MERGING AUCTION HOUSES

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Merging Auction Houses[#]

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Abstract

In this paper, we study the incentives for market concentration of (online and traditional) auction houses. Would sellers and buyers be better off if two separate auction houses merged? We suppose that each auction house has a separate clientele of sellers and buyers. Sellers value their (identical) units at 0, while buyers have independent private values. Each auction house uses an ascending auction or by revenue equivalence any auction mechanism that allocates units efficiently among those buyers at that auction house. If no buyers are lost upon the merger, we find that efficiency gains increase, but that the expected sellers' revenue increases by more than the efficiency gains, leaving the buyers worse off. This result extends Bulow and Klemperer's (1996) insight that the competition of an additional bidder increases auction revenue by more than the ability to commit to an optimal auction with one less bidder; in our model, the extra competition created by having all of the bidders bid against each other after the merger more than offsets any supply effects. With an example, we show that if buyers choose whether to participate or not, it is possible upon a merger that so many buyers are lost, the sellers are actually worse off. We conclude that without transfers from sellers to buyers, the merger may or may not be profitable for sellers.

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1 Introduction

How important is the number of buyers to a seller? Bulow and Klemperer (1996) showed that a seller with a single-unit values an extra buyer more than the ability to commit to an optimal selling mechanism. In particular, they show the seller earns more revenue using an ascending (English) auction with no reserve price among $N + 1$ bidders than the seller would earn using an optimal auction with only N bidders. We extend Bulow and Klemperer (1996) to show that a seller using an ascending auction who has many units to sell will get more expected revenue by selling more units as long as the increase in quantity supplied results in a proportional increase in the number of buyers participating in the auction.

We develop this result to address an interesting question. Suppose there are two auction houses (like Sotheby's and Christie's or eBay and Yahoo) and suppose each auction house has its own set of bidders.[1] In this environment, what is the incentive for market concentration? Ellison and Fudenberg (2002) report that Sotheby's and Christie's each have around a 45% market share in auctions of fine art, whereas eBay is by far the largest internet auction house, dominating Yahoo or Amazon. We show in a stark setting that the auction houses can earn more revenue for their sellers by merging, if their separate sets of buyers are merged. Note that we do not model how auction houses extract payments from transactions. Instead, we treat auction houses and sellers as identical, observing that many auction houses charge sellers to use their auction sites, but not buyers.

Our result may shed some light on why eBay has such a dominant position in internet auctions. But why don't we see a single dominant auction house for fine art? We show that when the auction houses merge—along with their separate sets of buyers—efficiency gains increase, but that the sellers' revenue increases by

[1] The assumption that each auction house has a separate set of bidders is justified if there exists a prohibitive opportunity cost of bidding at two separate auction houses. At a brick and mortar auction house, a bidder may want to be physically present, or for an online auction house there may be some cost to entry. For example, to bid at eBay one must create an account, which takes a few minutes of time. Another barrier eBay uses is the reputation of its bidders. Whenever a bidder wins an item, if it sends its payment in quickly it can request the seller to give it positive feedback. Alternatively, if the bidder did not send its payment in quickly, the seller may give the bidder negative feedback. Feedback, both positive and negative, is kept track of and is made available by eBay, so that a bidder without a sufficiently high rating may be prevented from bidding for a particular item. Since a bidder's rating in one auction house is not given at a competing auction house, a bidder may choose to participate only at one auction house. And though we have no evidence of whether or not buyers actually choose to participate only at one auction house, Goolsbee and Chevalier (2002) show that buyers do not arbitrage when buying books online from Amazon and Barnes and Noble.

more than the efficiency gains, thereby leaving the buyers worse off. We use this result to show that if buyer participation is endogenous, then upon the merger, buyer participation may decrease by enough so that the merger is not profitable for the sellers after all.

The industrial organization literature treats mergers of firms differently than we do. A common story in the industrial organization literature has a group of Cournot oligopolists merging. The key difference in this literature is that the oligopolists, even *before* the merger, share the same set of buyers in the market. In contrast, our model assumes each auction house has a separate set of buyers before the merger. It is still interesting to compare our results. Salant, Switzer, and Reynolds (1983) find that if an exogenous merger is imposed on a subset of Cournot olipopolists, those oligopolists can actually be worse off than before the merger. The firms left out of the merger respond to the merger by increasing output, so that through reduced output or reduced price the merged firms may suffer reduced profit. Nevertheless, if all of the firms in the oligopoly were to merge into a monopoly, they would unambiguously earn higher profit. In our paper, our results imply that whenever a group of separate auction houses merge—whether they be some subset or indeed all of the auction houses—their profit will increase, given there is no loss of buyers. On the other hand, we show with an example, that even if all of the auction houses merge, the sellers can be worse off if the buyers' choice to participate is endogenous. Another key difference in our study of auction house mergers and the mergers of Cournot oligopolists is our assumption on the strategic behavior of firms. Cournot oligopolists restrict supply in order to achieve higher prices and profits; we assume, however, that sellers use efficient auctions, not restricting supply. That is, upon the merger, all of the units from both auction houses will be sold. We make this assumption to stack the deck against a merger being profitable. Nevertheless, we still find that increased competition of the buyers more than offsets the increased competition of the sellers.

As far as we know, nothing in the auction literature treats mergers of auction houses. Rather, the literature in auctions on mergers addresses what happens when a group of bidders merges or forms a bidding ring. In a merger in a private value auction of a single unit, the merged group of bidders acts on the highest value or first-order statistic of the bidders in the merger, while each bidder outside the merger acts only on its private value draw. This creates an asymmetry between the merged bidders and the remaining bidders that makes solving for the equilibrium in a first-price auction challenging (for example, see Marshall, Meurer, Richard, and Stromquist, 1994). Some of the work on these types of mergers studies how the reduced competition may affect the auction revenues (for example, see Waehrer and Perry, 2001). In a merger in a common value setting, there is also the issue of how merged bidders can use all of their relevant pieces of information (see

Mares, 2000). The difference between a bidding ring and a merger is in how easily the bidders can share their private information. With a merger it is assumed that bidders have no problems sharing their private information, but work on bidding rings has recognized the bidders' incentives to share information (see McAfee and McMillan, 1992).

The paper that is most closely related to ours is Ellison and Fudenberg (2002). They consider a simple model where buyers and sellers simultaneously decide which auction house to go to, and then with their location fixed, participate in an efficient auction. They ask whether or not two auction houses can coexist in equilibrium, without individual buyers or sellers having an incentive to defect to the other auction house. They find that there cannot exist two auction houses of drastically different sizes, but that there can exist two auction houses of approximately the same size, where the difference in sizes that can coexist depends on the distribution of the buyers' values. In our paper, we take the locations of sellers and buyers as given, but then ask whether as a group the sellers in one auction house wish to merge with the sellers of the other auction house, bringing the buyers along with them. We find that the sellers always want to merge for a broad class of distributions of buyers' values whenever no buyers are lost upon the merger. But there is a problem if no buyers are lost: the gains to trade increase, but the sellers' profit increases by more than the gains to trade, leaving the buyers worse off. This means with endogenous buyer participation, so many buyers might be lost that sellers would be worse off and would choose not to merge. This has the same flavor as in trade theory: even though free trade between two countries generates gains to trade, without transfers from one country to another or without transfers to a country's sectors that suffer from free trade, trade agreements may never go through.

The outline of our paper is as follows. In section 2, we introduce our model and show that both total efficiency and total seller revenue rises if no buyers are lost. We also show with an example that individual sellers in one auction house may be worse off after the merger if the sellers in the other auction house do not compensate them. In section 3, we consider the buyers' incentives to participate in these auctions. We show that if no buyers are lost upon the merger, then the buyers are worse off after the merger. We also show with an example that if a buyer's entry decision is endogenous, then enough buyers may be lost upon the merger that sellers may actually be worse off after a merger. We conclude in section 4.

2 The Merging of Auction Houses Increases Total Efficiency and Seller Revenue if No Buyers Are Lost

There are two auction houses, $j = A$ and B. Each auction house has a separate clientele of buyers and sellers. Sellers in auction house j supply a total of M_j

identical units. We can view auction house j as being composed of one seller with M_j units or M_j sellers each with one unit. Either way, we assume that sellers value units at 0. Auction house j has N_j bidders, each of whom demands a single unit. We assume that $N_j > M_j$. As illustrated in Figure 1 below, in this section we will consider what happens when auction house A merges with auction house B, assuming that the merged house sells all of the units and all of the bidders participate.

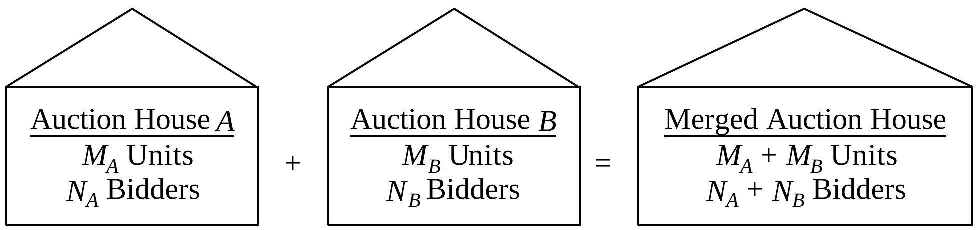

Figure 1: Auction House Merger

Throughout the paper we maintain an independent private values model: bidder n has a private value of v_n for a unit and each v_n is independently and identically distributed on an interval $[0, U]$ according to distribution function F. We assume that F is continuously differentiable with density f, which is everywhere positive on its support. Bidder n is risk neutral, seeking to maximize the expected difference between its value and the price it pays.

In this paper, we do not distinguish between auction houses and sellers. We view an auction house as the collection of sellers who sell their units in the auction house. Our assumption that auction houses and sellers are identical is valid if auction houses extract their payment from sellers, either charging entry fees to sellers for using their auctioning services or taking some percentage of the selling price.

We also assume that each auction house uses an *efficient* auction, where if there are M units sold those bidders who have the M highest values will each win one unit. For concreteness, we suppose that the auction house uses an ascending (English) auction with no reserve price, where efficiency obtains in equilibrium.[2] For simplicity, we do not model an optimal auction, though we note that all of

[2] In the ascending (English) auction of M units, the auctioneer will continuously raise the price beginning at price 0. Each bidder will depress a single button. A bidder can release its button at any price, and once a button is released it cannot be depressed again. The auction will stop at the first price p such that M or fewer buttons are depressed. Each bidder depressing a button at this price will win one unit and will pay the auctioneer p. It is a weakly dominant strategy for each bidder to depress its button at all prices below its value and to release its button if price exceeds its value.

proofs remain valid if we allow the auctioneer to use a positive reserve price (and the optimal auction in our symmetric, independent private values model is an ascending auction with an appropriately chosen reserve price). Our first result stems from the efficiency assumption:

Proposition 1: The total gains from trade increase if auction house *A* merges with auction house *B*.

Proof: An auction which sells M units to N bidders generates a total surplus of: $TS(M, N) = E\left\{\sum_{m=1}^{M} v^{m:N}\right\}$, where $v^{m:N}$ is the m^{th} highest (order statistic) of the N bidders' values. The proposition follows if the following inequality holds:

$$E\left\{\sum_{m=1}^{M_A} v^{m:N_A}\right\} + E\left\{\sum_{m=1}^{M_B} v^{m:N_B}\right\} < E\left\{\sum_{m=1}^{M_A+M_B} v^{m:N_A+N_B}\right\}.$$

To see that this inequality holds, the following argument is instructive. One way to generate random samples of N_A and N_B draws of bidder values from the distribution F is to first generate a random sample of $N_A + N_B$ draws from F and then randomly partition this sample, putting N_A values into subsample *A* and putting the remaining values into subsample *B*. Now consider a sample of $N_A + N_B$ draws, where the realizations are all different (any other sample does not occur with positive probability). With the merger, the highest $M_A + M_B$ valued buyers will be awarded units. This is the maximum surplus obtainable given these realized values. The only way the same surplus would arise if each auction house ran its own auction among the bidders in its subsample is if exactly M_A of the $M_A + M_B$ highest valued bidders were selected into subsample *A*. Any other partition would yield strictly less surplus since at least one bidder who does not have one of the highest $M_A + M_B$ highest values would obtain a unit. Since the probability that exactly M_A of the $M_A + M_B$ highest valued bidders would be selected into subsample *A* is less than one, the above inequality holds. *Q.E.D.*

Whether the auction houses wish to merge will depend not on whether the total surplus rises, but on whether the sellers' expected revenue rises. To address this, we first need to introduce a key function that we will use in our analysis: the marginal revenue of bidder *n* is defined as $MR(v_n) = v_n - \frac{1-F(v_n)}{f(v_n)}$. Those familiar with the auction literature will recognize this as Myerson's (1981) "virtual utility," but we prefer the Bulow and Roberts' (1989) interpretation that this is the marginal revenue a seller receives from bidder n. A seller cannot extract the full value from a buyer because the buyer has private information. An assumption that is common in the auction literature and that our analysis depends on is:

Assumption 1 (regularity condition): F is such that $MR(\cdot)$ is strictly increasing.

This assumption, which we maintain throughout the paper, will allow us to express the sellers' expected revenue in a convenient way for our analysis.

Revenue Equivalence Theorem (Myerson (1981), Engelbrecht-Wiggans (1988)): Consider *any* selling game with M units and N bidders. Then in *any* equilibrium such that the units are awarded efficiently and such that a bidder with value 0 has an expected payoff of zero, the expected sellers' revenue is:

$$SS(M, N) = ME\left\{v^{M+1:N}\right\} \tag{1}$$

And if $F(\cdot)$ satisfies the regularity condition, then we can also express sellers' expected revenue as:

$$SS(M, N) = E\sum_{m=1}^{M} MR(v^{m:N}) \,. \tag{2}$$

Proof: See appendix.

Equation (1) is simply the revenue from an ascending (English) auction. In this auction each bidder has a weakly dominant strategy of bidding sincerely (depressing the button until the price reaches its value and then releasing the button). In this equilibrium, the units will be awarded efficiently and the price will be the value of the $M + 1^{st}$ order statistic. A stronger version of revenue equivalence for single unit auctions was shown by Myerson (1981).[3] Engelbrecht-Wiggans (1988) extended revenue equivalence to multiple-unit auctions. Bulow and Roberts (1986) expressed revenue equivalence for a single-unit auctions in terms of marginal revenue, and Bulow and Klemperer (1996) used the regularity

[3] In stronger versions of revenue equivalence, the assumption of private values is not crucial nor are the assumptions of identical and regular distributions, but independence of the signals is crucial. Further, revenue equivalence across auction formats does not require an efficient allocation. More general versions of revenue equivalence say that each buyer's expected payment is a function solely of that buyer's probability of winning a good and the expected payment it makes when its value is at the lower end of its support. See Klemperer (1999) for more discussion on revenue equivalence.

condition to derive the formula for seller revenue given in equation (2). Bulow and Klemperer (1996) do not explicitly state the expression in (2) for $M > 1$ units, but certainly recognize it (see pages 592-593). Because our paper hinges on this result, we provide a proof, but we relegate it to the appendix since this is a standard result. The only innovation—and *innovation* is too strong a word—that we provide in our proof is expressing the formula for revenue in equation (2) using order statistics. By the Revenue Equivalence Theorem, the seller revenue is pinned down by the efficient assignment, so the seller revenue will be identical whether the efficient assignment results from an ascending (English) auction, a pay-your-bid auction, an $M + 1^{st}$ price auction, or individual sellers simultaneously running English auctions.[4]

We now state our main result.

Proposition 2: The total expected revenue increases if auction house A merges with auction house B.

Proof: Using equation (2), we need to show

$$E\left\{\sum_{m=1}^{M_A} MR(v^{m:N_A})\right\} + E\left\{\sum_{m=1}^{M_B} MR(v^{m:N_B})\right\} < E\left\{\sum_{m=1}^{M_A+M_B} MR(v^{m:N_A+N_B})\right\}.$$

This inequality can be demonstrated by applying the same argument used in the proof to proposition 1 to the random variables $MR(v_n)$ rather than v_n, noting from the regularity condition that $MR(v_n)$ is strictly increasing. *Q.E.D.*

Using proposition 2, we next state the following corollary, so that this result can be more easily interpreted in comparison to the Bulow and Klemperer (1996) result that a seller gets less expected revenue with an *optimal* auction among only N bidders than it does with an *efficient* auction among $N + 1$ bidders.

[4] In a pay-your-bid auction for M_j units, also known as a discriminatory auction, each bidder submits a sealed-bid. If a bidder submitted one of the highest M_j bids, it wins an object and pays its bid. If $M_j = 1$, then this is known as a first-price. In an $M_j + 1^{st}$ price auction, each bidder submits a sealed-bid. If a bidder submitted one of the highest M_j bids, it wins an object and pays the $M_j + 1^{st}$ highest bid. This type of auction is known as a Vickrey auction. Peters and Severinov (2001) and Bansal and Garg (2001) show that an efficient equilibrium is approximately obtained if each seller in the auction house simultaneously runs its own English auction and the bidders can bid across these auctions.

Corollary 1: If l is an integer with $l > 1$, then

$$SS(M, N) < SS(lM, lN).$$

The corollary indicates that the seller does better by increasing its supply if by doing so there is a proportional increase in bidders. The corollary results from proposition 2 by setting $M_A = M$, $N_A = N$, $M_B = lM–M$, and $N_B = lN–N$.

The result that a merger will take place relies on two key assumptions. The first assumption is that upon merging, the sets of buyers are merged as well: no buyers are lost after the merger. We relax this assumption in the next section. The second assumption is that sellers in the separate auction houses are able to share their gains to make all of the sellers in both auction houses better off. If, on the other hand, we view each auction house as a separate collection of sellers, and if each seller in an auction house with M units is only entitled to receive $1/M$ of revenue for each unit that it sells, then sellers in one auction house may be worse off after the merger even though total revenue increases. Thus, if sellers are unable to share the gains from merging, the merger will not be feasible. The following example illustrates this.

Example 1: Let the buyers' values be uniformly distributed on [0, 1] so that $F(v) = v$ and $f(v) = 1$. We will say that an auction house with M units uses a *proportional payment rule* if each seller receives $1/M^{\text{th}}$ of the revenue for each unit it supplies. Using equation (1), under the proportional payment rule each seller expects a price of $Ev^{M+1:N}$ per unit it supplies. With the uniform distribution, it can be shown that $Ev^{M+1:N} = \dfrac{N-M}{N+1}$. It follows that sellers in auction house A gain by the merger if and only if: $\dfrac{N_A - M_A}{N_A+1} \le \dfrac{N_A + N_B - M_A - M_B}{N_A + N_B + 1}$. This condition can be expressed as: $\dfrac{N_B}{M_B} \ge \dfrac{N_A+1}{M_A+1}$. Likewise, the sellers in B gain by the merger if and only if: $\dfrac{N_A}{M_A} \ge \dfrac{N_B+1}{M_B+1}$. If we think of N/M as a measure of the competition in an auction of M units to N bidders, then we can say that sellers in one auction house want the merger if and only if the competition in the other auction house is sufficiently high. If, for instance, $M_A = 1$ and $M_B = 2$, and $N_A = N_B$, then the seller in A will not want to merge with the sellers in B under the proportional payment rule. We summarize this example with the following remark:

Remark: Assuming that auction houses use a proportional payment rule, a merger may make some sellers worse off. If any seller can block the merger, it is possible that the two auction houses will not merge.

Compare our example with Ellison and Fudenberg (2002), who find that under the proportional payment rule, two auction houses *cannot* coexist if the ratio of buyers to units is too different. In contrast, our example shows that auction sites do not want to merge (so that they will coexist) if their ratio of bidders to units is too different. The difference in our models is that we ask whether as a group, the sellers in one auction house want to merge with the sellers in another auction house; whereas, Ellison and Fudenberg (2002) ask if any individual seller would defect from his auction house to sell his unit at the other auction house.

3 Bidder Participation

In the prior section, we have found that auction houses always have an incentive to merge if by doing so no buyers are lost and if the greater surpluses realized after the merger can be redistributed in such a way that no individual seller is worse off. In this section, we will show that under a stronger assumption than the regularity condition the buyers will be worse off after the merger, assuming no buyers are lost. We then will allow for bidders to choose whether to bid or not, where bidding will cost c.[5] In this environment, we will show with an example that enough bidders might be lost so that a merger is not profitable for the sellers. First, we make the following assumption:

Assumption 2 (Monotone Hazard Rate): Assume that $f(v)/[1 - F(v)]$ is strictly increasing in v.

Remark: Any distribution $F(\cdot)$ that has the monotone hazard rate necessarily satisfies the regularity condition. As examples, the uniform distribution has the monotone hazard rate, while the exponential distribution does not have the monotone hazard rate, but does satisfy the regularity condition.

We next show that buyers do not favor the merger.

Proposition 3: If F satisfies the monotone hazard rate and if no buyers are lost if auction house A merges with auction house B, then the buyer surplus decreases upon the merger.

[5] Levin and Smith (1994) model entry in the same way we do here: before bidding, a bidder must pay c to learn its private information.

Proof: Define buyer surplus in an auction with M units and N bidders as BS(M, N) = TS(M, N) – SS(M, N). From propositions 1 and 2, we have:

$$\begin{aligned} BS(M,N) &= E\left\{\sum_{m=1}^{M} v^{m:N} - MR(v^{m:N})\right\} \\ &= \sum_{m=1}^{M}\int_{0}^{U}\left[v-\left(v-\frac{1-F(v)}{f(v)}\right)\right] f^{m:N}(v)dv \\ &= \sum_{m=1}^{M}\int_{0}^{U}\frac{1-F(v)}{f(v)} f^{m:N}(v)dv \\ &= E\sum_{m=1}^{M}\Phi(v^{m:N}) \end{aligned}$$

where $\Phi(v)=\dfrac{1-F(v)}{f(v)}$ is the inverse of the hazard rate. The last equality follows because the hazard rate is monotone. In other words, $BS(M, N)$ is the expected value of the sum of the M lowest of the $\Phi(v)$'s. By the same logic used in the proof of proposition 1, $BS(M_A, N_A) + BS(M_B, N_B) > BS(M_A + M_B, N_A + N_B)$ since the merged auction selects out the $M_A + M_B$ lowest $\Phi(v)$'s, while the separated auctions sometimes selects higher $\Phi(v)$'s. *Q.E.D.*

The gist of proposition 3 is that upon the merger if all the buyers remain active, then the total buyer's surplus decreases. If buyer participation is endogenous, it then stands to reason that after the merger there may be less bidders than $N_A + N_B$. In the following example, we show that the loss of bidders may be severe enough that the merger will not take place.

Example 2: Again, let the buyers' values be uniformly distributed on [0, 1]. Let $c > 0$ be the cost a potential bidder must pay to draw its value from F. For an auction that sells M units efficiently, denote by $N(M, c)$ the number of bidders which participate in the auction. This can be calculated as the largest value of N such that buyers earn nonnegative profit:

$$\frac{BS(M,N)}{N} \geq c > \frac{BS(M,N+1)}{N+1}.$$

Note that $BS(M,N) = TS(M,N) - SS(M,N) = E\left\{\left(\sum_{m=1}^{M} v^{m:N}\right) - Mv^{M+1:M}\right\}$. Using that $Ev^{M+1:N} = \frac{N-M}{N+1}$ for the uniform distribution, it can be shown that $\frac{BS(M,N)}{N} = \frac{M(M+1)}{2N(N+1)}$. Whether the sellers will benefit by a merger will depend on the number of bidders that choose to participate. Below we consider two scenarios.

(*i*) Fewer bidders and sellers are better off: Let $M_A = M_B = 1$ and $\frac{1}{24} < c \leq \frac{1}{20}$. In this case, $N(1, c) = 4$ so that each auction house has 4 bidders without the merger, but only $N(2, c) = 7$ bidders after the merger. The merger results in losing one of the eight bidders, so that with the merger the expected price is 0.625, but without the merger the expected price is 0.6.

(*ii*) Fewer bidders and sellers are worse off: Let $M_A = M_B = 1$ and $\frac{3}{20} < c \leq \frac{1}{6}$. In this case, $N(1, c) = 2$ so that each auction house has 2 bidders without the merger, but only $N(2, c) = 3$ bidders after the merger. The merger results in losing one of the four bidders, so that with the merger the expected price is 1/4, but without the merger, the expected price is 1/3.[6]

[6] In this example, we have exploited the fact that the number of bidders in an auction is an integer. Now we show that if we drop this integer restriction, even with endogenous entry, the sellers always prefer a merger. Dropping the integer restriction, $N(M, c)$ is determined implicitly by: $c = \frac{M(M+1)}{2N(N+1)}$. In order for $N > M$, we assume that $c < 0.5$. Taking the derivative of the expected price the sellers receive $\frac{N(c,M)-M}{N(c,M)+1}$ with respect to M gives $\frac{-1}{N+1} + \frac{M+1}{(N+1)^2}\frac{2M+1}{2c(2N+1)}$. Substituting in $c = \frac{M(M+1)}{2N(N+1)}$ and simplifying gives $\frac{-1}{N+1} + \frac{1}{N+1}\left(\frac{2MN+N}{2MN+M}\right)$, which is positive since $N > M$. That is, the expected price is increasing in M, so that the sellers always want to merge.

It is also easy to derive scenarios where no bidders are lost upon the merger, so that proposition 2 implies that sellers would favor the merger. It follows that whether auction houses merge or not is ambiguous. Our conclusion from this example is that whether the sellers will want to merge will depend on how many bidders will be lost upon the merger. This conclusion is based on the buyers' surplus arising endogenously through the efficient auction—we have allowed for no transfers from sellers to buyers.

4 Conclusion

In this paper, we have shown that sellers in separate auction house with separate sets of bidders can increase their expected revenue if their auction houses merge, assuming that no bidders are lost upon the merger. We have also shown that the buyers' surplus would decrease upon the merger if no bidders are lost. Allowing for endogenous bidder participation, we showed with an example that whether seller revenue will increase upon the merger will depend on how many bidders are lost.

Another result obtained is that a seller who is thinking about increasing the quantity it sells should always do so if the increase in quantity attracts a proportional increase in bidders. But what if the seller faces a constant marginal cost of supplying additional units? To address this question we borrow a result from Ellison and Fudenberg (2002); they showed in their proposition 2 that: $\frac{TS(M,N)}{M} < \frac{TS(l\,M,\,l\,N)}{l\,M}$ where $l > 1$ is an integer. Assuming that the distribution of the buyers' values follows the monotone hazard rate, our proposition 3 implies that $\frac{BS(M,N)}{M} > \frac{BS(l\,M,\,l\,N)}{l\,M}$. Together, these imply that: $\frac{SS(M,N)}{M} < \frac{SS(l\,M,\,l\,N)}{l\,M}$. This result strengthens our corollary 1. Not only does seller revenue increase when supply is increased, but the seller revenue per unit increases as supply is increased. A seller with a constant marginal cost of production would want to increase supply as long as the increase results in a proportional increase in bidders. This result has some of the flavor of Bulow and Klemperer's (1996) result that a seller values competition more than its ability to commit to an optimal auction.

In our model for this paper, we have considered an efficient auction mechanism. If we would have allowed an auction house to use an optimal auction among its bidders, then the conclusion that the auction houses gain by merging would be trivial, when not allowing for endogenous bidder participation. The merged auction house can always replicate the separated auction procedures to get the same revenue. We view the efficient auction as stacking the deck against the sellers as far as possible, and even so, we have shown that additional competition of bidders brought about by the merger more than outweighs any additional competition by sellers (through increased supply) brought about by the merger.

To keep our paper simple, we have assumed an independent private values model. Our intuition is that our results would be even more pronounced in the affiliated environment of Milgrom and Weber (1982). In this affiliated environment Bulow and Klemperer (1996) show that an extra bidder stimulates revenue even more than in an independent private values model.

In this paper, we have assumed that sellers and the auction house they sell in are identical, so that the auction houses wanted to merge whenever this increased the sellers' expected revenue. We leave for future work to study how auction houses are middlemen between sellers and buyers, and whether an auction house's goal should be to maximize seller revenue (from which they can extract their payments for auctioning services).

5 Appendix

Proof of Revenue Equivalence Theorem: The following proof uses the techniques Myerson (1981) used in showing revenue equivalence in single-unit auctions. A nice textbook treatment of revenue equivalence for single-unit auctions is in Krishna (2002), and we follow that treatment somewhat. Consider any auction. Denote bidder n's equilibrium strategy by $s_n(v_n)$. Define the bidder's probability of a unit as $\overline{q}_n(s_n(v_n))$ and define the bidder's expected payment as $\overline{x}_n(s_n(v_n))$ given each player uses its equilibrium strategy. Then the bidder's expected payoff in equilibrium is: $\overline{q}_n(s_n(v_n))\,v_n - \overline{x}_n(s_n(v_n))$. Now define $q_n(v_n) = \overline{q}_n(s_n(v_n))$ and $x_n(v_n) = \overline{x}_n(s_n(v_n))$, so that the bidder's expected payoff in equilibrium is $U_n(v_n) \equiv q_n(v_n)v_n - x_n(v_n)$. Since no player wants to deviate in an equilibrium, it is necessary that $U_n(v_n) = \max_z \; q_n(z)v_n - x_n(z)$. Because $q_n(z)v_n - x_n(z)$ is an affine function of v_n, $U_n(v_n)$ is convex and absolutely continuous. Using an envelope theorem, we have $U_n'(v_n) = q_n(v_n)$ (Milgrom and Segal, 2002, provide such an envelope theorem). Since $U_n(v_n)$ is absolutely continuous, we may apply the Fundamental Theorem of Calculus to get: $U_n(v_n) = U_n(0) + \int_0^{v_n} q_n(t)\,dt$. Equating our two expressions for $U_n(v_n)$, we have $q_n(v_n)v_n - x_n(v_n) = U_n(0) + \int_0^{v_n} q_n(t)dt$. What we have shown is that for *any* equilibrium to *any* auction, each buyer's expected payment is a function only of the payoff the lowest type of that buyer obtains and the function that determines the probability that bidder gets the good. But under the hypothesis of the theorem, an efficient allocation determines the probability that the bidder obtain a good. In particular: $q_n(v_n) = \int_{v_{-n}} Q_n(v_n, v_{-n}) f(v_{-n}) dv_{-n}$, where Q_n (v_n, v_{-n}) is a function that equals 1 if v_n is among the M highest values of $v \equiv (v_n, v_{-n}) = (v_1, \ldots, v_N)$ and 0 otherwise. Setting $U_n(0) = 0$ which means that the expected payment is zero for a buyer who does not value a good at all, we have: $x_n(v_n) = q_n(v_n) v_n + \int_0^{v_n} q_n(t) dt$. The seller's expected revenue from bidder n is:

$$
\begin{aligned}
E[x_n(v_n)] &= \int_0^U q_n(v_n) v_n f(v_n) dv_n - \int_0^U \int_0^{v_n} q_n(t) f(v_n) dt dv_n \\
&= \int_0^U q_n(v_n) v_n f(v_n) dv_n - \int_0^U \int_{v_n}^U q_n(t) f(v_n) dv_n dt \\
&= \int_0^U q_n(v_n) v_n f(v_n) dv_n - \int_0^U q_n(t) \left(1 - F(t)\right) dt \\
&= \int_0^U MR(v) f(v_n) q_n(v_n) dv_n
\end{aligned}
$$

where we have switched the order of integration to get the second equality. Summing over each bidder's expected payment we have:

$$
\begin{aligned}
SS(M, N) &= \sum_{n=1}^{N} \int_{v_n} MR(v_n) q_n(v_n) f(v_n) dv_n \\
&= \sum_{n=1}^{N} \int_{v} MR(v_n) Q_n(v) f(v)\, dv \\
&= \int_{v} \sum_{n=1}^{N} MR(v_n) Q_n(v) f(v)\, dv \\
&= \int_{v} \sum_{m=1}^{M} MR(v^{m:N})\, f(v)\, dv \\
&= E\left\{ \sum_{m=1}^{M} MR(v^{m:N}) \right\}
\end{aligned}
$$

where the second to last equality uses the regularity assumption that says that a person with the m^{th} highest value has the m^{th} highest marginal revenue. This establishes equality (2) stated in the theorem. Equality (1) results by noting that one auction that satisfies the assumptions of the theorem (efficiency and that the a bidder with value 0 has an expected payoff of 0) is the ascending auction. In this auction each bidder will bid up to his value, and the auction will end as soon as the $M+1^{st}$ bidder drops out. Each of the M remaining bidders will win a unit and pay the auctioneer this price, $v^{M:N}$. *Q.E.D.*

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