A SUBSIDIZED VICKREY AUCTION FOR COST SHARING

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A Subsidized Vickrey Auction for Cost Sharing[*]

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Abstract

We introduce a subsidized Vickrey auction for cost sharing problems. Although the average, marginal, and serial cost sharing mechanisms are budget-balanced, they are not allocatively efficient and they do not induce players to truthfully reveal their values as a dominant strategy. The conventional Vickrey auction, on the other hand, is allocatively efficient and does induce truthful bidding as a dominant strategy, but also generates an overpayment. This paper modifies the conventional Vickrey auction so that some of the overpayment is used to subsidize additional production without upsetting the players' incentives to bid truthfully. Although this subsidized Vickrey auction is not allocatively efficient, it always Pareto dominates the conventional Vickrey auction and sometimes dominates other existing cost sharing mechanisms.

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1 Introduction

A group of players jointly produce a private good and decide how much everyone consumes and how much everyone pays. Examples include farmers sharing an irrigation system, office workers splitting a secretary's time, and divisions of a corporation using the same advertising department or training facility. In many respects, a cost-sharing problem is similar to a typical auction situation, where quantity is allocated among players who have different valuations. Unlike an auction, however, in a cost-sharing problem there is no opposing seller to allocate the goods and collect the payments: the players themselves design or choose a cost-sharing mechanism to accomplish these tasks.

In this paper, we study cost-sharing problems with independent private values. In this setting, many existing cost-sharing mechanisms—including the average, marginal, and serial mechanisms—are not allocatively efficient because the goods are not always allocated to the players who value them the most. Furthermore, many of these existing cost-sharing mechanisms do not induce players to truthfully reveal their values as dominant strategies. The question is then why not adopt the Vickrey (1961) auction to deal with cost-sharing problems since it induces truthful bidding as a dominant strategy and the resulting equilibrium is allocatively efficient. In an auction of goods by a seller, the Vickrey auction generates a profit for the seller because the buyers pay more than the production cost. In a cost-sharing problem, the Vickrey auction generates the same "profit." But with no opposing seller to collect this overpayment, the players have the thorny issue of what to do with this overpayment. Redistribution of the overpayment among the players could upset their incentives to bid truthfully and hence upset the allocative efficiency. And as we show with an example, if this overpayment is simply destroyed, then the Vickrey auction may give the players lower payoffs than they would get with other existing cost-sharing mechanisms.

There are mechanisms such as those studied by Arrow (1979), d'Aspremont and Gerard-Varet (1979a, 1979b) that forgo dominant strategies in order to achieve allocative efficiency and budget-balancedness; see Krishna (2002) for more on these mechanisms. However, if the players deem these mechanisms too difficult to implement, players may instead pursue ways to reduce the overpayment generated by the Vickrey auction while maintaining truthful bidding as a dominant strategy. Referring to modifications to his auction, Vickrey (1961, page 13) stated "However, it seems that all modifications that do diminish the [imbalance] of the scheme ... reintroduce a direct incentive for misrepresentation of the marginal value curves." One aim of our paper is to isolate those components of the Vickrey auction that ensure players do have the incentive to truthfully report their marginal values as dominant strategies. We find that allocative efficiency—though certainly a desirable property—is not among these components.

We introduce *perfect price discriminating* (PPD) mechanisms that operate as follows. Each player reports his marginal value curve or equivalently his piece of information which pins down his marginal value curve. The mechanism then generates a player specific supply curve for each player. A player receives the quantity where his reported marginal value curve intersects the supply curve generated for him, and he pays the area under the supply curve up to the quantity he wins. This payment rule is the analog of a perfectly price discriminating monopsonist buying an input. As we show in Proposition 1 if the supply curve generated for a player is independent of his reported marginal value curve, then it will be a dominant strategy for the player to report his true marginal value curve. We show that the conventional Vickrey (1961) auction can be reformulated as a PPD mechanism where the supply curve a player faces is the residual supply curve left over after the other players demands are satisfied. Since the residual supply curve faced by a player is independent of his own reported marginal value curve, the player has a dominant strategy to report his true marginal value curve, a well-known property of the Vickrey auction.

We then introduce a new PPD mechanism for cost-sharing, a mechanism we call the *subsidized Vickrey auction*. In the subsidized Vickrey auction, a player's supply curve is constructed independently of his reported marginal value curve in such a way that some of the overpayment he generates by displacing other players' bids is used to subsidize additional production. We show that when players have constant marginal values, the subsidized Vickrey auction always balances the budget and induces truthful bidding as a dominant strategy. Unlike the conventional Vickrey auction, this subsidized Vickrey auction is not allocatively efficient since the marginal cost will exceed everyone's marginal value. Although each individual player can be over-subsidized in the sense that the true marginal cost could be higher than his marginal value, the mechanism itself is feasible, always collecting enough payments to cover production costs. Even though the subsidized Vickrey auction is generally allocatively inefficient, we show that it Pareto dominates the conventional Vickrey auction. In some situations, it also Pareto dominates the average, marginal, and serial cost-sharing mechanisms.

Moulin and Shenker (1999) and Friedman and Moulin (1999) characterize the average, marginal, Aumann-Shapley, serial, and other cost-sharing mechanisms by some desirable properties in a cooperative framework. One such property, for example, is the demand-monotonicity property which says that the more quantity a player consumes the more he must pay, holding fixed the quantity consumed by the other players. Although many of the properties they consider certainly have strategic implications for the players, the focus of these papers is not on strategic behavior or equilibrium in noncooperative games. For a more recent paper in this cooperative vein, see Koster (2006) which has a more complete set of references to this literature. Moulin and Shenker (1992) consider some of these cost sharing mechanisms in a noncooperative game, and show that the serial cost sharing mechanism is dominance solvable. Their paper differs from ours in that their game has complete information, and the strategy space limits each player to choose a fixed quantity regardless of price. Our paper, on the other hand, considers players with private information and also admits players a richer strategy space, where players submit what amount to demand curves, being able to condition the quantity they want on the price. Moulin and Shenker (2001) consider a Clarke-Groves mechanism, and like what we do in this paper, wrestle with the tension between efficiency, budget-balance, and dominant strategies. But in their paper, consumption is binary: either a player consumes or does not. Furthermore, the cost function in Moulin and Shenker (2001) differs substantially from ours—their cost depends on the actual subset of players who consume—so that a direct comparison of their results to ours is not possible. Chen (2003), Chen and Razzolini (2005), and Razzolini, Reksulak, Dorsey (2004) run experiments on the average or serial cost sharing mechanisms in environments similar to the example we use in our paper where players have constant marginal values and costs are quadratic. However, the games they considered have either complete information or "limited information" in which the lab subjects know their own values, but the subjects are not told the distributions from which their opponents draw their values. We are not aware of research on cost sharing in which players have private information and play a noncooperative game. Outside of the literature on cost sharing, McAfee (1992) has some similar themes as our paper. In his paper, McAfee introduces an auction which maintains the dominant strategies of the Vickrey auction, and like our paper it deals with the budget imbalance problem by sacrificing some efficiency. Nevertheless, the model and application of his auction is much different from ours: his is a double auction with both strategic buyers and sellers, and each of his players demand or supply only a single-unit.

The next section specifies the cost-sharing game. Section 3 contains a simple example to illustrate the average, marginal, and serial cost mechanisms and the conventional Vickrey auction. In Section 4, we introduce PPD mechanisms for cost-sharing problems and represent the conventional Vickrey auction as a PPD mechanism. In Section 5, we reformulate a PPD mechanism as a subsidized Vickrey auction and investigate its properties. Section 6 contains our conclusions.

2 The Cost-Sharing Game

A set of $n \geq 2$ players, denoted as $N = \{1, \ldots, n\}$, jointly produce a private good and decide how much each player consumes and pays. The cost function $C(\cdot) : R_+ \to R_+$ is assumed to be strictly convex and twice continuously differentiable, and $C(0) = 0$ (so that only variable costs are considered, as typical in the literature). Let $c(\cdot) = C'(\cdot)$ denote the marginal cost function. An outcome $(\mathbf{q}, \mathbf{t}) = (q_1, \ldots, q_n, t_1, \ldots, t_n) \in R^n_+ \times R^n$ specifies player i's quantity q_i and player i's payment t_i for all $i \in N$. Outcome $(\mathbf{q}, \mathbf{t})$ is *feasible* if $t_1 + \ldots + t_n \geq C(q_1 + \ldots + q_n)$, and *budget-balanced* if $t_1 + \ldots + t_n = C(q_1 + \ldots + q_n)$.

We consider an incomplete information game with independent private values. Player i's type θ_i is a random variable with support Θ_i. Types are private information but the continuous distributions of types are common knowledge. Player i has a quasilinear utility that depends only on his type θ_i and his part of the outcome $(\mathbf{q}, \mathbf{t})$:

$$u_i(q_i, t_i|\theta_i) = V_i(q_i|\theta_i) - t_i.$$

For all $\theta_i \in \Theta_i$, player i's value function $V_i(\cdot|\theta_i) : R_+ \to R_+$ is assumed to be concave, strictly increasing, and twice continuously differentiable, with $V_i(0|\theta_i) = 0$. Let $v_i(\cdot|\theta_i) = V_i'(\cdot|\theta_i)$ denote player i's marginal value function. A simplifying assumption made throughout the paper is that $v_i(0|\theta_i) \geq c(0)$ for all $i \in N$ and all $\theta_i \in \Theta_i$. This assumption is harmless, serving only to eliminate some extraneous cases when the optimal production is zero.

A feasible outcome $(\mathbf{q}, \mathbf{t})$ is *Pareto efficient* if there does not exist another feasible outcome $(\mathbf{q}', \mathbf{t}')$ such that $u_i(q_i', t_i'|\theta_i) \geq u_i(q_i, t_i|\theta_i)$ for all $i \in N$, and $u_i(q_i', t_i'|\theta_i) > u_i(q_i, t_i|\theta_i)$ for at least one player $i \in N$. Outcome $(\mathbf{q}, \mathbf{t})$ is *allocatively efficient* if $\mathbf{q}$ maximizes the aggregate surplus:

$$\mathbf{q} \in \arg\max_{\mathbf{q}'} \sum_{i \in N} V_i(q_i'|\theta_i) - C\left(\sum_{i \in N} q_i'\right).$$

Observe that Pareto efficiency depends on both $\mathbf{q}$ and $\mathbf{t}$, but allocative efficiency depends only on $\mathbf{q}$.

A *cost-sharing mechanism* specifies a set of permissible bids B_i for each player $i \in N$, a quantity rule $\mathbf{q}(\cdot) : \times_{i \in N} B_i \to R^n_+$, and a payment rule $\mathbf{t}(\cdot) : \times_{i \in N} B_i \to R^n$. For each bid profile $\mathbf{b} = (b_1, \ldots, b_n) \in \times_{i \in N} B_i$, the mechanism selects the outcome $(\mathbf{q}(\mathbf{b}), \mathbf{t}(\mathbf{b}))$. A cost-sharing mechanism is feasible or budget-balanced if $(\mathbf{q}(\mathbf{b}), \mathbf{t}(\mathbf{b}))$ is feasible or budget-balanced, respectively, for all $\mathbf{b} \in \times_{i \in N} B_i$. Let $\mathbf{b}_{-i}$ denote the bid profile of player i's opponents. A bid b_i is *weakly dominant* for player i of type θ_i if for all $\mathbf{b}_{-i} \in \times_{j \neq i} B_j$,

$$b_i \in \arg\max_{b'_i \in B_i} u_i(q_i(b'_i, \mathbf{b}_{-i}), t_i(b'_i, \mathbf{b}_{-i})|\theta_i).$$

A cost-sharing mechanism induces a non-cooperative *cost-sharing game* of incomplete information, where players simultaneously choose their bids and then obtain the payoffs that result from the outcome the mechanism selects. For all $i \in N$, player i's strategy is a function that maps his types Θ_i into bids B_i. Player i's strategy is *weakly dominant* if it maps to a weakly dominant bid for all $\theta_i \in \Theta_i$.[1] In this paper, we sometimes consider a class of cost-sharing games where each B_i is the set of all possible marginal value functions: $B_i = \{v_i(\cdot|\theta_i) : \theta_i \in \Theta_i\}$. In such a cost-sharing game, player i is said to *bid truthfully* if for each $\theta_i \in \Theta_i$, his bid is $v_i(\cdot|\theta_i)$. For the conventional Vickrey auction and the subsidized Vickrey auction that we introduce in this paper, each player will have a weakly dominant strategy to bid truthfully. Consequently, bidding truthfully forms a Bayesian Nash equilibrium. Such an equilibrium is attractive because it does not depend on the distributions of players' types and requires only elementary equilibrium calculations.

3 A Motivating Example

In this section, we present an example to motivate our subsidized Vickrey auction for cost sharing. We first examine three prominent cost-sharing mechanisms and then show how the Vickrey auction works in this context. In the example, there are $n = 2$ players with independent private values. For $i \in \{1, 2\}$, player i draws his type θ_i from the uniform distribution on $[0, 1]$, and θ_i is player i's constant marginal value:

$$u_i(q_i, t_i|\theta_i) = \theta_i q_i - t_i.$$

[1] For expositional simplicity, we do not distinguish between *always optimal* and *weakly dominant* strategies as does Milgrom (2004), who further requires that weakly dominant strategies be unique.

The cost function is $C(q) = q^2/2$ with linear marginal costs $c(q) = q$. We first analyze three cost-sharing games induced respectively by the average, marginal, and serial cost-sharing mechanisms. In the case of a homogenous good, the Aumann-Shapley cost-sharing mechanism is identical to the average cost-sharing mechanism; see Moulin and Shenker (1992). See the same article and the references therein for more details about average, serial, and marginal cost-sharing mechanisms. In each of these three mechanisms, player i bids a quantity $b_i \geq 0$ and obtains the quantity he bid; i.e, $B_i = R_+$ and

$$q_i(\mathbf{b}) = b_i \quad \text{for } i \in \{1, 2\}.$$

The average, serial, and marginal cost-sharing mechanisms differ in their payment rules:

$$\begin{aligned} t_i^A(\mathbf{b}) &= q_i(\mathbf{b}) \cdot \frac{C\left(Q(\mathbf{b})\right)}{Q(\mathbf{b})} \\ &= q_i(\mathbf{b}) \cdot \frac{Q(\mathbf{b})}{2}, \end{aligned}$$

$$\begin{aligned} t_i^S(\mathbf{b}) &= \begin{cases} \frac{1}{2}C(2q_i(\mathbf{b})) & \text{if } b_i \leq b_j \\ \frac{1}{2}C(2q_j(\mathbf{b})) + C(Q(\mathbf{b})) - C(2q_j(\mathbf{b})) & \text{otherwise} \end{cases} \\ &= \begin{cases} q_i(\mathbf{b})^2 & \text{if } b_i \leq b_j \\ \frac{1}{2}Q(\mathbf{b})^2 - q_j(\mathbf{b})^2 & \text{otherwise,} \end{cases} \end{aligned}$$

$$\begin{aligned} t_i^M(\mathbf{b}) &= q_i(\mathbf{b})c(Q(\mathbf{b})) - \frac{1}{2}\left[Q(\mathbf{b}) \cdot c(Q(\mathbf{b})) - C(Q(\mathbf{b}))\right] \\ &= q_i(\mathbf{b})Q(\mathbf{b}) - \frac{1}{4}Q(\mathbf{b})^2, \end{aligned}$$

where $Q(\mathbf{b}) = q_1(\mathbf{b}) + q_2(\mathbf{b}) = b_1 + b_2$ is the aggregate quantity awarded. It is straightforward to verify that each of these mechanisms is budget-balanced.

In the corresponding cost-sharing games, player i's strategy maps his type θ_i to a bid $b_i(\theta_i) \geq 0$. For each cost-sharing mechanism, Table 1 gives the symmetric Bayesian Nash equilibrium that we have calculated and each player's *ex ante* payoff:

$$\int_0^1 \left[\theta_i b(\theta_i) - \int_0^1 t_i\left(b(\theta_i), b(\theta_j)\right) d\theta_j\right] d\theta_i,$$

where $b(\cdot)$ is the symmetric Bayesian Nash equilibrium and $t_i(\cdot,\cdot)$ is the payment rule associated with the mechanism.

Mechanism	Equilibrium Bidding Function $b(\theta_i)$	*ex ante* Payoff
Average	$\max\left\{2\sqrt{2}-3+\theta_i, 0\right\}$	0.094757
Serial	$\ln 2 - \ln(2-\theta_i)$.	0.096574
Marginal	$\max\left\{\frac{2}{3}\sqrt{15}-\frac{8}{3}+\frac{2}{3}\theta_i, 0\right\}$	0.098563

Table 1: Equilibrium bids and payoffs.

In the cost-sharing games induced by these three mechanisms, the equilibria do not involve weakly dominant strategies, and equilibrium outcomes are almost never allocatively efficient. To see that it is not weakly dominant for player i to bid as given in Table 1, suppose that player j bids 0 no matter what his type is. Then player i could improve his payoff by bidding the quantity where his marginal value equals marginal cost, noting player i will pay all of the costs. Allocative efficiency in this example requires that the player with the lower marginal value always wins zero quantity and the player with the higher value win the quantity where the marginal cost equals his marginal value, which in turn requires that $b_i(\theta_i) = \theta_i$ and $b_j(\theta_j) = 0$ whenever $\theta_i > \theta_j$. However, these bids occur with zero probability in the equilibria.

The idea of the Vickrey (1961) auction, which we call the conventional Vickrey auction in order to distinguish it from our subsidized Vickrey auction, is that players bid their marginal value curves, and then by construction, quantity is awarded to guarantee allocative efficiency presuming the players bid truthfully. Due to his ingenious payment rule, players do indeed have an incentive to bid truthfully, as Vickrey showed. Details specific to our example follow. Each player i bids $b_i \in [0,1]$ so that his reported marginal value curve is $v_i(q) = b_i$ for all $q \geq 0$. To guarantee allocative efficiency (and to settle the pesky matter of ties should both players report the same value), the quantity rule in the conventional Vickrey auction is:

$$q_i^V(\mathbf{b}) = \begin{cases} c_i^{-1}(b_i) & \text{if } b_i > b_j \\ \frac{1}{2}c_i^{-1}(b_i) & \text{if } b_i = b_j \\ 0 & \text{otherwise.} \end{cases}$$

In other words, the conventional Vickrey auction assigns all production to the player with the higher bid up to where the marginal cost is equal to his bid. The payment rule for player i in the conventional Vickrey auction involves an ingenious idea. First, find the efficient assignment of quantity in the absence of player i (or more precisely in this example, if player i reported his value at 0). Then, when quantity is awarded efficiently including player i, the conventional Vickrey auction has player i pay player j's marginal value for any quantity that j is displaced from and pay any costs of additional quantity produced. In particular, for this example,

$$t_i^V(\mathbf{b}) = \begin{cases} \int_0^{q_i^V(\mathbf{b})} \max\{b_j, c(z)\} dz & \text{if } b_i \geq b_j \\ 0 & \text{otherwise.} \end{cases}$$

Figure 1 below illustrates the winning player i's quantity and payment $(q_i^V(\mathbf{b}), t_i^V(\mathbf{b}))$ in the conventional Vickrey auction.

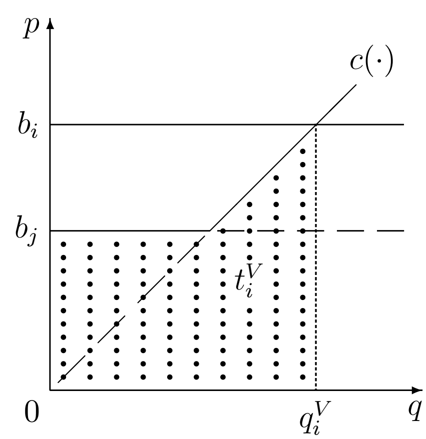

Figure 1: Winning player's outcome in the conventional Vickrey auction

We define $a_i(q) = \max\{b_j, c(q)\}$ as the *ask* curve that player i faces. Observe that the ask curve player i faces in no way depends on his own bid b_i, but rather on the bid of his opponent and the exogenously given marginal cost curve. As Figure 1 shows, the winning player i wins the quantity q_i^V where his reported marginal value curve intersects the ask curve, and pays the area under the ask curve for the quantity he wins. For the q^{th} marginal unit, player i will have to pay $a_i(q)$. With this interpretation, player i is just like a monopsonist who can perfectly price discriminate when buying his input in a market with (inverse) supply curve $a_i(\cdot)$. The best player i can do is to purchase the quantity where his marginal value curve intersects $a_i(\cdot)$, and this is guaranteed when player i bids truthfully. In other words, bidding truthfully is weakly dominant for player i. To put it another way, it is *weakly dominant* for player i to *bid truthfully* because by doing so, he wins all of the quantity that costs him less than his value and avoids winning any quantity that costs more than his value. This intuition is formalized in the proof of Proposition 1, given in the next section. Consequently, there is a symmetric Bayesian Nash equilibrium where each player i bids $b_i(\theta_i) = \theta_i$. The resulting *ex ante* payoff to each player is:

$$\int_0^1 \int_0^{\theta_i} \left[\theta_i^2 - \theta_j^2 - \int_{\theta_j}^{\theta_i} q \, dq \right] d\theta_j d\theta_i \;=\; \frac{1}{12} = 0.083333,$$

which is lower than that from any of the three cost-sharing mechanisms considered in Table 1. Observe that the winner's payment exceeds the production cost, and this overpayment is not accounted for in the players' utilities. In this truthful bidding equilibrium, the expected overpayment is

$$\int_0^1 \left[\int_0^{\theta_i} \frac{1}{2}\theta_j^2 d\theta_j + \int_{\theta_i}^1 \frac{1}{2}\theta_i^2 d\theta_j \right] d\theta_i = \;\frac{1}{12} = 0.083333,$$

which is equivalent to 0.041667 per player, or 50% of each player's *ex ante* payoff. In cost-sharing situations, players jointly produce the good: there is no opposing seller producing the good and pocketing this overpayment. One possible option for the players is to sell the rights for any overpayment to an outside party, which induces a budget-balanced mechanism. However, this option may not be practical in reality, which may be the reason that the Vickrey auction has not been considered much for cost-sharing problems.

The question that motivates this paper is whether the overpayment generated in the conventional Vickrey auction can be used to improve players' welfare without destroying their incentives to bid truthfully. Since direct refunds do not work, in section 5, we will modify the Vickrey auction in such a way that the overpayment subsidizes additional production, but without upsetting the players' incentives to bid truthfully. As we will show, this *subsidized Vickrey auction* performs quite well in this example: a player's *ex ante* payoff in the equilibrium is 0.104167, substantially higher than that from any of the prominent cost-sharing games considered in Table 1. Moreover, outside of this example, we will show that the subsidized Vickrey auction always outperforms the conventional Vickrey auction. Before introducing this subsidized Vickrey auction, in the next section we distill from the conventional Vickrey auction the key elements that ensure truthful bidding is a weakly dominant strategy.

4 Perfect Price Discrimination Mechanisms

In this section, we first describe a class of mechanisms, called *perfect price discrimination* (PPD) mechanisms, in which truthful bidding is a weakly dominant strategy. We then show that the conventional Vickrey auction is a PPD mechanism.

In a PPD mechanism, players bid or report their marginal value functions; i.e, $B_i = \{v_i(\cdot|\theta_i) : \theta_i \in \Theta_i\}$ is the set of allowable bids to player $i \in N$. From any bidding profile $\mathbf{b} = (b_1(\cdot), \ldots, b_n(\cdot))$, a non-decreasing *ask curve* $a_i(\cdot) : R_+ \to R_+$ is constructed for each player $i \in N$. Player i's part of the outcome is then determined as:

$$q_i(\mathbf{b}) \in \arg\max_{q_i} \int_0^{q_i} [b_i(z) - a_i(z)]\, dz, \tag{1}$$

$$t_i(\mathbf{b}) = \int_0^{q_i(\mathbf{b})} a_i(z) dz. \tag{2}$$

At this point, we stay agnostic about how these ask curves are constructed. Different ask curves induce different PPD mechanisms. Assume that $q_i(\mathbf{b})$ from (1) is well-defined. Then by (1), player i wins the quantity at which his bid curve and ask curve intersect, and by (2), player i pays the area under his ask curve up to the quantity he wins. We may interpret the ask curve as the inverse supply curve faced by a monopsonist who can perfectly price discriminate. Accordingly, we refer to such a mechanisms as perfect price discrimination (PPD) mechanisms. Figure 2 illustrates player i's outcome (q_i, t_i) in a PPD mechanism and

Proposition 1 gives a sufficient condition that ensures truthful bidding is a weakly dominant strategy.

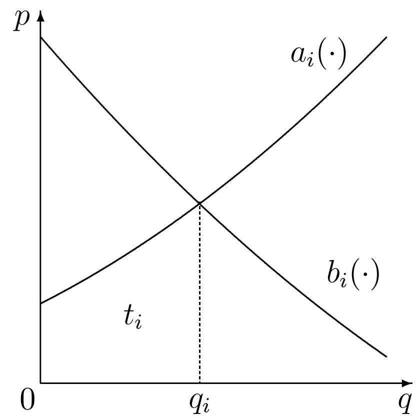

Figure 2: Player i's outcome in a PPD mechanism

Proposition 1 *In a PPD mechanism, if $a_i(\cdot)$ is independent of $b_i(\cdot)$, then bidding truthfully is a weakly dominant strategy for every player $i \in N$.*

Proof. Given that $a_i(\cdot)$ is independent of $b_i(\cdot)$, (2) implies that player i's bid can only affect his payment insofar as it affects the quantity he wins. This means that player i's payoff is bounded from above by

$$\max_{q_i} \quad V_i(q_i|\theta_i) - \int_0^{q_i} a_i(z)dz = \max_{q_i} \quad \int_0^{q_i} \left[v_i(z|\theta_i) - a_i(z)\right] dz,$$

which is achieved if player i bids truthfully due to (1). Since this is true for any $a_i(\cdot)$ that is independent of $b_i(\cdot)$, it is weakly dominant for player i to bid truthfully. ■

In the remainder of this section, we reformulate the conventional Vickrey (1961) auction as a PPD mechanism. To simplify the exposition, we separate the case when players have constant marginal values from the case when players have diminishing marginal values.

When every player has a constant marginal value, as in our example of section 3, every player submits a bid $b_i \in R_+$ (his reported marginal value). Given a bid profile $\mathbf{b} = (b_i, \mathbf{b}_{-i})$, the quantity and payment rules are:

$$q_i^V(\mathbf{b}) = \begin{cases} c_i^{-1}(b_i) & \text{if } b_i > \max_{j\neq i} b_j \\ \frac{1}{M} c_i^{-1}(b_i) & \text{if } b_i = \max_{j\neq i} b_j \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

$$t_i^V(\mathbf{b}) = \begin{cases} \int_0^{q_i^V(\mathbf{b})} \max\left\{c(z), \max_{j\neq i} b_j\right\} dz & \text{if } \ b_i \geq \max_{j\neq i} b_j \\ 0 & \text{if } \ b_i < \max_{j\neq i} b_j. \end{cases} \tag{4}$$

where M is the number of players tied with the highest bid. The tie breaking rule can be arbitrary since ties occur with zero probability in equilibrium. It follows immediately that the conventional Vickrey auction is a PPD mechanism with the following ask functions:

$$a_i^V(q) = \max\left\{c(q), \max_{j\neq i} b_j\right\}. \tag{5}$$

Since player i's ask function is independent of his own bid b_i, Proposition 1 applies, and truthful bidding is weakly dominant for every player. Therefore, there is an equilibrium where every player bids his true marginal value. The conventional Vickrey auction is feasible but not budget-balanced, since (4) implies that the winning player pays more than the production cost. On the other hand, the Vickrey auction is allocatively efficient since all production is awarded to the player with the highest marginal value at exactly the quantity where the marginal cost equals his marginal value.

When every player has a decreasing marginal value curve, formulating the conventional Vickrey auction as a PPD mechanism is slightly more complicated. In this case, every player $i \in N$ is allowed to submit a differentiable and strictly decreasing bid function: $b_i(\cdot) : R_+ \to R_+$. This bid is interpreted as a reported marginal value curve. Let $d_i(\cdot)$ be the demand curve associated with $b_i(\cdot)$:

$$d_i(p) = \begin{cases} b_i^{-1}(p) & \text{if } p \leq b_i(0) \\ 0 & \text{otherwise.} \end{cases}$$

Given the bid profile $\mathbf{b} = (b_1(\cdot), \ldots, b_n(\cdot))$, denote the corresponding demand profile by $\mathbf{d} = (d_1(\cdot), \ldots, d_n(\cdot))$. The aggregate demand is then the horizontal sum of individual demand curves: $d(p) = d_1(p) + \ldots + d_n(p)$.

Vickrey (1961) sets up a double auction with strategic buyers and sellers, and generates the market supply curve by aggregating the sellers' reports of their marginal costs. Accordingly, for a cost-sharing problem, we take the market supply curve to be the exogenous (inverse) marginal cost curve

$$s(p) = \begin{cases} c^{-1}(p) & \text{if } p \geq c(0) \\ 0 & \text{otherwise.} \end{cases}$$

Assume that there is a unique market equilibrium (p^*, q^*) where $s(p^*) = d(p^*) = q^*$. Likewise, assume for all $i \in N$ that there is a unique equilibrium (p^*_{-i}, q^*_{-i}) in the market without player i, such that $s(p^*_{-i}) = d_{-i}(p^*_{-i}) = q^*_{-i}$ where $d_{-i}(\cdot)$ is the aggregate demand by player i's opponents. The outcome in the conventional Vickrey auction is

$$q_i^V(\mathbf{b}) = s(p^*) - d_{-i}(p^*) \equiv d_i(p^*), \tag{6}$$

$$t_i^V(\mathbf{b}) = \int_{d_{-i}(p^*)}^{s(p^*)} \max\{b_{-i}(z), c(z)\}\, dz, \tag{7}$$

where $b_{-i}(\cdot)$ gives the height of $d_{-i}(\cdot)$. The left panel of Figure 3 illustrates player i's outcome in the conventional Vickrey auction, as defined by (6) and (7).

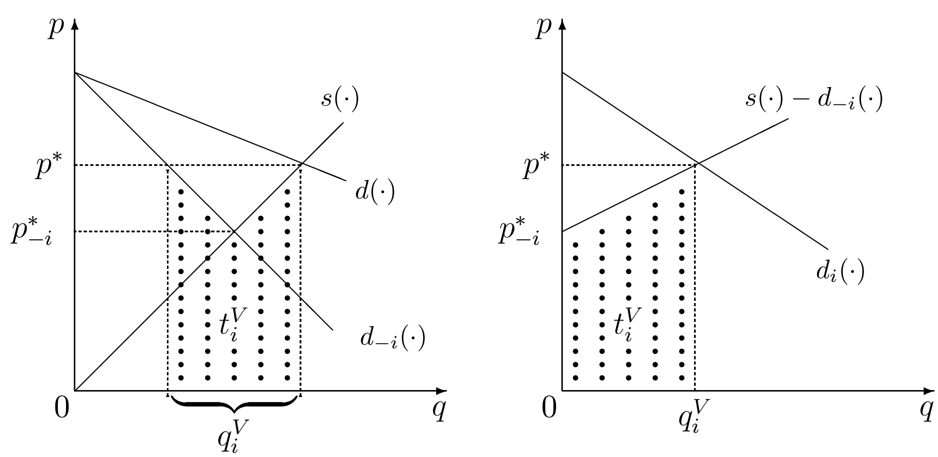

Figure 3: Player i's outcome in the conventional Vickrey auction

The left panel of Figure 3 closely resembles the scheme described by Vickrey (1961, pages 9-14), except that the supply curve in our paper is given by the marginal cost curve of production. We next reformulate the conventional Vickrey auction as a PPD mechanism. For any bid profile, the residual supply curve to player i is given by

$$r_i^V(p) = \begin{cases} s(p) - d_{-i}(p) & \text{if } p \geq p_{-i}^* \\ 0 & \text{otherwise.} \end{cases}$$

Our next proposition shows that the conventional Vickrey auction is a PPD mechanism with the inverse of $r_i(\cdot)$ as the ask function to player i in the PPD mechanism, establishing that the right panel of Figure 3 is equivalent to the left panel.[2]

Proposition 2 *For the case of $v_i'(\cdot|\theta_i)$, the conventional Vickrey auction is a PPD mechanism where for all $i \in N$, $B_i = \{v_i(\cdot|\theta_i) : \theta_i \in \Theta_i\}$, and $a_i(\cdot)$ is the inverse of $r_i^V(p)$.*

Proof. When $b_i(0) \leq p_{-i}^*$ this means that $p_{-i}^* = p^*$ and so $q_i^V(\mathbf{b}) = 0$ and $t_i^V(\mathbf{b}) = 0$ by (6) and (7). When $b_i(0) > p_{-i}^*$, due to the monotonicity and continuity of $b_i(\cdot)$ and $r_i(\cdot)$, the unique solution to (1) where $a_i(\cdot)$ is the inverse of $r_i^V(\cdot)$ is the unique intersection of $b_i(\cdot)$ and $a_i(\cdot)$. But because

$$d(p^*) = s(p^*) \Leftrightarrow d_i(p^*) + d_{-i}(p^*) = s(p^*) \Leftrightarrow d_i(p^*) = s(p^*) - d_{-i}(p^*) = r_i^V(p^*),$$

player i's demand and ask curves intersect at $d_i(p^*)$. The payment in the conventional Vickrey auction can be expressed as:

$$\int_{q^*-q_i^V}^{q^*} \max\{b_{-i}(q), c(q)\}\, dq = p^* q_i^V - \int_{p_{-i}^*}^{p^*} [s_i(p) - d_{-i}(p)]\, dp = \int_0^{q_i^V} a_i(z) dz,$$

as illustrated in Figure 3. This concludes the proof (illustrated in Figure 3). ■

Since player i's ask curve is independent of player i's bid, Proposition 1 implies that in the conventional Vickrey auction, bidding truthfully is a weakly dominant strategy, a well-known feature of the conventional Vickrey auction. To conclude this section, we collect the well-known properties of the conventional Vickrey auction in the following proposition.

[2] Indeed, the right panel of Figure 3 is just how Ausubel (2004) interprets the quantity and payment rule in the Vickrey auction: see his Figure 1.

Proposition 3 *The conventional Vickrey auction is feasible, but not budget balanced in general. It is weakly dominant to bid truthfully, and the resulting equilibrium outcome is allocatively efficient.*

5 A Subsidized Vickrey Auction

We now introduce the subsidized Vickrey auction for cost-sharing problems. For expositional convenience, we will consider separately the cases of constant and decreasing marginal values. To keep the notation from sprawling, we will use the same notation for these two cases.

5.1 Constant Marginal Values

Consider the case that for all $i \in N$, $v_i(q_i|\theta_i) = \theta_i$, where $\theta_i \geq 0$ is player i's constant marginal value. As in the conventional Vickrey auction, player i reports his marginal value: $B_i = \{v_i(\cdot|\theta_i) : \theta_i \in \Theta_i\} = R_+$. Given a bid profile $\mathbf{b}$, let $b_j = \max_{k \neq i} b_k$. Like in the conventional Vickrey auction, for player i to win any quantity he will have to bid at least b_j and by so doing will pay at least the unit price of b_j for any quantity he receives. But unlike the conventional Vickrey auction, the unit price will remain b_j beyond $q = c^{-1}(b_j)$ up until all of the overpayment that player i will generate by displacing bid b_j is exhausted. Specifically, define the quantity x_i implicitly by the following equation:

$$\underbrace{\int_0^{c^{-1}(b_j)} [b_j - c(z)]\, dz}_{\Omega_i} + \underbrace{\int_{c^{-1}(b_j)}^{x_i} [b_j - c(z)]\, dz}_{-\Psi_i} = 0, \tag{8}$$

where Ω_i and Ψ_i represent player i's overpayment and underpayment, respectively, should player i bid higher than b_j. We can then define the subsidized ask function faced by player i as:

$$a_i^{SV}(q) = \begin{cases} b_j & \text{if } q_i \leq x_i \\ c(q) & \text{otherwise,} \end{cases}$$

as illustrated in Figure 4 (where the diagonal represents the marginal cost $c(\cdot)$).

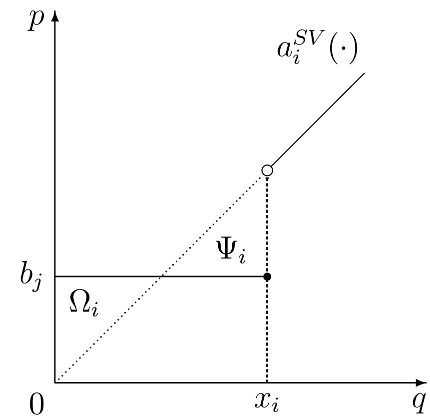

Figure 4: Player i's subsidized ask curve ($\Omega_i = \Psi_i$)

The *subsidized Vickrey auction* is then the PPD mechanism where for all $i \in N$, player i bids a marginal value and faces the subsidized ask function $a_i^{SV}(\cdot)$. Player i's outcome is given by (1) and (2) with his subsidized ask function $a_i^{SV}(\cdot)$. In the case of a tie for the highest bid, those submitting the highest bid will split x_i equally. Figure 5 illustrates two possibilities in determining player i's outcome, and the following proposition gives the properties of the subsidized Vickrey auction.

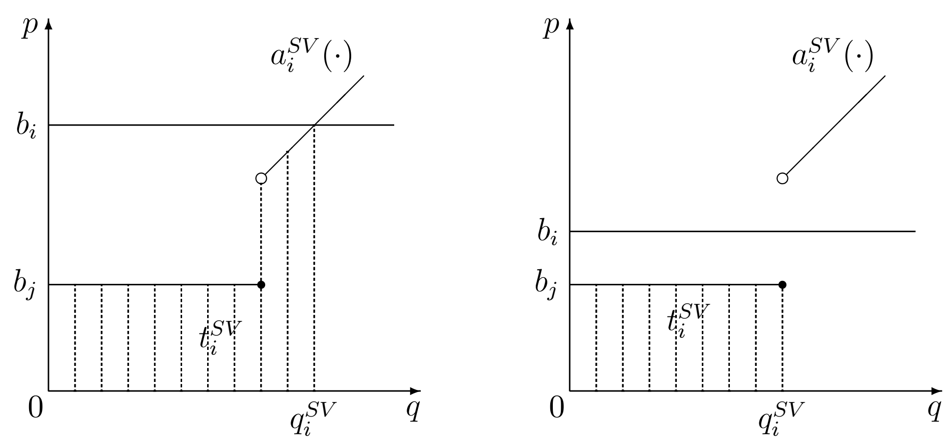

Figure 5: Player i's outcome in the subsidized Vickrey auction

Proposition 4 *In the case of constant marginal values, the subsidized Vickrey auction: (i) is budget-balanced, (ii) gives every player a weakly dominant strategy to bid truthfully, and (iii) in the truthful bidding equilibrium, Pareto dominates the conventional Vickrey auction.*

Proof. (i) Consider any bidding profile $\mathbf{b} = (b_1, \ldots, b_n)$, and denote by i the player with the highest bid. If $b_i = b_j \equiv \max_{k \neq i} b_k$, then the total quantity produced is equal to x_i and the winning players pay b_j per unit, so that by equation (8) the total payments collected are

$$x_i b_j = \int_0^{x_i} c(z)dz = C(x_i),$$

thereby balancing the budget. If $b_i > b_j$, then

$$\text{either } q_i^{SV}(\mathbf{b}) = x_i \quad \Leftrightarrow t_i^{SV}(\mathbf{b}) = x_i b_j = \int_0^{x_i} c(z)\, dz,$$

$$\text{or } q_i^{SV}(\mathbf{b}) > x_i \quad \Leftrightarrow t_i^{SV}(\mathbf{b}) = x_i b_j + \int_{x_i}^{q_i^{SV}} c(z)\, dz = \int_0^{q_i^{SV}} c(z)\, dz,$$

again applying equation (8). Either way, player i pays exactly the cost for the quantity he wins, so that the subsidized Vickrey auction is budget-balanced. (ii) By construction, the subsidized Vickrey auction is a PPD mechanism where every player's ask function is independent of this player's bid, so that Proposition 1 applies. (iii) Just as in the Vickrey auction, players who do not have the highest value win 0 and pay 0. The player with the highest value, however, faces an ask curve in the subsidized Vickrey auction that is below the ask curve he faces in the conventional Vickrey auction, either winning the same amount but paying less (as in the left panel of Figure 5) or winning more quantity at a cost below his value (as in the right panel of Figure 5). These events occur with probability one, thereby establishing the desired result. ■

Unlike the conventional Vickrey auction, the subsidized Vickrey auction is not allocatively efficient. In the equilibrium where every player bids truthfully, when the winner's value $\theta_i < c(x_i)$, as in the right panel of Figure 5, player i inefficiently wins x_i, instead of the efficient quantity $c^{-1}(\theta_i)$. Although the subsidized Vickrey auction may award too much to the winning player(s), in the equilibrium, quantity is never awarded to a player if he does not have the highest value.

We now return to the example considered in section 3. With the subsidized Vickrey auction described above, a player's *ex ante* payoff in the truthful bidding equilibrium is

$$\int_0^1 \left[\int_0^{\theta_1/2} \left(\theta_1 (2\theta_2) - \frac{(2\theta_2)^2}{2} \right) d\theta_2 + \int_{\theta_1/2}^{\theta_1} \left(\theta_1 \theta_1 - \frac{\theta_1^2}{2} \right) d\theta_2 \right] d\theta_1 \;=\; \frac{5}{48} = 0.104167,$$

which is higher than a player's *ex ante* payoff from the conventional Vickrey auction, and all three cost-sharing mechanisms we considered in Section 3.

5.2 Decreasing Marginal Values

Consider the case where $v_i(\cdot|\theta_i)$ is strictly decreasing for all $i \in N$ and all $\theta_i \in \Theta_i$. As in the conventional Vickrey auction, every player submits a downward sloping marginal value curve: $B_i = \{v_i(\cdot|\theta_i) : \theta_i \in \Theta_i\}$.

We next show how to construct the ask curves that will be used in the subsidized Vickrey auction. A subsidized ask curve is constructed so that some of the overpayments inherent in the conventional Vickrey auction are used to subsidize additional production. As shown in the left panel of Figure 3, player i's payment exceeds the marginal cost of production over the interval of $[d_{-i}(p^*), d_{-i}(p^*_{-i})]$. In the conventional Vickrey auction, for player i to obtain any quantity, he must force the market price above p^*_{-i}, which not only displaces other players from quantity they would otherwise win, but also expands the production. In the conventional Vickrey auction, player i must pay the social opportunity cost of the quantity he obtains. Since the other players value the quantity that player i displaces more than the production costs, this creates excess revenue over production costs. Specifically, we define player i's *overpayment function* as $\Omega_i(p) = 0$ for $p < p^*_{-i}$ and

$$\Omega_i(p) = \int_{d_{-i}(p)}^{d_{-i}(p^*_{-i})} [b_{-i}(z) - c(z)]\, dz, \text{ for } p \geq p^*_{-i}. \tag{9}$$

The following lemma summarizes some properties of $\Omega_i(\cdot)$.

Lemma 5 *For all $i \in N$, $\Omega_i(\cdot)$ is continuous and differentiable almost everywhere, and is strictly increasing over $\left[p^*_{-i}, b_{-i}(0)\right)$.*

Proof. *The continuity and differentiability of $b_{-i}(\cdot)$, $d_{-i}(\cdot)$, and $c(\cdot)$ imply that $\Omega_i(\cdot)$ is continuous everywhere and differentiable at all $p \notin \left\{p^*_{-i}\right\} \cup \{b_j(0) : j \neq i\}$, noting that kinks occur when aggregating the demands of the other players $j \neq i$. For $p \in [p^*_{-i}, b_{-i}(0))$, (9)* *implies that wherever it exists,*

$$\Omega_i'(p) = -d_{-i}'(p)[b_{-i}(d_{-i}(p)) - c(d_{-i}(p))] = -d_{-i}'(p)[p - c(d_{-i}(p))],$$

which is strictly positive due to $d_{-i}'(p) < 0$ *and* $p > c(d_{-i}(p))$ *for* $p > p_{-i}^*$. ■

We next show how to use the overpayment $\Omega_i(p)$ to offset some of the production costs. Specifically, for each $i \in N$, a subsidized supply function $s_i(p) \geq s(p)$ is created as follows (which is well-defined almost everywhere):

$$s_i(p) = \begin{cases} s(p) & \text{if } p \geq b_{-i}(0) \\ \text{solution to } s_i'(p) \cdot [c(s_i(p)) - p] = \Omega_i'(p) & \text{if } p \in [p_{-i}^*, b_{-i}(0)) \\ s(p) & \text{if } p < p_{-i}^*. \end{cases} \tag{10}$$

There are three different segments in player i's subsidized supply function. For $p < p_{-i}^*$ and $p \geq b_{-i}(0)$, $s_i(p)$ coincides with the actual supply curve (or marginal cost of production). But for $p \in [p_{-i}^*, b_{-i}(0))$, our next lemma shows $s_i(p)$ lies below and to the right of $s(p)$ far enough so that $\Omega_i(p)$ is exhausted (subsidizing production).

Lemma 6 *For all* $i \in N$, $s_i(\cdot)$ *is strictly increasing and for* $p \in [p_{-i}^*, b_{-i}(0))$,

$$\int_{q_{-i}^*}^{s_i(p)} \left[c(z) - s_i^{-1}(z)\right] dz = \Omega_i(p). \tag{11}$$

Proof. Because $s(\cdot)$ is the inverse of $c(\cdot)$, we have

$$s_i(p) = s(p) \Leftrightarrow c(s_i(p)) = p,$$
$$s_i(p) > s(p) \Leftrightarrow c(s_i(p)) > p.$$

Because $s_i(p) \geq s(p)$ for $p \in [p_{-i}^*, b_{-i}(0))$, Lemma 5 and equation (10) imply that $s_i'(p) > 0$ and $s_i(p) > s(p)$ for all p where $\Omega_i'(p)$ exists. For $p \in [p_{-i}^*, b_{-i}(0))$, integrating $s_i'(p) \cdot [c(s_i(p)) - p] = \Omega_i'(p)$ from p_{-i}^* up to p yields (11), doing a change of variable $p = s_i^{-1}(q)$ on the left hand side of (11) (Alternatively, differentiate both sides of (11) to get (10)). ■

The left hand side of (11) represents the underpayment by player i, which we denote by $\Psi_i(p)$. Figure 6 below illustrates player i's subsidized supply function $s_i(p)$ in the case of decreasing marginal values.

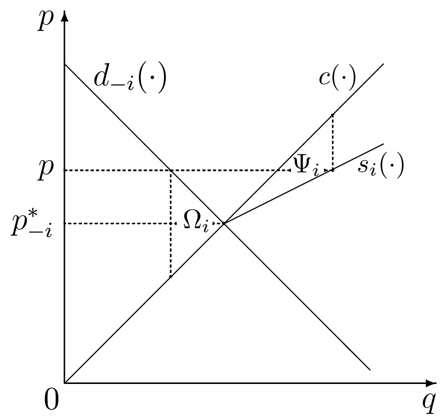

Figure 6: Player i's subsidized supply function ($\Omega_i = \Psi_i$)

Using the subsidized supply function $s_i(p)$, we define player i's residual supply function as

$$r_i^{SV}(p) = \max\{s_i(p) - d_{-i}(p), 0\}.$$

Then player i's subsidized ask function is the inverse of the residual supply player i is facing:

$$a_i^{SV}(q) = \begin{cases} p^*_{-i} & \text{for } q = 0 \\ r_i^{SV-1}(q) & \text{for } q > 0. \end{cases}$$

To summarize, the subsidized Vickrey auction is the PPD mechanism where every player i submits a downward sloping demand curve $b_i(\cdot)$ and player i's outcome is determined by $b_i(\cdot)$ and $a_i^{SV}(\cdot)$ according to (1) and (2). Now define the "market clearing price" p_i^{**} such that $d(p_i^{**}) = s_i(p_i^{**})$, which will differ across players because the subsidized supply functions differ across players. Our next lemma provides another way to derive player i's outcome in the subsidized Vickrey auction, which we state without proof since it is straightforward and resembles that for Proposition 2.

Lemma 7 *The subsidized Vickrey auction induces the following outcome rule: for all bid profile* $\mathbf{b}$*,*

$$q_i^{SV}(\mathbf{b}) = s_i(p_i^{**}) - d_{-i}(p_i^{**}) \equiv d_i(p_i^{**}) \tag{12}$$

and

$$t_i^{SV}(\mathbf{b}) = \int_{d(p_i^{**})-d_i(p_i^{**})}^{d(p_i^{**})} c(z)\, dz. \tag{13}$$

Observe from equation (13) that in some sense each player pays the production costs for the quantity he wins. The proof of our next proposition uses this feature to show that the subsidized Vickrey auction is indeed feasible.

Proposition 8 *In the case of decreasing marginal values, the subsidized Vickrey auction: (i) is feasible, (ii) is weakly dominant for every player to bid truthfully, and (iii) in the truthful bidding equilibrium, Pareto dominates the conventional Vickrey auction.*

Proof. Without loss of generality, assume $p_1^{**} \geq \ldots \geq p_n^{**}$. In the subsidized Vickrey auction, the total payment is

$$\begin{aligned}
\sum_{i=1}^{n} t_i^{SV}(\mathbf{b}) &= \sum_{i=1}^{n-2} \int_{d(p_i^{**})-d_i(p_i^{**})}^{d(p_i^{**})} c(q)\, dq + \int_{d(p_{n-1}^{**})-q_{n-1}^{SV}}^{d(p_{n-1}^{**})} c(q)\, dq + \int_{d(p_n^{**})-q_n^{SV}}^{d(p_n^{**})} c(q)\, dq \qquad (14)\\
&\geq \sum_{i=1}^{n-2} \int_{d(p_i^{**})-d_i(p_i^{**})}^{d(p_i^{**})} c(q)\, dq + \int_{d(p_n^{**})-q_n^{SV}-q_{n-1}^{SV}}^{d(p_n^{**})-q_n^{SV}} c(q)\, dq + \int_{d(p_n^{**})-q_n^{SV}}^{d(p_n^{**})} c(q)\, dq \\
&\;\;\vdots \\
&\geq \int_{d(p_n^{**})-Q}^{d(p_n^{**})-Q+q_1^{SV}} c(q)\, dq + \cdots + \int_{d(p_n^{**})-q_n^{SV}-q_{n-1}^{SV}}^{d(p_n^{**})-q_n^{SV}} c(q)\, dq + \int_{d(p_n^{**})-q_n^{SV}}^{d(p_n^{**})} c(q)\, dq \\
&= \int_{d(p_n^{**})-Q}^{d(p_n^{**})} c(q)\, dq \\
&\geq \int_0^Q c(q)\, dq = C(Q).
\end{aligned}$$

where $Q = \sum_{i=1}^{n} q_i^{SV}$ is the total amount awarded in the auction. The first equality is due to (13). The next inequality is the only subtle part of the proof. Observe that for each player i, by construction, $d(p_i^{**}) - q_i^{SV} \leq d(p^*) \leq d(p_i^{**})$, where p^* is the market clearing price in the conventional Vickrey auction. From this it follows that $d(p_n^{**}) - q_n^{SV} \leq d(p^*) \leq d(p_{n-1}^{**})$, showing that the first inequality above results by calculating the second to last integral of the strictly increasing marginal costs over a quantity interval of the same width q_{n-1}^{SV}, but for an interval closer to the origin (zero quantity). This process is repeated for each integral, and then the integrals are summed up to get the equality above. The last inequality results by again evaluating an integral of the same width, but closer to the origin, noting that $d(p_n^{**}) \equiv \sum_i d_i(p_n^{**}) \geq \sum_i d_i(p_i^{**}) \equiv Q$. Since the total payment is at least the total cost, the conventional Vickrey auction is feasible. (ii) By construction, the subsidized Vickrey auction is a PPD mechanism where every player's ask function is independent of this player's bid, so that Proposition 1 applies. (iii) No player is worse off in the subsidized Vickrey auction because the ask curve he faces in the subsidized Vickrey auction lies below the ask curve he faces in the conventional Vickrey auction. Indeed, any player that wins positive quantity in the Vickrey auction will be made strictly better off in the subsidized Vickrey auction because of the lower ask curve he faces. ■

Unlike the case of constant marginal values, in the case of diminishing marginal values the subsidized Vickrey auction is generally not budget-balanced. The inequalities in equation (14) are generally strict. In the event that one player, say 1, pushes the price p_1^{**} up so high that $p_1^{**} \geq b_j(0)$ for players $j > 1$, similar to what the left panel of Figure 5 shows for the cases of constant marginal values, the outcome will be budget balanced and allocatively efficient, and yet still Pareto dominate the outcome of the conventional Vickrey auction since the winning player will pay only the production costs and no more.

6 Conclusion

In many games of incomplete information, it is generally not possible to simultaneously achieve truthful bidding as a dominant strategy, budget balance, and allocative efficiency. Thus, choosing a mechanism to solve an allocation problem will involve tradeoffs. The Vickrey auction and Vickrey-Clarke-Groves mechanisms achieve allocative efficiency and dominant strategies, but are often plagued by budget imbalance. In this paper, we have shown that the budget surplus need not rule out altogether the Vickrey auction from use as a cost sharing mechanism. With some modifications, the Vickrey auction can be made to redistribute back the surplus to players, not in direct monetary rebates, but rather in terms of additional subsidized production. Although this subsidized Vickrey auction sacrifices some allocative efficiency, it still easily Pareto dominates the conventional Vickrey auction. We have shown in an example in a noncooperative game where players have uniformly distributed values, that our subsidized Vickrey auction also outperforms other cost sharing mechanisms, like the average and serial mechanisms. We do not believe that this ranking between our subsidized Vickrey auction and the average and serial cost sharing mechanisms holds universally in noncooperative games with incomplete information because by construction the subsidized Vickrey auction will grossly overproduce when players' values are close together with high probability. Looking at the performance of these cost sharing mechanisms in noncooperative games of incomplete information is a logical next step.

In the perfect price discriminating mechanisms introduced in this paper, we isolated the key feature in the conventional Vickrey auction that ensures truthful beating as a dominant strategy. Efficiency plays no role in whatsoever. Thus, efficiency can be sacrificed to alleviate budget imbalance while maintaining truthful bidding. In future research, we will explore whether this idea can help resurrect the Vickrey auction in other situations—like public goods, bidding rings (McAfee and McMillan, 1992), and partnership dissolutions (Cramton, Gibbons, and Klemperer, 1987)—where budget imbalance prevents its use.

Although we have limited discussion to environments with independent private values, it would not be difficult to adapt the subsidized Vickrey auction to the affiliated values of Milgrom and Weber (1982). Indeed in this setting, our subsidized Vickrey auction could be implemented with an ascending auction, similar to the Ausubel (2004) auction. Our subsidized Vickrey auction suffers the same drawbacks as conventional Vickrey auctions: players may want to merge, or a player may want to submit multiple bids under multiple identifies. Such behavior can compromise efficiency as described by Milgrom (2004) and Ausubel and Milgrom (2002).

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