# DETERMINING THE OPTIMAL SAMPLE SIZE FOR CONTINGENT VALUATION SURVEYS

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# Determining the Optimal Sample Size for Contingent Valuation Surveys*

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## ABSTRACT

Fundamentally, this paper is about the value of information.

Whenever a cost-benefit analysis has to be undertaken using benefits that are estimated from household survey data the size of the survey sample must be specified. The most obvious case is the valuation of environmental amenity improvements through contingent valuation (CV) surveys of willingness to pay. One of the first questions that has to be answered in the survey design process is "How many subjects should be interviewed?" The answer can have significant implications for the cost of project preparation.

Traditionally, the sample size question has been answered in an ad hoc way either by dividing an exogenously fixed survey budget by the cost per interview or employing some variant of a standard statistical tolerance interval formula. Neither of these approaches can balance the gains to additional sampling effort against the extra interviewing costs.

A better answer *is not* to be found in the environmental economics literature, though it can be developed by adapting a Bayesian decision analysis approach from business statistics. The paper explains and illustrates, with a worked example, the rationale for and mechanics of a sequential Bayesian optimization technique, which is applicable when there is some monetary payoff to alternative courses of action that can be linked to the sample data. In this sense, unlike pure valuation studies that are unconnected to a policy decision, investigators who use contingent valuation results directly in cost-benefit analysis have a hidden advantage that can be exploited to optimize the sample size. The advantage lies in the link between willingness to pay and the decision variable, the net present value of the prospective investment.

The core objective of the paper is practical. Readers without a statistical background can easily implement the method. An Appendix shows how, with just six key pieces of information, anyone can solve the optimal sample size problem in a spreadsheet. An automated spreadsheet algorithm is available from the authors on request. To run the program all the user has to do is enter the key data and then activate a macro that automatically computes the optimum number of additional observations needed to augment any initial "small" survey sample.

# INTRODUCTION

The contingent valuation (CV) literature devotes a great deal of attention to a number of aspects of survey design, but has had relatively little to say about sample size. For instance, Cooper's (1993) algorithm for deciding on the number of bid groups in a referendum CV survey and the number of interviews in each group takes the total sample size as exogenously set by the *deus ex machina* of research funding limits. When attention has been paid to sample size, the issue has either been to choose the number of observations needed to bring the sample mean of willingness to pay (WTP) sufficiently close to the (unknown) population mean, or to confidently test hypotheses about the effects on mean WTP of two or more alternative "treatments" in a survey instrument (such as question ordering or choice of payment vehicle). In their seminal book on CV, Mitchell and Carson (1989) suggest that, based on a simple statistical tolerance formula, sample sizes between 200 and 2500 are probably appropriate.[1]

Outside the journals, however, CV has become an operational tool for the analysis of the benefits of proposed investment projects and policies in the environmental and natural resource arenas. In this context, there is no particular virtue in striving for some pre-ordained degree of closeness to the true population mean of WTP. Rather, the point of the exercise ought to be defined in terms of the ultimate decision, determined, it is assumed, by cost benefit (CB) analysis. For a reliable CB analysis, no single sample size can adequately fit all investment decision settings, because reliability must relate to the possibility of making the wrong decision, not to the possibility of being more or less wrong only on the side of benefits.

Intuitively, what is involved is that increasing the sample size reduces the spread of the distribution of discounted net project benefits, thereby reducing the degree of overlap of that distribution and the point at which discounted net benefits are zero. Thus, increasing the sample size can reduce the probability that a bad (in the efficiency sense) project will be built or a good project turned down – but only at the cost of the additional sampling. So, cet par. we would expect the optimal sample size to be large the closer the initial estimates of discounted benefits and costs or the larger their absolute values.

The formalization of this common sense idea has its origins in Schlaifer's (1959, 1961) Bayesian decision analysis approach, which is discussed in an accessible way by a number of standard texts on the use of statistics in business decision making (Bonini *et al.* 1997; Jedamus and Frame 1969; Pfaffenberger and Patterson 1987; Lapin 1994; Winkler 1972). The second and third sections of this paper deals with this formal background. We then turn to an illustrative application using data on costs and benefits of a water quality improvement project in Brazil. That project is very briefly described and the basic WTP data set out. Then the results are examined. This includes looking at the sensitivity of optimal sample size to the size of expected net present value and contrasting the results with prescriptions from the standard methods referred to above. The final section contains some observations about applying the technique in practice.

**PROJECT RISK, THE VALUE OF INFORMATION AND LOSS-COST MINIMIZATION**

The decision analysis approach to sample design in general (Schlaifer 1959, 1961) and to statistical quality control applications in particular (Vaughan and Russell 1984, Russell, Harrington and Vaughan 1986) provide the keys to unlocking the optimal CV sample size problem. Somewhat loosely stated, the core concept involves finding the sample size that minimizes the sum of sampling costs and expected losses from making a mistaken decision. It combines prior subjective characterizations of the probability distribution of mean willingness to pay with data from an initial "small" CV sample of say, 250 cases, to decide whether an additional round of sampling should be undertaken, and, if so, how many subjects should be interviewed in that second round. Schlaifer (1961) calls this Bayesian "pre-posterior" decision making about the desirable sample size because a decision can be reached on the basis of partial information before actually doing any additional sampling.[2]

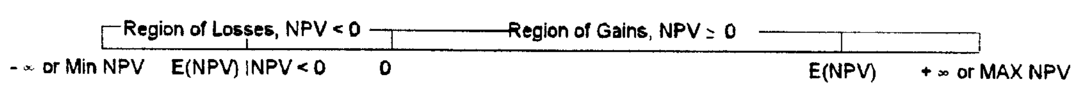
In the terminology of decision analysis, the CB decision is a two-action problem with infinite states of nature. The investment proposal can either be accepted, if in expectation it will yield a positive discounted net benefit, or rejected if it will not. Because many influences on NPV are random variables, so is NPV. Conceptually there are an infinite number of possible net benefit values, with an underlying probability density function; and in the probabilistic context of risk analysis, following this expected value decision rule has a quantifiable cost called the *cost of uncertainty*.

The cost of uncertainty is the expected opportunity loss of making the decision determined by the decision rule. That is, if the expectation E(NPV) taken over the entire NPV distribution is non-negative, the investment will be made. But if some portion of the NPV distribution falls below zero, actual losses are still possible. The cost of uncertainty can therefore be measured as the mean of that portion of the NPV distribution truncated from above at zero (the expected loss, given that a loss might indeed occur), multiplied by the probability of a negative NPV occurring.

If the project is not undertaken because the expected value of NPV is negative, the investment will not be made, thus foregoing any possibility of positive net returns. Symmetrically, the loss in this situation is the mean of that portion of the NPV distribution truncated from below at zero (the expected net gain foregone, given that a net gain might occur), multiplied by the probability of a positive NPV occurring.[3] The two opportunity loss situations are pictured below.

If the project is economically feasible its global mean NPV will be non-negative. The project will be undertaken so the region of possible opportunity loss is from negative infinity (or the minimum possible NPV) to zero:

**Case I: Project Feasible: Correct Decision is to Invest**




If the investment's expected NPV is negative it should not be undertaken, thus foregoing some possible gains lying in the region of opportunity loss from zero to plus infinity (or the maximum possible positive NPV):

**Case II: Project Infeasible: Correct Decisions is Not to Invest**

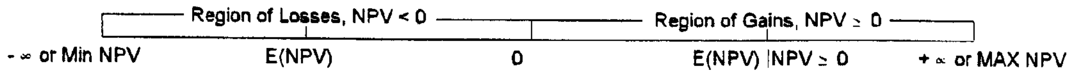



**Table 1** provides more formal definitions of the decision criterion, the probability of opportunity loss, the truncated mean loss and the cost of uncertainty. The cost of uncertainty ($E_{Loss,\,I}$ or $E_{Loss,\,II}$ in the table) is in part a function of the amount of prior subjective and sample information size on hand when the options are weighed to either invest immediately or wait and collect more information. At the point after a small initial sample of size $N_0$ is taken (or even before, when a prior guess is formed without any sampling at all) it represents the most the investor would be willing to pay to gather more information and eliminate all uncertainty about the project, which is why it is also called the expected value of perfect information, EVPI.

Additional sampling can never eliminate all uncertainty. But changes in $E_{Loss,I}$ (or $E_{Loss,II}$) with increases in sample size beyond the original small sample $N_0$ provide a measure of the gross benefit of the second stage of a sequential CV sampling scheme. Incremental CV samples with $\Delta N>0$ reduce the standard error of the CV mean estimate of project benefits (WTP), which transmits into a reduction in both the truncated mean loss in project NPV and the cumulative probability of that loss.

Only information about the factors that can have a significant impact on the project outcome reduce the cost of uncertainty in a meaningful way; and in most cases uncertainty about benefits will be a major influence (Vaughan *et al.* 1999, 2000a, 2000b). The *value of information* is the change in the cost of uncertainty occasioned by gathering additional information. The value of information must be compared with the cost of information. If the value exceeds the cost, it is worth doing additional sampling to gather more information; otherwise the project should be accepted or rejected on the basis of the information on hand.

## THE BAYESIAN METHOD

Given the costs of sampling, the core of the problem is to find optimal reduction in the cost of uncertainty, and hence the optimal size of the number of cases $\Delta N$ to augment an initial small sample of size $N_0$. In words, the steps required to find the optimal CV sample size via Bayesian decision analysis in a sequential approach are:[4]

(1) Postulate an a-priori guess about the expected value of WTP per household (or per person) and a reasonable opinion about the range in the expected value.[5]

(2) After the survey focus group sessions and the pre-test, draw a small initial referendum CV sample (e.g.: $N_0$ of around 250 observations, say 50 in each of 5 bid groups) and administer the final questionnaire. Calculate a sample mean WTP per household, the variance of the sample mean, and the standard error of the sample mean.[6] Approximate the population variance, $\sigma^2$, as the product of the initial sample size $N_0$ and the estimate of the variance of the sample mean.

(3) Do an initial economic project CB analysis to estimate the expected value of discounted net benefits, E(NPV), at baseline conditions. Determine whether the opportunity loss follows Case I (project acceptance) or Case II (project rejection) and locate the region of opportunity loss for NPV. Establish the parameters of a linear relationship between the expected value of NPV and the expected value of WTP by linear regression or by a shortcut method suggested later in this paper. The relation $E(NPV) = -\alpha + \beta\, E(WTP)$ is a key element in the method.

(4) Combine the prior guesses from Step 1 with the sample WTP information from Step 2, following a Bayesian formula to develop posterior estimates of the mean and standard error of WTP.[7]

(5) Using the posterior estimates from Step 4 as prior estimates, hypothetically increase the sample size from the base used in Step 2. Repeatedly compute the reduction in the variance of mean WTP that would result from sample augmentation over a range of sample sizes $\Delta N$ above the initial base $N = N_0$.

(6) Assume the expected value of NPV is normally distributed. Using the linear relationship

between NPV and WTP from Step 3, monetize the reduction in variance in the expected value of NPV losses associated with different degrees of augmentation of the original sample. These reductions in the *expected cost of uncertainty* ($E_{Loss,I}$ or $E_{Loss,II}$ as the case may be), from a second round of sampling represent the expected value of additional sample information, EVSI, or the benefits of sample augmentation.

(7) Over a range of hypothetical $\Delta$Ns above zero, numerically compare the *expected marginal value of information* contributed by an additional sample observation (i.e. successive changes in the *cost of uncertainty* obtained in Step 6) to the marginal cost of a sample interview. Find the sample size where the marginal value of information is approximately equal to the cost of an additional CV interview (for simplicity, assumed equal to the variable sampling cost and hence constant). The result is the *optimal (additional) sample size,* $\Delta N^*$. The total sampling effort $N_T$ will thus equal $N_0 + \Delta N^*$. The original small sample will be adequate if $\Delta N^*$ equals zero.

To formally develop the objective function, suppose priors have been formed regarding the mean and variance of WTP and an initial small sample has been taken (Steps 1 and 2 above). The "Conditional Value of Perfect Information" or CVPI function can be obtained from the linear relation in Step 3.[8] The horizontal axis intercept of the CVPI function is the break-even value $\mu_b$, given discounted costs equal to $-\alpha$. It can be found by setting E(NPV) to zero and solving the linear relation $-\alpha + \beta \bullet \mu_b = 0$ for $\mu_b = \alpha / \beta$. The slope, $\beta$, measures the decrease below zero in NPV for any WTP below the break-even value. So, for WTP $< \mu_b$, the CVPI function's dependent variable equals $\beta \bullet [\mu_b - E(WTP)]$ and for WTP $\geq \mu_b$ the CVPI function is zero.

We are now ready to calculate the required loss integrals. Schlaifer (1959, 1961) normalizes the extent of departure of the break even point from the sample mean WTP and computes a "unit loss integral" from the standard normal distribution (Table IV in Schlaifer 1961). Multiplying the value of the unit loss integral by $\beta$ representing the marginal contribution of the sample measurement (WTP) to the outcome (NPV) yields the expected loss of an (optimal) terminal action, ELTA. As previously mentioned in Table 1, the ELTA is also called the cost of uncertainty or the expected value of perfect information, EVPI, if the decision to invest is made immediately after taking the first small sample without gathering any additional information. That is:

$$ELTA = EVPI = \beta \bullet \sigma_1(\tilde{\mu}) \bullet L_N(|D|) \quad (1)$$

where:

$\beta$ Marginal contribution of WTP to NPV, in $ from Step 3..

$\sigma_1(\tilde{\mu})$ Standard error of the mean WTP posterior to taking a first small sample. The posterior can either be the standard error of the mean of that sample in the absence of a subjective prior (or under a very diffuse prior) or the posterior combination of a prior guess and the sample standard error. Specifically (Schlaifer, 1961, p.305; Paffenberger and Patterson, 1987, Ch. 23) the information contained in the prior distribution, $I_0$, is the reciprocal of the prior variance of the population mean, or $1/\sigma_0^2(\tilde{\mu})$, denoting the prior with a "0" subscript. The information in the first small sample, $I_{\bar{X}}$ is the reciprocal of the variance of the sample mean, or $1/\sigma^2(\bar{x})$. The posterior variance of the mean, $1/I_1$ is the sum $1/(I_0 + I_{\bar{X}})$ and the posterior standard error is the square root of that sum, or:

$$\sigma_1(\tilde{\mu}) = \sqrt{1/I_1} = [1/(I_0 + I_{\bar{X}})]^{1/2} = [1/(1/\sigma_0^2(\tilde{\mu}) + 1/\sigma^2(\bar{x}))]^{1/2}$$

$$= [(\sigma_0^2(\tilde{\mu}) \bullet \sigma^2(\bar{x})) / (\sigma_0^2(\tilde{\mu}) + \sigma^2(\bar{x}))]^{1/2}$$

$|D|$ The absolute value of the standardized difference between the break-even value of WTP, $\mu_b$, and the mean posterior to taking a first small sample, $E_1(\tilde{\mu})$. In Schlaifer's notation $|D| = |\mu_b - E_1(\tilde{\mu})| / \sigma_1(\tilde{\mu})$. For an expectationally profitable investment, D is negative, and represents, in effect, the "cushion" implied for the project decision by the mean of WTP from the prior and the original small sample estimate.

$E_1(\tilde{\mu})$ The mean posterior to taking a first small sample. It can either be the mean, $\bar{x}$, of that sample in the absence of a subjective prior or the posterior combination of a prior guess about the mean, $E_0(\tilde{\mu})$ and the sample mean. Using $I_0$ and $I_{\bar{X}}$ from directly above as weights:

$$E_1(\tilde{\mu}) = [I_0 \bullet E_0(\tilde{\mu}) + I_{\bar{X}} \bullet \bar{x}\,] \div [I_0 + I_{\bar{X}}\,].$$

$L_N$ Unit loss integral, or the expected value of the difference between the normalized random variable of interest, x, and D. For an expectationally profitable project, $L_N = \int (-D - x) f_N(x)\, dx$ (Bonini *et al.* 1997) where $x$ stands for all the possible negative values of WTP. Beyond $-D$ the probability of an opportunity loss is zero, so $L_N$ is not the mean loss of the truncated distribution; rather it measures the untruncated mean loss of the entire distribution (Jedamus and Frame 1969, p.97).

By taking a second sample and not acting immediately on the basis of the first small sample, it may be possible to reduce expected losses. The expected value of the new sample information, EVSI, is a function of the monetary value of the reduction in variance due to the second sample, or the reduction in the ELTA. To find the optimal size of a second sample, $\Delta N^*$, the function to be maximized includes the benefit of variance reduction and the sampling costs. The benefits are measured as the expected value of information obtained from a second sample of size $\Delta N > 0$, assuming the population variance of WTP is known or set equal to the variance obtained from the first sample. Analogous to EVPI, the expected value of the sample information, EVSI, is the value of the reduction in losses due to the reduction in variance brought about by taking more observations, $\Delta N$:

$$EVSI = \beta \bullet \sigma(\tilde{E}_1) \bullet L_N(|D_E|) \quad (2)$$

where:

$\sigma(\tilde{E}_1)$ Is the preposterior reduction in the standard error of the mean attributable to taking a second sample of size $\Delta N$. It is calculated as the square root of an information-weighted average of the posterior variance of the mean from above, $\sigma_1^2(\tilde{\mu})$, and the variance of the mean the new sample is presumed to produce, $\sigma^2/\Delta N$. To get $\sigma^2/\Delta N$, assume the population standard deviation (of individual observations, not the mean) $\sigma$, is approximately equal to the standard deviation from the first sample. The value of $\sigma^2$ can then be obtained as the product of the size of the first sample, $N_0 = 250$, and the variance of the mean WTP (see Table 3), or $\sigma^2 \approx N_0\, \sigma^2(\bar{x})$. Then, the Bayesian preposterior reduction in the standard error of the mean (Schlaifer 1961, p. 324; Paffenberger and Patterson 1987, p. 1108; Winkler 1972, p.364; Lapin 1994, p. 464) is just the square root of: [9]

$$\sigma^2(\tilde{E}_1) = \sigma_1^2(\tilde{\mu}) \left[ \frac{\sigma_1^2(\tilde{\mu})}{\sigma_1^2(\tilde{\mu}) + \sigma^2/\Delta N} \right]$$

$|D_E|$ Is the absolute value of the standardized difference between the break-even value of WTP, $\mu_b$, and the mean posterior to taking a first small sample, $E_1(\tilde{\mu})$, now using $\sigma(\tilde{E}_1)$ as the preposterior estimate of the decrease in the standard error of the mean WTP due to the new sample size $\Delta N$. In Schlaifer's notation $|D_E| = |\mu_b - E_1(\tilde{\mu})| / \sigma(\tilde{E}_1)$.

The costs of sampling are assumed to be a linear function of $\Delta N$, with fixed costs $K_s$ and unit variable costs $k_s$. Then, the full loss-cost function to be minimized with respect to $\Delta N$ is:

$$£ = \underset{\Delta N}{\text{Min}} : (\text{EVPI} - \text{EVSI}) + (K_s + k_s \Delta N) \qquad (3)$$

Once $N_0$ is chosen, EVPI and $K_s$ are constants.[10] Therefore minimization of £ is equivalent to maximizing a concentrated net benefit function £′ where EVSI represents the benefits of taking an additional sample of size $\Delta N$ and incurring total variable costs of $k_s \Delta N$. The expected net gain from (additional) sampling, ENGS, becomes:

$$£' = \underset{\Delta N}{\text{Max}} : \text{ENGS} = \text{EVSI} - k_s \Delta N = \beta \bullet \sigma(\tilde{E}_1) \bullet L_N(|D_E|) - k_s \Delta N \qquad (4)$$

EVSI is a function of $\Delta N$ because $\sigma(\tilde{E}_1)$ and $L_N(|D_E|)$ are nonlinear functions of $\Delta N$. The sample size that maximizes £′ with respect to $\Delta N$ has to be found numerically. In some cases finding the optimum will depend on making a good choice of the starting value for $\Delta N$ in the numerical search. Approximations to aid the search are discussed in Appendix 1. The next two sections demonstrate the application of the method using the concrete example of an investment project in Brazil.

## BENEFITS OF A WATER QUALITY IMPROVEMENT PROJECT IN BRAZIL

The parts of the Tietê River and its tributaries that flow through the São Paulo Metropolitan Area (SPMA) in Brazil are the most polluted bodies of water in the State. The Tietê enters the metropolitan area with acceptable water characteristics, but soon becomes anaerobic. From this point downstream, almost to the limit of the area, the large volume of domestic (80% of total organic load) and industrial waste dumped into the relatively small volume of river flow has made the river an open sewer canal for more than 80 kilometers. The water is too contaminated even for industrial use.

A proposed investment project contemplates the construction of additional sewage treatment plants and addition of capacity at existing plants to reduce the amount of oxygen-demanding organic material reaching the river.[11] The water quality results expected from the project, as measured by dissolved oxygen (DO), biochemical oxygen demand (BOD), and fecal coliform bacteria count, are modest, though at least anaerobic conditions will be eliminated under all but unusually low flows. The achievement of these improvements will be expensive. Estimated total discounted capital and operating costs over a thirty year project life are about 594.6 million Brazilian Reals (R$ of 1998).[12]

The major benefits of the project arise through the reduction of odors from the river as it becomes aerobic and the introduction of the possibility of enjoying some non-contact recreation (boating) on the cleaner river. The questionnaire to probe for WTP for these changes was designed, tested in focus groups, and refined under the direction of Robert Mitchell, co-author of a seminal book on CV (Mitchell and Carson, 1989). The key WTP question was of the referendum type and asked whether the responding household would be WTP one of the amounts 0.5, 2.0, 5.0, 12.0, or 20.0 (1998 Brazilian Reals) per month to have the projected water quality improvements. These payments would continue over 10 years.

For this illustration, a balanced random sub-sample of 250 observations was drawn from the actual 600 observation sample of Tietê CV survey interviews. Unlike the full sample, the small sub-sample was deliberately drawn to be representative of the spatial distribution of the respondents.[13] A size of 250 was chosen in order to have a reasonable minimum number of observations in each of the five bid groups in the referendum.

With referendum data, there are several competing alternative measures of the mean and its variance (Vaughan *et al.* 1999). A convenient way to bound the possibilities for our sample size problem is to choose a nonparametric mean that lies somewhere in between the upper and lower limits proposed by Boman *et al.*

(1999). **Table 2** presents our preferred measure for the small sample, an unequally weighted average of the Boman *et al.* upper(25% weight) and lower (75% weight) bounds, along with other candidate nonparametric means and their variances.[14]

The product of average benefits per household per month, twelve months per year and the number of beneficiary households (4.06 million in the initial year) produces a total annual gross benefit. Because of the way the CV question was asked, the benefits can only be counted over 10 years. Taking this timing into account and allowing for population growth, the present value of total benefits is functionally equivalent to the present value of total population times twelve months per year multiplied by monthly benefits per household.[15] Using the unequally weighted lower bound (75% weight) and upper bound (25% weight) mean of R$ 7.47 yields a present value of total CV-based benefits of R$ 101 million in shadow-priced terms.

## FINDING THE OPTIMAL CV SAMPLE SIZE FOR THE EXAMPLE PROJECT

Using actual data and the full Bayesian method involves forming a prior guesstimate about the mean WTP and the population variance, drawing an initial "small" sample, combining the sample estimates of mean and variance with the prior estimates to arrive at posterior estimates, and using those estimates to monetize the potential reduction in expected opportunity loss that might be gained by gathering more data and hence decide whether a larger sample would be optimal. The case study demonstration follows the structure of the spreadsheet algorithm for finding the optimal sample size provided in Appendix 2, which documents all of the calculation steps.[16]

### *Priors for the Parameters of the WTP Distribution*

First, assume that the population mean of WTP, $\mu$, is a random variable with a normal density function and choose a prior mean[17] Specifically, let the prior mean $E_0(\tilde{\mu}) = \hat{\mu}$ be 1% of the population's average income (Ardila *et al.* 1998, Choe *et al.* 1996), or R$ 8.28. To guesstimate the variability in mean WTP in advance of taking any measurements at all, Schlaifer's technique (1961, p. 301) asks the decision maker to speculate about what interval around the prior mean would give the guess an even (50 − 50) chance of being correct. Somewhat arbitrarily choosing an error of R$4.00 on either side of the prior says the true mean is as likely as not to fall between R$4.28 and R$12.28. From the standard normal distribution, the standardized value of $[\mu - E_0(\tilde{\mu})] / \sigma_0(\tilde{\mu})$ that demarcates 25% of the distribution's area is 0.67 so, solving $0.67 = 4.00/\sigma_0(\tilde{\mu})$, the prior for the population standard deviation of $\mu$ is 4.00/0.67 = R$5.97. This represents a weak or diffuse prior because the guess about the mean WTP has a relatively broad band of uncertainty and therefore $E_0(\tilde{\mu})$ has very little information content.

*Initial Sample Estimates of the Parameters of the WTP Distribution*

Table 2 gives the means and standard deviations for monthly household WTP estimated by four different non-parametric methods, using the special 250-household balanced subsample of the original Tiete project data. In this example we use the 75/25 Turnbull/ Paasche numbers (mean R$7.47 and population standard deviation R$13.23).[18]

*The Payoff Function*

The first key to implementing Schlaifer's approximation is the linear payoff function. It describes the relationship between the quantity measured by the sample and the payoff decision variable that depends on the sample information, which in this case are mean monthly household WTP and the expected value of NPV, respectively. This function is a compact summary of the CB analysis. Net present value is written as the linear relation $E(NPV) = -\alpha + \beta \cdot E(WTP)$. If the expected value of WTP from a CV survey is the only source of benefit, the intercept, $-\alpha$, represents the sum of discounted capital and operating costs of the investment. If there are any other sources of benefit that have not been estimated by CV they can be netted out of the discounted costs to get the intercept. The slope, $\beta$, is the marginal contribution to discounted net benefits of an increase in average WTP per household. For the case study $E(NPV \text{ in R\$}) = -594{,}653{,}964 + 100{,}988{,}487 \cdot E(WTP)$.[19]

Given this linear relationship between discounted profits and household WTP, if the sample mean WTP is normally distributed, the outcome variable, E(NPV) will also be normally distributed with mean E(NPV)

$= -\alpha + \beta \bullet E(WTP)$ and variance $VAR(NPV) = \beta^2 \bullet VAR[E(WTP)]$.[20] The break-even value that sets E(NPV) to zero is $\mu_b = \alpha/\beta$, = 594,653,964/100,988,487, or R$5.89. For any expectation of WTP less than $\mu_b$, opportunity losses in NPV will be incurred.

From Table 2 above it is clear that all of the nonparametric sample means other than the Turnbull lower bound mean WTP are above the break-even value, so for any of these estimates the correct decision is to invest. But, the sample mean is a random variable so there is some non-zero probability that it could be below the break even value. For example, the preferred measure, a 75-25 weighted combination of the lower and upper bound means from Table 2, is R$7.47, and its standard error is 0.84, putting the sample mean 1.88 standard errors above the break even value. The Kriström and Paasche means are even more distant from the break even value (R$3.31 and R$6.80 respectively in absolute terms and 3.28 and 4.96 standard errors respectively). Under these means the cumulative probability of a loss is clearly lower than it would be using the 75-25 weighted average to measure WTP and predict NPV.[21]

*The Distribution of Net Present Value*

This leads to the second key to Schlaifer's approach. It is that, given a successful project on average, each possible NPV loss has a probability associated with it. Centering the net benefits distribution on the most likely value of WTP, the observed sample mean, the loss probabilities are defined by the tail portion of a normal density function lying below $\mu_b$. The sum of the products of all the possible expected losses and their associated probabilities reveals the cost of uncertainty. **Figure 1** illustrates the superimposition of the linear relation representing the opportunity loss function (also called the conditional value of perfect information or CVPI function) on the normal E(WTP) distribution whose standard error is assumed known from the first small sample.

The distribution of gains and losses in Figure 1 is centered on the sample mean WTP of R$7.47, with an initial spread given by the mean standard error of the small sample, 0.84. Increasing the sample size decreases the amount of spread in the (assumed) normal density, thus decreasing the expected value of a loss, as demonstrated by the probability density function generated by a larger sample and a smaller standard error (the lightly shaded line in Figure 1). Unlike the the small sample's density, it has an infinitesimal amount of its area to the left of the break – even point of R$5.89.

Losses in NPV are shown as positive in the figure. The horizontal axis intercept of the CVPI function is the break-even value $\mu_b$, given discounted costs equal to $-\alpha$. It can be found by setting E(NPV) to zero and solving the linear function $-\alpha + \beta \bullet \mu_b = 0$ for $\mu_b = \alpha / \beta$. The slope of the CVPI function, $\beta$, measures the decrease below zero in NPV for any WTP below the break-even value. So, for WTP $< \mu_b$, the CVPI function's dependent variable equals $\beta \bullet [\mu_b - E(WTP)]$ and for WTP $\geq \mu_b$ the CVPI function is zero.

All else equal, higher discounted costs (a larger negative $\alpha$) shift the CVPI function to the right, raise the requisite break-even value, and, given the sample mean WTP, put more mass of the tail of the normal probability density under the non-zero part of the CVPI loss line. The expected opportunity loss or cost of uncertainty is the sum of the products of the normal density function to the left of the break-even value and the conditional value of perfect information to the left of the break-even value. Therefore, higher costs raise the cost of uncertainty, given the sample mean estimate of willingness to pay.

*Linear Sampling Cost Function*

The sampling cost function is linear, with a marginal (equals variable) cost per observation taken here to be R$89 in shadow-priced terms (Powers, 1981). Zero fixed costs for the second round of sampling are assumed.

## RESULTS

Under baseline initial conditions, including project costs and the diffuse prior, it is optimal to augment the initial sample size beyond 250 cases. The logit probability formula from Appendix 1 indicates that additional sampling should be done, and brute force exploration reveals that additional sampling can produce positive values for EVSI net of variable sampling cost. Numerical optimization using Excel's Solver routine from Appendix 2 returns a solution of 2243 cases for $\Delta N^*$. Adding in the initial sample of 250, the optimal sample size needed *en toto* is 2,493 cases.

*Comparing Optimal Sample Size with Some Standard Prescriptions*

The standard formula for the sample size required to have a (1-α)% chance of obtaining sample mean that is within E of the population mean, given the sample standard deviation, σ, is: $N = [(Z_{\alpha/2} \bullet \sigma)/E]^2$, where $Z_{\alpha/2}$ is the (1-α)% confidence interval statistic; and E is the acceptable size of the error in the sample mean (Pffaffenberger and Patterson 1987, p. 389). Mitchell and Carson (1989) replace σ/E by $\sigma/(\mu \bullet p) = V/p$ where V (=δ/μ) is the coefficient of variation, which they guess to around 2 and p is the desired maximum fractional error in the mean.

The original project work from which much of our data is drawn began with a required sample size estimate based on a 95 percent confidence interval and an allowable 10 percent sampling error for the sample average of household income – not the unknown household WTP. Using census data to estimate the desired mean and its standard deviation produced a required sample of 276 households.[22]

Another approach might be to construct a triangular distribution for WTP based loosely on information from Ardila *et al.* (1998) about estimated household WTP for sewer connections and water quality improvements. Suppose this distribution is anchored on the left at zero (the project is assumed to do no harm) and on the right at 3 percent of maximum income in the affected area, with the modal value of WTP taken to be one percent of mean income. Then, from the triangular distribution formulas, again requiring 95 percent confidence and allowing 10 percent error, the required same size would be 175.[23]

Quite a different estimate would be produced by using the Mitchell-Carson formula mentioned earlier, with the (guessed) coefficient of variation set to 2. For the same confidence interval and acceptable error percentage, the estimated required N is 1537. Even if we had the information from the initial sample to plug into the formula, it does not come close to the optimum. That is, using the sample mean of $R 7.47 and a population standard deviation of 13.25 from Table 2, and employing the same confidence level and allowable error, a total sample size of 1205 results.[24]

Thus, in this example, several variants of the standard method produce sample sizes from 175 to 1537.

These sample sizes are all inadequate. The optimization method based on the value of additional information recommends a total sample size of 2493, which is almost 1000 cases greater than the highest result from the standard tolerance method.

*What Happens As Project Economics Change?*

As the mean of WTP moves to the right, the overlap of the loss function and the distribution of NPV shrinks, and the value of additional information falls. In the example, if we had started with the Kriström intermediate estimate of per household WTP, the algorithm would have told us that no additional sampling was necessary.[25]

For contrast with improving on the baseline case via increases in mean WTP, consider, consider raising discounted costs by 25% (i.e. shifting the intercept of the linear net profit function from -$R 595 million to -$R 753 million). Analogous to a decreases in mean WTP, this brings the expected value of NPV very close to zero and puts half of the distribution of expected net returns into the negative region. Here, the optimal size of $\Delta$N will be at a maximum. **Table 3** shows the elements of a trial and error search for an optimum, assuming a diffuse prior for WTP, and **Figure 2** shows the optimum graphically.

The table and figure show that the response surface is very flat. The approximate optimum $\Delta$N is 6750 cases, but the gains to additional sampling diminish quickly after about 2000 cases. By inspection of the figure, the net gains from an additional sample of 4000 cases ($R 30,303,436) or even less are fairly close to the net gains at the optimum ($R 30,460,065). This is consistent with Schlaifer's (1961, Figure 21.5) numerical investigations which showed that moderate departures from the optimum number of cases (± 20% or even ± 30%) are likely to be inconsequential.

Similar exercises to shift the CVPI function less drastically to the left were conducted to see what general effect the location of the E(NPV) distribution relative to E(NPV) = 0 has on the optimal $\Delta$N. The mean of total discounted costs ($-\alpha$) was raised above the baseline by 5%, 10%, 15% and 20%, holding mean WTP constant at R$7.49.[26] **Table 4** shows the effect that the standardized distance of E(NPV) away from zero has on the optimal incremental sample size $\Delta N^*$ under diffuse priors, tight priors and total ignorance. One fundamental lesson that can be drawn from the table is that the optimal sample size is sensitive to the economics of the situation; the standard method, at best, comes within 60% of the optimum under the baseline conditions (i.e 1537 cases versus the optimum of 2493), but never does any better than this across a gamut of circumstances.

The results in Table 4 and other calculations not reported here suggest that, in this case, small samples suffice when sampling costs are high and the mean of the NPV distribution is over about 2.4 standard errors away from the break-even point of E(NPV)=0 because the decision has little downside risk. Then, there is no payoff in taking larger samples to shrink that risk by reducing the variance and further compressing the portion of the NPV distribution lying below zero. However substantial gains to increasing the sample size begin to emerge after the expected value of NPV falls below about 2.4 standard errors from zero. Although the algorithm does not explicitly incorporate Type II error, the fact that the required sample size increases as the gap between E(NPV) and the break-even point narrows provides protection against false acceptance of a mean WTP that justifies the project when in fact the true mean WTP would lead to the opposite conclusion.

Table 4 also shows that good prior estimates of the mean WTP and its spread can significantly reduce the amount of sampling effort needed to reach an optimum CV survey sample size for investment decisions. Unfortunately, given the state of the art, good prior estimates are probably unattainable, especially in developing countries.

## CONCLUDING OBSERVATIONS

Small CV survey samples are probably adequate for CB analysis when the fixed and variable costs of sampling are relatively high and the investment is extremely robust. If the investment has a probability of failure of less than one percent, it is not necessary to take large samples. In this sense the common perception that small CV samples will suffice is correct. If investment proposals are carefully screened and only the very best of them become candidates for final project analysis, massive CV sampling efforts to measure WTP more precisely are not worthwhile. However, investments with infinitesimal risks are rare.

At the other more common extreme, when the investment is borderline because nearly half of the NPV distribution falls in the negative quadrant, even though its mean NPV is barely positive, small (e.g. 250 observations) samples will always be inadequate. In this situation, which can be easily identified *a priori*, a search for the optimum number of *additional* cases needed to augment the small sample is recommended.

Like our worked example, many prospective investments fall somewhere in the middle ground between can't miss and borderline proposals. The existence of this grey area makes it risky to rely exclusively on any particular rule of thumb, be it for small, medium or large samples. But, in general, given an initial expectation for WTP and a service flow outcome so the time pattern and magnitude of gross benefits is held constant, the more costly the project the larger the sample that will be needed to justify it. This paper has shown how to make that general rule operational.

In many situations it will be a stretch to specify a prior guess as to mean WTP and its variance. Thus, while many CV studies have been done in developing country settings, to date they defy easy summarization (Ardila et al. 1998), and forming reasonable prior beliefs on the basis of fragmented and inconsistent past experience is difficult indeed. In fact, unless priors are reasonably accurate they will not contribute much information on WTP location and spread beyond what an initial survey sample contains, so the influence of relatively diffuse priors on the optimal decision will be trivial. Said another way, little will be gained from formulating wildly inaccurate or imprecise prior estimates; all the information content will be in the first small sample, $N_0$. The simplified sequential approach suggested in Appendix 2 mirrors those realities. So even without forming priors, it is possible to determine an optimal sample size.[27]

The Bayesian approach to optimal sample-size determination is relevant to benefits transfer and meta analysis efforts. The literature has been skeptical about the value of these uses of accumulated past experience to estimate the benefits of new projects (Brouwer and Spaninks 1999). The potential value of this kind of information has largely been ignored because researchers have focused mainly on the degree of correspondence between predictions of WTP generated from past studies and the actual mean WTP results from field work, working under the *as if* presumption that prior information would be used to replace new sampling.

This focus might be misplaced. Prior information need not be regarded purely as a substitute for new *in situ* CV survey sampling. The two are complementary because combining good prior predictions of WTP with actual survey samples can save a good deal of new sampling effort and money. The synthesis of past CV results to make accumulated contingent valuation WTP information transportable to new situations might pay off, if project analysis were commonly to involve the systematic use of an optimal Bayesian sample size protocol that currently must work under the handicap of total ignorance or diffuse priors.

To date, international lending institutions have not systematically followed reliable protocols for selecting CV survey sample sizes in their appraisal of prospective investments, and they are not alone. In this operating environment, new information has little value beyond its immediate contribution to the specific decision at hand, which is to economically justify a given investment project. The WTP data is used once and then forgotten. But this information could become more valuable if sample sizes were chosen in the future that take account of the expected opportunity loss the actual investor might incur. Then, there would be a good reason to take a longer range view about the value of information.

## APPENDIX 1

## APPROXIMATIONS TO INDICATE WHETHER MORE SAMPLING IS NEEDED AND THE SIZE OF $\Delta N$

Schlaifer (1961) relates the need for more sampling to the values of his *essential parameters of the problem of sample size*, labeled $Z_0$ and the previously defined $D_0$, and provides a nomogram (Figure 21.4, p. 332) that indicates whether it is worth taking a second sample, depending on the values of these parameters. The essential parameter $Z_0$ is a function of the marginal contribution to NPV of a change in WTP (i.e. $\beta$), the marginal costs of sampling (i.e. $k_s$) the population standard deviation of WTP (i.e. $\sigma$, approximated by the standard deviation, s, from the first sample) and the standard error of the mean WTP posterior to taking a first small sample (i.e. $\sigma_1(\tilde{\mu})$ ):

$$Z_0 = [\sigma_1(\tilde{\mu}) / \sigma] \bullet [\beta\sigma / k_s]^{1/3}$$

Since many readers may not have easy access to Schlaifer's book and the decision nomogram, we fit a logit probability model with a second-order index function to 197 pairs of $D_0$ and $Z_0$ points read from his Figure 21.4, coding the dependent variable as 1 if the nomogram recommended "Sample before Acting", and as 0 if it recommended "Act without Sampling". The model fit was reasonably good (pseudo $R^2$ of 0.80) with 182 correct predictions and 15 false predictions. As a substitute for the Schlaifer's figure, a decision to take an additional sample should be made if the predicted probability from the model is equal to or greater than 0.5:

$$\text{Prob.}_{\text{Sample}} = 1/ [1 + exp\,[2.2061 - 1.1255\, Z_0 - 4.6102\, D_0 + 0.\ 0066\, (Z_0)^2 + 4.8539\, (D_0)^2] \geq 0.5 \ ?$$

If the answer from evaluating the probability model (or Schlaifer's Figure 21.4) is "Sample before Acting" it will be necessary to search for the optimum, $\Delta N^*$. A good starting value for the grid search over $\Delta N$ can be found from a rough approximation to the optimum, $\Delta N_{Approx}$. using another simplification from (Schlaifer 1961, pp. 334-335):

$$\Delta N^* \approx \Delta N_{Approx} = [(\beta\sigma/k_s)^{1/3}]^{1/2} \bullet [\tfrac{1}{2}\, Z_0 / P_N\,(D_0)]$$

where $P_N\,(D_0)$ is the probability density of the standard Normal density function evaluated at $D_0$. The optimum size $\Delta N^*$ of the addition to the original small sample ($N_0 = 250$) either be found through trial and error by constructing crude fixed-step size grid search in the neighborhood of the initial guess, $\Delta N_{Approx}$. or by calling an optimization routine like Excel's Solver after specifying $\Delta N_{Approx}$. as the starting value.

## APPENDIX 2

## A SPREADSHEET PROGRAM FOR SAMPLE SIZE DETERMINATION

Anyone can implement the method developed in this paper using a spreadsheet algorithm in Quattro Pro that is available from the authors on request.[28] The optimization routine presumes that an initial small survey sample has already been taken, and asks whether it would be optimal to add to it in a second round of sampling, assuming no prior information is available. To run the program all the user has to do is click on an "Optimizer Macro" button to compute the optimum number of additional observations needed to augment an initial "small" survey, if any. The data entry and output results forms under the special case of totally diffuse priors are shown in **Table 2-1** below. The full set of spreadsheet instructions under the more general Bayesian structure appear in **Table 2-2** .

Under the assumption of no prior knowledge of Table 2-1, only six pieces of input information are needed: (1) the size of the initial small CV survey sample, (2) the expected value (mean) of willingness to pay (WTP) extracted from that sample, (3) the variance of mean WTP, (4) the average (equals marginal) cost of collecting a single survey observation, (5) the intercept of the linear CVPI function relating NPV to WTP and (6) the slope of the linear function, as explained in the body of the paper.

## TABLE 2-1. QUATTRO PRO MACRO: DATA INPUT FORM AND OPTIMAL RESULTS SUMMARY OUTPUT

***INSTRUCTIONS:***

***ENTER DATA IN BOX AT LEFT***

**DATA ENTRY**

| **STEP I. ENTER THE INITIAL SMALL SAMPLE DATA** | **Units** | **Data Entry** |
|---|---|---|
| Size of Initial "Small" Sample? | # of Cases | 250 |
| Sample Mean Willingness to Pay? | $/Household/Unit Time | $7.47 |
| Variance of Sample Mean? | $/Household/Unit Time | 0.7 |
| Sampling Cost per Household Interview? | $/Case | $89.00 |

**STEP II. SPECIFY THE LINEAR CVPI FUNCTION RELATING NPV TO WTP (NPV = α + β •MEAN WTP)**

| | | |
|---|---|---|
| Intercept (α) ? | $ Total Discounted Costs [Enter as Negative #] | -$594,653,984.00 |
| Slope (β) ? | # of Beneficiaries [Total Discounted] | 100,988,487 |

AND THEN

**RESULTS**

| | |
|---|---|
| **STANDARD ERRORS OF NPV AWAY FROM NPV = 0** | **1.89** |
| **SHOULD A SECOND SAMPLE BE TAKEN TO AUGMENT THE INITIAL SAMPLE?** | ***Probably Yes*** |
| **IF "Yes" CLICK ON THE BUTTON AT THE RIGHT TO RUN THE OPTIMIZER MACRO** | OPTIMIZER MACRO |
| Approximate Sample Size (Used as a Starting Value for Optimization) | 2,793 |
| **EXACT OPTIMUM** | **2,378** |

***HIT THE OPTIMIZER BUTTON***

***PROGRAM RETURNS THE OPTIMUM***

Note: This routine assumes the analyst has no prior knowledge about average WTP or its variance beyond what the initial "small" sample reveals. Neither the authors nor the Inter-American Development Bank warranty this program or the methods it employs.

**TABLE 2-2.**
**SPREADSHEET FORMULA LAYOUT FOR THE OPTIMUM SAMPLE SIZE CALCULATION**
**(Tietê Project Case at Baseline Costs Under Diffuse Prior Information, 1998 Brazilian Reals)**

| Row # | COLUMN A: Labels | COLUMN B: Labels | COLUMN C: Formulas | RESULT | COMMENT |
|---|---|---|---|---|---|
| 1-3 | **I. Form Priors** | | | | |
| 4 | *Prior Mean and Standard Deviation of WTP* | | | | |
| 5 | Prior Mean | $E_0(\tilde{\mu}) = \hat{\mu}$ | 0.01*828 | $8.28 | Guess the mean WTP. |
| 6 | Prior error @50% | e | 4 | $4.00 | Guess the variation in the mean covering a ±50% interval. |
| 7 | Prior error UL@50% | $\hat{\mu} + e$ | C5+C6 | $12.28 | Find the upper limit of the interval. |
| 8 | Prior error LL @50% | $\hat{\mu} - e$ | C5-C6 | $4.28 | Find the lower limit of the interval |
| 9 | Prior Upper Alt. U @+25% | $[\mu - E_0(\tilde{\mu})] / \sigma_0(\tilde{\mu})$ | NORMINV(0.75,0,1) | 0.6745 | Find the Standard Normal z statistic value for each tail outside the interval (i.e. each contains 25%) |
| 10 | Prior Standard Deviation of Population Mean | $\sigma_0(\tilde{\mu})$ | (C7-C5)/C9 | $5.9304 | Find the standard deviation of population mean WTP implied by the prior, based on the error limits, e. |
| 11 | Prior Variance of Population Mean | $\sigma_0^2(\tilde{\mu})$ | C10^2 | $35.1697 | Find the variance of mean WTP implied by the prior. |
| 12-13 | **II. Get Posterior Distribution from Normal Prior and Sampling Distributions, Sampling Variance Known** | | | | |
| 14-15 | *Sample Data* | | | | |
| 16 | Initial (or First) Sample Size | $N_0$ | 250 | 250 | Input value. Number of cases in initial small sample. |
| 17 | Expected Value of Sample Mean | $E(\bar{x}) = \mu$ | 7.47 | $7.47 | Input value. Calculate (Nonparametric) Mean WTP |
| 18 | Sample Variance | $s^2$ | C19^2 | $174.40 | Calculate variance of WTP from sample estimate of standard deviation of WTP immediately below. |
| 19 | Sample Standard Deviation | s | C16^0.5*C21 | $13.21 | Calculate sample estimate of standard deviation of WTP using standard error and square root of sample size, 250. |
| 20 | Variance of Sample Mean | $\sigma^2(\bar{x}) = s^2 / N_0$ | 0.449 | $0.70 | Input value. Calculate variance of sample Mean WTP. |
| 21 | Standard Error of Sample Mean | $\sigma(\bar{x}) = s / N_0^{1/2}$ | 0.670 | $0.84 | Input value. Calculate standard error of sample Mean WTP as square root of variance of mean WTP. |
| 22 | *Posterior Calculation* | | | | |
| 23 | Posterior Mean | $E_1(\tilde{\mu})$ | (C5*1/C11+C17*1/C20) / (1/C11+1/C20) | $7.49 | Compute posterior mean as a weighted average of prior and sample means based on quantity of information provided by each. See Rows 27 through 29 below. |
| 24 | Posterior Standard Error of Mean | $\sigma_1(\tilde{\mu})$ | (1/(1/C11+1/C20))^.5 | $0.83 | Compute posterior standard error of mean as a weighted average of prior and sample standard errors based on quantity of information provided by each. See Rows 27 through 29 below. |
| 25 | Posterior Variance of Mean | $\sigma_1^2(\tilde{\mu})$ | C24^2 | $0.68 | Compute as square of posterior standard error. |

| Row # | COLUMN A: Labels | COLUMN B: Labels | COLUMN C: Formulas | RESULT | COMMENT |
|---|---|---|---|---|---|
| 26 | *Quantity of Information* | | | | |
| 27 | In Sample Mean | $I_{\bar{X}}$ | 1/C20 | 1.43 | Relative information content. |
| 28 | In Prior Mean | $I_0$ | 1/C11 | 0.03 | Relative information content. |
| 29 | In Posterior Mean | $I_1$ | 1/C24^2 | 1.46 | Pooled information content |
| 30 | Check | $I_1 = I_0 + I_{\bar{X}}$ | C27 + C28 | 1.46 | |
| 31-32 | **III. Expected Profit After First Small Sample (i.e. Based on Posterior from II Above)** | | | | |
| 33 | Linear Profit Function Intercept | $\alpha$ | -594653984 | -$594,653,984 | Input data for intercept of linear relation between NPV and WTP, i.e. NPV=$\alpha$+$\beta$*WTP. Calculate outside from data generated by deterministic cost-benefit analysis model. |
| 34 | Linear Profit Function Slope | $\beta$ | 100,988,487 | $100,988,487 | Input data for slope of linear relation between NPV and WTP. Calculate outside from data generated by deterministic cost-benefit analysis model. |
| 35 | Expected Profit (NPV) | $\alpha + \beta \cdot E_1(\tilde{\mu})$ | C33+C34*C23 | $161,738,697 | Expected NPV at posterior baseline mean WTP of $5.83. |
| 36 | Break Even Value of WTP | $\alpha \div -\beta$ | C33/-C34 | $5.89 | Value of WTP that sets expected NPV to zero, given posterior mean WTP. |
| 37 | Standardized Loss | $\lvert D\rvert = \lvert\mu_b - E_1(\tilde{\mu})\rvert / \sigma_1(\tilde{\mu})$. | ABS(C36-C23)/C24 | 1.94 | Standardized distance between break-even WTP and posterior baseline mean. |
| 38 | Unit Normal Loss Integral | $L_N(D)$ | NORMDIST(C37,0,1,0) -C37*(1-NORMDIST(C37,0,1,1)) | 0.010051 | Unit Normal Loss Integral Factor (Schlaifer, 1961, Table IV, p. 370) |
| 39 | Expected Loss of Optimal Terminal Action | ELTA = EVPI = $\beta \cdot \sigma_1(\tilde{\mu}) \cdot L_N(\lvert D\rvert)$ | C34*C24*C38 | $839,505 | Expected loss of making an optimal "go" or "no-go" investment decision at this point without any additional sampling (i.e. based only on the initial priors and the original small sample N=250 cases). |
| 40-41 | **IV. Optimal New Sample Size (Depends on Data In Rows 46 to 64)** | | | | |
| 42 | Optimal New Sample Size | $\Delta N$ | Insert Approximate Trial Size to Initialize Optimization from C83 (2683) and then Optimize | 2243 | **Size of a hypothetical second sample to augment the initial sample of N=250. To find an optimum, iterate over alternative values of ΔN to find the sample size that maximizes the Expected Net Gain from a second sample (ENGS(ΔN) in Row 64 below.** |
| 43-45 | **V. Expected Value of Information from a New Sample vs. Cost of Sampling** | | | | |
| 46 | *Current Prior Set Former Posterior=New Prior* | | | | |
| 47 | Current Prior for Mean=Posterior From III | $E_1(\tilde{\mu})$ | C23 | $7.49 | Repeat of previously computed posterior value for new set of calculations. It now becomes a prior value in this step. |
| 48 | Current Prior for Standard Error of Mean | $\sigma_1(\tilde{\mu})$ | C24 | $0.83 | Repeat. Former posterior in III now a prior. |
| 49 | Current Prior for Variance of Mean | $\sigma_1^2(\tilde{\mu})$ | C25 | $0.68 | Repeat. Former posterior in III now a prior. |
| 50 | Variance of (Population) Mean at New Sample Size, Given Population Sigma Assumed Known and = s | $\sigma^2(\bar{x}) = \sigma^2 / \Delta N^2$ | C19^2/C42 | $0.08 | Key step. Standard error of the mean of the new sample. Used below to get revised posteriors in Rows 52 and 53. |

| Row # | COLUMN A: Labels | COLUMN B: Labels | COLUMN C: Formulas | RESULT | COMMENT |
|---|---|---|---|---|---|
| 51 | *New Posteriors* | | | | |
| 52 | Preposterior Reduction in Variance of Mean from ΔN | $\sigma^2(\tilde{E}_1)$ | C49* (C49 / (C49+C50)) | \$0.61 | Calculate updated change in variance due to a second sample of size ΔN using posterior from first sample as a prior and the new sample estimate from Row 50. |
| 53 | Preposterior Reduction in Std Error of Mean from ΔN | $\sigma(\tilde{E}_1)$ or $\sigma^*$ | C52^0.5 | \$0.78 | Square root of change in variance in Row 52 |
| 54 | \|D\| Absolute Value of Prior Standardized Loss from above | $\|D\| = \|\mu_b - E_1(\tilde{\mu})\| / \sigma_1(\tilde{\mu})$ | C37 | 1.94 | Repeat from above. Uses posterior 1 as new prior with 0 subscript |
| 55 | $\|D_E\|$ Absolute Value of Change in Standardized Loss due to ΔN | $\|D_E\| = \|\mu_b - E_1(\tilde{\mu})\| / \sigma(\tilde{E}_1)$ | ABS(C36-C47)/C53 | 2.04 | Uses New Posterior Standard Error of Mean to calculate standardized difference between break-even WTP and mean posterior to first sample of N=250. |
| 56 | Unit Normal Loss Integral | $L_N(D_E)$ | NORMDIST(C55,0,1,0) - C55*(1-NORMDIST(C55,0,1,1)) | 0.00755 | Unit Normal Loss Integral Factor for $D_E$ (Schlaifer, 1961, Table IV, p. 370) |
| 57 | Expected Value of Sample Information | EVSI (ΔN) | C34*C53*C56 | \$597,647 | Expected value of information in new optimal sample of ΔN* = 2243 |
| 58 | Unconditional Expected Terminal Loss | UETeL(N) | C39-C57 | \$241,859 | Terminal Loss after a new sample of ΔN=2243 is taken. Equal to ELTA before an additional sample (ie. at ΔN=0) minus the EVSI(ΔN) |
| 59 | *Sampling Costs* | | | | |
| 60 | Fixed Cost | $K_s$ | 0 | 0 | Set to zero to simplify. Include actual value here. |
| 61 | Marginal=Variable Cost | $k_s$ | 89 | \$89 | Input data. Example estimate is in 1998 Brazilian Reals( R\$) |
| 62 | Total Sample Cost | | C60+C61*C42 | \$199,680 | Multiply marginal sample cost by ΔN and add to fixed cost. |
| 63 | *Expected Net Gain from a Second Sample* | | | | |
| 64 | Expected Net Gain from a Second Sample of Size ΔN | ENGS(ΔN) | C57-C62 | \$397,967 | EVSI((ΔN*)* minus the total cost of taking an additional sample of size (ΔN*)*. This is the Objective to optimize over alternative values of (ΔN*)*. Use a grid search (see text) or, more efficiently, SOLVER in EXCEL, setting the TARGET CELL as C64, equal to MAX; by changing the (ΔN*)* cell, C42, subject to the constraint that C42 is ≥ a small positive number (e.g. 0.001) |

| Row # | COLUMN A: Labels | COLUMN B: Labels | COLUMN C: Formulas | RESULT | COMMENT |
| --- | --- | --- | --- | --- | --- |
| 65-67 | **VI. Addendum: Decide if Additional Sample is Necessary and Compute Approximate ΔN for Optimization Starting Value (See Appendix I)** | | | | |
| 68 | *D Value Standardized Loss* | D | C64 | 1.94 | **The First Essential parameter.** From Above |
| 69 | *Intermediate Components for Calculating $Z_0$.* | | | | |
| 70 | *First Component* | | | | |
| 71 | $\sigma_1(\tilde{\mu})$ From Small Sample $N_0$ | | C48 | $0.83 | From Above |
| 72 | $\sigma$ guess from sample data = s | | C19 | $13.21 | From Above |
| 73 | | $\sigma_1(\tilde{\mu}) / \sigma$ | C71/C72 | 0.06 | |
| 74 | *Second Component* | | | | |
| 75 | | $k_t$ | C34 | $100,988,487 | From Above |
| 76 | | $k_s$ | C61 | $89 | From Above |
| 77 | | $(k_t \sigma / k_s)^{1/3}$ | ((+C75*C72)/C76)^0.33 | 245.19 | |
| 78 | *Z Value* | $Z_0$ | C77*C73 | 15.36 | **The Second Essential Parameter** |
| 79 | | $\eta^*$ | ((1/C78*0.5)* (NORMDIST(C68,0,1,0)))^0.5 | 0.0446 | Crude approximation of the optimal ratio of: $n/((k_t \sigma / k_s)^{1/3})^2$ from Schlaifer |
| 80 | | $((k_t \sigma / k_s)^{1/3})^2$ | C77^2 | 60,116 | Denominator in ratio of $\eta = \Delta N / ((k_t \sigma / k_s)^{1/3})^2$ See Row 83. |
| 81 | Probability an Additional Sample Should be Taken | Logit Probability Model Approximation | +1/(1+exp(2.206+C78*-1.1255+C68*-4.6102+C78^2*0.006596+C68^2*4.8539) | 0.99 | Logit function fit to data from Schlaifer's Sample Decision Figure 21.4 |
| 82 | Take an Additional Sample Before Acting? | | IF(C81>0.5,"YES","NO") | Probably Yes | **Result from evaluating Logit model** |
| 83 | Quick Approximate Optimal ΔN (ignore if Answer to "Sample Before Acting?" is "NO" | | C79*C80 | 2683 | **Approximate solution for ΔN. Use in C42 above to initialize grid search or SOLVER optimization.** |

## TABLES AND FIGURES

### TABLE 1. FUNDAMENTAL DEFINITIONS

| | Case I<br>Correct Decision: Invest<br>$E(NPV) \geq 0$ | Case II<br>Correct Decision: Don't Invest<br>$E(NPV) < 0$ |
|---|---|---|
| **Decision Criterion: Global Mean NPV** | $E(NPV) = \int_{-\infty}^{\infty} NPV_i \bullet p_i \, d\,NPV$ | |
| **Probability of Opportunity Loss** | $F_{Loss\,I} = \int_{-\infty \text{ or Min}}^{0} p_i \, d\,NPV$ | $F_{Loss\,II} = \int_{0}^{+\infty \text{ or Max}} p_i \, d\,NPV$ |
| **Truncated Mean Loss** | $E_{T,I} = E(NPV \mid NPV < 0)$<br>$= \{\int_{-\infty \text{ or Min}}^{0} NPV_i * p_i \, d\,NPV\} / F_{Loss\,I}$ | $E_{T,II} = E(NPV \mid NPV > 0)$<br>$= \{\int_{0}^{+\infty \text{ or Max}} NPV_i * p_i \, d\,NPV\} / F_{Loss\,II}$ |
| **Cost of Uncertainty or Expected Value of Perfect Information or Expected Loss of a Terminal Action**[a] | $E_{Loss,\,I} = E_{T,I} \bullet F_{Loss\,I}$ | $E_{Loss,\,II} = E_{T,II} \bullet F_{Loss\,II}$ |

a These terms all appear in the literature and they all mean essentially the same thing. It may seem unnecessarily roundabout to express the cost of uncertainty as the product of a truncated mean and the fraction of the total probability distribution lying in the region of opportunity loss. However, this is necessary given the way the information is produced by the Crystal Ball Monte-Carlo simulation routine we used to verify the approximate solutions in the worked examples.

b The probability of occurrence of the $i^{th}$ NPV is represented as $p_i$ in the table.

## TABLE 2. SMALL SAMPLE NONPARAMETRIC MEANS

*(1998 R$ per Household per Month)*

| Estimator | Mean | Variance of Mean | Standard Error of Mean | Population Standard Deviation[a] |
|---|---|---|---|---|
| Lower Bound[b, e] | 5.75 | 0.45 | 0.67 | 10.61 |
| Weighted Lower (0.75) and Upper (0.25) Bound[e] | 7.47 | 0.70 | 0.84 | 13.23 |
| Intermediate Lower (0.50) and Upper (0.50) Bound[d, e] | 9.20 | 1.02 | 1.01 | 15.97 |
| Upper Bound[e] | 12.66 | 1.88 | 1.37 | 21.68 |

Notes:

a Approximation from the square root of the product of the variance of the mean and the sample size of 250 cases.

b Turnbull estimator originally proposed by Timothy C. Haab and Kenneth E. McConnell, 1997. "Referendum Models and Negative Willingness to Pay: Alternative Solutions," *Journal of Environmental Economics and Management*, 32; pp. 251-270.

c Proposed by William J. Vaughan and Diego J. Rodriguez, 2000. "A Note on Obtaining Welfare Bounds in Referendum Contingent Va;luation Studies," Unpublished IDB Working Paper. March.

d Originally proposed by Bengt Kriström, 1990. "A Non-Parametric Approach to the Estimation of Welfare Measures in Discrete Response Valuation Studies," *Land Economics*; 66, 2; May, pp. 135-39.

e Proposed by Mattras Bowman, Göran Bostedt and Bengt Kriström, 1999. "Obtaining Welfare Bounds on Discrete-Response Valuation Studies: A Non-Parametric Approach, *Land Economics*; 75, 2; May, pp. 284-94.

## TABLE 3. CRUDE STEP SEARCH FOR N* WITH E(NPV) NEAR ZERO

| | Trial Value for $\Delta N$: | 5750 | 6750 ≈ Optimum | 7750 |
|---|---|---|---|---|
| Change in SE of Mean | $\sigma(\tilde{E}_1)$ | 0.809321811 | 0.811879233 | 0.81379242 |
| Standardized Distance | $D_E$ | 0.0410 | 0.0409 | 0.0408 |
| Unit Normal Loss Integral | $L_N(D_E)$ | 0.378772051 | 0.378834534 | 0.378881025 |
| Value of Sample Information | EVSI | $30,957,867 | $31,060,815 | $31,137,831 |
| Total Sample Cost | $K_S + k_s \Delta N$ | $511,750 | $600,750 | $689,750 |
| Net Gain | ENGS | $30,446,117 | $30,460,065 | $30,448,081 |
| Marginal Gain | $\Delta EVSI / \Delta N$ | $120 | $88 | $67 |
| Marginal Cost | $k_s$ | $89 | $89 | $89 |

**TABLE 4. OPTIMAL INCREMENTAL SAMPLE SIZES, ΔN*, DEPENDING ON PRIORS AND INITIAL EVPI**

| Costs Relative to Baseline | 1.0 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 |
|---|---|---|---|---|---|---|
| Small Sample Standard Errors $\sigma(\bar{x})$ of E(NPV) from Zero[a] | 1.90 | 1.52 | 1.15 | 0.77 | 0.40 | 0.02 |
| **High Sampling Cost of R$89 Per Interview [b]** | | | | | | |
| Tight Prior [c] | 0 | 0 | 0 | 1996 | 4393 | 6530 |
| Diffuse Prior [c] | 2243 | 3411 | 4600 | 5657 | 6409 | 6715[d] |
| Total Ignorance | 2351 | 3507 | 4673 | 5697 | 6413 | 6687 |
| **Low Sampling Cost of R$30 Per Interview [b]** | | | | | | |
| Tight Prior [c] | 0 | 0 | 0 | 3994 | 7458 | 10729 |
| Diffuse Prior [c] | 3866 | 5656 | 7506 | 9160 | 10340 | 10821 |
| Total Ignorance | 4022 | 5799 | 7615 | 9219 | 10343 | 10774 |

Notes:

a. Does not reflect prior information.
b. All optima found using Microsoft Excel. Values differ slightly from optima found with Quattro Pro in Appendix 2, Table 2-1.
c. Prior guess of E(WTP) of R$8.28 with a prior ±50% error of R$0.50 for the tight prior and R$4.00 for the diffuse prior.
d. Exact optimum corresponding to the approximate optimum in Table 3 and Figure 2.

**FIGURE 1. Losses and Loss Probabilities for WTP Below Break-Even**

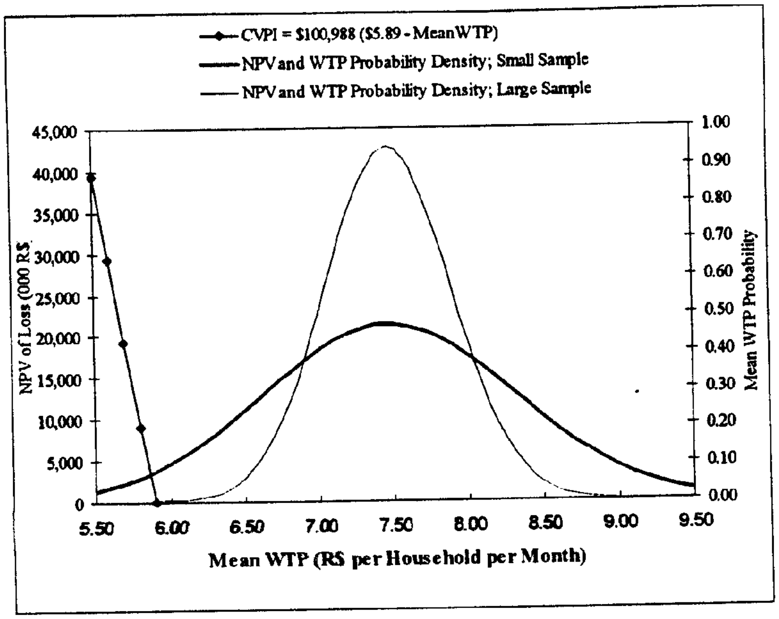
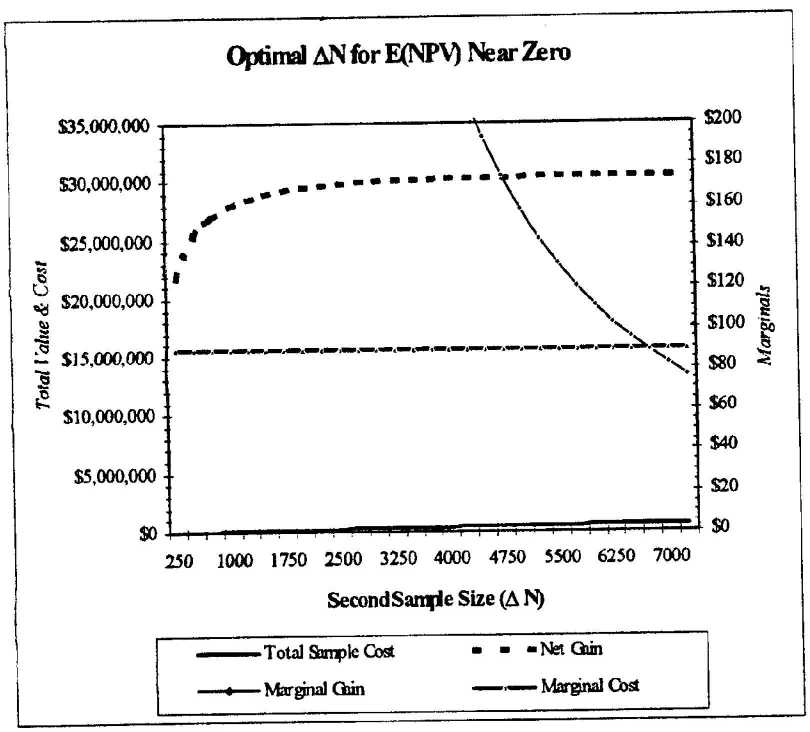



**Figure 2 The Expected Net Gain from Additional Sampling**



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## ENDNOTES

1. Mitchell and Carson (1989 Chapter 10, footnote 13, p. 225) assume a best guess of 2.0 for the coefficient of variation which drives the calculation.

2. In Winkler's (1972, p. 297) words, "This type of decision is called a *pre-posterior decision* because it involves the *potential* posterior distributions following the *proposed* sample." Winkler notes that pre-posterior analysis can be carried out repeatedly in sequential decision making. Our proposal involves a two-step sequence of taking an initial "small" sample and then doing a pre-posterior analysis that looks for the optimal number of surveys to add to the initial sample, which can either turn out to be zero or some positive number. Of course, in some circumstances the initial sample size itself may be suboptimal (too large), but then there will be no need to add to it.

3. Only if the probability distribution of NPV lies entirely in either the positive or negative domains will there be no cost of uncertainty, because you literally can't go wrong. In either of these extreme situations, case-specific sample estimates of willingness to pay may not even be necessary. If extreme upper and lower limits for willingness to pay can be posited a-priori, via benefits transfer or other past experience, and the investment either fails the CB test using the highest conceivableWTP or passes it using the lowest conceivable non-negative WTP value the investment decision might well be made without incurring sampling costs.

4. The focus here is on the choice of a sample for referendum CV, because this method seems to be used most often in practical, CB-related contexts. Of course, the same approach could be applied to CV data gathered using a direct revelation question format.

5. This is optional. See Appendix 2.

6. See Vaughan *et al.* 1999, for a discussion of the several methods available to produce estimates of mean WTP from referendum CV data. One of the nonparametric methods is most likely to be useful in this context.

7.If no prior is specified, only step 2 will be done at this point.

8. Opportunity losses are conditional because, after the first sample has been taken and the optimal act is chosen (in the case illustrated, invest because E(NPV)>0) they are conditional on that act. See Paffenberger and Patterson, p. 1069. For a full discussion that may be more accessible than Schlaifer's original book, see Winkler 1972.

9. The posterior formed by combining the prior with the small sample data takes on the role of the prior in this next step. The amount of revision in the variance is between the variance of the mean from the prior at this juncture, $\sigma_1^2(\tilde{\mu})$, and the preposterior variance after taking a second sample of size $\Delta N$. The latter is $[(\sigma_1^2(\tilde{\mu}) \bullet \sigma^2/\Delta N) / (\sigma_1^2(\tilde{\mu}) + \sigma^2/\Delta N)]$. Writing the difference and simplifying, the variance reduction is:

$$\sigma^2(\tilde{E}_1) = \sigma_1^2(\tilde{\mu}) - \left[\frac{\sigma_1^2(\tilde{\mu}) \bullet \sigma^2/\Delta N}{\sigma_1^2(\tilde{\mu}) + \sigma^2/\Delta N}\right] = \sigma_1^2(\tilde{\mu})\left[\frac{\sigma_1^2(\tilde{\mu})}{\sigma_1^2(\tilde{\mu}) + \sigma^2/\Delta N}\right]$$

The bracketed term on the right in the last expression can be viewed as a variance shrinkage factor (Lapin 1994, p. 1032). It approaches 1.0 as $\Delta N$ increases. In the limit increasing $\Delta N$ brings EVSI equal to ELTA through an equalization of $|D|$ and $|D_E|$, thus eliminating all uncertainty.

10. Throughout we assume the fixed costs of taking a second sample, $K_s$ are zero, because most of these costs (for consulting services, focus groups, questionnaire pretesting and design) would be incurred to obtain the initial sample of 250 cases.

11. A previous project, considered completed in 1998, resulted in a small improvement in river quality. This analysis takes 1998 quality as the base and treats the pre-1998 investment costs as sunk, but operation and maintenance costs as avoidable (i.e. the "without project" counter factual is to cease operation of the plants that are already in place because their benefits are negligible).

12. These are the shadow priced costs, discounted at an opportunity cost of capital of 12% (see Vaughan *et al.* 2000(a,b). The rate of exchange in 1998 was 1.14 Reals per U.S. dollar. The cost figure is net of other benefits attributable to the project because by cleaning the river it will allow an intrabasin transfer of water that will generate electrical energy.

13. The actual 600 observation referendum CV sample from our case study was unbalanced because it undersampled households living in districts that are contiguous to the river (31 percent in the sample, 61 percent from the metropolitan area census). Since households living in districts bordering the river are willing to pay significantly more on average for improved water quality than households in non-contiguous districts (R$6.07 per household per month versus R$4.51) the mean from the grand sample is a biased estimate of the population's average willingness to pay. We corrected for this by randomly drawing 250 observations from the grand sample using the constraint of the Census proportions, which meant that 152 of the available 184 available households living close to the river were included in the small sample, along with 98 of the 416 families living in more distant districts.

14. Vaughan and Rodriguez (2000) modify the nonparametric mean and variance formulas in Boman *et al.* (1999) by generalizing the lower bound variance formula from Haab and McConnell (1997). They demonstrate that there are legitimate intermediate measures lying in between the upper and lower bounds other than Kriström's intermediate mean which gives equal weight to each limit. The choice is a matter of subjective judgement; we prefer to weight the lower bound more heavily than Kriström's intermediate mean does. For the balance of the discussion, the approximately equal allocation of cases across bid levels is taken as given, ignoring the possibilities for variance reduction at any given total sample size that might be achieved by concentrating the bulk of the sample in the region of bid levels where $F_j = 0.5$.

15. Total benefits are the product of the present value of the total number of households in the metropolitan area over the *t* years of payment, $POP_t$ multiplied by the shadow price factor (spf = 0.78) and the resulting discounted sum multiplied by average household willingness to pay ($\overline{WTP}$) per month because we assume the latter is constant over time. Leaving the spf inside the summation so we can work directly with an average WTP that is not shadow priced:

$$\sum_{t=1}^{10} [POP_t \bullet 12 \text{ months/year} \bullet spf \bullet \overline{WTP}] / (1+r)^t \equiv \overline{WTP} \bullet \sum_{t=1}^{10} [POP_t \bullet 12 \text{ months/year} \bullet spf] / (1+r)^t.$$

16. The spreadsheet was successfully benchmarked using the example data in Schlaifer 1961. It was also independently replicated by a colleague to verify the cell formulas. The interested reader can safely duplicate the structure and insert his/her project data to compute an optimal sample size using the Bayesian approach. To get results under total ignorance, a separate spreadsheet is not needed; simply insert a very

large number in Row # 10 for the prior standard error of the mean. This will wash out the influence of the prior in all subsequent calculations.

17. In the treatment of the standard method, we had to form guesses about the mean WTP and the standard deviation of individual observations in the population. Here, we are speculating about the mean of all possible prior means and the spread in that (normal) prior distribution of hypothetical means. This explains the use of the notation $\sigma_0(\mu)$ rather than $\sigma_0$.

18. Similar calculations using any of the other means (e.g.: the Turnbull, Kriström or Paasche means) can easily be done by following the same structure.

19. The intercept for costs was shadow priced. The slope also incorporates a shadow price factor to allow the WTP to be expressed in terms of the original survey responses, without shadow pricing. Because WTP per household is on a monthly basis, population in every year has to be multiplied by a factor of 12 in addition to the shadow price factor. See note 14 above.

20. From the properties of the expectation and variance operators $E(\alpha + \beta X) = \alpha + \beta E(X)$. For example, see Paaffenberger and Patterson 1987, p. 208 and Little 1978, Chapter 10 on strictly linear relationships between random variables versus error propagation formulas.

21. The optimal augment to the initial small sample, $\Delta N$, depends on the initial loss probability, so using either of these higher mean WTPs would reduce $\Delta N$ compared to the recommendation based on the weighted 75−25 mean.

22. For details on this and the subsequent approximate, sub-optimal calculations, see Vaughan and Darling, 2000.

23. The formulas for a triangular distribution from $a$ (the minimum) to $c$ (the maximum) via $b$ (the mode) are:
Mean = $(a + b + c)/3$; Variance = $(a^2 + b^2 + c^2 - ab - ac - bc)/18$.

24. The standard formula, with the same confidence limit but a 6.9 percent error allowable would imply a sample size in the neighborhood of 2500.

25. The uncertainty about project desirability traceable to choice of econometric technique for dealing with referendum CV data is dealt with more fully in Vaughan et al., 1999.

26. Increasing project costs can be thought of as a proxy for decreasing E(WTP) or reducing the standardized distance between E(NPV) and zero at the initial sample size of $N_0 = 250$ cases.

27. Mechanically, all one does is insert a very large number for the prior variance and proceed with the optimization as usual. For a proof, see Vaughan and Darling (2000).

28. Contact William J. Vaughan by e-mail at williamv@iadb.org and ask for the "Sample Size Template." Indicate whether you will be using Quattro Pro versions 6, 7 & 8 or Quattro Pro Version 9. The template is not available in Microsoft Excel.