PRODUCTION EXTERNALITIES AND URBAN CONFIGURATION

by

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Abstract

Jacobs (1969) argues that uncompensated knowledge spillovers have played a crucial role in population agglomeration and thus in the generation of cities. We explore this idea formally by extending the Romer (1986) model of (inter-firm) externalities in production to an explicit spatial context. We postulate that knowledge spillovers between firms decrease with the distance between the firms. A general equilibrium model with households and firms residing in a linear or long, narrow city is constructed. The allocation of goods and factors, the locational choice of firm sites and household residences, as well as factor prices and land rents are all endogenously determined. The equilibrium urban configuration may be concentrated (with monocentric firm locations), dispersed (with completely mixed firm and household locations) or a combination (with incompletely mixed firm and household locations), depending on the population of firms as well as the transportation and firm-interaction parameters. Due to the distance-dependent production externalities, firms will be clustered together in any equilibrium. As a consequence, the duo-centric or any multi-centric urban configuration is never an equilibrium configuration. Moreover, except for a set of parameters of measure zero, the equilibrium urban configuration is unique.

JEL Classification: D51; R12

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1. Introduction

What are the driving forces behind the formation of cities? What are the roles of internal and external returns to scale in agglomerative activities? Since the second half of the 1980s, the Marshallian externality has been commonly used as the primary driving force for city formation. While the development of a spatial model with internal scale economies has reached a mature stage, previous studies of external returns either lack an explicit spatial structure or rely exclusively on numerical computation. Construction of a general-equilibrium spatial model elaborating analytically on the role of external scale economies in spatial agglomeration remains largely unexplored.

Almost three decades ago, Jacobs (1969) argued that ***uncompensated knowledge spillovers*** have played a central role in population agglomeration and thus in the generation of cities. Not until the mid-1980s did Romer (1986) and Lucas (1988) formalize this idea to create a revolution in their attempts to elaborate on the determinants of the endogenous rate of economic growth, giving birth to the so-called new growth theory . In their studies, an individual's knowledge or human capital generates a positive external effect on society's aggregate stock of knowledge or human capital, which in turns enhances each individual's production. Such a framework has fostered numerous research projects in the area of economic growth and development. In our paper, we explore this rich idea formally by extending the Romer (1986) model of positive externalities in production to an explicit spatial context.

Just how important are positive knowledge spillovers to spatial agglomeration? Abundant empirical evidence has lent strong support. For example, Glaeser *et al* (1992) and Henderson *et al* (1995) find significant knowledge spillovers both within and between industries, which are important to city employment growth and to the location decisions of industries. Using patent data, Jaffe *et al* (1993) conclude that knowledge spillovers are geographically concentrated in the sense that patents are more likely to cite previous patents from the same area. Yet more evidence is the formation of research parks, such as Boston's Route 128 and California's Silicon Valley [e.g., see a discussion by Saxenian (1996) with respect to the importance of knowledge spillovers between (vertically) integrated firms and by Krugman (1991) with respect to the external flows of technologies]. Moreover, in Rauch (1993) and Ciccone and Hall (1996), geographical concentration is shown to improve productivity significantly.

Despite its important implications for regional development and city growth, uncompensated knowledge spillovers have not been modeled formally in a general-equilibrium framework with location. This paper therefore provides, to our knowledge, the first attempt at such an endeavor. Specifically, we extend the Romer (1986) model of positive production externalities to allow for the inter-firm knowledge spillovers that decrease with distance between the firms. This geographically diminishing effect is measured by both Euclidean distance to the mean of the distribution of firms and an overall firm dispersion index. There are two important features of the Romer production externality: (i) it is of the external Marshallian type among firms; and (ii) its magnitude is based on the average or aggregation of the capital stocks of individual firms. We construct a general equilibrium model with households and firms residing in a linear or long, narrow city. We assume for tractability that each household or firm is required to occupy a fixed density of land.[1] In addition to using land, firms employ labor and capital to produce a single homogenous good, maximizing profits. Households choose workplaces and residences to maximize utility. In competitive equilibrium, the allocation of goods and factors, the locational choice of firm sites and household residences, and factor prices and land rents are all endogenously determined.

We consider the endogenous formation of urban configurations. The equilibrium configuration may be concentrated (with monocentric firm locations), dispersed (with completely mixed firm and

[1] Inelastic firm demand for land is a common simplifying assumption adopted in the literature. For example, see Ogawa and Fujita (1980), Fujita and Ogawa (1982) and Fujita and Thisse (1986).

household locations) or a combination (with incompletely mixed firm and household locations). The crucial determinants of the underlying urban configuration are: (i) the consumer commuting cost per unit distance to the firms, (ii) the degree to which knowledge spillovers become less effective as a result of overall firm dispersion, and (iii) the population of firms. Due to the distance-dependent production externalities, firms will be clustered together in any equilibrium. As a consequence, the duo-centric or any multi-centric urban configuration is ***never*** an equilibrium outcome. Moreover, except for a set of parameters of measure zero, the equilibrium urban configuration is ***unique***.

Two closely related papers are Ogawa and Fujita (1980) and Fujita and Ogawa (1982). In their pivotal work, Ogawa and Fujita (1980) incorporate a (strictly) convex inter-firm transactions cost schedule into a linear city model to generate the urban land use patterns. Under a Leontief production technology in labor and land (with no capital), firms minimize the real resource costs of transactions, thus determining the types of urban configuration. In this framework, multi-centric urban configurations cannot arise in equilibrium. Fujita and Ogawa (1982) extend their previous study by considering a "locational potential function" in which a weighted average of pairwise Euclidean distances between firms has a negative effect on firms' profit. In contrast to the transactions cost arguments, this paper focuses on the externality of business agglomeration. Regardless of the various types of urban configurations, the locational potential function is always (strictly) concave in distance over business areas, thus implying a (strictly) convex penalty cost for firm dispersion. Using numerical analysis, they find that multi-centric urban structures are possible and multiple equilibria are present.

Our paper differs significantly from both papers: (i) we formalize the knowledge spillover idea *a la* Romer in which knowledge spillovers are regarded as uncompensated factor inputs in firms' production (rather than via transactions cost or firm profit independent of aggregate capital usage); (ii) we follow closely the Romer convention to allow both Euclidean distance to the mean of the distribution of firms and an overall firm dispersion index to affect the productivity of inter-firm knowledge spillovers (rather than an integral of pairwise distance between firms); and, (iii) we permit factor substitution (rather than using a Leontief production function). Notably, by developing a generalized Romer framework of knowledge spillovers rather than the locational potential function, we can conduct the analysis ***analytically*** to obtain the general equilibrium urban configuration without relying on numerical examples. Since we consider externalities of firm agglomeration with a strictly convex penalty for firm dispersion, our model structure in this regard is relatively closer to Fujita and Ogawa (1982). Yet, a multi-centric urban configuration cannot arise in equilibrium.[2] Our results regarding the absence of a multi-centric city and the uniqueness of spatial equilibrium are obviously in contrast with findings in Fujita and Ogawa (1982), in the more recent sequels using models of product differentiation [e.g., see Fujita and Krugman (1997)], and in the spatial competition models with an active land market [e.g., see Fujita and Thisse (1986)].[3]

Another related paper is Palivos and Wang (1996), who consider knowledge spillovers in a monocentric city setting with endogenous growth. Their paper focuses on determining the optimal paths for output and population growth and the contrast between decentralized and socially optimal outcomes. They treat the monocentric urban configuration as exogenously given and assume unified household-firm units. In contrast, our paper examines the economic and geographical interplay between households and firms and, as a consequence, determines the equilibrium urban configuration

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[2] As argued in Section 4.4 below, multi-centric equilibria can never be generated in models like ours, independent of the convexity or concavity properties of the penalty or transactions cost schedule. The key to the generation of multi-centric equilibria in Fujita and Ogawa (1982) is the fixed factor proportions and the separability of the locational potential function that compares firm locations pairwise to obtain the overall penalty for a firm by integration. In contrast, our penalty functions are explicitly non-separable into a pairwise form and the external factor is allowed to affect the factor proportions.

[3] In contrast to our perfect competition framework, Fujita and Krugman (1997) consider monopolistic competition whereas Fujita and Thisse (1986) employ a Nash equilibrium concept with oligopoly. Neither incorporates Romer externalities nor allows firms to compete for land use.

endogenously.

It is useful to summarize and highlight the main contributions of our paper relative to the existing literature. First, we model precisely the source of external returns to scale via Jacobs-Romer knowledge spillovers between firms. Second, we consider an explicitly spatial structure with locational choice of residence, job site and production site within a perfectly competitive, general-equilibrium framework with capital, labor and land as production inputs. Third, we allow both distance to the mean and the overall dispersion of firms to affect the degree of knowledge spillovers and formally model it by applying index number theory. Finally, our analytical results provide testable implications for rent gradients and spatial configuration with respect to commuting, population size and spillover parameters, which may be useful for empirically distinguishing this theory from others.

The organization of the remainder of the paper is as follows. In Section 2, we present the basic environment and the model structure. We then describe the equilibrium concept in Section 3. Section 4 studies the formation of completely mixed, monocentric and incompletely mixed urban configurations and establishes the parameter values for which these various configurations are equilibria. We also show that no multi-centric configuration can emerge in equilibrium. In Section 5, we prove the existence and the uniqueness (almost everywhere) of spatial equilibrium and characterize the relationships between commuting and dispersion parameters and the resulting urban configurations. We conclude the paper in Section 6. An appendix contains all proofs.

2. The Model

Consider a linear city spread over a featureless "long-narrow" line represented by $\Omega \equiv [-1,1]$, with uniformly distributed land. There is a continuum of firms of mass M and a continuum of households of mass N (with N + M = 2, N > 0, M > 0). Each household occupies a unit density of land. For simplicity, we assume that there is an ***absentee landlord***, who owns all the land (of measure two) and consumes no land. In addition to using land, firms hire capital and labor to undertake the production of a single homogeneous composite consumption good. While the absentee landlord spends the entirety of the rental income from firms and households for composite good consumption, each household chooses workplace and residence to maximize utility - which will turn out to be the same as maximizing net income for consumption. Both factor and goods markets are perfectly competitive. Both factors are fully employed in equilibrium.

The central feature of the model is the influence of the uncompensated inter-firm knowledge spillovers in production. An individual firm employs capital (K), labor (L) and land to produce goods using a constant returns to scale technology which exhibits a Cobb-Douglas form. Due to the presence of uncompensated knowledge spillovers a la Romer (1986), the aggregate capital stock (of all firms located in the city, denoted $\overline{K}$) has a positive effect on the individual production of each firm. In contrast with Romer (1986), we allow the magnitude of this positive externality to diminish with distance. Thus, in addition to a relative distance measure, we incorporate a dispersion measure - the more concentrated firms are, the more effective knowledge spillovers will be.

Let z be the location index, $z \in \Omega \equiv [-1,1]$. Denote the density of firms at location z under a particular urban configuration τ (to be determined in equilibrium) by $m_\tau(z)$. We then denote the ***mean location*** of firm sites as $\mu = \int_{z\in\Omega} z \cdot m_\tau(z) dz$ and the ***overall dispersion*** of firm sites as σ_τ, to be defined next. To suit our needs, we require this overall dispersion index to be (i) absolute (invariant to adding a constant to every firm's location), (ii) decomposable (into subgroups with subgroup consistency),[4] and (iii) symmetric (to the mean location). The measures in the Kolm-Pollak class satisfy these properties [see Kolm (1976) and Pollak (1979)]. The simplest among these is an absolute deviation measure:

[4] For a comprehensive discussion and mathematical characterization of the decomposability property (of poverty indices), the reader is referred to Foster and Shorrocks (1991). Of course, in the context of urban economics considered herein, decomposability is defined in terms of firm clusters rather than income groups.

$$\sigma_\tau = (2/M)\int_{z\in\Omega} m_\tau(z)\,|z-\mu|\,dz \qquad (1)$$

where the constant multiplier 2 is incorporated so that the maximum overall dispersion is normalized to unity.[5] For example, this overall dispersion index is simply measured by the ratio of the area of firm sites to the total area of the linear city if firms form a connected set in Ω and are uniformly distributed within that connected set. When firms locate in every location, $\sigma_\tau = 1$; when firms are cluster within an interval [-q, q], $\sigma_\tau = 2q/2 = q < 1$.

Now let $Q(z) = 2-(z-\mu)^2-\varepsilon\sigma_\tau^2 > 0$ measure the ***degree of effectiveness of interactions*** between a particular firm z and the others in the linear city given a configuration of type τ, where $\varepsilon \in (0,1)$ indicates the degree of penalty on overall dispersion of firms and the second term specifies a quadratic cost function in terms of the distance between a particular firm site and the mean site. Thus, one may regard our Q function as a proxy (with the first and second moments) for the (locational) distribution of firms. More importantly, it captures the Romer convention in which externalities enter the system based on an average or aggregation of individual measures. Further, its simplicity enables us to obtain analytical results without relying on numerical examples. Since Q is (strictly) concave in z, the penalty for firm dispersion is (strictly) convex in z. This property is analogous to the transactions cost setup in Ogawa and Fujita (1980) and the locational potential function setup in Fujita and Ogawa (1982).[6]

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[5] We could use the variance measure or any other measure in the Kolm-Pollak class, but choose the absolute deviation measure for its analytical simplicity. For symmetry, we focus primarily on the case when $\mu = 0$ and thus $\sigma_\tau = (4/M)\int_0^1 m_\tau(z)\,z\,dz$.

[6] More specifically, 2-Q may be regarded comparable to the transactions cost $\tau T(x)$ in Ogawa and Fujita (1980), whereas Q is similar to the locational potential function F(x) in Fujita and Ogawa (1982). Under Leontief production, the locational potential function setting in Fujita and Ogawa (1982) can be rewritten in a functionally equivalent form as a transactions cost depending on an exponential measure of distance between firms, which can be compared directly with that in Ogawa and Fujita (1980) using a linear measure of distance. Yet, such an equivalence cannot be established with more general production functions such as the Cobb-Douglas form.

Therefore, with a density s(z) of land, a firm located at $z \in \Omega \equiv [-1,1]$ is able to produce goods under the following production technology:

$$Y(z) = AK^{\alpha}L^{\beta}[Q(z)\overline{K}]^{1-\alpha-\beta}D(z)^{1-\alpha-\beta} \tag{2}$$

where $\alpha, \beta \in (0,1), \alpha+\beta \in (0,1)$ and D(z) is the effective land input given by D(z) = min {1, S(z)} with $S(z) = s(z) \;\forall\; s(z) \geq 1$ and $S(z) = 0 \;\forall\; s(z) < 1$. Thus, the efficient use of land is at s(z) = 1, which simplifies the analysis greatly - the model is not tractable otherwise. The production technology is constant returns to scale with respect to all private factor inputs {K, L, D}, which is important in justifying zero profit and computing the land rent in a way consistent with the existing literature (see the discussion of the bid rent function below).

Both the locational potential function approach in Fujita and Ogawa (1982) and our consideration of the Q function regard firm agglomeration as an external economy. However, it is important to note that not only the form of this external factor but also its interaction with the production factor inputs differ sharply between the two papers. In particular, in our framework, Q is nonseparable from K and L and can affect the factor proportion K/L in equilibrium.

Take output as the numéraire. Let *R(z)* denote the land rent at location z and let r and w denote the rental cost of capital and the wage rate, respectively. Each firm seeks to maximize its profit under the production technology specified in (1):

$$\max_{\{K,L,z\}} \; \pi = AK^{\alpha}L^{\beta}[Q(z)\overline{K}]^{1-\alpha-\beta} - rK - w(z)L - R(z) \tag{PF}$$

Given constant returns to scale in private factors, and thus zero profit in equilibrium, the conventionally defined ***bid rent function*** of firm is: $R_F(z) = \max_{K,L} \; AK^{\alpha}L^{\beta}[Q(z)\overline{K}]^{1-\alpha-\beta} - rK - w(z)L$.[7] Under perfect competition, the land rent facing each firm will be equal to the bid rent in equilibrium. Free mobility

[7] Thus, it is important to have the power associated with unit land input be 1-α-β in the production function specified in (2).

implies that capital rental is constant across locations; however, wages may vary with different locations. Taking the constant capital rental r and the labor wage schedule {w(z)} as given, firms make locational choice of the production site facing the trade off between the land rent and labor costs and the external benefit from knowledge.

The first-order conditions with respect to K, L and location z are, respectively,

$$\alpha \frac{Y}{K} = r \tag{3}$$

$$\beta \frac{Y}{L} = w(z) \tag{4}$$

$$-2(1-\alpha-\beta)\frac{Y}{Q(z)}\;(z-\mu) \;=\; R_F'(z) + w'(z)L \tag{5}$$

where "primes" represent derivatives of functions. Obviously, (3) and (4) equate the marginal products of capital and labor with the corresponding factor prices. Equation (5) is a locational equilibrium condition for firms, stating that the knowledge-spillover gain from moving marginally closer to a central location is exactly offset by the increased land rent and labor cost.[8]

All households are identical in every respect. Each is endowed with one unit of labor and receives no disutility from work (and thus supplies one unit of labor inelastically). Each household has a utility $\hat{U}(c,h)$ where c is composite good consumption and h is residential land consumption. Assume $\hat{U}(c,h) = U(c) \;\forall\; h \geq 1$ and $\hat{U}(c,h) = 0 \;\forall\; h < 1$ where $U(c)$ is strictly increasing and concave in c with U(0) = 0; this implies each optimizing household will consume exactly one unit of land. Denote by I(x,z) the net income of a household residing in x while working at z. This household earns a wage of w(z), incurs a linear commuting cost of t|x-z| and pays land rent R(x) on its one unit of land consumption. Given the assumptions on utility, its object is to maximize consumption (that is equal to

[8] This is parallel to the Muth (1969) condition in the conventional urban economics framework.

the net income):

$$\max_{\{x,z\}} \quad I(x,z) \; = \; w(z) - t|x-z| - R(x) \tag{PC}$$

Locational no-arbitrage requires that each household must reach a constant net income for any pair of work and residential locations, $I(x,z) \; = \; I_0$, under which a household's bid rent is defined as: $R_C(x) = \max_z \; \{ w(z) - t|x-z| \;\; s.t. \;\; I(x,z) \; = \; I_0 \}$. Hence, from (PC) under perfect competition, this means:

$$R_c(x) \; = \; w(z) - t|x-z| - I_0 \tag{6}$$

for all x where consumers live. Under this condition, a representative household (x,z) has no incentive to change either employment or residential location, due to the fact that the incremental benefit from changing location exactly offsets the incremental cost. Should this condition fail to hold, there will be a positive measure of land unoccupied in equilibrium.

3. Equilibrium

We are now prepared to define the concept of equilibrium with location. Under perfect competition with constant returns, each firm earns zero profit in equilibrium. Combining with (3) and (4), we have:

$$Y \; = \; \frac{1}{1-\alpha-\beta} R_F(z) \tag{7}$$

Substituting (7) into (5) to eliminate Y and applying the definition of Q yield an ordinary first-order differential equation for R_F with respect to z. By integration, one can express firm's bid rent as a function of the wage rate, the location index, and other exogenous parameters:

$$R_F(z) = \frac{2-(z-\mu)^2-\varepsilon\sigma_\tau^{\,2}}{w(z)^{\beta/(1-\alpha-\beta)}} \Lambda_\tau(z_\tau) \tag{8}$$

where $\Lambda_\tau(z_\tau) = R_F(z_\tau)\, w(z_\tau)^{\beta/(1-\alpha-\beta)} / [2-(z_\tau-\mu)^2-\varepsilon\sigma_\tau{}^2]$, depending on a reference point (z_τ) and the endogenous urban configuration (τ) to be determined.

An explanation of each of the equilibrium conditions follows the definition.

Definition 1: A ***competitive spatial equilibrium*** is a list of quantities for each location z {K(z), L(z), Y(z)}, prices {r(z), w(z), R(z)} and population densities {M(z), N(z)} for $z \in \Omega \equiv [-1,1]$ such that the following conditions are satisfied:

(i) profit maximization: (3) and (4);

(ii) land rent:

$$R(z) = Max \ \{ R_F(z), R_C(z), 1 \}$$

$$R(z) = R_F(z) \qquad if \quad M(z) > 0$$

$$R(z) = R_C(z) \qquad if \quad N(z) > 0$$

$$R(-1) = R(1) = 1$$

where households' and firms' bid rent functions are given by (6) and (8);

(iii) zero profit: (7);

(iv) land market clearance:

$$M(z) + N(z) = 1, \quad \forall \ z \in \Omega$$

(v) labor market equilibrium:

$$\int_{z \in \Omega} L(z)M(z)dz = N$$

(vi) population balance:

$$\int_{z \in \Omega} M(z)dz = M$$

$$\int_{z \in \Omega} N(z)dz \ = \ N$$

Part (i) is a firm's optimization conditions given perfect competition. Part (ii) defines the equilibrium land rent as the upper envelope of the two bid rent functions R_F and R_C. Since both the demand for and the supply of land at the boundary are completely inelastic, the equilibrium land rent is indeterminate. Thus, we normalize the boundary land rent to unity in order to obtain a unique equilibrium rent schedule. Part (iii) specifies zero profit under perfect competition, and (iv) implies no vacant land in the city. Part (v) equates aggregate labor demand with labor supply. It may be more fully spelled out by "local labor market equilibrium" conditions:

$\int_{z \in B} [N(J^{-1}(z)) - L(z)M(z)]dz \ = \ 0$, where B is a measurable subset of Ω, $N(x)$ is the population density of consumer residing at x, and the job-selection function $J(x)$ is defined such that the net wage, $w(z) - t\,|x - z|$, for a consumer residing at x is maximized with respect to job site z.[9] Part (vi) ensures that firms and households are all located. We focus on the case of ***symmetric*** urban configurations, as in all previous studies. Thus, it suffices to examine the equilibrium conditions on the right half of the city where $z,\ x \ \geq \ 0$.

4. Endogenous Determination of Urban Configuration

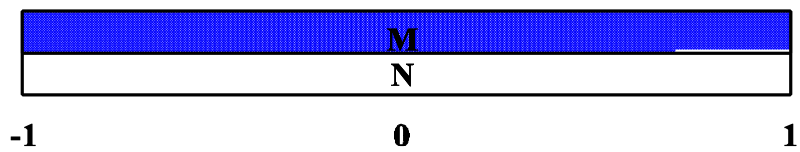
In this section, we determine the endogenous formation of urban configurations. The equilibrium configuration may be concentrated (with monocentric firm locations), dispersed (with completely mixed firm and household locations), a combination (with incompletely mixed firm and household locations) or multi-centric (with more than one firm cluster). While the first three cases are all possible, we will show that the latter configuration cannot be an equilibrium outcome.

[9] The job-selection function is parallel to the "commuting pattern" function in Fujita and Ogawa (1982). It is written over intervals since a single point has zero measure. As we will see in Section 4 below, the symmetric nature of the model implies it is not necessary to solve the job selection function in characterizing the equilibrium.

4.1. The Completely Mixed Urban Configuration

A ***completely mixed*** urban configuration (denoted τ = C) is one in which firms and workers occupy every location along the linear city (see Figure 1).

Figure 1: Completely Mixed Urban Configuration



In this case, the mean location for firms is at the center, $\mu = 0$, and the measure of firm dispersion attains its maximum, $\sigma_C = 1$.[10] Thus, the degree of effectiveness of interactions is: $Q(z) = 2 - z^2 - \varepsilon$.

Moreover, every consumer works where they live. As a consequence, there is no need to differentiate households' from firms' locations, so x and z can be interchanged, and labor market equilibrium implies $M(z)L(z) = N(z)$ for all $z \in \Omega$. The bid rent functions, R_F and R_C, must be identical and thus,

$$\begin{cases} R_F(z) = R_C(z) \\ R_F^{/}(z) = R_C^{/}(z) \\ R_F^{//}(z) = R_C^{//}(z) \end{cases} \qquad z \in \Omega \tag{9}$$

Under the properties associated with the completely mixed urban configuration, consider the following condition on exogenous parameters where $B_1(\varepsilon, N) = \frac{2}{1-\varepsilon}\left[1 + \frac{\beta}{1-\alpha-\beta}\frac{1}{1+I_0(\varepsilon,N)}\right]^{-1}$ and I_0 will be specified below with $\frac{\partial I_0}{\partial \varepsilon} > 0$ and $\frac{\partial I_0}{\partial N} < 0$,

Condition C: (completely mixed urban configuration) $B_1(\varepsilon, N) \leq t$.

This condition requires that the unit commuting cost be sufficiently large while the penalty for firm

[10] In this case, $m(z) = M/2$ for all z and straightforward integration of (1) gives the overall dispersion measure.

dispersion due to less effective knowledge spillovers be sufficiently low.

Proposition 1: *Under Condition C and* $\frac{1-\alpha-\beta}{\beta}\frac{N}{2} < 1$, *there is a competitive spatial equilibrium with a completely mixed symmetric urban configuration in which firms and households occupy every location in the linear city and equilibrium land rent, net income and wage (at z = 0) are given by the following:*

$$R(z) = \frac{2-z^2-\varepsilon}{1-\varepsilon}\left[\frac{1+I_0(\varepsilon,N)}{R(z)+I_0(\varepsilon,N)}\right]^{\frac{\beta}{1-\alpha-\beta}} \tag{10}$$

$$I_0(\varepsilon,N) = \left[1-\frac{1-\alpha-\beta}{\beta}\frac{N}{2}\right]w(0;\varepsilon,N) \tag{11}$$

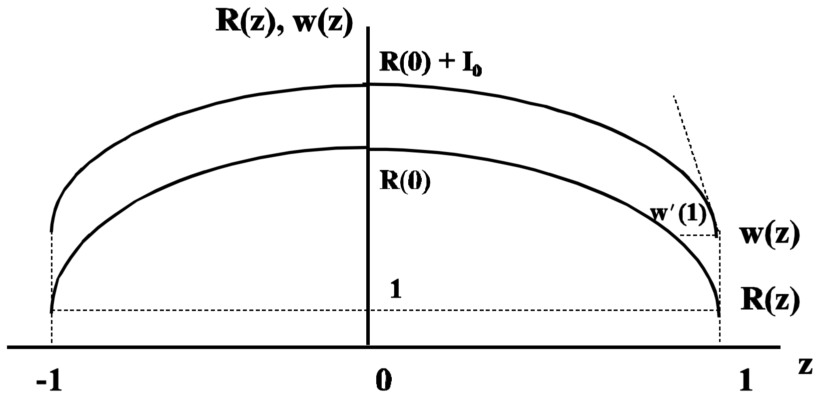
$$\varphi_0(\varepsilon,N)\, w(0;\varepsilon,N)^{\frac{1-\alpha}{\beta}} = 1+w(0;\varepsilon,N)\,(1-\frac{1-\alpha-\beta}{2}\frac{N}{2}) \tag{12}$$

where $\varphi_0(\varepsilon,N) = [\frac{1-\alpha-\beta}{\beta}\frac{1-\varepsilon}{2-\varepsilon}\frac{N}{2}]^{\frac{1-\alpha-\beta}{\beta}}$, with $\frac{\partial\varphi_0}{\partial\varepsilon}<0$ and $\frac{\partial\varphi_0}{\partial N}>0$.

Proof: See the Appendix. ||

Equations (10)-(12) jointly determine equilibrium land rent, wage at z = 0 and the endogenous net income earned by each household, I_0. The values of the other endogenous variables follow from (3), (4), (7) and (8) accordingly. In Figure 2, we graph the equilibrium land rent and wage schedules in the linear city. In particular, we illustrate that firms near the city edge are penalized for locating far from the mean location (zero), but pay lower rent and wages.

Figure 2: Equilibrium Wage and Land rent Schedules - Completely Mixed



Importantly, under this urban configuration, the righthand side of Condition C measures the slope of the wage schedule at z = 1 and thus it implies: $\max_z |w^{\prime}(z)| = |w^{\prime}(z)|_{z=1} \leq t$. Therefore, given Condition C, no household has incentive to work away from home - by working for a firm closer to the city center, the incremental gain from a higher wage is dominated by the induced cost of commuting. This together with (6) ensures that for each household, its equilibrium choice of residence is the same as its workplace, resulting in an urban configuration in which firms and households are mixed in every location and equilibrium commuting costs are minimized at zero.

Proposition 1 indicates that this completely mixed urban configuration emerges when commuting is sufficiently costly and the penalty for firm dispersion is sufficiently low (Condition C). While the intuition behind costly commuting is straightforward, that regarding the firm dispersion penalty deserves further comment. When this penalty is high, it is more likely that a firm may reside in a small cluster, rather than spreading over the entire linear city. As a consequence, firms are more willing to pay higher wages to attract workers residing in the outskirts to commute to work. We relegate more complete discussion of this possibility to the next subsection.

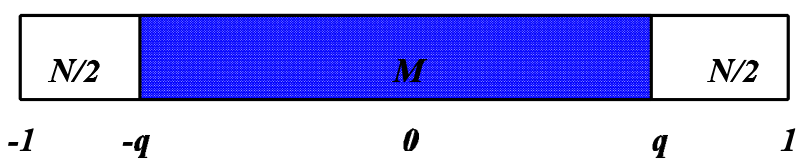
Straightforward comparative statics show that a greater penalty for firm dispersion due to less effective knowledge spillovers (a larger ε) results in an increase in the equilibrium bid rent, equilibrium wage and equilibrium net income. Intuitively, when there is a greater penalty for firm dispersion, the relative disadvantage for outlier firms becomes less severe (since this penalty is imposed uniformly). Thus, outlier firms invest more and, by factor complementarity (in the Pareto or Edgeworth sense), both land rent and wage increase. Since labor supply is fixed, the resultant increase in labor demand must then be offset by a further increase in the unit cost of labor, implying a higher net income for households.

Notably, due to the linear commuting cost schedule, the wage function and the household's bid rent function always maintain a fixed relationship as depicted in Figure 2 or equation (6). Thus, in the rest of the paper, we will restrict our attention to the analysis of the equilibrium land rent, leaving the equilibrium wage aside for the sake of brevity.

4.2. The Monocentric Urban Configuration

We turn next to examine the case of a ***monocentric*** urban configuration (denoted τ = M) in which all firms locate toward the city center within an interval [-q, q] $(0 < q < 1)$ while households reside in the outskirts [-1, -q] and [q, 1] (see Figure 3).

Figure 3: Monocentric Urban Configuration



In this case, M = 2q, $\mu = 0$ and $\sigma_M = 2q/2 = q$.[11] Thus, we can express q in terms of N: $q = 1 - \frac{1}{2}N$ and the degree of effectiveness of interactions can be computed as: $Q(z) = 2 - z^2 - \varepsilon q^2$. Moreover, at q (and by symmetry at -q), the firm and consumer bid rent functions must be identical: $R_F(q) = R_C(q)$.

[11] Under the monocentric configuration, $m(z) = M/(2q)$ for all $z \in [-q, q]$ (and zero otherwise). Thus, utilizing (1), the overall dispersion measure becomes q.

Define $B_2(\varepsilon, N) = \dfrac{(1-N/2)^2[2+(3+\varepsilon)tN/2]-tN}{[2-(1+\varepsilon)(1-N/2)^2](1-N/2)}$, where B_2 is strictly increasing in ε.

Consider the following condition,

Condition M: (monocentric urban configuration) $B_2(\varepsilon, N) \geq t$.

This condition requires that the unit commuting cost is sufficiently small while the penalty for firm dispersion is sufficiently high.

Proposition 2: *Under Condition M and* $\dfrac{2\beta}{1-\alpha} < N \leq 2\left(1-\sqrt{\dfrac{2}{3+\varepsilon}}\right)$, *there is a competitive spatial equilibrium with a monocentric symmetric urban configuration in which all firms are clustered toward the city center [-q, q] while households reside in the outskirts of the linear city, where* $q = 1-\dfrac{1}{2}N$ *, the bid rent schedule for households in the outskirts is* $R_C(x) = 1 + t(1-|x|)$ *and the bid rent schedule for firms within the cluster [-q, q] is given by:*

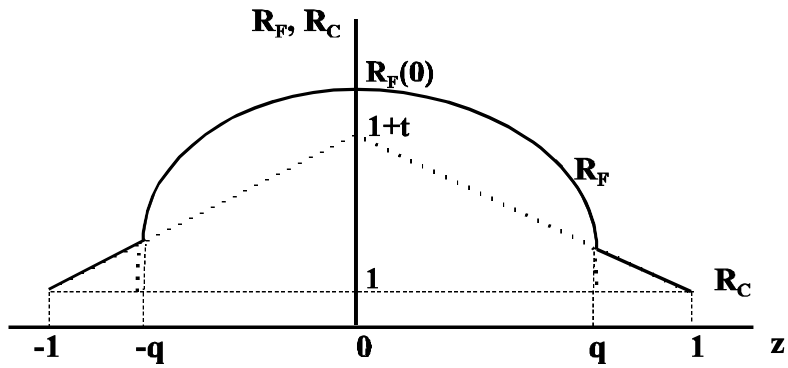
$$R_F(z) = \frac{(2-z^2-\varepsilon q^2)[1+t(1-q)]}{[2-(1+\varepsilon)q^2]\left\{1+t(q-z)/[1+I_0+t(1-q)]\right\}^{\beta/(1-\alpha-\beta)}} \tag{13}$$

where $1+I_0+t(1-q) = \dfrac{\beta}{1-\alpha-\beta}\dfrac{(1+tN/2)(1-N/2)}{N/2}$ *is increasing in t and decreasing in N.*

Proof: See the Appendix. ||

It is clear from (6) and (13) that both firm and consumer bid rent functions decrease in the distance of the plant/housing site from the city center, the origin (see Figure 4).

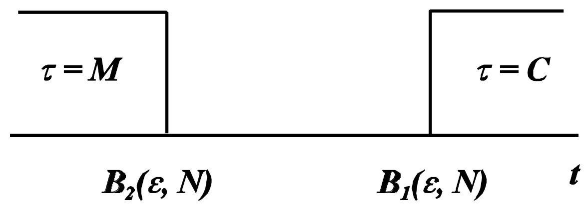
Figure 4: Equilibrium Land rent Schedules - Monocentric



Notably, the firm bid rent function is concave and maximized at $R_F(0)$, whereas the consumer bid rent function is linear with a slope of an absolute value of t and its values range from 1 to 1+t. The two bid rent schedules intersect at locations q and -q, i.e., $R_F(q) = R_C(q) = R_F(-q) = R_C(-q) = 1 + \frac{t}{2}N$. As shown in the Appendix, Condition M implies that $R_F(0) \geq R_C(0) = 1+t$ and $|R'_F(z)|_{z=q} \geq |R'_C(z)|_{z=q}$. These properties together guarantee that $R_F(z) \geq R_C(z)$ for all $z \in [-q, q]$, which ensure that households have no incentive to move to any location within the firm cluster. Moreover, since $|R'_C(x)| = t$ for all $x \in [-1, -q] \cup [q, 1]$, the utility gain from moving toward the center to save commuting costs is exactly offset by the loss due to a higher rent; households are indifferent about residing at any location in the outskirts. Therefore, a spatial equilibrium with the monocentric urban configuration is obtained.

Proposition 2 suggests that when commuting cost is very low, firms are concentrated to take advantage of knowledge spillovers and households commute and receive a high wage to offset the travel costs. Notice that it can be shown that $B_2(\varepsilon, N) > B_1(\varepsilon, N)$ for all $\varepsilon \in (0,1)$ and $N \in (0, 2)$. Thus, for various values of the unit commuting cost, t, we can determine the associated equilibrium urban configuration as illustrated in Figure 5.

Figure 5: Determination of Equilibrium Urban Configuration

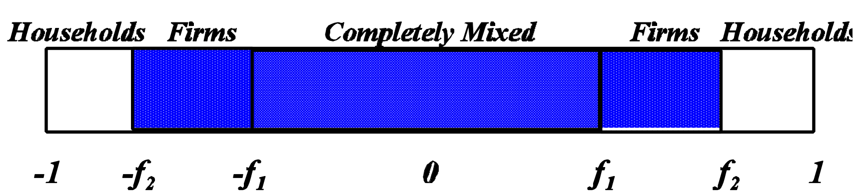


Intuitively, when the unit commuting cost is sufficiently high, the completely mixed urban configuration emerges as the unique equilibrium outcome; when the unit commuting cost is sufficiently low, the unique equilibrium urban configuration turns out to be monocentric. A question remains is what urban configuration arises in equilibrium and whether the equilibrium urban configuration is unique if the unit commuting cost is moderate [i.e., $B_2(\varepsilon, N) < t < B_1(\varepsilon, N)$] .

4.3. The Incompletely Mixed Urban Configuration

We next illustrate the possibility that the urban configuration is ***incompletely mixed*** (denoted τ = I) in the sense that firms locate over the area [-f_2 , f_2] while households reside both toward the city center [-f_1 , f_1] and in the outskirts [-1, -f_2] and [f_2 , 1], where $0 < f_1 < f_2 < 1$ (see Figure 6).

Figure 6: Incompletely Mixed Urban Configuration



Thus, firms and households are completely mixed only in the area around the city center. While households reside in disconnected regions, all firms locate in a connected region around the city center.

In this case, $\mu = 0$ and $\sigma_I = 2f_2/2 = f_2$.[12] Also, we have: $Q(z) = 2 - z^2 - f_2^2$.

The equilibrium land rent function is such that

$$R(z) = R_F(z) = R_C(z) \qquad \forall\ z \in [-f_1\ ,\ f_1]$$

$$R(z) = R_F(z) > R_C(z) \qquad \forall\ z \in (-f_2\ ,\ -f_1) \cup (f_1\ ,\ f_2) \tag{14}$$

$$R(z) = R_C(z) > R_F(z) \qquad \forall\ z \in [-1\ ,\ -f_2) \cup (f_2\ ,\ 1]$$

and $R_F(-f_2) = R_C(-f_2) = R_F(f_2) = R_C(f_2)$. Now consider the following condition on exogenous parameters,

Condition I: (incompletely mixed urban configuration) $B_2(\varepsilon, N) \le t \le B_1(\varepsilon, N)$.

This condition requires a moderate unit commuting cost and a moderate penalty for firm dispersion.

Proposition 3: *Under Condition I, there is a competitive spatial equilibrium with an incompletely mixed symmetric urban configuration in that firms locate over the area $[-f_2\ ,\ f_2]$ while households reside both toward the city center $[-f_1\ ,\ f_1]$ and in the outskirts $[-1, -f_2]$ and $[f_2\ ,\ 1]$ for $0 < f_1 < f_2 < 1$. The land rent schedule in the outskirts $[-1, -f_2]$ and $[f_2\ ,\ 1]$ is* $\mathrm{R}(z) = R_C(z) = 1 + t(1 - |z|)$ *and the land rent schedule over the area $[-f_2\ ,\ f_2]$ is $R(z) = R_F(z)$ with $R_F(z)$ solving:*

$$R_F(z)\,[R_F(z) + I_0]^{\frac{\beta}{1-\alpha-\beta}} = (2 - z^2 - \varepsilon f_2^2)\,\Lambda_I(f_2) \tag{15}$$

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[12] The calculation of the overall dispersion measure is similar to that under the monocentric configuration except that in this case, $m(z) = M/(2f_2)$ for all $z \in [-f_2\ ,\ f_2\ ]$.

where $\Lambda_I(f_2) = \dfrac{1+t(1-f_2)}{2-(1+\varepsilon)f_2^2}[1+I_0+t(1-f_2)]^{\frac{\beta}{1-\alpha-\beta}}$ *and* f_1, f_2 *and* I_0 *solve jointly* ${}_F(f_1) = 1+t(1-f_1)$ *and*

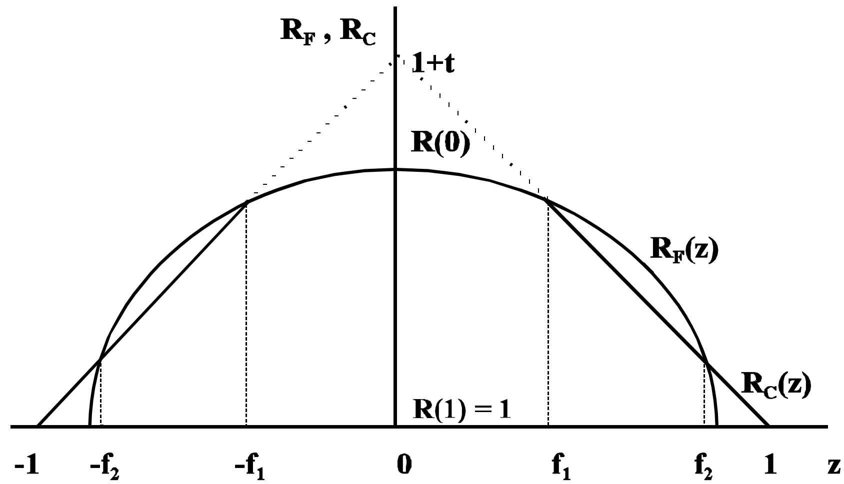
$$\frac{\beta}{1-\alpha-\beta}\int_{f_1}^{f_2}\frac{R_F(z)}{1+I_0+t(1-z)}\,dz = \frac{1}{2}N(1-f_2) \tag{16}$$

$$\frac{\beta}{1-\alpha-\beta}\int_{0}^{f_1}\frac{R_F(z)}{I_0+R_F(z)}\,dz = \frac{1}{2}Nf_2 \tag{17}$$

Proof: See the Appendix. ||

Recall that locational equilibrium requires the equilibrium land rent schedules to satisfy (14). In analogy to the monocentric case, the household bid rent function in the outskirts [-1, $-f_2$] and [f_2, 1] is characterized by a linear function with a slope of t in absolute value. Since firms and households are completely mixed in the central cluster [$-f_1$, f_1], their bid rent must be identical, as given by (15). As shown in the Appendix, locational equilibrium implies: (i) $\max_{z\in[-f_1,f_1]}|w'(z)| = |w'(z)|_{z=f_1} \le t$ and (ii) $|R_F'(f_2)| \ge |R_C'(f_2)| = t$. The former is similar to the completely mixed case, guaranteeing no household in the central cluster desires to work away from home. The latter is parallel to the monocentric case, ensuring no firm has an incentive to locate in the outskirts and no households wants to reside in [$-f_2$, $-f_1$] $\cup$ [f_1, f_2]. From (15) and the fact that $|R'(z)| = |w'(z)|$, conditions (i) and (ii) can be combined into: $|R_F'(f_1)| \le t \le |R_F'(f_2)|$, which holds under Condition I. Of course, from the household bid rent, we have $|R_C'(x)| = t$, so households are indifferent about residing at any location x $\in$ [-1 , $-f_2$] $\cup$ [f_2, 1]. Figure 7 plots the land rent schedules in the incompletely mixed city and illustrates the above arguments.

Figure 7: Equilibrium Bid Rent Schedules - Incompletely Mixed

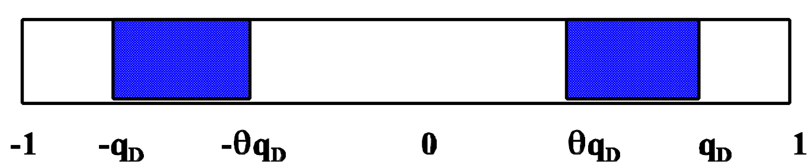


This proposition states that when the unit commuting cost and the penalty for firm dispersion are moderate, the urban configuration is incompletely mixed - it is neither completely dispersed (as in the completely mixed case) nor completely concentrated (as in the monocentric case).

4.4. Can the Duocentric Urban Configuration be an Equilibrium Outcome?

In the end, we would like to ask if there exists any ***multi-centric*** urban configuration, such as the ***duocentric*** city (denoted τ = D) in which firms are divided into two disconnected clusters $[-q_D, -\theta q_D]$ and $[\theta q_D, q_D]$ while households reside either around the center $[-\theta q_D, \theta q_D]$ or in the outskirts $[-1, -q_D]$ and $[q_D, 1]$, where $0 < \theta < 1$ and $0 < q_D < 1$ (see Figure 8).

Figure 8: Duocentric Urban Configuration



Under the duocentric urban configuration, there are two subgroups of firms - the left cluster and the right cluster. Our overall dispersion index is decomposable in firm clusters and hence still appropriate in this case.[13] Straightforward calculation shows that $m(z) = M/[2(1-\theta)q_D]$ for all $z \in [-q_D, -\theta q_D] \cup [\theta q_D, q_D]$ and thus $\sigma_D = (1+\theta)q_D$. We can therefore measure the degree of effectiveness of interactions, Q_D, as:

$$Q_D = \left[2 - z^2 - \varepsilon(2v)^2\right] \tag{18}$$

where $v = [(1+\theta)q_D]/2$ represents the distance of the within-the-cluster mean location from the global mean location (0).

In spatial equilibrium, we have:

$$R(z) = R_C(z) > R_F(z) \qquad \forall\ z \in (-\theta q_D\ ,\ \theta q_D)$$

$$R(z) = R_F(z) > R_C(z) \qquad \forall\ z \in (-q_D\ ,\ -\theta q_D) \cup (\theta q_D\ ,\ q_D) \tag{19}$$

$$R(z) = R_C(z) > R_F(z) \qquad \forall\ z \in [-1\ ,\ -q_D) \cup (q_D\ ,\ 1]$$

and $R_F(-q_D) = R_C(-q_D) = R_F(q_D) = R_C(q_D)$ and $R_F(-\theta q_D) = R_C(-\theta q_D) = R_F(\theta q_D) = R_C(\theta q_D)$. Thus, there is a crucial difference between the incompletely mixed and the duocentric urban configurations: in the latter case, the bid rent for firms in $(-\theta q_D , \theta q_D)$ is ***strictly less*** than that for households and hence the equilibrium land rent equals the household bid rent within this central cluster where no firm locates.

Proposition 4: *In competitive spatial equilibrium, the duocentric symmetric urban configuration cannot emerge.*

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[13] We would like to point out that this is a new application of index numbers to the urban economic context. Specifically, we regard firm clusters as subgroups and define decomposability according.

Proof: See the Appendix. ||

Importantly, under our knowledge spillover setup, a firm's production penalty for distance from the average location of firms is strictly increasing and strictly convex, implying that it is disadvantageous for firms to be separated spatially into different clusters. Given such a duocentric configuration, a firm will always move to the average location of firms; the penalty, land rents, and wages are all lower there. As a consequence, a duocentric city in which firms are grouped into two disconnected clusters cannot be an equilibrium outcome. By similar arguments, any multi-centric urban configuration can be ruled out as a spatial equilibrium configuration. It is important to note that the proof (by contradiction) of Proposition 4 relies on mutually contradictory slope conditions. This argument remains valid with a linear or strictly concave penalty. Thus, multi-centric urban configurations can never arise in equilibrium in a model with Jacobs-Romer production externalities, regardless of the concavity/convexity property of the Q function.[14] The multi-centric equilibrium configurations generated by Fujita and Ogawa (1982) appear to be a consequence of the separable form of the spatial interaction function rather than its convexity/concavity properties. Specifically, the properties of the function that compares firm locations pairwise rather than its integral seem important. Here we have explicitly assumed a non-separable form of penalty.

5. Further Discussion

In the previous section, we determine endogenously the underlying urban configuration, depending crucially on the spatial primitives of the model. Utilizing Propositions 1-3, we obtain:

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[14] When Q is linear, $w'(\theta q_D) > 0$ implies $R'_F(\theta q_D) < 0$; when Q is strictly convex (when the penalty is strictly concave), $w'(q_D) < 0$ implies $R'_F(q_D) > 0$. Either case leads to a contradiction.

Theorem 1: (Existence) *For any commuting cost and dispersion penalty parameters, there is a competitive spatial equilibrium.*

Then, the result in Proposition 4, in conjunction with Propositions 1-3, enables us to conclude:

Theorem 2: (Uniqueness) *Almost surely in commuting cost and dispersion penalty parameters, there is a unique competitive spatial equilibrium associated with a symmetric urban configuration which is completely mixed, monocentric or incompletely mixed.*

Proposition 5: (Characterization of the Urban Configuration) *The almost surely unique competitive spatial equilibrium possesses the following properties:*

(i) *a completely mixed symmetric urban configuration emerges when the unit commuting cost is sufficiently high, whereas a monocentric urban configuration arises when such a cost is sufficiently low;*

(ii) *a sufficiently large knowledge-spillover penalty on the overall dispersion of firms causes the formation of a monocentric symmetric urban configuration, but a completely mixed symmetric configuration disappears;*

(iii) *a sufficiently large population mass of firms induces a completely mixed symmetric urban configuration.*

The *uniqueness* of spatial equilibrium and the fact that *no* multi-centric urban configuration is an equilibrium contrast with the existing literature, such as the locational potential function framework of Fujita and Ogawa (1982), the more recent sequels using models of product differentiation [e.g., see Fujita and Krugman (1997)] and spatial competition models [e.g., see Fujita and Thisse (1986)]. Our uniqueness property is primarily due to the strong agglomerative force from knowledge spillovers among firms. Of course, if $t = B_1(\varepsilon, N)$, the completely mixed and the incompletely mixed urban configurations can co-exist; if $t = B_2(\varepsilon, N)$, the monocentric and the incompletely mixed urban configurations can co-exist. These knife-edge cases, however, require specific combinations of some exogenous parameters, which have zero measure in the entire parameter space. Therefore, our uniqueness property holds almost surely. This sharp contrast with Fujita and Ogawa (1982) is mainly due to the nonseparability of our Q function and the fact that the external factor is allowed to affect the factor proportion under our framework.

Interestingly, we show that the Romer-type production externalities with a strictly convex penalty for firm dispersion and distance from the average firm location are sufficient to lead to the formation of a city in which firms are *always clustered together* in any spatial equilibrium. This eliminates the possibility of any multi-centric urban configuration, suggesting that when positive knowledge spillovers are the primary agglomerative forces, multi-centric cities cannot emerge under perfect competition.[15]

Parts (i) and (ii) of Proposition 5 are straightforward (as discussed in Section 4 above), corroborating with findings in Ogawa and Fujita (1980).[16] Part (iii) of Proposition 5 deserves further comment. Notice that each household or firm occupies a unity density of land and the masses of households and firms (N and M, respectively) sum to two. Thus, the result is derived via the effect of N on $B_1(\varepsilon, N)$ specified in Condition C. For a sufficiently large mass of firms, the relative commuting costs are low even if all households reside in the outskirts of the city. Hence, commuting is likely to occur in equilibrium and the completely mixed urban configuration is unlikely to arise as an equilibrium outcome.

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[15] Of course, by incorporating matching externalities [e.g., see Abdel-Rahman and Wang (1997)], it is possible to generate a multi-centric urban structure.

[16] This is by treating our penalty parameter ε the same as the transactions cost parameter τ in Ogawa and Fujita (1980).

6. Concluding Remarks

Based on the Romer-type production externality, our paper has developed a general equilibrium framework under which the unique equilibrium urban configuration (completely mixed, monocentric or incompletely mixed) is determined analytically, depending on the population of firms, the commuting cost and the firms' knowledge spillover parameters. We show that the incorporation of distance-dependent production externalities is sufficient to ensure that firms are always clustered together in any competitive spatial equilibrium, ruling out the possibility of multi-centric urban configurations, in contrast with findings in Fujita and Ogawa (1982) and in more recent sequels employing models of product differentiation or spatial competition.

Along these lines, there are a few straightforward extensions. First, one may relax the assumptions of a fixed supply of labor and fixed demands for land. Second, one can investigate the usefulness of index number theory for measuring dispersedness of firms and quantifying the externality. Third, one may revisit our work using a discrete population model *á la* Berliant and Fujita (1992). The main purpose of these exercises is to check the robustness of the absence of multi-centric urban configuration and the uniqueness of competitive spatial equilibrium. Of course, they are accomplished at the expense of increased complexity, making analytical results less likely.

Moreover, it may be of interest to examine the welfare properties of competitive spatial equilibrium. In particular, the presence of uncompensated knowledge spillovers may lead to a sub-optimal equilibrium - in equilibrium firms fail to account for the positive production externality and thus under-invest compared to the optimum. An intriguing question is whether such inefficiencies are lower under one urban configuration relative to others. Furthermore, one may add an externality to the consumer utility via local congestion or neighborhood effects. In the former case, a more dense population has a negative influence on household utility, which serves as an additional force for dispersion. In the latter case, the distance-dependent positive externality makes the incompletely mixed urban configuration (in which households are not clustered together) less likely to emerge. Finally, our findings provide empirically testable hypotheses regarding (i) the shape of the rent density, (ii) the locations of firms and consumers, and (iii) comparative statics describing the dependence of the urban configuration (measured as a discrete variable) on commuting, population and production-spillover parameters. Those regarding production spillovers may also be compared with the empirical localization externality measured by Rosenthal and Strange (1998). These extensions are left to future work.

Appendix

Proof of Proposition 1:

Recall that $\mu = 0$ and $\sigma_C = 1$. Since $R_F(1) = R_C(1) = 1$, pick $z_C = 1$ as the reference point. From (8), we have $\Lambda_C(1) = w(1)^{\beta/(1-\alpha-\beta)}/(1-\varepsilon)$, which can be substituted back into (8) and (9) to yield:

$$R(z) = R_F(z) = \frac{2-z^2-\varepsilon}{1-\varepsilon}\left[\frac{w(1)}{w(z)}\right]^{\frac{\beta}{1-\alpha-\beta}} \tag{A1}$$

$$\frac{R_F(0)}{R_F(1)} = R(0) = \frac{2-\varepsilon}{1-\varepsilon}\left[\frac{w(1)}{w(0)}\right]^{\frac{\beta}{1-\alpha-\beta}} \tag{A2}$$

Since x = z under this configuration, (6) implies $R(1) = w(1) - I_0 = 1$ and thus,

$$w(1) = 1 + I_0 \tag{A3}$$

Similarly, we have:

$$R(0) = w(0) - I_0 \tag{A4}$$

Substitution of (6) with x = z into (A1) yields (10). Next, labor market clearance under the completely mixed urban configuration implies: L = N/2, which can be combined with (4), (7) and (9) to generate:

$$\frac{N}{2} = \frac{\beta}{1-\alpha-\beta}\frac{R(0)}{w(0)} \tag{A5}$$

Substituting (A4) into (A5) gives:

$$w(0) = \frac{I_0}{1 - \frac{1-\alpha-\beta}{\beta}\frac{N}{2}} \tag{A6}$$

or, equivalently, (11). In order for w(0) > 0, we need:

$$\frac{1-\alpha-\beta}{\beta}\frac{N}{2} < 1 \tag{A7}$$

(A6) together with (A2)-(A4) can be used to solve for w(0), w(1), R(0) and I_0. By tedious manipulation, the system reduces to a single equation in w(0), (12). The solution exists under the condition specified in (A7) and is unique. Once the solution of w(0) is obtained, one can get w(1), R(0) and I_0, and, from $R(z) = w(z) - I_0$ and (A1), the equilibrium wage and land rent schedules {w(z)} and {R(z)} are pinned down. Straightforward comparative statics show that an increase in ε or a decrease in N rasies both w(0) and I_0(ε, N). Also from $R(z) = w(z) - I_0$, $R'(z) = w'(z)$ and hence the equilibrium wage and rent schedules have identical slope at every point z with w(z) uniformly above R(z) by a constant I_0. Tedious but straightforward differentiation implies:

$$\frac{dR}{dz} = \frac{\frac{-2z}{1-\varepsilon}(1+I_0)^{\frac{\beta}{1-\alpha-\beta}}[R(z)+I_0]^{1-\frac{\beta}{1-\alpha-\beta}}}{\{\frac{1-\alpha}{1-\alpha-\beta}R(z)+I_0\}}$$

In order to support the completely mixed urban configuration, it is required that no household has incentive to work away from home because by working for a firm closer to the city center, the incremental gain from a higher wage is dominated by the induced cost of commuting. This is guaranteed by:

$$|w'(z)| = |R'(z)| \le |R'(z)|_{z=1} = \frac{2}{1-\varepsilon}\frac{1}{\frac{\beta}{1-\alpha-\beta}\frac{1}{1+I_0(\varepsilon,N)}+1} \le t$$

The above arguments together with (6) ensure a locational equilibrium choice of workplace and residence under the completely mixed urban configuration. Q.E.D.

Proof of Proposition 2:

Given the properties of the monocentric urban configuration, we can follow arguments similar to those in the Proof of Proposition 1 to use (6) and (8) to obtain:

$$w(q) = 1 + I_0 + t(1-q); \quad w(0) = 1 + I_0 + t \tag{A8}$$

$$R_C(q) = R_C(1) + t(1-q) = 1 + t(1-q) \tag{A9}$$

$$R_F(z) = \frac{2 - z^2 - \varepsilon q^2}{[1 + I_0 + t(1-z)]^{\frac{\beta}{1-\alpha-\beta}}} \Lambda_M(q) \tag{A10}$$

where $\Lambda_M(q) = \frac{1+t(1-q)}{2-(1+\varepsilon)q^2}[1+I_0+t(1-q)]^{\frac{\beta}{1-\alpha-\beta}}$ and (A8) is derived from equating the net income of households residing at x = q or 0 and x = 1 (the urban fringe). Differentiating (A10) at z = q implies that $R_F^{/}(q) < 0$ if (but not only if) $N \le 2\left(1-\sqrt{\frac{2}{3+\varepsilon}}\right)$. Labor market clearance implies:

$\int_0^q L(z)\,dz = N/2$ and for the case of L(z) = L, we have L = (N/2)/(1-N/2). This can be substituted into (4) and (7) (at locations 0 and q), in conjunction with $R_F(q) = R_C(q)$ and (A9) to generate:

$$\frac{N/2}{1-N/2} = \frac{\beta}{1-\alpha-\beta}\frac{R_F(0)}{1+I_0+t} = \frac{\beta}{1-\alpha-\beta}\frac{1+t(1-q)}{1+I_0+t(1-q)} \tag{A11}$$

Substituting (A11) into (A10) gives (13). Utilizing q = 1-N/2 and manipulating (A11) to eliminate I_0, we obtain:

$$R_F(0) = 1+\frac{1}{2}tN+\frac{1-\alpha-\beta}{\beta}\frac{N}{2}t \tag{A12}$$

In order for $R_F(0) > R_C(0) = 1 + t$, we need: $\frac{2\beta}{1-\alpha} < N$. Condition M implies $|R_F^{/}(z)|_{z=q} \geq |R_C^{/}(x)|_{x=q} = t$. These together guarantee that $R_F(z) \geq R_C(z)$ $\forall$ z $\in$ [-q, q] and $I(x,z) = I_0$ $\forall\ x \in [-1, -q] \cup [q, 1]$ and $z \in [-q, q]$, which ensure locational equilibrium in this urban configuration. Q.E.D.

Proof of Proposition 3:

Following arguments similar to those in the Proof of Proposition 2, we manipulate (6) and (8) using the properties of this urban configuration to obtain:

$$w(z) = 1 + I_0 + t(1-z) \quad \forall\ z \in [f_1, f_2] \tag{A13}$$

$$R_C(z) = 1 + t(1-z) \quad \forall\ z \in [f_1, 1] \tag{A14}$$

$$R_F(z) = \frac{2 - z^2 - \varepsilon f_2^2}{[1 + I_0 + t(1-z)]^{\frac{\beta}{1-\alpha-\beta}}} \Lambda_I(f_2) \quad \forall\ z \in [f_1, f_2] \tag{A15}$$

and (15) $\forall$ z $\in$ [0, f_2], where $\Lambda_I(f_2) = \frac{1 + t(1-f_2)}{2-(1+\varepsilon)f_2^2} [1 + I_0 + t(1-f_2)]^{\frac{\beta}{1-\alpha-\beta}}$. Differentiating (A15) gives:

$$R'_F(z) = -R_F(z)\left[\frac{2z}{2-z^2-\varepsilon f_2^2} - \frac{\beta}{1-\alpha-\beta}\frac{t}{1+I_0+t(1-z)}\right] \tag{A16}$$

Since all households residing at x $\in$ [f_2, 1] commute to work with firms at z $\in$ [f_1, f_2], labor market clearance implies: $\int_{f_1}^{f_2} L(z)\,dz = (1-f_2)N/2$; thus, $\int_0^{f_1} L(z)\,dz = f_2 N/2$. Substituting (4), (6), (7), (14) and (A13) into these market clearing conditions, we obtain (16) and (17). Utilizing (A15), we can combine (16) and (17) with $R_F(f_1) = R_C(f_1) = 1 + t(1-f_1)$ to solve for the equilibrium values of f_1, f_2 and I_0. In order to support this urban configuration, it is required that in spatial equilibrium, (i) no household in the central cluster desires to work away from home (a condition similar to that in the completely mixed case) -

$$\max_{z \in [-f_1, f_1]} |w'(z)| = |w'(z)|_{z=f_1} \leq t \tag{A17}$$

and (ii) no firm has incentive to locate in the outskirts and no household wants to reside in $[-f_2, -f_1] \cup [f_1, f_2]$ (a condition that appeared in the monocentric case) -

$$|R_F'(f_2)| \geq |R_C'(f_2)| = t \tag{A18}$$

Utilizing (14) and the fact that $|R'(z)| = |w'(z)|$, we can combine (A17) and (A18) into:

$$|R_F'(f_1)| \leq t \leq |R_F'(f_2)| \tag{A19}$$

Substitution of (A16) into (A19) yields:

$$G(f_1; \bullet) \leq t \leq G(f_2; \bullet) \tag{A20}$$

where $G(f_i; \bullet) = [1 + t(1-f_i)]\left[\dfrac{2f_i}{2 - f_i^2 - \varepsilon f_2^2} - \dfrac{\beta}{1-\alpha-\beta}\dfrac{t}{1+I_0+t(1-f_i)}\right]$. As both f_1 and f_2 approach unity, $G(f_1) = B_1$ and (A20) reduces to Condition C; when f_1 approaches zero, $G(f_2) = B_2$ and (A20) reduces to Condition M where we set $f_2 = q$. Under Condition I, (A20) holds for any $0 < f_1 < f_2 < 1$, thus ensuring the incompletely mixed urban configuration in spatial equilibrium. Q.E.D.

Proof of Proposition 4:

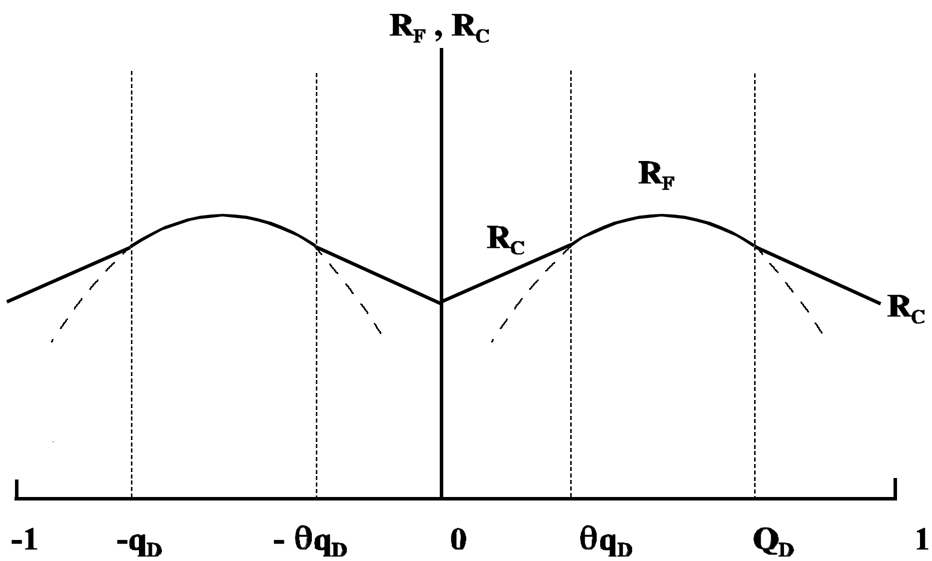
Equations (6), (8) and (18), together with the boundary condition, yield the bid rent functions for firms:

$$R_F(z) = \frac{2 - z^2 - \varepsilon(2\nu)^2}{w(z)^{\beta/(1-\alpha-\beta)}}\Lambda_D \tag{A21}$$

where $\Lambda_D(q_D) = \dfrac{1+t[1-2\nu/(1+\theta)]}{2-[1-2\nu/(1+\theta)]^2-\varepsilon(2\nu)^2}\left[1+t(1-\dfrac{2\nu}{1+\theta})+I_0\right]^{\frac{\beta}{1-\alpha-\beta}}$. Differentiating (A21) leads to:

$$R'_F(z) = -\frac{\Lambda_D}{w(z)^{(1-\alpha)/(1-\alpha-\beta)}}\left\{2z\,w(z)+\frac{\beta}{1-\alpha-\beta}\,w'(z)[2-z^2-\varepsilon(2\nu)^2]\right\} \tag{A22}$$

Suppose that equilibrium with the duocentric urban configuration exists. Then, the land rent schedules satisfying the properties in (19) can be characterized by the diagram below:



That is, the following slope conditions must be satisfied:

$$R'_F(\theta q_D) > R'_C(\theta q_D) = w'(\theta q_D) > 0 \tag{A23}$$

$$R'_F(q_D) < R'_C(q_D) = w'(q_D) < 0 \tag{A24}$$

However, (A22) and the last inequality of (A23) together imply: $R_F(\theta q_D) < 0$, contradicting to the first inequality of (A23). Thus, the duocentric urban configuration cannot emerge in equilibrium. Q.E.D.

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