CHOREOGRAPHIC WAYS OF KNOWING AS GENERATIVE SITES FOR STEM LEARNING, DESIGN, AND ANALYSIS

By

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This dissertation is dedicated to Rhea Lynn Vogelstein. May you continue to move and grow in the world as the joyful, curious, and loving person you are becoming.

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TABLE OF CONTENTS

Page

LIST OF FIGURES

Figure Page

CHAPTER 1

DISSERTATION WRAPPER

Introduction: A reflection on how I got here, what I think I've learned, and why it means so much

Nine years ago, I was wrapping up my college education, where I double majored in dance and mathematics, trying to figure out where I fit into the world and how I would exist outside of the structured supports in college that enabled me to regularly work in the two disciplines I enjoyed most. I really loved being a student, being given time and space to explore what I loved in the Ailey dance studios late at night when the lights of 9th avenue looked like twinkling stage lights reflected into the studio and in an empty Fordham classroom with white board space waiting to be filled with color coded lines of proof and algebraic logic. Imagining life beyond the comforts of school was challenging, there did not seem to be a place where I could continue to develop my interests as rigorously and fully as I had in college.

I also felt like I did not quite fit into either world on their own. I had been given many subtle and not so subtle hints throughout my college career that I was not really cut out to be a professional dancer. While I received good grades in my dance classes, where my ability to learn combinations of dance movements quickly, ask detailed questions about movement execution, and catch any inconsistencies in teachers' descriptions of choreography at a fine grain level were appreciated, these were not qualities in my professors' eyes that meant I would be successful as a dancer. I was "smart," and good enough to keep up with the technical asks of these classes, but not necessarily good enough at executing these movements at the level expected of someone outside of the classroom. I was always told I needed to acquire better lines (become more flexible and

limber, something my body seemed to resist), lose more weight (something else my body and soul struggled to accomplish), or once my advisor and supposed mentor told me, "there might be some companies in Texas, I don't know their names, that might be interested in you." And with respect to pursuing a career in mathematics, I had spent one summer as a math research assistant but had no clear sense of how to enter that profession, what it entailed, or if I even wanted to.

My senior year began with participation in a required course entitled "Senior Seminar," in which we as dance majors were exposed and asked to think about alternative career paths to being a dancer (e.g., arts administration or lighting design for the stage). At the time, I was quite offended by the existence of the course. After spending three years intensely immersed as students in one of the most prestigious contemporary dance schools in the world (The Ailey School), dreaming of our careers as professional dancers (which is what brought us to this school and kept us there), we were now being not so subtly told to consider other lines of work, that many of us were not cut out to be dancers. The reality of this uncertain future was hard to swallow, but looking back on this course now I am very grateful that it existed and that I participated in it because it was in this seminar that I first articulated how I identified, connected, and envisioned learning through both dance and mathematics:

> Throughout my entire college career I have confounded many people merely by telling them that I am pursuing a BFA in Dance along with a BS in Mathematics. Most people look at these two areas of study as if they are oil and water; they will never mix. I, however, see dance and math in a symbiotic relationship, each enhancing my understanding of the other. They are both the study of patterns, one in a physical manifestation and the other in an abstract one. This past spring I was pleasantly surprised to learn about the opening of MoMath right here in New York City. MoMath is a new museum dedicated to teaching mathematical concepts in fresh, innovative and creative ways for learners of all ages. The museum will open on December 15th and I would love to be the person who brings

> movement as a learning tool to an exhibit one day. I am a firm believer that the creativity of the arts allows for more open minded and fertile work in academic fields, while classic academic knowledge allows artists to delve deeper into their own work. Most dancers do not even know the untapped mathematical knowledge they have stored in their bodies and minds. As an avid learner of both of these areas of study the connection is clear to me and I can only imagine what will happen if this unlocked potential is explored in dancers and students of math everywhere (Vogelstein, 2012)[1].

I return to this early writing because this is where some of the most consequential seeds for this dissertation were planted. Reading these words, I am brought back to my younger self, a person bubbling over with enthusiasm for bringing the two seemingly disparate parts of herself into a cohesive whole with the intent that others would blossom after experiencing the connections I so intimately felt.

I so desperately wanted to find a place where my expertise and competencies could be appreciated and nurtured. At the time I was convinced that the newly opened Museum of Mathematics (MoMath) was the place for this to occur. And while working at MoMath (my first job after college) changed my life in many ways (it was where I met my husband Jake and my best friend Amanda and first tested my designs for interdisciplinary dance and math learning) it was also an extremely toxic work environment where my intellectual independence and creative

[1] While well received by my professor at the time, not all of my peers agreed that there would be generative connections across these disciplines. Most vividly I remember a classmate of mine vehemently discrediting these ideas because she had failed her statistics class the previous semester. She was adamant that if her dance expertise was mathematical then it would have translated to her math class and she would not have struggled as much as she did. I do not remember how I responded to her claims, although I do remember feeling attacked both personally and professionally in the moment. Now I can see how differing assumptions about "transfer," communities of practice, and learning more broadly swirled and collided in this heated moment (if I recall one or both of us ended up in tears by the end) and I have tremendous empathy for a student who saw obtaining a minor in statistics as accumulating needed cultural capital and was pushed out of the spaces she needed to be in in order to obtain those skills. And here I was saying that she had what she needed inside of her to succeed mathematically, so when she didn't and if I was right maybe that meant there was something wrong with her. While at the time all I could see was my own identity and self-worth being attacked, I now see how she might have felt similarly. This reminds me that the generative and potentially liberatory prospects of this line of work are not guaranteed only because *I* find them exciting and expansive.

enthusiasm was taken advantage of and simultaneously stifled around every corner. At the time I didn't know much about what being a professor meant, both in terms of the day-to-day experience of the profession or what the path to becoming one entailed. I didn't even know the terms "scholar" or "academic." And I certainly didn't know that there was a field called the Learning Sciences where my ideas and dreams could blossom. At the time all I knew was that I needed to find work, I didn't feel like I fit in anywhere, and I was really interested in questions about how people learned math and how people learned more broadly.

In some ways this story could be evidence of leaky pipelines in both the disciplines of dance and mathematics, representing me as a water droplet that, whoops, fell out of both. However, I'm reminded of my mentor Tesha's righteous anger at this prevalent metaphor in STEM education because I was not and am not a water droplet whose movement was left to the whims of rejection by others. I didn't just passively fall out of a hole. In fact, these separate spaces were not places that affirmed my dignity, my self-worth, or creative potential and it was a good thing that I found community elsewhere.

At the end of this chapter of my life, as the grad school door at Vanderbilt is about to hit my butt on the way out, I am so grateful to have found a home in the Learning Sciences. And yes, I do really feel at home here in terms of comfortability, acceptance, and even tensions with relatives that I disagree with on certain things (it's not perfect, nothing is). Needless to say, I am overwhelmed, some might say "verklempt[2]," at the place I am today, at the fact that this is my dissertation you are reading right now, and at the prospect of a lifelong career deepening understandings and relationships to collaboratively design for learners to expansively engage in STEM through physical, choreographic forms of participation.

[2] This is a Yiddish term meaning overcome with emotion, commonly used by bubbies (grandmothers) when their children or grandchildren reach life milestones and accomplishments.

Motivation: Why bring choreographic ways of knowing to the forefront of STEM learning?

My history and identity as a dance artist inform my political commitment to positioning dance as a form of research and dancers as collaborative research partners. Dance has too often been cast as an artistic and physical pursuit separate from the superior, thinking brain. However, how we move and feel in our bodies is an important part of learning and sensemaking, and cutting off these resources limits what and how we can learn in STEM and other learning environments. In this dissertation, I focus on choreographic practices as processes of inquiry to challenge social narratives that cast embodied and artistic ways of knowing as antithetical to rigorous, scientific methods of knowledge production. For example, part of Chapter 2 demonstrates how a group of four 8th grade girls manipulating a large square sheet of Mylar developed a complex conceptualization of reflections as consisting of the coordination of two pairs of pairs. They came to this understanding by attending to how their choreography looked and felt; by privileging these girls' aesthetic sensibilities and axiology, this work illuminates the novel sensemaking approaches dance afforded these girls in their mathematics learning.

One of the ways intelligence has been reified as a cerebral disembodied pursuit is through the positioning of mathematics as a prestigious, pure discipline of platonic ideals separated from the messy corporeal world we live in. However, research in embodied mathematics has worked to put the body back into the equation (Stevens, 2012). While recent work in the past few decades has supported a general argument that the body is a supportive resource for mathematics learning, the different ontological commitments across this work has made for a field that conceptualizes the body's role in cognition in very different ways. Conceptualist work in embodied mathematics (Goldin-Meadow et al., 2001; Lakoff & Núñez, 2000) theorizes that gestures and other basic

embodied experiences (e.g., over and under) lead cognition in an individual's development of mathematical image schema. In contrast, interactionist theorists (Hall et al., 2014; Hall & Nemirovsky, 2012; Nemirovsky et al., 2012) have expanded cognition from the mind out through the body to the social and tool-laden environment. Unlike conceptualist theories of embodiment, interactionist perspectives theorize the body itself as the means and ends of learning, and not as a reflection of internalized, abstract thought. While this work also stemmed from gesture analysis, it differs in that it foregrounds how sensemaking is grounded in *making* gestures when the materiality of the body comes into contact with the materiality of the world (Streeck, 2009; Goodwin, 2017) and when people take different perspectives (Cassell & McNeill, 1991; Gerofsky, 2010). In contrast to the conceptualist perspective that foregrounds how the mind draws on the body, interactionist approaches foreground the physical and social world as providing rich sites for learning that highly structure and constitute the learning itself.

This dissertation builds off of conceptualist work in embodied cognition by bringing mathematical and computational learning and knowing into the creation and enactment of ensemble choreography through practices of physical research. This transforms mathematical objects and perspectives in two important ways. First it rescales mathematical objects to be created and formed by groups of movers, taking them out of the small paper-scale of most mathematics learning environments (Hall et al., 2014; Ma, 2016) and into a body-based danceable scale for a group. This means that the ontology of mathematical objects, specifically geometric objects as analyzed in Chapters 2 & 3, changes. For example, a square is transformed from a definitional object to a unit of production (De Freitas & Sinclair, 2014). This leads to the second transformation in that creating, maintaining, and manipulating mathematical objects at this scale now becomes a necessarily ensemble achievement (Hall, 1996; Kelton & Ma, 2020; Ma, 2017; Ma & Hall, 2018).

Designing for and supporting ensemble mathematical activity through choreographic practices extends this work by transforming the ontology of mathematical objects once again from ensemble achievements to ensemble performances. For example, a reflection is no longer shown by its composite image but by a repeatable, performative process of coordinating the movement of vertices (Chapter 2). Moving inside of mathematical concepts affords new noticings of their composition (Kirsh, 2010; Kremling et al., 2018). Focusing on ensemble choreographic practices for STEM learning also foregrounds how these forms of full body participation can invite learners to bring sensemaking resources typically not seen as relevant to STEM learning into these environments (Champion, 2018; Lerman, 2011; Solomon et al., 2022) and affords group-centered ways of being to dominate participation (Brady et al., 2016; Hahn & Jordan, 2017; Vossoughi et al., 2020).

In centering choreographic inquiry processes such as the practice of physical research that the dancers I collaborate with engage in (Chapter 3), I also center collective practices in the design and analysis processes of this research (Chapter 4). For example, as discussed in Chapter 4, my dance collaborators and I have been thinking through a phenomenon we are calling "choreographies of care," where small groups demonstrate their care for each other in a larger group context by moving and responding to each other's movements. This is important in STEM learning environments where what counts as learning can often center hyper-individualism, normative and simplified views of correct thinking, and competitiveness. Such STEM environments typically ignore potential resources and cultural assets for creative production.

Across three studies and empirical papers, this dissertation focuses on the ways embodied, choreographic sensemaking practices can expand the means and ends of STEM learning with respect to mathematics and computation. I explore choreographic practices as processes of inquiry

because (1) historically, embodied and artistic ways of knowing have been cast as antithetical to rigorous, scientific methods of knowledge production, and (2) practices for developing rule-based improvisational choreography map generatively onto mathematical and computational concepts. I care strongly about how embodied theories of learning, informed by the expressive and artistic practices of dancers and choreographers, can reframe *what* is learned in STEM environments, *how* it can be learned collectively, and *who* is involved in expanding the pedagogical and design implications of this work. This work invites learners to draw upon epistemic resources typically left out of STEM contexts when not centering aesthetic and ethical collective imaginings.

Contributions: New insights from the research, relations, and reflections that grew from this work

Chapter 2 traces different approaches by young students and STEM educators to reenact ensemble choreography and illuminates how reenacting choreography can function as a design method, mathematical activity structure, and method of Interaction Analysis. These exploratory interviews demonstrate the mathematical and computational disciplinary potential of choreographic reenactment and development by showing how ensemble learning can hybridize practices from dance and mathematics. Additionally, this paper highlights how manipulating large-scale props as an ensemble supports the development of intercorporeality (Hahn & Jordan, 2017) amongst participants moving together as they become a new entity that is larger than the sum of its parts. And finally, this paper demonstrates how in the analysis of ensemble activity sometimes we need to supplement traditional methods of Interaction Analysis (Hall & Stevens, 2015; Jordan & Henderson, 1995) with ensemble activity. Leveraging reenactment as both design inspiration, activity structure, and method of analysis shows how engaging in the creation of ensemble

choreography can support design research throughout the entire process, which is expanded in Chapter 4.

Chapter 3 highlights the generative ways a quartet of professional dancers engaged in the same mathematical reenactment activity as described in the second chapter. I traced these dancers' professional practice after the first study through a two-year ethnographic study of their local dance company. The primary discovery of this work was a practice the dancers refer to as "physical research," which demonstrates how bodies moving in response to each other can support agentic forms of learning that foreground interpersonal relations. Analyzing physical research across contexts demonstrates the expansiveness of choreographic practices for supporting ensemble discoveries in hybrid disciplinary activities. In addition, this paper contrasts a quartet of professional dancers and 8th grade girls both trying to reenact a phrase of triangle folding choreography. The girls set their prop down and stopped moving to debate what kind of a triangle they had folded and in turn did not see embodied sensemaking resources as relevant once their inquiry was labeled as mathematics. Although the dancers set their prop down at one point in their inquiry, their conjectures and understandings were only real once they could enact them as an ensemble. This paper questions who and when learners view embodied ways of knowing as relevant to disciplinary learning. By better understanding physical research as a choreographic practice of inquiry with generative mathematics learning potential, the analysis from this chapter set me up for the collaborative design and analysis work described in Chapter 4.

The analysis from Chapter 3 led to the research shared in Chapter 4, in which two of the dancers from the ethnographic project joined our design team to collaboratively create and co-analyze a movement and computation art camp for middle schoolers. We approached designing for computational learning as a process of translation taking into account that programming can

be challenging to learn, as computers are made to produce a single interpretation of code as a language, while humans as expressive communicators can in turn see multiple ways of interpreting the language of others. We leveraged the rule structures of constructing improvisational-based choreography with the expansive interpretation used in enacting these "choreographic scores." Our analytic collaboration made visible how choreographic methods can create small emergent groupings among students, as responding to movements made in close proximity supported the development of emergent duets, trios, and quartets within a larger group of movers. Engaging in physical research helped learners see others' movements as contributions to respond to physically, grounding their contributions in an ethos of care and collectivity embedded within the computational structures students physically generated together. Chapter 4 in this dissertation focuses on the processes of interdisciplinary design and analysis that enabled us to see how learners took up practices from physical research in this computational learning environment. Specifically, this analysis focuses on our history as an interdisciplinary team across the disciplines of dance and the learning sciences to demonstrate how our relationships supported the development of hybrid design and analysis practices and can open up possibilities for practices in design research.

This dissertation demonstrates how choreographic ways of knowing can serve as generative sites for STEM learning, design, and analysis. Together, these papers contribute theoretical insights to embodied learning theory by demonstrating how attending to aesthetics and physical expression through choreographic practices can introduce generative relations as well as mathematical and computational discoveries for learners from various communities.

References

Brady, C., Weintrop, D., Anton, G., & Wilensky, U. (2016, February). Constructionist learning at the group level with programmable badges. In *Proc. Constructionism Conf* (pp. 61-68).

Cassell, J., & McNeill, D. (1991). Gesture and the poetics of prose. *Poetics Today*, *12*(3), 375-404.

Champion, D. N. (2018). *The STEAM dance makerspace: A context for integration: An investigation of learning at the intersections of STEM, art, making and embodiment* (Doctoral dissertation, Northwestern University).

De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.

Gerofsky, S. (2010). Mathematical learning and gesture: Character viewpoint and observer viewpoint in students' gestured graphs of functions. Gesture, 10(2), 321-343.

Goldin-Meadow, S., Nusbaum, H., Kelly, S. D., & Wagner, S. (2001). Explaining math: Gesturing lightens the load. *Psychological science*, *12*(6), 516-522.

Goodwin, C. (2018). *Co-operative action*. Cambridge University Press.

Hahn, T., & Jordan, J. S. (2017). Sensible objects: Intercorporeality and enactive knowing through things. *Intercorporeality: Emerging socialities in interaction*, 267-288.

Hall, R. (1996). Representation as shared activity: Situated cognition and Dewey's cartography of experience. *The Journal of the Learning Sciences*, *5*(3), 209-238.

Hall, R., Ma, J. Y., & Nemirovsky, R. (2014). Rescaling bodies in/as representational instruments in GPS drawing. In *Learning technologies and the body* (pp. 124-143). Routledge.

Hall, R., & Nemirovsky, R. (2012). Introduction to the special issue: Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences*, *21*(2), 207-215.

Hall, R., & Stevens, R. (2015). Interaction analysis approaches to knowledge in use. In *Knowledge and Interaction* (pp. 88-124). Routledge.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *The journal of the learning sciences*, *4*(1), 39-103.

Kelton, M. L., & Ma, J. Y. (2020). Assembling a Torus: Family Mobilities in an Immersive Mathematics Exhibition. *Cognition and Instruction*, 1-30.

Kirsh, D. (2010). Thinking with the body.

Kremling, J., Arndt, N., Roman, A., Zilm, M., & Straehler-Pohl, H. (2018). Dancing on the junction of mathematics and bodily experience. *The Mathematics Enthusiast*, *15*(1), 201-227.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from* (Vol. 6). New York: Basic Books.

Lerman, L. (2011). Hiking the Horizontal.

Ma, J. Y. (2016). Designing disruptions for productive hybridity: The case of walking scale geometry. *Journal of the Learning Sciences*, *25*(3), 335-371.

Ma, J. Y. (2017). Multi-party, whole-body interactions in mathematical activity. *Cognition and Instruction*, *35*(2), 141-164.

Ma, J. Y., & Hall, R. (2018). Learning a part together: Ensemble learning and infrastructure in a competitive high school marching band. *Instructional Science*, *46*(4), 507-532.

Nemirovsky, R., Rasmussen, C., Sweeney, G., & Wawro, M. (2012). When the classroom floor becomes the complex plane: Addition and multiplication as ways of bodily navigation. *Journal of the Learning Sciences*, *21*(2), 287-323.

Solomon, F., Champion, D., Steele, M, & Wright, T. (2022). Embodied Physics: Utilizing dance resources for learning and engagement in STEM. *Journal of the Learning Sciences, 31*(1), 73-106.

Stevens, R. (2012). The missing bodies of mathematical thinking and learning have been found. *Journal of the Learning Sciences*, *21*(2), 337-346.

Streeck, J. (2009). *Gesturecraft: The manu-facture of meaning* (Vol. 2). John Benjamins Publishing.

Vogelstein, L. (2012). *A Justification of the Math-Dance Connection.* [Unpublished senior seminar paper in Charmaine Warren's course]. Fordham University & The Ailey School.

Vossoughi, S., Jackson, A., Chen, S., Roldan, W., & Escudé, M. (2020). Embodied Pathways and Ethical Trails: Studying Learning in and through Relational Histories. *Journal of the Learning Sciences*, 1-41.

CHAPTER 2

REENACTING MATHEMATICAL CONCEPTS FOUND IN LARGE-SCALE DANCE PERFORMANCE CAN PROVIDE BOTH MATERIAL AND METHOD FOR ENSEMBLE LEARNING

Introduction

There has been growing attention to the body's role in learning and teaching mathematics, including how mathematical concepts are grounded in embodied metaphor (e.g., Núñez et al. 2006); how mathematical activity engages the acting body (e.g., Gerofsky 2010, 2016; Hall and Nemirovsky 2012); how to design learning environments that invite productive embodiment (e.g., Abrahamson and Lindgren 2014; Sinclair et al. 2013); and how we can manage the challenge of capturing and analyzing data that include multi-modal, multi-body, mobile human interaction in mathematical activity (e.g., Hall and Stevens 2015; Ma 2017; Marin and Bang 2018; Shapiro et al. 2017).

Foraging and dissecting public media to design for creative re-use

This study developed in a design practice that uses *foraging* and *dissection* to create environments for collective exploration and learning. Grounded in various disciplines, we "forage" in public media to find cultural performances that we feel resonate with disciplinary content [e.g., the late Hans Rosling's popular TED talks offering "The best stats you've ever seen" (Rosling 2006) as used in Kahn and Hall (2016)]. Treating these performances as found objects with open-ended possibilities for re-use, we next design activities that invite teachers and students to "dissect" records of performances (i.e., video, audio, and text) to ask how they were made, explore how

disciplinary concepts might act as useful lenses, and create their own performances with similar materials [e.g., using Gapminder tools and open-source data to build model-based stories about world health and wealth (Kahn and Hall 2016)]. We propose that foraging and dissecting performances or objects found in public media can support a productive design practice in Learning Sciences research.

This article illustrates our approach using cultural performances with an enormous international audience. The opening and closing ceremonies of the 2016 Rio Olympic Games included performances that Brazilian choreographer Deborah Colker (http://www.ciadeborahcolker.com.br/the-company) co-designed with Brazilian dance companies and indigenous communities. Colker and her collaborators intended to offer an image of Brazil's cultural diversity and national identity:

> The most important thing is the possibility to mix styles and ideas and aesthetics, dance and music and energy. We are a new country, with contemporary ideas, not just what foreign people think about Brazil: pretty women and football. Brazil is this and so many other things. It is an amazing place for contemporary dance and music and film and fashion and art (Harss 2016).

Our initial viewing of the Rio performances brought to mind collective learning and dynamic geometric formations at a "walking scale" (e.g., Hall et al. 2014), but none of the press coverage we found took this perspective. Our foraging and dissection created an open-ended design space for creative re-use, with materials that expressed the complex history and diversity of Brazilian culture. Our purpose was not to alter the narrative intended by the original creators, but instead to honor and open up complexities in their work by asking participants to explore how dancers might have created the performance, making explicit use of props, multi-person, ensemble reenactments, and mathematical descriptions.

In this sense, we re-used (Eglash 2004) an existing cultural performance for a narrower investigative purpose. We view this re-mixing of cultural activities (Brazilian dance and mathematics education research) as a source of design innovation to create new cultural activities, much as our earlier work with GPS drawing or walking-scale, ensemble geometry supported new forms of mathematical activity. Our approach mixes different cultural practices and meanings (Bar et al. 2016) to create something with novel mathematical potential. The activities we explore here are creatively displaced from the global, Olympic stage into classrooms and laboratory spaces in a university, but the resulting activity is different from each. We designed a *dissection* environment that put participants' bodies into relations with material and space approximating that of the Rio dancers, but we also asked them to consider the activity from a mathematical perspective. We deliberately created something *between* existing cultural practices to elicit new cultural activities, however unpredictable, as embodied responses.

Ensemble mathematical learning while reenacting choreographed performance

Recognizing mathematical structures in this large-scale, ensemble performance, we decided to use video from it to foster and study *ensemble learning* in mathematical activity. By ensemble learning we mean learning that is fundamentally collective and performative, where learners recognize the need to act together [e.g., competitive high school marching bands (Ma and Hall 2018)].

A well-established literature on peer interaction and small-group mathematics learning (Webb 1991; Cohen 1994) investigates joint problem-solving activity (Teasley and Roschelle 1993; Cohen and Lotan 1997); how and why such interactions can fail (Barron 2003); and how to design tasks "worthy" of groups (Lotan 2003). In contrast, by focusing on *ensemble learning* (Ma

and Hall 2018), we foreground situations where doing things together is *necessary* for both performance and learning. For instance, in this study, using dissection and reenactment to discover how to "flip" a shiny plastic sheet (made of Mylar) to make a "wave" required active participation of all members of the quartet, in interaction with the physical possibilities of the sheet as a prop. These kinds of learning phenomena may be much more common and powerful than our field has yet to fully grasp. They also connect to related work on *collective mathematics*, where social and interaction structures of groups are used in generative ways to produce and explore mathematical structures (see Brady et al. 2013, for several distinctive forms).

We followed earlier design research to foster highly engaging forms of ensemble activity and learning (e.g., Hall et al. 2014; Ma 2017; Headrick Taylor 2017). In designing an interview protocol for this study, we asked how viewing the Rio performances could engage participants in discussions about how they were carried out. If quartets *reenacted* what they found in the Rio recording or created their own performances, could this lead to ensemble learning in mathematical activity?

We chose selections from the Olympic television broadcast that struck us as having significant mathematical possibilities. The YouTube Olympic Channel's broadcast is at https://www.youtube.com/watch?v=N_qXm9HY9Ro; our interviews used a 3-min passage: 11:36 to 14:40. In this tribute to Brazilian *gambiarra* (creating beauty from almost nothing), mathematical structures animate the simplest of props to create striking visual effects (Fig. 1). The performance operates at three levels—*quartet, assembly*, and *whole*—all depending on ensemble action beyond the human individual. These levels are represented in Fig. 1a–d, increasing in scale from quartet to whole. The *quartet*, defined by the prop and quartet (right column of Fig. 1), creates geometric shapes (e.g., folded triangles, Fig. 1a, b; and stretched squares, Fig. 1d) that are seeds

of higher-level actions. These seeds sprout mid-level *assemblies* that form and dissipate (e.g., foursomes of quartets produce geometric figures in their own right, Fig. 1b; and waves propagate in and across sections, Fig. 1c). Finally, there is the highest-level *whole*, where forms suggesting crystalline structure, liquid waves, and vortex motion emerge and disintegrate as the performance unfolds in time.

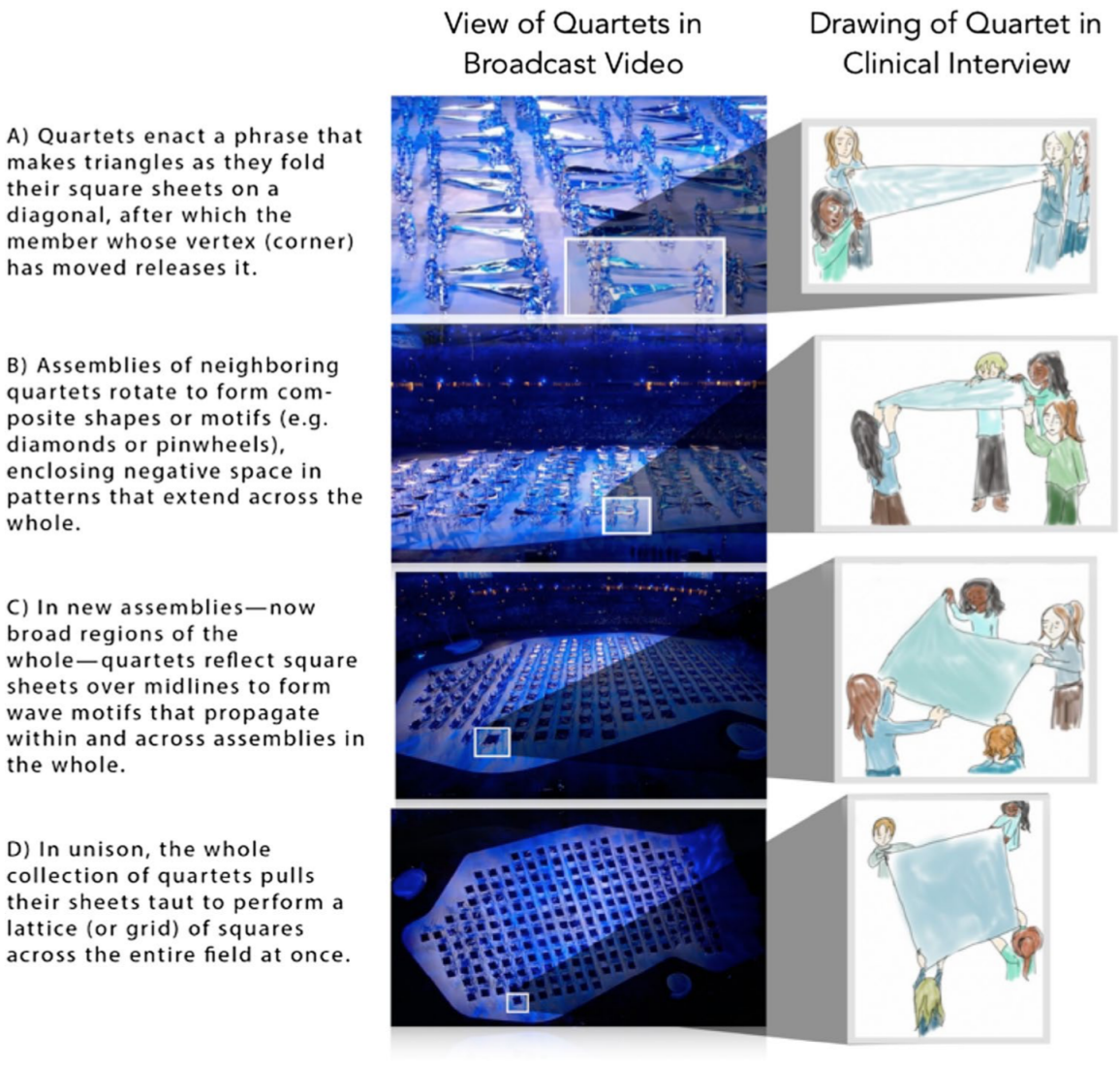

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Figure 1. Ensemble performance at increasing scales, in selection from the opening ceremony

Mathematical structures can be identified at all three levels of the performance, and each level draws its significance and impact from the others. Thus, the performance has a coherent integral unity, although we conjectured that the *quartet* level would be a manageable entry point for participants to engage with the performance by enacting some of its key aspects. The quartets can be described as employing the dihedral group D4 of symmetries of the square for aesthetic effects, extending this basic set of movements in artistically and mathematically suggestive ways. Rotations and reflections, but also translations, form a substantial part of the vocabulary of movements with the prop. Further added to this set of actions are contractions to a point and folds over the prop's lines of symmetry. Moreover, the physicality of the prop and its reflective surface embodies the notion of "sidedness" and orientation (to both points and planes) that is backgrounded by the mapping to the dihedral group as typically taught in mathematics.

Early pilot interviews showed how important it was for quartets to have physical props that were replicas of those used in the actual performance (Vogelstein et al. 2017). We conducted our interviews in a space that facilitated alternation between "viewing" and "doing" (i.e., the quartets could easily watch, reenact, and re-watch performances in the videos). We expected quartets would make discoveries about what they could do as an ensemble, coordinating their movements and exploring the expressive possibilities of the system we came to call "people-plus-prop." Below, we describe our methods, then turn to close analysis of quartets as they make sense of the Rio performance by viewing, reenacting, and creating performative phrases of their own. This analysis is both an example of design research using creative re-use of foraged material (the Rio recordings) and a comparative exploration of how ensemble learning can unfold and generate a growing repertoire of talk and co-operative action. What is learned, we argue, emerges in processes that are larger than individuals (i.e., the ensemble), are shaped by material engagements with things and

the surrounding space (e.g., props similar to those used in Rio), and produce experiences of "we" or "with" (i.e., intercorporeality) in new cultural activity that mixes existing disciplinary practices (here, dance and mathematics).

Methods

Designing a space for video-elicited co-operative action

Video-elicited interviews

Our first source of data is a series of 90-min video-elicited interviews, using footage from the Rio opening ceremony as described above. Quartets for the interviews were composed of participants who knew each other and shared disciplinary backgrounds. For this article, we focus on two quartets, one consisting of 8th grade girls and the other of scientists who have transitioned into middle school Science, Technology, Engineering, and Mathematics (STEM) teaching.

The interviews invited quartets to view the Rio recordings and engage in ensemble mathematical activity (Ma and Hall 2018) using a 7′ × 7′ square sheet similar in both size and material to the prop used by the Rio performers. The interview had four parts:

1. Quartets watched and discussed the video.
2. They engaged a series of challenges:
    a. Fold the sheet into a triangle.
    b. Fold it into a triangle *like the Rio performers*.
    c. Flip (reflect) it *like the Rio performers*.
3. They were asked about mathematics and prompted to describe both the performance and their experiences with the prop mathematically.
4. They were invited to create their own performances with the sheet, exploring what they could do with the prop, noting any limitations of working with it, and reflecting on their overall experience.

We positioned four cameras to capture quartets' interactions from above, from two adjacent sides of the room, and from the table where participants sat for discussions during the interview (see Fig. 2). Top and side views of reenactments were particularly helpful when analyzing quartets' work, as these corresponded to camera angles used in the Olympic broadcast. We also ran screen capture software on the computer that displayed the video, so when participants used it to study parts of the Rio performance their interactions with the video and each other were captured.

Figure 2. Five camera positions (including the side view that produced this figure)

Video-elicited interaction analysis reenactment session

Our second source of data was an Interaction Analysis (IA) session involving the three authors and three other doctoral students. The room was arranged like the room in which the interviews were conducted. We recorded this session using a single camera (side view).

We include a video record of our IA session as a source of data for two reasons. First, it illustrates how we used *our own* reenactments to supplement conventional approaches of IA (e.g., Jordan and Henderson 1995) to analyze our data (the work of a quartet of STEM educators). While some proponents of IA have recommended choral reading of transcripts with careful attention to emphasis (Erickson 2004), we know of no published analyses in which analysts used their own bodies, working with physical materials similar to those used by people in a target video, to explore the structural organization of multi-person co-operative activity. We invited our participants to reenact Rio performances with the expectation that this would lead to discoveries, and we did the same for phases of our own analysis of their work.

Our second reason for analyzing video from our IA session is to illustrate how we made discoveries about the people-plus-prop system itself. Reenactments in the IA session helped explain how the STEM educators created a "cool" phrase consisting of two reflections, which they later could not replicate. But cycles of viewing and reenacting *also* led our IA quartet to discover and systematize what could be created with four bodies moving in coordination with the prop. In this sense, reenactment was instrumental (and we think necessary) both for understanding what quartets did in the video record and for exploring what *could* be done with the quartet-plus- prop system.

Comparative and multi-modal interaction analysis

Analyses of both data sources were conducted iteratively using IA methods focusing closely on multimodal aspects of gaze, gesture, and coordinated movement of bodies (Goodwin 2017; Hall and Stevens 2015; Kendon 1990; Streeck et al. 2011). Video records were first logged for content, with particular focus on how quartets cycled between viewing a performance and repeatedly reenacting parts of it as an ensemble. As we developed an understanding of these processes of "viewing and doing," we selected segments for detailed analysis. We treated co-operative action by the people-plus-prop system as a unit of analysis that was capable of more than any of its members, alone; was shaped by engaging with material and setting; and produced collective experiences [i.e., intercorporeality (Meyer et al. 2017)] that were relevant for disciplinary practices (in both dance and mathematics).

Since multi-body, ensemble performances were necessary for the reenactments we analyze, we use transcription sparingly, instead presenting sequences of images we call "toon strips" (see also Laurier 2014) to show bodies moving sequentially through performance phrases. In reducing the video data to identify and analyze patterns of action by the people-plus-prop system, we were pushed to develop other representational tools as well. One such tool is agent-based pseudocode as used in Sect. 3.2 to model the performance phrases improvised by a quartet in the later stages of the interview. These representations complement the toon strips, highlighting patterns in quartets' coordinated movements.

Results

In this section we analyze three episodes of ensemble activity that emerged from foraging and dissecting the Rio performance, which highlight the diversity of work produced in the

interviews and indicate the potential of our approach. First, a quartet of 8th grade girls reenacting two different performative elements of the Rio video shows how dissection can put canonical epistemologies on shaky ground. Next, a quartet of STEM educators illustrates how the prop itself was a key participant in the quartet-plus-prop system. Last, our own reenactments of a performance phrase produced by the STEM educators demonstrate the potential of reenactment as an extension of IA methods.

Dissection and reenactment can provoke fundamental questions about knowing and interpreting mathematical figures

In this section we analyze a quartet of 8th grade girls (Ava, Octavia, Taylor, and Tracy), who engaged intensely in reenacting phrases from the Rio performance but encountered fundamental challenges to their mathematical understandings along the way. We focus on the second stage of the interview (2c and then 2b) to show the differences in their reasoning when working as an *ensemble with* the prop versus as a collection of *individuals around* the prop.

Figure 3. 8th grade girls enact their final Wave performance

Dissecting performances leads to discoveries about ensemble coordination

The girls found it challenging to reenact Rio performance phrases whose fluid delivery made them deceptively simple. Asked to flip (reflect) the square like the Rio performers, the girls called this "the Wave" and determined that Rio performers enacted two roles, "overs" and "unders", to "flip the cover" (Fig. 3). But coordinating this phrase was more complex than expected. The girls found it challenging to reenact Rio performance phrases whose fluid delivery made them deceptively simple. Asked to flip (reflect) the square like the Rio performers, the girls called this "the Wave" and determined that Rio performers enacted two roles, "overs" and "unders", to "flip the cover" (Fig. 3). But coordinating this phrase was more complex than expected.

After viewing the Rio video, all four girls immediately had ideas for how to reenact the Wave, shouting, "I know how to do it!" However, as soon as they picked up the prop, conflicting ideas surfaced and they were unable to reenact it. A first challenge came with role mapping. All agreed they needed two "unders" and two "overs", but it took work to negotiate (a) the orientation of their reenactment, (b) their role pairings, and (c) the actions for each role.

After the group pulled the sheet taut, Tracy began moving towards Taylor (Fig. 4a), but both Taylor and Ava immediately waved her off. Taylor explained why Tracy had to walk back: "no! ...those two people *((gesturing to Ava and Octavia[3]))* need to sit down and we go *((lifting her corner and walking towards Ava))*". Taylor rejected Tracy's move, which implied an orientation for the Wave and a pairing of roles that differed from Tracy's idea (Fig. 4b). In her verbal and

[3] Transcript conventions include the following: Turns at talk are labeled with identified speakers. EMPHATIC utterances are shown in upper case. *((Activity descriptions))* appear within double parentheses and in italics.

gestural explanation, Taylor assigned Ava and Octavia the role of "unders", grouping herself and Tracy as "overs".

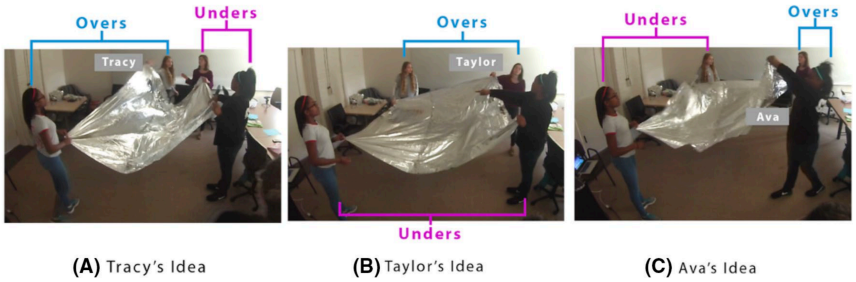

Figure 4. Different ideas for Wave orientation and pairings

Ava then interrupted Taylor to point towards Tracy and Octavia announcing, "no, we're going that way", agreeing with Tracy's original orientation but changing the role pairings (Fig. 4c).

Once they assigned roles, determining how to execute the Wave remained a challenge. The panning camera in the Rio recording made it difficult to follow a single quartet, so the girls viewed several quartets in "slow motion," tracking individual performers. After several re-viewings, the group came to a consensus that the "unders" moved under the sheet and did not remain stationary. Ava explained this as an operation between partners— "The goal is to end up in your partner's place, so like say me and Octavia are partners, I'm gonna end up where she is, she's gonna end up where I am" (Fig. 3c). In Ava's proposal, each "under" was paired with her opposite "over" (Fig. 5).

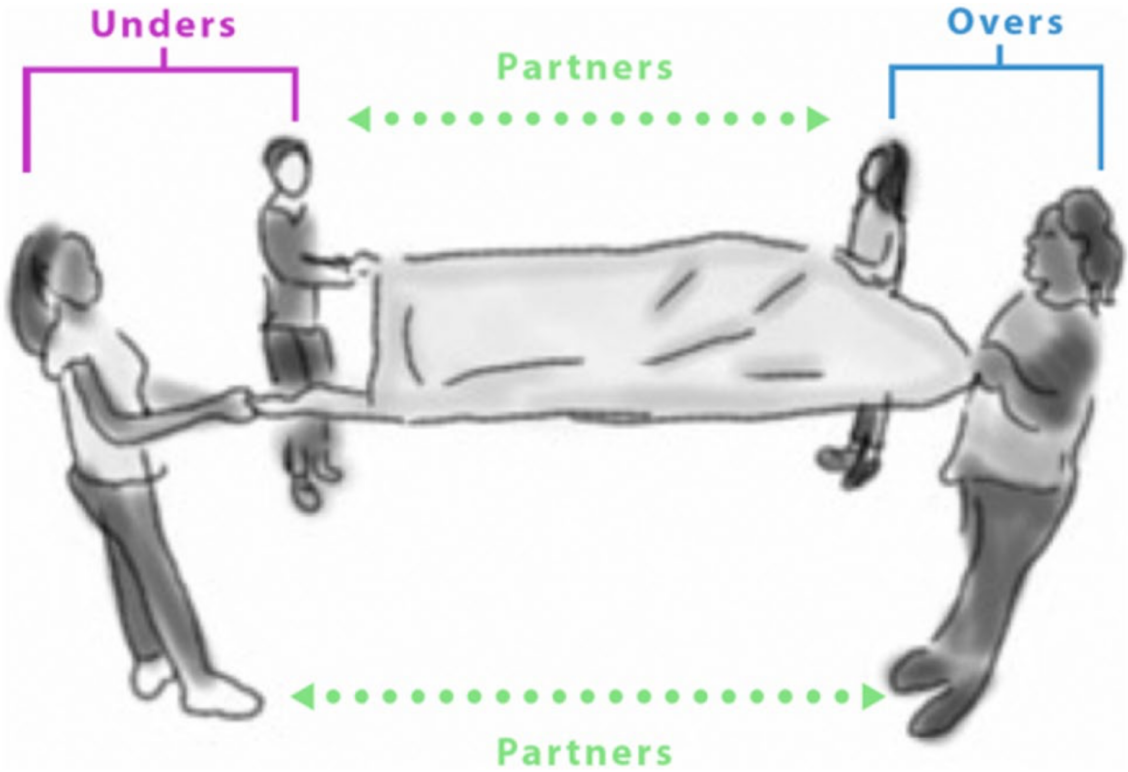

Figure 5. Two groupings for coordinating the Wave.

Focusing on paired roles helped, but Ava and Octavia each thought the other role needed to move first:

> Octavia: No, you guys have to flip it over
> Ava: We can't flip it over until you go under
> Octavia: Y'all have to flip it when we're going at the same time, you have to be like WHOOO *((jumps up and sweeps right arm up and over her head))*

As they talked through their expectations for the Wave, Octavia offered a new solution to the coordination problem—moving "at the same time"—and demonstrated an aesthetic element ("whooo"), showing the importance of movement quality (in addition to timing) when performing the ensemble phrase.

> While reenacting the Wave, the girls dissected important elements of the performance in an iterative process of viewing and doing the phrase together. By coordinating the two roles ("overs" and "unders") and attending to partner relations, they managed to reflect the square over a midline by acting together (i.e., "overs" had to "whooo" together, and each "over" had to switch places with her "under" partner). The enacted Wave was intercorporeal—not an individual accomplishment, but something learned and performed as an ensemble.

Negotiating an epistemic stance on the products of ensemble performance

A contrasting set of challenges arose for the girls when they assessed their reenactment of the Triangle folding phrase from the Rio performance. Like other quartets in our study, the girls were unsure whether the triangle they produced (an isosceles right triangle) was the same as in the Rio video, since the camera angle made folded shapes look like scalene right triangles (see Fig. 1). Whereas in the Wave the girls struggled to coordinate reenactment; here, they struggled to make sense of the shape they made in terms familiar to them from school mathematics.

Assigning mathematical terms to the triangle they produced revealed the girls' different ideas about the role of perspective in determining the nature and appropriate description of geometric figures. To better assess their work, the girls laid their triangle on the ground and began an extended (6.5 min) effort to determine the kind of triangle it was. They invoked a variety of mathematical concepts and terms (e.g., hypotenuse, congruent, isosceles, right triangle, 45°, 90°), but for them, the meanings depended on the perspective one took on the shape. For example, Tracy insisted from the start that they had folded a right triangle. Positioning herself at a leg of the triangle (position A in Fig. 6) she argued, "see if you look from this angle, it's 90 degrees." Ava followed Tracy to position A and for a moment agreed: "That is 90." But once she returned to her previous position at the hypotenuse (position B), she strongly disagreed, invoking an epistemic stance (Goodwin 2007) linked to what properly happens in school:

> Ava: IN MATH, when you get a math problem there are some problems where it looks like this *((standing at position A in* Fig. 6*)*), it will be like number one from that angle *((moving to position B))* number two from this angle, you're not going to turn the math book and say it's a right angle, you're gonna say that's an isosceles.

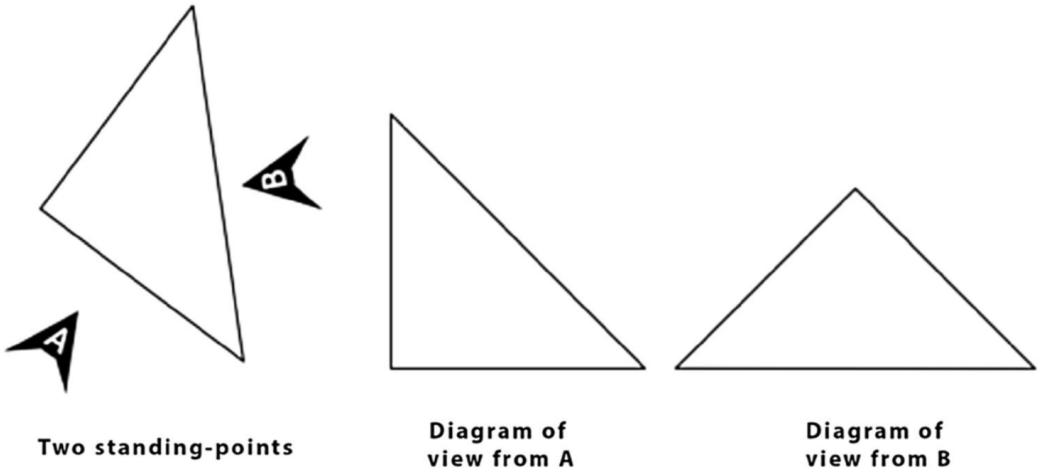

Figure 6. Ava's two standing points, and her two corresponding whiteboard diagrams

Ava appealed to a different activity ("in math") to argue they should classify the triangle depending on orientation. When Tracy remained unconvinced, Ava repeated her argument with support from Octavia, who invoked "they" as an author of mathematical figures in textbooks (Rowland 1999):

> Octavia: You're not gonna look at it like that because when they give it to you they don't put the 90 up here ((*points to the top of a triangle in the diagram view from B)*) [...] They don't put the 90 up here. They DO NOT DO THAT in books.

In this discussion, Ava invoked an interpretive genre, how one reads figures "IN MATH," and Octavia added an authorial "they" whose intentions are definitive and can be inferred from the orientations of figures in books. Throughout this episode, two epistemic frames collided— (1)

conventional representations of geometric figures that "they" give "in math" and (2) the perspectives of viewers or enactors of an ensemble performance. The scale of the folded prop and its placement on the floor (compared to the board) disrupted any privileged orientation that would enable them to see it as a textbook figure. The girls were free to move around the prop and change their perspective (in fact they had to choose a vantage point explicitly), so no "intended" orientation could be invoked. This crisis of interpretation was negotiated with high energy but also evident enjoyment[4] by the group, even as Tracy rejected the necessity of adopting a "textbook approach" to the prop.

The prop plays an essential role in creating and sustaining a space for ensemble reasoning

In reenacting the Wave, the girls remained physically connected, grasping the prop even as they debated about what to do with it. Though they quickly reached agreement on the role of "overs" and "unders" to "flip the cover", working out the timing and aesthetic quality for these roles took much longer, involving multiple slow motion viewings of the video; pointing at the screen to specific performers within a chosen quartet; and reenacting newly-gleaned information as an ensemble. The prop created an attentional space within which the girls acted and reasoned *as an ensemble* to resolve difficult challenges. In contrast, in the Triangle episode, once the girls set the sheet down they became disconnected and largely worked individually, talking over one another and using incompatible schemes of reasoning.

[4] Instead of responding to the underdetermined tasks in the interview by asking "what are we supposed to do?" or complaining that they "didn't get it", as one might do in a "school" context, the girls dove into the work whole-heartedly throughout the interview.

Props can act as partners in generating ensemble mathematical performances.

In this section we analyze a quartet of STEM educators (Damien, Katherine, Monica, and Tanesha) in the final stage of the interview, focusing on a sequence of the ensemble's work that led to a surprising discovery-in-action. First, we describe patterns in their improvisational work as they developed an increasingly complex performance. And second, we show how this work set the stage for a discovery and the surprise at their own achievement. Our analysis illustrates how collective experiences with dissection can fuel collaborative improvisation to explore the performative potential of a people-plus-prop system.

Emergent structure can appear in the creative explorations of people-plus-props

The STEM educators first negotiated a shared goal: to create what Katherine called a "dancy" performance that would be "fancier" than a static shape. In pursuing that goal, the quartet improvised fluidly: each member "took the lead" in different phases while others responded to new ideas enthusiastically, building upon each other's contributions. Moreover, the quartet became more attentive to emergent patterns in their movements and actions with the prop, which they engaged increasingly as a kind of dynamic partner. Shifts in the ideas they explored and in leaders enabled us to delineate sub-episodes that produced elements of an unfolding performance. We briefly describe the focal action of each sub-episode and notate the progressive changes in the group's choreographed phrases using *agent-based pseudocode* to model the emergent structures in their improvisation.

Figure 7. Enacting Katherine's idea: each member raised their corner in sequence; then they all rotated one position to the left, together

SE1: Like a Dance Monica responded to Katherine's "dancy" idea, suggesting a simultaneous up-down wave of the sheet, followed by a group rotation. This had a satisfying look and feel, but Katherine then suggested *sequential* waving followed by Monica's in-unison rotation. The effect of this on the sheet was appealing, and the *sequential* approach was adopted (Fig. 7):

First, simultaneous (Monica)

```
ask actors [ raise-corner ]
ask actors [ lower- corner ]
ask actors [ rotate-one-position-to-left ]
```

Then, sequential (Katherine)

```
(Figure 7)
while [ n < 4 ]
[
   ask actor n
     [ raise-corner lower-corner ]
   set n n + 1
]
ask actors [ rotate-one-position-to-left]
```

Figure 8. At the beginning of Tanesha's proposal Monica did this*, then an* in-and-out*; then Katherine* physically *repeated it.*

SE2: Ins and Outs Tanesha then proposed a new move that she called "ins and outs." In articulating this idea she created two distinct, encapsulated action units. With the term "in-and-out" she introduced a motion toward the center of the prop, creating a sag, and then back out. She also encapsulated the movement of the prior sub-episode (raising and lowering one's corner) calling it "a *this*" and demonstrating it with her own corner (Fig. 8).

Encapsulating the prior phrase as a "this" and defining an "in and out"

```
to this
   raise-corner
   lower-corner
end

to in-and-out
   forward-towards-center
   back-away-from-center
end
```

Encapsulated actions in Tashena's new proposal (Figure 8)

```
while [ n < 4 ]
[
   ask actor n
     [    this
          in-and-out ]
   set n n + 1
]
ask actors [ rotate-one-position-to-left ]
```

To make her proposal comprehensible, Tanesha thus created a partial enactment of the idea to invite the quartet to take it up. In the people-plus-prop system, this went beyond making a "bid for the floor," to making a "bid to join in." Without enacting an idea as an ensemble, the group could not fully comprehend or assess it.

Interestingly, Tanesha was in fact disappointed by the result when the group enacted her proposal. The prop offered a kind of *resistance* to her aesthetic idea, as its airy materiality made it

difficult to execute the *in-and-out* rapidly. Nevertheless, the other members of the group took up the proposal and built upon it.

SE3: Billowing Katherine suggested a change that returned the group to simultaneous action. By modifying Tanesha's *in-and-out* to be done by all actors together, a new effect appeared:

```
ask actors [ this ]
ask actors [ in-and-out ]
```

When they did a *this* in unison and then moved in toward the center, the prop billowed attractively. This became a new "thing" (or phrase), which the group executed in the "dancy" framework (Fig. 9):

New encapsulated action (billow)
(Figure 9)

```
to billow
   ask actors [ this ]
   ask actors [ in-and-out]

end
```

New phrase (dancy billow)

```
billow

ask actors [ rotate-one-position-to-left ]
```

Once this phrase became established, the group could then consider its visual effects—referring to the prop as "like a big balloon"; wondering, "what this would look like from the outside"; and recalling a similar billow when playing with a parachute.

SE4: Inspired by Shared Memories of a Parachute After Monica suggested they treat the prop like a gym class parachute, Katherine suggested a performance where pairs of participants switched places under the billowing chute. As the group enacted Katherine's proposal to switch corners and places, they reached a new level of complexity (Fig. 10).

```
let pair1 actors 1, 3
ask actors [ raise-corner ]
ask pair1 [ forward-toward-center release-corner grab-(other)-corner
          forward-away-from-center turn-around ]
ask actors [ lower-corner ]
```

This new phrase not only introduced novel actions and patterns, it also involved coordinating action at a new level—pairs. As we saw with the 8th grade girls, acting in pairs can be challenging. Moreover, the phrase prompted new terms, so that *humans, places*, and *corners* could be described independently. The members of each pair switched "places" *and also* switched "corners."

During these sub-episodes the quartet produced a rapid flow of talk and co-operative action, appearing to build a form of "intercorporeality" (Meyer et al. 2017; Hahn and Jordan 2017) that allowed them to play with different instantiations of phrases (e.g., simultaneous, sequential, raises and lowers, and ins-and-outs) as they focused on small performative changes that produced consequential aesthetic outcomes. It is also important to note that as these ensembles developed such capacities, they expressed feelings of pleasure at the aesthetic qualities of multi-party enactment.

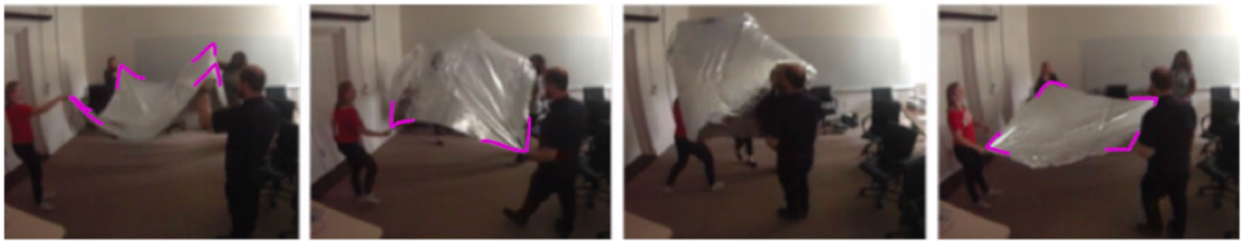

Figure 9. The quartet did a this *and then an* in-and-out *all together*

Figure 10. Katherine and Damien switched places and corners

Figure 11. Katherine and Damien tried to switch places but not corners

Ensemble enactment with the active participation of a prop can generate mathematically rich performances

SE5: Switch Places Not Corners After the success of the parachute-inspired performative phrases, Damien suggested a variation where the pairs would switch *only* places, keeping hold of their corners. He conveyed this idea to the group and his pair-partner Katherine through words and gestures. He claimed this would result in an operation of reflection on the sheet itself: "it will twist, it will fold over as it's ballooning."

```
let pair1 actors 1, 3
ask actors [ raise-corner ]
ask pair1 [ forward-toward-center
            forward-away-from-center
            turn-around ]
ask actors s[ lower-corner ]
```

This phrase failed, as the prop *refused to cooperate* (Fig. 11). Unlike the mild aesthetic resistance the prop put up in sub-episode 2, here the performance stopped as the sheet tangled when pair1 attempted their interchange. In working to make sense of what had occurred, the group immediately expanded their repertoire of terms and references: Damien observed (expecting to be understood) that "oh, wait, we don't have a *go-under* for that."

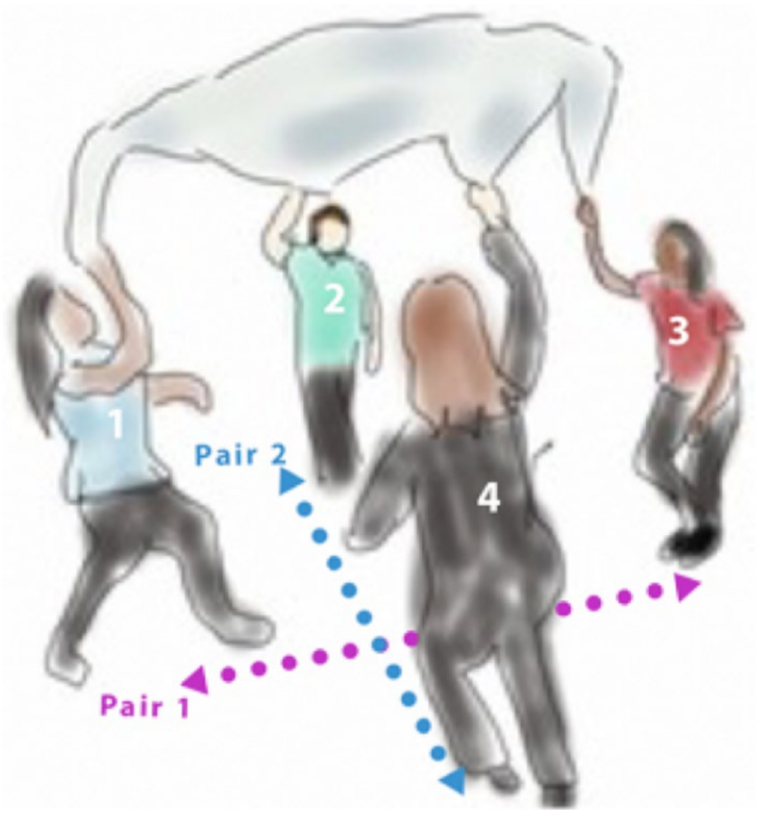

Figure 12. The "cool" performance as described by the pseudocode in SE 6

SE 6: The "Cool" Performance The failed attempt at *switching places, not corners* involved only one pair at a time. In response, Monica made a suggestion consistent with the flow of ideas in the session: "We could all do it." In context, this meant changing from *sequential* to *simultaneous* pair action. Thus, the revised phrase would be (Fig. 12).

```
let pair1 actors 1, 3
let pair2 actors 2, 4
ask actors [ raise-corner ]
ask pairs [ switch-places ]
```

Before they could enact this idea as a group, the pairs felt the need to articulate and clarify their coordinated motion. Monica and Damien took the explanatory lead, gesturally tracing

pathways to suggest Monica's and Katherine's trajectories "under" the sheet. Monica clarified for Tanesha: "I go under you," as she gestured up and over her head, depicting the sheet flipping over. Tenesha completed the thought, "ok, and flip." In these preparatory communications, the group went from talking about a single reflection to talking about two simultaneous reflections, where the sheet would flip as each diagonal pair crossed over and under each other.

Stunningly, this performance phrase worked: the prop billowed as pairs crossed beneath it, and it unfurled beautifully as the actors reached their new positions and pulled it taut (Fig. 13). They found their performance "cool" but were puzzled by their success (Damien said, "I'm not sure even what happened") and attempted in vain to repeat the performance. Their struggles to describe and understand what they had achieved as an ensemble indicate the leap in complexity involved in this last phrase.

In the next section, we analyze a quartet of researchers attempting to make sense of this "cool" performance. We convened this session because the footage of sub-episode 6 baffled us as well for several months when we approached it with traditional IA methods. Following the logic of ensemble thinking and in an effort to explore the potential of reenactment as a method, we assembled a group to *reenact* the STEM quartet's performance.

Reenactment can be a method to make discoveries and systematize ensemble performances mathematically

In this section we analyze a group of learning scientists working together to understand what the STEM educators from the preceding section created. As with the interviewees, these researchers were organized as a quartet; they viewed the video recording of the STEM teachers and then reenacted it, building their own performance repertoire together. Our analysis supports

several provisional findings about how reenactment fosters ensemble mathematical activity and learning, as well as how reenactment can be used as a method alongside other IA tools.

We were initially drawn to this episode both mathematically and aesthetically (Vogelstein et al. 2017). On rewatching the STEM educators' exploration we noticed they described a performance of two simultaneous reflections. From the top view, they transformed the prop through a 180° rotation: it ballooned up, spun around, and was drawn back down into a square (bottom row of Fig. 13). The side camera gives a contrasting perspective on the performance, clearly showing each diagonal pair crossing each other and "switching places not corners" (top row of Fig. 13). This diagonal double-reflection pattern cannot be seen from above.

Mathematically, a theorem of transformational geometry says the composition of two reflections produces a rotation; and the two diagonal reflections executed by each pair "switching places not corners" should indeed rotate the prop 180°. However, as the last section revealed, the large and billowy form of the prop can complicate operations on it, particularly reflections across diagonals. Achieving *two* of these reflections simultaneously was thus a feat with both theatrical and mathematical interest. We were taken with this beautiful and novel (at least to us) way of *enacting* a mathematical theorem, and we were also curious why the quartet was *unable* to replicate their initial success.

Despite being drawn to this performance, we did not immediately try to reenact it. We discussed the episode as a problematic artifact for several months prior to the IA session described here, and even presented it as a puzzle at a conference, garnering suggestions for how to represent the performance, none of which involved reenactment. Thus it was not our initial instinct to turn to reenactment; yet this move turned out to be pivotal in our understanding both of this particular episode and of the broader potential of this design space.

We first describe what the IA quartet discovered about the STEM teachers' performance of the "cool" phrase. We next describe discoveries of technique, and how these led the IA quartet to "systematize" how a double reflection might be accomplished.

Figure 13. Side view and top view of the STEM educators' "cool" performance

Reenactment drives discovery

The IA participants' initial understanding of the STEM educators' performance influenced how they structured the reenactment session. First, they organized the space by assigning their bodies to the bodies of the STEM teachers in a manner that facilitated coordination (e.g., Vogelstein, wearing red, took the role of Monica, who was wearing red in the video, and moved to the position Monica occupied in the side-view video the IA session used). After viewing the "cool" phrase video, the IA quartet began a series of explorations of possible enactments: a single reflection, then two consecutive single reflections, and finally two simultaneous reflections. Thus, when they returned to the video record of the STEM educators, they had a better understanding of

how pathway crossings in the video compared to their own successful and unsuccessful ensemble enactments. By the end of this phase of work, the IA participants felt they had a "good enough" explanation of what STEM educators had done—a double reflection of the sheet, but with "mistakes" and "corrections" that the educators had never noticed, leading to their failure to replicate their own "cool" phrase. This explanation was discovered as the IA quartet made and noticed their own mistakes and corrected them during reenactments.

Figure 14. The IA quartet reflected the sheet to explore what STEM teachers might have done. On the left, they paid close attention to twists at their corners; on the right, they used "slo mo" and "freeze" to focus on their trajectories

During this exploration, the IA quartet would slow down or "freeze" a reenactment in order to examine the state of the sheet and their own "paths" as they attempted two simultaneous reflections. As shown in the left image of Fig. 14, corners/participants in the IA quartet planned their paths carefully, using talk and gesture to describe and select from among the alternative ways of passing "under" the billowing sheet and "around" other moving corners/persons. As shown in the right image, they also did this by slowing down during enactments (e.g., pausing while holding the sheet above their heads). They noticed that some paths created "twists", visible in the prop and felt when their thumbs holding each corner went from "up" to "down" (Fig. 14). The IA quartet repeated their enactments multiple times while searching for paths in which diagonal pairs walked

past each other to "switch places, not corners," as done by the STEM educators. Their ensemble reenactments were supported by defining and repeating operations on the sheet until they produced an explanation-in-action of the STEM educators' work they found satisfactory.

Discoveries can be systematized using reenactment

The IA participants' ensemble reenactments were a *method*, driving multiple levels of analysis and supplementing more corporeally passive forms of IA (e.g., sitting, watching film, and discussing what individuals notice in the data). The IA quartet discovered together how facets of interaction with the sheet contributed to make the "cool" phrase possible (e.g., "billowing" the sheet above enables people/corners to cross "under" a line of symmetry or go "around" one another during a reflection). Then, as they encountered trouble or nuances (e.g., twists in the sheet as thumbs moved from "up" to "down"), they began to ask questions about the quartet-plus-prop as an expressive system—what *could* be made with this system? Their discoveries served both to explain troubles encountered by the STEM teachers and to support their own conceptualization of the system *as* a system that could produce a range of phenomena, including ensemble performance of a continuous double reflection that avoided "twists" or shifts in grip (up/down). As the session proceeded, the group's explanations tended toward finding a *formal* description of the quartet-plus-prop system. In contrast to an approach to transformational geometry in which one maps *vertices* of a polygon and "knows" the *body* of the figure will follow obediently, ensemble performance made it clear that the figure as a whole, and hence also the corners held by the members of the quartet, needed to be considered along with their orientation. A form of emergent mathematization thus appeared as the group explored the range of phenomena that the prop could produce: a system made up of phrases enacted with their moving bodies that included the double-reflection as one phenomenon among many.

Figure 15 shows a sequence of movements from a reenactment late in the IA session, as the quartet refined and stabilized a double reflection phrase that avoided the trouble that the STEM educators encountered (and repaired, without evident awareness). Much as we found with reenactments attempted by interviewees, stabilizing the performance phrase shown in Fig. 15 required a variety of inventions: changing the pace of action to make phenomena visible (e.g., "slo mo" enactments to understand the source of "twists"); attuning to other members' actions (e.g., as soon as one member began to raise a corner the rest of the quartet followed suit and immediately began the next reenactment); inventing and using new terms to refer to emergent units of ensemble activity (e.g., a "trivial solution" designated a rotation in which nobody crossed any one in the ensemble performance); and using gesture at various scales to describe what just happened or propose new actions for the ensemble (e.g., pointing along trajectories to be traversed).

Figure 15. Using reenactment to find and stabilize double reflections that did not produce "twists". This performance produced one of four alternatives the quartet explored while systematizing a double reflection phrase producing a 180 degree rotation of the sheet.

What distinguishes the IA quartet, by comparison with the students and STEM educators, is the extent to which they sought a systematic, exhaustive description of what could be done with

the quartet-plus-prop, by generating and comparing multiple courses of reenactment. They did this by noticing, naming, and executing phrases that deployed their bodies in slightly different ways (e.g., using different paths to go "under" or "around" the sheet and each other). Of the three quartets in this article, this group came closest to understanding their own bodies in the quartet-plus-prop as a *mathematizable system* of objects and operations (transformations). Of course, this is only one mode of understanding the quartet-plus-prop system, and we take up differences across quartets in the Discussion.

Discussion

Foraging and dissection as a design practice

This paper illustrates an approach to learning environment design, in which we *forage* among rich cultural performances to devise systems of people-plus-props that can foster ensemble mathematical learning. We then engage groups of participants in an interview protocol that invites them to *dissect* these cultural performances, both by reenacting aspects of them and by exploring the expressive potential of the people-plus-prop system. In these interviews, we have seen how dissection can lead to a hybrid activity, in which neither dance nor mathematics leads, but something new is enacted that features elements of both. We have noted the exuberance with which ensembles took up this performance space (Ma and Munter 2014) in their playful and creative re-use of the recordings and prop. Whether in reenactment or open choreographic exploration, the challenge of executing ensemble performances involved complex interactions in which participants utilized talk and action to codify coordinated movements and to respond to material affordances and constraints of the prop.

Considering the 8th grade girls, we see that familiar practices of viewing and doing were disrupted in this hybrid space, forcing the ensemble to invent new practices. Fundamentally, the girls negotiated shared approaches to *parsing* the Rio performance in order to reenact it. Their sense that they had "got it," after an initial viewing of the Wave quickly evaporated when they were challenged to enact the phrase. This led them to discover new units for action (e.g., role pairs they called "overs" and "unders," interaction partners across roles, and the aesthetic and procedural unit of their "whoo") in ensemble performances that engaged and produced familiar mathematical objects in new ways. In the Triangle challenge, their attempt to assess what they produced as an ensemble provoked an epistemic clash over the appropriate terms to import from school mathematics.

The STEM educators' work showed how dissection activities can support creative re-use of the people-and-prop system in a relatively short timeframe. Our analysis traced the quartet as they developed a performance repertoire (e.g., their *in-and-out* move, the "dancy" framework, and alternating use of sequential and simultaneous phrases) and built an intercorporeal sense of "we" acting in relation to the prop. Their performance phrases illustrate an important aspect of the hybrid space: the knowledge that can be built there is neither the "mathematics" of mathematicians (or schools) nor the "dance" of dancers. Their ensemble performance created new mathematical entities and relations that were aesthetically pleasing (e.g., a "cool" phrase in which the sheet becomes a "balloon" to enable reflections) but difficult to stabilize or explain. The STEM educators' final phrase exceeded their own capacity to describe or reproduce it, but we see its genealogy in emergent patterns of their collective improvisation.

Themes from both of these groups were underscored and carried forward in the IA session. Faced with the challenge of understanding the STEM educators' actions, we encountered an

epistemic barrier reminiscent of the 8th grade girls, as we initially used traditional methods of video-based IA. However, by entering the hybrid space ourselves, we experienced the power of reenactment and were finally able to gain analytic purchase on the "cool" performance. Like the STEM educators, we began to develop terms and techniques to support ensemble action (e.g., use of "slo mo" enactment to analyze and identify alternative ways to flip). We, too produced knowledge-in-use (Hall and Stevens 2015) shaped by mixing cultural practices—our emergent description of the people-plus-prop dance performance *as a sys*tem involved conceptions that fell outside of the mathematics of transformational (2D) geometry and symmetry groups. Formally describing this system required enactment and language that went beyond existing concepts or techniques we could import from either mathematics or dance.

Ensemble learning in mathematical activity

Analyzing the multi-modal interactions of quartets as they reenacted or created performance phrases using the prop provided insights into a form of ensemble learning, which by design blended aspects of dance and mathematics. While most would agree that dance involves ensemble learning (i.e., dancers *must* perform together to learn performances), the necessity of multi-body interaction is less clearly established for mathematics learning.

Considering the patterns explored by quartets in this study as *hybrid* forms of ensemble activity composed of mathematics and dance, we identify new and potentially generative mappings between performative structures and mathematical structures. Interpreted mathematically, performance phrases that return the prop to its initial visual position evoke elements of familiar symmetry groups of polygons. Specifically, they correspond to members of D4, the dihedral group of symmetries of the square.

From a mathematical perspective, the subgroup structure of D4 is rich and important, including two subgroups of order 4 (isomorphic to Klien-4) that are generated by reflections over the diagonals and the horizontal-vertical midlines, respectively. Key properties of these reflections are expressed in secondary mathematics teaching through the concept that "two reflections produce a rotation" (here a 180° rotation). Aesthetically, these transformations were enacted in the performances of many of the quartets. For example, the STEM educators organized their "Cool" phrase by coordinating two diagonal reflections, formed when two pairs exchanged places; while the 8th grade girls achieved the "Wave" with a reflection over a midline of the prop, also produced as two pairs exchanged places.

From an ensemble performance perspective, a focus on orchestrating pairs' movements provided logistical supports for quartets to build up and coordinate complex actions. It offered an interpersonal structure—exchanging places with a partner—within the quartet that made building ensemble performance more tractable. This was particularly visible in the work of the 8th grade girls described above in their "Wave." They identified "over" and "under" roles and assigned partners across roles. When these partners exchanged places, the quartet reflected the square prop over a midline.

While beyond the scope of this paper, these exchanges of partner positions can be described from the perspective of academic mathematics as *transpositions*. Just as partner exchanges were basic phrase units for the quartets' explorations in our interviews, transpositions are basic units for analyzing the structure of mathematical groups (Lederman 1957). We believe this will be a fertile design space for continuing studies of ensemble learning and teaching of mathematics.

The promise of ensemble learning remains as an open question. In a creative re-use of the Rio recordings, we designed a hybrid activity in which people had to work together to make (enact

or reenact) the dynamic shapes visible in the Rio performance record. Without co-operative action in ensemble performance, there would be no dynamic shapes—if four members of the quartet did not pull the edges of the prop taut, it was not possible to make or transform a square or a triangle. Similarly, without coordinating the motion of all members of the quartet, a line of symmetry or reflection could not be realized. This is a basic material affordance for ensemble mathematical learning using the quartet- plus-prop in this environment. In addition, the paths and orientations of vertices under simple geometric transformations became highly consequential, a facet usually backgrounded in paper-scale mathematics. The prop's material push-back (e.g., tangles in the sheet) also led to important mathematical discoveries for both the STEM educators and IA quartets. If the object of analysis in our clinical interview (the Rio performance) was a case of ensemble learning in dance, then so too was what our quartets did together during reenactment or creative enactment. And if the objects and relations of mathematical practice had to be produced by people moving together in the interview, what they learned how to do together was also a form of ensemble mathematics.

Reenactment as a method in interaction analysis (IA)

There is a further, methodological contribution to consider in the case of the IA quartet. Without fully realizing it, we too were drawn to the necessities of ensemble learning. Early on, we struggled to make progress with conventional approaches to IA, which tend to be relatively passive (e.g., analyzing a video record while seated). If they focus on bodies at all, they do so from a perspective "outside" the recorded activity (Hall and Stevens 2015). Working with the physical prop to *design* the clinical interview protocol, we made discoveries about what was possible, including what Rio performers managed to do beautifully. But we only later realized that our

analysis of ensemble learning in mathematical activity might also benefit from ensemble activity. By reenacting what we thought we saw in the video record, we were able to generate new insights both about what quartets *actually* did (these details *might* be available to passive viewing, but we struggled to find them) and what *could* be done with the quartet-plus-prop system. This suggests that reenactment as a form of ensemble learning can also be used effectively as a supplemental method in IA. If participant observation (one aspect of ethnography) supports deeper understanding in IA (Goodwin 2017; Hall and Stevens 2015), then reenactment as a set up for ensemble learning might also augment more traditional methods of IA. We offer methods of reenactment in IA as another contribution of this article, which we believe can be quite powerful.

References

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. *The Cambridge Handbook of the Learning Sciences, 2*, 358–376.

Bar, F., Weber, M. S., & Pisani, F. (2016). Mobile technology appropriation in a distant mirror: Baroquization, creolization, and cannibalism. *New Media and Society, 18*(4), 617–636.

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences, 12*(3), 307–359.

Brady, C., White, T., Davis, S., & Hegedus, S. (2013). SimCalc and the networked classroom. In *The SimCalc vision and contributions* (pp. 99–121). Dordrecht: Springer.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research, 64*(1), 1–35.

Cohen, E. G., & Lotan, R. A. (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. New York: Teachers College Press.

Eglash, R., Croissant, J. L., Di Chiro, G., & Fouche, R. (2004). *Appropriating technology: Vernacular science and social power*. Minneapolis: University of Minnesota Press.

Erickson, F. (2004). *Talk and social theory: Ecologies of speaking and listening in everyday life*. Cambridge: Polity Press.

Gerofsky, S. (2010). Mathematical learning and gesture: Character viewpoint and observer viewpoint in students' gestured graphs of functions. *Gesture, 10*(2), 321–343.

Gerofsky, S. (2016). Approaches to embodied learning in mathematics. In L. D. English, & D. Kirsher (Eds.), *Handbook of International Research in Mathematics Education* (pp. 60–97). New York: Routledge.

Goodwin, C. (2007). Participation, stance and affect in the organization of activities. *Discourse and Society, 18*(1), 53–73.

Goodwin, C. (2017). *Co-operative action*. Cambridge: Cambridge University Press.

Hahn, T., & Jordan, J. S. (2017). Sensible objects: Intercorporeality and enactive knowing through things. In C. Meyer, J. Streeck & J. S. Jordan (Eds.), *Intercorporeality: Emerging socialities in interaction* (pp. 267–288). New York: Oxford University Press

Hall, R., & Nemirovsky, R. (2012). Introduction to the special issue: Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences, 21*(2), 207–215.

Hall, R., Ma, J. Y., & Nemirovsky, R. (2014). Re-scaling bodies in/as representational instruments in GPS drawing. In V. Lee & M. Linn (Eds.), *Technology and the body: Perspectives from the learning sciences* (pp. 112–131). New York: Routledge.

Hall, R., & Stevens, R. (2015). Interaction analysis approaches to knowledge in use. In A. A. diSessa, M. Levin & J. S. Brown (Eds.), *Knowledge and interaction: A synthetic agenda for the learning sciences* (pp. 72–108). New York: Routledge.

Harss, M. (2016, August 8). For this choreographer, the Olympics are the zenith. *New York Times*. Retrieved from https://www.nytim es.com/2016/08/08/arts/dance/deborah-colker-choreographer-rio- olympics-opening-ceremony-dance.html. Accessed 4 Feb 2019.

Headrick Taylor, K. (2017). Learning along lines: Locative literacies for reading and writing the city. *Journal of the Learning Sciences, 26*(4), 533–574.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *The Journal of the Learning Sciences, 4*(1), 39–103. Kahn, J., & Hall, R. (2016). Getting personal with big data: Stories with multivariable models about health and wealth. *Paper*

presented at the annual meeting of the American Educational Research Association. Washington, D.C.

Kendon, A. (1990). Conducting interaction: Patterns of behavior in focused encounters. New York: Cambridge University Press.

Laurier, E. (2014). The graphic transcript: Poaching comic book grammar for inscribing the visual, spatial and temporal aspects of action. *Geography Compass, 8*(4), 235–248.

Lederman, W. (1957). *Introduction to the theory of finite groups*. Edinburgh: Oliver and Boyd.

Lotan, R. A. (2003). Group-worthy tasks. *Educational Leadership, 60*(6), 72–75.

Ma, J. Y., & Munter, C. (2014). The spatial production of learning opportunities in skateboard parks. *Mind, Culture, and Activity, 21*(3), 238–258.

Ma, J., & Hall, R. (2018). Learning a part together: Ensemble learning and infrastructure in a competitive high school marching band. *Instructional Science 46*(4), 507–532.

Ma, J. Y. (2017). Multi-party, whole-body interactions in mathematical activity. *Cognition and Instruction, 35*(2), 141–164.

Marin, A., & Bang, M. (2018). "Look it, this is how you know:" Family forest walks as a context for knowledge-building about the natural world. *Cognition and Instruction, 36*(2), 89–118.

Meyer, C., Streeck, J., & Jordan, J. S. (2017). *Intercorporeality: Emerging socialities in interaction*. New York: Oxford University Press. Núñez, R., Edwards, L., & Matos, J. F. (2006). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39*(1–3), 45–65.

Rosling, H. (2006). The best stats you've ever seen [Video file]. Retrieved from https://www.ted.com/talks/hans_rosling_shows_the_best_stats_you_ve_ever_seen?langu

age=en. Rowland, T. (1999). Pronouns in mathematics talk: Power, vagueness and generalization. *For the Learning of Mathematics, 19*(2), 19–26.

Shapiro, B. R., Hall, R., & Owens, D. A. (2017). Developing and using interaction geography in a museum. *International Journal of Computer Supported Collaborative Learning, 12*, 377–399. Sinclair, N., de Freitas, E., & Ferrara, F. (2013). Virtual encounters: The murky and furtive world of mathematical inventiveness. *ZDM—The International Journal on Mathematics Education, 45*(2), 239–252.

Streeck, J., Goodwin, C., & LeBaron, C. (Eds.). (2011). *Embodied interaction: Language and body in the material world*. New York: Cambridge University Press.

Teasley, S. D., & Roschelle, J. (1993). Constructing a joint problem space: The computer as a tool for sharing knowledge. In: S. P. Lajoie & S. J. Derry (Eds.), *Computers as cognitive tools* (pp. 229–258). Hillsdale, NJ: Lawrence Erlbaum Associates.

Vogelstein, L., Brady, C., & Hall, R. (2017). *Mathematical reflections: The design potential of ensemble performance*. In P. Blikstein, & D. Abrahamson (Eds.), Proceedings of the 2017 conference on interaction design and children (pp. 583–588). New York: Association for Computing Machinery

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education, 22*(5), 366–389.

CHAPTER 3

MATHEMATICAL PHYSICAL RESEARCH: MATHEMATICAL AGENCY IN THE PRACTICES OF PROFESSIONAL DANCERS

Introduction

Although mathematics is commonly understood as a disembodied, Platonic discipline, scholars in mathematics education have argued for the importance of recovering the body for mathematics learning (Hall & Nemirovsky, 2012; Stevens, 2012). Lakoff and Núñez (2000) traced the discipline of mathematics backwards to embodied semantic primitives. By linking the form of the human body to the mathematics that we have today, Lakoff & Núñez argued that embodied resources for sensemaking were fundamental to the historical development of mathematics, even if not to contemporary mathematics teaching or learning. An entrenched history in math education has positioned the body as a container, with the sole purpose of transporting a thinking brain, which means that inviting learners to ground their mathematical sensemaking in their bodies is not always an invitation that is accepted, taken up, or trusted. While the body is an omnipresent resource for learners in all contexts, it is commonly dismissed or erased in formal learning environments, as learners routinely do not see their everyday embodied activities as mathematically relevant (Moschkovich, 2002; Nasir et al., 2008). Thus, it is insufficient to invite learners to use their bodies for mathematical sensemaking without better understanding culturally and historically laden practices that foreground the body as the primary site for sensemaking. As more research explores the multimodal and embodied ways learners can make sense of disciplinary ideas (Hall & Nemirovsky, 2012; Ma, 2017; Marin et al, 2020), a question remains: how can we design math learning activities in which learners have reason to trust their bodies as resources? In this paper I

propose that looking to the practice of professional dancers may provide answers, as their sensemaking centers on embodied forms of collective noticing as ever present and crucial to exploring how they can express and make sense of mathematical entities.

Motivation

A group of four 8th grade girls were participating in an exploratory mathematics activity: I asked them to reenact a moment from the opening ceremony of the 2016 Rio Olympics in which performers folded large flexible silver squares into triangles. The girls folded their own similar square prop into a triangle, but disagreed about what kind of a triangle they had folded. They placed their large triangle on the ground and engaged in an enthusiastic debate as to whether it was a right triangle (with a 90-degree angle) or an isosceles triangle (in which two edges have equal length). Ava started the debate standing fixed at one edge of their triangle, "IN MATH [...] you're not going to turn the math book and say it's a right triangle, you're gonna say that's an isosceles." Octavia supported Ava's argument for team isosceles, "when they give it to you they don't put the 90 up there ((pointed to the angle on the floor Tracy argued was 90 degrees)) [...] They don't put the 90 up there. THEY DO NOT DO THAT in books."

Here, Ava and Octavia (all names used in this paper are pseudonyms) used school-based mathematical epistemologies cultivated from paper-scale geometry to reason about their large, walking-scale triangle, deferring to a fixed understanding of what "they" do "in math." This was perfectly reasonable. The implicit norms in mathematics textbooks correlate orientation with geometric properties by deferring authority to the invisible authors of these books, allowing for quick recognition. These sensemaking resources employed "in math," however, limited the girls' engagement with the mathematical properties being discussed, as they accepted a static,

decontextualized understanding of mathematical rules, even for a mathematical object they had made by moving together at walking scale (Vogelstein, Brady, & Hall, 2019). The moment these girls labeled their activity as "math," they turned away from their embodied sensemaking resources both within themselves and each other.

The ubiquity of our bodies, as inseparable from our existence, makes them a fundamental resource for how we make sense of the world around us. Thus, if we want to ground mathematics learning with these powerful resources, we must leverage the body as a powerful source of reasoning which is not a trivial task. In this paper I examine a contrasting case to the one described above to show how professional dancers in the company, Novel Tectonics, engaged in this same activity in a way that linked their mathematical reasoning to their embodied practice called "physical research," building mathematical discoveries from agentic and embodied forms of reasoning. By engaging in their practice of physical research, these dancers took up mathematical reasoning in expansive ways. The 8th grade girls wanted to do this, but did not have a toolkit to support this kind of reasoning and thus were far more constrained. The practice of physical research has important structures to support collective mathematical sensemaking that links embodied noticings to ensemble mathematical learning in a way that foregrounds participants' individual and collective creativity and agency.

In this paper I analyze the response of professional dancers (in conversation with 8th grade students' response) to an embodied mathematical activity I created to intentionally support learners' recruitment of everyday, embodied reasoning. This activity was presented in a double stimulation interview (Engeström, 2007) in which participants were presented with novel tools (large prop and video recording from Rio) to accomplish a task (reenact folding choreography from the 2016 Rio Olympic Opening Ceremony), designed to provoke observably creative

solutions for study (Vogelstein et al., 2019). Building mathematical learning from intuitive sensemaking means that learners see these forms of reasoning as valuable and part of their everyday lives and not stuck in the isolated context of mathematics (Hall & Jurow, 2015; Nasir et al., 2008). In contrast to the episode of 8th grade girls' reasoning about their mathematical choreography, professional dancers framed their engagement in this context as a practice of inquiry, flexibly leveraging embodied sensemaking resources to explore what was expressively possible within the people-plus-prop system. Instead of reaching a solution quickly (inverting the relationship between speed and intelligence), these dancers slowly created new possibilities and ideas together. The dancers referred to this practice as physical research, recruiting how they participate in a dance environment to this hybrid movement and mathematics context. In physical research participants explore their unknown expressive potential as an ensemble through an iterative process of creating and responding to collective, full-body movement. This foregrounds artistic agency and collective persistent inquiry (Lerman, 2014; Sengupta-Irving & Agarwal, 2017). The dancers' engagement in mathematical physical research in the interview points to new possibilities for expansive mathematics learning through their embodied mathematical sensemaking. To that end I explore how the practices of professional dancers can broaden our conceptualizations of epistemic mathematical practices by asking the following research questions:

1. How did dancers engage in processes of physical research in their professional context and what forms of participation did this support?
2. How did dancers engage in physical research to support mathematics reasoning in the context of the Rio interview and what forms of participation did this support?

I address these questions by using an ethnographic study (Lave, 2011) to trace the cultural practice of physical research from the dance studio to the double stimulation interview (Engeström, 2007). Use of physical research in the interview setting shows how dancers leveraged this practice for mathematics learning. I operationalize learning as a socially distributed process of changes to participation in a community of practice (Lave & Wenger, 2011) and directly take up recent methodological calls in the learning sciences to contextualize analyses of disciplinary learning in the history of participants' relations as mediated by embodied practices (Gutiérrez & Rogoff, 2003; Vossoughi et al., 2020). Dancers' use of mathematical physical research offers a new way to think about the designs for mathematics teaching and learning that place everyday resources of the body on par with symbolic or graphical resources in conventional approaches to mathematics education (Gerofsky, 2010).

Theoretical Framework

Ensemble Learning

Ensemble learning (Ma & Hall, 2018) references activities that are done with others that cannot be learned alone. These activities are necessarily a collective endeavor in which participants recognize the need to learn as a group, such as a high school marching band preparing for a competition or professional dancers engaging in an exploratory choreographic process where the group explores what they can express collectively (Lerman, 2014). Analytically, this necessitates a shift in the unit of analysis from the individual to the ensemble. This does not mean that individuals and their contributions are erased; rather, this shift contextualizes individual

contributions in how they inform the group's work together and highlights the coordinated efforts of distributed learning (Danish et al., 2020; Ma, 2017).

Ensemble mathematical activity can expand *what* is possible to learn in mathematics contexts by changing the scale and modality of geometric objects, thus re-scaling the body's relationship to geometric figures and changing the modality of engagement. In traditional paper-scale geometry, learners maintain a bird's eye view and an authoritarian relationship to manipulating small objects. The representational infrastructure of mathematics changes when the scale and materiality of geometric objects change (e.g., 7' x 7' shiny silver Mylar square) so that manipulating geometric objects now requires the participation of a group. This disrupts canonical embodied routines and creates new possibilities for modal engagements, understanding, and participation in mathematics learning (Hall, et al., 2014; Ma, 2016; Vogelstein et al., 2017 & 2019). For example, in the Rio interview, dancers choreographed a way to find the midpoint of one edge of the prop by having two people hold adjacent corners with one hand to create a taut edge for their other hand to slide along. When the two hands met, they were in the middle of the edge and had found the midpoint. This is an excellent example of how modalities of engagement shifted from using a ruler to first measure the entire length and then divide it in two, to two hands sliding towards each other. This led to a conceptualization of midpoint that highlighted the relationship between the defining points, enacting the same travel time from end points to midpoint. This relational understanding of a mathematical concept was built from the coordination of moving bodies and perspectives.

Embodied Mathematics

Ensemble activity foregrounds the importance and necessity of *doing* things together since these activities cannot be done by someone on their own, which highlights the body as a central to learning. How the body is used in learning processes (embodied learning), however, has been theorized differently by many scholars. Stevens (2012) outlined two theoretical perspectives characterizing the majority of embodied mathematics literature: conceptualist and interactionist. He argued that the conceptualist stance positions the body as revealing cognition in the brain (Alibali & Nathan, 2012; Lakoff & Núñez, 2000) and the interactionist position commits to seeing cognition in interaction (Goodwin, 2000; Nemirovsky et al., 2012). I take an interactionist perspective which aligns with Kirsh's (2010) notion of physical thinking, in which studying a choreographer revealed how physically enacting movements affords opportunities for observations and actions that could not be accessed through simulation or without direct participation. Notably, becoming mathematical entities through exploratory dance research can bring relations into existence in meaningful ways (Kremling et al., 2018).

When geometric figures are re-scaled and necessarily manipulated by an ensemble, noticings while moving become consequential for learning as learners gain an intrinsic perspective of the object they constitute together. For example, when the dancers slid their hands towards each other to find their midpoint, they began to notice how this action leveraged the midpoint as an anchor for the folding sequence that followed. Coordinating both intrinsic and extrinsic perspectives add to the depth of mathematics learning in these environments by emphasizing how components coordinated together constitute complex mathematical objects. Embodiment constituted *in* the movement of ensembles shows how being immersed inside and outside of mathematical phenomena supports the coordination of multiple perspectives which leads to new

forms of engagement and sensemaking that positions learners as agentic and actionable mathematics thinkers.

Intercorporeality & Relationality

Expanding possibilities in mathematics education through studying ensemble learning from an interactionist and embodied perspective highlights how individuals' movements are understood in relation to the whole group composition. Participants gain a view of mathematical structures from their intrinsic perspective, while also engaging in mathematical sensemaking with others as they are forced to coordinate distributed perspectives and participation to create objects together. Thus, how groups learn to reason as a group through the development of new relations is foregrounded in these learning environments (Sengupta-Irving & Agarwal, 2017).

Leading work in the learning sciences has called for research that values new forms of relationality as outcomes of learning (DiGiacomo & Kris D. Gutiérrez, 2016; Vossoughi et al., 2020), expanding what it means to take changes in participation as evidence of learning (Lave & Wenger, 1991). As an intimately connected practice enacted with an ensemble, physical research supports the development of *intercorporeality* (Meyer et al., 2017), or a sense of withness. Hahn and Jordan (2017) argued that evidence of intercorporeality is a change in participants' identities from I's to we. Embodied anticipations begin to stabilize as reciprocal responsibility develops, which also supports empathy and *intercorporeal trust*. Empathy and trust are important sensibilities to cultivate in learning environments, and are commonly absent from mathematics contexts (Cooper, 2010).

Tracing the practice of physical research over the years has shown deep reciprocal relations between these dancers, foregrounding intercorporeal trust as a fundamental aspect of their practice.

Thus, when these dancers entered into the Rio interview, they were able to draw upon well-established relations of trust in each other to both explain what the Rio dancers did and to create new performance elements. Analysis of the dancers' developed sense of withness, and a substantial repertoire of language and body practices for engaging with this mathematical task importantly expands understandings of what mathematics learning can look like and result in.

Methods

This paper traces the practice of physical research through ethnography and a double stimulation interview. The 90-minute interview was designed as a Vygotskian double stimulation experiment (Engeström, 2007), which allows researchers to observe participants' inventions in response to novel situations such as choreographing geometric transformations with a large Mylar square. Initial analysis of professional dancers engaging in this interview prompted two years of ongoing ethnographic observations to better understand the practice of physical research in the context of the dance studio and how the dancers leveraged it in the interview.

Participants & Research Context

The primary participants consisted of 10 professional dancers and the artistic director, Clarke, of a contemporary dance company, Novel Tectonics, located in a mid-sized city in the American Southeast. Almost all of the dancers moved from all over the country to work in the artistic "research lab" Clarke created where dancers explored how they could collectively embody new ideas. While foregrounding the exploratory nature of dance is not unique to Novel Tectonics, it is noteworthy that these dancers moved to a new city for a job that was not enough to make ends meet. Clarke described this artistic community as follows: "We are Black, White, Latinx. We are

gay, straight, bisexual, Christian, Jewish, believers in science, and atheists." The dancers' diversity was reflected in physical research by leveraging bold movement proposals that reflected their unique perspectives. Dancers have expressed that leveraging their artistic agency and allowing them to express possibilities they could not explore on their own drew them to this community (Vogelstein, 2020).

Data Sources & Analysis

Ethnographic Data

Novel Tectonics rehearsed 3 hours a day, 5 days a week from September to May. During the 2017-2018 & 2018-2019 seasons, I observed 1-2 rehearsals a week for a total of 50 observed rehearsals. Data collected during rehearsals included field notes that describe observations of how these dancers engaged in cycles of physical research as well as video recordings of their work. Field notes were collected as a means to capture events and impressions from being present as an observer, supplementing the inherently limited view of the camera lens, while video recordings captured the fast-moving bodies of the dancers to allow for micro-ethnographic Interaction Analysis (IA) of how multimodal aspects of gaze, gesture, and coordinated movements were used in interaction (Hall & Stevens, 2015).

Double Stimulation Interview Data

The Rio interview was designed as a double stimulation experiment (Engeström, 2007) in which participants were presented with a primary stimulus (the Rio Olympic reenactment task) to make their uptake of a secondary stimuli, representational supports or tools (large square Mylar prop and video recording from Rio), observable. As a whole, the interview consisted (1) of

showing a group of 4 people a performance from the 2016 Rio Olympic opening ceremony, (2) asking them to reenact parts of the performance using a similar prop, (3) having them discuss the relevance of mathematics in this activity, and (4) allowing participants to choreograph their own performance with the prop. 4 cameras recorded the interview (2 cameras on adjacent walls, 1 on the ceiling, and 1 on the table), approximating the different angles used in the viewed recording of the Rio performance. Analysis of the dancers' participation in this interview shows how they leveraged consequential embodied sensemaking resources by engaging in physical research.

Findings

How these dancers engaged in physical research professionally

This analysis demonstrates how engaging in the practice of physical research allowed the dances to collectively engage in expansive, agentic mathematical sensemaking in which, unlike the 8th grade girls, mathematical authority resided in their own seemingly simple observations. Analyses of the first year of ethnographic data illuminated the basic structure of physical research in practice (Vogelstein, 2020) across multiple contexts. In the rehearsal studio, cycles of physical research were enacted through making and responding to physical proposals between the choreographer and dancers to develop "movement vocabularies" and then "sculpt" them through proposals to change what had been established. Framing any movement enacted as a proposal meant that movements were always thought of as something that could be responded to, ideas that others could and should build off of. Choreographic proposals for sculpting existing movements were generally verbal prompts to alter what the dancers had created (e.g. enact these moments but in a diagonal formation) from an extrinsic perspective, to which the dancers used their bodies to

respond to these new arrangements by making new proposals and responding to each other physically from an intrinsic perspective until something had formed, a referenceable movement vocabulary, that the choreographer could continue to sculpt (Figure 16).

This model of physical research helps illuminate how these dancers participated in this practice, proposing ideas with their bodies that they responded to until something had solidified that could be observed and then sculpted. As they engaged in this practice year-in and year-out, their ability to work together as an ensemble also developed as proposals and responses blended into trusting movements together:

> I feel like it happens physically because […] there is also like a trust that like the body like has some understanding of what's happening just by the person in the room being in the room. You know, I feel like there's something that is exchanged in that, that chain of sharing that is more than verbal. […] Maybe it's the way that Maddie makes a proposal and I then, you know, follow up. […] It's still improvisational but there's still a sense of knowing. You know, like if I end up behind Maddie and my arms are in front of her then she knows that if she wanted to she could like thrust all of her weight back and I could fall safely. […] And it's through the doing and the redoing I think too (Darius, Interview March, 2019).

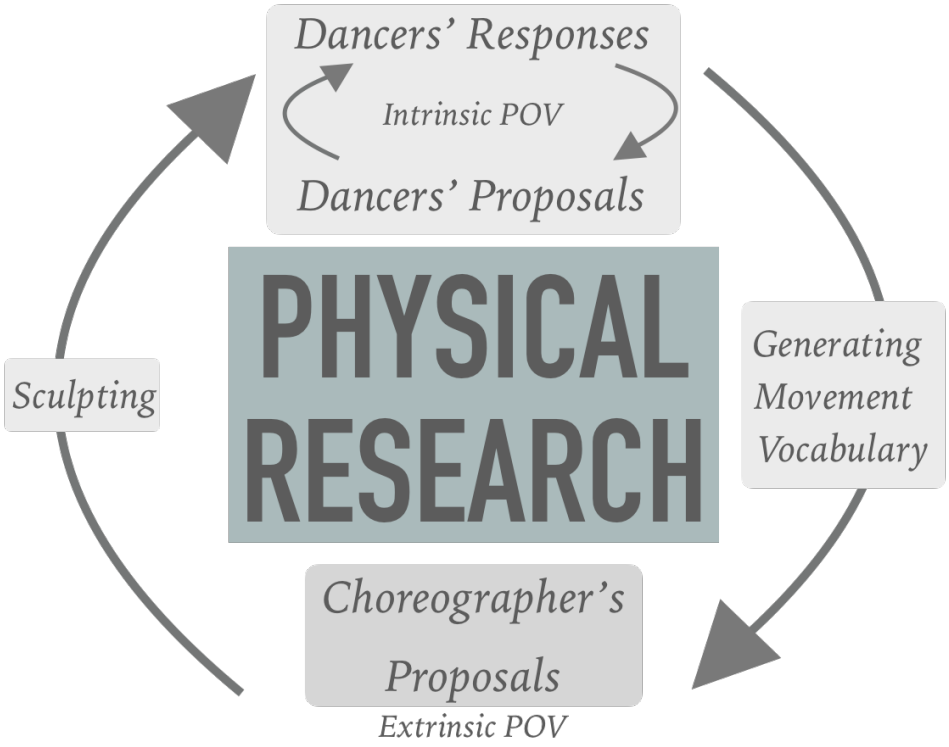

Figure 16. Model of physical research

Committed to the notion that people's physical presence affects what is possible, Darius described physical research as a process developed by moving in response to and with others. He linked this to the relationships with his colleagues, where trust supported them to exchange ideas and move in new ways together without words. He emphasized, "doing and redoing," in which long-term trajectories of participation support the development of intercorporeal trust as an innate part of physical research. In the next section I use this ethnographically-grounded model of physical research to analyze these dancers' collective mathematical sensemaking in the Rio interview to show how physical research can support agentic forms of mathematical reasoning that can also foster new forms of relationality through trust as a valued outcome of learning.

How these dancers engaged mathematics in physical research in the interview

In this episode, four dancers from Novel Tectonics (Cathy, Darius, Faith, & Maddie), participated in an experimental hybrid movement and mathematics activity in which they were working on reenacting choreography from the opening ceremony of the 2016 Rio Olympics that involved a quartet folding a large silver flexible square sheet of plastic into a triangle. Over the course of a 20-minute process of physical research, the dancers iteratively refined their understanding of the referent choreography by exploring together what was visible in the video record and what was possible to create with the prop together.

By engaging in this mathematical activity through their collective practice of physical research, the dancers continuously explored possibilities with the people-plus-prop system (quartet and square prop), making new discoveries until they were "satisfied" and stopped. Framing their activity as physical research meant that the dancers continued to redefine how they could fold the prop into the desired shape. Their engagement focused on exploring possibilities rather than perfecting the execution of a set sequence of movements, remaining open to opportunities for changes through new proposals. In the following analysis I outline the dancers' physical research trajectory as it led them to discover three different embodied understandings of the concept of midpoint and how the resources involved in cycles of physical research supported this engagement.

Physical Research of a Triangle: Three different ensemble coordinations used to find a midpoint

Noticings from the video record became crucial to the ensemble's engagement in physical research as they coordinated their extrinsic perspective of the performance from re-watching the video with their intrinsic perspective as a part of a group manipulating a similar prop. These

intuitive sensemaking resources, comparing observable configurations, were easily taken up in their inquiry. For example, early on the group identified that the camera angle used to broadcast the performance captured observable triangles with one 90-degree angle and three sides of different lengths (a right scalene triangle). The dancers proceeded to fold the square prop on the floor to prove that creating the desired triangle was possible. On the floor, the first proposed movement was a grab at the midpoint of one edge, to which another dancer responded by using this as a fulcrum to fold her corner up and over across the midline of the object; this was the first way they found this midpoint together.

In addition to coordinating observations from the video with their own roles in the choreography, the dancers also responded to how they manipulated the prop together, engaging with it as a fifth member of the group that needed to be supported. Once they had folded a right scalene triangle on the floor, they transitioned to a standing up configuration, sculpting their movement vocabulary. Once they stood up, the dancers quickly discovered that they needed to actively pull corners to tighten edges in order to make folds along the same lines as they had on the floor, reconfiguring the supports the floor had passively provided. However, now that active force was needed at the corners of the square, nobody could reach to grab a midpoint. Cathy and Faith creatively slid their hands along the taut edge between them using the symmetry of their movements and the length of their wingspans to find the midpoint between them. This second movement vocabulary for establishing a midpoint was found by responding to the new configurations and movement vocabulary they continuously developed.

After choreographing a triangle-folding pattern standing up, utilizing points that created tension to fold across straight edges, the dancers had solidified a new movement vocabulary but some were still not satisfied. Faith and Maddie made a proposal to sculpt this vocabulary by

incorporating a visual formation from the Rio performers' "hit" in the video that they had not yet created. Faith's proposal to sculpt, "Yeah let's do the dumpling airbag," (Figure 17A) came from an extrinsic perspective on the choreography, positioning Faith as the choreographer in this moment. The "dumpling-airbag" referred to a formation where all four performers started with their prop extended as a square and all walked to the center together, puffing the prop into a dumpling-like shape below their waists. Observing and naming this formation allowed the movement to be referenced and reenacted as a group. Maddie led the group in enacting this formation, coordinating their movements with a verbal cue, "So, dumpling," (Figure 17B). Creating the dumpling formation was the group's response to Faith's sculpting proposal and once formed, Cathy's new intrinsic perspective led to a new way to find the midpoint, "oh and then find our edge here," (Figure 17C & D) as opposed to pulling it tight as they had before. The dumpling folded the prop without a need to stretch edges taut and in fact folded each edge in half so that the midpoints between corners now puffed out between members, making it easy to grab as the edges collapsed in half. Confirming the utility of this formation, Faith followed up by announcing to the group, "this is how they do it," (Figure 17E). This comment solidified the generation of a new movement vocabulary, setting the group up for another process of sculpting as they continued in their cycles of physical research.

This example of the development of finding the midpoint of an edge, distributed amongst the dancers, shows how engaging in cycles of physical research structured participation so as to make small, intuitive noticings (e.g., all of the Rio performers gathered together at one point) about the aesthetics of movements consequential and productive for inquiry. One small proposal became important as it was amplified by seeding responses from other members distributed in the group as individual wonderings became whole group enactments (from I's to we's). The relations of trust

enabled deeper understandings of what can be done and at the same time the relational structure of the mathematical object (the multiple ways the prop could be folded across a midpoint). These forms of reasoning are typically not available when this sort of mathematics is engaged with at paper-scale under circumstances in which individuals manipulate entire geometric figures on their own, foregrounding the resultant state and not relations and pathways in movement. It is also important to note that proposals to sculpt in this process of physical research supported a process of mathematical engagement in which discoveries were being made, even 17 minutes into a 20-minute inquiry process, constantly fueling new ideas, formations, and understandings from the dancers.

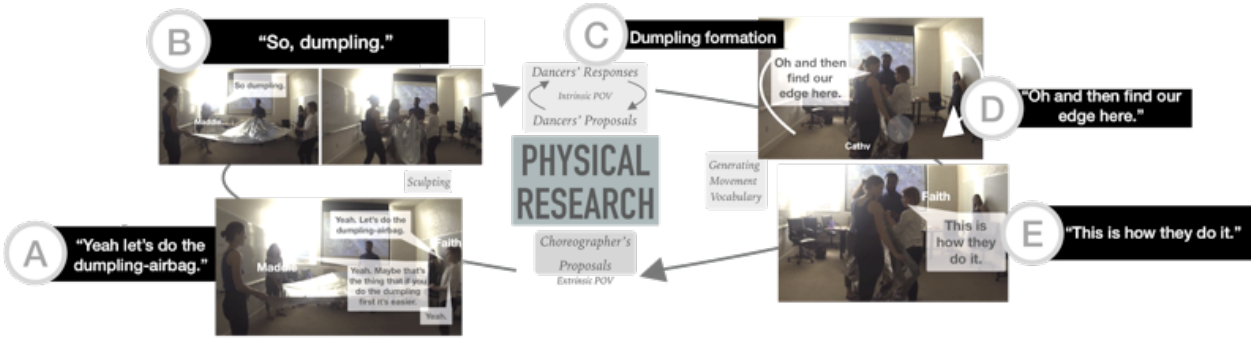

Figure 17. Mapping the dancers' mathematical discovery that midpoints can emerge through symmetric movements onto the preliminary analytic model of physical research

This analysis demonstrates how these dancers' engagement in physical research in the Rio interview supported the use of everyday sensemaking resources for agentic mathematical reasoning. Simple noticings of measurement, orientation, and configurations led to the development of complex embodiments of three different ways midpoints could be found and used as anchors for symmetric transformations. Additionally, this analysis shows how physical research structured the use of these sensemaking practices to support new relations between learners

because as the dancers engaged in this activity their fluency in responding to each other's movement proposals increased through the development of intercorporeal relationality and trust.

Discussion & Significance

To concretize the importance of this paper, I return to the opening example. Although Ava and Octavia's intuition when folding the prop into a triangle was to recruit disembodied forms of reasoning used, "in math," their eagerness to engage in an extended debate about the properties of a triangle and resultant conclusion. Eventually Tracy persuaded Ava by showing that in the creation of the triangle one of the corners came from a square and since it was not altered in the process of folding, it must have remained 90 degrees. Tracy's insistence and Ava's change in reasoning points to students' willingness to engage their intuitive, embodied sensemaking skills, although not always right away. Imagine what kind of mathematical participation these girls could have engaged in if they had exercised their agentic forms of embodied sensemaking from the beginning as physical research supported the dancers' mathematical reasoning or had trusted each other and built off of each other's proposals, as the triangle in question was actually both right *and* isosceles.

This paper contributes to work in mathematics education to better understand practices (such as physical research) that support embodied, everyday sensemaking resources learners already possess as relevant to mathematics inquiry. The observations the dancers made and acted on utilized basic embodied sensemaking resources (e.g., mirroring slow movements, noticing visible differences in formations) that the 8th graders could have enacted, yet it was engaging in the practice of physical research, a practice that had cultivated their ability to work as a collective, that enabled these dancers to engage in such deep and generative mathematical inquiry.

Studying these dancers' engagement in physical research, both in the dance studio and in the interview, shows the development of a responsive collective agency through the making and responding to proposals as an ensemble. This enactment of agency shifted the relations between the dancers, supporting collective and embodied forms of mathematical inquiry and imagining. By illuminating the structures of activity that supported physical research over time, this analysis both challenges normative conceptions of mathematical learning as mediated primarily with pencil-and-paper and illuminates possibilities to guide current and future designs of innovative and inclusive embodied mathematics learning environments. Questions about how to design to support these collective practices so that learners see their everyday, embodied sensemaking resources as mathematically relevant remain. Drawing from these initial findings, I have begun the next phases of this work, co-designing and co-analyzing mathematics activities with the dancers of Novel Tectonics, using their practice of physical research to structure forms of participation for students that are promising in initial analyses.

References

Alibali, M. W. and Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences*, 21: 247–286.

Cooper, B. (2010). In search of profound empathy in learning relationships: Understanding the mathematics of moral learning environments. *Journal of Moral Education*, *39*(1), 79-99.

Danish, J. A., Enyedy, N., Saleh, A., & Humburg, M. (2020). Learning in embodied activity framework: a sociocultural framework for embodied cognition. *Int. J. Comput. Support. Collab. Learn.*, *15*(1), 49-87.

DiGiacomo, D. K., & Gutiérrez, K. D. (2016). Relational equity as a design tool within making and tinkering activities. *Mind, Culture, and activity*, *23*(2), 141-153.

Engeström, Y. (2007). Putting Vygotsky to work: The change laboratory as an application of double stimulation. In *The Cambridge companion to Vygotsky*. Cambridge University Press.

Gerofsky, S. (2010). Mathematical learning and gesture: Character viewpoint and observer viewpoint in students' gestured graphs of functions. *Gesture*, *10*(2-3), 321-343.

Goodwin, C. (2000). Action and embodiment within situated human activity. *Pragmatics*, 32: 1489–1522.

Gutiérrez, K., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice. *Educational researcher*, *32*(5), 19-25.

Hahn, T., & Jordan, J. S. (2017). Sensible objects: Intercorporeality and enactive knowing through things. In *Cognition in 3E: Emergent, Embodied, Extended* (pp. 37-54). Springer, Cham.

Hall, R., & Jurow, A. S. (2015). Changing concepts in activity: Descriptive and design studies of consequential learning in conceptual practices. *Educational Psychologist*, *50*(3), 173-189.

Hall, R., & Nemirovsky, R. (2012). Introduction to the special issue: Modalities of body engagement in mathematical activity and learning. *Journal of the Learning Sciences*, *21*(2), 207-215.

Hall, R., Ma, J. Y., & Nemirovsky, R. (2014). Rescaling bodies in/as representational instruments in GPS drawing. In *Learning technologies and the body* (pp. 124-143). Routledge.

Hall, R., & Stevens, R. (2015). Interaction analysis approaches to knowledge in use. In *Knowledge and Interaction* (pp. 88-124). Routledge.

Kirsh, D. (2010). Thinking with the body. In *Proceedings of the annual meeting of the cognitive science society* (Vol. 32, No. 32).

Kremling, J., Arndt, N., Roman, A., Zilm, M., & Straehler-Pohl, H. (2018). Dancing on the junction of mathematics and bodily experience. The Mathematics Enthusiast, 15(1), 201-227.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from* (Vol. 6). New York: Basic Books.

Lave, J. (2011). *Apprenticeship in critical ethnographic practice*. University of Chicago Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University.

Lerman, L. (2014). *Hiking the horizontal: Field notes from a choreographer*. Wesleyan University Press.

Ma, J. Y. (2016). Designing disruptions for productive hybridity: The case of walking scale geometry. *Journal of the Learning Sciences*, *25*(3), 335-371.

Ma, J. Y. (2017). Multi-party, whole-body interactions in mathematical activity. *Cognition and Instruction*, *35*(2), 141-164.

Marin, A., Taylor, K. H., Shapiro, B. R., & Hall, R. (2020). Why Learning on the Move: Intersecting Research Pathways for Mobility, Learning and Teaching. *Cognition and Instruction*, 1-16.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical thinking and learning*, *4*(2-3), 189-212.

Meyer, C., Streeck, J., & Jordan, J. S. (Eds.). (2017). *Intercorporeality: Emerging socialities in interaction*. Oxford University Press.

Nasir, N., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. Review of Research in Education.

Nemirovsky, R., Rasmussen, C., Sweeney, G. & Wawro, M. (2012). When the classroom floor becomes the complex plane: Addition and multiplication as ways of bodily navigation. *Journal of the Learning Sciences*, 21: 287–323.

Sengupta-Irving, T., & Agarwal, P. (2017). Conceptualizing perseverance in problem solving as collective enterprise. *Mathematical Thinking and Learning*, *19*(2), 115-138.

Stevens, R. (2012). The missing bodies of mathematical thinking and learning have been found. *Journal of the Learning Sciences*, *21*(2), 337-346.

Vogelstein, L. (2020) Physical Research: Professional Dancers Exploring Collective Possibilities in a Solidifying Substrate. Proceedings of the International Conference of the Learning Sciences, 2020, 737-739.

Vogelstein, L., Brady, C., & Hall, R. (2017, June*).* Mathematical Reflections: The Design Potential of Ensemble Performance. In *Proceedings of the 2017 Conference on Interaction Design and Children* (pp. 583-588).

Vogelstein, L., Brady, C., & Hall, R. (2019*).* Reenacting mathematical concepts found in large-scale dance performance can provide both material and method for ensemble learning. ZDM Mathematics Education 51(2).

Vossoughi, S., Jackson, A., Chen, S., Roldan, W., & Escudé, M. (2020). Embodied pathways and ethical trails: Studying learning in and through relational histories. *Journal of the Learning Sciences*, *29*(2), 183-223.

CHAPTER 4

INTERDISCIPLINARY COLLABORATION IN DESIGN RESEARCH: A PROCESS OF COMPOSING ACROSS DESIGN, ANALYSIS, AND RELATIONS

One of the motivations for the development of design research methods in the learning sciences was to "propose an alternative conception of a domain" (Cobb et al., 2003, p. 11) to support expansive practices in learning environments. In this paper, we argue that one consequential way to engage in design research methods towards this aim is to work interdisciplinarily in both the design and analysis phases of our work, as shifts in design practices necessitate shifts in analysis practices. As we demonstrate, working interdisciplinarily can expand disciplinary practices and conceptions by not only bringing new perspectives to established design practices (e.g. engaging dancers in traditional DBR methods) but by disrupting and shifting design practices (e.g. leveraging choreographic methods for design to explore possibilities for computational learning as designers and for youth as learners) which importantly can lead to new analysis practices (e.g. using choreographic methods of inquiry to understand expansive participation). Our primary aim is to demonstrate that when interdisciplinarity in the design process changes both design processes and the designs themselves, this necessitates changes to analysis processes in order to see how new values and learning begin to grow in these spaces. Thinking differently about design entails thinking differently about analysis. In this paper we share the story of how our interdisciplinary team of dancers, choreographers, and learning scientists designed and analyzed an activity we called the Soup Game. We describe the Soup Game as a case from our collaboration that shows the most overlap across our disciplinary perspectives in choreography and computational modeling. Our findings foreground how dance-based

perspectives and practices can be generative for youth learning and how these same practices are linked to disciplinary concepts and practices in computational modeling. By sharing the story of the Soup Game from ideation through analysis, we demonstrate how interdisciplinary, collaborative design research can disrupt existing methods for design and analysis by foregrounding collective ways of knowing and being in relation to each other.

Motivation for Creating an Interdisciplinary Design Research Collaboration

Motivations from the literature: Traditions in design research & participatory design research point to the importance of interdisciplinary collaboration throughout research phases

The Learning Sciences is grounded in commitments to designing innovative and ecologically relevant learning environments, bringing new possibilities for learning into the world in order to study their theoretical and practical implications (Brown, 1992; Cobb et al., 2003; Collins, 1992). Methods for design experiments and design based research (DBR) developed from educational researchers' desires, "to transform classrooms from academic work factories to learning environments that encourage reflective practice among students, teachers, and researchers" (Brown, 1992, p. 174). Imagining transformative possibilities for learning and bringing them to life to not only expand what is possible in learning environments but our conceptualizations and theories of learning, position methods of design research as supporting innovative and exploratory research in the Learning Sciences.

In order to push past what had been done and how learning was understood, scholars who initiated the development of methods for design research emphasized the necessity of collaboration

across expertise as a way to expand research possibilities. Cobb and colleagues claimed, "The crucial determinant in any type of design experiment is that the team collectively has the expertise to accomplish the functions associated with developing an initial design, conducting the experiment, and carrying out a systematic retrospective analysis" (Cobb et al., 2003, p.11-12). According to these scholars, embedding design experiments in classrooms meant that a research team needed expertise in "the design and analysis of classroom learning environment, professional teaching communities, and schools as institutions" (p. 12). The expansive aims of design research were met methodologically in creating interdisciplinary research teams that worked from the initial design phase through the implementation and analysis phases. However, Cobb and colleagues (as well as other early leading scholars in the development of design research methodologies (Brown, 1992; Collins, 1992)) conceptualized educational researchers themselves to hold the necessary expertise needed to conduct design research. Diversity in expertise for these scholars spanned nuances and variations in educational research expertise and not in other disciplines that could further support their aim to "propose an alternative conception of a domain" (p. 11).

Research methods for expanding the means and ends of domain learning have been challenged by methodological innovations in the field's recent history, as scholars have explicitly challenged normative ideas of power and voice in the design research process (Bang et al., 2016; Bang & Vossoughi, 2016; Gutiérrez & Jurow, 2016). In particular, participatory design research (PDR) foregrounds the importance of designing for subject-subject relations both in design partnerships and desired outcomes from designs (Bang & Vossoughi, 2016). Specifically, this work has called into question who is invited and positioned as a researcher in design research collaborations, working with educators (McWilliams, 2016; Zavala, 2016), youth (Ehret & Hollett, 2016), parents (Booker & Goldman, 2016), and community members (Bang et al., 2016; Jurow et

al., 2016; Vakil et al., 2016) in this process. With respect to cultivating relationships between collaborators in the design process, important innovations in PDR are tied to "the ways that accomplish role re-mediations that disrupt normative forms of imagining and decision making" (Bang & Vossoughi, 2016, p. 178). Thus, highlighting who is involved in design processes consequentially impacts what can be imagined and created in design and how learning is conceptualized in these spaces in activity and analysis.

Work in PDR (Bang & Vossoughi, 2016) and other collaborative design traditions (Goldman et al., 2022) has illuminated the bidirectionality in re-mediating role relations between Learning Scientists and other partners through engaging in practices of co-design. By bringing community stakeholders into spaces and practices for designing learning environments, however, what is designed for has shifted more than the practices for designing themselves. These collaborations have impacted problem scoping by demonstrating how research collaborations with participants embedded in research contexts can illuminate "what is consequential for communities [which is] critical for the study and design of learning" (Jurow et al., 2016, p. 210). This is partially due to the fact that ethically engaging in collaborations with participants outside of academic settings means that important negotiations around aims and resources occur: "to be productive partners, we have to be transparent, realistic, and respectful about the constraints of time, financial compensation, language, and relevance, making sure that the process of collaboration is not burdensome" (Jurow et al., 2016, p. 220). While the commitments for conducting PDR open up opportunities to change methods of knowledge production through collaborative design work, the logistics of developing these relations have challenged the expansion of these methods.

While expanding methods of interdisciplinary design has proven challenging, some scholars have pointed to learning from other communities' design practices as an important starting

point. Zavala (2016) argued for a need to study design practices in grassroots communities because Learning Scientists' design expertise is commonly not embedded in the activity systems they design in and for:

> What is needed in order to understand the value of PDR are ethnographies of design activity that will assist learning scientists in reflecting upon the limits and possibilities of how they come about design, what voices are included and excluded, and how the entire research process is shaped, including the questions that orient the research all the way to its outcomes (Zavala, 2016, p. 247).

Zavala's proposal is a humbling one for researchers enculturated into a field marked by a strong association with design, asking Learning Scientists to look to and learn from other communities of designers to reimagine and shape our own design research practices from ideating through implementation, and analysis. Thus, Zavala argued not only for working with new collaborators in processes of design research, but learning from their design practices to expand and re-mediate what our own practices for design can look like.

Bang and colleagues' community based design work (2016) showed a complementary approach to engaging with existing design practices outside of design research traditions. This work demonstrated how embedding researchers and resources in partners' community contexts can support needed shifts in guiding epistemologies for knowledge production in joint design processes. Specifically, their demonstration of the development of axiological innovations, changes in "theories, practices, and structures of values, ethics, and aesthetics—that is what is good, right, true, and beautiful" (Bang et al., 2016, p. 28.) points to consequential ways to transform design research practices in the development of "creative, deliberate, and consequential interventions" (Bang et al., 2016, p. 39).

The work shared in this paper rests at the intersection of the traditions described above as a way to demonstrate what can be learned and expanded upon methodologically from engaging in practices of design research across disciplinary practices of research in both the ideational and analysis phases of design. Specifically, we worked as an interdisciplinary team of researchers and designers across our respective fields of Learning Sciences and Dance/Choreography with the goal of collectively expanding the means and ends of computational learning for youth. Throughout our collaboration we explicitly interrogated our own design practices and have leveraged those reflections in this paper. Our interdisciplinary team expands the vision for interdisciplinarity as proposed by early scholars in design research, as dancers and choreographers have not historically been positioned as relevant contributors to research on the design of STEM-based learning environments. This collaboration reflects the directionality of many PDR and co-design partnerships by inviting dance collaborators to join an existing Learning Sciences design research team as consultants and convening in university spaces. However, the relational history that grounded this interdisciplinary partnership was rooted in a two-year ethnographic study in which Lauren engaged in understanding Rebecca & Curtis's professional research practices as dancers and choreographers (Vogelstein, 2020). Thus, the invitation to work interdisciplinary in a process of design research came from positioning all collaborators as designers in their own rights and researchers in both the design and analysis phases. In this paper we look at how "relational dynamics shape processes of partnering and the possible forms of learning that emerge in and through them" (Bang & Vossoughi, 2016, p. 174) through both the design and analysis phases of design research. We demonstrate how this collaboration made participatory design research a matter of analysis as well as design.

Motivations from our shared histories: Building a foundation of seeing and trusting each other's research practices as generative together

As described in the previous section, methodological innovations in DBR and PRD have pushed learning scientists to re-scope who they collaborate with and how those collaborations can shift what is designed and how it supports new conceptualizations of learning. In this section we describe more specifically the motivation for convening our particular design team as an interdisciplinary research team across practices in the learning sciences and dance and choreography. This contextually specific motivation is important as it highlights how our relational history mediated our initial goals as design collaborators. These goals then supported the development of our axiological innovation around the use of a compositional lens for the design and analysis of collective, physical, computational learning environments.

One important way to situate this collaboration is inside of a larger DBR project. Corey and Lauren had been working on a grant aimed at developing innovative ways for middle schoolers to learn mathematics and computational concepts together in the context of a one-week coding and art summer camp. In the first iteration of our design work, we (Lauren as a doctoral student & Corey as her advisor) foregrounded the importance of physical-based embodied activity in learning to interpret and write code for agents to execute in an agent-based modeling (ABM) environment (NetLogo (Wilensky, 1999)). We developed what we called the Stadium Card activities as a means to introduce code syntax with important properties of immobile agents referred to as patches (e.g., their color and location). Participants stood in a grid each holding a stack of large colored squares to represent their patch color and responded to code the teacher as the observer would relay to them. We conjectured that this activity would support observable links between verbalized, syntactically correct code, individuals' interpretations of code through actions, and their collective

visual creations. We hoped that learners would start to see code as performable and dynamic. We conjectured and hoped students would take up this view of code as performance by choreographing final projects that were either physical performances akin to the stadium card activity or digital performances that emphasized the collective enactment needed to create digital images in an agent-based coding environment. While our initial analysis demonstrated that this activity supported generative opportunities for learners to "take the patch perspective" in interpreting code into physical actions, only a few students tried to create complex performances in this context which proved almost too challenging to break down into actionable steps and translate into readable code (Vogelstein & Brady, 2019). Overall, this first iteration showed promise in leveraging the links between physically enactable code for individuals and what a group could create or perform together (Brady, 2021). However, viewing code as performable to generate collective, computational expressivity did not take off in any meaningful way for learners in this first iteration.

While the primary impetus that brought us together came from Corey & Lauren's iterative DBR work, it is important to note that this was not in fact the beginning of our relationships across disciplines (dance and learning sciences) and contexts (formal research relations and friendships). During this time, Lauren was engaged in a two-year ethnographic study of professional dancers' collective inquiry practices, observing Rebecca & Curtis in their professional capacity as dance and choreographic artists. She chose to study this dance company because they framed choreography as a collective inquiry practice, which is not how all dance spaces conceptualize the choreographic process. The primary finding from this research was a mechanistic description of the practice this dance company referred to as "physical research." Describing physical research foregrounded how this group of dancers used structured forms of improvisation to create and manipulate sequences of movements by responding to each other's movements as opportunities to

generate new ideas as they moved in relation to each other (Vogelstein, 2020). The emphasis on responsivity is key to this practice. For example, even when Rebecca[5] prompted the dancers to engage in solo research, exploring the embodiment of contradictions they held in their bodies, the dancers still moved in response to each other. Solos quickly became duets and duets became trios as dancers found themselves in close proximity to others, responding to what each was doing physically in shared space. When Curtis slid to the floor and ended up near another dancer's head, he turned his gaze down tenderly to her head, kissed it, and then proceeded to move her head as her body responded in kind to his manipulations (also see Figure 19). This rigorous form of research emphasized the generative possibilities of exploring a group's expressive potential when members' participation was respected through response, such that participants had their ideas responded to as well as responded to others' ideas.

Our invitation to Rebecca and Curtis to collaborate on the design of new movement-based computational activities foregrounded how their research practices allowed for forms of participation that had eluded us in our previous design iterations. Corey and Lauren saw Rebecca and Curtis's expertise in practices of physical research as generativity linked to their hope for developing conceptualizations of computation as dynamic, collective performances. Rebecca and Curtis's methods for developing group-based movement performances went beyond what Lauren and Corey had been able to imagine and support on their own. For example, in the development of the Stadium Card activities Lauren & Corey enacted the activities with groups twice, first with the grant advisory board and second with the math teachers who led the camp. These enactments led to important changes in the activity structure (Vogelstein & Brady, 2019), however, our ideas

[5] During Lauren's two years studying practices of physical research, she observed Rebecca in two choreographic residencies with the company. These were three-week periods in which Rebecca's position as a dancer in the company changed and she led the group as the choreographer, guiding their research with a generous and perceptive eye.

for the activity were still confined in a way that only allowed for participants to enact a single, correct interpretation of code. Although what that interpretation created at the collective level could be surprising to participants, their ability to respond to each other's actions and make new proposals for enactable code was limited.

While the Learning Scientists on our team saw the dancers as engaging in research practices that supported the kinds of practices they hoped to see in their own design, Curtis and Rebecca saw Corey and Lauren's work as an extension of their own commitments and curiosities about dance and learning. Having participated in Lauren's ethnographic project, Rebecca and Curtis experienced what it felt like for their sensemaking abilities to be taken seriously as they were positioned not only as subjects of research but also as intellectual partners through research practices outside of their expertise. Social science research practices are not neutral engagements, and have a history in which practitioners have used their power to do harm to subjects and participants (Bang & Vossoughi, 2016; Tuck & Yang, 2012). Thus, it should not be taken for granted that Lauren, Rebecca, and Curtis's engagement in this initial ethnographic work deepened their trust in one another and helped surface similar shared commitments (e.g., combatting deficit notions of dance as a corporeal, intuitive, frivolous, and unintelligent pursuit).

Our collaboration was built off of existing and developing understandings of each other's design and inquiry practices. Before we convened as a collaborative design research team, Curtis and Rebecca had developed understandings of what Learning Sciences research processes could entail and the kinds of findings that could emerge, while Lauren and Corey had developed understandings of dance-based research practices. These relational foundations set us up to begin a design research collaboration in which our different forms of disciplinary expertise were (1) needed to expand conceptualizations of the domain of computational learning (Cobb et al., 2003)

and (2) understood as different research practices to engage in together (Zavala, 2016). The participants not affiliated with a university institution identified and were positioned as researchers in their own right from the beginning of our design work.

In the rest of this paper, we focus on the development of an axiological innovation, taking a compositional stance towards design and analysis, which allowed us to engage in new practices that created space for and illuminated youth creativity and expressiveness in computational learning. We first describe the concept of composition across choreographic ways of knowing and computational modeling ways of knowing that developed from our partnership in order to demonstrate the practices that informed this understanding in both our design and analysis phases of collaboration. We then share reflections on what it meant to take up choreographic practices of inquiry as rigorous academic research and expansive forms of participation for young learners through an analysis of how we attended to composition in our design and analysis phases of collaboration. In the end, we offer this analysis of our interdisciplinary design partnership as a case study to demonstrate the expansive potential and challenges in broadening where expertise on learning and design come from in the study of learning through design based research.

Composition: generative point of contact across disciplines first used in design and cultivated through analysis

In collaborating across design practices in choreography and computation, we came to see important relations between processes of composing and the resultant compositions created as generative for both discovery and pedagogy. The term composition generally refers to ideas about creation and part-whole-relations and refers both to creations as well as the creative process across disciplines. For example, as a noun, a composition can refer to a piece of music, a painting, a novel,

a water molecule, or a mathematical function. As a verb the term can refer to the act of creating one of these nouns as something formed from the arrangement of smaller components: notes, brush strokes, words, chemical elements, or mathematical functions. Each of these different disciplinary uses of the term composition are associated with different values and aesthetics, different socially constructed norms for how to arrange components to create something beautiful and new. These axiologies are a reflection of how processes of composing are related to the compositions they create.

In this section we describe our different disciplinary uses of how the term composition is used in our respective choreographic practices and computational modeling learning environments. We argue that we came from two communities that are distinguished by their relationship between composing and composition that were grounded in but distinct from the mainstream of their core disciplines. We also demonstrate how these close relationships between composing and composition reveal important axiological commitments that fuel discovery and pedagogy. Attending to composition as a concept across disciplines was generative because it acted as lens which helped us better understand the relationships between design practices across these disciplines.

Choreographic Composition: Traditional and locally situated practices

The notion of composition in choreography entails creating both individual movement elements and allowing them to come together into a group-level enactment. In traditional choreographic processes of composing a single choreographer *sets* a movement vocabulary *on* a group of dancers by telling them what movements to execute in order to achieve a certain visual arrangement of moving bodies—the composition—for an audience to appreciate. For example, at

the beginning of Alvin Ailey's (1960) iconic choreographic work *Revelations*, Ailey clustered the dancers in a flock with winged arms until he had one dancer elongate and stretch out his arms into an open "V" shape (Figure 18). In this moment of composition, Ailey chose to have a single dancer expand his arms out, framing this movement with the uniformity of the other dancers' stillness, which allowed for the outstretched "V" shaped arms of a single dancer to be foregrounded in that moment. Traditional composition in choreography denotes how movements are arranged in ways that change over time to create contrasts in formations and movement dynamics for an audience, as dictated by a choreographer and performed by dancers.

Figure 18. A noteworthy compositional moment from Alvin Ailey's (1960) Revelations in which a group of dancers move in unison to stretch their arms out in wing shaped curves (left image) until the tallest dancer in the center straightens his arms to accent a moment of silence in the music (right image).

In traditional choreographic composing processes, the agency and evaluative power rests solely with the choreographer. This has positioned dancers at the service of the choreographer's vision without any agency in terms of shaping the composition in the composing process (Duffy & Beaty, 2019). In limiting dancers' agency, many traditional processes of choreographic composing can strip dancers of their dignity. When dancers are only seen as serving a choreographer's vision for an audience, if a dancer cannot physically execute what a choreographer asks of them then this becomes the dancer's failure and not the choreographer's. In fact, frequently dancers are positioned as an impediment to making a choreographer's vision come to life. Dancers'

bodies have to answer to the choreographer's vision, and the choreographer, who is only accountable to a future audience, thus can often have no reciprocity towards the dancers.

While the structures of traditional choreographic composing practices do not necessitate the mistreatment of dancers, the lack of agency they imbue in dancers means that all too often this is the case. In fact, Rebecca & Curtis joined the dance company they were working with at the time of this study because the composing practices engaged in by that community were constructed explicitly to combat the dehumanizing effects of many traditional choreographic composing practices. In contrast with traditional compositional approaches, practices of physical research distributed composing agency amongst the choreographer and dancers so that movement vocabularies come from the dancers' bodies and are not dictated to them. In fact, while many traditional composing processes focused solely on the composition being created, practitioners of physical research found that by focusing on the process of composing in terms of dancers' agency and relationality they could create more interesting and compelling compositions. While this does not guarantee that healthy relationships are developed between a choreographer and dancers in the composing process, as choreographers can still be exploitative of what dancers offer in the process, it does create more promising conditions for composing processes that respect the humanity, artistic agency, and perspectives of collaborators. Rebecca and Curtis aimed to bring the reciprocity they found in their professional dance relations to their work in educating youth. Thus, for Rebecca and Curtis, the choreographic composing practices they enacted and were interested in engaging with youth focused on disrupting traditional choreographic composing practices by leveraging dancers' ingenuity in the process.

One of the primary distinctions of physical research as a choreographic process of composing is that compositional moments were discovered through the sensitivities of the

participants and not on a predetermined cue. By predominantly using improvisational techniques, physical research-based composing practices focus on finding interesting moments that arise when dancers move together in such a way that their movements are seen as proposals that others need to respond to. For example, in one engagement in physical research, James stood with his hands at his mouth (Figure 19A) and Curtis responded by walking towards James with his left hand grabbing his own face (Figure 19B) until they made contact and started responding to each other's movements as a duet together (Figure 19C). This compositional moment was felt by Curtis and James in how they attuned to each other to move in response to each other and it was observed by the choreographer, Rebecca. She was able to observe this duet framed by one other dancer standing and the rest of the dancers moving slowly on the ground. Understanding the emergence and establishment of this compositional moment through improvisation meant seeing Curtis's movement choice (Figure 19B) as a response to James's initial proposal (Figure 19A) while also holding these movements in sight of the full composition created by all of the dancers in the space together. Within a process of physical research, Rebecca as the choreographer looked for configurations like this that stood out to her, and she worked with dancers to continually create conditions for finding similar moments and making them even better.

Figure 19. The development of a duet as a compositional moment in an improvisational engagement in physical research.

Physical research is important because it is a choreographic process of composition that foregrounds dancer agency and responsivity as a process of inquiry. Instead of being beholden to a master vision for compositions, physical research composing processes create compositions that are the result of an emergent order that arises through relations of reciprocity, mutual trust, and active embodied listening. By attending to the sensitivities and relations of participants inside developing compositions in the composing process, in the end an audience sees an enactment of relations that draw their vibrant meaning from what it feels to be inside of the composition. The

audience—instead of being the consumer and primary occasion for a composition to come together—is positioned as receiving a gift in witnessing what has been developed between artists in the composing process.

Computational Modeling Composition: Descriptive and exploratory modeling practices

In computational modeling the notion of composition refers to how computational rules governing individual computational elements relate to aggregate patterns. In particular, the kind of computational modeling that Corey and Lauren have a history engaging in is agent-based modeling (ABM) using the software NetLogo (Wilensky, 1999). In this environment, composing entails creating computational rules for agents (mobile agents called turtles and immobile agents called patches), which result in the emergence of visual arrangements when they are 'run'. The visual feedback in the NetLogo world, and the resultant compositions, makes it distinctive as a computational environment.

In the field of computational modeling there is a division that we see as analogous to the division between traditional dance choreography and the choreography driven by physical research. On one side, descriptive modeling begins from a pre-established phenomenon that the modeler aims to represent computationally. The goal of this process is to refine and discover the computational rules that accurately represent and reproduce the target phenomenon. Once this underlying compositional structure is uncovered, the modeler can then learn more about how this phenomenon came to be. For example, in watching a flock of birds fly in a V shape, it might look like the flock is following a leader bird at the front. However, in developing computational rules for all of the birds in a flock to follow in order to fly in a V shape, it turns out they all follow the same rule with no leader. In composing a descriptive model, when ideas do not produce the desired

phenomenon, they are positioned as bugs that need to be discarded or fixed. Similarly, to traditional choreographic composing processes the power of agency and evaluation lies solely in the modeler as they converge on a chosen phenomenon akin to a choreographer's vision.

While descriptive modeling starts with a compositional goal and works to unearth the correct rules to enact said phenomenon, processes of composing in exploratory modeling start with speculatively creating computational rules and exploring the compositional possibilities they can generate, individually or together. We see exploratory modeling as entailing similar composing processes to physical research. The aim of exploratory modeling is to discover what a certain infrastructure or group of agents can create together. In comparison to the convergent nature of descriptive modeling, exploratory modeling is a composing process that is open-ended and divergent in nature. Composing processes of exploratory modeling are not evaluated by approximations to an ideal, but rather are judged by their generativity and surprise. When a modeler comes across a surprise compositional moment, it becomes amplified rather than being viewed as a bug that needs to be fixed or rid of in processes of descriptive modeling. Although processes of composing in both descriptive and exploratory modeling involve creativity the fixed or open-ended compositional goals differ greatly and are important to note (Wilensky & Rand, 2015).

Composing Processes and Compositions Across Disciplines: Physical research and exploratory modeling

As designers across these disciplines, we made generative connections (1) between traditional choreographic composing processes and descriptive computational modeling practices, and (2) between physical research and composing processes in exploratory computational

modeling. In particular, with respect to the latter pair the relationship between composing processes as discovering compositional possibilities was salient. Physical research as a composing process foregrounds dancers as participants who make choices in response to others' movement proposals within improvisational structures and tasks that can be altered and changed. The development of compositions as enactable and executable movements by individuals to create a visual whole together relates to processes of composing in exploratory modeling, in which computational rules for agents to execute are used to explore emergent patterns and visuals at the aggregate level. In both of these practices, processes of composing are tightly linked to emergent compositions. The agency of what a collective can make is closely related to the experience of making it together.

The history of physical research as a practice that pushed against the lack of dancer agency (and oftentimes dignity) in traditional choreographic composing processes provides new ways of viewing descriptive and exploratory modeling. While computational agent agency might seem like a nonsense idea, since computational agents can only execute code in a particular way, how modelers view and frame agent agency might have important consequences. For example, viewing computational rules as proposals to respond to as someone enacting those rules or viewing them is a dramatic shift from interpreting them as commands to be strictly followed, which influences both the process of composing and the composition as an outcome of this process. The power to create becomes distributed amongst the collective of dancers and a choreographer or agents and a modeler, instead of concentrated in an individual. As Curtiss shared early on in our collaboration about participating in processes of physical research, "I feel like our relationship to the choreographer is like we're partners […] it's different than what I think the standard is because Rebecca [the choreographer] is like openly and knowingly being unknowing in the room" (Curtis,

August 2018). In thinking across composing processes and compositional views across disciplines in our collaborative design and analysis work, we sought to explore how the views and practices in physical research and exploratory modeling could generatively support learners' computational expressivity and learning as collective practices.

Bringing Composing Processes from Physical Research and Exploratory Modeling Together to Design for Youth Computational Learning and Expressivity

The four authors of this paper came together as co-designers to sculpt activities for middle-schoolers in a one-week art and coding summer camp that would allow learners to engage in expressive physical and digital forms of computational thinking. In particular, this paper focuses on the design and analysis of one activity for the camp, an activity we called the Soup Game in which campers developed movements to be components (ingredients) in an emergent piece of choreography that they would enact together (the soup), inventing names and rules for these movements (e.g., students decided that "walking" meant everyone "take 10 steps forward"). In this section we describe how we laid the foundation for collaboratively designing this activity together by deepening our understandings of each other's design practices. By first engaging in composing processes based in physical research and exploratory modeling we were able to see how each other's composing processes could be generative for our own disciplines.

Although we began this collaborative work by engaging in physical research and exploratory computational modeling processes, our initial design ideas for the camp followed a more descriptive modeling trajectory. It was only when we let physical research practices, specifically Rebecca and Curtis's pedagogical expertise, lead our designs that we were able to truly embrace a compositional stance for our design work that combined exploratory modeling and

physical research. Our eventual stance highlighted the importance of foregrounding campers' agency in developing their own computational rules, rather than being given pre-set rules (or even "primitives" to combine) by a leader or choreographer. We argue that leveraging these ideas surrounding composition in our co-design work led to support new values and aesthetics for computational learning.

Foundations for convening as an interdisciplinary design team

As mentioned in previous sections, we convened as an interdisciplinary design team in the second iteration of a DBR study after we had already started to get to know each other through a separate two-year ethnographic study. Corey & Lauren's invitation to Rebecca & Curtis to join their design team was rooted in a deep respect for Rebecca & Curtis's physical research practices and an initial conjecture that these practices could foreground ideas around code as an expressive form of performance for learners. Lauren initially framed Curtis & Rebecca's involvement as collaborators who "are going to do some consulting work on our camp project, helping us with the design of the camp curriculum that leverages choreography as a way to think about computation and mathematics" (personal communication 5/2/19). At the same time, Rebecca & Curtis saw this opportunity as beneficial towards broadening the utility of their professional practices outside of dance studios to other learning spaces where they saw dance practices as relevant and generative. Rebecca & Curtis had an important level of trust and respect for Corey & Lauren's ongoing research, along with an understanding that their knowledge could help further the project's agenda to broaden valued forms of learning in STEM spacers for youth, spaces where they had not felt welcomed themselves.

Additionally, Curtis & Rebecca understood this consultant work as economically beneficial in a time period after their company's season came to a close and before their arranged summer work began. Like most professional dancers in the United States, Rebecca & Curtis at the time were freelance artists that needed to supplement their work as performers with many other short-term jobs. One of the primary jobs they supplemented their income with was work as dance educators. They each had a deep history in dance education working outside of their performance jobs, and thus their freelance work both allowed them to join this research team and established them as informed pedagogical partners.

The initial motivations to collaborate grew from a shared desire to leverage the power of collective forms of physical expression as expansive and generative for developing engaging and creative learning pathways that disrupt traditionally confining structures in STEM spaces. We have identified three of our commitments that supported this shared endeavor: (1) centering movement as both the means and ends of learning and not as an accommodation or merely a means to a more cerebral end (Marin, Taylor, Shapiro, & Hall, 2020), (2) explicitly positioning embodied ways of knowing as intelligent, valid, and rigorous, and (3) rescoping what research means, to include moving in response to and with others. These three commitments all surfaced prior to our first day in a university conference room designing for children's computational learning. They were first observed, felt, and discussed in the foundational phase of our relationship building and allowed us to begin designing together from a shared starting point that from the outset challenged dominant notions of what research and design practices could entail.

Spending time in each other's compositional processes as part of design: Deepening our understandings of each other's design practices with new compositional perspectives

We began this design collaboration from a place of unknowing and authentic questioning. Lauren & Corey did not claim to know exactly how computational learning and physical research practices overlapped or if it was even possible for a group of newcomers to engage in generative processes of physical research. This motivated the necessity of playing across these modalities, taking turns leading the group, to demonstrate how our enactments together were needed in order to find generative connections across disciplines. Beginning from a place of unknowing that required the participation of a group to explore possible answers is also a fundamental aspect of physical research as an inquiry practice (Vogelstein, 2020), so from the start we were building from similar axiological commitments across disciplines.

We took turns leading the group in each other's design practices, which foregrounded how new compositional perspectives could generate discoveries and support learning in our respective disciplines. While the foundation of our relationships enabled us to enter into this design process with a sense of how our different research practices could work together, these initial understandings were based on close observations of each other and not in first hand experiences engaging in each other's work. Although it was not a linear process, gaining first-hand experience engaging in each other's practices proved important in that it helped us understand connections between composing processes and compositions in physical research and exploratory modeling.

During this time, the learning scientists dipped their toes into physical research and the dancers were introduced to NetLogo, which allowed each team member to share their own practices and engage in each other's with an eye towards how these new tools might transform their own practices. Curtis & Rebecca led Lauren & Corey in improvisational warm-ups and the

enactment of movement scores together. Movement scores are rule structures that can generate choreography when people make choices within them, such as walking around a room and maintaining equidistant spacing between two others in a larger group. As the initiators of this collaboration, Lauren & Corey's stance to this experience was to try and understand how these choreographic practices could relate to exploratory computational modeling. We were excited about the prospects of using physical research practices in our collaborative design work because we could see that Rebecca & Curtis were really good at getting people to explore a space of possibilities as a group in ways that had challenged and eluded us in the past. For Lauren & Corey, moving inside of these movement structures as a group emphasized the possibilities in dynamic, collective enactments as directly related to the responsiveness of engaging in exploratory computational modeling.

Reversing relations, Corey led Rebecca & Curtis[6] in a NetLogo introduction through exploratory modeling, allowing them to explore how to create and direct movable agents (turtles) on the computer. They started by making as many turtles as they wanted, having them walk forward, and then began to create new compositions from questions and ideas these first lines of code and arrangements inspired. One of the exciting and promising aspects of this was that fairly quickly it became clear that Curtis & Rebecca's choreographic composing processes could be generative in this digital, computational environment. Their stance to learning NetLogo was one in which they searched for how this environment could be useful for their own compositional thinking.

This time in each other's design practices was driven by the question: What can we make together? While this question is not necessary for all generative collaboration, it was for ours

[6] Lauren was out of town that day and could not join.

because it got to the heart of an exploratory modeling stance. This stance emphasizes that what is possible to create is unknown. This approach stretched the expressivity of both disciplinary borders. To accept improvisation (yes, and…) means breaking free of conventional choreographic composition and descriptive (fitting) practices in ABM.

Working across disciplines in this way meant that we kept the respect for our individual expertise intact while also leaving room for shifts in how we engaged in our processes individually and collectively. We started generating ideas for movement scores that had to be accountable to conceptualizing with simple enough code for a beginner to express similar ideas in NetLogo. And we started using simple NetLogo code to develop visually compelling compositions in NetLogo that could also spark interesting ideas for physical movement scores. This back and forth, and melding of seeing these two disciplinary spaces as provocations for each other led us to hone in on what we thought were generative movement prompts (e.g., wander, gather, align, spread). We outlined multiple physical and digital responses to these prompts and explored them as proof of their generativity in both of these modalities.

Our initial connection between the use of movement scores in physical research and computational modeling processes was to think of the movement prompts we generated as verbs that could act as computational primitives. We conjectured from this work that having learners respond to the movement prompts we generated would be best to seed their expressive explorations. We had shown through our own explorations that there were multiple, generative interpretations of these verbs both physically and digitally and thus was a fertile space for youth. This interpretation, however, was more in line with traditional processes of choreographic and descriptive computational modeling processes. What we did not understand at first is that we could only see these movement primitives as physically and digitally generative because we spent the

time discovering and developing them ourselves. We downplayed our own learning through engaging in these exploratory composing processes.

Staying in line with the core commitments of physical research as a composing process, however, meant that we needed to foreground participant agency. In physical research, the dancers need to develop the movement vocabulary from their own bodies, which is also present in exploratory modeling composing processes. Handing students movement primitives to describe themselves turned our exploratory stance into a more traditional compositional process. While descriptive computational modeling approaches are generative on their own in that discipline specific primitives are much more expressive than lower-level, discipline-general language primitives (Sengupta et al., 2013; Wilkerson-Jerde, 2015), this stance fundamentally conflicts with the composing processes utilized in physical research and to a certain extent exploratory computational modeling. The inertia of traditional composing processes proved harder to break free from than we initially thought. In the end we chose to have learners develop their own vocabulary rather than provide one. Our next design challenge was to think about how to support learners to develop their own vocabulary. It turns out, dancers could help us with this immensely. As described in the next section, it wasn't until we went back to centering practices of physical research that our design work returned to its initial aims, emphasizing the exploratory nature of enacting collective forms of physical and digital computational expressivity that leveraged learner agency and ownership.

Compositional convergence in the Soup Game as a designed activity: Surfacing new values for computational participation and learning

As designers we began with a close hold on the movement rules students would enact physically and digitally; however, when we began to open up our thoughts on pedagogy, we came to realize that developing ideas for movement rules is an important part of the process. This turn from assembling primitives to participants developing primitives was a shift occasioned by mapping exploratory modeling at the level of primitives to physical research. This shift led us to develop the Soup Game, the activity we implemented in the camp.

The Soup Game, a rule-based dance game, was originally suggested by Rebecca & Curtis as a generative activity structure that would enable learners to develop and enact their own movement rules first as individuals and then as a collective with different compositional formations. The structure of the game is as follows: players develop movement rules that each have two components, (1) a name and (2) an action (it did not matter which one came first in the development of a movement rule), and then a leader calls out the names and the group enacts the actions together as they are called out. In their pedagogical practices, Rebecca and Curtis played this game when working with dance students as a way to demonstrate how complex choreography can be generated quickly in a way that allowed students to compose by (1) making a variety of movement rules, (2) calling them out in different orders, and (3) enacting the movement rules as they were called out. When playing this game with students, the speed at which interesting choreography can be developed is meant to lower the pressure and anxiety of developing choreography. This game also gives students an opportunity to stand up as leaders in a choreographic setting by getting to create and then call out the movement rules. Curtis & Rebecca

used the Soup Game to support students in a collective composing process where they played a role in the emergence of compositions both from inside and outside of the composition.

The Soup Game as it was enacted in our camp was very similar to the original game Rebecca & Curtis used in their pedagogical practices and it provided campers the opportunity to develop choreography quickly as an ensemble while co-developing an understanding of computational concepts (e.g., code specificity, agent interpretation, procedures, and conditional statements). Playing the Soup Game involved first enacting a facilitator defined and student named rule, "Craziness" which meant to spread out while always moving, and then having participants generate a set of other rules to enact together. The game cycled between developing rules and trying them out as a leader (first the teacher and then students) called out the names of the movement rules in new sequences. As each enactment session occurred the group accumulated more movement rules to enact together and as they were enacted in different orders at different times, they created new compositional formations.

Similarly to how Rebecca & Curtis had used this activity to help young dancers experience how complex choreography can be generated from simple movement rules, we emphasized in our new Soup Game the pedagogical importance of setting the bar low for developing initial movements rules. This is why we began with a facilitator defined rule, spread out while always moving. While there is a lot of freedom in enacting this score, participation is not too complex, and there is a definite structure inherent in it: (1) continue to make spatial choices keeping yourself apart from others, (2) stay in the defined active space, and (3) do not stop moving. Beginning from a place where a simple idea to spread out while moving, a common pedestrian activity, could generate compelling choreography came from our shared commitment that any movement can be considered dance and choreography. Engaging in this design work together explicitly brought

forward our dance lineages for the first time as we discussed our connections to postmodern perspectives on dance that pushed against centuries of narrowly defining dance as an elite and distant art form solely performed by specific, disciplined bodies (Barnes, 2011)[7]. We leveraged this postmodern view of dance in order to position all learners and their movement ideas as valid and generative in the design of this activity. So, while our Soup Game design emphasized learners generating movement rules themselves, we also thought it was important to seed the environment with a first movement rule that emphasized the generativity and deceptive complexity of simple pedestrian movements.

Our design for the Soup Game positioned learners' compositional agency in the enactment of movement rules as well as emphasizing the importance of agency in the generation of movement rules. By beginning the activity with a facilitator generated movement rule (spread out while always moving) we aimed to set the bar low for participation in enactment and rule generation as a way to invite learners to contribute. With the emphasis on learners generating movement rules to enact together, we kept the foundation of composing processes in exploratory modeling and physical research at the forefront, necessitating collective enactment to explore what it possible to express as a group. Our conjecture was that this open-ended and exploratory compositional stance could and should happen early for learners and not be deferred later until they had acquired basic skills or technical training. While we understand the benefits for engaging in more traditional, descriptive modeling practices, we think learners should experience both kinds of composing processes and that an exploratory process does not need to follow a descriptive process. We were

[7] The dance world is a vast and heterogeneous space filled with different kinds of dance that are entangled with different kinds of commitments. Not all dancers and choreographers position the choreographic process as a form of research, many choreographic practices are built on achieving platonic-like forms and replicating sameness. Some see dance more as a competitive sport than an expressive medium. Thus, it was important that Lauren shared similar dance-based sensibilities, commitments, and values with Rebecca & Curtis because just both having dance histories was not sufficient for engaging in this work together at the level of intimacy we achieved.

interested in how an exploratory compositional stance could support learners' computational understandings and expression in this medium by foregrounding the necessity of collective enactment as a form of discovery. In the next section we share how we came to see and understand youths' responses to participation in this activity and how we once again needed to think expansively across our disciplinary practices in order to do so.

Analyzing Compositions: Learning to see composing processes in video records of student participation as emergent choreographies

In the Soup Game we asked participants to explore computational concepts and expressivity through the development of group-based emergent choreography, and thus in our process of trying to understand how campers engaged in this activity we needed to treat all aspects of their participation as sites for creative work in search of emergent choreographies. In line with postmodern dance traditions in which any movements can be danced, we chose to treat our data (video records) as works of dance. This led us to approach analysis as a process of exploring the entailments of this perspective. We wanted to investigate the question, "If the data is a dance, then what does it mean to make sense of, analyze, or respond to it?" Drawing from practices of physical research meant that we could view the video record from the perspective of an audience lucky enough to witness the participants' choreographic work. We could interpret their actions as inquiry that produced emergent choreographies, and we could respond to their work by taking it as a prompt to develop new choreographies—responding as dancers to their movement proposals within our own processes of physical research.

Coming to understand the video record as dance created from a composing process based in physical research meant that we needed to find ways to attend to the sensitivities and relations

among participants within the emergent composition captured by our cameras. Exploring the entailments of the phrase "If this is a dance then..." also allowed us to make methodological decisions consequential to our analysis, and ground these decisions in pre-existing dance practices. We could think about how dancers and choreographers use video recordings of dances in their own practice as rationales for our own approaches to analyzing these video recordings. While we explored many dance-based practices for video analysis, the two we focus on involve treating video recordings as choreography by (1) analyzing them through a compositional lens and (2) re-setting them as movement scores to be danced by other performers (in particular, ourselves). Instead of treating the video record as a piece of traditional choreography in which audience members or analysts are the occasion and target for the performance, we used postmodern dance-based perspectives on the composing process to attend to the choices campers made when playing the Soup Game. And instead of treating this performance as a piece of predetermined, traditional choreography that could be re-set or replicated exactly, we treated participants' movements as proposals that could inspire new compositions as responses. In the rest of this section, we describe how we conceptualized the video record as a capture of our participants' processes of physical research and how we engaged with it by responding to it physically, in a new (second-order) process of physical research. We demonstrate how maintaining an exploratory, compositional stance in our collaborative analysis enabled us to identify new forms of computational practices and expressivity that centered on participant relationality. We also discuss how the dance-centered findings have interpretations and applications relevant to modeling practices as generative sites for interdisciplinarity.

If this is a dance then we can view dancers making bold compositional choices and see "flares" propagate through a group

By attempting to finish the sentence, "If this is a dance then..." we began to explore the entailments that came from analytically positioning students' movements and participation in the Soup Game as emergent choreography. This meant understanding the viewable record as both a composition and an emerging process of composing. In particular, we said, if this (piece of video recording as data) is a dance that is an emergent composing process of physical research, then we can view students' movement choices as proposals that others responded to with their own movement choices, creating movement ideas together. Physical proposals are a way of enacting movements in an inquiry-based way, asking with one's body, "What do you think of this idea?" Physical responses are a sign of respect: engaging with another's idea in a way that continues the inquiry and replying with one's body, "Your idea made me think of this. What do these new movements afford together?" We became very interested in what viewing students' participation through this lens afforded, in keeping with our developing sense of physical research, in which, "Framing any movement enacted as a proposal meant that movements were always thought of as something that could be responded to, ideas that others could and should build off of" (Vogelstein, 2021, p. 5). Taking a physical research perspective resonated with our commitments from traditions in Interaction Analysis (IA) (Hall & Stevens, 2015; Jordan & Henderson, 1995) that work to view "learners as sensible, capable, and act[ing] with intelligence and ingenuity" (Marin et al., 2019) through methods of micro-analysis and close viewings of interaction.

In our initial viewings of youth participation in the Soup Game, it was clear that how students chose to participate in this environment (e.g., running when everyone else walked) would be disciplined or controlled in many traditional learning environments. In this dance-based

environment and through the compositional lens we used to view their participation, students' actions were instead interpreted as purposeful, generative, and not needing to be controlled or reprimanded. For example, when everyone was enacting the movement rule "spread out while always moving" by walking around slowly, Keith chose to spread out while running, lifting his knees up, and pumping his arms up and down (Figure 20). This was a bold movement proposal that emphasized variability in the locomotor nature of different movements, as Keith's running contrasted with the walking surrounding him. Curtis noted during our analysis process that this proposal (and those of other students) was very similar to the practices of professional dancers, "it's like literally things that, choices that we would make in an improvisation" (Curtis, 7/22/20). Seeing his own artistic instincts reflected in students' participation underscored the importance of this reframing: the Soup Game was a context for learning where "disruptions" like Keith's running were welcomed and viewed as generative. In the Soup Game, high knee running or even spinning while spreading out in space were taken up by other students as compelling movement ideas and viewed in our interdisciplinary analysis as beautiful compositional choices akin to those of professional dancers.

The existence and generativity of students' bold movement proposals is noteworthy itself, as divergent physical actions in most learning environments are commonly interpreted as distracting, off topic, and needing to be disciplined (Lindberg et al., 2021). Activity structures to support classroom management in many learning environments define focus and attentiveness as still and quiet bodies because this is the dominant picture of cognition in individuals; moreover, in a classroom making noise and taking up space can detract from what others can see, hear, or contribute to a group. In traditional classroom group activity structures, one person needs to contribute at a time, which casts spontaneous and bold "bids" or proposals as rude interruptions.

Even from a traditional choreographic compositional stance, dancers choosing to deviate from the scripted movements are often reprimanded. Our view of the Soup Game as a composing process of physical research, however, foregrounded how this environment afforded multiple loud forms of simultaneous participation that were the basis for generative expression and learning. Viewing learners' participation as a process of composing in physical research led to a consequential reframe—interpreting disruptive movement choices not as distractions but as contributions that could generate useful ideas. Disruptions were seen as openings that could be responded to by others, in order to bring about new ideas and relations as movements spread throughout the group.

We refer to the spread of movement ideas as flares, which struck us for their beauty compositionally as well as their generativity for computational learning and expression. Flares entailed initial movement proposals that stood apart from others' interpretations as bold and noticeable and then propagated throughout the space as others responded and adapted these ideas. An example of two simultaneous flares is depicted in Figure 20. This particular sequence of events began with two important movement proposals shown in Figure 20A. While enacting the movement prompt for craziness (spread out while always moving), Keith (blue) responded by running with high knees, while Marissa (orange) made the bold proposal to spin. Both high-knee running and spinning contrasted with the predominant walking interpretation of the movement rule. As she continued her exploration, Marissa responded to Keith's proposal by trying on his high-knee running flare (Figure 20B-E). And a third student, Hunter (in white), picked up Marissa's spinning flare in his investigation (Figure 20B). Hunter sustained the spinning flare and continued to develop it into the next movement rule, which required jumping. His spin-and-jump movement is shown in Figure 20F.

These relations among the students had clear parallels for Curtis and Rebecca to the work of dancers in a composing process of physical research: making bold movement proposals and responding to others' proposals with new movements. New ideas and physical relations, such as a running duet and spinning jumps, developed when participants attended to each other's movements, choosing to enact and transform them in an emerging shared vocabulary of movement. As with a group of dancers, flares were common in the Soup Game, and they offered evidence of how students valued one another’s physical contributions by “trying on” and transforming their movements.

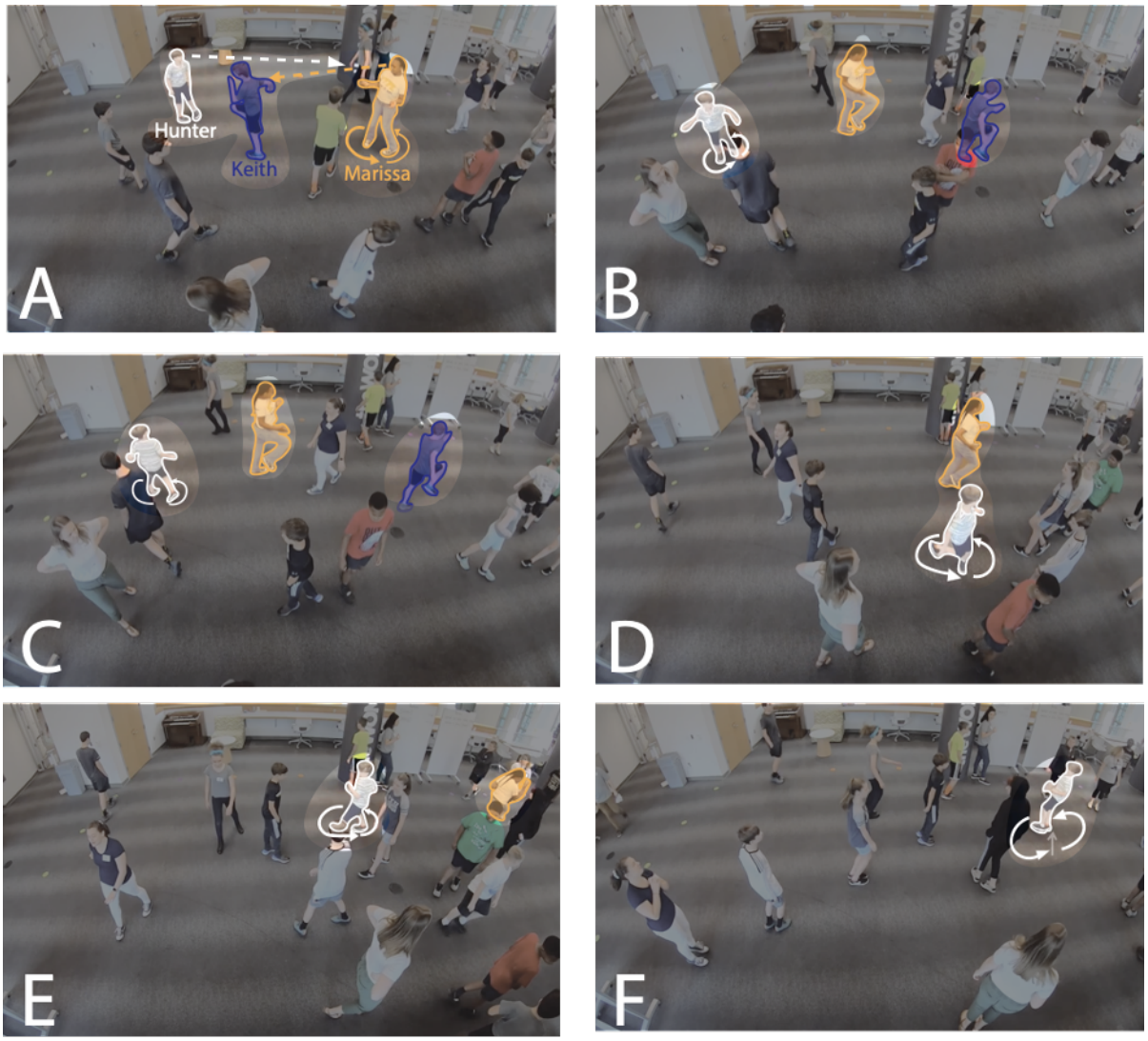

Figure 20. Flares propagated through the Soup Game when students saw others' movements as proposals (A) to be responded to (B) developed (C, D, & E) and then transformed (F).

Stretching and exploring a variety of interpretations of a rule to be enacted also aligns closely with practices in exploratory computational modeling. While our discussions around the design of the Soup Game emphasized the importance of agency in the development and naming of movement rules, the idea of a flare foregrounds generativity that can surface even after movement rules are "set," when they are enacted. Defining a movement rule is one opportunity to explore what might be and moments of collective enactment offer continued opportunities to

explore the limits of how to interpret it. In the Soup Game, the exploratory part of exploratory computational modeling happened during all phases. While enacting or running the computational rules, participants continually explored what was possible to create individually and collectively through their physical proposals and responses. Flares were an embodied form of appreciation and idea generation and stabilized as a participant structure over the course of the camp, supporting learners as they developed and enacted increasingly complex computational movement ideas.

Learning to see flares through interdisciplinary analysis processes

We viewed learners' participation in the Soup Game as consisting of sequences of movement choices that were purposeful and valid. Together these physical contributions created a cohesive whole composed of synchronous and contrasting components that produced patterns which we could observe. Engaging in joint traditions of video analysis from choreography and Interaction Analysis (IA) (Hall & Stevens, 2015; Jordan & Henderson, 1995) enabled us to attend to flares by attending to how patterning developed in the record. In this section we describe processes of viewing the video record as an emergent piece of choreography that allowed us to see the flares described above.

Once we appreciated that the students' contribution to the Soup Game was exceptional and followed interaction patterns more typical of dance than of classrooms, we recognized that our analytic approach would have to be responsive to that dimension of the students' work. Our analysis of video recordings from the Soup Game drew on traditions from choreographic composition and IA in order to better understand how disruptive forms of participation were generative for participants' collective expressive explorations. Bridging seemingly incommensurable video analysis traditions has shown to be generative in helping learning scientists understand a more complex and nuanced understanding of sensemaking in interaction.

In particular DeLiema and colleagues found that utilizing video analysis methods from different traditions in educational research, "led to a more expansive sense of what was relevant to the participants, opened up new and transparent considerations about data selection, and amounted to a novel conjecture about the focal construct" (DeLiema et al., 2021, p. 28). Our viewing practices that extended outside of educational research to choreographic research allowed us to see patterns made by small groups playing the Soup Game in which students tried out what others were doing so that bold movement ideas could be spread and iterated throughout the space.

Patterning is a key noticing tool from the discipline of dance that we utilized to identify the development of solos, duets, trios, and quartets within the Soup Game. By patterning we mean the qualities of movements that led to noticings of emergent groupings of bodies in space. For dancers, patterning affords noticings of how certain configurations of bodies in space can communicate different relations (Lerman, 2014). Thus, noticing and devising patterning inside of choreography is a form of communication in dance. In utilizing patterning in our analysis, this meant attuning to commonalities of how students used space, time, and physicality in the emergence of group formations and relations.

We attended to patterning as a way to identify what is referred to in IA as "hot spots" (Ma, 2016; Jordan & Henderson, 1995), focusing our close noticings on how solos, duets, trios, and quartets emerged and dispersed in enacting movement rules in the Soup Game. We did this in individual viewings and memoing as well as collective viewing sessions. When we came together for collective viewing sessions, we played the "stop game" a common form of collective viewing in IA in which a hot spot is viewed and anyone watching can call for the video to be stopped to re-watch a moment together in order "to ground assertions about what is happening on the tape in the material" (Jordan & Henderson, 1995).

In our collective viewing sessions, we chose to first view the video record without sound to focus our attention on patterning in the video record. Watching video without sound is fairly common in IA and disrupts practices that can easily privilege verbal utterances in the analysis of interaction. It is also a common dance-based practice used for similar purposes, when a choreographer or dancer wants to strictly focus on the physical forms being created by the moving bodies in the recording. Sometimes a choreographer will view video recordings without sound when preparing for a rehearsal or a dancer might view a video in silence when learning new choreography to focus on the physicality. When viewing the Soup Game in silence as an emergent piece of choreography, one of the first things that we noticed was Marissa & Keith's running duet, "the way they're running and that spreads between the two of them" (7/22/20). In this moment (Figure 20B & 20C) we attuned to how Marissa and Keith moved in unison together across the space and the spatial change that developed as they began to spread out while moving in the same direction.

Once a patterning (in this case an emergent duet) was noticed we then looked for the initiation of this formation which in most cases involved tracking lines of sight[8]. Similar to how Kendon (1990) identified facing formations by attending to how overlapping attentional spaces in interaction projected from the lower half of a person's body (o-space), we attended to where students were facing while enacting movements together. However, we primarily looked for lines of sight, instead of where feet were pointed, as evidence of observing a movement proposal before responding with one's own body (for example figure 20A). These lines of sight were found after

[8] There were other moments during the soup game when students would do the exact same movements in synchrony (e.g., curl up in a ball on the floor hugging one's knees and roll back and forth) where there is no evidence of a sight line that signifies one student saw another before enacting the same movement. These initiations of flares suggest some kind of felt experience in space akin to what Hollett and colleagues (2021) describe through their analysis of mobile architectures.

identifying similarities in students' movements to confirm the visibility of a proposal before a response was made.

Solomon and colleagues' (2022) used similar evidence in their analysis of high-school girls improvising the sensational essence of different periodic elements to show how movement genres (e.g., ballet, krumping, stepping) spread throughout a group. Lines of sight were used to identify the initiations of movement propagation throughout the shared space, which we call a flare. Unlike Kendon's maintained and negotiated shared o-space, however, students did not need to maintain a shared attentional space in the Soup Game for a flare to develop. These formations developed more akin to what Marin (2020) identified as tandem formations[9] in that people moving together did not necessarily face each other (e.g., they could be facing the same direction with one in front of the other such as Marissa & Keith in Figure 20B). Yet even without facing each other, movers can still create formations and energy between them that impact the shared environment and compositional whole (Hollett et al., 2021)[10]. Thus, in using a compositional lens for close viewings of video recordings we were able to identify the development of emergent groupings by attending to how movement ideas spread throughout the space.

The development and accumulation of our noticings began by attending to how the running duet stood out from the dominant form of walking in the group's enactment. When we went backwards in time to trace lines of sight as initiating the development of this duet we found another

[9] Marin first identified tandem formations in her dissertation (2013) in which she also described variations of tandem formations for groups larger than two people. In her most recent articulation of tandem formations (Marin, 2020) she only describes tandem formations as duets, for two people walking and not facing each other.

[10] It is interesting to note that there are histories and connections to various dance practices across the authors cited in this section, however, none connect their noticings and analysis to dance-based ways of viewing video recordings. Solomon and colleagues (in press) have experience as dancers and dance educators, working with dance educators as facilitators of their design work. Marin (2020) "grew up in a family of artists, dancers, and musicians. I spent many hours observing my family members observe others as they learned choreography, engaged in musical improvisation, and performed in front of audiences" (p. 287). And lastly Land (Hollett et al., 2021) spent decades as the mother of ballet conservatory students immersed herself in their world of dance. While these histories are mentioned in their papers, how they impacted their analysis processes is yet to be uncovered although mentioning these histories and identities seems important to how they understand their work.

duet, a spinning duet that Curtis pointed out showed how "for a moment they [Marissa & Hunter spinning] are like the two anchoring things" in the group composition. Although only briefly, this spinning duet stood out compositionally as an anchor or counterpoint to the composition as a whole. This relationship between individuals' movement proposals and their development into small group relations as duets stood in direct conversation with noticings at the group and collective level. This compositional viewing allowed us to attend to how the group created a consequential context for the flare to exist. The contrast in movement proposals between an individual and the group helped the idea of spinning, for example, become noticeable through this contrast, both in the enactment of the game and in our analysis of it. This noticeableness of a bold movement proposal, the contrast of individual from group, then became something for participants to benefit from as it generated new responses and allowed for movements to develop in conversation with how the movement rule at hand was being enacted.

Incorporating the proposal and response relations inherent in physical research as a choreographic process of inquiry allowed the Soup Game to be taken up as an activity that changed how students saw each other. Students saw each other's bold movement proposals as ideas to be taken up and tried out themselves. This is a distinct way of seeing peers and their contributions as a way of demonstrating respect for new ideas. As Hunter illustrated with his spinning jumps, physically observing others and enacting their ideas afforded new noticings and movement ideas. As analysts, take this to mean that flares offer one form of evidence of creative group-level thinking because of the responsive and collective nature of their compositional organization. The compositional possibilities explored through the emergence of flares also act as expressive ways to stretch the meanings of movement rules. Our analysis of flares focused on students' participation in the Soup Game as a collective and expressive endeavor. In the next section, we demonstrate

how we engaged in a form of re-setting and reenactment that allowed us to make noticings in a video record as a collective and expressive endeavor ourselves.

If this is a dance then we can view participants' movements as proposals and we can respond to them as dancers by re-setting their movements on ourselves

A second response to finishing the sentence, "If this is a dance, then..." entailed responding physically by composing a newly related composition ourselves through a process dancers and choreographers refer to as re-setting. One way that dancers better understand the internal workings of a piece of choreography is by enacting or performing it themselves. Dances are adapted through re-setting processes, as the work is re-set on new dancers in a new context. Thus, when we took our data to be a video record of a dance, that meant we could re-set it on ourselves as dancers in response to the original record.

Re-setting is a dance practice that frequently relies on video recordings of performances in order to perform a piece again. Sometimes a dance is re-set on a new cast and sometimes after a long time away from a work a piece is re-set on the same dancers. In general, re-setting practices use video as an extrinsic perspective on a dance that is coupled with dancers' enactments in the process of re-setting as an intrinsic perspective. Switching back and forth between these two perspectives balances valuing the record as the choreographic object and creating space to meet the needs of new dancers. Traditional processes of re-setting are linked to traditional practices of choreographic composing and are tethered to video records with the goal of creating as exact a replication as possible (e.g., to preserve a legacy such as the heavily protected Balanchine trust). In traditional re-setting practices dancers' intrinsic perspectives are utilized only in service of recreating certain compositional elements (e.g., a new dancer needing to taking larger steps to

move across the stage if the piece was originally set on a taller dancer) and often adjustments are positioned as failings on the new dancers' part.

Although some ways of re-setting are in line with traditional choreographic practices, others are more responsive to dancers' agency and emergent ideas such as the practices in physical research. For Rebecca & Curtis, re-setting is a practice that is more of a process of reinterpretation, in which video is positioned as a series of proposals to respond to and these responses allow performers to attend to what can be created anew in relation to what arose before as the people and context for the work evolves (Vogelstein & Hall, 2020). This is a disruptive and generative form of re-setting as part of physical research. By attending to the opportunities in the present and emphasizing what can be newly formed, changes in dancers and contexts over time can be reframed as opportunities rather than shortcomings.

The analysis practices that placed ourselves moving in relation to the video record and each other allowed us to not only see, but also to feel how formations made by small groups playing the Soup Game were developed, maintained, and transformed. The insight gained through re-setting helped us attend to how the individuals in these small groups developed reciprocity towards each other. We refer to these close relations enacted physically as choreographies of care; how coordinated movements in response to each other can support and enact caring relations. These relations struck us for their intimate compositions as well as the new forms of collective participation they supported in this learning environment.

We understand care to be an interactional accomplishment that makes others' acts not only possible, but more comfortable (Spruill et al., 2021). These interactions commonly involve affective gestures and the maintenance of close proximity (Goodwin & Cekaite, 2018; Spruill et al., 2021). In this analysis we operationalize care as an embodied stance:

> Bodies work with other bodies to produce stance, or alignment to the current activity, in human action - in proposals about how to engage with the other and in responses to such proposals. Stances are intrinsically multiparty, metamodal intercorporeal displays - actions that entail multiple concurrent entanglements of bodies (Goodwin & Cekaite, 2018, p. 123).

Goodwin & Cekatie's analyses drew on intimate family relations, demonstrating care routinely embodied in the home. Our analysis looks at embodiments of care at an art and coding camp in which youth engaged in interdisciplinary choreographic and computational activities. To demonstrate how choreographies of care were enacted in the Soup Game, we show how a group of four boys collectively aligned to the activity by volunteering each other to contribute through the maintenance of close proximity and affective pushes and pulls. In attending to the boys' choreographic proposals and responses, we demonstrate how they supported each other as individual contributors whose collective participation was recognized and supported by the teacher. We find this significant because it provides evidence that (embodied forms of) social caring can open epistemic agency in learning environments (Appleby et al., 2021).

In this particular episode the boys amplified each other's ideas for the larger group by volunteering each other on two consecutive occasions as they contributed to the definition of a new movement rule in the Soup Game. Spatially, the group participated from within a clump configuration. They faced inwards towards each other, close to the teacher, but behind her field of vision (Figures 21A-C). In this inward-facing quartet their formation ebbed and flowed, maintaining a tight shared space as each of the boys migrated closer to another and then backed up a bit, gently poking and jabbing at each other (Figure 21A, B, G, & L). As with the bold movement proposals analyzed in the prior section, this small group behavior might be seen as problematic in a traditional classroom space. Specifically, these physical forms of participation, standing behind the teacher, facing one's peers instead of the teacher, and even gently pushing and

shoving each other would traditionally be seen as disruptive forms of off task behavior in many learning environments. Instead, we came to see how these boys' activity enacted caring relations towards each other. As remarkable, the teacher was able to pick up on the boys' closeness, working with their expressive formation to engage them as a group in the task of generating a new movement rule for the Soup Game.

This short episode began with the boys' physical proposal in a clump behind the teacher (Figure 21A-C) and she responded to the boys' spatial proposal by turning to face them and asking for "an action or a button" (Figure 21D). Justin responded to this proposal by subtly pointing to Evan, which the teacher picked up on, as she then called on Evan (Figure 21E). She responded to the playfulness she observed by offering Evan a new proposal, "Justin volunteered you. You can name it after him if you want" (Figure 21F). We interpreted the teacher's offering as giving Evan a simple and fun way to participate that responded to the playful dynamic within this group. At this stage of the activity, when a student was called on, they could either contribute a movement rule or the name of a movement rule ("An action or a button"), and naming was a significantly easier way to contribute. Inviting Evan to participate in this way by naming an action after Justin invited him to participate in a fairly easy way in relation to the quartet (as well as inviting him to playfully retaliate against Justin for calling him out). Instead of supplying his friend's name as a light hearted response, however, Evan offered a movement rule as a more serious contribution (Figure 21F). While playfulness, especially in a group of boys of color, might be interpreted as not wanting to engage sincerely in an activity, this did not stop the teacher from inviting their participation and then Evan rejected the teacher's proposal for an easier pathway to participation. He responded confidently with a new movement action. Justin's small gesture in response to the

teacher led to an amplification of a supportive group ready to meaningfully contribute to the activity.

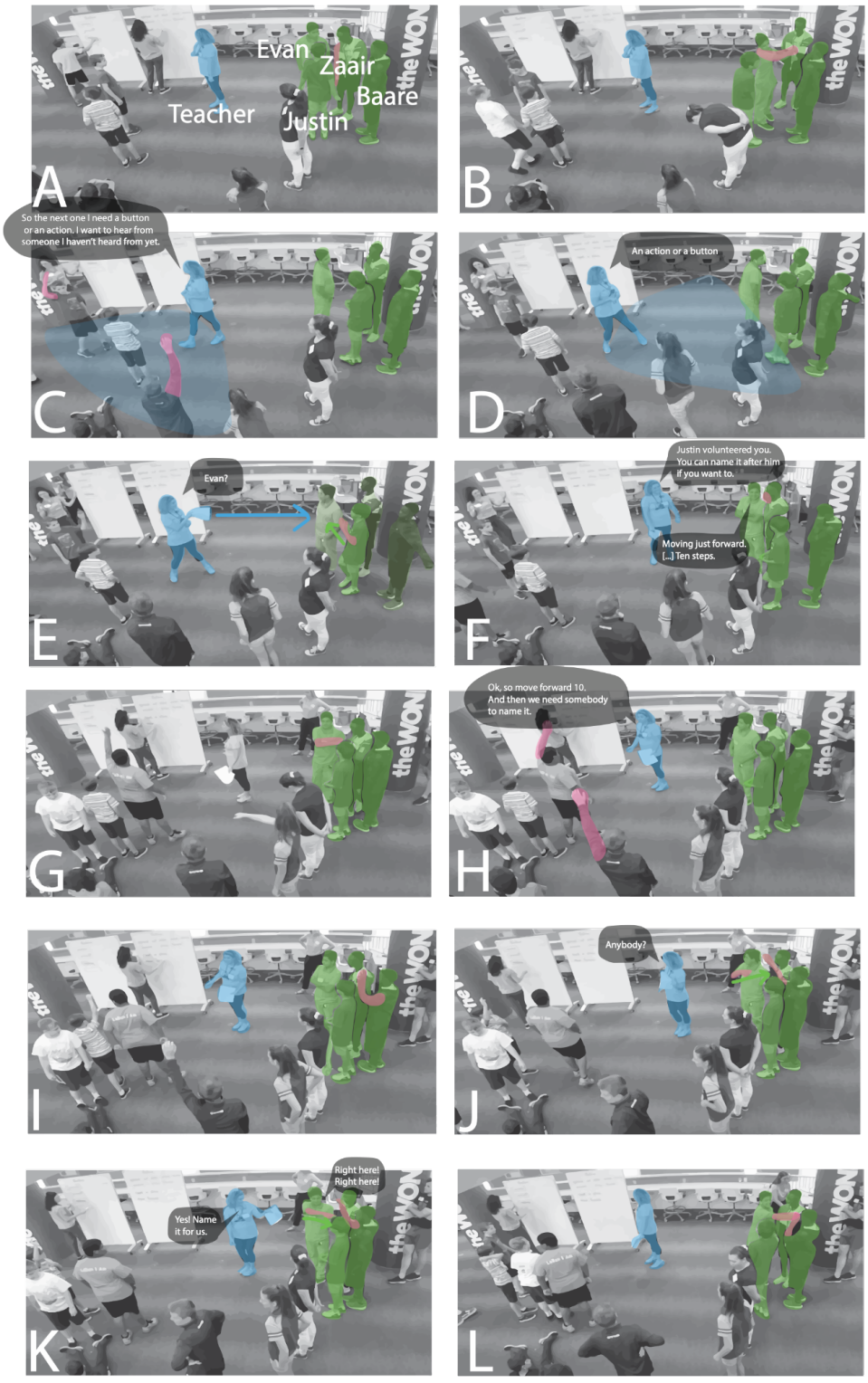

Figure 21. Choreographies of care supported two moments of encouragement and contribution from a group of four boys, generating a movement rule (A-G) and naming it (H-L)

After being pushed himself to contribute to a movement rule, Evan reciprocated and helped Baare garner the teacher's attention to name the movement he had contributed. When the teacher asked for someone to name Evan's movement, Baare raised his hand (Figure 21I), but the teacher was facing away from the boys and did not see this bid to contribute. At this moment, Baare's attempted action was shut down, it was not possible. Evan responded to Baare's proposal by pointing to Baare and calling for the teacher's attention, "Right here! Right here!" (Figure 21K). When she heard Evan, the teacher turned around and proposed that Barre name the movement, "Yes! Name it for us" (Figure 21K). Now Baare's action was possible and more easily achieved as a result of Evan's caring intervention. This sequence of amplification through volunteering exemplified how care was generously passed from one boy in this group to the next (from Justin to Evan and then from Evan to Baare). This sequence attuned our noticings to how care and trust in these boys' relationships was physicalized in these moments (e.g., playfully pushing each other). Most importantly, this analysis was the impetus for defining the idea of choreographies of care, showcasing how care is an embodied, coordinated, and relational action.

As described in this section, choreographies of care demonstrated one of the affordances of allowing learners to leverage their full bodies in collective forms of participation as this enabled them to further develop their relations with each other as a quartet that could participate within the larger group. The ties between these boys' physical participation showed the careful coordination of caring interactions that enabled new forms of participation. These boys simultaneously remained closely aware of each other, while responding to the larger group, and caring for each other's contributions physically and verbally.

Additionally, in leading the Soup Game, the teacher engaged this group of four students as a quartet, by building upon the subtle proposals the group made (so subtle that our initial analyses

missed them). On inviting participation from "someone new," the teacher pivoted her body toward the huddle of boys (Figure 21C-D). Shifting the definition of a participant from an individual to a group, she directed an invitation to that quartet (Figure 21D), and took up a small gesture Justin made toward Evan (Figure 21E) as a nomination of him as a volunteer and as a representative of the group. As she addressed the boys, their quartet tightened (Figure 21F-G), enabling her to maintain her attention on them as a unit. This supported them in adding to the activity and also deepened the ties between them, which lasted throughout the camp.

In a way, the teacher was coming to understand the full group as a complex system and in doing so started to see how small groups within that larger system could participate at the level of the small group and not the individual. Analytically we came to view the small group of boys within the larger group of students as a mid-level configuration (Levy & Wilensky, 2008), a small group of individuals used to reason about complex systems. Attending to mid-level relational configurations in this way shows promise that mid-level thinking can be generated for how groups of individuals learn together and not just how individuals reason about complex phenomena on their own. Pedagogically we conjecture that the teacher's ability to see this group as a mid-level entity meant that she was able to see interest even when students' visible physicality might not be read on the surface as attentive to the activity at hand.

Learning to sense choreographies of care by composing new relations ourselves

We viewed learners' participation in the Soup Game as emergent choreography that could be re-set on ourselves as dancers, serving as a series of proposals for us to respond to and feel through the creation of new compositional relations amongst ourselves. We engaged in joint traditions of video-based reenactment analysis from choreography and IA (Erickson, 2004; Vogelstein et al., 2019), which enabled us to see choreographies of care in our collaborative

analysis of the Soup Game. In this section we describe processes of responding to the video record physically. By re-setting participants' choreography on ourselves we came to feel and see the choreographies of care as described above.

We chose to re-set the observable choreography in our data as a process of re-setting aligned with methods from physical research and found important connections with practices in IA. Developments in methods of reenactment in IA have pushed analysts to consider what is newly felt and noticeable when analysts put themselves in similar configurations as a generative response to observable, recorded interactions. Erickson's (2004) work on choral transcript readings encouraged analysts to engage in an oral form of reenactment that foregrounds the rhythmic coordination of talk in interaction. Physical uses of reenactment as a method in IA have foregrounded the coordination of ensemble activity for analysts in a way "that our analysis of ensemble learning...might also benefit from ensemble activity" (Vogelstein et al., 2019, p. 345). Physical-based methods of IA reenactment have also shown to generate new configurations, acting as processes of composing that can create related, but new compositions. These new compositions have helped analysts gain insight into other related and possible interactional achievements (Marin et al., 2020, Vogelstein et al., 2019). The forms of reenactment in IA necessitate the enactment of groups as a form of collective sensemaking about groups, foregrounding what a collective can notice together.

In particular, the concept of a choreography of care came from focusing on the episode in which a group of four boys of color engaged in developing a movement rule for the Soup Game (Figure 21). Curtis was the first one to identify these boys as a hot spot in the data because of how he resonated with what he saw in the record:

> And there is also something that is striking about them all being people of color gathering together like that, that's really noticeable

> to me. And the fact that there is some like confidence that is gleaned from the physical, like you know, from the physical space [...] And I feel that like, you know, in improvisational practice, like when I'm improvising with a group of people over a course of time there are people that I strangely like gravitate towards always. [...] When I first brought this moment up I was flabbergasted, awestruck, and had like watched it 15 times [...] and it still warms my heart (Curtis, 9/29/20).

The episode struck Curtis for many reasons. Compositionally, seeing a quartet clumped together was striking and personally, as a man of color himself seeing that this quartet consisted of four boys of color stood out to him, "that's really noticeable to me." The proximity and physical movements between these boys also led Curtis to conjecture that the boys gained confidence from these interactions, which he related to his own inclination to improvise near and with certain dancers in his own practice. Finally, his emotional response, "flabbergasted, awestruck, [...] warms my heart," helped sustain this attachment through many viewings by himself and eventually sharing it with our collective. Curtis's personal resonance with this hot spot was one of the motivations to get us up and moving with and in relation to the boys in the quartet.

Before responding to the video record physically, our initial viewings left us with questions about when and how these boys volunteered each other to contribute to the activity. We chose to try and answer these questions by moving in response to the video record. As a quartet of analysts responding to a quartet of participants, we assigned each member of our team to one of the four boys we were focusing on. Rebecca suggested that we first focus on responding to participants' upper body movements in order to begin to attune to moving with each other:

> Another thought that I had, which is kind of an abstracted version would be if we all like stay in the frame whether you're seated like the way we are now and we used the gesture, like we're reenacting the gesture of the person just in relationship to the [video], without worrying about the angles of our bodies or anything. That way we

> all enact gesture together towards the screen to see what that feels like (Rebecca, 9/29/20).

Rebecca's choreographic proposal was to create a new quartet amongst ourselves in relation to the quartet we were responding to in the video record. Using Rebecca's proposal for a movement score we moved in tandem with our respective participants, responding to their gestures by amplifying them in our own bodies as we also moved in coordination with each other as a new quartet (Figure 22). Moving in a new spatial relation (mediated by the grid pattern in Zoom) with newly enacted, similar, gestures, allowed us to "lean into the distortion [that] magnifies the synchronies and asynchronies" (Curtis, 9/29/20) from this new compositional arrangement. The choreographic move to recompose our bodies with respect to each other and the video record allowed for new relational noticings when moving together and when viewing our movements as a choreography relative to participants in the video record.

We found that moving with each other in tandem with the quartet helped us feel and then see how these boys coordinated movements in sequence to achieve their contributions as a group. For example, Justin's pointing gesture to Evan was small, subtle, and not obvious from the angle of the camera that captured this interaction. In fact, upon our first series of viewings of this moment we did not understand how Justin volunteered Evan or how the teacher knew he had done so. It was not until we tasked ourselves with physically responding to these boys' movements in synchrony with their video recording, that Curtis's responses to Justin and Lauren's responses to Evan made this pointing gesture noticeable. Lauren and Curtis's new movements in a new spatial arrangement helped us feel and see how Justin pointed to Evan who quickly responded by playfully pushing Justin's hand away (Figure 22A). This small, playful moment only became clear when we enacted these movements together, generating new noticings through our own process of composing through physical research.

Figure 22. Reconstructed images[11] (also depicted in Figure 21E & K respectively) from our physical research using colors to highlight the one-to-one movement relations and compositional relations across the participants in the data and our own enacted gestures following their movements (Lauren & David in red, Corey & Zaair in orange, Rebecca & Baare in green, and Curtis & Justin in yellow.)

[11] In comparison to the recording we were responding to (Figure 21) the referent video has been enlarged to see the quartet better. In addition, corresponding colors and line segments were used to show how movements were performed in synchrony with the participants in the video and the quartet formation was recomposed in the Zoom grid. It becomes clear, for example, in the bottom image that on the left David pointed to Baare and on the right (because of the mirroring effect on Zoom) Lauren pointed away from Rebecca.

Responding to the video record with our own proposals through a process of re-setting allowed us to notice new relations in the record through creating new relations amongst ourselves as analysts. For example, it was clear from watching the record that Evan pointed to Baare to get the teacher's attention (Figure 4K). However, in responding to this moment together physically, Lauren & Rebecca in relation to Evan & Baare could feel and newly see how Evan pointed to Baare at the same moment Baare raised his hand. We felt ourselves attuning to each other's movement in that moment, moving in synchrony, and could see in our amplified gestures how our hands moved at the same time (Figure 5B). This helped us get a more nuanced sense of how closely attuned these boys were to each other in their movements. As a process of physical research, we did not physically respond to the video record in order to create a performance for some new, external audience; we were engaging in this process to notice how it felt ourselves to be moving in response to the record and each other. We used this process to leverage our own bodies moving in relation to each other to amplify the sensitivities between participants as enacted in the record.

Responding to recordings of the Soup Game through a process of re-setting a quartet's choreography onto ourselves as a group foregrounded how these boys responded to each other's movements as a way to sustain their close relations as a group. We needed the physical and spatial amplification of their movements by re-setting them in order to better sense how attuned they were to each other. Responding to others' proposals with care such that someone else's next proposal is more comfortable and easier to enact is at the heart of physical research composing practices. When Justin subtly pointed to Evan or Evan told the teacher to call on Baare, these boys lowered barriers to contributing to the generation of a movement rule in the Soup Game. Contributions became playful interactions amongst the group that were then shared out to everyone else. As designers and analysts this was striking because of how it changed our understanding of who could

participate in this game, small groups could do what we had imagined individuals would do. For the boys, their small group became a space for enacting and receiving care. And for us as analysts and the teacher pedagogically, smaller groups became entities to reason with and about. Foregrounding the choreographic nature of students' participation in the Soup Game highlighted the multiple forms of collective reasoning this design supported.

Discussion & Conclusion

Foregrounding composing as a distributed process to create emergent compositions highlighted innovative practices in both dance and modeling, and indicated potential connections between these practices that fueled design and analysis. Conceptualizations of composing and composition across disciplines surfaced at four levels: (1) in the practices of dance and modeling, (2) in the design of the camp, (3) in the students' enactment of camp activities, and (4) in the analysis of these enactments. Our collaborative design experience led us to recognize the multiple, generative connections across disciplinary practices, which fueled our design work. The strength of this design work and the exciting ways youth engaged in these activities also led us to work as an interdisciplinary team in the analysis process, highlighting the necessity of conducting interdisciplinary analyses. Following the connections from our analytic work led to this paper, writing through how innovations in design are linked to methods of analysis. In this discussion we reflect on the story of our interdisciplinary collaboration across computational modeling in the learning sciences and choreographic methods of inquiry in dance to discuss how it can help us better understand how interdisciplinarity can (1) highlight generative cross-disciplinary concepts (e.g. composing and composition), (2) necessitate innovations linked across design and analysis

practices, and (3) turn our attention toward the importance of developing of new and trusting partnerships across disciplines.

Composing and composition

The initial findings from our interdisciplinary analysis show that youth took up forms of participation that were meaningful in and across dance-based choreographic practices and learning sciences-based computational modeling practices. As described in the previous section, we attended to flares and choreographies of care as noteworthy forms of participation in the Soup Game that emphasized the emergence of small group relations in the context of larger collective explorations. These are two examples of how youth engaged in the Soup Game in ways that reflected discipline specific ideas and participation structures across our two disciplines (choreography and computational modeling). Together our findings suggest that interdisciplinary work does not have to situate one discipline in service of another, this work can allow for forms of participation that are relevant in multiple disciplines simultaneously.

Flares propagated throughout the Soup Game as youth made movement proposals that were responded to by others so that movement ideas were enacted and transformed in new ways. This form of participation resonated with practices in physical research and exploratory computational modeling. One of the key practices in composing processes of physical research is dancers making proposals through movements and responding to others' proposals with new movements. So, when Keith started to run while spreading out and always moving and Marissa responded by following him to form a running duet across the space (Figure 3A-D), Keith made a physical proposal and Marissa responded in kind. Together they formed a running duet in which they traveled together across the room. As Curtis recognized, bold movement proposals and

responses like running when everyone else chose to walk are “choices that we would make in an improvisation” (Curtis, 7/22/20).

At the same time, this enactment of running while spreading out and always moving engaged a fundamental part of exploratory computational modeling. Changing one variable in a computational rule (in this case the speed at which one spread out while moving) as a way of exploring what a rule can express is fundamental to exploratory modeling. Starting to attune to compositional elements that can be manipulated, such as the speed and direction one moves in, can support the development of new rules and compositions from an exploratory stance. The concept of a flare highlights how building off of what can visibly be created to develop new visual arrangements is an important part of both physical research and exploratory modeling.

In processes of exploratory computational modeling, however, changes to compositions are made after viewing a composition and not in a composition itself. A modeler will run the code for a model, see what it creates, and then make changes to the computational rules that govern its creation. In the case of a flare, participant agency is foregrounded such that changes can be made and tried out while a model is running, as participants enact computational rules. This has pushed us to reconsider how a computational modeler might think about agent agency in exploratory modeling processes, viewing agents as more agentive than digital agents are traditionally viewed. So, while a flare is grounded in generative disciplinary practices in both choreography and computational modeling, looking at these practices in conversation with each other can also lead to new ideas about the original practices themselves.

The enactment of choreographies of care also demonstrated connections to disciplinary practices in dance and modeling. While flares demonstrate how movement elements can be generated in composing processes of physical research, choreographies of care illuminate how

attuning to others' movements can support caring relations between participants. Moving in relation to certain members of a larger collective is common when engaging in physical research as Curtis described, "when I'm improvising with a group of people over a course of time there are people that I strangely like gravitate towards always" (Curtis, 9/29/20). Thus, the close group formation of the boys enacting choreographies of care speaks to similar sensitivities amongst dancers themselves. As demonstrated in our analysis, this close proximity and playful movements supported forms of participation as a group, not as individuals, such as Evan and Baare jointly grabbing the teachers' attention verbally ("Right here! Right here! Figure 4K) and visibly (pointing and hand raise Figure 4K). Developing sustained relations in a small group, allows dancers in practices of physical research to move as one, anticipating each other's movement proposals and in the Soup Game this meant participants could jointly support collective contributions to the group.

In computational modeling, identifying mid-level arrangements and sometimes even enacting them is an important part of reasoning about how aggregate patterns emerge from agents' interactions (Levy & Wilensky, 2008). As a quartet, the group of boys acted as a mid-level entity, a smaller group within a larger group, that was recognized by the teacher and able to participate in the Soup Game. We conjecture that being able to see and reason about mid-level groupings can be generative pedagogically, as this kind of sensemaking enabled the teacher leading the Soup Game to engage with these boys as a group able to contribute to the larger group together. Additionally, attending to and nurturing these small groups can be one way to support caring relations among learners who have demonstrated how care is co-constructed in such intimate ways.

The descriptions of flares and choreographies of care as initial findings from our interdisciplinary design research demonstrate that emergent and contingent formations can be

worthy of disciplinary sophistication, as these episodes showed powerful relations to practices in both modeling and dance. We initially convened as an interdisciplinary design team with a cursory idea of how particles from physical research might be related to practices in computational modeling and unsure if newcomers to dance would even be able to engage in practices from physical research. Youth participation in the Soup Game suggests that it is possible for youth coming to these disciplinary spaces to engage in activities in multiple ways that show the seeds for two disciplinary practices at once. In traditional modeling and dance learning environments, learners are asked to first acquire component skills before any exploratory work can be understood as disciplinary relevant (e.g., you have to master certain dance moves before you can choreograph or you have to be able to create certain aggregate phenomena before you can begin an exploratory model). The Soup Game as an activity, however, suggests that learners' social tendencies can produce the seeds of practices in multiple disciplines (e.g., dance and modeling). This shift towards cultivating practices rather than skills can accommodate seeing the resources that learners bring socially to an activity as being the beginnings to develop interdisciplinary practices.

Youth already have the capacity and willingness to develop caring relations with their peers and community and "when a learning community collectively asks what is being cared for and explicitly engages youths' caring practices, youth's identities, experiences, cultures, and histories become assets" (Spruill et al., 2021, p. 273). We have observed in our data that this intentional practice of seeing and allowing oneself to be seen in collective embodiment can be a practice of care and respect. We hope to continue understanding how this important pedagogical practice can support students in empowering each other through a shared embodied understanding of "my learning is your learning is our learning."

Design and analysis

Supporting the new forms of participation described in our findings (flares and choreographies of care) and understanding them analytically meant that changes were made in both our design and analysis practices. Our design process foregrounded engaging in and supporting inquiry practices from both physical research and exploratory computational modeling by spending time leading each other in our disciplinary practices. What we could create by enacting movement rules together and watching computational agents follow similar movement rules helped us see how our processes for composing and making sense of compositions across disciplines were related. More specifically, the sub-communities within our respective disciplines that we associated with (e.g., practitioners of physical research as distinct from traditional choreographic processes) shared similar values that guided our inquiry. And our choice to associate with these sub-communities was motivated by shared desires to support humanistic and dignity affirming spaces for inquiry. Both of these practices necessitated collective enactments in order to discover what can be created compositionally. We focused on the exploratory nature of these composing processes in our design of the Soup Game. The generativity in collective enactment was linked to participant spontaneity and ingenuity in the development and enactment of movement rules.

Designing to support new forms of participation meant that we also needed new methods in order to see what these new forms of participation entailed. We were only able to see and conceptualize flares and choreographies of care as generative forms of participation because of the interdisciplinary shifts in our analysis practices. Since the Soup Game was designed to have participants explore computational expressivity through the creation of emergent choreographic compositions, we needed a way to view their participation as emergent compositions. Thus, we

proposed treating the recordings of the Soup Game as emergent choreographies. If data is viewed as a dance then there are discipline specific ways we can make sense of and respond to it. We leveraged tools from choreographic practices and IA as forms of close micro-interactional analysis to engage with viewing the record as a work of dance. Utilizing these multi-disciplinary analysis practices enabled us to attend to how emergent groupings developed as youth made bold movement proposals and responded to them. We also responded to the recording as a prompt for our own physical research, moving in relation to participants in the record and ourselves as a new emergent composition. In our new composition, new relations and movement coordinations were felt and seen in ways that allowed us to understand how caring relations supported collective participation in the Soup Game. Interdisciplinary collaborations across phases of design research can be consequential because if you change how you design learning environments, what you are designing for shifts, and then how you are able to see and make sense of these learning spaces must adjust as well.

Partnership

The consequential shifts in our interdisciplinary design and analysis processes were consequentially supported by the growing partnership we nurtured. We consider the companionship in our collaboration to be an axiological innovation. Our practices would not have shifted in the ways they did without the relations cultivated in this partnership. The long-term relationship that we developed, beginning in ethnographic understandings of our design practices, laid the groundwork to engage in such a meaningful collaboration. The collaboration we tended to was one that we sustained through the design and analysis phases of our design research and beyond, becoming both an important value in and outcome from our work.

By identifying our partnership as a valuable part of our collaboration we were able to engage in research that not only engaged our distinct disciplinary practices, but pushed how we understood our areas of expertise. We established a shared enterprise through our design research that was reciprocal personally and disciplinarily, in that one group of people or discipline was not positioned as serving another. Through the lens of composition as part of our design and analysis process Lauren & Corey came to see how exploratory computational modeling can lay a generative foundation for newcomers to computational modeling. We found that by amplifying agent agency through collective embodiments, a seemingly contradictory idea in agent-based modeling, we can support computational expressivity for learners. And Rebecca and Curtis came to see the pedagogical power of their physical research practices in a new light. Interpreting interactions from choreographic-based compositional perspectives foregrounded how care and respect can be co-constructed through simple proposals and responses.

In order to support this partnership, we had to navigate challenges to support it financially. The economics of this work are crucial. We work from a commitment that choreographic ways of knowing are valid and rigorous forms of research and thus choreographers should be paid for their work as researchers. While this might seem obvious and unnecessary to express explicitly, we believe it is extremely important to raise because too often artists' work is positioned as culturally ornamental and artists are expected to accept payment in "opportunities" and "exposure." Logistically we supported this long-term collaboration through graduate fellowship opportunities Lauren procured and a significant portion of Corey's startup funds. Additionally, supporting long term partnerships across disciplines also means navigating different timelines for processes of composing. The lifecycle of projects for many dancers operates on shorter timescales than the lifecycle of academic research projects. This meant that even with financial resources to support

our interdisciplinary collaboration, we had to work around our dance collaborators' frequently changing schedules and availability because this work was secondary to their financial stability. When working with collaborators across disciplines it is important to take into account that it can be challenging to find convergence in different expectations and work commitments entrenched in different disciplines.

Developing and sustaining caring relations amongst ourselves required renegotiating trust at every new phase of our collaboration, even when we felt close and comfortable in the relationships we had developed. For example, in the design phase Corey & Lauren invited Rebecca & Curtis to join their design research as collaborators because of their choreographic expertise. Yet it was Rebecca & Curtis's pedagogical expertise that ultimately impacted the design of the Soup Game the most. We take this to be evidence that we were deepening trust as more facets of ourselves became recognized and used in our collaboration. Even though the design phase of our collaboration opened up space for us to engage in each other's design practices, returning to those practices in our analytic collaboration did not always happen right away. For example, on the day in which we re-set the choreographies of care observed in the data, Rebecca made three bids for us to start moving over the course of half an hour before we actually started re-setting the work. We find this noteworthy because even though at that time we had developed close and supportive relationships, we did not always see or pick up on each other's proposals right away to disrupt sedentary methods of collaboration.

Trust and care were constantly renegotiated through these vulnerable processes of learning. As Appleby and colleagues (2021) demonstrate in their analysis of social caring in a cohort of physics learners, "Ideas can seem outrageous, not worth serious attention and effort to grasp, but caring and respect for the people who are thinking those ideas might contribute to our stability in

productive inquiry" (p. 14). Being bold enough to question methods of inquiry that were well established in our fields to explore new and developing ways of seeing and doing research can be inspiring and also intimidating. For example, proposing to view a piece of educational research data as a piece of improvisational choreography could be easily shot down or it could feel awkward doing in a non-dance setting. However, the trust and generosity we worked to continually renegotiate and develop supported our ability to make bold proposals ourselves and (eventually) respond in generative ways.

Ultimately, we found a lot of self-similarity in the kinds of learning we desired to design for and the kinds of learning we supported within our collaboration. While we aimed to support learners to experience how collective embodiments can allow for new understandings of computational expressivity, we also attended to how our own design and analysis processes needed to come from our own collective choreographic explorations. While we attended to how learners developed reciprocity through making bold movement proposals and responding to them in kind, our collaboration also stemmed from making and responding to bold proposals in a caring fashion. We turned our respect for the power of collective expressive embodiment, the phenomenon of interest, inwards to the idea of partnership. Through the case of our collaboration, we demonstrate that design research without deep partnership has limits to what it can create, do, and see. There is a need for methodological innovation and we argue that innovation needs to be accompanied by a shift in values that are core to the foundations of design research as an interdisciplinary enterprise. Thus, if we change the nature of our interdisciplinary partnerships, we are also altering the nature of it at its core (an axiological innovation).

Developing new understandings of expansive design practices and their analytic entailments through interdisciplinary collaboration has much to offer scholarship that utilizes

methods of design research. While design research scholars have pushed the field to expand our conceptualizations of design, our case study demonstrates how changes in design practices necessitate changes in analysis practices, emphasizing how these cycles of design research are inextricably linked and supported by the relations cultivated through these practices. Through the work, we experienced what it meant to value the beauty in choreographic composition as both process and product in design research. This work foregrounds what Bang & Vossoughi (2016) characterize as important to PDR, designing to support relations between learners for their own sake, while also attending to how this changes how learning happens and what can be learned. Our story demonstrates how attending to processes of composing and emergent compositions in both the design and analysis phases of our collaboration helped us leverage connections across disciplinary practices and push boundaries towards supporting learning environments (both in our design process and the result of our design process, for youth) where new forms of participation and relationality can be valued.

References

Appleby, L., Dini, V., Withington, L., LaMotte, E., & Hammer, D. (2021). Disciplinary significance of social caring in postsecondary science, technology, engineering, and mathematics. *Physical Review Physics Education Research*, *17*(2), 023106.

Bang, M., Faber, L., Gurneau, J., Marin, A., & Soto, C. (2016). Community-based design research: Learning across generations and strategic transformations of institutional relations toward axiological innovations. *Mind, Culture, and Activity*, *23*(1), 28-41.

Bang, M., & Vossoughi, S. (2016). Participatory design research and educational justice: Studying learning and relations within social change making.

Brady, C. (2021). Patches as an expressive medium for exploratory multi-agent modelling. *British Journal of Educational Technology*, *52*(3), 1024-1042.

Banes, S. (2011). *Terpsichore in sneakers: post-modern dance*. Wesleyan University Press.

Booker, A., & Goldman, S. (2016). Participatory design research as a practice for systemic repair: Doing hand-in-hand math research with families. *Cognition and Instruction*, *34*(3), 222-235.

Brown, A. (1992). "Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings." *The journal of the learning sciences* 2.2: 141-178.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, *32*(1), 9-13.

Collins, A. (1992). Toward a design science of education. In E. Scanlon & T. O'Shea (Eds.), *New directions in educational technology* (pp. 15-22). New York: Springer-Verlag. Originally published as Collins, A. (1990, January). *Toward a design science of*

education. Technical Report No. 1, Center for Technology in Education. New York: Bank Street College of Education.

DeLiema, D., Hufnagle, A., Rao, V. V., Baker, J., Valerie, J., & Kim, J. (2021). Methodological innovations at the intersection of video-based educational research traditions: Reflections on relevance, data selection, and phenomena of interest. *International Journal of Research & Method in Education*, 1-18.

Duffy, A. B., & Beaty, A. (2019). Flexibility of artistic roles and shared ownership between dance educators and students in choreography and performance. Research in Dance Education, 20(2), 130-147.

Ehret, C., & Hollett, T. (2016). Affective dimensions of participatory design research in informal learning environments: Placemaking, belonging, and correspondence. Cognition and Instruction, 34(3), 250-258.

Erickson, F. (2004). *Talk and social theory: Ecologies of speaking and listening in everyday life*. Cambridge: Polity Press.

Goldman, S. R., Hmelo-Silver, C. E., & Kyza, E. A. (2022). Collaborative Design as a Context for Teacher and Researcher Learning: Introduction to the Special Issue. Cognition and Instruction, 40(1), 1-6.

Goodwin, M. H., & Cekaite, A. (2018). Engagements of care entailing touch. *Embodied Family Choreography: Practices of Control, Care, and Mundane Creativity.* Routledge.

Gutiérrez, K. D., & Jurow, A. S. (2016). Social design experiments: Toward equity by design. *Journal of the Learning Sciences*, *25*(4), 565-598.

Hall, R., & Stevens, R. (2015). Interaction analysis approaches to knowledge in use. In *Knowledge and Interaction* (pp. 88-124). Routledge.

Hollett, T., Peng, X., & Land, S. (2021). Learning with and beyond the body: The production of mobile architectures in a ballet variations class. *Journal of the Learning Sciences*, 1-30.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *The journal of the learning sciences*, *4*(1), 39-103.

Kendon, A. (1990). Spatial organization in social encounters: The F-formation system. *Conducting interaction: Patterns of behavior in focused encounters*.

Kirsh, D. (2010). Thinking with the body.

Lerman, L. (2014). *Hiking the horizontal: Field notes from a choreographer*. Wesleyan University Press.

Levy, S. T., & Wilensky, U. (2008). Inventing a "mid level" to make ends meet: Reasoning between the levels of complexity. Cognition and Instruction, 26(1), 1-47.

Lindberg, L., Marin, A., & Silverman, Z. (2021). Improvisational Dance as Enactive Cognition: What Do Novice Dancers Teach Us about Embodied Cognition in Dance?. *Journal of Dance Education*, 1-12.

Ma, J. Y. (2016). Designing disruptions for productive hybridity: The case of walking scale geometry. *Journal of the Learning Sciences*, *25*(3), 335-371.

Marin, A. (2020). Ambulatory sequences: Ecologies of learning by attending and observing on the move. *Cognition and Instruction*, *38*(3), 281-317.

Marin, A. M. (2013). *Learning to attend and observe: Parent-child meaning making in the natural world* (Doctoral dissertation, Northwestern University).

Marin, A., Faulstich, E. N. O., Bordeaux, C., Clark, C., Gravell, J., Lindberg, L., Morales, D., Rodriguez, L., & White Eyes, R. (2020) The embodied and relational dimensions of

video as substrate. Proceedings of the International Conference of the Learning Sciences, 2020, 1474-1475.

Marin, A., Taylor, K. H., Shapiro, B. R., & Hall, R. (2020). Why learning on the move: intersecting research pathways for mobility, learning and teaching. Cognition and Instruction, 38(3), 265-280.

McWilliams, J. (2016). Queering participatory design research. *Cognition and Instruction*, *34*(3), 259-274.

Sengupta, P., Kinnebrew, J. S., Basu, S., Biswas, G., & Clark, D. (2013). Integrating computational thinking with K-12 science education using agent-based computation: A theoretical framework. Education and Information Technologies, 18(2), 351-380.

Solomon, F., Champion, D., Steele, M, & Wright, T. (In press). Embodied Physics: Utilizing dance resources for learning and engagement in STEM. *Journal of the Learning Sciences*.

Spruill, N. C., Hennessy Elliott, C., Della Volpe, D., & Alcantara, K. (2021). Engineering Care: How Two Young Women of Color Establish Positional Identities in a Robotics Space. *Journal of Pre-College Engineering Education Research*, *11*(1), 256-275.

Tuck, E. & Yang, K. W. (2012). Decolonization is not a Metaphor. *Decolonization: Indigeneity, education & society*, *1*, 1-40.

Vakil, S., McKinney de Royston, M., Suad Nasir, N. I., & Kirshner, B. (2016). Rethinking race and power in design-based research: Reflections from the field. *Cognition and Instruction*, *34*(3), 194-209.

Vogelstein, L. (2021) Mathematical Physical Research: Mathematical agency in the practices of professional dancers. Proceedings of the International Society of the Learning Sciences Annual Meeting, 2021.

Vogelstein, L. (2020) Physical Research: Professional Dancers Exploring Collective Possibilities in a Solidifying Substrate. Proceedings of the International Conference of the Learning Sciences, 2020, 737-739.

Vogelstein, L., & Brady, C. (2019). Taking the patch perspective: A Comparative analysis of a patch based participatory simulation. In *Proceedings of the 2019 Conference on Computer Supported*

Vogelstein, L., Brady, C., & Hall, R. (2019). Reenacting mathematical concepts found in large-scale dance performance can provide both material and method for ensemble learning. *ZDM*, *51*(2), 331-346.

Vogelstein, L., & Hall, R. (2020). The push that never made sense to me: the substrate of dancers' professional intrinsic and extrinsic vision. In D. Keifert & N. Enyedy (Chairs), Analytic designs: Goodwin's substrates as a tool for studying learning [Symposium]. Proceedings of the International Conference of the Learning Sciences, 2020, 1471-1478.

Wilensky, U. 1999. NetLogo. http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computer-Based Modeling, Northwestern University. Evanston, IL.

Wilensky, U., & Rand, W. (2015). An introduction to agent-based modeling: modeling natural, social, and engineered complex systems with NetLogo. Mit Press.

Wilkerson-Jerde, M., Wagh, A., & Wilensky, U. (2015). Balancing curricular and pedagogical needs in computational construction kits: Lessons from the DeltaTick project. Science Education, 99(3), 465-499.

Zavala, M. (2016). Design, participation, and social change: What design in grassroots spaces can teach learning scientists. Cognition & Instruction, 34(3).