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This paper studies price setting within a chain of grocery stores, using a scanner database that contains observations of retail prices for 435 products within 75 stores over 121 weeks. We find price dispersion within the chain. Although price dispersion is pervasive 75% of the prices are equal to the modal price. The mode changes frequently: 35% of the modes change in an average week. This suggests that the distribution of prices may react relatively fast to aggregate shocks. Stores differentiate themselves by the prices of relatively few items. Typically most prices in the store are at the mode of the cross sectional price distribution, some are above the mode and some are below the mode. The probability of a price change is 3.6% when the price is at the mode and 76.2% when the price is not at the mode. We explain the apparent attraction to the mode in terms of a model in which price discreteness plays an important role but there is no inertia. We also find that the probability of a price change is higher when the deviation from the mean of the cross sectional price distribution is large. But unlike conventional wisdom, the probability of a price change is higher for young prices.

Key Words: Price Discreteness, Price Dispersion, Price Changes, Price Rigidity.

JEL CODES: E00, L11

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1. INTRODUCTION

This paper studies price setting within a chain of grocery stores, using a scanner database that contains observations of retail prices for 435 products within 75 stores over 121 weeks.

We summarize the data with the following questions in mind. How should we measure price rigidity in a chain? What determines the probability of a price change? Does the law of one price hold among stores that belong to the same chain? If not why does it fail?

Questions regarding price changes and price rigidity are central to macroeconomics. Accordingly the macro literature focuses on price changes, including models that stress menu costs (Barro [1972], Sheshinski and Weiss [1977]), the cost of paying attention (Mankiw and Reis [2007]), and time dependent models (Taylor [1980], Calvo [1983]). Whereas, the focus of the industrial organization literature is on price dispersion (the cross sectional variations in the price of the same good) with models that stress product differentiation (Chamberlin [1933]), search costs (Reinganum [1979], Rob [1985]), and demand uncertainty (Prescott [1975]).

The empirical literature also tends to focus on one issue at a time. Bils and Klenow (2004), Klenow and Krvystov (2005), Dhyne et al. (2005), Nakamura and Steinsson (2007) study aspects of price change while Pratt, Wise and Zeckhauser (1979) and Lach (2002) study price dispersion. Here we study both aspects of price setting and exploits the connection between the two aspects.

Maybe the most important connection arises when attempting to measure price rigidity. Most of the micro price studies assume that the store has a unique optimal price and measure price rigidity by the frequency of price changes in a store. This measure may be relevant to the ability of the average store to react to store specific shocks but may be misleading if we are interested in the ability of prices to react to a macro shock like doubling the money supply. For example, suppose that there are two stores: Store 1 posts the price 0.9 in odd periods and the price 1.1 in even periods while store 2 posts the price 0.9 in even periods and 1.1 in odd periods. In this example 100% of the prices are changed every period but the price distribution may be rigid, if stores do not change their price setting behavior in response to an aggregate shock: From a macro point of view, doubling the money supply will only be neutral if the stores oscillate between 1.8 and 2.2; otherwise, there will be real effects.

The distinction between the rigidity of the price distribution and the rigidity of individual prices is sharp in a Prescott (1975) type model in which sellers are indifferent among prices in the equilibrium range. The Prescott model was used in Eden (1994) to study the issue of money non-neutrality. In this model money surprises are non-neutral but anticipated changes in the money supply are neutral as long as the price distribution is flexible. The behavior of individual prices is of little consequences. Prices may appear rigid when the distribution is flexible and prices may appear flexible when the distribution is rigid.

In our data, price dispersion is pervasive but the fraction of prices that are equal to the mode is large (75% on average) and quite stable over weeks. We therefore characterize the entire price distribution by its mode. We look at the mode price for the 435 products and found that in an average week 35% of modes change. This suggests that the distribution of prices is rather flexible.

Our discussion of the mode is different from the discussion in Eichenbaum, Jaimovich and Rebelo (EJR, 2008) who use the quarterly mode for each product-store cell to evaluate price rigidity. They find that the product-store mode is rigid and argue that price rigidity is important even though individual prices change often. Our conclusion about the importance of price rigidity is different because we look at a different mode.

To better understand the difference between EJR and our approach we consider a variation of the above example in which the mode is defined. We assume that nominal aggregate demand is trendless and fluctuates around the mean. There are three stores that post the price 1 in most weeks but from time to time change it to 0.9 or 1.1. We distinguish between two cases: (a) In any given week only one store may post a price that is different from 1 and
(b) Stores move in concert and always post the same price. In both cases the store-product quarterly mode is 1 and does not change over time. Thus, prices are rigid if we use the EJR approach. But the weekly cross sectional mode may tell a different story. In the first case, it does not change over time while in the second case it does. Since we assume that nominal aggregate demand fluctuates, we may say that in the first case most of the prices fail to react to aggregate shocks while in the second case they do react to aggregate shocks.

Similar to the EJR mode, our cross sectional mode looks like an attractor: In our data the weekly frequency of price change decreases by about 70% if the price is equal to the mode of the cross sectional distribution. This holds even when we control for the average of the distribution.

To account for this observation and other transition probabilities, we employ a model in which price discreteness plays an important role and there is some price rigidity of the Calvo (1983) type. In this model the chain manager sets a price menu with relatively few alternatives and stores choose their price out of this menu. Stores' preferences do not depend on past history and are consistent with the steady state cross sectional distribution of price ranks. Surprisingly the model accounts for the transition probabilities quite well. Thus, in our model, the attractor property arises simply because the mode is by definition relatively "large" and store preferences are consistent with the steady state price ranks distribution.

As was mentioned above, we interpret the finding that 35% of the modes change in an average week as evidence against the rigidity of the price distribution. We should stress that this is evidence against price rigidity as defined by the class of models that assume a fixed cost for changing prices (as in Barro [1972] and Sheshinski and Weiss [1977]) or a restriction on the ability to change prices (as in Calvo [1983]). It cannot be used as evidence against models that assume a cost that depends on the size of the price change as in Rotemberg (1982). To appreciate the difference, consider the case in which the money supply doubles and all agents change their prices by 50%. In this case price rigidity is important even when all agents changed their price immediately after the increase in the money supply.

Motivated by the need to distinguish between competing models, we describe the data in a way that can be used to distinguish between various hypotheses. Here is a short summary of our main findings.

1. The frequency of mode changes is high and highly correlated with the frequency of price changes. This suggests a flexible price distribution that reacts to aggregate shocks.
2. There is a substantial amount of price dispersion across stores within the chain at the same point in time. The amount of a product's price dispersion is positively correlated with its average price and its share in the chain's revenue.
3. Stores differentiate themselves by the total number of products with a price above (or below) the chain's mode price. We find that stores vary their individual product prices, but maintain roughly the same number of prices above/below the mode. Therefore customers can easily predict the ranking of stores when they rank stores by the basket of all goods, but will find it hard to predict the ranking of stores when the ranking is by individual products.

4. Recently set prices and modes are more likely to change.

The last finding is similar to the findings in Campbell and Eden (CE, 2007) and Nakamura and Steinsson (2008). It holds even when controlling for heterogeneity among products and stores.

We start with a description of the data. Next, we focus on several stylized facts about price dispersion and price changes. The fourth section constructs a model which is capable of describing some of these facts. The final section tests the model's predictions alongside other state and time dependent theories. The conclusion contains a discussion of the type of model that may explain our main findings.

2. DOMINICK'S FINER FOODS (DFF) DATABASE

The Dominick's Finer Foods (DFF) database arose through a joint venture between the Chicago-area supermarket chain and the University of Chicago Graduate School of Business. The partnership started in September of 1989 to document a price randomization experiment, but continued even after the experiment was completed in 1995. The data make use of scanner technology which electronically records items as they were purchased in each store. The observations were aggregated weekly to give the transaction price, quantity sold, and profit margin for about 3,500 products in 100 store locations. Each product is labeled by a unique Universal Product Code (UPC) which documents the product's manufacture, size, and type. For instance, a 6pk of Coke has a different UPC than a 6pk of Pepsi, a 6pk of Diet Coke, or a 2 liter Coke. The data does not contain a complete summary of UPCs sold by the chain, but covers nearly 30% of all dollar sales.

To date, several papers have used the DFF sample to study pricing. Most recently, Midrigan (2007) and Kehoe and Midrigan (2007) study menu costs effects for regular and sale prices. Chevalier, Kashyap and Rossi (2003) address demand's impact on retail prices, quantity sold, retail margins, and wholesale prices, during holidays. Earlier papers by Dutta, Bergen, and Levi (2002) and Peltzman (2000) examine the response of prices to cost shocks. These studies test various aspects of price change, but typically ignore the chain aspect of the data by choosing price data from a single store.

When dealing with the DFF source, the major decision one must make is what to do with missing information which occurs when no units were sold or when a UPC was temporarily replaced by another UPC denoting a new size, flavor, color or even holiday packaging. For instance, Mint Flavored M&Ms replace regular M&Ms during the Christmas season. There are several ways to handle these gaps. Chevalier, Kashyap and Rossi (2003) fill the UPCs with their replacements in order to maximize the sample period, while Midrigan (2006) and Peltzman (2000) take observations from other stores in order to obtain the maximum number of products. Since we want weekly data within many stores, we fill missing observations using the data around the gap. First, we drop any UPC with a gap longer than a month. Second, if the price is the same on both sides of the gap, we assume it did not change during the gap, and fill the "missing" price from the previous week's price. Next, we look at gaps in which prices are not the same on both sides. We replace all single week gaps with previous week's price and drop gaps that are longer than a week. Finally, any UPC-Store cell with unfilled gaps are dropped from the sample.

Keeping this filling process in mind, there is a tradeoff between the length of the sample and the number of unfilled gaps. We chose a period of 121 weeks that maximizes the number of balanced price observations. This yields a sample of 435 UPCs within 75 stores. The sample period spans the final two years of the sample thus avoiding the randomized pricing experiment which occurred in the first few years of the study.

2.1 Summary Statistics

For reporting purposes, the UPCs have been aggregated into 15 basic categories. The first two columns of Table 1 provide the number of UPCs and price observations in each category, whereas the final column lists the fraction of "sale price" observations, i.e. temporary price reductions.[2] The original data does contain an indicator denoting sale prices. In addition to these indicators, we define a sale price as any price that decreased and returned to its original level within two weeks. Clearly evident from the last column of the table is the importance of sales prices in the DFF data. A product is on sale 25.6% of the time and there is a great deal of variation across categories suggesting that sales are more common in certain groups of products

[2] Throughout the rest of the paper, the term "sales" will only refer to temporary price reductions. We will be explicit when we address the quantity of goods sold or the revenue of a product.

such as soft drinks. The average frequency of sales for products other than soft drinks is about 21% that is still higher than the frequency reported by Nakamura and Steinsson (2007) [3].

Plotting the prices of two sample products from Store 56 in Figure 1 illustrates the difference between sale and regular (No sales) prices. The price of Nestea Iced Tea (64 oz.) is relatively stable while the price of Diet 7-UP (24pk) is variable. Starting with Nestea Iced Tea, we see two periods of fluctuation (weeks 17-37 and 74-89) followed by relatively steady prices. When sale prices are replaced by the regular (No sales) prices that preceded them, we see that the regular price is relatively consistent over time, but has some variation. On the other hand, Diet 7-UP is in almost constant fluctuation, with more regular prices and sale prices.

In each week we have 32,625 prices and 435 modes (the most common price over the 75 stores). Table 2 describes the fraction of prices and modes that are changed in an average week. 38% of the prices and 35% of the modes change in an average week.

As expected, sale prices greatly increase the frequency of price change. When we replace sales with their regular price, prices change only 7.7% of the time and modes changes only 5.3% of the time. The second result gleaned from Table 2 is the relative flexibility of newly posted prices. 57.1% of young prices with age less than or equal to 3 weeks and 52% of young modes change in an average week. This is about 50% higher than the frequency of a price change when all prices are included which suggests that the probability of a price change decreases with age. We will examine this finding in detail, later.

Figure 2 plots the fraction of price and mode changes over weeks. The fraction of price changes is always higher than the fraction of mode changes but the difference is small. Moreover, the correlation between the two variables is very high (0.989). We will incorporate these features when attempting to model price setting in the chain.

[3] Nakamura and Steinsson (2007) find that 14% of unprocessed food and 13% of processed food observations are sales.

Table 1: Sample Statistics

Category	# of UPCs	# of Price Observations	% of Sale Observations
Analgesics (ana)	3	27,225	16.5
Cheeses (che)	60	544,500	22.9
Cookies (coo)	66	598,950	23.8
Crackers (cra)	18	163,350	27.2
Dish Detergent (did)	8	72,600	18.2
Front End Candies (fec)	36	326,700	12.0
Frozen Dinners (frd)	5	45,375	19.9
Frozen Entrees (fre)	24	217,800	21.9
Frozen Juices (frj)	17	154,275	27.2
Fabric Softeners (fsf)	4	36,300	20.9
Laundry Detergents (lnd)	6	54,450	15.9
Oatmeal (oat)	20	181,500	9.5
Refrigerated Juices (rfj)	45	408,375	23.7
Soft Drinks (sdr)	114	1,034,550	38.6
Soaps (soa)	9	81,675	15.5
All Products	435	3,947,625	25.6
All Products Excluding Soft Drinks			20.8

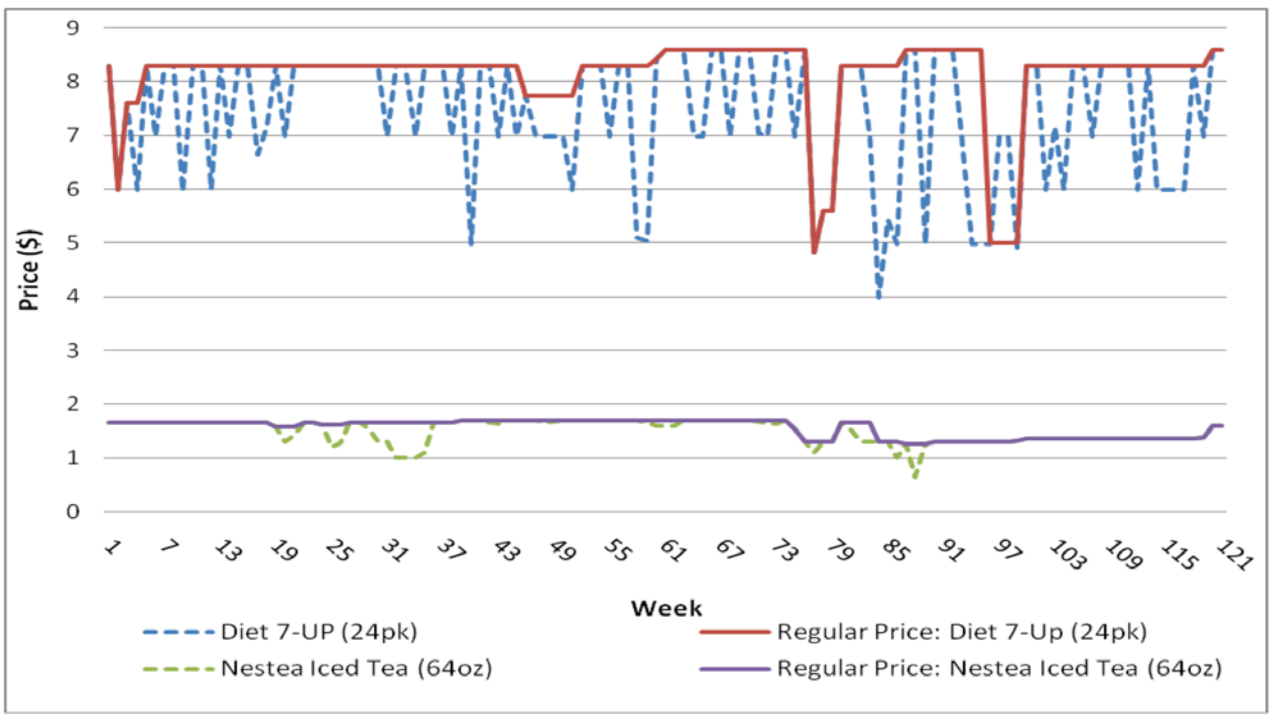

Figure 1: Prices of Two Sample Cells

Table 2: Price and Mode Change Frequencies

	Prices					Modes			
	-		No Sales			-		No Sales	
Cat.	All	Young	All	Young		All	Young	All	Young
ana	26.2	43	6.2	14.8		23.8	34.0	3.1	10.5
che	37.6	55	8.5	21		35.1	49.8	5.8	16.4
coo	31.8	48.9	6.9	15.6		28.6	42.2	4.6	12.1
cra	37.3	49	6.9	19.2		33.4	42.5	3.6	15.0
did	27.1	46.2	2.8	12.7		24.4	40.4	1.2	7.5
fec	15.7	43.2	5.1	13.6		14.7	40.2	4.1	11.7
frd	29.2	45	4	21		27.4	39.4	1.2	19.2
fre	29.9	47.2	3.9	17.9		25.4	39.6	1.4	12.1
frj	42.4	53.4	10	19.3		39.3	47.0	6.8	14.6
fsf	27.1	44.3	3.1	9.4		25.4	37.9	2.1	10.5
lnd	29.3	52.6	10.3	25.7		23.9	40.7	8.0	17.9
oat	15.6	36.7	3.6	12.3		13.3	28.3	2.6	8.4
rfj	39.7	60.2	11.3	23		35.0	52.4	8.4	18.4
sdr	56.7	67	9.3	19.1		53.8	63.7	6.7	18.1
soa	20.8	45.5	4.2	10.5		20.4	39.8	2.5	18.5
All	38	57.1	7.7	18.8		35.0	52.0	5.3	15.8

Notes: All numbers are the fraction of prices (modes) that are changed in an average week. The first column is the abbreviated product category (the full names are in Table 1). The second column is the frequency of price changes for all prices. Then we have the frequency for young prices (less than or equal to 3 weeks) only. The next two columns repeat the calculations after replacing sale prices with their regular prices. The last four columns repeat the calculations for the modes.

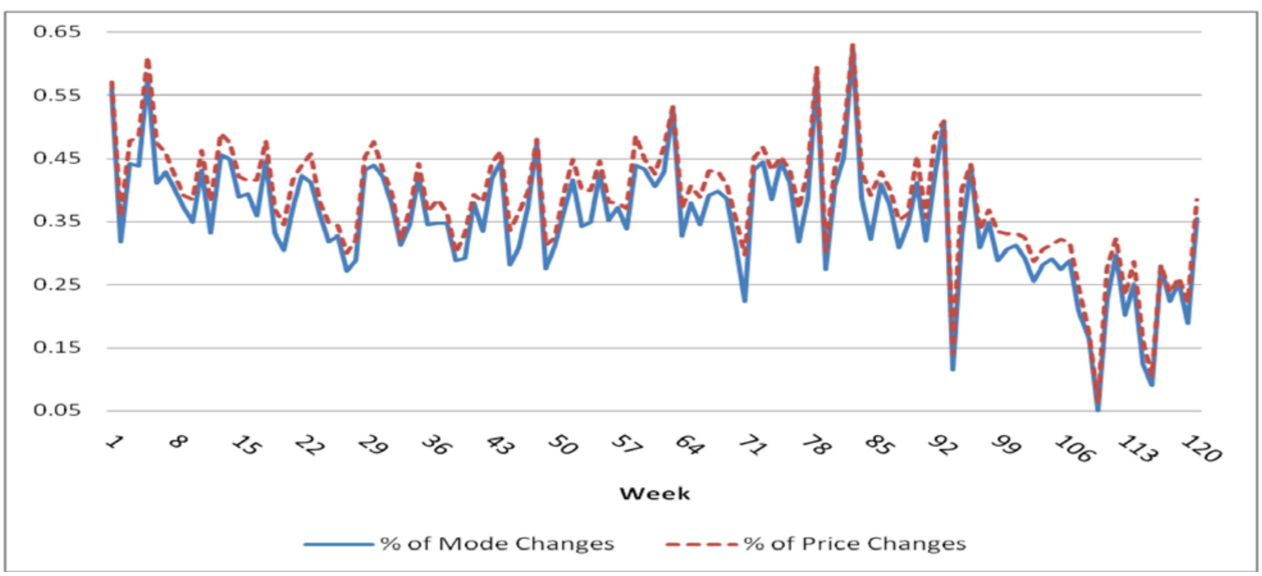

Figure 2: The Frequency of Price and Mode Changes Over Weeks

3. PRICE DISPERSION

The law of one price strictly holds in only 21% of the UPC-Week cells. Or in other words, price dispersion occurs in 79% of the UPC-Week cells. The frequency of price dispersion varies across product categories. In cookies, crackers, and analgesics, dispersion occurs in only about 50% of the cells, compared to frozen dinners, laundry detergents, and oatmeal, where dispersion occurs in 100% of the cells.

Table 3 contains four sets of price dispersion measures. The first column in each pair is the simple average, whereas the second is a weighted average where the weight for each UPC is its share in the chain's revenues. The first pair of columns is the number of different prices in an average UPC-Week cell. When looking at all prices (All), the average is 5.7 different prices per cell and 6.8 per cell when weighting by revenue share. The next pair is the percentage of prices equal to the mode (the most common price for the UPC-Week cell). The simple average is 75%, and the weighted average is 71%. The third pair is the average percentage price difference between the highest and the lowest store. The simple average is 18% and the weighted average is 19%. Note that a buyer who plans to buy all 435 items and spend according to their share in revenues can save 18.9% by going from the most expensive store to the cheapest but can save only 17.5% when spending an equal amount on each item. The last pair of columns is the standard deviation of the log of the price. The simple average of the standard deviation is 3.4%. The weighted average is 3.8%. Note that the measures of price dispersion are higher when weighting the UPCs by revenue shares suggesting more price dispersion in items that generate relatively more revenues.[4]

The finding that high weight items tend to have more price dispersion poses a challenge to various hypotheses. It looks like evidence against "cost of paying attention" models because for items with more revenues the cost of not paying attention is likely to be higher. The same goes for menu cost models in general since changing the price to the optimal one is likely to be more important for items that generate more revenues. It also poses a challenge to the hypothesis that we search more for high weight items and as a result there is less price dispersion in these items.

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[4] We also calculated the price dispersion measures after replacing sale prices with regular prices. The removal of sales prices tends to reduce the degree of price dispersion. For instance, the simple average number of prices is 4.8 per cell indicating that sales decrease the number of different prices in an average UPC-Week cell.

Table 3: Measures of Price Dispersion

	# of Prices		% at Mode		Ln(highest/lowest)		SD of Ln(Price)	
Cat.	-	Weighted	-	Weighted	-	Weighted	-	Weighted
ana	3.6	3.8	90	90	13	13	2.8	2.6
che	5.1	5.5	76	74	14	14	2.7	2.7
coo	4.2	4.4	86	86	8	8	1.5	1.5
cra	4.0	3.8	86	87	8	6	1.5	1.2
did	4.3	4.6	69	66	11	11	2.6	2.9
fec	2.9	2.7	90	91	16	14	2.9	2.7
frd	5.1	4.9	60	60	20	20	4.4	4.5
fre	6.2	6.3	65	65	20	19	3.7	3.6
frj	5.3	5.4	68	68	18	17	3.6	3.5
fsf	5.6	5.7	63	62	14	15	4.4	4.3
lnd	7.5	8.8	58	56	16	18	3.8	3.9
oat	5.7	5.8	60	60	19	19	3.8	3.7
rfj	8.7	9.0	62	61	32	33	6.1	6.2
sdr	6.9	8.8	76	69	22	23	4.2	4.8
soa	5.1	5.5	60	56	11	12	3.5	3.8
All	5.7	6.8	75	71	18	19	3.4	3.8

Notes: The numbers reported are averages per UPC-Week cells. The first column is the category name abbreviated. The full name of each category is in Table 1. We then have four pairs of columns. In each pair the first is the simple average and the second is a weighted average. The weights are the fractions in the chain's total revenue. The first pair is the number of different prices in an average UPC-Week cell, the second pair is the percentage of prices at the mode, the third pair is the log of the ratio of the highest price to the lowest price times 100, the last pair is the standard deviation of the log of price times 100.

To examine this hypothesis further, we compute the correlation between price dispersion measures and UPC characteristics. The matrix in Table 4 describes the correlations between our price dispersion measures, the UPC's share in revenue (weight) and its average price. It illustrates that our different measures of price dispersion are all positively correlated with the product’s weight and the average price (note that the percentage at the mode is inversely related to price dispersion).[5] We also note that the correlation between the weight and the average price is high (0.75). Thus UPCs with high average prices generate more revenues. In the Appendix, we address a difficulty in comparing the standard deviation measures of price dispersion across various studies. Although various studies use different deflators, we find that they are equivalent under the mean squared deviation definition of the variance.

[5] The correlations are higher in absolute value when sale prices are replaced by their regular price (not reported here).

Table 4: Correlation of Price Dispersion Measures with UPC characteristics

	# Prices	At Mode	H/L	SD(Price/Avg)	Weight	Ln(Price)
# Prices	1					
At Mode	-0.71	1				
H/L	0.72	-0.46	1			
SD(Price/Avg)	0.68	-0.38	0.93	1		
Weight	0.61	-0.44	0.18	0.22	1	
Ln(Price)	0.45	-0.40	0.15	0.23	0.75	1

Notes: The number of observations (UPCs) is 435. All correlations are significant at the 1% level. The variables are: the number of prices in an average week (# prices), the percentage of prices at the mode, the log of the ratio of the highest to the lowest price (H/L), the standard deviation of the log of relative price, the share in revenue (weight) and the average log of the price.

Some of the correlations in Table 4 are different from the findings in Pratt, Wise and Zeckhauser (1979).[6] They should be further investigated because as was said before they pose a challenge to various hypotheses. The hypothesis that we search more for high weight items may be rescued if the weight in the consumer budget is different from the weight in the chain's revenue. For example, imagine that all households purchase two things: Cookies and paper towels. There are many different kinds of cookies (recorded as different items), but only one kind of paper towel. It is possible that each household spends more on cookies than on paper towels but paper towels generates the highest revenues for the chain. If this is the case there may be more search for cookies. We think however that the weight of the item in the chain's revenue is a good proxy for its weight in the average household's budget.

Under this assumption, we may expect more search for high weight items and therefore the price of these items should be closer to the competitive price. To examine this hypothesis, we use a cost measure provided in the data to calculate the average percentage markup for each UPC. Although the cost measure is based on the historical cost of purchase rather than the current cost of buying the good, we think that when averaging over weeks it is not a bad proxy for the true cost. The average percentage markup across all UPCs is 28%, which falls to 25% when weighted. The correlation between the markup and the weight is -0.55 and similar to the other reported correlations is highly significant. This suggests that high weight items do indeed have lower markups on average.

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[6] Pratt, Wise and Zeckhauser (1979) find a positive relationship between the mean of the price of the good and the standard deviation (both measured in dollar terms). Their regression implies a negative relationship between the mean and the coefficient of variation. We find that the opposite is true.

We thus need a model in which high weighted items have more price dispersion and low markups. A Prescott (1975) type model of price dispersion may provide an explanation. In the monopolistic version of the Prescott model, the chain looses expected revenues if there is not sufficient price dispersion. We may thus think of price dispersion as desirable from the chain's point of view (more on this later). But because of returns to scale in the price setting activity, it may be much easier for a chain to determine a single price per week at the chain's headquarters. It may therefore be the case that the cost of implementing price dispersion is less for low weight items than the benefits and as a result we have less price dispersion in these items. To get low markups for high weight items, we may assume high demand elasticities for these items, possibly because consumers are willing to search more for high weight items. Note that the main difference between a Prescott type model and the “cost of paying attention” type model is that in the Prescott type price dispersion is desirable while in the attention deficit type price dispersion is the result of mistakes.

We now turn to describe the important role of the mode in the cross sectional distribution of the chain's prices.

3.1 The Mode

From Table 3, we see that the number of different prices in an average UPC-Week cell is much less than the number of stores. This suggests that many stores post the same price. In what follows, we rank price by the number of stores that post them. The most common price (the mode) is ranked 1, the second most common price is ranked 2 and so on. Figure 3 describes the percentage of prices at the five most common prices as a function of time. The fraction of stores that post the most common price (the mode) is 75% on average and fluctuates between 62 and 83 percent. The average fraction of prices at the second most common price is 12% with a range between 8-15 percent. On average, 95% of the prices are at one of the five most common prices.

As can be seen from Figure 3, the deviations from the law of one price occur in all weeks: The fraction of prices at the mode is always less than 83%. Moreover, Table 3 shows that deviations from the law of one price occurs in all product categories: The percentage of prices at the mode ranges from 58 to 90 percent. Does a similar finding hold for stores or do certain stores alway deviate from the mode while others do not? To examine this potential "store effect", we ranked stores by a score, defined as the net number of prices above the mode (the number of prices above the mode minus the number of prices below the mode) expressed as a percentage of the total number of prices in the store.

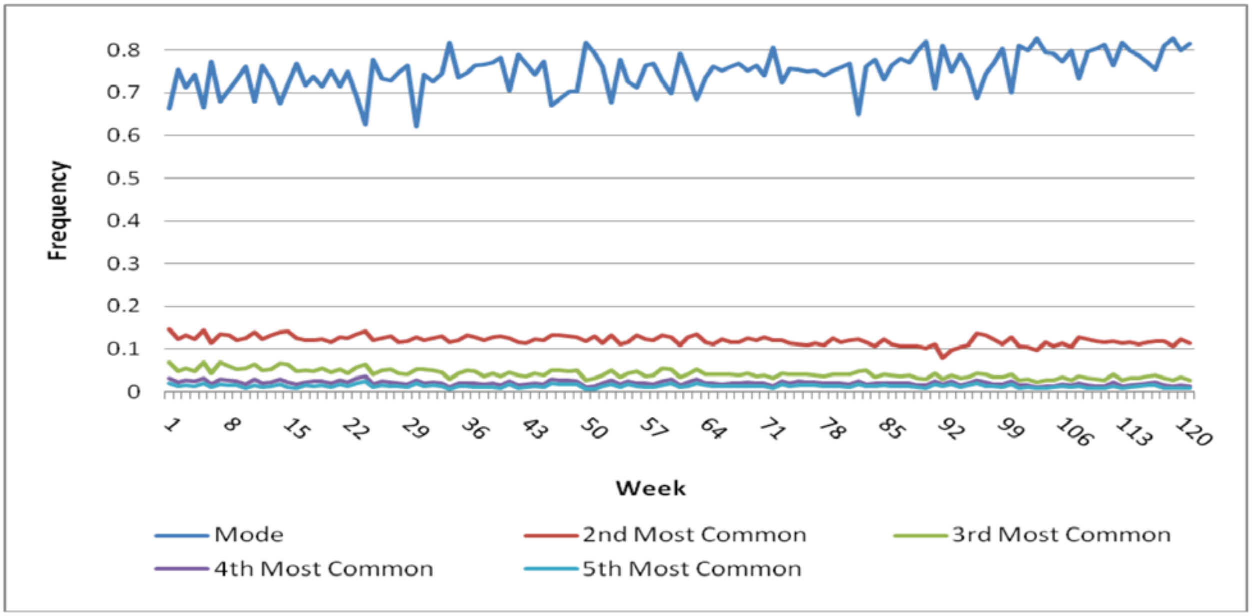

Figure 3: Percentage of Prices at the Most Common Weekly Prices Over Weeks

Figure 4 describes each store's average score, the standard deviation of its score and its average price change frequency where all averages are per week. To make the graph easy to interpret, stores were sorted according to their score. Note the difference between the two most divergent stores. The most expensive store (ranked 1) has a score of 29%, whereas, the cheapest store (ranked 75) has a score of -43%. In addition, the standard deviation of the scores has a *U* shape suggesting that predicting the price on the basis of the store’s score is more difficult for divergent stores. We will discuss the predictability issue shortly. The frequency of price changes is almost a straight line implying similarity in the standard measure of price stickiness across stores.

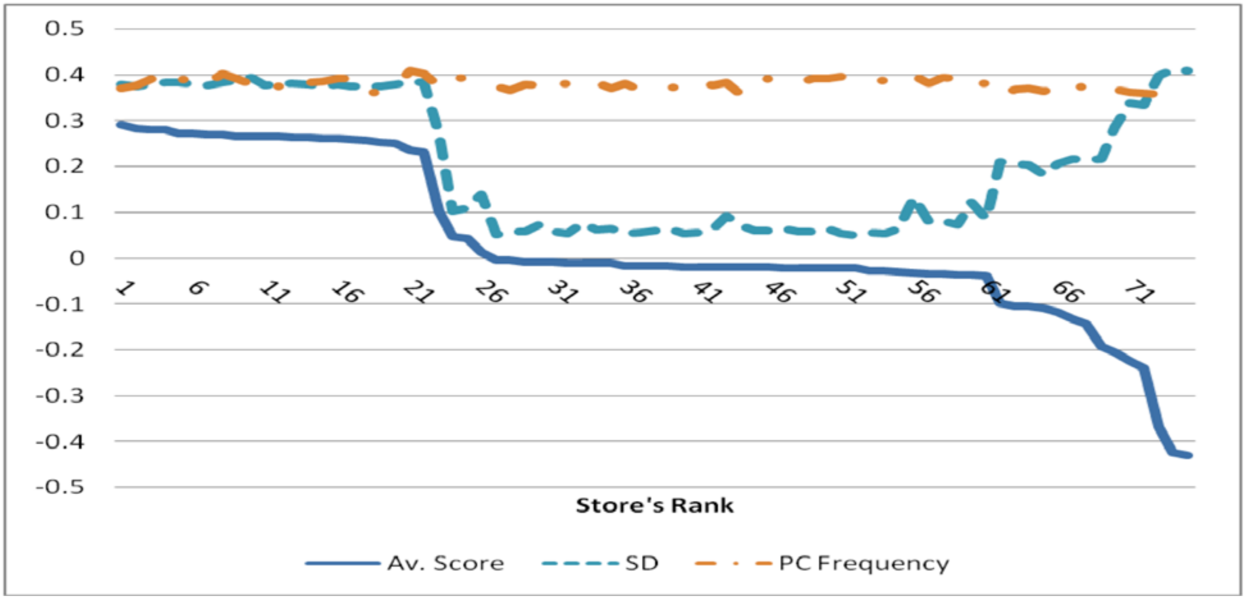

Figure 4: Average Scores, its Standard Deviation and the Frequency of Price Change by Store.

Using the same score sorting, Figure 5 describes the percentage of prices above the mode, at the mode and below the mode. The graphs suggest a division into three price groups: High (rank 1-22), medium (rank 23-61) and low (rank 62-75). The first high ranking group has on average 60% of the prices at the mode, 33% above the mode and 7% below the mode. The medium group has on average 86% of the prices at the mode, 6% above the mode and 8% below the mode. The low group has on average 67% of the prices at the mode, 6% above the mode and 27% below the mode.[7]

[7] We also tried a different grouping of stores based on the description in the data set (http://research.chicagogsb.edu/marketing/databases/dominicks/index.aspx). According to this description the corporate headquarters construct four general price strategies: High, Medium, Low, and Club Fighters. The broad strategies are then handed down to zone managers who provide more specific guidelines to their stores. We found that stores within each strategy are similar, but in all cases there are group outliers and sometimes the strategies are indistinguishable from each other. For instance, the "Low" and "Medium" strategy stores are more often in the mode (83% and 85% respectively) than the prices of stores that belong to the "High" and "Club Fighters" strategies (63% and 65% respectively). In our scoring method there are no outliers and three groups can be easily distinguished. We therefore find it easier to use our scoring method for describing the data.

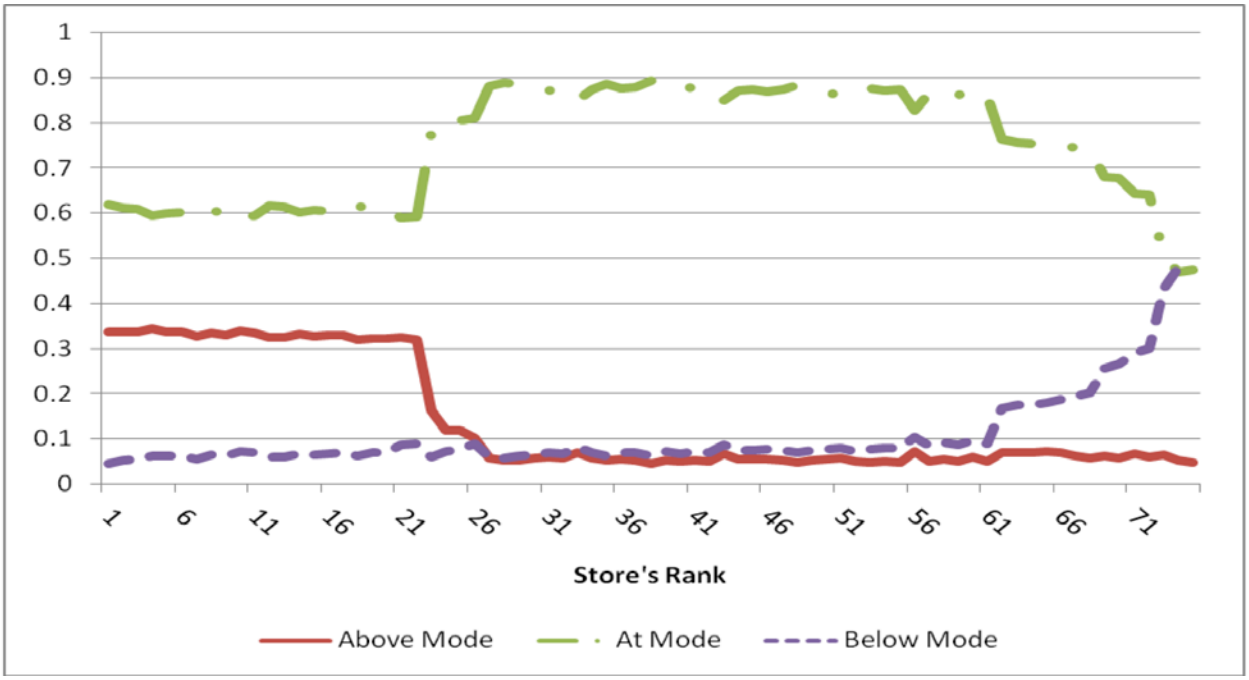

Figure 5: Fraction of Prices at the Mode, Below the Mode and Above the Mode by Store.

Figure 6 graphs a different measure of the divergence from the mode: the percentage price difference of an unweighted basket of the 435 UPCs. We kept the original (score) sorting for comparison. As can be seen from the Figure, the average percentage difference is almost monotonic indicating that this second measure is almost identical the score's ranking. The Figure graphs the average price difference and its standard deviation for each store. The high-ranked group of stores is on average 1.2% more expensive than the mode and the standard deviation for this group is on average 2.6%. The second group is on average 0.3% cheaper than the mode, with an average standard deviation of 1%. The low-ranked group is on average 1.9% cheaper than the mode with an average standard deviation of 2.9%. Once again the lowest and highest price groups have a larger standard deviation, giving the graph a U shape.

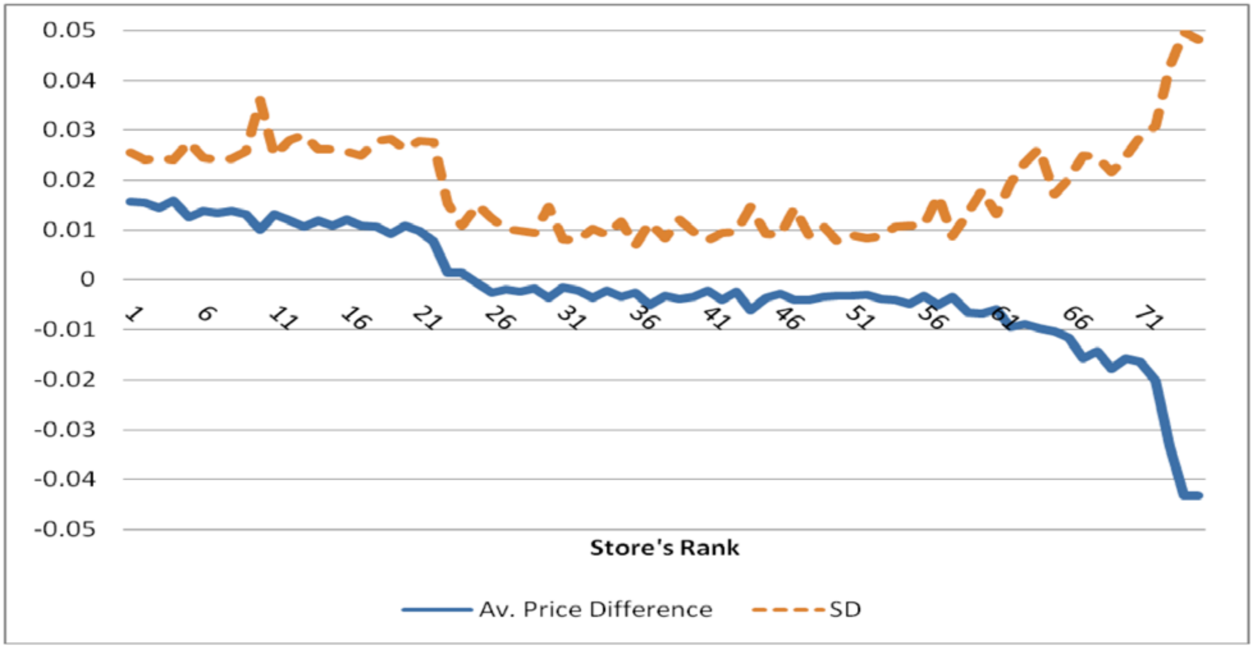

Figure 6: Average Percentage Difference From The Mode and Its Standard Deviation

3.2 Rank Correlations

We have shown that all stores have some prices above and below the mode, but there are also stores (those with high scores) that are likely to have more prices above the mode. To gain some further insight, we now turn to the question of predictability. We may think of a buyer who observes in week 1 the prices of a UPC, say a particular toothbrush, in all the 75 stores. With this information, the buyer can rank the price of this toothbrush in each store from highest (store 1) to lowest (store 75). Can the buyer predict the toothbrush price ranking in subsequent weeks using only the rank in week 1?

To answer this question, we computed the correlation between the “UPC Price Ranking” in week 1 and the price ranking of the same UPC in week t ($t = 1,...,121$). The graph labeled UPC ranking in Figure 7 reports the average correlations between the UPC price rankings. For the average or representative UPC, the correlation between its ranking in week 1 and its ranking in week 2 is 0.36. The correlation remains at approximately this level until week 5 where it jumps to 0.77. The correlation then drops (in week 6) to 0.25 and then decline slowly reaching the level of 0.2 in week 121.[8] When taking an average over weeks ($t = 2,...,121$), we find a correlation (between the rankings of an average UPC in an average week with its ranking in week 1) of 0.26. This average masks considerable variations across UPCs. We can find a UPC with an average correlation of -0.52 and a UPC with an average correlation of 0.97. The standard deviation of the correlations in the average week is 0.34.[9]

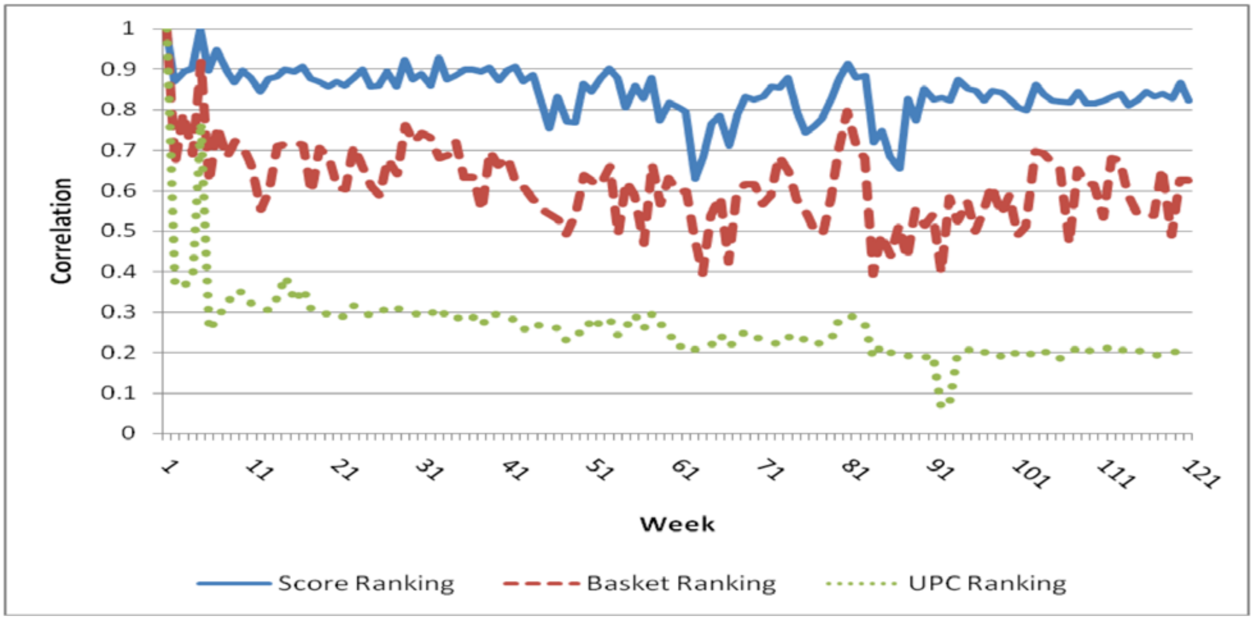

Figure 7: Correlation With Rank Measures in Week 1 and Week $t = 1,...,121$

Our findings are consistent with Lach (2002) who finds a low correlation between the product price ranking in the first month of his sample and subsequent months; however, we get a different picture when we adopt the point of view of a buyer who is interested in buying one unit from each of the 435 items in our sample and rank stores according to the price of this basket. The line labeled "Basket Ranking" in Figure 7 describes the predictability of this ranking. The correlation between the basket rankings in week 1 and week 2 is 0.67. The correlation then declines slowly and hovers around 0.60. The average correlation (across weeks) is 0.61.[10]

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[8] We were puzzled by the high correlation in week 5. To check whether there is something special about week 5, we computed the same graph when using the ranking in week 2 rather than in week 1 as the explanatory variable. We found that the jump in week 5 does not occur in this case and we think that it simply occurred by chance.
[9] We also computed the average of the correlations weighted by the shares in the chain's revenues. The results were similar with an average correlation (over weeks) of 0.28.
[10] We also computed the correlation when ranking stores by a weighted basket (using the shares in the chain's revenues for weights). The results were similar with an average correlation of 0.8.

Finally, we compute the correlations between rankings based on the store's net number of prices above the mode, labeled "Score Ranking" in Figure 7. Compared to the basket rank, the score rank is more predictable. The score rank correlation between week 1 and week 2 is 0.86 and declines slightly reaching 0.82 in week 121. The average correlation (across weeks) is 0.84.

We thus conclude that the store's rank is highly predictable when ranking stores by the price of the basket of the goods in our sample or by the net number of prices above the mode. On the other hand, the ranking of individual items is harder to predict. One possible explanation is that stores try to avoid stock-outs that may occur when recent demand for the product was unexpectedly high and a new shipment is not expected soon. The store may therefore choose to increase the price of items with relatively high probability of being stocked out and decrease the prices of low stock-out probability items (possibly by making a sale). This explanation is consistent with Aguirregabiria (1999) who found a significant and robust negative effect of inventories at the beginning of the month on current price.

We now turn to a model that may be used to assess the degree of price stickiness in the chain.

4. A MODEL

As was said in the introduction, we need a model to judge the importance of price rigidity. Price rigidity may be important even if 100% of them are changed every week but the size of the changes is small and not enough to clear markets. In addition we need to distinguish between the stickiness of the cross sectional distribution of prices and the stickiness of individual prices. The first is important to judge the ability of the chain to react to aggregate shocks and the second is important to judge the ability of individual stores to react to idiosyncratic shocks. We adopt a Calvo type model to measure these two types of price stickiness.

The model also helps us to better understand some of the main features of our data. We found that the fraction of prices at the mode, the second most common price, the third most common price etc. is quite stable. However, this finding does not arise from persistently higher prices for particular products in particular stores. Although price dispersion is common, the distribution of prices is highly discrete and in particular the mode of the distribution is “large”. We want to understand the implication of the discrete nature of the distribution on the transition probabilities from and to the mode and on the frequency of sales.

Following the data description in the Chicago GSB files, we assume that pricing decisions are made on two levels: The chain level and the store level. The chain level attempts to adjust the distribution of prices to aggregate shocks, while the store level is concerned with store specific shocks. This division of responsibilities arises because of increasing returns to scale in collecting information and making decisions about the effects of aggregate shocks.

The assumption that pricing decisions are made on two levels is also consistent with a Prescott type model. In this class of models only the distribution of prices is determined in equilibrium and sellers are indifferent among prices in the equilibrium range. We may therefore assume that the chain manager specializes in setting the distribution of prices while individual stores choose their location in the distribution to accommodate store specific shocks.

We also assume that the chain manager values uniformity because the chain is formed to economize on search costs. Therefore the chain manager chooses a highly discrete price distribution. For example, he may allow three prices: 0.95, 1 and 1.05. Stores with high inventories (that were accumulated because sales were unexpectedly low) choose 0.95. Stores with low inventories choose 1.05 and stores with inventories in the normal range choose 1. Thus, by presenting the stores with a discrete menu of prices the chain manager balances between two objectives: uniformity and stock-out avoidance.

4.1 The Price Setting Process

We proceed with the assumption that price setting decisions are made on two levels: the chain level and the store level. The chain manager hands the store managers a menu of $n+1$ prices. The menu in week t has the prices P_{it} where $i=1,...,n+1$ and each of the store managers chooses a price out of this menu whenever he can make a price setting choice. We assume that when the store manager chooses the week t price he knows the choice of other store managers and the mode of the week t cross sectional price distribution. In week t a fraction x_{it} of the stores post the price P_{it} where $i=1,...,n+1$. We choose indices such that $x_{1t} \geq x_{2t} \geq ... \geq x_{n+1t}$. Thus the first price, P_{1t}, is the most common price (the mode of the distribution), P_{2t} is the second most common price and so on. For tractability we assume a steady state distribution of price ranks: $x_{it} = x_i$ for all i and t. Note that although the number of price alternatives $n+1$ and the distribution of price ranks do not change over time dollar prices may change.

At the beginning of each week, stores observe the realization of a random variable that determines which of the $n+1$ prices they want to post. We view the choice of the store as the choice of an index (index i) or a rank: The store may choose the first (most common) price offered by the chain manager, the second price and so on. In the steady state the distribution of preferred ranks is consistent with the steady state distribution: A fraction x_1 of the stores prefers the first price (the mode) and in general a fraction x_i of the stores prefers rank i. Finally, we abstract from store heterogeneity and assume that the store's preferred rank does not depend on its previous history. We thus think of the stores as participating in a lottery that treats all the participants symmetrically: The lottery assigns rank i to a fraction x_i of the stores.

The assumption of history independence is a strong assumption. It may however be quite realistic if the store chooses its price rank on the basis of the beginning of the week level of inventories and shipment arrives in the middle of the week. In this case the store manager will use both the price and the quantity shipped to reach his target level of inventories at the beginning of next week and there is little reason to expect serial correlation in the beginning of week inventories.

As in the Calvo model, agents may not be able to change their prices but whenever they can change prices there is no cost of doing so. We assume that the chain manager changes the weekly menu with probability Γ. If the chain manager changes the menu all stores change their prices. When the chain manager does not change the menu (with probability $1-\Gamma$) he allows a (randomly selected) fraction γ of the stores to change their prices and choose another price that is on the menu. The store manager may choose not to make a price change even when he is allowed to do so.

At the risk of repetition, we now describe the price setting process in terms of Eichenbaum's fairy parable. We assume a chain level fairy that arrives with probability Γ. If the chain level fairy arrives, the chain manager changes the menu of all the $n+1$ prices and all stores change prices. If not, the chain manager does not change the menu of prices. There is also a store level fairy that arrives whenever a chain level fairy does not arrive. The store level fairy randomly selects a fraction γ of the stores. Stores that got selected may change their price and choose another price that is on the menu. Those stores which are not selected cannot change their price. Figure 8 illustrates.

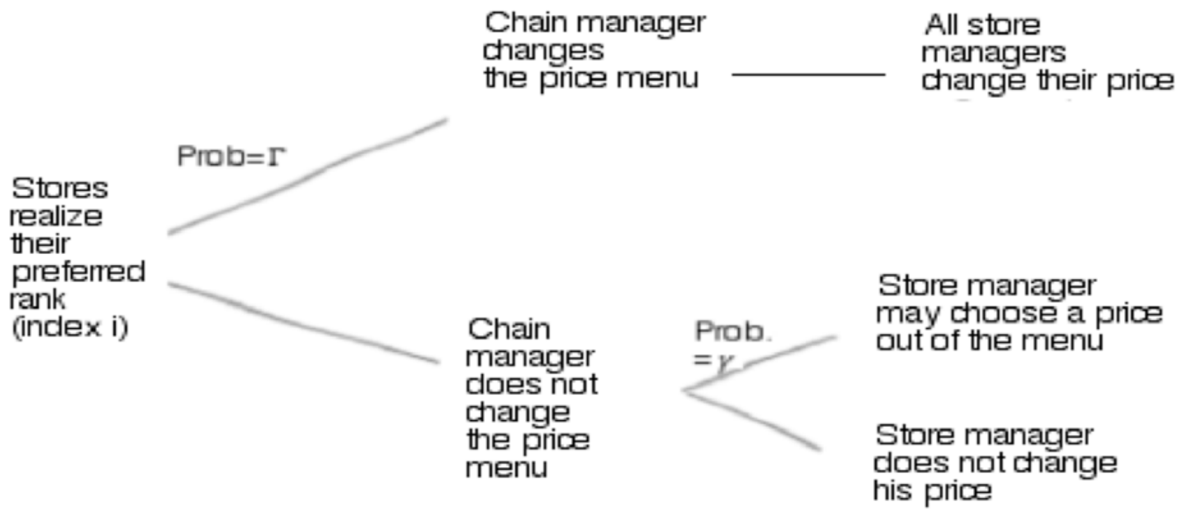

Figure 8: Model's Sequence of Events

Since the chain manager changes the price menu whenever he is allowed to do so we have:

(1) $P_{it} \neq P_{it-1}$ with probability Γ and $P_{it} = P_{it-1}$ with probability $1-\Gamma$.

We use lower case letters p_{jt} to denote the price posted by store j in week t. As was said before, a store manager will change his price if the chain manager changes the price menu. Since a store may also change its price when the menu does not change, the probability of a price change for a store that posted a price indexed i in week $t-1$ is:

$\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{it-1}) = \Gamma + (1-\Gamma)\gamma(1-x_i)$. The unconditional probability is:

$\phi = \Pr ob(p_{jt} \neq p_{jt-1}) = \Gamma + (1-\Gamma)\gamma S$, where $S = \sum_{i=1}^{n+1} x_i(1-x_i)$. This leads to:

(2) $p_{jt} \neq p_{jt-1}$ with probability ϕ and $p_{jt} = p_{jt-1}$ with probability $1-\phi$.

We use $\phi = \Gamma + (1-\Gamma)\gamma S$ to solve for the conditional probability that the store level fairy will arrive:

(3) $$\gamma = \frac{\phi - \Gamma}{(1-\Gamma)S}$$

In our sample, in an average week the chain changes a fraction $\Gamma = 0.35$ of the modes. The frequency of price change is: $\phi = 0.38$. We also estimated $S = 0.33$. Plugging these numbers in (3) leads to $\gamma = 0.14$.

Price rigidity may be measured by the probabilities that the two fairies will arrive. This is a two dimensional vector (Γ, γ). The stickiness of the distribution is measured by Γ which is 0.35 in our sample. The stickiness of individual prices may be measured by the probability that a store is allowed to change its price by: $\Gamma + (1-\Gamma)\gamma$. This is equal to 0.44. It should be noted that in our model the probability that a store is allowed to change its price is greater than the fraction of stores that actually change their price in a given week: $\Gamma + (1-\Gamma)\gamma > \phi$. This is because there are relatively few prices on the menu and as a result some stores that are allowed to change their price do not change it.

4.2 Conditional probabilities

We now turn to the effect of being at the mode on the probability of a price change. A store is at the mode if its last week price is equal to the current week mode ($p_{jt-1} = P_{1t}$). While this might seem like an innocuous distinction, there is a large difference in the probability of a price change between stores that are at the mode and stores that are not. A store that is at the mode changes its price with probability 3.6%, but that probability drastically increases to 76.2% when the store is not at the mode.

This difference in transition probabilities arises in our model simply because, by definition, the mode is "large". To build some intuition, consider the following parable. There are 6 cities. A large city, say New York, with 75% of the total population and 5 cities of equal size each with 5% of the total population. Individuals learn their preferred location at the beginning of each period. The preferred location does not depend on their current location or previous history and the distribution of preferences is consistent with the steady state distribution: 75% of the population prefers New York and 25% of the population prefers one of the 5 smaller cities (5% per city). We now let some of the agents costlessly change their location. The agents who are allowed to change their location choose New York with probability 0.75 and any of the other 5 cities with probability 0.05. The probability of a location change for an agent who is allowed to change his location is therefore 1 - 0.75 = 0.25 if he currently resides in New York and 1 - 0.05 = 0.95 if he resides in one of the small cities. Thus those who are at the mode of the location distribution are much less likely to change their location even when past history does not matter for the location choice. Based on this idea, we can use the model to see whether the large effect of being at the mode on the price changing probability can be explained just by the "size" of the mode or does it require additional assumption like stickiness at the mode.

We assume that a store can be in the mode $(p_{jt-1} = P_{1t})$ only if the price menu did not change. The probability that a store is in the mode is therefore $(1-\Gamma)x_1$. A store that is at the mode will change its price rank if it is allowed to do it (with probability γ) and if its preferred rank is different from the mode. Under the assumption that history is not relevant, the probability that the store prefers the first price (the mode) is $x_1 = 0.75$. Therefore the probability that a store that is already at the mode will change its price if it is allowed to do so is $1-x_1$. We compute the conditional probability of a price change using Bayes' rule and the following probabilities[11]: $\Pr ob(p_{jt-1} = P_{1t}) = (1-\Gamma)x_1$ and $\Pr ob(p_{jt} \neq p_{jt-1} \cap p_{jt-1} = P_{1t}) = (1-\Gamma)x_1\gamma(1-x_1)$. This leads to:

(4) $$\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{1t}) = \frac{(1-\Gamma)x_1\gamma(1-x_1)}{(1-\Gamma)x_1} = \gamma(1-x_1)$$

In our sample: $\gamma(1-x_1) = (0.14)(1-0.751) = 0.035$.

The average fraction of stores that posts a non-mode price is $x = (1-x_1)/n$. In our sample $n = 4.6$ and $x = 0.054$. The probability of a price change for a store that is not in the mode is:

(5) $$\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t}) = \frac{\Gamma + (1-\Gamma)(1-x_1)\gamma(1-x)}{\Gamma + (1-\Gamma)(1-x_1)} = 0.726$$

The difference between (5) and (4) is:

(6) $$\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{1t}) - \Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t}) = -0.691$$

We have used the data to estimate five parameters: ϕ, Γ, S, x_1, x. Using these parameters we calculated steady state conditional probabilities under the assumption that history does not matter. We may therefore measure inertia relative to these predictions. We say that there is inertia at the mode when the estimated frequency of a change in price for stores that are at the mode is less than the prediction of the model. If we find inertia we may conclude that history matters in the expected direction: A store that is at the mode is more likely to prefer the mode than a store that is not at the mode. In terms of the cities parable, the probability that you prefer living in New York is higher if you are already living in New York. The opposite of inertia is a

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[11] $\Pr ob(A \mid B) = \Pr ob(A \cap B) / \Pr ob(B)$

preference for change: A store has preference for change if the probability that it prefers its current price rank is less than the steady state fraction of stores at that price rank.

To check for inertia we computed the conditional frequencies[12]:

(7) $Freq(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{1t}) = 0.036$; $Freq(p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t}) = 0.762$

These frequencies are very close to the predicted values in (4) and (5) suggesting that history does not matter much. If at all, there may be slight preference for change for stores that are at the mode.

The probability that a store that was not at the mode will move to the mode is:

(8) $$\Pr ob(p_{jt} = P_{1t} \mid p_{jt-1} \neq P_{1t}) = \frac{\Gamma x_1 + (1-\Gamma)(1-x_1)\gamma x_1}{\Gamma + (1-\Gamma)(1-x_1)} = 0.545$$

We derived (8) by using Bayes' law and $\Pr ob(p_{jt-1} \neq P_{1t}) = \Gamma + (1-\Gamma)(1-x_1)$;

$\Pr ob(p_{jt} = P_{1t} \cap p_{jt-1} \neq P_{1t}) = \Gamma x_1 + (1-\Gamma)(1-x_1)\gamma x_1$.

The probability of moving from one non-mode price to another non-mode price is:

(9) $$\Pr ob(p_{jt} \neq P_{1t} \cap p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t}) = \frac{(1-x-x_1)\left[\Gamma + (1-\Gamma)(1-x_1)\gamma\right]}{\Gamma + (1-\Gamma)(1-x_1)} = 0.215$$

To derive (9) we used Bayes' law and: $\Pr ob(p_{jt-1} \neq P_{1t}) = \Gamma + (1-\Gamma)(1-x_1)$;

$\Pr ob(p_{jt} \neq P_{1t} \cap p_{jt} \neq p_{jt-1} \cap p_{jt-1} \neq P_{1t}) = \Gamma(1-x-x_1) + (1-\Gamma)(1-x_1)\gamma(1-x-x_1)$.

Table 5 describes the results. The model accounts for the transition probabilities quite well.

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[12] $Freq(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{1t})$ was computed as follows. We first created a dummy that takes the value 1 when $p_{jt-1} = P_{1t}$ and zero otherwise. We then drop all observations for which this dummy is equal zero to get a subsample of stores that are in the mode. We then compute the fraction of stores for which $p_{jt} \neq p_{jt-1}$ in the subsample. Other conditional frequencies were computed in a similar way.

Table 5: Model predictions and conditional frequencies estimates

ϕ - Frequency of Price Change	0.38	
Γ - Frequency of Mode Change	0.35	
x_1 - Proportion of Stores at Mode	0.75	
x - Proportion of Stores at a Randomly Selected Non Mode Price	0.05	
γ - Fraction of Stores that can change their price when the mode does not change	*0.14*	
A store can change its price with Prob. = $\Gamma + (1-\Gamma)\gamma$	*0.44*	
Conditional Probabilities	Predicted	Actual
(a) Moving from the mode: $\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} = P_{1t})$	*0.035*	0.036
(b) Changing prices when not in the mode: $\Pr ob(p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t})$	*0.726*	0.762
Difference = (a) - (b)	*-0.691*	-0.726
Moving to the mode: $\Pr ob(p_{jt} = P_{1t} \mid p_{jt-1} \neq P_{1t})$	*0.545*	0.519
Moving from a non-mode price to another non-mode price $\Pr ob(p_{jt} \neq P_{1t} \cap p_{jt} \neq p_{jt-1} \mid p_{jt-1} \neq P_{1t})$	*0.215*	0.243

4.3 The frequency of sale prices

The frequency of sale prices in the DFF data is much higher than the frequency of sale prices in the ERIM data studied by CE. As can be seen from Table 1, sale prices are 25.6% of all price observations. The comparable number for the ERIM sample is 3.4%. In both papers, a sale occurs when a decline in the price reverts to the exact same price within two weeks.

The difference may be explained by the fact that the ERIM data is a sample of all the stores in the city (only some of which belong to chains) while all the stores in the DFF data belong to the same chain. Our model of price setting in a chain assumes that the store manager can choose a price out of a menu with relatively few alternatives. A store manager who decided to reduce his price because he accumulated “too much” inventories or because he wishes to experiment with a lower price is therefore much more likely to go back to exactly the same price if he is part of a chain.

To elaborate, consider an independent store that currently posts the price p_t and chooses to experiment by reducing its price for two weeks. After the two-week period, it uses the accumulated information about the demand for the product and changes the price to the expected profit maximizing level $p_{t+2} = p^*$. We now compare the experience of the independent store to the experience of a hypothetical similar store that belongs to a chain and has to choose its price from a menu with few discrete alternatives: $P_1,...,P_{n+1}$. Suppose that it starts the experiment with a price $p_t = P_i$ and suppose further that the menu did not change in the two weeks period. After the experiment, the store concluded that the expected profit-maximizing price is p^*. It will definitely go back to P_i if $p^* = P_i$. It may also return to P_i if $p^* \neq P_i$ and there is no other price on the menu that promises a higher expected profit than P_i. Thus for the independent store $p_{t+2} = p_t$ if $p_t = p^*$ and $p_{t+2} \neq p_t$ otherwise. For the store that belongs to a chain, there is a set of optimal prices A_i such that $p_{t+2} = p_t = P_i$ if $p^* \in A_i$ and $p_{t+2} \neq p_t$ otherwise. Since typically the set A_i will have many elements, the probability that the store that belong to the chain will go back to exactly the same price is likely to be higher. This may explain why sales defined as a price reduction that reverts to its exact same level within two weeks, are much more common in our sample than in the CE sample.

Our explanation of the relatively high frequency of sales and the importance of the mode relies on the discreteness of the price menu. As we shall see, this discreteness also play an important role in explaining price changes.

5. PRICE CHANGES

The above model assumes that history does not matter and the only thing that can predict price changing behavior is whether the current price is at the mode or not. We now use this model as a benchmark (the null hypothesis) and examine whether other variables can explain why some prices are changed and others do not. We first describe two standard determinants of price changes.

5.1. Time Dependency: The hazard function

In Table 2, we showed that the frequency of price and mode changes is much higher for younger prices. Figure 9 explicitly describes this relationship between the frequency of price changes and age: the hazard function. As we can see, 65% of the 1 week-old prices were changed. The frequency of the 2 week-old prices falls to 35%, after which the slope of the hazard function flattens considerably. By the fifth week, the frequency is reduced to 22%, and reaches 10% at the age of 15. The hazard for mode changes is almost identical to that of price changes suggesting that the declining hazard phenomenon is not driven only by store specific shocks. This negative correlation is different from the prediction of traditional models. In the Calvo

(1983) model, there is no relationship between age and the probability of price change. Standard menu cost models suggest an increasing hazard function since a store has little to gain from changing a new price.

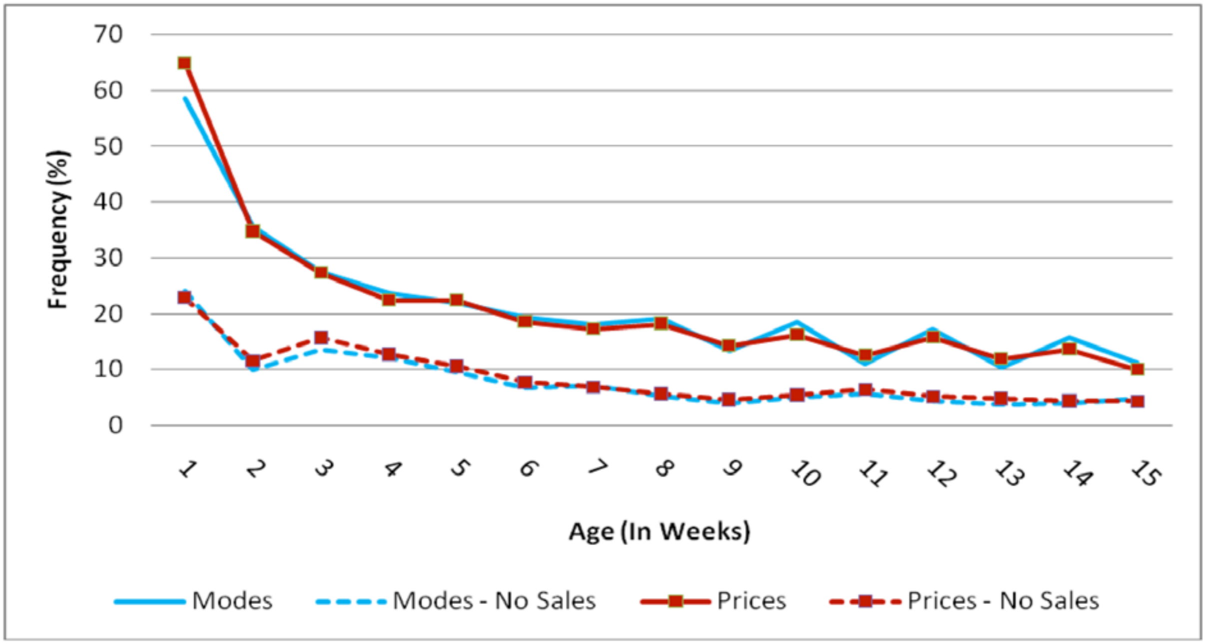

Figure 9: Hazard Function for Price and Mode Changes

It is well known that heterogeneity can lead to a decreasing hazard function. For example, a worker that has been unemployed for a long time has a lower chance of finding employment. This empirical fact is often explained by assuming that workers are heterogeneous. The argument is that workers who are unemployed for a long time are more likely to belong to the chronically unemployed type. By analogy it can be argued that an old price is more likely to belong to the rigid type. To see if the decreasing hazard in our case is due to heterogeneity between cells, we look at the slope of the hazard function in each Store-UPC cell, i.e. a 6pk of Coke at Store 101. Figure 10 describes the within cell correlation between age and the frequency of price change. As can be seen, a clear majority of cells have a negative relationship with an average correlation of -0.23.

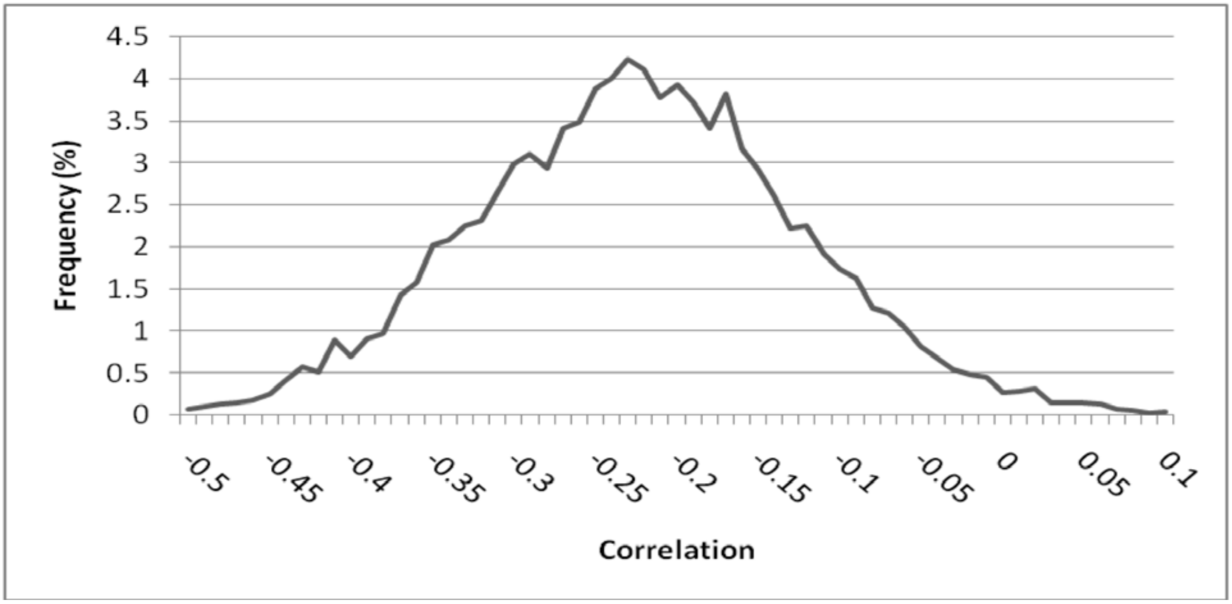

Figure 10: Within Cell Correlation Between Age and the Price Change Dummy

Table 6 uses the approach developed by CE to determine whether these negative correlations are significantly different from zero. This is done by comparing the frequency of change for young prices (less than or equal to 3 weeks) with the frequency of change of old prices. We find that in 93.2% of the cell the frequency of price change is higher for younger prices and the hazard function is therefore negatively sloped. Moreover, the slope is significantly negative for 59.4% of the cells, compared to the 2.7% with a significantly positive relationship. Even when sale prices are replaced, the null can be rejected for 39.3% of cells. The results suggest that the time relationship seen in Figure 9 is not only a result of store or product heterogeneity.

Table 6: Slopes of Within Cell Hazard Functions

	Prices								Modes						
	All Prices				No Sales				All Modes				No Sales		
Cat.	< 0	Sig. < 0	Sig. > 0		< 0	Sig. < 0	Sig. > 0		< 0	Sig. < 0	Sig. > 0		< 0	Sig. < 0	Sig. > 0
ana	100	85	0		88	42	0		100	67	0		80	46	0
che	95	57	0		95	56	1		97	42	0		87	49	1
coo	100	68	0		83	21	1		97	50	0		82	24	1
cra	99	48	0		90	46	0		89	33	0		88	43	0
did	100	90	0		74	31	5		100	100	0		77	35	1
fec	83	67	16		60	25	22		83	64	17		60	25	21
frd	100	57	0		89	67	1		100	20	0		94	77	4
fre	100	87	0		78	41	2		100	58	0		59	45	3
frj	95	53	0		91	26	0		94	41	0		89	31	1
fsf	100	81	0		57	16	0		100	25	0		46	32	0
lnd	100	66	0		97	62	0		100	33	0		98	44	0
oat	93	79	5		77	30	9		95	65	5		80	30	8
rfj	98	72	0		94	47	0		98	64	0		95	47	0
sdr	86	36	3		91	44	1		83	42	3		91	42	1
soa	89	89	11		75	44	16		89	89	11		75	42	19
All	93	59	3		86	39	3		92	51	3		83	39	3

Notes: Entries are in percentage terms. The first column is the percentage of (Store-UPC) cells with a negative correlation between the frequency of price change and age. The second column is the percentage of cells with negative correlation that is significantly different from zero. The third column is the percentage of cells with significantly positive correlation. The following columns repeat the calculations when sale prices are replaced by regular prices and for modes instead of regular prices.

5.2 State Dependency: Relative Price

According to the state dependent approach in Barro (1972), Sheshinski and Weis (1977), Caplin and Spulber (1987), and Caplin and Leahy (1991), stores keep their real (relative) price in a certain range (Ss band) and change their nominal price whenever the real price reaches an upper or a lower critical value (S or s). The real price is often computed by using an average price of the product as a deflator. Theory thus predicts that firms will exhibit an “attraction” to the average that is greater than the “attraction” to the mode. Indeed, in the absence of price discreteness, the mode should have no effect on the probability of a price change once the average is held constant. We now turn to examine this issue.

There is a difficulty in using our data for examining the role of the average price because the other stores in our data belong to the same chain and thus may not properly represent all the other stores in the relevant market. In what follows, we abstract from this problem and assume that the average price of the other stores that belong to the chain is a good proxy for the average price in the relevant market.

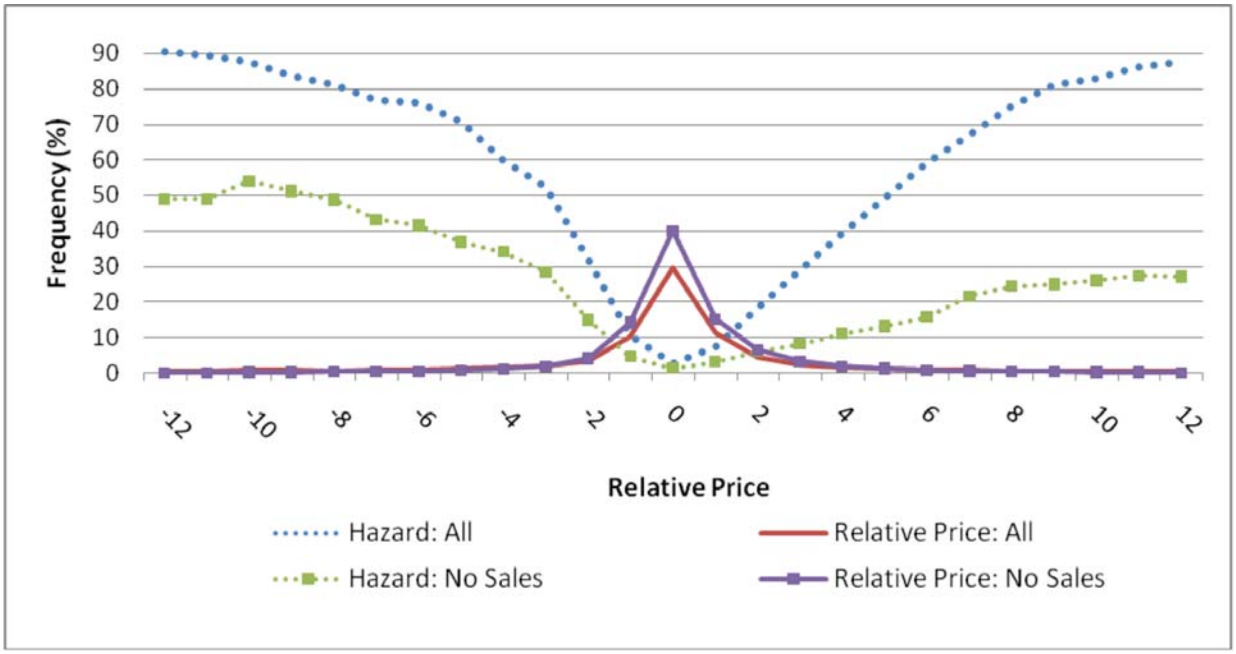

Figure 11: The Hazard as a Function of Relative Price

We define the relative price as logarithm of the cell's nominal price in the previous week divided by the current average price. We have also normalized this ratio with the store's mean to eliminate the bias from high-low pricing rules. To illustrate the normalization by an example, we consider a store that follows a low price strategy and on average posts a price that is 5% less than the mean. Suppose now that the store's last week price was equal to the average price in the current week. In this case, its relative price according to our normalization is 1.05. Note that the relative price is the price gap that would remain if the store did not change its price.

Figure 11 plots the relationship between relative price (on the horizontal axis) and the frequency of price change (on the vertical axis). This is done for all prices (All) and after replacing sale prices by their regular prices (No Sales). The bottom lines present the distribution of relative prices, while the higher lines give the conditional frequency of price change. Looking at the hazard for all prices, we see that the relative price is a good predictor of the frequency of price change and as a price diverges in either direction, the frequency of a change increases. The effect of the relative price is much less dramatic after replacing sale prices by their regular price. For example, when the store's price is 10% higher than the average price the frequency of price change is more than 80% when considering All Prices and less than 30% when considering the No Sales prices.

6. FORECASTING PRICE CHANGE

In the previous sections, we have discussed several variables that may affect a store’s decision to change its price. We now estimate a forecasting model that will allow us to judge the contribution of each variable to the explanation of price changes. The forecasting model employed is a marginal probit regression. It takes a binary dependent variable denoting whether a price changed during the current week. If the price had changed, the observation takes the value “1”, otherwise it is “0”.

In Table 7, we start with simple regressions that only use categories, store dummies and one of three age specifications: linear, quadratic and cubic. The table also reports the results when using the data with All Prices and when "sale prices" are replaced with "regular" prices (No Sales). In each column, the effect of age on the probability of a price change is negative and significant; however, the effect is much smaller when we replace sales prices by regular prices. To illustrate the size differences between the three age specifications, Figure 12 graphs the results. The negative slopes of all lines show that the probability of a change declines as a price ages. However there are significant differences in magnitudes between the estimates that use All Prices and those that use the No Sales data.

Table 7: The Effect of Age on the Probability of a Price Change

	All Prices			No Sales		
Age	-0.029	-0.042	-0.075	-0.002	-0.005	-0.009
	(-253)	(-399)	(-442)	(-236)	(-296)	(-241)
Age^2		0.001	0.003		0.001	0.001
		(385)	(74)		(227)	(133)
Age^3			-0.001			-0.001
			(-38)			(-91)
Observations	3,847,720	3,847,720	3,847,720	3,847,720	3,847,720	3,847,720
Pseudo R-squared	0.139	0.150	0.169	0.062	0.075	0.087

Notes: Coefficients are estimated using a probit regression. Marginal effects from the mean have been reported. T-statistics are listed in parenthesizes. Errors have been clustered by store to account for possible within-group correlations. Category and store dummies are included but have not been reported. All calculations without sales have had the sales prices replaced by their original regular price. All coefficients are significant at the 1% level.

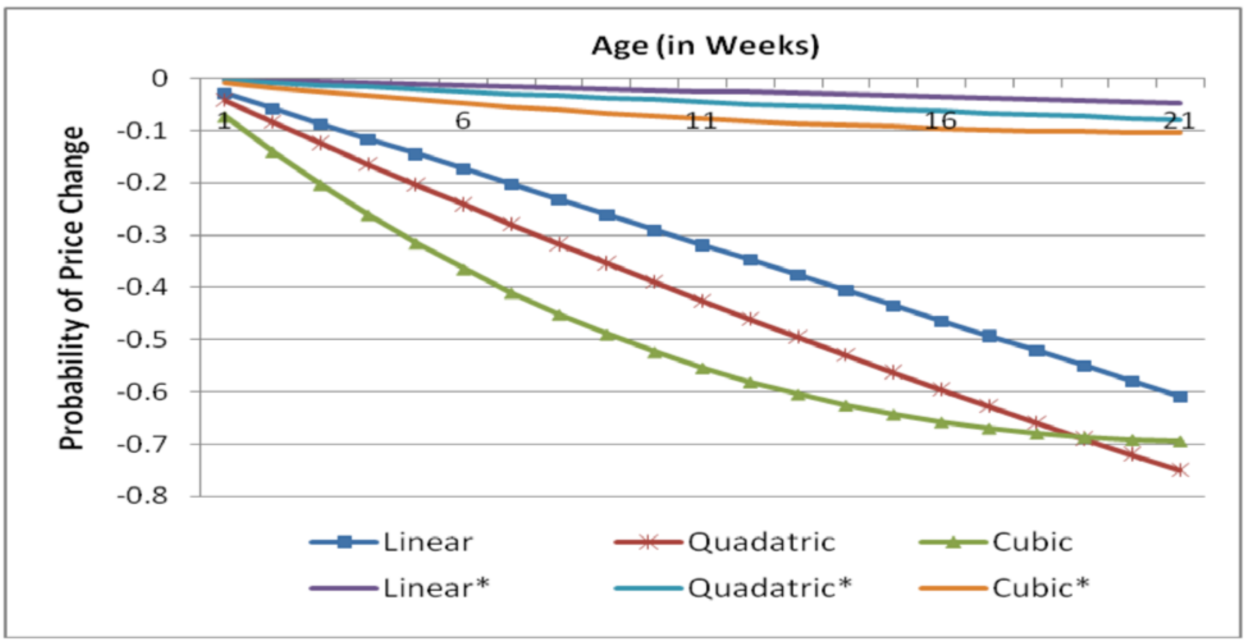

Figure 12: Implied Hazard Function Under Various Specifications ("*" denotes No Sales data.)

We also repeat this same experiment for our relative price variables in Table 8. We illustrate the different relative price specifications in Figure 13. As expected, the larger the absolute value of the relative price the larger is the probability of a price change. Once again, the magnitudes when looking at the data with no sales are much smaller.

Table 8: The Effect of Relative Price on the Probability of a Price Change

	Probability of a Price Change					
	All Prices			No Sales		
Relative Price^2	8.578	8.681	12.13	0.632	0.400	1.121
	(51)	(54)	(104)	(26)	(17)	(47)
Relative Price		0.177	-0.323		-0.196	-0.273
		(16)	(-15)		(-37)	(-26)
Relative Price^3			11.03			1.132
			(92)			(30)
Observations	3,847,720	3,847,720	3,847,720	3,847,720	3,847,720	3,847,720
Pseudo R-squared	0.305	0.305	0.345	0.036	0.04	0.056

Notes: Coefficients are estimated using a probit regression. Marginal effects from the mean have been reported. T-statistics are listed in parenthesizes. Errors have been clustered by store to account for possible within-group correlations. Category and store dummies are included but have not been reported. All calculations without sales have had the sales prices replaced by their original regular price. All coefficients are significant at the 1% level.

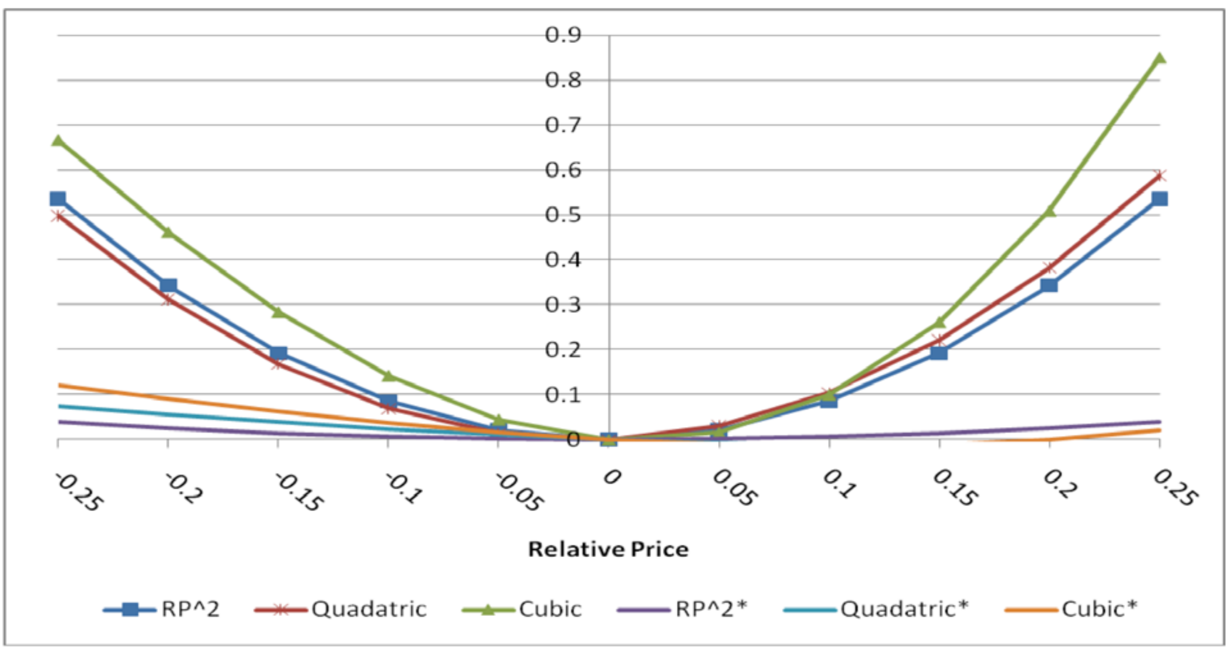

Figure 13: The Effect of Relative Price on the Probability of a Price Change Under Various Specifications. ("*" denotes No Sales data.)

Figure 13 puts a question mark on the practice of substituting regular prices for sale prices. Because we still wish to capture the effect of sales within the multivariate regression, we estimate the regressions with and without a sale dummy denoting whether the last week price was a "sale" price. In addition, we introduce two additional variables: The percentage of prices changed within the store, and a backward looking dummy variable denoting whether the previous price was equal to the current mode. The percentage of prices changed within the store is relevant when there is increasing returns to scale in the price changing activity.[13] The mode dummy is relevant if price discreteness is important and the store chooses its price out of a price menu with relatively few alternatives.

Table 9: The Probability of a Price Change

	Probability of Price Change					
Age	-0.022	-0.015	-0.016	-0.043	-0.027	-0.029
	(-116)	(-31)	(-30)	(-147)	(-34)	(-33)
Age^2	0.0002	0.0001	0.0001	0.0004	0.0002	0.0002
	(88)	(21)	(24)	(141)	(32)	(32)
Relative Price^2	6.888		2.31	7.828		2.454
	(48)		(17)	(48)		(16)
% of Price Changes in Store	0.367	0.208	0.215	0.243	0.114	0.114
	(92)	(12)	(13)	(68)	(6)	(7)
Price[t-1]=Mode[t]		-0.753	-0.723		-0.76	-0.735
		(-104)	(-84)		(-94)	(-81)
Coming off Sale Week	0.47	0.379	0.389			
	(249)	(31)	(34)			
Pseudo R-squared	0.442	0.628	0.659	0.363	0.588	0.623

Notes: Coefficients are estimated using a probit regression. Marginal effects from the mean have been reported. T-statistics are listed in parenthesizes. Errors have been clustered by store to account for possible within-group correlations. Category and store dummies are included but have not been reported. All coefficients are significant at the 1% level.

Table 9 summarizes the results of the various specifications of the price change equation. The first column reports the regression with all the variables except the mode dummy. The

[13] See Lach and Tsiddon (1996, 2006) and Midrigan (2006) for a more in-depth discussion of this possibility.

coefficients of all the variables have the signs suggested by traditional theory except for the age variables which imply a declining hazard.[14]

The second column replaces the relative price variable with the mode dummy. The coefficient of the mode dummy is -0.753 and highly significant. As a result of replacing the relative price variable with the mode the Pseudo R-squared went up from 0.442 to 0.628. The third column combines both the relative price variable and the mode dummy. The size of the relative price variable is now a third of its original value. The coefficient of the mode dummy hardly changes and it is still highly significant. We also tried other specification including a linear and cubic terms for the relative price variable, but this did not affect the importance of the mode. In the absence of price discreteness the coefficient of the mode dummy is expected to be zero. This supports the hypothesis that price discreteness is an important part of the explanation of price changes.

This hypothesis gains additional support from the fact that the coefficient of the mode dummy is very close to the prediction of our model that is based on price discreteness. In Table 5, we calculated the difference between the price changing probability for a price that is at the mode and a price that is not at the mode. Given our estimates of the underlying parameters of the model, the difference should be -0.691. When we estimated the difference in the conditional frequencies directly from the data, we found it to be -0.716. Both of these numbers are close to the regression's coefficient of the mode dummy suggesting that the size of the coefficient is robust to the inclusion of other variables in the regression.

The mode dummy also lowers the age coefficient. Its absolute value is now about 70% of its original level, but it is still highly significant and the implied hazard function is declining.

There may be a problem with using the Sale dummy in the regression. This is because our definition of a sale uses information about the future (a decline in the price that returned to exactly the same price) and this is not consistent with the forecasting nature of our exercise. We therefore repeat, in the last three columns of Table 9, the previous specifications without the sale dummy. The coefficient of the mode dummy does not change when the sale dummy is removed. Consistent with the finding of Table 7, the age coefficients increase in absolute value (about

[14] We also ran this regression with an additional linear term of the relative price variable. In this case the effect of a decrease in the relative price has a small positive effect, the rest of the results hold.

twice the coefficients in the first three columns). Consistent with the finding in Table 8 the relative price coefficient is also larger but the difference is relatively small. The removal of the sale dummy reduces the coefficients of the "percentage of price changes in the store".

We now turn to the forecast of mode changes. In addition to the age of the mode, we included the percentage of stores that posted the mode last week as an additional explanatory variable. Including this variable allows for a feedback from store managers to the chain manager. As can be seen from Table 10, the age coefficient is in the range of -2.3 to -2.8 percent. The coefficient of the percentage of stores at the mode is also negative (-4.3%) suggesting that when more stores are at the mode the chain manager is less inclined to change it. This is consistent with the hypothesis that the chain manager takes the price choices of the stores as a signal about demand and when stores deviate from the mode he is more inclined to change the menu.

Table 10: The Probability of a Mode Change

	Probability of a Mode Change	
Mode Age	-0.0279	-0.0230
	(-33.21)	(-28.28)
Mode Age^2	0.0003	0.0003
	(23.25)	(20.34)
% of Stores at Mode		-0.4310
		(-42.02)
Observations	50,674	50,674
Pseudo R-squared	0.173	0.203

Notes: Coefficients are estimated using a probit regression. Marginal effects from the mean have been reported. T-statistics are listed in parenthesizes. UPC and week dummies are included but have not been reported. All coefficients are significant at the 1% level.

7. CONCLUSIONS AND DISCUSSION

We argued that from a Macro point of view the rigidity of the price distribution may be more important than the rigidity of individual prices. In the DFF chain the fraction of prices that are at the mode is large and quite stable over weeks. We therefore characterized the price distribution by its mode. We find that in an average week 35% of the modes do change and the fraction of mode changes is highly correlated with the fraction of price changes. This suggests that fixed costs for changing the price distribution are not important.

In most of the UPC-Week cells there are more than one price but the price distribution is highly discrete: In an average week the number of distinct prices is much less than the number of stores indicating that many stores post the same price for the same item. This may occur because the chain balances its desire for uniformity with the need to react to store specific shocks.

The discreteness property of the distribution is important for understanding the transition probabilities from and to the mode. We use a simple model in which price decisions are made on two levels: The chain manager selects a menu of prices and the store manager chooses a price out of this menu. Stores' preferences do not depend on history and conform to the steady state distribution. Surprisingly this rather mechanical model captures the transition probabilities in the data quite well and explains why the probability of a price change depends critically on whether the price is at the mode or not.

The stock-out avoidance motif in Aguirregabiria (1999) may account for some of our findings. Aguirregabiria considers the problem of a retailer who face demand uncertainty that may lead to stock-outs. In his model, individual stores use prices to reduce the stock-out probability: They post a relatively high price when the level of inventories is low and a low ("sale") price when the level of inventories is high. We may assume that the chain manager values uniformity and therefore requires the stores to choose out of a menu with relatively few alternatives. Thus, in this framework price discreteness may be used to achieve a balance between the desire to minimize stock-outs and the desire to minimize price dispersion.

The stock-out avoidance motive for changing prices may also account for the observation that young prices are more likely to change. To illustrate assume that the store reached a low level of inventories in some good and reacts by (a) increasing the price of the good and (b) order a new shipment. The increased price lower the probability of a stock-out until the new shipment arrives. When the new shipment arrives there is no longer a need for the high price and as a result a price change occurs. Similarly, when the store reached a high level of inventories it may reduce the price and this may reduce inventories quite fast leading to another price change. Replacing sale prices with regular prices diminishes the effect of age on the probability of a price change because sales are an integral part of the story.

Our findings are also consistent with Prescott type models in which price dispersion arises as a result of demand uncertainty. The Uncertain and Sequential Trade (UST) model in Eden (1990) is a version of the Prescott (1975) model in which trade occurs sequentially and sellers must make irreversible selling decisions before they know the realization of demand. In this UST model, there exists a unique equilibrium with price dispersion both for the competitive and the monopoly cases. The monopoly case may be more relevant here. The fact that there exists a unique (non degenerate) distribution of prices that maximizes expected profits says that the expected profits when using a single price are lower than the expected profits when using the optimal price distribution. Dana (2001) illustrates the desire to have a (non degenerate) price distribution in a rigid price version of the Prescott model.

In the UST model, cheaper goods are sold first. This abstraction is of course unrealistic. We do not observe that in each week relatively cheap goods are stocked-out and relatively expensive goods are not sold at all. Instead most stores sell some quantity from each UPC regardless of the posted price. To account for the data we must therefore add some friction to the UST model.

Search costs are one such friction missing in the UST model. In the presence of search costs, it is no longer the case that a buyer who shops for a basket of goods will choose to go to another store simply because the price of say toothbrushes is relatively expensive. We may also expect that the store may want to insure prospective buyers that they can find the standard basket of goods sold by supermarkets. The store may therefore try to avoid stock-outs by increasing the price of low inventories items and decreasing the price of high inventories items. This may explain why we observe both high and low price items in the same store. It may also explain why predicting the price of a basket of goods in a given store is easy relative to predicting the price of an individual item. An alternative explanation may assume that the store is trying to discriminate between informed and uninformed buyers as in Varian (1980).

Our findings may also help explain the interesting observation made by Chevalier, Kashyap and Rossi (2003). Using the DFF data set, they observe that prices do not rise during periods of high demand. This may occur in a Prescott type model, if demand uncertainty is low during periods of high demand. To see this point, consider a perishable good that is demanded with probability 0.5 in a "regular" week and is demanded with probability 1 in a "peak demand" week. Assume that whenever the good is in demand buyers are willing to pay a high reservation price for it. The cost of the good to the supermarket chain is 5 dollars. In a competitive environment in which stores make zero profits on average, the store will charge a price of 10 dollars in a "regular" week in which it makes a sale with probability 0.5. The store will charge a price of 5 dollars in a "peak demand" week in which it makes a sale with probability 1. Thus we may observe that the price in a "peak demand" week is lower than the price that we observe in a "regular" week when the good is actually sold. The intuition is that in high demand periods the probability of making a sale is higher on average and therefore the average price is lower. This explanation can be tested by comparing measures of price dispersion during holidays and other "peak demand" weeks to the measures during "regular weeks". We plan to address this interesting issue in another paper.

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APPENDIX: THE STANDARD DEVIATION OF RELATIVE PRICES

We address a difficulty in comparing the standard deviation measures of price dispersion across various studies. Although various studies use different methods, we find that they are equivalent under the mean squared deviation definition of the variance but yield somewhat different results when using the unbiased estimator of the variance. Eden (2001), Lach (2002) and Ahlin and Shintani (2007) focus on the variance of real prices. Eden (2001) for example, use $R_{ijt} = \frac{P_{ijt}}{P_{it}}$, where P_{ijt} = the price of good i in store j in week t and $P_{it} = (1/75)\sum_{j=1}^{75} P_{ijt}$ = the average price of good i in week t. When taking the log of the relative price, the variance does not depend on the deflator as long as it is common to all the stores in the cell and can be treated as a constant. In detail, the average of the log of relative price is:

$\ln R_{it} = (1/75)\sum_{j=1}^{75} \ln(R_{ijt}) = (1/75)\sum_{j=1}^{75}\left(\ln(P_{ijt}) - \ln(P_{it})\right) = 0$. The variance of relative price is therefore:

$$Var_{it}^{\ln R} = (1/75)\sum_{j=1}^{75}\left(\ln(R_{ijt}) - \ln R_{it}\right)^2 = (1/75)\sum_{j=1}^{75}\left(\ln(P_{ijt}) - \ln(P_{it}) - \ln R_{it}\right)^2 = (1/75)\sum_{j=1}^{75}\left(\ln(P_{ijt}) - \ln P_{it}\right)^2 = Var_{it}^{\ln P}$$

The same equivalence applies to Lach (2002) and Ahlin and Shintani (2007) who deflate by the CPI rather than by the average price in the Product-Week cell. Next, some studies report the variance over the entire sample while we took the average in the Week-Product cell and then averaged over cells. Under the mean squared definition, the order in which the average squared deviation is computed does not matter. As we said above, these equivalence results hold for the mean squared deviation definition of the variance. In many computer programs (like Excel), the unbiased estimator of the variance is used and therefore different measures and different ways of computing the average leads to different results. We computed several possible measures and found that the correlation between them is high (around 0.97). Here, we report one measure only based on the variance of the log of the relative price.