A NONPARAMETRIC MEASURE OF CONVERGENCE TOWARD PURCHASING POWER PARITY

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A Nonparametric Measure of Convergence Toward Purchasing Power Parity*

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Abstract

It has been claimed that the deviations from purchasing power parity are highly persistent and have quite long half-lives under the assumption of a linear adjustment of real exchange rates. However, inspired by trade cost models, nonlinear adjustment has been widely employed in recent empirical studies. This paper proposes a simple nonparametric procedure to evaluate the speed of adjustment in the presence of nonlinearity, using the largest Lyapunov exponent of the time series. The empirical result suggests that the speed of convergence to a long-run price level is indeed faster than what was found in previous studies with linear restrictions.

Keywords: Mean reversion; Nonlinear time series; Nonparametric regression; Purchasing power parity puzzle; Real exchange rates.

JEL classification: F31; F41

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1 Introduction

Since Rogoff's (1996) observation on the volatile yet extremely persistent real exchange rate, the mean reversion to long-run purchasing power parity (PPP) has attracted considerable attention from researchers. To measure persistence, the half-life of deviations from PPP has been frequently employed as a quantity of interest. Using the empirical evidence provided by Frankel (1986), Diebold, Husted and Rush (1991) and Lothian and Taylor (1996), Rogoff claimed the consensus of three to five-year half-lives of deviations. However, as recently pointed out by Taylor (2001), nonlinearity might possibly be a source of large half-life estimates, since it could cause upward bias if a linear model were incorrectly employed in the estimation.

The nonlinear adjustment of deviations from PPP can mainly be justified by the presence of trading costs, including transportation costs, insurance costs, information costs and tariffs and non-tariff barriers. As emphasized by Obstfeld and Rogoff (2000), trade costs most likely play a central role in the persistence of international price differentials, as well as in many other empirical puzzles in international macroeconomics. For this reason, estimating nonlinear time-series models has become a very popular approach among the recent studies on the real exchange rates adjustment dynamics (e.g., Michael, Nobay and Peel, 1997, Obstfeld and Taylor, 1997, O'Connell, 1998, Sarantis, 1999, Taylor and Peel, 2000, Baum, Barkoulas and Caglayan, 2001, and Taylor, Peel and Sarno, 2001). One difficulty regarding the nonlinear models is that, unlike the traditional linear approach, the interpretation of results in terms of the persistency of PPP deviations is not straightforward, since the rate of adjustment becomes slower as deviations become smaller by construction. One may report the shape of nonlinear impulse response functions to investigate the difference between linear and nonlinear models. However, such nonlinear impulse response functions can be defined in several different ways (see Gallant, Rossi and Tauchen, 1993, Koop, Pesaran and Potter, 1996, and Potter, 2000), and their evaluation usually relies on simulation methods.

The purpose of this paper is to propose a simple persistency measure of PPP deviations based on the largest Lyapunov exponent of the nonlinear adjustment process of real exchange rates. While this measure is certainly not a unique measure of convergence, there seem to be some advantages in PPP applications. First, estimation of the measure is straightforward and no simulation method is required. Since this approach does not rely on computer-intensive methods, the measure can be obtained even if the sample size is very large. Second, it is similar to a half-life measure in the sense that it can be interpreted as the half-lives of the locally linearized nonlinear processes. By definition, it corresponds to an exact half-life concept if the true process is linear. This measure is therefore convenient for assessing the effect of nonlinearity in comparison with the previous results of linear half-lives of PPP deviations available in the PPP literature. Third, the measure is estimated using the nonparametric regression technique without specifying the parametric functional form. In consequence, the method is robust to very general nonlinearity in the adjustment process.

The remainder of the paper is organized as follows: Section 2 introduces the nonlinear adjustment process of real exchange rates and proposes a nonparametric convergence measure. The finite sample properties of the proposed measure are also investigated by a Monte Carlo simulation. In Section 3, the proposed measure is applied to two different data sets, the annual historical exchange rate series originally constructed by Lee (1976) and quarterly series during the current float. Comparison of the results with those based on a conventional linear measure is also provided. Finally, some concluding remarks are made in Section 4.

2 Linear and Nonlinear Adjustments Toward PPP

2.1 Motivation

Let q_t be the (log of the) real exchange rate series defined by

$$q_t = s_t + p_t^* - p_t \quad (1)$$

where s_t, p_t^*, and p_t are the (log of the) nominal exchange rate, the (log of the) foreign price level and the (log of the) domestic price level, respectively. The conventional approach in investigating the speed of convergence to PPP is to employ the following linear autoregressive model of order one (AR(1)),

$$q_t = \rho q_{t-1} + \varepsilon_t \quad (2)$$

where $0 < |\rho| < 1$ and ε_t is a white noise.[1] For annual data, the half-life of deviations from PPP (τ) is the number of years required for the initial deviation from the long-run level to dissipate by half under zero future shocks. Suppose the long-run PPP level ($E[q_t] = 0$) as the starting point q_0 with an initial shock $\delta > 0$. From $\delta/2 = |q_\tau| = |\rho|^\tau \delta$, the half-life is then given by $\tau \equiv \ln(1/2)/\ln|\rho|$ where absolute value is introduced to allow oscillation.[2] Since the denominator

[1] For absolute prices, long-run PPP (or the law of one price) implies that the mean of the process is zero. While a non-zero mean is allowed for price indexes, the constant term is excluded in the AR(1) model for illustrative purposes.

[2] In PPP applications, the estimated AR(1) coefficients are almost always positive, suggesting no need for this absolute value transformation.

$\ln|\rho|(\approx |\rho| - 1 = |q_t/q_{t-1}| - 1$ for a small value) can be interpreted as the speed of adjustment (in absolute value), τ becomes greater than unity only if the speed of adjustment is slower than that of the AR(1) model with $\rho = 0.5$. As ρ approaches unity, the speed of adjustment $\ln|\rho|$ approaches zero from the left, and half-life τ approaches infinity, implying the absence of convergence toward PPP. In practice, the half-lives are estimated by

$$\widehat{\tau} = \frac{\ln(1/2)}{\ln|\widehat{\rho}|} \tag{3}$$

where $\widehat{\rho}$ is an OLS estimator of ρ in (2).

By construction, the speed of adjustment, as well as the half-life, does not depend on the initial level of real exchange rate (q_0) or the size of deviations (δ) in the linear AR(1) model. This feature can graphically be seen by the shape of the impulse response function in Figure 1. The time needed for the initial deviation δ to become $\delta/2$ (τ) is identical to the time for $\delta/2$ to become $\delta/4$ (τ'). However, because arbitrage for each good depends on the relative size of international price differentials and trade costs, the speed of adjustment is likely to be slower when the deviation from PPP is smaller.[3]

Theoretical models of exchange rates with trade costs have been developed by many researchers, including Dumas (1992), Sercu, Uppal and Van Hulle (1995) and Betts and Kehoe (1999). For example, in an extreme case with a single good, the dynamics of real exchange rates can be described

[3] Assumption of a constant speed of adjustment is still appropriate in many other applications. For example, in nuclear physics, half-life is often used to characterize radioactive materials. Since the probability of decay of an atom is constant, the proportion of survived nuclei in a fixed period of time is constant. Therefore the half-life does not depend on the total number of initial nuclei.

by the following threshold autoregressive (TAR) model:

$$q_t = \begin{cases} c + \rho(q_{t-1} - c) + \varepsilon_t & \text{if } q_{t-1} > c; \\ q_{t-1} + \varepsilon_t & \text{if } -c \le q_{t-1} \le c; \\ -c + \rho(q_{t-1} + c) + \varepsilon_t & \text{if } q_{t-1} < -c; \end{cases} \tag{4}$$

where $0 < \rho < 1$. The threshold parameter c can be interpreted as transaction cost in a simple "iceberg" model (Sercu, Uppal and Van Hulle, 1995), and the model implies the fast price adjustment outside the band because of the arbitrage opportunities in trading the goods.[4] Such a TAR model has been estimated by Obstfeld and Taylor (1997) and O'Connell (1998) and has been used in Taylor (2001) to illustrate the problem of misspecification with the linear half-life measure. While the idea of a discrete threshold is appealing for the analysis of deviations from the law of one price, the aggregation of many goods with different costs of arbitrage generally suggests gradual change in the speed of adjustment for PPP deviations. For this reason, a class of smooth transition autoregressive (STAR) models has been popularly employed in recent studies, including Michael, Nobay and Peel (1997), Sarantis (1999), Taylor and Peel (2000), Baum, Barkoulas and Caglayan (2001), and Taylor, Peel and Sarno (2001).

In general, smooth nonlinear adjustment of the real exchange rates can be described by the following nonlinear AR(1) model:

$$q_t = m(q_{t-1}) + \varepsilon_t \tag{5}$$

where $m(q_{t-1})$ is a nonlinear conditional mean function. However, it is well-known that the shape of

[4] Here we are excluding the possibility of instantaneous arbitrage or $\rho = 0$.

nonlinear impulse response function depends on the initial conditioning variable q_0 (or the history, in the case of higher-order AR models). In addition to q_0, the shape of the impulse response also depends on the size and the sign of current and future shocks. For example, when $q_0 = 0$, the impulse response shown in Figure 2 implies that it takes more time for $\delta/2$ to dissipate by half (τ') compared to the time from δ to $\delta/2$ (τ). This fact makes interpretation of persistency or defining nonlinear half-life complicated. Gallant, Rossi and Tauchen (1993) have proposed a (j-period) nonlinear impulse response based on the difference between the (j-step ahead) expectation conditioned on q_0 and an expectation conditioned on $q_0+\delta$ that can be evaluated by a combination of nonparametric conditional density estimation and Monte Carlo integration. However, each impulse response still depends on q_0 and δ. Taylor and Peel (2000) and Taylor, Peel and Sarno (2001) employed this definition of nonlinear impulse response and reported half-lives of their estimated STAR model for several different δ's but for a particular q_0. Koop, Pesaran and Potter (1996) and Potter (2000) proposed another definition of nonlinear impulse response which can be interpreted as a generalization of Gallant, Rossi and Tauchen's definition. In either case, computation involves the Monte Carlo integration method. In addition, consideration of sampling variability or estimation error makes evaluation of the nonlinear impulse responses or the half-lives even more difficult. Below, we consider an alternative measure of persistency based on the largest Lyapunov exponent of the time series.[5]

[5] Potter (2000, footnote 10) also mentioned using the Lyapunov exponent as a possible alternative to his generalized impulse response function.

2.2 Lyapunov Exponent of Nonlinear Time Series

The largest Lyapunov exponent is a measure of stability of a dynamic system in terms of the sensitive dependence on initial conditions. For the nonlinear AR(1) model (5), the Lyapunov exponent is defined by

$$\lambda \equiv \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} \ln |Dm(q_{t-1})| \tag{6}$$

where $Dm(q_{t-1})$ is the first derivative of the conditional mean function. Suppose two different initial conditions q_0 and q_0' with small difference δ ($q_0' = q_0 + \delta$). Then, λ is the average growth rate of difference between two trajectories $\{q_t\}_{t=0}^{\infty}$ and $\{q_t'\}_{t=0}^{\infty}$. The Lyapunov exponent is often used to define a chaotic system because two trajectories diverge for such a system. On the other hand, for a stable system with a steady state, the Lyapunov exponent can be interpreted as an average rate of convergence. Since the denominator of the linear half-life τ is the speed of convergence $\ln|\rho|$, we can construct an analogous measure of persistency for a stable nonlinear system by replacing $\ln|\rho|$ with λ, or $\tau^* \equiv \ln(1/2)/\lambda$. It should be noted that τ^* is identical to τ under the linearity assumption, since $Dm(q_{t-1}) = \rho$ for all t. As an example, let us consider the TAR model (4). Since the model implies $Dm(q_{t-1}) = \rho$ outside the band and $Dm(q_{t-1}) = 1$ inside the band, λ is the average of $\ln\rho$ and $0(= \ln 1)$ weighted by the time spent in each region (which depends on c and ε_t). If the probabilities of q_t being in two regions are equal, then τ^* for the TAR model will be twice of $\tau^*(= \tau)$ for the linear AR model with the same ρ in terms of this particular measure of persistency.[6]

[6] Taylor (2001) employed a different concept of convergence for the TAR model. He defines a half-life of the model as $\ln(1/2)/\ln|\rho|$ regardless of the size of c.

In general, the nonlinear AR model (5) can be estimated by using the nonparametric regression technique without the specification of the functional form as in the TAR or STAR models. To estimate the Lyapunov exponent from data, Nychka, Ellner, Gallant and McCaffrey (1992) have proposed a sample analogue estimator based on the nonparametric method. Following this idea, we estimate τ^* by

$$\widehat{\tau}^* = \frac{\ln(1/2)}{T^{-1}\sum_{t=1}^{T}\ln\left|\widehat{Dm}(q_{t-1})\right|} \tag{7}$$

where $\widehat{Dm}(q_{t-1})$ is a nonparametric estimator of the first derivative of $m(q_{t-1})$ in (5) and T is the sample size. This measure of convergence is not an exact half-life since half-life can be defined in many different ways, similarly to the nonlinear impulse responses discussed in the previous subsection. However, it is a half-life-like measure in the sense that it can be interpreted as the half-lives of the locally linearized nonlinear processes. Suppose a first-order Taylor series expansion of (5) at the initial point, or $q_1' - q_1 = Dm(q_0)(q_0' - q_0) = Dm(q_0) \times \delta$. By repeating the expansion at each local point and using absolute values, we have

$$\delta/2 = |q'_{\tau^*} - q_{\tau^*}| = |Dm(q_{\tau^*-1})| \times \left|q'_{\tau^*-1} - q_{\tau^*-1}\right| = \cdots = \prod_{t=1}^{\tau^*} |Dm(q_{t-1})|\,\delta,$$

and $\tau^{*-1}\ln(1/2) = \tau^{*-1}\sum_{t=1}^{\tau^*}\ln|Dm(q_{t-1})|$ implies $\tau^* = \ln(1/2)/\left(\tau^{*-1}\sum_{t=1}^{\tau^*}\ln|Dm(q_{t-1})|\right)$. For this reason, we simply call $\widehat{\tau}^*$ as a nonparametric half-life measure despite the fact that it is not the half-life for a certain δ and q_0.

In principle, any nonparametric estimator can be used for the derivative estimation. In this paper, we employ a class of kernel-type regression estimators called a local polynomial regression estimator (see Fan and Gijbels, 1996). There are several advantages of local polynomial regression over the Nadaraya-Watson regression estimator. First, it reduces the bias of the Nadaraya-Watson estimator. Second, it adapts automatically to the boundary of design points and no boundary modification is therefore needed. Third, and most importantly for our purpose, it is superior to the Nadaraya-Watson estimator in the context of derivative estimation. As explained in Fan and Gijbels (1996, p.77), the local polynomial of order two, or local quadratic smoother, is preferable for first derivative estimation for the same reasons.

The local quadratic estimators at point x can be obtained by minimizing the weighted least squares criterion $\sum_{t=1}^{T}\left(q_t - \beta_0 - \beta_1(q_{t-1} - x) - \beta_2(q_{t-1} - x)^2\right)^2 K_h\left(q_{t-1} - x\right)$ where $K_h(u) = K(u/h)/h$, $K(u)$ is a kernel function and h is a smoothing parameter (or bandwidth). The first derivative estimator $\widehat{Dm}(x)$ is given by the second element of the solution

$$\widehat{\beta} = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{y},$$

where

$$\mathbf{X} = \begin{bmatrix} 1 & (q_0 - x) & (q_0 - x)^2 \\ \vdots & \vdots & \vdots \\ 1 & (q_{T-1} - x) & (q_{T-1} - x)^2 \end{bmatrix},$$

$\mathbf{y} = (q_1, \ldots, q_T)'$ and $\mathbf{W}$ =diag$\{K_h(q_0 - x), \ldots, K_h(q_{T-1} - x)\}$. We compute the first derivatives at each data point q_{t-1} and substitute into (7) to obtain $\widehat{\tau}^*$.

It is now common practice to report the confidence intervals for $\widehat{\tau}$ in the linear model to consider sampling variability. For example, to evaluate the precision of the half-life, Cheung and Lai (2000) reported both asymptotic and bootstrap confidence intervals for $\hat{\tau}$ while Kilian and Zha (2002) used Bayesian confidence intervals. In the empirical section, we also report the confidence interval for the nonparametric measure $\hat{\tau}^*$ based on the asymptotic distribution of the local quadratic estimator of the Lyapunov exponent derived in Shintani and Linton (2001).[7] In Murray and Papell's (2002) study, they employed a median-unbiased estimator of ρ and reported that confidence intervals of $\hat{\tau}$ included infinity in many cases, which implies some possibilities of a unit root. For a unit root process, the linear measure $\hat{\tau}$ is consistent in the sense that half-life estimates diverge to infinity as the sample size increases. In Shintani and Linton (2001), it is shown that the Lyapunov exponent based on the local quadratic regression converges to zero when the true process is a random walk, or $m(q_{t-1}) = q_{t-1}$ and an iid error in (5). This implies $\hat{\tau}^*$ is also consistent in the sense that it diverges to infinity for a unit root case.

2.3 Simulation

We conduct a small Monte Carlo simulation to investigate the finite sample performance of the proposed measure of persistence. We first consider the TAR model (4) as a true process. We generate the artificial data from (4) with $\varepsilon_t = N(0, 1)$, $\rho = 0.5$ and the varying threshold parameter c, then estimate both nonparametric and linear measures with the sample sizes $T = 100$ and 200. Each experiment is replicated 10,000 times and the performance of two measures is evaluated by the relative distance from the true half-life (τ^*) of (4). For the nonparametric measure, the Gaussian kernel function with several values of the smoothing parameter h is employed.

[7] For comparison, we report the confidence interval of the linear measure $\hat{\tau}$ based on the limit distribution of the rate of convergence $\ln|\hat{\rho}|$ instead of that of AR parameter $\hat{\rho}$ in the empirical section.

The upper half of Table 1 reports results based on the simulation with $T = 100$. First, when $c = 0$, (4) reduces to a simple linear model with AR parameter 0.5 and the true half-life is one year. As expected, since there is no misspecification with linear measure in this case, the bias of the linear measure is smaller than that of the nonparametric measure. However, the variability of both estimates is almost identical. Second, when c increases, the performance of linear measure $\widehat{\tau}$ worsens as a result of misspecification. The upward bias becomes more severe when (4) has a longer true half-life. When the true half-life is as large as 5.70, unit roots are obtained for several cases which result in infinite half-life estimates (The median is 8.62). On the other hand, the nonparametric measure $\widehat{\tau}^*$ does not suffer from upward bias observed in the linear measure. While the nonparametric estimates depend on the selection of the smoothing parameter to a certain degree, the small sample bias, as well as variability in the estimates, is found to be considerably less than those of linear measure. The lower half of Table 1 reports results with $T = 200$. When $c = 0$, both nonparametric and linear measures become closer to the true half-life compared to $T = 100$ case. When c increases, the upward bias of linear measure is larger than those for $T = 100$ case, while relatively accurate estimates are obtained with nonparametric measure with a smaller smoothing parameter.

Second, we consider the following simple STAR model:

$$q_t = q_{t-1} - q_{t-1}\left[1 - \exp\left\{-q_{t-1}{}^2\right\}\right] + \varepsilon_t \tag{8}$$

with $\varepsilon_t = N(0, \sigma^2)$. This class of model has been employed in Taylor, Peel and Sarno (2001) and others. For this model, we control the true half-life (τ^*) by varying the dispersion parameter σ. All other values for the simulation are identical to those in the TAR case. The results are presented in Table 2. In general, the results are somewhat similar to the TAR results. Nonparametric measure is closer to the true half-life than the linear measure. For large σ, the performance of the nonparametric measure is satisfactory. For small σ, nonparametric measure shows the downward bias. However, upward bias of the linear measure is larger compared to that of the nonparametric measure.

In summary, the simulation result supports Taylor's (2001) discussion that inappropriate linear specification may result in large half-live estimates if there is nonlinearity in the adjustment process. The nonparametric measure, in contrast, seems to be robust for such a nonlinearity.

3 Empirical Results

3.1 Data

In this section, the persistence of PPP deviation is reinvestigated using the nonparametric half-life measure. Two different data sets are used for the analysis. The first data set is the long-horizon annual real exchange rate series originally constructed by Lee (1976) and later extended by Murray and Papell (2002), using the sample period 1900 to 1996. Countries under consideration are Canada, France, Italy, Japan, the Netherlands and the U.K. All the series are WPI-based real exchange rates with the U.S. dollar used as the numeraire currency. The well-known caveat of using the long-horizon data is that it includes both fixed and float exchange rate periods. The second data set we consider consists of the real exchange rates under the current float period, and it presumably suffers less from the effect of the regime shift. We utilize the data used in Murray and Papell (2002) which consists of quarterly CPI-based real exchange rates of twenty countries from 1973:1 to 1998:2.[8]

3.2 Cointegrating Rank Test

As a preliminary analysis, we investigate the possibility of nonstationarity of the real exchange rates as many previous studies could not reject the unit root hypothesis. Rather than conducting the unit root test for every single series, we utilize a multivariate test, as it is invariant to the choice of the numeraire currency. More specifically, the system consists of U.S. dollar-based real exchange rates for all countries (other than the U.S.) instead of the real exchange rates for all bilateral pairs. Such an approach is employed by Taylor and Sarno (1998) with the cointegrating rank test of Johansen (1991). In addition to the commonly used Johansen likelihood ratio (LR) test, we employ a nonparametric cointegrating rank test proposed by Shintani (2001). Table 3 reports the results from both data using one lag in the estimation. If the long-run PPP holds, we should find the full cointegrating rank and no unit root. However, for the annual series, the result implies four unit roots based on the LR test and three unit roots based on the nonparametric test. It implies that the PPP is unlikely for about half the countries considered. Furthermore, for the quarterly series, the number of nonstationary elements is greater than those found in the annual data. These observations are consistent with the former studies that found unit roots in the real exchange rates. However, it should be noted that both the LR test and the nonparametric test are linear cointegrating rank tests. They are therefore not valid when the true process has a nonlinear structure.

[8]The U.S. dollar is used as a base currency. See Murray and Papell (2002) for the data construction.

3.3 Estimation of Two Convergence Measures

Let us now turn to the nonparametric half-life estimation of the real exchange rates. For the kernel function $K(u)$ required in the estimation of the nonlinear autoregressive model, the Gaussian kernel is employed. The smoothing parameter h is selected by minimizing the residual squares criterion (RSC) given in Fan and Gijbels (1996, p.118), which is known to be a consistent selection method for the local polynomial regression. For the heteroskedasticity and autocorrelation consistent (HAC) variance estimation required for the standard error, we employ the QS kernel with a lag window parameter selected by the optimal selection method of Andrews (1991).

For the annual data set, the nonparametric half-life estimates with the 95 percent confidence intervals are provided in Table 4. In the same table, the results using the conventional linear measure are also reported for comparison. On the whole, the nonparametric half-lives do not differ much from the linear half-lives except for the U.K. On one hand, quite similar values between the two measures are obtained for Canada, France and Italy. On the other hand, somewhat shorter half-lives are obtained with a nonparametric measure for Japan and the Netherlands. It is interesting that the largest reduction in half-life is observed in the case of the U.K. The half-life based on the conventional linear measure is 4.84 years. This number is indeed very close to 4.6 years of half-life implied by Frankel's (1986) study of the long-horizon dollar/pound real exchange rates (see Rogoff, 1996, p. 656). By employing the nonparametric measure, the half-life is reduced to 2.64 years with a substantially smaller confidence interval.

Even if there is only a moderate difference between the nonparametric and linear half-life estimates, it does not imply that the adjustment process is well approximated by the linear process.

This point become clearer if we plot the nonparametric estimates of the rate of adjustment at each data point. Figure 3 shows the estimated speed of adjustment for six countries. Obviously, none of them are flat. More importantly, it shows faster adjustment when the deviations from the long-run level are large. The notable fact is that we have not imposed any parametric restriction to obtain a structure such as the STAR model. It implies that the data is favorable to our conjecture of the presence of trade costs as a source of nonlinearity.

For the quarterly data set, both results based on nonparametric and linear measures are reported in Table 5. For the conventional linear measures, slightly shorter half-lives are obtained than those based on the long-horizon data. The only exception is Canada with fairly long half-life point estimates. The median half-life based on the linear measure is 2.52 years compared to 3.01 years obtained from the long-horizon annual data. At the same time, the wide confidence intervals of linear measure show the uncertainty of the point estimates. Indeed, infinity is included for seventeen out of twenty countries, which implies the difficulties of excluding the possibility of a unit root. These observations are consistent with the former findings in the literature as well as with the result of the linear cointegrating rank test in this paper.

When the nonparametric measure is employed, half-lives again become shorter than those based on the long-horizon data. However, the most notable finding is that the nonparametric method provides smaller half-life estimates than the corresponding linear estimates for all the countries except Canada. The median of the nonparametric half-life measure is 1.44 years and the median of the difference between the nonparametric and linear measure is 0.99 years (the average half-life and difference are 1.53 and 1.08 years, respectively, when Canada is excluded). On average, about a 40 percent reduction in half-lives is observed by introducing nonlinearity into the adjustment process. With respect to the precision of the point estimates, the confidence intervals of nonparametric half-lives are considerably shorter than those for the linear half-lives. In some cases, 95 percent upper bounds for the nonparametric half-lives are indeed lower than corresponding point estimates based on the linear measure. In contrast to the linear measure, infinity is excluded from all the confidence intervals of nonparametric measure, again with the exception of Canada.

4 Conclusion

This paper has introduced a nonparametric convergence measure of PPP deviations which allows for general nonlinear real exchange rate adjustment. If the nonlinearity in the adjustment process is a possible pitfall in understanding the PPP puzzle as discussed in Taylor (2001), our nonparametric measure seems to be a reasonable way to evaluate the average speed of adjustment.

The simulation result on the finite sample properties of the nonparametric measure is found to be encouraging. When the true process has TAR or STAR structure, half-lives based on the linear measure suffers from upward bias because of the misspecification. In contrast, the nonparametric measure provides relatively accurate estimates for both linear and nonlinear adjustment process.

The proposed measure is applied to two different real exchange rates data sets. When the annual historical data is used, the nonparametric method yields more than two years of reduction in the half-life of U.K./U.S. real exchange rates compared to the linear estimate of 4.84 years. When the current float data is used, a one-year reduction from the linear estimates is observed on average in twenty countries. These empirical results suggest that the speed of reversion is indeed faster than in previous studies with the linear assumption. Furthermore, the nonparametric measure yields a shorter confidence interval than that of linear measure. In case of the former, infinite half-lives are excluded from the intervals for almost all cases.

The shorter half-lives obtained in this paper may, to some degree, reconcile the persistency issue with the long-run PPP. While we suspect that the nonlinearity is caused by the presence of trade costs, our nonparametric methods cannot identify the source of nonlinearity. For further investigation of this issue, it seems that analysis based on disaggregated prices offers useful information. One approach is to use price indexes of both traded and nontraded goods along the line suggested by Engel (1999, 2000). The other approach is to measure the persistency of good-by-good international price differentials. The latter approach is currently being pursued by the author (Crucini and Shintani, 2002).

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Table 1
Finite Sample Performance of Convergence Measures with TAR Model

Threshold Parameter (c)		0.00	0.50	1.00	2.00	5.00
True Half-Lives (τ^*)		1.00	1.41	1.84	2.70	5.70
	(1) $T = 100$					
Nonparametric	$h^* = 0.3$	0.88	1.24	1.60	2.40	4.93
Measure ($\widehat{\tau}^*$)		(0.25)	(0.35)	(0.46)	(0.79)	(2.93)
	$h^* = 0.4$	0.89	1.31	1.77	2.76	5.72
		(0.25)	(0.35)	(0.47)	(0.83)	(3.15)
	$h^* = 0.5$	0.90	1.36	1.88	3.01	6.33
		(0.25)	(0.35)	(0.49)	(0.89)	(3.44)
Linear		0.97	1.53	2.25	4.05	∞
Measure ($\widehat{\tau}$)		(0.25)	(0.38)	(0.57)	(1.26)	(–)
	(2) $T = 200$					
Nonparametric	$h^* = 0.2$	0.92	1.25	1.57	2.28	5.03
Measure ($\widehat{\tau}^*$)		(0.19)	(0.27)	(0.37)	(0.60)	(1.99)
	$h^* = 0.25$	0.93	1.29	1.66	2.52	5.77
		(0.19)	(0.27)	(0.36)	(0.61)	(2.18)
	$h^* = 0.3$	0.93	1.34	1.77	2.76	6.39
		(0.19)	(0.26)	(0.36)	(0.60)	(2.31)
Linear		0.98	1.56	2.32	4.34	12.81
Measure ($\widehat{\tau}$)		(0.18)	(0.27)	(0.40)	(0.87)	(5.22)

Note: Mean of estimated half-lives with standard deviation in parenthesis. The smoothing parameter for the nonparametric measure is $h = h^* \times$range. 10,000 replications.

Table 2
Finite Sample Performance of Convergence Measures with STAR Model

Dispersion Parameter (σ)		0.30	0.25	0.20	0.15	0.10
True Half-Lives (τ^*)		1.11	1.30	1.66	2.45	4.15
	(1) $T = 100$					
Nonparametric	$h^* = 0.3$	1.10	1.32	1.66	2.21	3.17
Measure ($\hat{\tau}^*$)		(0.25)	(0.33)	(0.45)	(0.67)	(1.10)
	$h^* = 0.4$	1.20	1.47	1.86	2.48	3.55
		(0.28)	(0.36)	(0.48)	(0.70)	(1.16)
	$h^* = 0.5$	1.29	1.58	2.01	2.67	3.82
		(0.30)	(0.38)	(0.51)	(0.74)	(1.24)
Linear		1.61	1.99	2.55	3.44	5.08
Measure ($\hat{\tau}$)		(0.37)	(0.47)	(0.65)	(1.00)	(1.86)
	(2) $T = 200$					
Nonparametric	$h^* = 0.2$	1.05	1.25	1.58	2.13	3.21
Measure ($\hat{\tau}^*$)		(0.17)	(0.23)	(0.33)	(0.50)	(0.84)
	$h^* = 0.25$	1.09	1.32	1.68	2.31	3.50
		(0.17)	(0.24)	(0.34)	(0.50)	(0.84)
	$h^* = 0.3$	1.15	1.41	1.82	2.50	3.76
		(0.19)	(0.25)	(0.34)	(0.50)	(0.83)
Linear		1.65	2.05	2.66	3.65	5.56
Measure ($\hat{\tau}$)		(0.25)	(0.33)	(0.45)	(0.70)	(1.30)

Note: Mean of estimated half-lives with standard deviation in parenthesis. The smoothing parameter for the nonparametric measure is $h = h^* \times$range. 10,000 replications.

Table 3
Multivariate Linear Cointegrating Rank Test

$\mathcal{H}_0$:Cointegrating Rank (Unit roots)	Johansen LR Test		Nonparametric Test	
(a) Annual Data: 1900-1996				
0 (6)	130.21*	[102.14]	353.65*	[241.33]
1 (5)	78.37*	[76.07]	219.73*	[182.07]
2 (4)	45.21	[53.12]	136.56*	[132.22]
3 (3)	23.30	[34.91]	77.48	[89.76]
4 (2)	7.95	[19.96]	33.04	[55.22]
5 (1)	2.40	[9.24]	13.56	[27.51]
(b) Quarterly Data: 1973:1-1998:2				
14 (6)	74.38	[102.14]	44.09	[241.33]
15 (5)	54.95	[76.07]	29.05	[182.07]
16 (4)	38.26	[53.12]	16.80	[132.22]
17 (3)	21.79	[34.91]	8.80	[89.76]
18 (2)	11.71	[19.96]	4.23	[55.22]
19 (1)	4.27	[9.24]	0.80	[27.51]

Notes: Numbers with asterisks imply that the null hypothesis is rejected at the 5 percent significance level. Numbers in brackets are 5% critical values.

Table 4
Persistency of PPP Deviations
(Annual Data: 1900-1996)

Country	Nonparametric Measure			Linear Measure		
	$\hat{\tau}^*$	95% CI		$\hat{\tau}$	95% CI	
1. Canada	3.10	[2.09,	5.98]	3.03	[1.81,	9.37]
2. France	1.41	[1.08,	2.05]	1.36	[0.89,	2.87]
3. Italy	2.53	[1.53,	7.40]	2.47	[1.52,	6.61]
4. Japan	6.14	[3.78,	16.28]	6.50	[3.28,	426.50]
5. Netherlands	2.21	[1.50,	4.18]	2.99	[1.77,	9.62]
6. United Kingdom	2.64	[1.88,	4.40]	4.84	[2.58,	39.47]

Note: QS kernel with optimal lag window (Andrews,1991) is used to construct confidence intervals for nonparametric measure.

Table 5
Persistency of PPP Deviations under the Current Float
(Quarterly Data: 1973:1-1998:2)

Country	Nonparametric Measure			Linear Measure		
	$\widehat{\tau}^*$	95% CI		$\widehat{\tau}$	95% CI	
1. Australia	2.64	[1.52,	9.74]	3.42	[1.38,	∞]
2. Austria	1.19	[0.80,	2.29]	2.35	[1.16,	∞]
3. Belgium	2.16	[1.31,	6.03]	3.12	[1.40,	∞]
4. Canada	32.22	[5.99,	∞]	20.00	[3.16,	∞]
5. Denmark	0.98	[0.70,	1.61]	2.59	[1.23,	∞]
6. Finland	2.27	[1.33,	7.64]	2.84	[1.30,	∞]
7. France	0.94	[0.67,	1.54]	2.47	[1.17,	∞]
8. Germany	1.08	[0.70,	2.32]	2.36	[1.13,	∞]
9. Greece	1.28	[0.90,	2.16]	2.56	[1.22,	∞]
10. Ireland	0.91	[0.64,	1.61]	1.60	[0.85,	13.61]
11. Italy	1.75	[1.08,	4.57]	2.37	[1.14,	∞]
12. Japan	2.76	[1.78,	6.12]	3.78	[1.73,	∞]
13. Netherlands	1.53	[0.93,	4.45]	2.22	[1.09,	∞]
14. New Zealand	2.09	[1.34,	4.78]	2.25	[1.09,	∞]
15. Norway	0.44	[0.30,	0.83]	1.87	[0.95,	89.15]

(continued)

Table 5 (Continued)

Country	Nonparametric Measure			Linear Measure		
	$\widehat{\tau}^*$	95% CI		$\widehat{\tau}$	95% CI	
16. Portugal	2.25	[1.42,	5.37]	3.85	[1.65,	∞]
17. Spain	2.54	[1.58,	6.41]	3.65	[1.63,	∞]
18. Sweden	0.43	[0.26,	1.21]	3.27	[1.43,	∞]
19. Switzerland	0.63	[0.47,	0.92]	1.19	[0.67,	4.94]
20. United Kingdom	1.35	[0.87,	3.00]	2.06	[1.02,	∞]

Note: See note of Table 4.

Fig. 1.
Linear Impulse Response and Half-Life

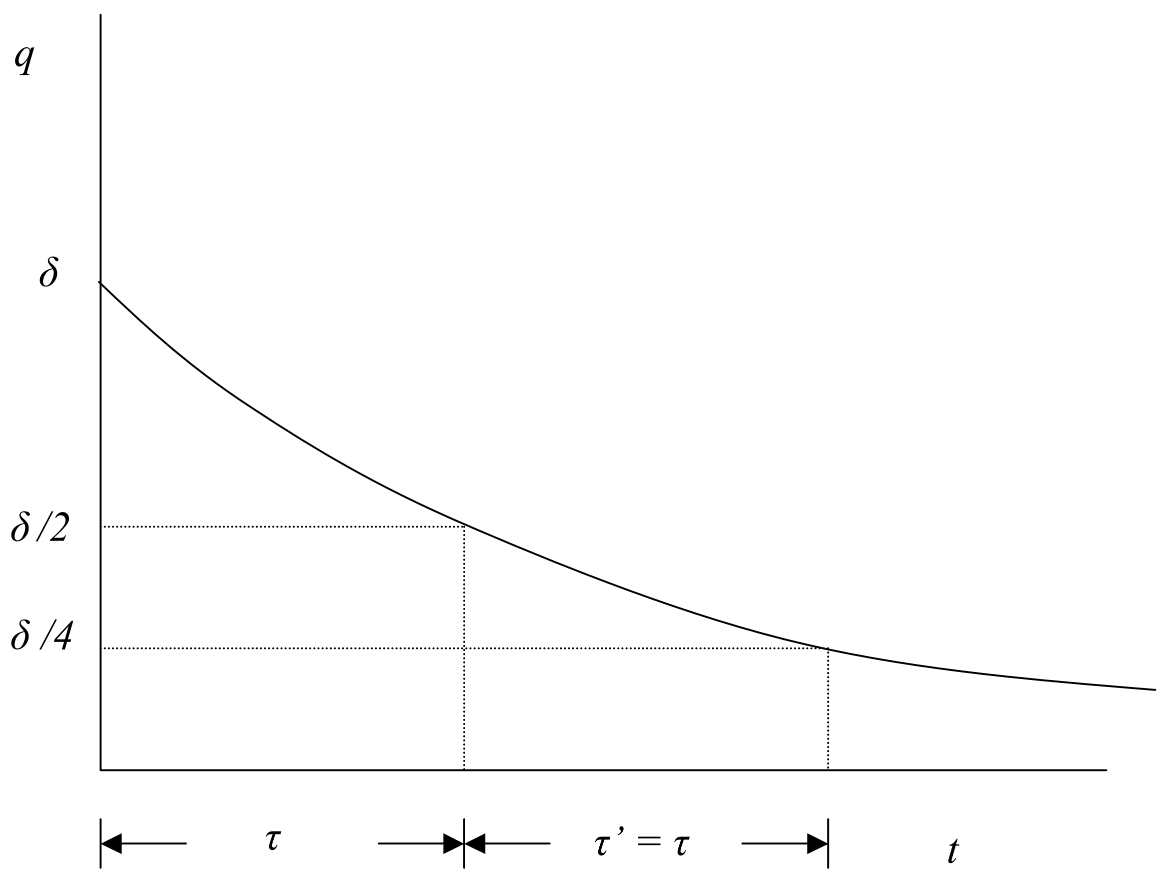

Fig. 2.
Nonlinear Impulse Response and Half-Life

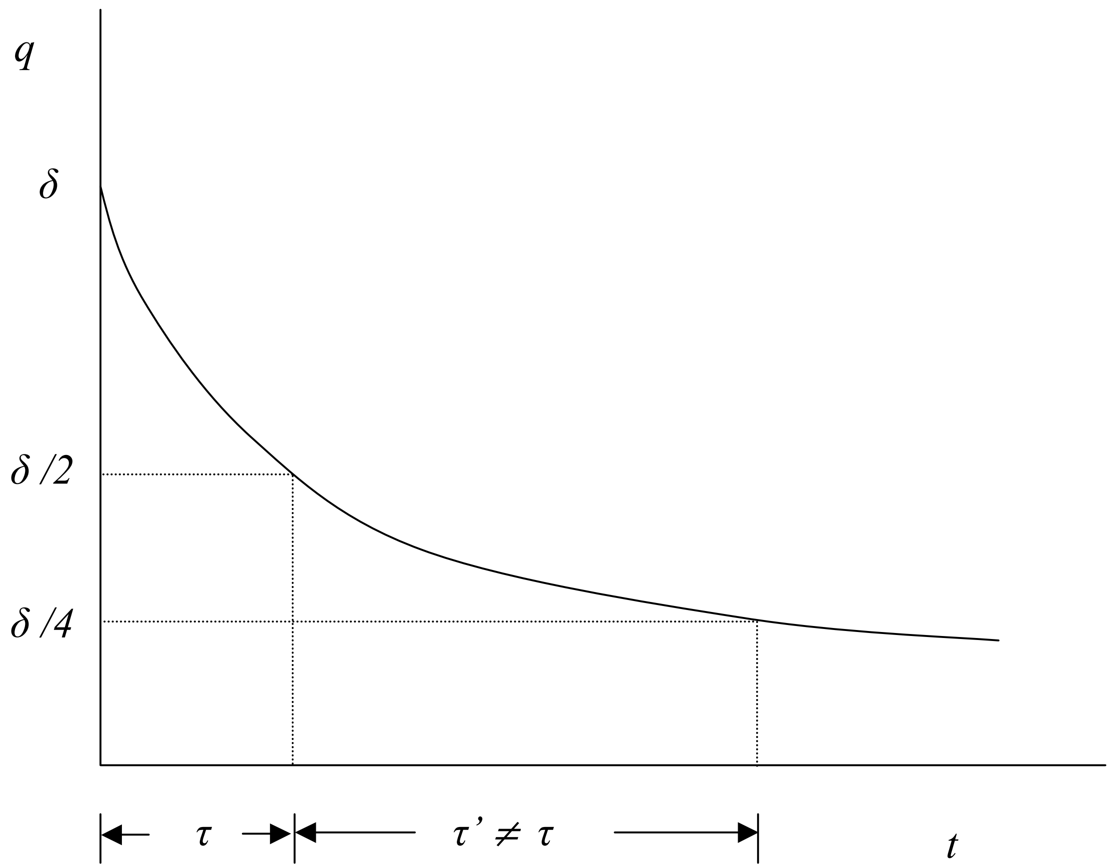

Fig. 3. The Rate of Convergence