ECONOMIC STRUCTURE, POLICY OBJECTIVES, AND OPTIMAL INTEREST RATE POLICY AT LOW INFLATION RATES

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Abstract

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In this article, the optimal interest rate rule generated by Svennson's (1997) dynamic model is used to determine the impact that a number of key structural characteristics have on the downward flexibility of interest rates at low rates of inflation. The potential impact of preferences for inflation stability, relative to output stability, on the monetary authority's ability to use expansionary interest rate policy is also considered. Estimates of the model for six countries provide evidence of the quantitative significance of the theoretical results. The empirical results are used to identify which monetary authorities are likely to be the most severely constrained in the event of an economic downturn. The size of the contraction that would be required for the interest rate constraint to bind is estimated for each country in the sample.

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Keywords: interest rate rule, monetary policy rule, Taylor rule.

1. Introduction

The achievement of low and predictable inflation rates has been the primary objective of monetary policy in many countries for the past twenty years. However, the attainment of this objective may raise new difficulties for the conduct of monetary policy. Because low inflation expectations are likely to be accompanied by low nominal interests rates, and the nominal interest rate necessarily has a zero lower bound, central banks may be contrained in their ability to use stabilizing interest rate policy in periods of business cycle contraction.[1].

Concern over the implications that the achievement of price stability may have for the implementation of expansionary monetary policies is not a recent development. Keynes (1936) himself was concerned with this issue. Although the high inflation rates of the 1970s and 1980s caused problems associated with the zero interest rate bound to be viewed primarily as a point of academic interest, the success of central banks in controlling inflation during the 1990s led to a revival of interest in this issue. Summers (1991) and Fischer (1996) have argued that a low positive rate of inflation, rather than zero inflation, is the appropriate long run target for monetary policy. Fischer has suggested that a target rate of 2 percent, which would result in inflation rates fluctuating between 1 and 3 percent, should be sufficient to allow central banks to generate negative real interest rates in periods of recession, should this become necessary.

The results of simulations by Fuhrer and Madigan (1997) and Reifschneider, Williams, Sims, and Taylor (2000) support Fisher's suggested 2 percent inflation target for the United States. However, this does not necessarily mean that 2 percent is an appropriate target for all low inflation countries. A priori one would expect that differences in economic structure, policy preferences, and the magnitude of economic disturbances might require inflation targets to differ across countries. In this article

[1]Under the assumption that there are no storage costs associated with holding money, the rate of interest on money cannot fall below zero. This point was originally, and more generally, made by Fisher (1930)

I use Svennson's (1997) dynamic macroeconomic model to study the impact of cross-country differences in structure and preferences on the choice of inflation target. In order to provide some quantitative insight into the implications of a 2 percent inflation target, I use coefficient estimates from Svennson's model to conduct a counter-factual experiment for a sample of six countries. The objective of this experiment is to determine whether monetary policy in these countries would have been constrained by the zero interest rate bound over the period 1982 to 1996 if a 2 percent inflation target had been achieved. The countries included in this study are Canada, France, Germany, Italy, the United Kingdom, and the United States.

In the theoretical model, the transition function of the policy authority's programming problem is described as a first-order difference equation in order to obtain an optimal response function that, like the original Taylor rule, contains no lagged endogenous variables. In order to preserve consistency between the theoretical results and their empirical application, the endogenous variable may be lagged only once in the relevant estimation equation. Because the statistically significant lag-length is generally positively related to the frequency of the data employed, annual data is used to estimate country-specific reduced-form equations and interest-rate response function in this study.[2] The interest rate response functions I estimate therefore describe the monetary policy implemented in each country in terms of the ex post average annual relationship between the domestic interest rate, inflation, and the output gap that the policy generated.

The rest of this article is organized as follows. A modified version of Svensson's (1997) dynamic model is introduced in Section 2 and used to derive the optimal interest rate rule. In Section 3, the sensitivity of the optimal interest rate rule to differences in structure and preferences across countries is analyzed. Estimation of the structural equations for the six countries included in this study is undertaken in Section 4. Quantitative implications of low inflation rates for the effectiveness of

[2]Using quarterly data, Rudebusch and Svensson (1999) have found that the statistically determined transition function is a fourth-order difference equation.

counter-cyclical interest rate policy are obtained in Section 5. The impact of model and parameter variation is discussed in Section 6. A brief summary of the results obtained may be found in Section 7.

2. The Optimal Interest Rate Rule

For the purposes of deriving the optimal interest rate rule, I use a modified version of the model employed by Svensson (1997). The economic structure is summarized by the following reduced-form equations:

$$\pi_{t+1} = \alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = \beta_0 + \beta_1 y_t - \beta_2 (i_t - \pi_t - r^*) + \mathbf{b}_3 \mathbf{x}_t + \eta_{t+1} \tag{2}$$

where π_t is the inflation rate in period t, y_t is the output gap, i_t is the nominal interest rate, r^* is the equilibrium real interest rate, and $\mathbf{x}_t = (x_{1t}, x_{2t}, x_{3t}, ...)$ is a column-vector of exogenous and predetermined variables that have an impact on the magnitude of the output gap. The variables ε_{t+1} and η_{t+1} represent random disturbances to inflation and the demand for goods, respectively, which are not contemporaneously observable. As in Svensson's original model, each time period t is assumed to have a duration of one year.

The model employed by Svensson is a special case of (1) and (2) in which $\alpha_0 = \beta_0 = r^* = 0$ and $\alpha_1 = 1$.[3] Removing some of the restrictions that Svensson originally imposed in his model allows (1) and (2) to represent a wider class of models and also accommodates country-specific differences in economic structure. For example, the addition of the $\mathbf{x}_t$ vector to (2) allows variables such as exchange rates, which Ball (1999b) has found to be important for efficient interest rate management in open economies, to be introduced into the model. In this study, decisions about which variables to include in the $\mathbf{x}_t$ vector were made on a purely empirical basis.

[3] McCallum (1997) has pointed out that imposing the restriction $\alpha_1 = 1$ leads to dynamic inconsistency in this model when the policy authority sets interest rate policy to minimize the variation of nominal income. Tests conducted as part of the empirical application discussed in Section 4, strongly rejected this parameter restriction for every country in the sample.

One potentially controversial feature of the model used here is that (1) describes a backward-looking Phillips curve. Recently, the desire to use an aggregate supply equation that can be derived from an explicit microeconomic optimization problem has led some authors to use a 'New Keynesian' Phillips curve in place of (1). In this new version of the Phillips curve, expected future inflation either replaces or supplements expected current inflation as a determinant of the current inflation rate.[4] In this study, I chose not to incorporate future expected inflation into (1) for two reasons. First, as Mishkin (1999) has pointed out, the models from which forward-looking Phillips curves are derived have the implication that the policy authority need not act pre-emptively to control inflation. However, one of the lessons that policy-makers learned from the experiences of the period under study was precisely that pre-emptive action was necessary given the lags in the economy's response to policy changes. Second, the empirical evidence on the significance of expected future inflation as a determinant of the current inflation rate is mixed and the results seem to be quite sensitive to the estimation method used. Using quarterly US data, Fair (1993) and Fuhrer (1997) obtain estimates for the forward-looking expectations component that are not significantly different from zero; other estimates for the US range between statistically significant coefficients of 0.28 to 0.42.[5] Overall, the empirical results indicate that the coefficient on the forward-looking expectation component is low and this, together with Levin, Wieland, and Williams' (1999) finding that the inclusion of a forward-looking inflation element does not significantly improve the performance of the simple rules suggests that (1) is a parsimonious representation of a reasonable generic structural model.[6]

Following Svensson (1997), I assume that the policy authority's objective is to stablilize inflation around the long-run inflation target π^* and the output gap around

[4]See, for example, Rotemberg and Woodford (1997,1999) and Svensson (2000).

[5]Rudebusch (2000) provides a summary of the estimation results obtained in a variety of studies.

[6]Levin, Wieland, and Williams (1999) used US data in their study. It is possible that the US results are not representative and that expected future inflation may be of greater importance in determining the rate of inflation in other countries.

zero. The policy authority's one-period loss function is then given by:

$$L(\pi_t, y_t) = \frac{1}{2}\left\{(\pi_t - \pi^*)^2 + \lambda y_t^2\right\} \tag{3}$$

where λ is the relative weight assigned to output stabilization. With period-by-period losses given by (3), the policy authority's intertemporal loss function is:

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau} L(\pi_{t+\tau}, y_{t+\tau}) \tag{4}$$

where δ is the policy authority's discount rate, and E_t denotes that the expectation of future losses is conditioned on the information available at time t.

Given that the policy authority views the short-term interest rate i_t as its control variable, the policy authority's objective is to set i_t so as to minimize (4). From (1) and (2) it is evident that the policy authority faces a two-period control lag. Following Svensson (1997), the policy authority's problem can be formulated as

$$V(\pi_{t+1|t}) = \min_{y_{t+1|t}} \left\{\frac{1}{2}\left[(\pi_{t+1|t} - \pi^*)^2 + \lambda y_{t+1|t}^2\right] + \delta \mathrm{E}_t V(\pi_{t+2|t+1})\right\} \tag{5}$$

subject to

$$\pi_{t+2|t+1} = \alpha_0 + \alpha_1 \pi_{t+1} + \alpha_2 y_{t+1}$$

where the notation $z_{t+1|t}$ denotes the value that the variable z is expected to take on in period $t+1$ conditional on the information available in period t. Once the optimal value of $y_{t+1|t}$ has been obtained, the optimal level of i_t can be inferred from (2).

Because the period loss function (3) is quadratic and the constraint is linear, $V(\pi_{t+1|t})$ must be a quadratic polynomial. Let $V(\pi_{t+1|t})$ be given by

$$V(\pi_{t+1|t}) = k_0 \ + \ k_1(\pi_{t+1|t} - \pi^*) \ + \ \frac{k_2}{2}(\pi_{t+1|t} - \pi^*)^2. \tag{6}$$

Using (6) to replace $V(\pi_{t+2|t+1})$ in (5) and taking the derivative of the expression in braces with respect to $y_{t+1|t}$ results in the first-order condition

$$y_{t+1|t} = -\frac{\delta \alpha_2 k_1}{\lambda} \ - \ \frac{\delta \alpha_2 k_2}{\lambda}\left[\pi_{t+2|t} \ - \ \pi^*\right] \tag{7}$$

where

$$k_1 \;=\; \frac{\lambda\delta\alpha_1 k_2[\alpha_0 - (1-\alpha_1)\pi^*]}{\lambda(1-\delta\alpha_1) + \delta\alpha_2^2 k_2} \tag{8}$$

$$k_2 \;=\; \frac{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)] \;+\; \sqrt{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)]^2 \;+\; 4\delta\alpha_2^2\lambda}}{2\delta\alpha_2^2}. \tag{9}$$

Details of the solutions for k_1 and k_2 are provided in Appendix 1.

From (1), $\pi_{t+2|t}$ can be expressed as

$$\pi_{t+2|t} \;=\; \alpha_0(1+\alpha_1) \;+\; \alpha_1^2\pi_t \;+\; \alpha_1\alpha_2 y_t \;+\; \alpha_2 y_{t+1|t}. \tag{10}$$

Substituting (10) into (7) reveals that the solution to (7) is

$$y_{t+1|t} = -\,\frac{\delta\alpha_2\,[k_1 + (1+\alpha_1)\alpha_0 k_2]}{\lambda + \delta\alpha_2^2 k_2} \;-\; \frac{\delta\alpha_2 k_2 \alpha_1^2\,\pi_t}{\lambda + \delta\alpha_2^2 k_2} \;-\; \frac{\delta\alpha_2^2 k_2 \alpha_1\, y_t}{\lambda + \delta\alpha_2^2 k_2} \;+\; \frac{\delta\alpha_2 k_2\,\pi^*}{\lambda + \delta\alpha_2^2 k_2}. \tag{11}$$

Substituting (11) into (2) and solving for the interest rate i_t yields the optimal interest rate rule

$$i_t \;-\; \pi_t \;=\; \bar{K} \;+\; g_1\;[\pi_t - \pi^*] \;+\; g_2\; y_t \;+\; \mathbf{g}_3\;\mathbf{x}_t \;+\; r^*. \tag{12}$$

where

$$\bar{K} \;=\; \frac{\beta_0}{\beta_2} \;+\; \frac{\delta\alpha_2\,[k_1 + (1+\alpha_1)\alpha_0 k_2 + k_2(\alpha_1^2 - 1)\pi^*]}{\beta_2(\lambda + \delta\alpha_2^2 k_2)} \tag{13}$$

$$g_1 \;=\; \frac{\delta\alpha_2 k_2 \alpha_1^2}{\beta_2(\lambda + \delta\alpha_2^2 k_2)} \tag{14}$$

$$g_2 \;=\; \frac{[(\alpha_1 + \beta_1)\delta\alpha_2^2 k_2 + \lambda\beta_1]}{\beta_2(\lambda + \delta\alpha_2^2 k_2)} \tag{15}$$

$$\mathbf{g}_3 \;=\; \frac{\mathbf{b_3}}{\beta_2}. \tag{16}$$

The interest rate rule given in (12) is immediately recognizable as a variant of the two-parameter Taylor rule first introduced by Taylor (1993).[7]

[7]The interest rate rule (12) reduces to Taylor's original two-parameter rule when $\bar{K} = \mathbf{b}_3 = 0$. One set of parameter restrictions for which Taylor's original two-parameter rule is also the optimal interest rate rule is $\alpha_0 = \beta_0 = \mathbf{b}_3 = 0$ and $\alpha_1 = 1$.

3. Differences in Structure and Preferences

It is apparent from (12) that the optimal interest rate response to observed inflation and output gaps depends on the structural characteristics of each economy as well as the relative importance that the national policy authority assigns to price and output stability. Policy authorities in countries where relatively small interest rate changes elicit strong counter-cyclical responses are less likely to find monetary policy to be seriously constrained by the zero interest rate bound in periods of contraction. In this section, I use the results derived above to establish the relationship between economic structure, policy preferences, and the magnitude of the optimal interest rate response when inflation is low.

The concern about the constraint posed by the zero interest rate bound has arisen as economies have achieved low inflation rates. In the context of the inflation-targeting model employed here, it is useful to define what is meant by 'low inflation rates' in terms of the inflation target π^*. In particular, I will treat a low inflation environment as one in which the average annual inflation rate is equal to the inflation target so that $\pi_t = \pi^*$ for all t. Under this assumption, (12) becomes

$$i_t \;=\; \bar{K} \;+\; g_2\, y_t \;+\; \mathbf{g}_3\, \mathbf{x}_t \;+\pi^* \;+\; r^*. \tag{17}$$

In order to be successful, inflation targeting policies must be consistent with the long-run properties of the underlying economy. The structural equations (1) and (2) have long run implications that can be used to further simplify (17). A natural way to represent long-run equilibrium in an economy described by (1) and (2), is to require that $\pi_t = \pi_{t-1} = \pi^*$, $y_t = 0$, and $(i_t - \pi_t - r^*) = 0$.[8] Imposing the first two constraints on (1), and the second and third constraints on (2), yields

$$\pi^* \;=\; \frac{\alpha_0}{(1-\alpha_1)} \tag{18}$$

[8] Requiring that $\pi_t = \pi_{t-1}$ can be thought of as employing NAIRU as the equilibrium concept for inflation, rather than price level stability, which would impose the more stringent requirement that $\pi_t = \pi_{t-1} = 0$.

$$\beta_0 \;=\; \mathbf{b}_3\mathbf{x}^* \tag{19}$$

where $\mathbf{x}^*$ is composed of the long-run equilibrium values of the components of the vector $\mathbf{x}_t$.[9] Using (18) to simplify (13) and then substituting the resulting expression, along with (19) into (12) yields the average interest rate response rule[10]

$$i_t \;=\; g_2\, y_t \;+\; \pi^* \;+\; r^*. \tag{20}$$

3.1 Structural Differences

The relationship between structural characteristics and the magnitude of interest rate changes can be obtained by differentiating (20) with respect to the structural parameters α_1, α_2, β_1, and β_2. Differentiating (20) yields the following results:

$$\frac{\partial i_t}{\partial \alpha_1} \;=\; \frac{\alpha_0}{(1-\alpha_1)^2} \;+\; \left\{\frac{\delta\alpha_2^2 k_2[\lambda+\delta\alpha_2^2 k_2] \;+\; \lambda\delta\alpha_2^2\left(\frac{\partial k_2}{\partial \alpha_1}\right)}{\beta_2[\lambda+\delta\alpha_2^2 k_2]^2}\right\} y_t \tag{21}$$

$$\frac{\partial i_t}{\partial \alpha_2} \;=\; \left\{\frac{\lambda\delta\alpha_1\alpha_2[2k_2+\alpha_2\left(\frac{\partial k_2}{\partial \alpha_2}\right)]}{\beta_2[\lambda+\delta\alpha_2^2 k_2]}\right\} y_t \tag{22}$$

$$\frac{\partial i_t}{\partial \beta_1} \;=\; \frac{y_t}{\beta_2} \tag{23}$$

$$\frac{\partial i_t}{\partial \beta_2} \;=\; -\left\{\frac{[(\alpha_1+\beta_1)\delta\alpha_2^2 k_2+\lambda\beta_1]}{\beta_2^2(\lambda+\delta\alpha_2^2 k_2)}\right\} y_t \tag{24}$$

where

$$\frac{\partial k_2}{\partial \alpha_1} \;=\; \frac{\lambda\alpha_1}{\alpha_2^2}\left\{1 \;+\; \frac{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]}{\{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]^2+4\delta\alpha_2^2\lambda\}^{1/2}}\right\} \tag{25}$$

$$\frac{\partial k_2}{\partial \alpha_2} \;=\; \frac{1-2k_2}{\alpha_2} \;+\; \left\{\frac{2[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]\delta\alpha_2^2+4\delta\alpha_2^2\lambda}{\{2\delta\alpha_2^3[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]^2+4\delta\alpha_2^2\lambda\}^{1/2}}\right\}. \tag{26}$$

It is straightforward to show that in (21)–(24), all of the expressions in braces are positive for all permissible values of δ and λ. The impact of changes in structure on

[9]If strict price level stability were used to characterize long-run equilibrium, then the long run inflation target would have to be $\pi^* = 0$, which, in turn, would require that $\alpha_0 = 0$ in (1).

[10]Note that $\mathbf{x}_t = \mathbf{x}^*$ was used to obtain (20) from (17).

the interest rate reduction needed to stabilize output is unambiguous for α_2 and β_1, and β_2. Equation (1) indicates that inflation is more responsive to changes in the output gap the larger is α_2. The implication of (22) is that countries whose inflation rates are less responsive to changes in the output gap will be less likely to find their monetary policies constrained by the zero interest rate bound.

The parameter β_1 measures the degree of persistence in the output gap from one period to the next. From (23) it is apparent that the optimal interest rate response to a contractionary output gap is increasing in β_1. According to (2), the output gap responds to interest rate changes with a one-period lag. This being the case, effective use of interest rate policy in the past reduces the current output gap and, when this gap is highly persistent, also has a stabilizing impact on future output. It is therefore beneficial for countries with highly persistent output gaps to be aggressive in the implementation of counter-cyclical interest rate policy. As a consequence, the lower interest rate bound will be of greater concern in countries where economic contractions tend to be highly persistent.

The parameter β_2 reflects the responsiveness of the output gap to changes in the real interest rate in the IS equation (2). The expression in (24) shows that, for a given output gap, the magnitude of the policy authority's optimal interest rate response is smaller the more responsive is the output gap to interest rate changes. It follows that the zero interest rate bound is less binding the larger is β_2.

In (22)–(24), the qualitative impact of structural differences on the optimal interest rate policy is independent of the sign of the output gap. This is not true of difference in the degree of inflation persistence, α_1. The degree of inflation persistence not only affects the optimal response to a given output gap, but also the long-run inflation target π^*. In particular, (18) indicates that the appropriate long-run inflation target is inversely related to α_1. Thus the reduction in interest rates needed to counteract a contractionary output gap may be partially or completely offset by the need to increase the interest rate in order to achieve a lower inflation target, dampening the policy authority's interest rate response to y_t. If however, the output gap is positive, the interest rate increase in response to the output gap is amplified by the need to achieve a lower inflation target.

3.1 Differences in Policy Preferences

In addition to structural differences, countries may also exhibit differences in the objective functions upon which policy decisions are based. In the loss function (4) that is used to represent the policy authority's objectives, there are three possible dimensions across which preferences may differ. National policy authorities may differ in (i) their choice of inflation target π^*, (ii) the importance they assign to output variation relative to inflation variation λ, and (iii) the rate at which they discount the future δ. In order for an inflation target to be sustainable in the long run, it must be consistent with the underlying economic structure. In the context of the model employed here, such long-run consistency causes the appropriate inflation target to be fully determined by the structural parameters α_0 and α_1. Thus in the context of this analysis, differences in π^* reflect differences in the underlying structure of the economy rather than differences in national policy objectives. The discussion below therefore focuses on the impact of variations in δ and λ on the optimal interest rate response.

Differentiating (20) with respect to δ and λ yields the following results:

$$\frac{\partial i_t}{\partial \delta} \;=\; -\;\frac{\alpha_2^2[(\alpha_1+\beta_1)\alpha_2^2\delta k_2 \;+\; \lambda\beta_1]}{\beta_2(\lambda+\delta\alpha_2^2k_2)^2}\left\{k_2+\delta(\frac{\partial k_2}{\partial \delta})\right\}y_t \tag{27}$$

$$\frac{\partial i_t}{\partial \lambda} \;=\; -\;\alpha_1\delta\alpha_2^2\left\{k_2-\lambda(\frac{\partial k_2}{\partial \lambda})\right\}y_t \tag{28}$$

where

$$k_2+\delta(\frac{\partial k_2}{\partial \delta}) \;=\; \frac{\lambda\alpha_1^2+\alpha_2^2}{2\alpha_2^2} \;+\; \frac{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]\delta\alpha_2^2 \;+\; 2\delta\alpha_2^2\lambda}{2\delta\alpha_2^2\{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]^2+4\delta\alpha_2^2\lambda\}^{1/2}} > 0 \tag{29}$$

$$k_2-\lambda(\frac{\partial k_2}{\partial \lambda}) \;=\; \frac{1}{2}\left\{1 \;+\; \frac{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2) \;+\; 2\lambda]}{\{[\delta\alpha_2^2-\lambda(1-\delta\alpha_1^2)]^2+4\delta\alpha_2^2\lambda\}^{1/2}}\right\} > 0 \tag{30}$$

It is evident from (30) that a low rate of time preference (i.e., high δ) decreases the optimal interest rate response. Because interest rate changes affect output with a lag, and output is subject to random disturbances, too strong a response to current output may increase the magnitude of future output deviations. Consequently, monetary authorities with discount factors close to one prefer to be more cautious in the implementation of interest rate changes. By contrast, more aggressive use of interest rate policy is optimal for monetary authorities who place a higher value inflation performance relative to output stability (i.e., who are characterized by low λ). The zero interest rate bound can therefore be expected to be of greater concern to policy authorities who have a high rate of time preferece and also more concerned with inflation performance than with output stability.

The analysis in this section has provided some insights into the qualitative relationship between optimal interest rate policy and cross-country differences in economic structure and preferences. The question that remains to be addressed is whether the zero interest rate bound is likely to constrain monetary policy in practice. It is this empirical question that is the focus of the remainder of this article.

4. Estimation of the Structural Equations

The structural parameters in (1) and (2) affect the flexibility of interest-rate policy at low inflation rates. Assessing the quantitative significance of the effects derived in Section 3, requires the estimation of the structural equations (1) and (2) for each of the six countries included in this study.

4.1 Parameter Estimates

In order to preserve consistency between the theoretical and the estimated efficiency criteria, equations (1) and (2) were estimated using annual data for each country in the sample. The estimation period for Canada, France, the United Kingdom, and the United States begins in 1975 and ends in 1996. The estimation period for Germany also begins in 1975 but it ends in 1995. The German data set was truncated at 1995 because including 1996 introduced serious end-point problems. Equations (1) and (2) did not find strong support in the Italian data. For Italy, the quality of the estimation results deteriorated steady as the sample was extended beyond 1992. Rather than drop Italy from the sample altogether, I elected to include the Italian results for the period of best fit, which is 1973-92. For countries other than Italy, the year 1975 was chosen as a starting point to eliminate possible estimation problems associated with the abandonment of the Bretton Woods system in early 1973. Unfortunately, choosing 1975 as the initial date does not eliminate other sources of structural disturbance, such as the impact of the OPEC oil price increases which strongly influenced short-term Phillips curve relationships in most countries until the early 1980s. Furthermore, for the European countries, the financial turmoil surrounding the ratification of the Maastricht treaty in late 1992 appears to have caused some temporary changes in structural relationships. In the German data, the impact of German unification is also clearly discernable.

In order to keep the estimation equations as close to their theoretical counterparts as possible, dummy variables were used to deal with the above-mentioned changes in structure. Every effort was made to avoid introducing structural dummies after 1982 to ensure that the estimated parameter values correspond to the time period which Taylor identifies as being associated with interest rate policies that follow Taylor rules. The variables used for the estimations were obtained from the International Monetary Fund's International Financial Statistics. Following Taylor's (1993a) example, the output gap was calculated as the deviation of the natural log of annual real GDP from its trend which, for the purposes of this study, is assumed to be deterministic and linear.[11] All other variables were pre-tested for order of integration

[11] In a more recent article, Taylor (1998a) uses a Hodrick-Prescott (HP) filter to obtain a quarterly GDP trend series for the United States. The likelihood that the results obtained here might be sensitive to the construction of the output gap was assessed using a Wald test on the slope coefficient obtained by regressing the standardized linear-trend gap on the standardized HP gap. The test results indicate that, for all countries except Canada, there is no significant difference in the two annual output gap series. In Canada's case, the linear-trend gap results in larger output gap values

using Augmented Dickey-Fuller tests. Perron's (1989) procedure was applied in those cases where structural change in the data generating process was suspected. The null hypothesis of a unit root was rejected at a significance level of at least 10% for all of the non-output variables needed to estimate (1) and (2).[12] The presence of a significant deterministic trend was rejected at the 5% level for these variables.[13] The parameter estimates obtained using OLS to estimate (1) and (2) are reported in Tables 1 and 2, respectively. Details of the variable definitions, unit root tests, and the estimation results may be found in Appendix 3.

The dates given in column 1 of Table 1 identify the time periods over which the estimated values, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are free of structural changes. In Table 2, these dates specify stable periods for $\hat{\beta}_1$ and $\hat{\beta}_2$. In each table, the t-statistic associated with the parameter estimate is given in parentheses below the estimated value. The 95% margin of error is provided in the column immediately to the right of each estimate.[14]

The country-specific components of $\mathbf{x}_t$ that were used to estimate (2) are described in Table 3. The variables representing x_{31} and x_{32} were chosen by the following method. I used the characteristics of each country to identify a set of variables that might be expected to have a significant influence on output and/or inflation. For instance, in Canada's case, US output, prices, and interest rates, and the Canada/US

than does the HP gap. The decision to use the linear-trend gap for annual Canadian GDP data is supported by Serletis (1992).

[12]Note that the unit root hypothesis could not be rejected at the 10% level for the French nominal interest rate. However, it is the real interest rate that is needed to estimate (2), and this (composite) variable is I(0) at the 10% level.

[13]As noted above, the sample period is characterized by a number of significant changes in the economic environment. For most of the countries in this study, the 1980's were a transition period in which countries were wrestling with the results of the oil price increases. Perron's (1989) Model A captures the impact of the oil price shocks in the form of a shift in the mean of the inflation and/or interest rate processes in France, Italy, the United Kingdom, and the United States. The significance of this shift may very well decline as the sample period lengthens with the passage of time.

[14]The 95% confidence interval for $\hat{\alpha}_i$ is given by $\hat{\alpha}_i \pm 1.96\hat{\sigma}_{\alpha_i}$ for $i = 1, 2$.

TABLE 1

Parameter Estimates for Equation (1)

	$\hat{\alpha}_1$	$1.96\ \hat{\sigma}_{\alpha_1}$	$\hat{\alpha}_2$	$1.96\ \hat{\sigma}_{\alpha_2}$
Canada	0.4964	0.1615	0.1324	0.0883
1982-96	(6.4580)		(3.1500)	
France	0.9810	0.1336	0.2933	0.2417
1981-94	(15.6500)		(2.5862)	
Germany	0.5841	0.2297	0.1979	0.0963
1975-95	(5.3652)		(4.3882)	
Italy	0.7008	0.1831	0.6615	0.3935
1981-92	(8.2714)		(3.6308)	
U.K.	0.4900	0.0865	0.1943	0.1505
1981-96	(11.953)		(2.7246)	
U.S.A.	0.5062	0.1694	0.1820	0.1662
1982-95	(6.3355)		(2.3206)	

exchange rate were all likely candidates. For European countries like France and Italy, German output, prices, and interest rates, as well as the value of the domestic currency relative to the dmark were in the intial variable set. I then ran a series of regressions for each country and retained only those variables whose coefficients were significant at the 5% level. For Canada and the United States, dummy variables analogous to those employed in (29) were introduced into (30). For both the US and Canada, only the dummy variable associated with x_{31} was found to be significant at the 5% level.

4.2 Parameter Invariance

There are doubtless many ways that one might think of conducting an empirical assessment of the practical importance of the zero interest rate bound. The method I have chosen, which will be described in detail in the next section, takes the form of a

TABLE 2

Parameter Estimates for Equation (2)

	$\hat{\beta}_1$	1.96 $\hat{\sigma}_{\beta_1}$	$\hat{\beta}_2$	1.96 $\hat{\sigma}_{\beta_2}$	$\hat{\beta}_{31}$	1.96 $\hat{\sigma}_{\beta_{31}}$	$\hat{\beta}_{32}$	1.96 $\hat{\sigma}_{\beta_{32}}$
Canada	0.9386	0.1062	0.7311	0.2991	-0.1192	0.0588		
1983-96	(18.838)		(5.2084)		(-4.3245)			
France	0.9693	0.2668	0.4204	0.1650	-0.9121	0.3410	0.0559	0.0423
1981-94	(7.9179)		(5.5516)		(-5.8293)	0.0804	(2.8755)	
Germany	0.6315	0.1155	0.8563	0.2892	0.2597	0.2073	-0.0668	0.0384
1975-89	(11.812)		(6.3956)		(2.7056)		(-3.7586)	
Italy	0.6628	0.2001	0.4037	0.1457	-0.5990	0.3222		
1976-92	(7.1038)		(5.9444)		(-3.9880)			
U.K.	0.6663	0.2493	0.1468	0.1012	-1.5219	0.5278	-0.5155	0.3386
1975-96	(5.6662)		(3.0746)		(-6.1123)		(-3.2278)	
U.S.A.	0.4480	0.2466	0.8541	0.2750	-0.4838	0.3330	-0.3888	0.1797
1982-96	(3.8708)		(6.5169)		(-3.0960)		(-4.6114)	

Table 3

Country-Specific Components of $\mathbf{x}_t$

Canada	$x_{1t} = q^{us}_{t-1}$ = lagged Canada/US real exchange rate x_{2t} = none
France	$x_{1t} = \pi^{ger}_{t-2}$ = German inflation rate, lagged two periods $x_{2t} = \Delta e^{fus}_{t-1}$ = lagged % Δ nominal franc/dollar exchange rate
Germany	$x_{1t} = \Delta Y^{us}_{t-2}$ = US output growth, lagged two periods $x_{2t} = \Delta e^{gus}_{t-1}$ = lagged % Δ nominal dmark/dollar exchange rate
Italy	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation x_{2t} = none
U.K.	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation $x_{2t} = \Delta Y^{ger}_{t-1}$ = lagged German output growth
U.S.A.	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation $x_{2t} = y^{ger}_{t-1}$ =lagged German output gap

counter-factual policy experiment. In order for the results of such an experiment to be at all meaningful, the parameter estimates must be invariant to changes in the underlying policy parameters. The problem of policy-based parameter invariance, often referred to as the Lucas critique, arises because (1) and (2) are reduced-form equations whose parameters may be composites of the economy's structural (invariant) parameters and the policy authority's behavioural parameters, δ and λ. While the Lucas critique must always pertain to reduced-from equations in theory, the question of interest from the point of view of this study is whether such parameter invariance is of empirical significance.

An empirical approach to dealing with the issue of potential parameter invariance has been suggested by Hendry (1988). This approach regards parameter invariance as a theoretical possibility which may or may not be of empirical significance in the context of a particular study. Ericsson and Irons (1995) illustrate a method of testing for the empirical significance of the Lucas critique which is particularly appropriate in this study, given size of the data set and the constraints that the theoretical structure places on the specification of the estimating equations. The idea behind their methodology is to test whether changes in the processes generating the explanatory variables lead to significant changes in the parameter estimates. The test methodology is composed of two steps. The first step involves careful modelling of the individual processes generating the variables included in the estimating equation. In the second step, these marginal processes are introduced into the original estimating equation. An F-test is then used to determine whether introducing information about how a particular variable changes over time has a significant impact on the parameter estimate associated with that variable. Failure to reject the null hypothesis (that the estimated parameters are statistically invariant) is interpreted as empirical support for the assumption of policy-based parameter invariance. The results obtained by applying this test to each of the countries included in this study indicate that all of the parameters estimated on the basis of (1) and (2) are statistically invariant for Canada, France, the United Kingdom, and the United States. The results for Germany and Italy are less satisfactory. In the case of Italy, the null hypothesis is rejected for two coefficient estimates, $\hat{\alpha}_1$ and $\hat{\beta}_1$. The results for Germany are even weaker with the null hypothesis being rejected for the parameters $\hat{\alpha}_1$, $\hat{\beta}_1$, and $\hat{\beta}_{32}$. These results indicate that the values reported for Germany and Italy in Tables 1 and 2 must be interpreted with caution. Details of the estimated marginal processes and the invariance test results are provided in Appendix 3.

5. A Counter-Factual Empirical Assessment

During the 1970s and 1980s interest rates were so high that there was little reason to be concerned that the zero interest rate bound might contstrain the use of interest rate policy in periods of economic contraction. The recent success that many central banks have had in reducing average annual inflation rates to the two per cent level has caused the lower bound on interest rates to become a matter of concern. Although the analysis in Section 3 identifies the circumstances under which monetary policy may be more or less constrained by the lower interest rate bound, the theoretical analysis alone cannot provide any insight into the empirical relevance of this constraint.

One way to obtain an empirical assessment of the practical significance of the zero interest rate bound is through simulations.[15] Alternatively, one might conduct a counter-factual experiment in which one asks what would have happened in some past period if, during that period, interest rates had been as low as they are now. I have chosen this second approach. In this section I estimate the annual interest rate levels that would have been optimal for six countries over the period 1976 to 1996 if inflation rates had been held steady at 2% during this period. I also provide an estimate of the maximum annual contractionary gap that could have been eliminated using interest rate policy alone.

The estimated optimal interest rate level can be computed using (17). It is evident that in addition to parameter estimates of all of the structural coefficients, computation of the optimal interest rate requires the choice of specific values for the discount factor δ, the preference parameter λ, the inflatin target $\pi*$, and the long-run equilibrium interest rate $r*$.[16] The calculated optimal interest rate levels were found to be insensitive to variations in δ over the range $0.75 \leq \delta \leq 099$, I therefore followed common practice and set $\delta = 0.99$. Stuart (1996) provides evidence that the average lon-run real interest rate level in the industrialized countries is close to 3.5% for the period under study here. I therefore set $r^* = 0.035$ in (12). The inflation target, π^* was set equal to 0.02 for all countries. Determining an appropriate value for the preference parameter λ is more problematic because it is concievable that countries may differ quite substantially in the relative weights they assign to inflation variation relative to output variation. In order to allow for a fairly wide range of preferences, I calculated the values of $\bar{K}$ and g_2 over the range $0.25 \leq \lambda \leq 50$ then used the average

[15]For simulations of the impact of low inflation rates on the effectiveness of US monetary policy see, for example, Fuhrer and Madigan (1997), Orphanides and Wieland (1998), and Reifschneider, Williams, Sims, and Taylor (2000).

[16]The complete set of parameter estimates used to compute (17) may be found in Appendix 2. Tables 1 and 2 report only a subset of these estimates.

of the values obtained to compute the optimal interest rate reported in Table 4.

Whether the zero interest rate bound is likely to constrain exapansionary monetary policy depends on the magnitude of business cycle contractions. In order to determine which countries are more likely than others to face conditions in which a zero rate of interest is a binding constraint, I use (17) to obtain a measure of the output gap that would have accompanied an interest rate of 0%. The formula for this measure is obtained by setting $i_t = 0$ om (12) and solving for y_t. This value, which is reported in the row labeled 'max gap = y_t^*' is obtained as follows:

$$y_t^* \;=\; \frac{-[\hat{K} \;\; + + \;\mathbf{g}_3 \;\mathbf{x}_t \;\; + \pi^* \; + \; r^*]}{\hat{g}_2} \tag{31}$$

where $\hat{K}$ and $\hat{g}_2$ are average values of $\bar{K}$ and g_2, respectively, over the range $0.25 \leq \lambda \leq 50$. The values of y_t^* reported in Table 4 were calculated with $\delta = 0.99$, $\pi^* = 0.02$, and $r^* = 0.035$.

For each country, the row labeled 'optimal i_t' in Table 4 reports the annual interest rates that would have been required to hold inflation at a target level of 2% (with $\delta = 0.99$, $r^* = 0.035$, and $0.25 \leq \lambda \leq 50$). It is evident that there are three countries — Canada, France, and the UK — for whom the zero interest rate floor may have represented a binding constraint on monetary policy over the sample period. According to Table 4, France would have found its ability to implement expansionary interest rate policy to be constrained for two thirds of the sample period. Only during the late 1980s and early 1990s would the Bank of France been free to implement the optimal monetary policy. For the Bank of England, on the other hand, expansionary monetary policy would have been constrained from 1982 to 1984 and again from 1991 to 1993. For Canada, the calculated optimal interest rates are negative for the period 1993 to 1995.[17] For the remaining three countries, the actual output gap (given in the row labeled 'GDP gap') lies well above the maximum output gap that monetary policy could address without breaching the zero interest rate bound.

[17]The results for Canada for the year 1982 are anomalous. The regression results indicate that the Canadian output gap was positively related to the real interest rate in 1981 and 1982. Because of the positive relationship between the real interest rate, an interest rate reduction would have been required to stabilize output even though the output gap was positive.

TABLE 4

Optimal Interest Rates and Maximum Output Gaps

(percentages; $\delta = 0.99$, $\pi^* = 0.02$, $0.25 \leq \lambda \leq 50$)

	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Canada															
GDP gap = y_t	8.42	1.51	0.98	3.45	4.46	4.08	4.61	5.72	4.50	0.62	-4.83	-7.72	-9.16	-8.33	-10.17
max gap = y_t^*	4.89	-3.04	-3.18	-2.39	-1.60	-1.34	-2.10	-3.14	-3.67	-5.65	-5.65	-5.65	-5.65	-5.65	-5.65
optimal i_t	-3.09	12.67	11.81	16.04	18.38	18.22	17.92	18.01	15.73	8.13	1.06	-2.68	-4.56	-3.48	-5.86
France															
GDP gap = y_t	0.99	-0.41	-1.20	-1.43	-1.04	-0.91	1.39	3.46	3.84	2.52	1.76	-1.81	-3.95	-1.20	-1.84
max gap = y_t^*	1.05	1.91	1.48	-0.09	-0.16	1.03	-1.44	-1.66	-1.28	0.97	-0.06	1.09	0.81	1.33	0.94
optimal i_t	-0.14	-6.32	-7.28	-3.65	-2.41	-5.29	7.70	13.93	13.93	4.22	4.89	-7.88	-12.93	-6.87	-7.57
Germany															
GDP gap = y_t	-5.11	-5.83	-5.90	-6.25	-3.08	-2.64	-1.34	-0.51	3.24	5.93	5.64	0.60	-0.60	-0.22	
max gap = y_t^*	-6.76	-9.15	-7.80	-9.58	-11.20	-13.49	-12.23	-10.61	-18.20	-19.89	-17.42	-9.34	-9.87	-10.58	
optimal i_t	1.27	2.55	1.46	2.56	6.25	8.35	8.38	7.76	16.49	19.86	17.74	7.64	7.13	7.97	
Italy															
GDP gap = y_t	0.67	-1.02	-1.03	-1.11	-0.90	-0.48	0.85	1.08	0.54	-0.87	-2.61				
max gap = y_t^*	-5.48	-5.54	-5.68	-5.73	-5.75	-5.92	-5.88	-5.81	-5.71	-5.72	-5.66				
optimal i_t	12.93	9.49	9.77	9.70	10.21	11.45	14.17	14.52	13.16	10.20	6.42				
UK															
GDP gap = y_t	-5.00	-3.50	-3.31	-1.73	0.36	2.95	5.73	5.78	4.07	-0.02	-2.66	-2.53	-0.78	-0.70	-0.70
max gap = y_t^*	2.66	0.66	-1.14	-2.35	-3.19	-5.89	-6.78	-3.94	-1.04	1.06	2.08	0.79	-2.60	-1.96	-3.63
optimal i_t	-34.78	-18.91	-9.86	2.82	16.09	40.14	56.78	44.11	23.19	-4.91	-21.53	-15.08	8.26	5.69	13.32
USA															
GDP gap = y_t	-5.02	-3.73	-0.25	0.33	0.68	1.18	2.50	2.48	0.76	-2.94	-2.24	-1.70	-0.23	1.43	1.30
max gap = y_t^*	-13.90	-19.10	-21.92	-22.75	-23.37	-22.66	-21.65	-19.35	-17.04	-13.36	-9.83	-9.65	-14.53	-16.77	-17.69
optimal i_t	4.80	7.95	11.32	12.10	12.59	12.76	12.98	11.82	9.69	6.00	4.70	4.86	7.89	9.91	10.38

6. Model and Parameter Variation

6.1 Model Variation

The version of Svensson's model employed in this study differs from other applications in a number of ways. In Svennson's original model, the Phillips curve has $\alpha_0 = 0$ and $\alpha_1 = 1$. Rather than imposing these parameter restrictions, I have allowed all parameters to be determined by the data. Although the results I obtain support the unrestricted model, one might still argue in favor of Svensson's original specification on theoretical grounds. In particular, adherents of the accelerationist Phillips curve might reasonable argue that inflation would, in the absence of stabilizing monetary policy, exhibit a unit root and that it is the impact of monetary policy on observed inflation outcomes that causes the inflation process to be stationary. If one accepts this argument, then the optimal monetary policy should be derived with $\alpha_0 = 0$ and $\alpha_1 = 1$ in (1). Another feature of the model specification that sets it apart from those typically used in empirical applications of Svensson's model is the inclusion of the country-specific $\mathbf{x}_t$ vector in (2).[18] The intercept term β_0 is made necessary by the inclusion of $\mathbf{x}_t$ variable to ensure that in long run equilibrium output gap implied by (2) is zero.

The model specification I have employed results in a somewhat more general interest rate response function than the two-parameter Taylor rules that have more commonly been employed. Deriving the optimal interest rate rule under the constraints $\alpha_0 = \beta_0 = \mathbf{b_3} = 0$ and $\alpha_1 = 1$, yields the two-parameter Taylor rule:

$$i_t \; - \; \pi_t \; = \; \gamma_1 \; [\pi_t - \pi^*] \; + \; \gamma_2 \; y_t \; + \; r^*. \tag{32}$$

where

$$\gamma_1 \; = \; \frac{\delta\alpha_2\chi}{\beta_2(\lambda + \delta\alpha_2^2\chi)} \tag{33}$$

$$\gamma_2 \; = \; \frac{[(1+\beta_1)\delta\alpha_2^2\chi + \lambda\beta_1]}{\beta_2(\lambda + \delta\alpha_2^2\chi)} \tag{34}$$

[18] Although Svennson (1997) does analyze a model in which an $\mathbf{x}_t$ vector of additional variables is included in the aggregate demand equation, such variables have not typically been included in other empirical applications.

TABLE 5

Optimal Interest Rates and Maximum Output Gaps

(percentages; $\alpha_0 = \beta_0 = \mathbf{b_3} = 0$, $\alpha_1 = 1$, $\delta = 0.99$, $\pi^* = 0.02$, $0.25 \leq \lambda \leq 50$)

	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Canada															
GDP gap = y_t	8.42	1.51	0.98	3.45	4.46	4.08	4.61	5.72	4.50	0.62	-4.83	-7.72	-9.16	-8.33	-10.17
max gap = y_t^*	5.80	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90	-3.90
optimal i_t	-2.49	7.63	6.88	10.36	11.79	11.25	11.99	13.57	11.84	6.37	-1.30	-5.37	-7.41	-6.23	-8.82
France															
GDP gap = y_t	0.99	-0.41	-1.20	-1.43	-1.04	-0.91	1.39	3.46	3.84	2.52	1.76	-1.81	-3.95	-1.20	-1.84
max gap = y_t^*	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27	-2.27
optimal i_t	7.91	4.50	2.59	2.02	2.97	3.28	8.87	13.89	14.81	11.62	9.78	1.12	-4.07	2.60	1.03
Germany															
GDP gap = y_t	-5.11	-5.83	-5.90	-6.25	-3.08	-2.64	-1.34	-0.51	3.24	5.93	5.64	0.60	-0.60	-0.22	
max gap = y_t^*	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	-6.42	
optimal i_t	1.12	0.51	0.44	0.15	2.86	3.24	4.36	5.06	8.27	10.58	10.33	6.02	4.98	5.32	
Italy															
GDP gap = y_t	0.67	-1.02	-1.03	-1.11	-0.90	-0.48	0.85	1.08	0.54	-0.87	-2.61				
max gap = y_t^*	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18	-2.18				
optimal i_t	7.18	2.92	2.90	2.68	3.24	4.30	7.65	8.24	6.87	3.30	-1.07				
UK															
GDP gap = y_t	-5.00	-3.50	-3.31	-1.73	0.36	2.95	5.73	5.78	4.07	-0.02	-2.66	-2.53	-0.78	-0.70	-0.70
max gap = y_t^*	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21	-1.21
optimal i_t	-17.27	-10.42	-9.56	-2.38	7.12	18.93	31.56	31.79	24.01	5.40	-6.60	-6.02	19.44	2.29	2.32
USA															
GDP gap = y_t	-5.02	-3.73	-0.25	0.33	0.68	1.18	2.50	2.48	0.76	-2.94	-2.24	-1.70	-0.23	1.43	1.30
max gap = y_t^*	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22	-8.22
optimal i_t	2.14	3.01	5.33	5.72	5.95	6.27	7.17	7.16	6.01	3.54	4.04	4.36	5.35	6.46	6.37

$$\chi \;=\; \frac{\left[\delta\alpha_2^2 - \lambda(1-\delta)\right] \;+\; \sqrt{\left[\delta\alpha_2^2 - \lambda(1-\delta)\right]^2 \;+\; 4\delta\alpha_2^2\lambda}}{2\delta\alpha_2^2}. \tag{35}$$

Setting $\pi_t = \pi^* = 0.02$ and $r^* = 0.035$ in (32) yields the following optimal interest rate response function

$$i_t \;=\; 0.055 \;+\; \gamma_2 y_t. \tag{36}$$

The magnitude of the output gap that would have accompanied an interest rate of 0% may be obtained directly from (36) by setting $i_t = 0$ and then solving for y_t. Thus, from (36) $y_t^* = -(\gamma_2 y_t)/0.055$. The optimal interest rate and maximum output gap measures that are consistent with (36) are given in Table 5. As in Table 4, the values in Table 5 are calculated with $\delta = 0.99$ and $0.25 \leq \lambda \leq 50$. Note that when the optimal output gap is computed on the basis of (36), it becomes time-invariant.

A comparison of the estimated counter-factuals given in Tables 4 and 5 indicates some sensitivity to model variation with respect to the magnitude of the maximum output gap and the timing of negative optimal interest rates. The qualitative results, however, are not greatly affected.

6.2 Parameter Variation

Empirical studies of monetary policy in the post Bretton Woods period have typically employed quarterly data. Consequently, there are relatively few annual parameter estimates available against which to compare the estimates in Tables 1 and 2. A recent article by Orphanides and Wieland (2000) is a rare exception. Orphanides and Wieland provide annual estimates of α_2, β_1, and β_2 for the Eurozone and the United States over the period 1976-1998. Orphanides and Wieland use an accelerationist Phillips curve where, by assumption, $\alpha_0 = 0$ and $\alpha_1 = 1$. Averaging the parameter estimates given in Tables 1 and 2 for France, Germany, and Italy yields $\alpha_2 = 0.38$, $\beta_1 = 0.75$, and $\beta_2 = 0.56$, all of which are close to the values that Orphanides and Wieland obtain for the Eurozone as a whole. However, there is less agreement with regard to the parameter estimates obtained for the United States. In particular, Orphanides and Wieland's estimates of α_2 and β_2 differ significantly from those obtained here; the β_1 estimates, on the other hand, are virtually identical. Using OECD data Orphanides and Wieland obtain $\alpha_2 = 0.39$ and $\beta_2 = 0.32$ whereas my parameter estimates are $\alpha_2 = 0.18$ and $\beta_2 = 0.85$. In order to test the sensitivity of the US results given in

Table 6

US Counter-Factuals: Alternative Parameter Values

$\alpha_0 = \beta_0 = \mathbf{b_3} = 0$, $\alpha_1 = 1$, $\alpha_2 = 0.39$, $\beta_1 = 0.47$, $\beta_2 = 0.32$

$\delta = 0.99$, $\pi^* = 0.02$, $0.25 \leq \lambda \leq 50$

	1982	1983	1984	1985	1986	1987	1988	1989
GDP gap = y_t	8.42	1.51	0.98	3.45	4.46	4.08	4.61	5.72
max gap = y_t^*	-3.21	-3.21	-3.21	-3.21	-3.21	-3.21	-3.21	-3.21
optimal i_t	-3.09	-0.88	5.07	6.07	6.66	7.51	9.79	9.74
	1990	1991	1992	1993	1994	1995	1996	
GDP gap = y_t	4.50	0.62	-4.83	-7.72	-9.16	-8.33	-10.17	
max gap = y_t^*	-3.21	-3.21	-3.21	-3.21	-3.21	-3.21	-3.21	
optimal i_t	6.80	0.47	1.67	2.59	5.11	7.95	7.73	

Table 5, I have recalculated the optimal interest rate and maximum output gap values for the United States using Orphanides and Wieland's estimates.[19] The results are reported in Table 6.

There are a number of obvious differences between the US counter-factuals given in Tables 5 and 6. First, the estimated maximum output gap is considerably smaller when Orphanides and Wieland's parameter estimates are used. Second, according to Table 6, the optimal US interest rate would have been negative in 1982 and 1983 if a 2% inflation target had been achieved in those two years. In Table 5, by contrast, the optimal interest rate is always non-negative. Finally, the variability of the optimal interest rate is somewhat higher in Table 6 than in Table 5. Clearly, the results are sensitive to changes in the magnitudes of the parameter estimates. However, given the size of the variation in the α_2 and β_2 estimates employed in this sensitivity analysis, the qualitative results are remarkably robust. It seems fairly safe to say that from 1984-1996, a 2% inflation target would not have caused US monetary policy to be constrained by the zero interest rate bound.

[19] In the model that Orphanides and Wieland (2000) estimate, they set $\alpha_0 = 0$, $\alpha_1 = 1$, and $\mathbf{b_3} = 0$, which are also the assumptions that I impose on the model in Section 6.1. It is for this reason that I use the values given for the US in Table 5, rather than those given in Table 4, to assess the impact of parameter variation on the estimated US counter-factuals.

7. Conclusion

In this article I used Svennson's (1997) dynamic model to determine the impact that a number of key structural characteristics have on the downward flexibility of interest rates at low rates of inflation. The theoretical results indicate that monetary authorities will be more constrained by the zero interest rate bound (i) the more responsive is inflation to the output gap, (ii) the higher is the persistence of the output gap over time, and (iii) the lower is the responsiveness of aggregate demand to changes in the interest rate.

A counter-factual experiment was used to assess the quantitative implications of adopting, and successfully achieving, a 2% inflation target in six countries. The results of this experiment indicate that the zero interest rate floor would have been binding for three of these countries. For Canada, France and the UK, an optimal interest rate reponse would have required violation of the zero interest rate bound on several occasions, and far too often to be considered a rare event. These results suggest that a target inflation rate of 2% may not be sufficiently high to prevent the zero interest rate floor from binding in all countries.

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Appendix 1. Determination of k_1 and k_2

Solutions for k_1 and k_2 can be obtained by applying the envelope theorem to (5) and (6). Using (6) to replace $V(\pi_{t+2|t+1})$ in (5) and taking the derivative of the expression in braces with respect to $\pi_{t+1|t}$ yields

$$V_\pi(\pi_{t+1|t}) \;=\; (\pi_{t+1|t} - \pi^*) + \delta\alpha_1\{k_1 + k_2(\pi_{t+2|t} - \pi^*)\}. \tag{A.1}$$

Using (1) and (7), $\pi_{t+2|t}$ can be expressed as

$$\pi_{t+2|t} \;=\frac{\lambda\alpha_0 - \delta\alpha_2^2 k_1}{\lambda + \delta\alpha_2^2 k_2} \;+\; \frac{\delta\alpha_2^2 k_2 \pi^*}{\lambda + \delta\alpha_2^2 k_2} \;+\; \frac{\lambda\alpha_1}{\lambda + \delta\alpha_2^2 k_2}\,\pi_{t+1|t}. \tag{A.2}$$

Substituting (A.2) into (A.1) results in

$$V_\pi(\pi_{t+1|t}) \;=\; \left[1 \;+\; \frac{\delta\alpha_1^2\lambda k_2}{\lambda + \delta\alpha_2^2 k_2}\right](\pi_{t+1|t} - \pi^*) + \frac{(\alpha_1 - 1)\delta\lambda k_2}{\lambda + \delta\alpha_2^2 k_2}\,\pi^* + \frac{\lambda\delta\alpha_1(k_1 + \alpha_0 k_2)}{\lambda + \delta\alpha_2^2 k_2}. \tag{A.3}$$

Differentiating the conjectured solution for $V(\pi_{t+1|t})$, given by (6), with respect to $\pi_{t+1|t}$ yields

$$V_\pi(\pi_{t+1|t}) \;=\; k_1 + k_2(\pi_{t+1|t} - \pi^*). \tag{A.4}$$

Using (A.3) to identify the coefficients in (A.4) produces

$$k_1 \;=\; \frac{\lambda\delta\alpha_1 k_2[\alpha_0 - (1 - \alpha_1)\pi^*]}{\lambda(1 - \delta\alpha_1) + \delta\alpha_2^2 k_2} \tag{A.5}$$

$$k_2 \;=\; 1 \;+\; \frac{\delta\alpha_1^2\lambda k_2}{\lambda + \delta\alpha_2^2 k_2} \tag{A.6}$$

It is evident from (A.5) and (A.6) that solving for k_2 identifies both k_1 and k_2. Rearranging (A.6) yields the quadratic polynomial

$$\delta\alpha_2^2 k_2^2 \;+\; [\lambda - (\lambda\alpha_1^2 + \alpha_2^2)\delta]k_2 \;-\; \lambda \;=\; 0. \tag{A.7}$$

Solving (A.7) for k_2 yields

$$k_2 \;=\; \frac{\left[\delta\alpha_2^2 - \lambda(1 - \delta\alpha_1^2)\right] \;+\; \sqrt{\left[\delta\alpha_2^2 - \lambda(1 - \delta\alpha_1^2)\right]^2 \;+\; 4\delta\alpha_2^2\lambda}}{2\delta\alpha_2^2}. \tag{A.8}$$

Only the positive root of (A.7) is a solution for k_2 because, from (A.6), k_2 must equal 1 for all non-zero values of δ and α_2 when $\lambda = 0$; this condition is not satisfied by the negative root.

Appendix 2. Details of Empirical Procedures

A2.1 Estimation of Equations (1) and (2)

Annual data from the International Monetary Fund's International Financial Statistics (IFS) on CD-ROM was used to estimate (1) and (2) for each country. In each case, the output gap y_t was constructed as the deviation of the natural log of real GDP (IFS line 99b.r) from its linear trend value in period t. For all countries except Germany, the inflation rate π_t was measured as the change in the natural log of the GDP deflator from period $t-1$ to period t. National GDP deflators were constructed as the ratio of nominal GDP (IFS line 99b.c) to real GDP. The Consumer Price Index (IFS line 64) was used to measure German inflation because real GDP figures are not available for Germany prior to 1979. The interest rate employed was a short-term market rate (IFS line 60bs for France and 60b for all other countries). Two types of dummy variables were used, step dummies (SD) and pulse dummies (PD). The dates associated with each dummy indicate the years for which the value of the dummy was set equal to 1. For example, SD7579 indicates that the step dummy has a value of 1 from 1975 to 1979, inclusive, and a value of zero in evey other year. All other, country-specific variables used in estimating equation (2) are defined below, immediately following the equation in which they appear.

Estimation results for (1) and (2) are reported below on a country-by-country basis. In each case, variables without a superscript are domestic variables. Foreign variables are identified by a superscript composed of the first three letters of the relevant country's name. In the case of the United States and the United Kingdom, the superscript is composed of the intials US and UK, respectively. The estimation period for each country is given in parentheses beside the country name. The coefficient of determination R^2 is given immediately following each equation. The F-statistics associated with Lagrange multiplier tests for first and second-order serial correlation are also reported. Neither the first-order statistic, LM(1), nor the second-order statistic, LM(2), is significant at the 10% level for any country.

1. Canada (1975-1996)

$$\begin{array}{ccccccccc} \pi_t & = & 0.013606 & + & 0.030769\ \text{SD7981} & + & 0.496440\ \pi_{t-1} & + & 0.132408\ y_{t-1} \\ & & (3.5564) & & (5.3572) & & (6.4581) & & (3.1480) \end{array}$$

$$R^2 = 0.941842 \qquad \text{LM}(1) = 0.0019 \qquad \text{LM}(2) = 0.0319$$

$$
\begin{array}{rlllll}
y_t & = & 0.012089 & + \; 0.938622\, y_{t-1} & + \; 1.817638(\mathrm{SD8182})(i-\pi)_{t-1} \\
 & & (2.0241) & (18.8376) & (5.7809) \\
 & - & 0.731140(i-\pi)_{t-1} & - \; 0.103863\mathrm{SD8182} & - \; 0.119246\, q^{us}_{t-1}\mathrm{SD8390} \\
 & & (5.2084) & (5.3949) & (4.3245)
\end{array}
$$

$$R^2 = 0.978682 \qquad \mathrm{LM}(1) = 0.0040 \qquad \mathrm{LM}(2) = 0.5422$$

The variable q^{us} is the real Can/US exchange rate calculated using the Canadian and US GDP deflators and the average bilateral Can/US exchange rate (IFS line rf).

2. France (1975-1996)

$$
\begin{array}{rllll}
\pi_t = & -\;0.008442 & + \; 0.014131\mathrm{SD7880} & + \; 0.980997\pi_{t-1} & + \; 0.293297 y_{t-1} \\
 & (1.8181) & (2.4450) & (15.6500) & (2.5862)
\end{array}
$$

$$R^2 = 0.956566 \qquad \mathrm{LM}(1) = 1.1982 \qquad \mathrm{LM}(2) = 0.7075$$

$$
\begin{array}{rlllll}
y_t & = & 0.043040 & + \; 0.969312 y_{t-1} & - \; 0.805115(\mathrm{SD7580}) y_{t-1} \\
 & & (6.3031) & (7.917920) & (2.3338) \\
 & - & 0.420430(i-\pi)_{t-1} & - \; 0.912145\pi^{ger}_{t-2} & + \; 0.055866\Delta e^{us}_{t-1} \\
 & & (5.5516) & (5.8293) & (2.8755)
\end{array}
$$

$$R^2 = 0.892232 \qquad \mathrm{LM}(1) = 0.3981 \qquad \mathrm{LM}(2) = 2.3183$$

The variable Δe^{us} is the change in the natural log of the average nominal France/US exchange rate (IFS line rf).

3. Germany (1975-1995)

$$
\begin{array}{rlll}
\pi_t = & 0.0111584 & + \; 0.584158\pi_{t-1} & + \; 0.197923 y_{t-1} \\
 & (2.9272) & (5.3652) & (4.3382)
\end{array}
$$

$$R^2 = 0.795353 \qquad \mathrm{LM}(1) = 2.8107 \qquad \mathrm{LM}(2) = 1.3212$$

$$
\begin{array}{rlllll}
y_t & = & 0.009774 & + \; 0.631462 y_{t-1} & - \; 0.856270(i-\pi)_{t-1} \\
 & & (1.8456) & (11.8118) & (6.3956) \\
 & + & 0.259669\Delta Y^{us}_{t-2} & - \; 0.066758\Delta e^{us}_{t-1} & + \; 0.055489\mathrm{SD9092} \\
 & & (2.7056) & (3.7586) & (8.8869)
\end{array}
$$

$$R^2 = 0.969949 \qquad \mathrm{LM}(1) = 0.0071 \qquad \mathrm{LM}(2) = 0.0166$$

The variables ΔY^{us} and Δe^{us} denote the change in the natural logarithm of US real GDP and the change in the natural log of the average Germany/US nominal exchange rate (IFS line rf), respectively.

4. Italy (1973-1992)

$$\begin{aligned}
\pi_t = {} & 0.016422 + 0.161620\text{SD7380} + 0.700819\pi_{t-1} \\
& (1.8290) \qquad (4.5825) \qquad (8.2714) \\
& + 0.661509 y_{t-1} - 0.873030(\text{SD7380})\pi_{t-1} - 1.236094(\text{SD7379}) y_{t-1} \\
& (3.6308) \qquad (3.7122) \qquad (3.7879)
\end{aligned}$$

$R^2 = 0.963075$ LM(1) = 1.0512 LM(2) = 0.8403

$$\begin{aligned}
y_t = {} & 0.031946 + 0.662790 y_{t-1} - 0.403751(i-\pi)_{t-1} \\
& (4.9140) \qquad (7.1038) \qquad (5.9444) \\
& - 0.0598952\pi^{ger}_{t-1} - 0.057373\text{PD75} \\
& (3.9880) \qquad (5.9666)
\end{aligned}$$

$R^2 = 0.885715$ LM(1) = 0.0891 LM(2) = 2.2659

5. United Kingdom (1975-1996)

$$\begin{aligned}
\pi_t = {} & 0.022138 + 0.490036\pi_{t-1} + 0.0194312 y_{t-1} + 2.541687(\text{SD7580}) y_{t-1} \\
& (5.9378) \qquad (11.9527) \qquad (2.7246) \qquad (8.9847)
\end{aligned}$$

$R^2 = 0.960926$ LM(1) = 0.1893 LM(2) = 0.4003

$$\begin{aligned}
y_t = {} & 0.061433 + 0.666267 y_{t-1} - 0.146825(i-\pi)_{t-1} \\
& (5.7983) \qquad (5.6662) \qquad (3.0746) \\
& - 1.521862\pi^{ger}_{t-1} - 0.515498\Delta Y^{ger}_{t-1} \\
& (6.1123) \qquad (3.2278)
\end{aligned}$$

$R^2 = 0.884930$ LM(1) = 1.2712 LM(2) = 2.6031

6. United States (1975-1996)

$$
\begin{aligned}
\pi_t \;=\; & \underset{(4.2851)}{0.016689} + \underset{(4.8616)}{0.024307\mathrm{SD7781}} + \underset{(6.3355)}{0.506153\pi_{t-1}} + \underset{(2.3206)}{0.181972y_{t-1}} \\
& - \underset{(2.1264)}{0.015952\mathrm{PD96}}
\end{aligned}
$$

$$R^2 = 0.929046 \qquad \mathrm{LM}(1) = 1.0396 \qquad \mathrm{LM}(2) = 0.8335$$

$$
\begin{aligned}
y_t \;=\; & \underset{(6.2078)}{0.0404026} + \underset{(3.8708)}{0.447961y_{t-1}} - \underset{(6.6169)}{0.854082(i-\pi)_{t-1}} \\
& - \underset{(3.0960)}{0.483859\pi^{ger}_{t-1}} - \underset{(4.6114)}{0.388817y^{ger}_{t-1}} + \underset{(3.2164)}{0.718136(\mathrm{SD7581})(i-\pi)_{t-1}}
\end{aligned}
$$

$$R^2 = 0.875167 \qquad \mathrm{LM}(1) = 0.0010 \qquad \mathrm{LM}(2) = 2.3053$$

A2.2 Invariance Tests

The invariance tests conducted using the procedure described in Section 4.2 of the main text are summarized in Tables A2.1 and A2.2. The variables used to model the marginal processes in (1) and (2) are given in Table A3.1. Note that marginal processes are specified only once. Variables used as regressors for more than one country are specified in the section pertaining to their country of origin if they are used for domestic estimation or, if used only for foreign countries, in the section pertaining to the first country for which the variable is used.

The results of the invariance tests for the parameters α_1, α_2, β_1, β_2, β_{31}, and β_{32} are presented in Table A3.2. The calculated values of the test statistic and the distributions of the statistic under the null hypothesis are reported. An asterisk appended to the test statistic in Table A2.2 indicates rejection of the null hypothesis that the estimated parameter value is invariant at the 5% level of significance.

A2.3 Unit Root Tests

The results of the unit root tests undertaken are reported in Table A2.3. Augmented Dickey-Fuller tests were used for variables which did not contain any apparent structural breaks. F-tests were used to determine the appropriate form of the test equation. In those cases

Table A2.1

Marginal Processes

		Regressors[a]	R^2
Canada:	π_{t-1}	π_{t-2}, SD7981, SD8689YT_{t-1}	0.929097
	y_{t-1}	y_{t-2}, SD8292, SD8292YT_{t-1}, SD8292YT^3_{t-1}	0.942910
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, $(i-\pi)^{us}_{t-1}$, SD8182$(i-\pi)^{us}_{t-1}$, PD80	0.770946
	q^{us}_{t-1}	q^{us}_{t-2}, Δq^{us}_{t-2}, π^{us}_{t-3}, SD9195	0.859076
France:	π_{t-1}	π_{t-2}, y_{t-2}, SD8292π_{t-2}, PD94	0.954917
	y_{t-1}	y_{t-2}, SD7579YT_{t-1}, SD8790YT_{t-1}, SD9294YT_{t-1}	0.829176
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, SD7881(t-1), SD7881	0.912695
	Δe^{us}_{t-1}	Δe^{us}_{t-2}, $\Delta e^{ger/us}_{t-1}$	0.874298
Germany:	π_{t-1}	π_{t-2}, π_{t-3}, SD7981, PD86	0.922911
	y_{t-1}	y_{t-2}, SD7579, SD8285, SD9092	0.941247
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, i^{us}_{t-1}, i^{us}_{t-2}, PD86, SD7580(t-1), SD9092	0.919230
	ΔY^{us}_{t-2}	i^{us}_{t-3}, YT^{us}_{t-2}, PD82, PD91	0.762180
	Δe^{us}_{t-1}	SD7881, SD7881(t-1), PD86	0.613645
Italy:	π_{t-1}	π_{t-2}, SD7380π_{t-2}, SD7380π^{ger}_{t-2}	0.907503
	y_{t-1}	y_{t-2}, $(i-\pi)_{t-1}$, SD7980, SD8890	0.944798
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, SD8092π_{t-2}, PD92, PD75	0.943125
U.K.:	π_{t-1}	π_{t-2}, PD76, SD7980	0.906777
	y_{t-1}	y_{t-2}, SD8188, SD8188YT_{t-1}, SD9192	0.880105
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, PD76, SD8090	0.919834
	ΔY^{ger}_{t-1}	SD8591ΔY^{ger}_{t-2}, SD7576(t-1), SD7576, SD8182, PD93	0.771210
U.S.A.:	π_{t-1}	π_{t-2}, π^{jap}_{t-2}, SD7781, SD8791	0.895109
	y_{t-1}	y_{t-2}, y^{jap}_{t-3}, Δe^{jap}_{t-2}, PD82	0.884877
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, g^{ger}_{t-2}, SD7581, SD7581(t-1)	0.870782

[a]The variable YT_{t-1} is the trend value of the natural logarithm of real GDP at time $t-1$ and $e^{ger/us}_{t-1}$ is the natural logarithm of the DMark cost of one US dollar at time $t-1$. All other variables in the table are as defined in Section A3.1.

Table A2.2

Results of Invariance Tests

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_{31}$	$\hat{\beta}_{32}$
Canada	0.6065	0.6585	2.2230	2.6700	0.9197	
	$F(2,15)$	$F(4,13)$	$F(4,11)$	$F(3,11)$	$F(4,9)$	
France	0.7066	0.5033	2.6200	1.4300	0.7013	1.7879
	$F(4,10)$	$F(4,10)$	$F(4,7)$	$F(3,9)$	$F(4,8)$	$F(2,9)$
Germany	8.1250*	1.7541	5.2069*	2.3687	2.3423	10.1126*
	$F(4,13)$	$F(4,12)$	$F(2,10)$	$F(5,8)$	$F(4,8)$	$F(4,7)$
Italy	4.2509*	1.2995	7.9041*	0.5894	1.4539	
	$F(4,9)$	$F(4,8)$	$F(4,9)$	$F(3,11)$	$F(4,10)$	
U.K.	1.3805	1.1566	0.7504	0.6986	1.7876	1.2447
	$F(2,15)$	$F(4,12)$	$F(4,11)$	$F(2,14)$	$F(4,10)$	$F(3,13)$
U.S.A.	0.6885	0.2189	2.2571	2.3773	0.5222	1.5000
	$F(3,13)$	$F(4,10)$	$F(4,9)$	$F(4,10)$	$F(4,11)$	$F(4,10)$

where visual inspection of the data suggested the presence of structural change, Perron's (1989) procedure was employed. In particular, Perron's Model A was used to allow for a shift in either the unit root process or the time trend. Perron has shown that the critical values of the test statistic depend on the time period in which the structural break occurs. The year of the break and the proportion λ of the total observations occurring prior to the break are given in the third column of Table A2.3. The absence of an entry in this column indicates that there was no apparent break in the data over the sample period.

The test statistics obtained on the basis of Augmented Dickey-Fuller tests and Perron's test procedure are given in the last column of the table under the heading ADF/ADFP. It is evident from the reported results that all of the variables employed in estimating (1) and (2) are I(0) at a level of significance of at least 10%. In Table A2.3, significance of the test statistic at the 1% and 5% level is denoted by ** and *, respectively. Where there are no asterisks, the significance level of the test statistic is 10%. The presence of a deterministic time trend was rejected at the 5% level of significance for all variables.

Table A2.3

Unit Root Tests

	var.	lags	break/λ	ADF/ADFP
Canada	π	0		$\tau_\mu = -2.6352^*$
	i	1	$1992/\lambda = 0.9$	$\tau_\lambda = -3.8287^*$
	q^{us}	1		$\tau_\mu = -3.7908^*$
France	π	0		$\tau = -1.7342$
	$i - \pi$	0	$1989/\lambda = 0.27$	$\tau_\lambda =$ -3.6447
	Δe^{us}	0		$\tau = -3.0588^{**}$
Germany	π	0		$\tau_\mu = -2.7336$
	i	0		$\tau_\mu = -4.3628^{**}$
	ΔY^{us}	0		$\tau_\mu = -3.4638^*$
	Δe^{us}	0		$\tau = -3.0048^{**}$
Italy	π	0	$1983/\lambda = 0.55$	$\tau_\lambda = -5.8452^{**}$
	i	0		$\tau_\mu = -2.8172$
U.K.	π	0	$1980/\lambda = 0.27$	$\tau_\lambda = -4.3998^{**}$
	i	1	$1979/\lambda = 0.23$	$\tau_\lambda = -4.2279^*$
	ΔY^{ger}	0		$\tau = -2.0055^*$
U.S.A.	π	0	$1981/\lambda = 0.31$	$\tau_\lambda = -3.7160$
	i	1		$\tau_\mu = -2.9393^\dagger$
	π^{jap}	0		$\tau_\mu = -8.4581^{**}$
	Δe^{us}	0		$\tau = -3.1183^{**}$

†In order to achieve this result, the beginning of the sample period was expanded to 1960. All other results reported in the table correspond to the estimation periods for each country as noted in Section A2.1.