquad k' = (1\text{-}\delta)\left\{ k + s - \int_{\omega \in Z_B} \mathbf{r} c_b d\omega \right\} \tag{6b}$$

$$(1+\mu)m' = \left\{ m + w - \int_{\omega \in Z} \mathbf{p} c_m d\omega \right\} \tag{6c}$$

Equation (5) *and* $c_b, c_m \geq 0$.

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14 At this juncture, recall the transformations: $w_t \equiv W_t/z_t$ and $p_t \equiv P_t/z_t$. In view of this, the stationary environment is one in which the nominal wage, W, and the price level, P, grow at the common rate μ.

where V is the household's value function, $k \equiv k(\omega,h)_t$, $k' = k(\omega,h)_{t+1}$, $m' = m_{t+1}$ and D is the CES valuation of goods in the set Z. To simplify the notation all time subscripts are suppressed. Condition (6*a*) is the consumer's objective function and (6*b*) describes the evolution of the household's inventory of goods. The household augments its current inventory holdings, k, through its goods' income s and depletes them through bartering for goods in Z_B. Analogously equation (6*c*) is the law of motion for the household's accumulated money balances. Consider,

Lemma 1. (*Household Behavior*)

Each consumer's optimal behavior is described by,

$$k = m = 0 \tag{7a}$$

(*A*) *If* $w/\mathbf{p} \geq [(1\text{-}x)/x](s/\mathbf{r})$ *then for all* $\omega \in Z$,

$$c(\omega) = (1/\alpha x)[(w/\mathbf{p}) + (s/\mathbf{r})] \tag{7b}$$

(*B*) *If* $w/\mathbf{p} < [(1\text{-}x)/x](s/\mathbf{r})$ *then,*

$$c(\omega) = c(\omega)_b = (1/\alpha x^2)(s/\mathbf{r}) \qquad \forall\omega\in Z_B \tag{7c}$$

$$c(\omega) = c(\omega)_m = [1/(\alpha(1\text{-}x)x](w/\mathbf{p}) \qquad \forall\omega\in Z_M \tag{7d}$$

Proof. All proofs are presented in Appendix B.

The environment confronting each household is stationary and non-stochastic, implying the absence of a precautionary saving's motive. With positive discounting, consumers optimally set their inventory, k, and cash, m, holdings to zero in steady-state [equation (7*a*)]. Consumers seek uniform consumption levels of each of the differentiated products in Z, as each good enters symmetrically into their strictly concave utility functions. However, this might not always be possible and this is the key to the distinction between cases *A* and *B* in the Lemma. For instance in part *B*, given that $s/\mathbf{r} > [x/(1\text{-}x)](w/\mathbf{p})$, the consumer is relatively 'goods rich.' Under these circumstances, equation (7*c*) shows that for any pair of goods $\omega_1 \in Z_B$ and $\omega_2 \in Z_M$, $c(\omega_1) = (1/\alpha x^2)(s/\mathbf{r}) > c(\omega_2) = [1/\alpha x(1\text{-}x)](w/\mathbf{p})$. Here the problem of the double coincidence of wants prevents the household from using his real good's income, $s/\mathbf{r}$, to obtain uniform levels of consumption (by affecting a simultaneous reduction in $c(\omega_1)$ and an increase in $c(\omega_2)$). In contrast, in the cash abundant case: $w/\mathbf{p} > [(1\text{-}$

$x)/x](s/r)$, uniform consumption levels *are* possible. In this case Lemma 1 shows that $c(\omega_1) = c(\omega_2) = \{(w/\mathbf{p}) + (s/\mathbf{r})\}/(\alpha x)$, indicating that consumers simply spread out their periodic real incomes: $\{(w/\mathbf{p})+(s/\mathbf{r})\}$ across all the matches that provide them with utility.

Lemma 2 describes the consumer demand functions at a firm, ω, that posts prices $q = (p,r)$ given that other firms post $\mathbf{q} = (\mathbf{p},\mathbf{r})$ *a.e.*[15] Define: $\hat{\rho} \equiv [((1-x)/x)(r/s)(w/p))]^{\gamma}$ and consider,

Lemma 2. *(Consumers' demand functions)*

(A) If $w/\mathbf{p} \geq [(1-x)/x](s/\mathbf{r})$ *then,*

$$c(\omega) = (1/\alpha x)[(w/\mathbf{p}) + (s/\mathbf{r})]\{\chi_A(\mathbf{r}/r)^{\gamma} + (1-\chi_A)(\mathbf{p}/p)^{\gamma}\} \tag{8a}$$

where $\chi_A = 1$ *if* $\omega \in Z_B$ *and* $r \leq (p/\mathbf{p})\mathbf{r}$; *otherwise* $\chi_A = 0$.

(B) If $w/\mathbf{p} < [(1-x)/x](s/\mathbf{r})$ *then,*

$$c(\omega) = (\alpha x^2(1-x))^{-1}\Big\{x(1-\chi_B)(w/\mathbf{p})+(1-x)\chi_B(s/\mathbf{r})\Big\}\Big\{\chi_B(\mathbf{r}/r)^{\gamma}+(1-\chi_B)(\mathbf{p}/p)^{\gamma}\Big\} \tag{8b}$$

where $\chi_B = 1$ *if* $\omega \in Z_B$ *and* $r \leq \hat{\rho}(p/\mathbf{p})\mathbf{r}$; *otherwise* $\chi_B = 0$.

(C) (Financing) If $s > 0$, *then,*

$$c(\omega)_b = \begin{cases} c(\omega) \\ (1/\alpha x^2)(s/\mathbf{r}) \\ 0 \end{cases} \qquad as\ r \begin{cases} < \\ = \\ > \end{cases} [\chi_C+(1-\chi_C)\hat{\rho}](p/\mathbf{p})\mathbf{r} \tag{8c}$$

where $\chi_C = 1$ *if* $w/\mathbf{p} \geq [(1-x)/x](s/\mathbf{r})$ *and* $\chi_C = 0$ *otherwise.*

The demand functions in Lemma 2 are easily recovered and take standard constant elasticity forms. For instance let $(w/\mathbf{p}) \geq [(1-x)/x](s/\mathbf{r})$ (case *A*) and let $\omega \in Z_B$. First, assume that the terms of goods-for-goods trading are favorable, so that $r < (\mathbf{p}/p)\mathbf{r}$. In this case, the demand function is $c(\omega) = c(\omega)_b = (1/\alpha x)\{(w/\mathbf{p})+(s/\mathbf{r})\}(\mathbf{r}/r)^{\gamma}$, indicating that the consumer procures ω through barter alone. Here, the pertinent price is the barter trading price $(\mathbf{r}/r)$. Alternatively, if $r > (\mathbf{p}/p)\mathbf{r}$, monetary exchange is now more

[15] Each firm is negligible in the continuum, implying that, strictly speaking, it does not make sense to evaluate the demand functions for a specific product. In order to obtain meaningful demand functions we, in essence, posit a 'large' firm which controls $\varepsilon > 0$ of the commodity space and which sets prices q. Consumer behavior is then derived in the limit $\varepsilon \to 0$, in which case the firm recognizes that its choice of q has a negligible effect on each consumer's wealth.

favorable, $(\mathbf{p}/p)$ is the relevant price, and the consumer uses only cash to finance the transaction: $c(\omega) = c(\omega)_m = (1/\alpha x)\{(w/\mathbf{p})+(s/\mathbf{r})\}(\mathbf{p}/p)^{\gamma}$.

A similar interpretation holds for, case B wherein $w/\mathbf{p} < [(1-x)/x](s/\mathbf{r})$. However, since $\hat{\rho} > 1$, the terms of monetary trade might have to be quite attractive before a consumer with an abundant supply of goods switches to the cash only financing of a good in Z_B.

IV. Pure Barter Exchange

In the *pure barter equilibrium* (*PBE*) all trade involves the exchange of goods for goods and money is not valued. Each period, workers receive their remuneration in terms of their employers output alone and, upon payment, search for trading partners. In order to establish the existence of a steady-state symmetric Nash equilibrium, our analysis proceeds as follows. We first *assume* that money is valueless and derive each seller's best-response given: (*i*) the consumer demand functions in Lemma 2 and (*ii*) both the prices and labor contracts offered by other firms. We then solve for the symmetric steady-state full-employment *PBE* and finally check that no agent optimally accepts cash, which, in this case, is trivial.

Firm's Behavior.

We determine the best response behavior of and arbitrary firm, indexed ω, conditional on the demands presented in Lemma 2; given values of $\mathbf{q} = (\infty,\mathbf{r})$ and $\mathbf{v} = (\mathrm{s},0)$ offered by other firms and a given level of employment per firm L (assuming that money is valueless).[16]

Firm ω matches with a set $\hat{Z}_0$ of employed customers, where $\sigma[\hat{Z}_0] = \alpha L$. It is convenient to identify each of these households with their current employer, denoted ω'', since it is the goods that they bring with them to market that ultimately determine firm ω's payoff. Parallel to the definitions in Section III for households, we can define $\hat{Z} \equiv \{\omega''\in\hat{Z}_0\colon \omega\in\Omega(\omega'')^*\}$, as the set customers that match with firm ω and who desire its product (ω). We partition $\hat{Z}$ into two subsets, $\hat{Z}_B$ and $\hat{Z}_M$, which represent, respectively, matches satisfying the double coincidence of wants and the household's (but not the owner's) single coincidence of

16 We distinguish the *ex ante* per-firm employment level L from its *ex post* full-employment equilibrium value $l^* = L = 1$.

wants. The measures of these sets are: $\sigma[\hat{Z}] = \alpha xL$, $\sigma[\hat{Z}_B] = \alpha x^2 L$ and $\sigma[\hat{Z}_M] = \alpha x(1\text{-}x)L$. The owner of firm ω maximizes her lifetime utility $\hat{V}$,

$$\hat{V}(\hat{k}) = max_{\{\hat{c},l,s,r\}} [\hat{c} + \alpha Lx^2 rc] + \beta\hat{V}(\hat{k}) \tag{9a}$$

$$s.t., \quad \hat{k}' = (1\text{-}\delta)[\hat{k} + f(l) - sl - \alpha x^2 Lc - \hat{c}] \tag{9b}$$

$$(s - \mathrm{s})l \geq 0 \tag{9c}$$

$$\hat{k} \geq sl \tag{9d}$$

where $\hat{k}' = \hat{k}(\omega)_{t+1}$, $\hat{k} = \hat{k}(\omega)_t$, and both time subscripts and the argument ω are suppressed. As a consequence of symmetry, the firm's relative price is $r = r(\omega,\omega'')$ for $\omega'' \in \hat{Z}_B$ and $r = 0$ otherwise.

In (9a) the owner of firm ω derives utility by consuming her own product ($\hat{c}$) in conjunction with goods in $\hat{Z}_B$ acquired after bartering with households.[17] Equation (9b) is the evolution equation of the firm's inventory holdings: any output not used to pay workers is either consumed by the owner, sold to other households, or else is stored for the future. Condition (9c) is the worker's participation constraint. The firm must offer a contract that offers a good's payment of at least s to be accepted by workers. The inequality (9d) reflects the absence of capital markets; all payments to workers are financed from beginning of period inventory holdings. The first-order and Benveniste-Scheinkman conditions are:

$$\hat{c}: \quad 1 - \beta(1\text{-}\delta)\hat{V}_k \leq 0 \text{ and } \hat{c} \geq 0 \text{ and } \hat{c}[1 - \beta(1\text{-}\delta)\hat{V}_k] = 0 \tag{10a}$$

$$l: \quad \beta(1\text{-}\delta)f' = \mathrm{s}\ (=s) \tag{10b}$$

$$s: \quad -\hat{V}_k + \psi = 0 \tag{10c}$$

$$r: \quad \alpha Lx^2 c[\gamma\beta(1\text{-}\delta)\hat{V}_k/r - (\gamma\text{-}1)] = 0 \tag{10d}$$

$$\hat{k}: \quad \hat{V}_k' = \beta(1\text{-}\delta)\hat{V}_k + \mu_B \tag{10e}$$

where $\hat{V}_k \equiv \partial\hat{V}(k)_t/\partial k_t$, $f' \equiv df(l)/dl$, and ψ, μ_B are the Lagrange multipliers on the constraints (9c) and (9d) respectively. The complementary slackness conditions reflect the possibility that the firm might, after

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17 Goods ω and ω'' are exchanged only if the double coincidence of wants is satisfied (i.e., only if $\omega'' \in \hat{Z}_B$). In equation (9a) the value of goods acquired by the owner from trading with households (in utility terms) is: $rc\alpha x^2 L$. It is derived as: $\int_{\hat{Z}_B} \hat{c}(\omega'')d\omega'' = \int_{\hat{Z}_B} r(\omega,\omega'')c(\omega)d\omega'' = rc\int_{\hat{Z}_B} d\omega'' = rc\alpha x^2 L$. The first equality follows from the identity that income equals expenditure: $\hat{c}(\omega'') \equiv r(\omega,\omega'')c(\omega)$. The second follows from symmetry ($r = r(\omega,\omega'')$ for all $\omega'' \in \hat{Z}_B$) and the third from the law of large numbers, $\sigma[\hat{Z}_B] = \alpha x^2 L$).

paying workers, optimally exchange all of its residual output with consumers and set $\hat{c}(\omega) = 0$. Condition (10*b*) says the firm hires workers up to the point at which the marginal benefit of labor equals its marginal cost (all measured in terms of real output). The other first-order conditions (10*c*) and (10*d*) and the envelope condition (10*e*) possess similar routine interpretations.

Steady-State Equilibrium

In a symmetric steady-state equilibrium with full-employment, the numbers of workers per firm is equalized ($L = l = 1$), each firm sets a common price ($\mathrm{r} = r = r^*$), and all firms offer the same payment to workers: $\mathrm{s} = s = s^*$. Also, in the *PBE*, cash is valueless ($p^* = \infty$) and money wages are not paid to workers $w^* = 0$. In order to avoid the tedious duplication of results in the boundary case $\hat{c} = 0$, in which the owner trades away all of her residual output, consider:

Condition U. *Let* $\beta \leq \gamma/\{\gamma + (1\text{-}\delta)(1\text{-}\gamma)\}$

Condition *U* ensures that owners discount the future sufficiently rapidly that it is optimal, at the margin, for them to consume unsold output beyond that required to pay for next period's labor. We assume throughout that Condition U holds.

Theorem 1. (*Pure Barter Equilibrium: PBE*)

Under Condition U a unique symmetric steady-state PBE exists. It is described by,

$$l^* = 1 \tag{11a}$$

$$\mathrm{v}^* = \{w^*, s^*\}, \text{ where } w^* = 0 \text{ and } s^* = \beta(1\text{-}\delta)f'(1) \tag{11b}$$

$$p^* = \infty \text{ and } r^* = \gamma/(\gamma\text{-}1) \tag{11c}$$

$$\hat{c}^* = f(1) - \beta(1\text{-}\delta)f'(1)[1 + (r^*/(1\text{-}\delta))]/r^* > 0 \tag{11d}$$

$$c(\omega)^* = (s^*/r^*)(1/(\alpha x^2)) \ \forall\omega \in Z_B \text{ and } c(\omega)^* = 0 \text{ otherwise.} \tag{11e}$$

$$\hat{k}^* = s^* \text{ and } \hat{m}^* \leq M_O. \tag{11f}$$

Equation (11*b*) says that workers are hired up to the point at which the value of their payment, s^*, equals the net value of their marginal product (adjusted by $\beta(1\text{-}\delta)$, reflecting discounting and the depreciation of inventory). Equation (11c) determines equilibrium pricing. The condition $r^* = \gamma/(\gamma-1)$ is standard in models of monopolistic competition. It equals each consumer's common marginal rate of substitution between all goods in their consumption set. With $p^* = \infty$, it is neither optimal for workers to exchange their labor for money nor for firms to trade their goods for money. Given symmetric pricing, each household uniformly allocates his periodic real income s^*/r^* among all commodities that satisfy the double coincidence of wants $\omega \in Z_B$. From (11b) and (11c) real income is,

$$(s^*/r^*) = [(\gamma-1)/\gamma]\beta(1-\delta)f'(1) \tag{12}$$

In (12) the term $1/r^* = (1-\gamma)/\gamma < 1$ is the wedge between workers' real incomes and their (suitably) discounted marginal product that arises by virtue of each firm's monopoly power. As $\gamma \rightarrow \infty$ consumers regard all goods as close substitutes. In this case each firm's monopoly is minimal and and both real incomes, s^*/r^*, and the relative prices, r^*, converge to their 'competitive' values (equations (11c) and (12)).

V. Monetary Exchange Under Steady-State Inflation

Although barter only trading is always an equilibrium, our model also admits monetary equilibria. Two cases may be distinguished. First, in the *pure monetary equilibrium* (*PME*) cash is used on one side of every transaction (goods and labor). Second, in the *mixed trading equilibrium* (*MTE*) monetary exchange and barter coexist.[18] Which of these two exchange regimes pertains depends crucially upon the parameter, $\Delta \equiv (1+\mu)(1-\delta)$, which captures the comparative advantage of barter relative to monetary exchange: barter is more attractive the lower is the rate of depreciation of goods, δ, and the higher is the rate of monetary growth μ.

The basic strategy used to prove the existence of a steady-state equilibrium and to characterize its properties, is essentially identical to that used for the *PBE* in Section IV. The main difference is ruling out the possibility, in the *PME*, that a firm will 'defect' from the proposed equilibrium and offer its

18 The MTE considered here is quite distinct from the "mixed-monetary equilibrium" (MME) analyzed by Kiyotaki and Wright (1993). There a mixed-strategy equilibrium is invoked in which each agent is indifferent between accepting and rejecting money as long as the population of agents accept it with a specific critical probability. As we explain below, the MTE is a *pure-strategy* equilibrium and it emerges only in specific regions of the parameter space.

employees a contract that includes both goods and cash payments, which workers optimally accept. Unlike fiat money, goods *are* intrinsically valuable.

Firm's Behavior

We determine the best-response of an arbitrary firm, indexed ω, conditional upon the consumer demand functions presented in Lemma 2; given values of $\mathbf{v} = (\mathrm{w,s})$ and $\mathbf{q} = (\mathbf{r,p})$ offered, *a.e.*, by other firms, and a given level of aggregate employment per firm of L.

With the lump sum cash transfer from the authorities, if the firm employs l workers at a wage W, its cash balances evolve as,

$$\hat{M}_{t+1} = [\hat{M}_t + \mu M_0(1+\mu)^t + \int_{\omega''\in\hat{Z}} P_t c_{mt} d\omega - W_t l], \tag{13}$$

where: $\mu M_0(1+\mu)^t$ is the nominal value of the periodic cash transfer and c_{mt} is the (money financed) demand for the firm's product, ω, by households, $\omega''\in\hat{Z}$. (It is determined using Lemma 2 and equation (5), as: $c(\omega)_m = c(\omega) - c(\omega)_b$). The firm augments its money holdings through cash sales to consumers and depletes them through money wage payments to workers (Wl). Using the transformations, $\hat{m}_t \equiv \hat{M}_t/z_t$, $p_t = P_t/z_t$ and $w_t = W_t/z_t$ in conjunction with the measure $\sigma[\hat{Z}]$, equation (13) becomes:

$$(1+\mu)\hat{m}_{t+1} = (\hat{M}_{t+1}/z_{t+1})(z_{t+1}/z_t) = [\hat{m}_t + \mu M_0 + \alpha x L p_t c_{mt} - w_t l] \tag{14}$$

Given the evolution constraint, (14), and the measures $\sigma[\hat{Z}_B]$ and $\sigma[\hat{Z}_M]$, the owner of firm ω solves:

$$\hat{V}(\hat{k},\hat{m}) = max_{\{\hat{c},l,w,p\}}\ [\hat{c} + \alpha L x^2 r c_b + \beta\hat{V}(\hat{k}',\hat{m}')] \tag{15a}$$

s.t.,

$$\hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p[(1\text{-}x)c(\omega_1)_m + xc(\omega_2)_m - wl], \qquad \omega_1\in\hat{Z}_M \textit{ and } \omega_2\in\hat{Z}_B \tag{14'}$$

$$\hat{k}' = (1\text{-}\delta)[\hat{k} + f(l) - sl - \alpha x L[(1\text{-}x)c(\omega_1) + xc(\omega_2)] - \hat{c}], \qquad \omega_1\in\hat{Z}_M \textit{ and } \omega_2\in\hat{Z}_B \tag{15b}$$

$$U[D] \geq (1\text{-}\beta)V_0 \tag{15c}$$

$$\hat{k} \geq sl \tag{15d}$$

$$\hat{m} \geq wl \tag{15e}$$

where, $\hat{m}' = m(\omega)_{t+1}$, $\hat{k}' = \hat{k}(\omega)_{t+1}$, $\hat{k} = \hat{k}(\omega)_t$, and the time subscript is suppressed from $\hat{c},c,p,l$, and w.

The possibility of barter implies that owners can derive utility by consuming their own product as well as goods acquired after trading with households (15*a*). Equation (14′) re-states the law of motion describing the evolution of the firm's money holdings. Notice that in (14′) households in $\hat{Z}_B$ and $\hat{Z}_M$ may well finance their purchases differently: members of the former set may use cash and goods while members of the latter must use cash. Equations (15*a*)-(15*d*) are exact analogous of their barter counterparts (9*a*)-(9*d*). In (15*c*) - the participation constraint - $U[D]$ is the periodic utility derived by the firm's employees from the contract ν, given that other firms set prices $\mathbf{q} = (\mathbf{p},\mathbf{r})$ *a.e.*[19]

At this juncture, it is important to emphasize that inequality (15*e*) is an *ex post finance* constraint, which arises due to the absence of capital markets. Correctly interpreted it is *not* an *ex ante* cash in advance constraint (restricting both the *means* of payment and exchange). The reason is that firms have the option of paying workers in terms of their own output (which workers can use to barter for goods with other firms). The object of the present exercise is to circumscribe the conditions under which this latter possibility either is or is not optimally utilized.

Steady-State Equilibrium.

In a symmetric steady-state equilibrium with full employment, employment per firm is equalized ($l = L = l^* = 1$), each firm sets a common price $p = \mathbf{p} = p^*$, and all firms offer the same contract $\nu = \boldsymbol{\nu} = \nu^* = (w^*,s^*)$. In addition, in the *PME* workers are not paid in goods, $s = \mathbf{s} = s^* = 0$, while in the *MTE* both barter and monetary exchange coexist ($w^* > 0$ and $s^* > 0$). Theorem 2 establishes the existence of monetary equilibria.

Theorem 2. (*Monetary Equilibria*)

Given condition U and denoting $\Delta \equiv (1+\mu)(1-\delta)$. *There is a unique stationary symmetric monetary equilibrium a.e.,*

(*A*) *If* $\Delta < 1$, *it is a PME characterized by,* $w^* > 0$ *and* $s^* = 0$,

(*B*) *If* $\Delta > 1$, *it is a MTE characterized by,* $w^* > 0$ *and* $s^* > 0$.

(*C*) *If* $\Delta = 1$, *there is a unique PME.*

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19 The term D is the consumer's valuation of the basket of goods acquired during the period. Formally, $D = \{\alpha x^2 c(\omega_1)^{(1-1/\gamma)} + \alpha x(1-x)c(\omega_2)^{(1-1/\gamma)}\}^{\gamma/(\gamma-1)}$, where $\omega_1 \in Z_B$ and $\omega_2 \in Z_M$.

Once again Condition U ensures that $\hat{c} > 0$ in either regime. In part (*a*) the condition that $\Delta < 1$ implies the comparative advantage of monetary exchange relative to barter. Here, the rate of inflation is not so high that firms optimally offer their employees both cash and goods payments. However, this is not so in (*b*), as $\Delta > 1$ and, as a consequence, $s^* > 0$.

In the knife-edge case $\Delta = 1$, neither monetary exchange nor barter has a comparative advantage. Accordingly, firms and workers are indifferent to any contract $\nu = (w,s)$, offering workers (equilibrium) utility V_0^*, provided that $w \geq p^*[(1-x)/x](s/r^*)$. The reason is that, under these circumstances, (*i*) households secure uniform consumption levels of all goods in their consumption set $\Omega^*(h)$ (Lemma 1) and (*ii*) at the margin, money wage payments, w, and payments in kind, s, are equally costly to the firm..

VI. Characterization of the PME and the MTE

In this Section we characterize formally the properties of the *PME* and *MTE* described in Theorem 2 and discuss the implications of our results.

Theorem 3. (*The PME and the MTE*):

(*A*) *In any symmetric steady-state monetary equilibrium:*

$$l^* = 1 \quad (16a)$$

$$\nu^* = \{w^*, s^*\}, \text{ where } M_0 = \hat{m}^* = w^* l^* \quad (16b)$$

$$r^* = \gamma/(\gamma-1) \quad (16c)$$

(*B*) *If* $\Delta \leq 1$, *then in the PME*

$$s^* = 0 \quad (17a)$$

$$p^* = M_0(1+\mu)r^*/[\beta f'(1)] \quad (17b)$$

$$\hat{c}^* = f(1) - \{\beta(1-\delta)f'(1)\}/[r^*\Delta] > 0 \quad (17c)$$

$$c(\omega)^* = (w^*/p^*)(1/\alpha x) \ \forall \omega \in Z \quad (17d)$$

$$\hat{k}^* = 0. \quad (17e)$$

(C) *If* $\Delta > 1$, *then in the MTE*

$$s^* = \beta(1-\delta)f'(1)\left\{\frac{x}{x + (1-x)\Delta^{1-\gamma}}\right\} > 0, \tag{17a'}$$

$$p^* = \left\{\frac{M_0(1+\mu)r^*}{\beta f'(1)}\right\}\left\{\frac{x\Delta^{\gamma-1} + (1-x)}{(1-x)}\right\} \tag{17b'}$$

$$\hat{c}^* = f(1) - \left\{\frac{\{xr^*/(1-\delta)\} + \{x + (1-x)\Delta^{-\gamma}\}}{r^*[x + (1-x)\Delta^{1-\gamma}}\right\}\beta(1-\delta)f'(1) > 0, \tag{17c'}$$

$$c(\omega_1) = c_b^* = (s^*/r^*)(1/\alpha^*x^2) > c(\omega_2) = c_m^* = (w^*/p^*)[\alpha x(1-x)], \text{ where } \omega_1 \in Z_B \text{ and } \omega_2 \in Z_M \tag{17d'}$$

$$\hat{k}^* = s^* > 0. \tag{17e'}$$

The competitive labor market assumption, in conjunction with full wage and price flexibility, implies that all workers are employed in any putative symmetric equilibrium (16*a*). Moreover, in a monetary equilibrium, the money stock is optimally held across each of the periods. Indeed, with $m(h) = 0 \ \forall h \in H_0$, firms hold all of the money balances at the end of each period and in an amount just sufficient to cover next period's wage bill. Notice that the barter trading price is $r^* = \gamma/(\gamma-1)$ as was the case for the *PBE* (16*c*).

Inspection of (17*b*) and (17*b'*) indicates that the price level is simply proportional to the (initial) stock of money M_0. Further examination of the system of equations (17) and (17') reveals that money is neutral, as the real variables in the model are independent of M_0. This finding, familiar in Walrasian models, stands in sharp contrast to much of the earlier search-money literature. Like Shi (1997) this results arises from the assumptions that money is divisible and because agents can hold both cash and goods. These features sever the somewhat artificial link between the supply of money and the 'fraction' of money traders, assumed in the economy in the earlier literature.

From (16*b*),(16*c*) and (17*b*) it follows that each household's real income in the *PME* is:

$$(w^*/p^*) = [(\gamma-1)/\gamma](1-\delta)\beta f'(1)/\Delta \tag{12'}$$

As in equation (12) the term $(\gamma-1)/\gamma < 1$ stems from the monopolistically competitive structure. Notably, (12') differs from the real income obtained in the *PBE* (12) only in the inclusion of the factor $1/\Delta$, reflecting the (possible) depreciation of goods necessarily stored under barter and the deleterious effects of anticipated inflation. A comparison of (17*c*) and (12′) indicates that money is not superneutral. An increase in the monetary growth rate, μ, re-distributes wealth from households to the owners of firms. It follows that there is no rate of inflation that is unequivocally Pareto optimal. A similar finding is obtained by Casella and Feinstein (1990), in which the monetary infusion is applied to one of two separate sectors. The underlying nominal variables are easily recovered. For instance: $P_t^* = p^*(1+\mu)^t$, indicating a constant steady-state rate of inflation equal to the monetary growth rate μ.

In the *PME* workers are not paid in goods, $s^* = 0$, and thus cannot subsequently engage in barter. This implies that in equilibrium the value of the relative price r^* is inconsequential for the payoff accruing to any given firm and, indeed, that witnessing a worker with goods for sale is an 'out-of-equilibrium' event. In the proof of Theorem 3 we show that: (*i*) if $r^* \geq \gamma/(\gamma\text{-}1)$ it is optimal for each firm to set $s^* = 0$ and that (*ii*) if $s^* = 0$, it is weakly dominant for each firm to set $r^* = \gamma/(\gamma\text{-}1)$. This establishes the *PME* as a sequential equilibrium under the elimination of weakly dominated strategies.

The *MTE*, perhaps not surprisingly, possesses many properties in common with the *PBE* described earlier and the *PME* described above. For the purposes of the present discussion, the key feature of the equilibrium is that $s^* > 0$ *and* $w^* > 0$, implying that monetary exchange and barter coexist. Given that $\gamma > 1$ and $\Delta = (1+\mu)(1\text{-}\delta) > 1$, it is easily seen from (17*a*′) that: $ds^*/d\mu > 0$. Thus, further increases in the rate of expansion of the money supply (and hence the rate of inflation) raise the steady-state volume of barter transactions. This finding is consistent with the commonly observed patterns of exchange under hyperinflation in which barter emerges, as sellers accept goods and cash payments and in which workers receive part of their remuneration in terms of their employer's output (Holtfrerich (1986) and Huang (1948) offer pertinent evidence on exchange patterns for the German and Chinese hyperinflations respectively).

This non-superneutrality result differs from those obtained by Casella and Feinstein and by Shi (1997). In Casella and Feinstein, an increase in the monetary growth rate affects the relative bargaining power of buyers and sellers under a *given* exchange protocol. Absent lump sum re-distributive taxation, this tends to improve the steady-state welfare of sellers relative to buyers. Shi considers endogenous exchange patterns and uncovers an interesting *trading opportunity effect*. This arises since each household fails to recognize the trading externality arising from its choice of money holders in the family. An increase in the money growth rate encourages households to trade money away by increasing this fraction (which promotes economic activity). In our model, the non-super-neutrality result stems from the fact that we endogenize *both* the medium of exchange and the means of factor payments. At higher rates of inflation, each firm optimally adjusts the terms of its contractual offer to workers by substituting away from cash payments towards (less costly) payments in kind.

As we have seen earlier (Lemma 1), consumers seek to spread their periodic real incomes uniformly across all goods they contact and desire. In view of this, the result in ($17d'$) that: $c_b^* > c_m^*$, reflects the distortionary effects of (hyper)inflation on steady-state consumption patterns. For sufficiently rapid rates of monetary growth (in which $\Delta > 1$), consumers substitute away from those goods they can procure through cash payments alone toward those that they can obtain through barter. Manipulation of ($17b'$) in conjunction with the other first-order conditions gives: $(c_m^*/c_b^*) = \Delta^{-\gamma} < 1$. Recall that all goods are close (perfect) substitutes as $\gamma \to \infty$. In this case, variety is inconsequential and, provided $\Delta > 1$, households can drive their consumption of c_m^* close to zero with little utility loss (i.e., $lim_{\gamma\to\infty}(c_m/c_b) = lim_{\gamma\to\infty} \Delta^{-\gamma} = 0$).

As in the *PME* money is neutral and a once and for all anticipated increase in M_0 simply raises all prices in direct proportion without real effects. Equation ($17b'$) implies, $\partial^2 p^*/\partial M_0 \partial\mu \propto [(1-x) + \gamma x\Delta^{\gamma-1}] > 0$. This says that increases in the initial money stock, M_0, have proportionately greater effects on the price level, p^*, the greater is the rate of inflation μ. This behavior is often imposed as a key assumption in the hyperinflation literature. Here it is derived endogenously. It arises because, as the volume of monetary transactions decline with μ, a given monetary infusion (M_0) is used to procure ever fewer goods. The term $x\gamma\Delta^{\gamma-1}$ appears, as it reflects the rate at which households are willing to abandon cash-financed consumption and switch to barter. In the case of perfect substitutes, then $\gamma \to \infty$ and hence $\gamma\Delta^{\gamma-1} \to \infty$. Here, even small differences in μ have a dramatic effect on the volume of barter transactions and hence upon the sensitivity of the price level, p^*, to the money stock M_0.

Casella and Feinstein (1990) obtain a similar result to the one reported above, but for quite different reasons. Their model is characterized by *predetermined* (monetary) exchange patterns, overlapping generations of different search vintages (corresponding to a buyer's duration in the market), and in equilibrium a maximal such vintage (at which point a buyer's money holdings have atrophied to a point of obsolescence).

They show that increases in the monetary growth rate lead to discrete changes in this maximal vintage and hence to discrete changes in the steady-state population of buyers in the market. As this occurs, the average time buyers hold cash is shortened and the velocity of circulation increases. As a consequence of this chain of events, any new (one off) injection of cash has a proportionately greater effect on prices the greater is the rate of money creation. In contrast, our finding is a direct consequence of endogenous adjustments to the aggregate volume of monetary and barter transactions undertaken in equilibrium. This latter mechanism is precluded in Casella and Feinstein, since an exogenous exchange role for money is prescribed *a priori*.

Turning now to workers' real incomes. These are,

$$(w^*/p^*) + (s^*/r^*) = [(\gamma-1)/\gamma]\beta(1-\delta)f'(l^*)\left\{ \frac{x + (1-x)\Delta^{-\gamma}}{x + (1-x)\Delta^{1-\gamma}} \right\} \tag{12''}$$

It is instructive to consider the limit, $\mu \rightarrow \infty$. Consider,

Theorem 4. *As the rate of monetary growth becomes arbitrarily large (i.e., as $\mu \rightarrow \infty$), the MTE converges to the PBE described in Theorem 1.*

In particular, from (12″),

$$lim_{\mu\rightarrow\infty} (w^*/p^*) = 0$$

$$(s^*/r^*)_{PBE} = lim_{\mu\rightarrow\infty}\{(s^*/r^*) + (w^*/p^*)\} = [(\gamma-1)/\gamma]\beta(1-\delta)f'(l^*)$$

indicating, from equation (12), that each worker's real income converges to that of the *PBE*: $(s^*/r^*)_{PBE}$. However, note that for any finite rate of inflation the monetary component of the real wage is strictly positive: $w^*/p^* > 0$ (provided of course cash is still valued). The continued circulation of money is a consequence of each household's preference for consumption variety. Even if μ is extremely large, small holdings of real money balances allow workers to secure an additional $\alpha x(1-x)$ goods relative to the basket they could obtain using barter alone (i.e., if $w^* = 0$). By virtue of their relative scarcity of these goods in the household's consumption basket, they possess extremely high marginal utilities of consumption and command a commensurately high 'willingness to pay.'

VII. Welfare Analysis

In this Section we compare the welfare properties of the *PBE* and *PME* described above. (In order to ensure the conditions of Theorem 2 are satisfied we assume throughout that $\Delta \leq 1$.). Up to this point, the measures of owners, households, and product varieties are normalized as: $\sigma[\hat{H}_0] = \sigma[H_0] = N = 1$ and it has been assumed that $\alpha(1) = \alpha_0 < 1$. It is instructive to consider an economy in which limited consumption variety is not an intrinsic feature of the environment. For this purpose, fix the measure of contacts at $\alpha(N) = \alpha_0 < \infty$ and set $M(N) = M_0 N$, let $\sigma[\hat{H}_0] = \sigma[H_0] = N$, and consider the limit $N \to \infty$. Under this re-scaling, the equilibrium properties of the model remain unchanged (in particular the values of employment and money holdings per firm remain at $l^* = 1$ and $\hat{m}^*$ per firm respectively). Theorems 1 and 3 may be used to compute each agent's steady-state lifetime discounted utility in the *PBE* (B) and the *PME* (M),

$$V^{*B} = U\left[(\alpha_0 x^2)^{1/(\gamma-1)}(s^*/r^*)\right](1-\beta)^{-1} \tag{18a}$$

$$\hat{V}^{*B} = \left[f(l^*) - (s^*/r^*)l^*\left\{1 + \frac{\delta r^*}{(1-\delta)}\right\}\right](1-\beta)^{-1} \tag{18b}$$

$$V^{*M} = U\left[(\alpha_0 x)^{1/(\gamma-1)}(w^*/p^*)\right](1-\beta)^{-1} \tag{18c}$$

$$\hat{V}^{*M} = \left[f(l^*) - (w^*/p^*)l^*\right](1-\beta)^{-1} \tag{18d}$$

Using equations (12) and (12′), it is readily verified that periodic real incomes in the *PBE* and in the *PME* are: $(s^*/r^*) = \beta(1-\delta)f'(l^*)$ and $(w^*/p^*) = (s^*/r^*)/\Delta$ respectively. In order to better understand the role played by trade frictions, α_0, and by the problem of the double coincidence of wants, $x^2 \leq x \leq 1$, it is instructive to first examine the benchmark case in which goods are perfectly storable ($\delta = 0$) and in which there is no monetary growth ($\mu = 0$). In this case $\Delta = 1$ and, as a result, $(w^*/p^*) = (s^*/r^*)$. Consider,

Theorem 5. (*Welfare properties of the PBE and the PME with* $\delta = \mu = 0$).

(*A*) $\hat{V}^{*B} = \hat{V}^{*M}$

(*B*) *For finite* α_0 *if* (*a*) $x < 1$, *then* $V^{*B} < V^{*M}$ *and* (*b*) $x = 1$, *then* $V^{*B} = V^{*M}$.

(*C*) (*Convergence*) $lim_{\alpha_0 \to 0}\ V^{*B} = lim_{\alpha_0 \to 0}\ V^{*M} = \bar{u}/(1-\beta)$, *where* $\bar{u} = lim_{D \to \infty} U(D)$.

Given that $\mu = \delta = 0$, then owners are equally well off in either the *PBE* or the *PME*. This is natural: they have no preference for consumption variety and under the conditions of the Theorem, there is neither an intrinsic disadvantage of barter (depreciation of inventory) nor of monetary exchange (inflation). However, Theorem 5 shows that even with $(s^*/r^*) = (w^*/p^*)$, workers' welfare levels are strictly lower in the *PBE* than in the *PME* whenever $\alpha_0 < \infty$ and $x < 1$. The drawback of barter exchange is that the problem of the double coincidence of wants stymies the *variety* of the resultant consumption basket (which may be seen by comparing: $\alpha_0 x > \alpha_0 x^2$ in equations (18*a*) and (18*c*)). However, if $x = 1$, agents are 'generalists' in consumption. Accordingly, all trades are beneficial and hence are consummated in the equilibrium.

Part (*iii*) of the Theorem is important, as it illustrates in a precise way the crucial role played by search frictions in the model. Thus, the *PBE* and the *PME* converge in welfare terms as trade frictions vanish, $\alpha_0 \to \infty$. Analogous welfare findings are reported in Kiyotaki and Wright (1993) for their *PME* and *PBE*. The key difference between our respective approaches that whereas their model is one of *temporal* frictions ('waiting times') our model is one of *atemporal* frictions (limited consumption variety). The welfare properties of the model in which $\delta > 0$ and $\mu > 0$ are intuitive. An increase in the depreciation rate of goods, δ, lowers the steady-state welfare of both households and firms in the *PBE*, leaving welfare levels in the *PME* unchanged. Similarly, an increase in the monetary growth rate is deleterious (to households) in the *PME*, but irrelevant in the *PBE* since money is not valued.

VIII. Conclusions and Suggestions for Further Work

In this paper we develop a monopolistically-competitive macroeconomic model, with decentralized search and trade. We study monetary and barter exchange, in an environment wherein the strong restrictions on storability and divisibility of goods and money, made in the earlier search literature, are relaxed. The resultant structure proves highly tractable and is used to examine endogenous trading patterns, pricing, inflation, and the effects of monetary policy.

We believe that the framework admits a number of interesting extensions. In future work we intend to incorporate a variety of assets (including share holdings and dividend payments) as well as a credit market. This exercise expands the scope of instruments at the government's disposal and permits a much richer analysis of the effects of monetary policy. The lack of a precautionary savings motive and the model's complete symmetry lead to a simple degenerate distribution of cash balances ex post, with firms holding all of the money in the economy at the end of each period. If instead, we assume that households are subject either to idiosyncratic taste shocks or shocks to their endowment of human capital, a *non-degenerate* cash distribution would emerge in equilibrium. In this case it would be of interest to solve for the distribution and to explore the effects of monetary policy on it. Finally, the explicit inclusion of firms is significant. This feature provides a natural forum for admitting endogenous capital accumulation by firms and thus for exploring the links between inflation and growth. These are enduring and important issues in monetary theory, but have proven to be difficult subjects of study when viewed under the conceptual lens of extant search theory.

Appendix A

We demonstrate that a suitable choice of a Wicksell preference/production structures ensures only household-firm trades arise in equilibrium, thus endogenizing the main features of assumption (3*a*). For this purpose, assume that there are $J \geq 3$ separate classes of goods and types of households indexed $j = 1,2,\ldots,J$. Within each class normalize the measure of firms and households to unity. Assume that households in class j: (*i*) consume only goods that belong to class $j+1$ (all modulo J) and (*ii*) possess the skills necessary for working in any firm in class j. Assume that the owner of a firm in class j derives utility from either their own good or goods in class j-1. Under this schema, household-household trades do not arise in equilibrium. A household that owns inventory in class j desires goods in class $j+1$. However, households who work for firms in class $j+1$ desire goods in class $j+2$ and so on. Inter-firm trades do not arise for similar reasons. However, household-firm trades may arise. A household in class j desires goods in $j+1$ and the owner of a firm that produces goods in class $j+1$ desires goods in $(j+1) - 1 = j$.

Appendix B

Proofs of Lemmas 1 and 2 and Theorems 1-5.

Lemmas 1 and 2. Consumers' Behavior.

Consider the recurrence relation, $V = U + \beta V'$ in (6*a*) [recall, $V = V(k_t,m_t)$ and $V' = V(k_{t+1},m_{t+1})$]. Assume that $s > 0$ and that $w > 0$ (if $sw = 0$ the problem is trivial). Partition the set of firms Ω_0 as Ω and Ω'' with measures (1-ε) and ε. Firms in Ω post prices $\mathbf{q} = (\mathbf{p},\mathbf{r})$ and those in Ω'' post prices $q = (p,r)$. The household matches with $\mathcal{Z}_0$ firms. Define $Z_0 \equiv \mathcal{Z}_0 \cap \Omega$ and $Z''_0 \equiv \mathcal{Z}_0 \cap \Omega''$. With a slight abuse of notation, define: $Z \equiv \{\omega: \omega \in Z_0 \cap \Omega(h)^*\}$; $Z_B \equiv \{\omega \in Z: \omega \in \Omega^*(h)$ *and* $\omega' \in \hat{\Omega}(\omega)^*\}$ and $Z_M \equiv \{\omega \in Z \setminus Z_B: \omega \in \Omega^*(h)\}$. Similarly, define: Z'', Z''_B, Z''_M for the set Z''_0. The measures of these sets are: $\sigma[Z_B] = \alpha x^2(1\text{-}\varepsilon)$, $\sigma[Z_M] = \alpha x(1\text{-}x)(1\text{-}\varepsilon)$, $\sigma[[Z''_B] = \alpha x^2(\varepsilon)$, and $\sigma[Z''_M] = \alpha x(1\text{-}x)(\varepsilon)$. Consider the constraints,

$$c(\omega') - c(\omega')_b \geq 0 \text{ and } c(\omega')_b \geq 0 \text{ for all } \omega' \in Z_B \cup Z''_B \qquad [\mu(\omega')_B] \qquad \text{(B1)}$$

where μ_B are non-negative multipliers. The first-order Benveniste-Scheinkman conditions are:

$$c(\omega): \qquad \Lambda c^{-1/\gamma} = \mathbf{p}(1+\mu)^{-1}\beta V_M \qquad \omega \in Z_M \qquad \text{(B2)}$$

$$c(\omega): \qquad \Lambda c^{-1/\gamma} = \mathbf{p}(1+\mu)^{-1}\beta V_M - \mu_B \qquad \omega \in Z_B \qquad \text{(B3)}$$

$$c(\omega)_b: \qquad -\beta(1\text{-}\delta)V_k\mathbf{r} + \mathbf{p}\beta V_M - \mu_B = 0 \qquad \omega \in Z_B \qquad \text{(B4)}$$

k: $kV_k(1 - \beta(1-\delta)) = 0$ (B5)

m: $mV_m(1 - \beta(1+\mu)^{-1}) = 0$ (B6)

$c(\omega)''$: $\Lambda c''^{-1/\gamma} = p(1+\mu)^{-1}\beta V_M,$ $\omega \in Z''_M$ (B7)

$c(\omega)''$: $\Lambda c''^{-1/\gamma} = p(1+\mu)^{-1}\beta V_M - \mu''_B,$ $\omega \in Z''_B$ (B8)

$$c(\omega'')_b: \quad \left.\begin{array}{r} -\beta(1-\delta)V_k r + p(1+\mu)^{-1}\beta V_M - \mu''_B \le 0 \\ c''_b \ge 0 \end{array}\right\} \textit{Comp.}, \ \omega'' \in Z''_B \qquad \text{(B9)}$$

where $\Lambda \equiv [\partial U/\partial D](D)^{1/\gamma}$. With $\beta < 1$, $(1-\delta) \le 1$, and $(1+\mu)^{-1} \le 1$, then from complementary slackness conditions (B5) and (B6): $k = m = 0$. This follows as $V_m > 0$ and $V_k > 0$, from (B2)-(B4). Using this, (6*a*),(6*b*), and the measures of Z_B and Z gives: $(1/\alpha x^2)(s/\mathbf{r}) = c_b$ and $(1/\alpha x)(w/\mathbf{p}) = (1-x)c+x(c-c_b)$.

(i) Lemma 1

Assume that $\varepsilon = 0$. There are two cases to consider. (*a*) Let $(w/\mathbf{p}) \ge (s/\mathbf{r})[(1-x)/x]$. If $\mu_B > 0$, then: $c(\omega) = c(\omega)_b$ from complementary slackness. Also (B2) and (B3) give: $c(\omega_1) < c(\omega_2)$, where $\omega_1 \in Z_M$ and $\omega_2 \in Z_B$. Constraints (6*b*) and (6*c*) imply: $c_b = (s/\mathbf{r})/(\alpha x^2) > c_m = (w/\mathbf{p})/(\alpha x(1-x))$. This contradicts (*a*). It follows that $\mu_B = 0$ and that $c = c(\omega)\ \forall \omega \in Z_M \cup Z_B$. Constraints (6*b*) and (6*c*) give: $c_b = (s/\mathbf{r})/\alpha x^2$ and $w/\mathbf{p} = \alpha x c - \alpha x^2 c_b$. In turn this yields, $c = \{(w/\mathbf{p})+(s/\mathbf{r})\}/(\alpha x) \ge c_b$. (*b*) Let, $(w/\mathbf{p}) < (s/\mathbf{r})[(1-x)/x]$. If $\mu_B = 0$, then the previous argument gives: $\{(w/\mathbf{p})+(s/\mathbf{r})\}/(\alpha x) \ge c_b = (s/\mathbf{r})/(\alpha x^2)$ a contradiction. So $\mu_B > 0$, implying that: $c(\omega) = c_b\ \forall \omega \in Z_B$. The constraints (6*b*) and (6*c*) then yield: $c(\omega_1) = (w/\mathbf{p})/(\alpha x(1-x)) < c(\omega_2) = (s/\mathbf{r})/(\alpha x^2)\ \forall \omega_1 \in Z_M$ and $\omega_2 \in Z_B$.

(ii) Lemma 2

From (B2) and (B7) and from (B3) and (B8),

$(c(\omega)/c(\omega''))^{-(1/\gamma)} = (\mathbf{p}/p)$ *where* $\omega \in Z_M$ *and either* $\omega'' \in Z''_M$ *or* $\omega'' \in Z''_B$ *and* $c_b'' = 0$. (B10)

$(c(\omega)/c(\omega''))^{-(1/\gamma)} = (\mathbf{r}/r)$ *where* $\omega \in Z_B$ *and* $\omega'' \in Z''_B$ *and* $c_b'' > 0$. (B11)

The demand functions reported in part (B) of the Lemma are derived as follows. Consider the limit $\varepsilon \to 0$, in which case $c(\omega)$ is determined by Lemma 1 for all $\omega \in Z_M \cup Z_B$. First consider, $w/\mathbf{p} < [(1-x)/x](s/\mathbf{r})$. The argument used to prove Lemma 1 implies, $\mu_B > 0$ and $c = c_b = (s/\mathbf{r})/(\alpha x^2)$ (all $\omega \in Z_B$) $> c_m = (w/\mathbf{p})/[\alpha x(1-x)]$ (all $\omega \in Z_M$). Equations (B10) and (B11) give:

$$c_b'' \equiv c(\omega'') = c_b(\mathbf{r}/r'')^{\gamma} \qquad \text{for all } \omega'' \in Z_B'' \text{ if } c_b > 0 \tag{B12}$$

$$c_m'' \equiv c(\omega'') = c_m(\mathbf{p}/p'')^{\gamma} \qquad \textit{Otherwise} \tag{B13}$$

Also, since $\mu_B'' \geq 0$, $c_b'' \geq c_m''$. The R.H.S of (B12) is decreasing in r''. Let, $\hat{\rho} \equiv \{((1-x)/x)(s/\mathbf{r})(\mathbf{p}/w)\}^{\gamma}$ as in the text. Then, using (B12), (B13), in conjunction with Lemma 1 gives,

$$c(\omega'') = (s/\alpha\mathbf{r}x^2)(r/r'')^{\gamma} \qquad \textit{if } r'' \leq \hat{\rho}(\mathbf{r}/\mathbf{p})p'' \textit{ and } \omega \in Z_B'' \tag{B14}$$

$$c(\omega'') = (w/\alpha\mathbf{p}x(1-x))(p/p'')^{\gamma} \qquad \textit{if } \omega'' \in Z_M \textit{ or if } r'' > \hat{\rho}(r/p)p'' \textit{ and } \omega \in Z_B'' \tag{B15}$$

Equations (B14)-(B15) are compactly written by defining the indicator function χ_B as is done in Lemma 2. Case (A) follows analogously. Finally, part (*C*) follows from the complementary slackness condition reported in (B9). If $s = 0$, the consumer's demand functions are derived directly from (6*b*), (6*c*), (B2) and (B7) with $c_b = 0$. Likewise if $w = 0$, then (6*b*), (6*c*), (B3), and (B8) are used.‖

Theorem 1 (The PBE)

Given the stationary values (s,r), equations (10) in the text uniquely define the representative firm's best response behavior. Since $f(l)$ is strictly concave, there is a unique value s^* at which point: $\beta(1-\delta)f'(l^*) = s^*$ and $l^* = 1$. If $\hat{c} > 0$, then complementary slackness gives, $r^* = \gamma/(\gamma-1)$ as the unique best response for r. However, under Condition U, $\hat{c} = 0$ is impossible. This follows as: $\beta\{\gamma + (1-\delta)(\gamma-1)\}/\gamma \leq 1$ and the strict concavity of $f(\cdot)$ implies:

$$\hat{c}/l^* = f(l^*) - \beta f'(l^*)\{\gamma + (1-\delta)(\gamma-1)\}/\gamma \geq f(l^*)/l^* - f'(l^*) > 0, \tag{B16}$$

establishing the uniqueness of s^*, r^*, l^*. Equation (10*e*) implies that $\mu_B > 0$. Hence, from complementary slackness and (9*d*), $k^* = s^* l^*$. Finally, Lemma 1 gives consumers' equilibrium demands as: $c(\omega)^* = (s^*/r^*)/(\alpha x^2)$ for all $\omega \in Z_B$.‖

Theorems 2 and 3. (Monetary Exchange)

(*i*) *The PME.*

Let $\Delta \equiv (1-\delta)(1+\mu) < 1$. We first establish that with $\Delta < 1$ there is a stationary ***PME*** and characterize its properties (parts (*A*) and (*B*) of Theorem 3). Given that $s^* = 0$, the basic proof of the uniqueness of the symmetric steady-state equilibrium is virtually the same as that used in Theorem 1 above, once obvious adjustments are made to the first-order conditions analogous to (10) reported in the text. The only caveat is that we must prove that it is not optimal for a firm to defect from the proposed equilibrium and to offer workers $s > 0$, contrary to the Theorem. Let $(s^*, r^*, \hat{c}^*, w^*, p^*, l^*)$ be the values reported in parts (*A*) and (*B*) of Theorem 3. In the proposed equilibrium the firm is assured a periodic utility,

$$\hat{c}^* = f(l^*) - (w^*/p^*)l^* > 0 \tag{B17}$$

Consider an arbitrary firm, $\omega \in \Omega_0$, that sets $q = (p,r)$ and offers $s > 0$ (if $s = 0$, there is nothing to prove). The first-order conditions for the firm's problem, evaluated in steady state, are easily derived with the aid of the recurrence relation: $\hat{V} = \hat{c} + \beta \hat{V}'$ (equation (15*a*)). After manipulation,

$$\hat{c}: \qquad \hat{V}_k = 1/\beta(1-\delta) \tag{B18}$$

$$l: \qquad f(l)' = s\hat{V}_k + w\hat{V}_m \tag{B19}$$

$$w: \qquad -\hat{V}_m + \psi/p^* = 0 \tag{B20}$$

$$s: \qquad -\hat{V}_k + \psi/r^* = 0 \ (s > 0) \tag{B21}$$

$$p: \qquad \{(\gamma/p) - (\gamma-1)\beta(1-\delta)\hat{V}_m/\Delta\} = 0 \tag{B22}$$

where, $\hat{V}_k \equiv \partial\hat{V}/\partial k$ etc., and ψ is the non-negative multiplier on the worker participation constraints, $w/p^* + s/r^* \geq w^*/p^*$. Simple manipulation of these conditions, noting $r^* = \gamma/(\gamma-1)$ gives,

$$p/p^* = \Delta < 1 \tag{B23}$$

$$\beta(1-\delta)f'(l)/r^* = w^*/p^* \tag{B24}$$

$$\psi > 0 \Rightarrow w/p^* + s/r^* = (w^*/p^*) \tag{B25}$$

However, (12′) says that,

$$\beta(1-\delta)f'(l^*)/(r^*\Delta) = w^*/p^* \tag{B26}$$

implying that $l^* \geq l$, as $\Delta < 1$ and $f(\cdot)$ is strictly concave. Under the proposed defection, the firm's steady-state periodic payoff is derived from: $k = sl = (1-\delta)\{f(l) - \hat{c} - \alpha x l^* c_m\}$ and $c_m = c^*(p^*/\mathbf{p})^\gamma$ as,

$$\hat{c} = f(l) - [sl/(1-\delta)] - \alpha x l^* c^* (p^*/\mathbf{p})^\gamma \tag{B27}$$

$$= f(l) - [sl/(1-\delta)] - l^*(w^*/p^*)\Delta^{-\gamma} \tag{B28}$$

where, (B28) follows from (B27), since $p^*/\mathbf{p} = \Delta^{-1}$ and $c^* = (1/\alpha x)(w^*/p^*)$. Finally, comparing (B28) and the steady-state payoff (B17), gives:

$$\hat{c}^* - \hat{c} = \{f(l^*) - f(l)\} + l^*(w^*/p^*)\{\Delta^{-\gamma} - 1\} + s/(1-\delta) > 0 \tag{B29}$$

The inequality in (B29) follows since, $l^* \geq l$, $(1/\Delta)^\gamma \geq 1$, and $s > 0$. This establishes that it is strictly sub-optimal for the firm to defect from the proposed equilibrium and to offer $s > 0$ given that $r^* = \gamma/(\gamma-1)$. Finally, we show that setting $r^* = \gamma/(\gamma-1)$ is a weakly dominant strategy. For this purpose, suppose that $\varepsilon' > 0$ workers are endowed at the beginning of each period with goods alone. In this case, the relative price r^* is well defined and is easily shown to be determined by (12*c*) in the text. The equilibrium in the modified game converges, as $\varepsilon' \to 0$, to that described in parts (*i*) and (*ii*) of Theorem 3. This establishes it as a sequential equilibrium under the elimination of weakly dominated strategies.

(*ii*) *Theorem 2 and parts (A) and (B) of Theorem 3.*

Given employment per firm L, prices $\mathbf{q} = (\mathbf{p},\mathbf{r})$ and labor contracts $\mathbf{v} = (\mathbf{w},\mathbf{s})$, the owner of firm ω solves the following program:

$$\text{(P)} \qquad V(\hat{k},\hat{m}) = max_{\{\hat{c},l,v,q\}}\ [\hat{c} + \alpha L x^2 r c_b + \beta V(\hat{k}',\hat{m}')] \qquad \text{(B30}a\text{)}$$

$$s.t., \qquad \hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p[(1-x)c(\omega_1)_m + xc(\omega_2)_m - wl], \quad \omega_1 \in \hat{Z}_M \ and\ \omega_2 \in \hat{Z}_B \qquad \text{(B30}b\text{)}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L[(1-x)c(\omega_1) + xc(\omega_2)] - \hat{c}], \quad \omega_1 \in \hat{Z}_M \ and\ \omega_2 \in \hat{Z}_B \qquad \text{(B30}c\text{)}$$

$$U[D] \geq (1-\beta)V_0 \qquad \text{(B30}d\text{)}$$

$$\hat{k} \geq sl \qquad \text{(B30}e\text{)}$$

$$\hat{m} \geq wl \qquad \text{(B30}f\text{)}$$

The basic strategy of proof is virtually identical to that used above. The only caveat is that - as indicated by Lemma 2(*C*) - there is a discontinuity in the means used by consumers to *finance* their purchases from firm ω. This must be dealt with before the firm's best-response function is derived. For this purpose we introduce a convexification that avoids the discontinuity and ensures that households and firms accrue payoffs at least as great as without it. Call this extended program (P*). We show that the solution of the extended program (P*) is also a solution of (P).

Consider firm ω and recall that $\hat{Z}_B$ is the set of the firm's customers that satisfy the double coincidence of wants. The convexification takes the following form. We assume that the firm assigns to each $\omega'' \in \hat{Z}_B$ the indicator: $I(\omega,\omega'') \in \{0,1\}$. If $I = 0$ the household must finance all their purchases using goods, while if $I = 1$ the customer must use money. The firm chooses: $\theta(\omega) \equiv \text{Prob}[I(\omega,\omega'') = 0 \mid \omega'' \in \hat{Z}_B]$. Under this scheme, the firm's receipts are continuous in θ and its prices q. The firm's demand functions are:

(*a*) *If* $(\mathbf{w}/\mathbf{p}) \geq [(1-x)/x](\mathbf{s}/\mathbf{r})$, *then:*

$$c(\omega)_b = (1/\alpha x)\{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\}(\mathbf{r}/r)^{\gamma} \qquad \textit{If } \omega \in Z_B \textit{ and } I(\omega) = 0 \qquad \text{(B31}a\text{)}$$

$$c(\omega)_m = (1/\alpha x)\{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\}(\mathbf{p}/p)^{\gamma} \qquad \textit{Otherwise} \qquad \text{(B31}b\text{)}$$

(*b*) *If* $(\mathbf{w}/\mathbf{p}) < [(1-x)/x](\mathbf{s}/\mathbf{r})$

$$c(\omega)_b = (1/\alpha x^2)\{\mathbf{s}/\mathbf{r}\}(\mathbf{r}/r)^{\gamma} \qquad \textit{If } \omega \in Z_B \textit{ and } I(\omega) = 0 \qquad \text{(B31}c\text{)}$$

$$c(\omega)_m = (1/\alpha x(1-x))\{(\mathbf{w}/\mathbf{p})\}(\mathbf{p}/p)^{\gamma} \qquad \textit{Otherwise} \qquad \text{(B31}d\text{)}$$

Under the convexification, the firm solves:

$$\text{(P*)} \qquad \hat{V}(\hat{k},\hat{m}) = max_{\{\hat{c},l,\nu,q,\theta\}} [\hat{c} + \alpha L x^2 r\theta c_b + \beta\hat{V}(\hat{k}',\hat{m}')] \qquad \text{(B32}a\text{)}$$

$$s.t., \qquad \hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p[(1-x\theta)c_m - wl], \qquad \text{(B32}b\text{)}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L[(1-x\theta)c_m + x\theta c_b] - \hat{c}], \qquad \text{(B32}c\text{)}$$

$$U[D] \geq (1-\beta)V_0 \qquad [\psi l] \qquad \text{(B32}d\text{)}$$

$$\hat{k} \geq sl \qquad [\mu_B] \qquad \text{(B32}e\text{)}$$

$$\hat{m} \geq wl \qquad [\mu_M] \qquad \text{(B32}f\text{)}$$

$$1 \geq \theta \geq 0. \qquad [\mu_\theta] \qquad \text{(B32}g\text{)}$$

where c_m and c_b are given by equations (B31).

Where ψ, μ_B,μ_M are non-negative Lagrange multipliers on the constraints (B32*d*)-(B32*f*) and μ_θ is a multiplier on the constraint $1 \geq \theta$, ensuring that the mixing probability cannot exceed unity. To show that P^* implements P, let $(\mathbf{w}/\mathbf{p}) \geq [(1-x)/x](\mathbf{s}/\mathbf{r})$. In Program (P) if:

(*a*) $r < \mathbf{r}(p/\mathbf{p})$ then $c(\omega) = (1/\alpha x)\{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\}(\mathbf{p}/p)^{\gamma}$ for $\omega \in Z_M$ and $c(\omega) = c(\omega)_b = \{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\}(\mathbf{r}/r)^{\gamma}$ for $\omega \in Z_B$. Program (*P*) gives:

$$\hat{V}(\hat{k},\hat{m}) = [\hat{c} + \alpha L x^2 r c_b + \beta\hat{V}(\hat{k}',\hat{m}')] \qquad \text{(B33}a\text{)}$$

$$\hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p[(1-x)c_m - wl], \qquad \text{(B33}b\text{)}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L[(1-x)c(\omega_1) + xc(\omega_2)] - \hat{c}], \qquad \omega_1 \in \hat{Z}_M \textit{ and } \omega_2 \in \hat{Z}_B \qquad \text{(B33}c\text{)}$$

In Program (P*) set $\theta = 1$,

$$\hat{V}(\hat{k},\hat{m}) = [\hat{c} + \alpha L x^2 r c_b + \beta\hat{V}(\hat{k}',\hat{m}')] \qquad \text{(B34}a\text{)}$$

$$\hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p[(1-x)]c_m - wl], \tag{B34b}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L[(1-x)c_m + xc_b] - \hat{c}], \tag{B34c}$$

Comparing (33) and (34) shows $(P^*) \Rightarrow (P)$.

(*b*) $r > \mathbf{r}(p/\mathbf{p})$ then in (P) $c(\omega) = c_m = (1/\alpha x)\{(\mathbf{w}/\mathbf{p})+(\mathbf{s}/\mathbf{r})\}(\mathbf{p}/p)^{\gamma} > c_b = 0$, $\forall\omega\in Z_B\cup Z_M$. In this case,

$$\hat{V}(\hat{k},\hat{m}) = [\hat{c} + \beta\hat{V}(\hat{k}',\hat{m}')] \tag{B35a}$$

$$\hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p c_m - wl], \tag{B35b}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L c_m] - \hat{c}], \tag{B35c}$$

In (P*) set $\theta = 0$.

$$\hat{V}(\hat{k},\hat{m}) = [\hat{c} + \beta\hat{V}(\hat{k}',\hat{m}')] \tag{B36a}$$

$$\hat{m}'(1+\mu) = [\hat{m} + \mu M_0 + \alpha x L p c_m - wl], \tag{B36b}$$

$$\hat{k}' = (1-\delta)[\hat{k} + f(l) - sl - \alpha x L c_m] - \hat{c}], \tag{B36c}$$

(*c*) $r = \mathbf{r}(p/\mathbf{p})$, then in (P) $c(\omega) = (1/\alpha x)\{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\}(\mathbf{p}/p)^{\gamma} \geq c_b$. In (P*) setting $\theta = \{(1/x)(\mathbf{s}/\mathbf{r})\}/\{(\mathbf{w}/\mathbf{p}) + (\mathbf{s}/\mathbf{r})\} \leq 1$ implements *P*. Similar arguments for $(\mathbf{w}/\mathbf{p}) < [(1-x)/x](\mathbf{s}/\mathbf{r})$ establish (P*) $\Rightarrow$ (P).

The first-order conditions for (*P**) are readily derived.

$\hat{c}$: $\quad 1 - \hat{V}_k\beta(1-\delta) \leq 0$ and $\hat{c} \geq 0$ with, $[1 - \hat{V}_k\beta(1-\delta)]\hat{c} = 0$ (B37*a*)

l: $\quad \beta(1-\delta)\hat{V}_k f(l)' = s\hat{V}_k + w\hat{V}_m$ (B37*b*)

r: $\quad \alpha x^2 L\theta c_b\left\{\dfrac{\gamma(1-\delta)\beta\hat{V}_k}{r} - (\gamma-1)\right\} = 0$ (B37*c*)

p: $\quad \alpha x(1-\theta x)\beta V_m L(1+\mu)^{-1}\left\{\Delta((V_k/V_m)(\gamma/p)) - (\gamma-1)\right\} = 0$ (B37*d*)

θ: $\quad \left.\begin{array}{ll}\alpha x^2 L[c_b\{r - \beta(1-\delta)V_k\} + \beta(1-\delta)c_m\{\hat{V}_k - \hat{V}_m p/\Delta\} - \mu_\theta] & \leq 0 \\ \theta & \geq 0\end{array}\right\}$ *Comp.* (B37*e*)

w: $\quad -\hat{V}_m w + \psi\Lambda D_w \leq 0$ *and* $s \geq 0$ *(comp.)* (B37*f*)

s: $\quad -\hat{V}_k s + \psi\Lambda D_s = 0$ *and* $w \geq 0$ *(comp.)* (B37*g*)

m: $\quad m = wl = \alpha x L(1-x)p c_m$ (B37*h*)

k: $\quad k = sl = (1-\delta)\{k + f(l) - sl - \alpha x L[(1-x)c_m + xc_b] - \hat{c}\}$ (B37*i*)

where: $\Lambda \equiv [\partial U/\partial D](D)^{1/\gamma}$, $D_w \equiv \partial D/\partial w$ and $D_s \equiv \partial D/\partial s$.

There are three cases to consider.

(*i*) Let $\Delta < 1$.

Condition U implies that $\hat{c} > 0$, in which case: $V_k = 1/[\beta(1-\delta)]$. Assume that $\hat{s} > 0$ and that $(w^*/p^*) > [(1-x)/x](s^*/r^*)$ [if $s^* = 0$, there is nothing to prove]. Equations (B37*d*) and (B37*f*)-(B37*g*) yield, respectively, $\Delta V_k/V_m = (p^*/r^*) = V_k/V_m$, which is a contradiction. Now suppose that, $0 < (w^*/p^*) < [(1-x)/x](s^*/r^*)$. In this case (B37*f*)-(B37*g*) give: $V_m/V_k = (D_w/D_s) = [((1-x)p^*s^*)/(r^*w^*x)]^{(1/\gamma)}(r^*/p^*)$. Equations (B37*c*) and (B37*d*) imply, $(r^*/p^*)\Delta = V_m/V_k$. It follows that: $1 > \Delta = [((1-x)p^*s^*)/(r^*w^*x)]^{(1/\gamma)} > 1$, which is a contradiction. Consequently, whenever $\Delta < 1$ then $s^* = 0$ is the only candidate equilibrium. Part (*a*) of the proof establishes the existence of the *PME* under these circumstances. Thus (P^*) implements (P) with $s = \mathbf{s} = s^* = 0$ and $\theta'' = 0$.

(*ii*) Let $\Delta > 1$.

In any putative symmetric steady-state equilibrium: $L = l = l^*$, $\mathbf{r} = r = r^*$ etc. Suppose that $\Delta > 1$, condition U is satisfied, and that contrary to claim that $\theta'' < 1$ and $\hat{c}^* > 0$. Then $\mu_\theta = 0$ from complementary slackness. Also, using (B37*c*) and (B37*d*) in (B37*e*) gives,

$$(c_b^* - c_m^*)(r^* - 1) \leq 0$$

which implies, $c_b^* \leq c_m^*$ as $r^* > 1$. Thus,

$$((w^*r^*)/(s^*p^*)) \geq \{1-x\}/(x)$$

Also, (B37*f*) and (B37*g*) give,

$$(w^*/s^*)\beta(1-\delta)V_m = (1 - x)/(x) \tag{B38}$$

Hence,

$$[(w^*r^*)/(s^*p^*)]\Delta = \{1-x\}/(x) \leq [(w^*r^*)/(s^*p^*)] \tag{B39}$$

which is a contradiction, as $\Delta > 1$. Thus, $\theta'' < 1$ and $\hat{c}^* > 0$ is not optimal. Tedious manipulation of the first order conditions shows that under Condition U, $\hat{c}^* > 0$. Thus, in any putative equilibrium $\theta'' = 1$ and $\hat{c}^* > 0$. It is straightforward to verify that the expressions reported in Theorem 3 are the unique solutions to the optimality conditions (B37) for (P^*). This is also a solution to Program (P) at $r = (\mathbf{r}/\mathbf{p})p$ and $s/\mathbf{r} \geq (w/\mathbf{p})[x/(1-x)]$.

(*iii*) $\Delta = 1$. In this case, the arguments used in part (*a*) of the proof show that $s^* = 0$ uniquely defines a stationary symmetric *PME*.‖

Theorems 4 and 5.

Theorem 4 is a direct consequence of Theorems 1 and 3, noting that: $lim_{\mu \to \infty} \Delta^{-(\gamma-1)} \to 0$, since $\Delta > 0$ and $\gamma - 1 > 0$. Theorem 5 follows from Theorem 1 and 3 and equations (18).‖

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Figure I

The Time Sequence