Introducing Linear Measurement in the Primary Grades

Jennifer Abbey Nicol

Vanderbilt University

Abstract

Linear measurement is a complex concept. In order to become proficient in linear measurement one must understand and perform numerous components. The following lessons introduce primary students to linear measurement using research proven methods to increase understanding. The lessons are created to capitalize on students’ misconceptions. The following paper is divided into three main sections. The first section is “Lesson Goals and Criteria.” In this section the general ideals and standards used to guide the formation of the lessons are outlined. This paper examines the problem selection, introduction of problem, exploratory activity, and discussion. The second section contains four lessons as well as four papers of “Explanation and Justification” to accompany each lesson. The four lessons concentrate on measuring techniques, importance of the standard unit, a footstrip similar to a yardstick, and the use of the phase “part of” to represent a fraction. The final section, “Lesson Components,” outlines the components considered in the creation of each lesson. All lessons give weight to learners and learning, learning environment, curriculum and instructional strategies, and assessment. The paper outlines and explains to educators lessons for introducing linear measurement to primary students.

**Lesson Goals and Criteria**

There are four main sections to each lesson: the selection of the problem, the introduction of the problem to the class, the exploratory activity, and the final discussion. Each one of these components are different throughout the four lessons, however, many standards were used in the creation of all four lesson plans. The standards and goals were used to guide during the process of constructing each individual lesson. The standards and goals for the four lessons are detailed below.

Problem Selection

The selection of problems is a task which requires thought and deliberate choices. All problems must require “cognitive engagement” from the students (Henningsen & Stein, 1997). If a task does not require a student to cognitivately engage in order to create a solution then the class time is not being used wisely. Every student in the class should be simulateneuously challenged, even if they are each grableling with a different aspect of the task. Drawing on current research, “tasks that scaffold, [have] the appropriate amount of time, [model] high-level performance, and [continue] to pressure students for explanation and meaning result in the most amount of student engagement” (Henningsen & Stein, 1997). The four lessons comprising this section on distance introduction meet all of these requirements.

Each task is designed to build on the previous task allowing for the teacher to scaffold student thinking. The teacher must monitor the amount of time on task as well as model high-level performances. The time on task will vary with each individual class and group. Finally, a task and discussion must push students to understand and explain each solution. Each activty and discussion are designed to help students reach new conclusions. Solutions to problems are never provided by the teacher; rather, the tasks are created to allow the student to discover the problem or issue and then suggest a solution. This process of encountering a problem and then creating solutions will require an increased level of cognitive engagement from all students. It is vital that the teacher engages the students in this process. It is the role and responsibility of the teacher to build on student thinking in order to increase cognitive engagement for all students of all abilities.

The lessons and exploratory activities also avoid aspects of lessons which lower cognitive engagment. Tasks where “the challenging aspects of the tasks are removed during the implementation phase,” too much or too little time is devoted, lack of prior knowledge, poor classroom management, and removal of challenging aspects result in the least amount of engagement (Henningsen & Stein, 1997). During implication, one also needs to be careful and purposeful not to remove the challenge of a task while attempting to ensure that all students have sufficent background knowledge to access the problem. This is a delicate balance to find. With the selections of problems, it is important to make sure that all students are able to access and relate to the problems. Students need to understand the issue or concern in the problem in order to be able to understand why the class is working to solving it as well as which solutions would be helpful. The exploratory tasks are designed to revolve around an issue all students are able to access. In order to ensure this, a teacher must explain to the students the problem or issue which requires the object to be measured. However, the teacher should not model the measuring because it would remove the challenging aspects of the tasks. The lesson plans reflect this important balance. It is ceratinly not necessary, but sometimes helpful, to choose problems that “involve real world contexts” (Jackson & Cobb, 2010). The choice for real world problems will potentially allow more students to engage in the problem and therefore develop sophisticated thinking. The lessons in the introduction of distance draw on real world situations with a fun twist for the interests of primary grade students.

Introduction of Problem

When introducing problems one must ensure that students “experience the task scenerio as real” (Jackson & Cobb, 2010). A student must understand there is an issue which requires a solution. In other words, the students need to understand there is a problem which must be solved. This requires students to be familiar with the situation (Jackson & Cobb, 2010). In order to account for this, one needs to “support all students’ understanding of the cultural suppositions inherent in the task scenario” (Jackson & Cobb, 2010). A task must be accessible to all students. In order for students to be able to understand the issue which needs to be solved, they must first understand all components of the problem. The task of explaining the problem is done in the introduction. In order to do so effectively, one must understand the previous knowledge and cultural understandings of the class. The introducation will potentially change from class to class.

Similarly, as a teacher, one must also “support students’ development of situation-specific imagery of the mathematical relationships described in the task statement” (Jackson & Cobb, 2010). It is easy to assume that all students are able to access the situation described in the math problem, however, this is often not true. One must carefully analyze the problems posed for any aspect that may need to be discussed in order to ensure that every students is able to understand all components of the task. These needs will change each year depeneding on the experiences of the students in the class.

When introducing tasks, one needs to always do so conceptually. Conceptual explanations are “grounded in conceptions of the situations” (Thompson, Philipp, Thompson, Boyd, 1994). The introduction of a task will consistently ground the “numbers” of the problem in the context of the situation rather than separating the computation from the situation. This practice will provide students with many skills, which will allow them to think of mathematics in a useful context that extends beyond the classroom. When conducting classroom discussions one must push students to discuss the problems in the same manner in which they are introduced, conceptually not procedurally.

Exploratory Activity

After the introduction of the tasks, students will work either individually or in small groups on an activity to explore the posed problem. If the students are in groups, the sizes will vary depending on the task. When students are in groups, two aspects must be addressed. The first aspect is “explicit negotiation with students’ productive norms of communication and argumentation in small groups” (Jackson & Cobb, 2010). The second aspect of small groups is the teacher’s “role in supporting students’ development of small group relationships in which no student is established as either a social authority or a mathematical authority” (Jackson & Cobb, 2010). It is not productive if everyone in a group is referencing one person as an expert. Students must communicate and collaborate together to discuss multiple sophisticated mathematical approaches to problems.

The most important aspect of the exploratory activity is the opportunity for students to “actually be involved in doing mathematics- to explore interesting mathematical situation, to look for patterns, to make conjectures, and to make logical arguments for their conjectures” (Lappan, 1993). Students must be able to explore the posed problem, discover the issue, and formulate the solutions. The solutions are often formed with “self-invented methods” (Whitenack, Knipping, Novinger, & Underwood, 2001). Through this exploration students are able to truly explore the many dimensions of the posed problems. This exploration will allow students to form deeper understandings and build stronger connections.

Discussion

One of the most influential aspects of a mathematics lesson is the classroom discussion. Discussion is a collaborative process orchestrated by the teacher and class; “the discourse of a classroom is formed by students and the teacher and the tools with which they work” (Ball, 1991). However, as a teacher and the head of the class, it is important to be cautious and deliberate because “teachers play a crucial role in shaping the discourse of their classrooms through the signals they send about the knowledge and ways of thinking and knowing that are valued” (Ball, 1991). It is easy to assign value to particular answers, however, if one does so at an inappropriate time, even unknowingly, it can form student thinking and limit productive discussions. A teacher must think critically before assigning value to “better” answers for this will greatly affect student thinking. Initially, student discussions are often designed to gather and propose solutions. It would not be helpful to place value on solutions at this beginning stage; however, eventually, students must understand that some solutions are more sophisticated and efficient than others. The teacher should introduce this understanding deliberately. Placing value on solutions can be appropriate and beneficial but must be done intentionally in order to most benefit students.

In order to ensure that these discussions are productive it is important to monitor students’ problem solving techniques while they are in groups. The information gathered will inform choices regarding student solutions discussed as a group. The class discussions should largely be shaped by student solutions found during the exploratory activity. These solutions can serve as a starting point for discussion. Some solutions may also serve as models of efficient or sophisticated solutions.

Once the solutions to be discussed are selected, as a class, it is important to discuss the problems in the context of the student’s thinking. When the students’ explanations are stopped after the correct answer is given one “[misses] opportunities to gain insight into students’ thinking” (Ball, 1991). This insight is vital to understand students’ level of understanding. It is also crucial that students are able to see other students’ thinking modeled in order to allow them to develop more diverse and potentially more sophisticated thinking strategies. To help students understand that some solutions are more efficient and sophisticated one needs to be deliberate with the order in which students present their solutions. The order solutions are presented can greatly influence students’ interpretation of their important and efficiency.

During discussion times, students will reflect on the mathematical process in order to help them develop more sophisticated mathematical thinking strategies. It is important to understand that children “do not happen to spontaneously begin reflecting at the same moment” (Cobb, Boufi, McClain, Whitenack, 1997). Instead, reflection is “supported and enabled by participating in discourse” (Cobb et al., 1997). It is the teacher’s job to create this reflective discourse in his/her classroom; “participation in reflective discourse supports and enables individual reflection on, and reorganization of, prior activity, it does not cause it, determine it, or generate it” (Cobb et al., 1997). In order for relfective discourse to be effective one must orchastrate the dicussion in a manner which will allow students to understand one another’s solutions. This practice is often most important when a student or group of students presents a soltuon different or more sophisticated than others created.

Reflective dicourse will not only allow students to develop flexibility and efficency with mathematical tasks but it will also help them form skills which will serve them greatly in mathematical courses in the future. It is the teacher’s responsibilty to initiate reflective shifts in discourse. This “requires considerable wisdom and judgment on the teacher’s part” (Cobb et al., 1997). It is very important that one conducts a meaningful discussion rather than allowing the students to simply “infer what the teacher wants them to say and do.” (Cobb et al., 1997). This again relates to the caution of placing value on some solutions over others. Helping students to develop the ability to engage in reflective dicourse is argueably one of the most difficult and beneficial aspects of teaching mathematics.

The last significant aspect of a mathematical discussion is the mannor in which students explain their answers. Three main skills are beneficial in order for students to do so in the most productive manner. Firstly, students must explain their answers conceptually not procedurally or calculationally. Similarlly to the introduction of the task, students must talk about the “numbers” of the problem in the conext of the situation rather than isolated and distanced. Second, students must also explain all steps they took in constructing their answer. While doing so, students should explain their thinking behind each choice made. Finally, students should understand which comments are productive and helpful to the group. It is important that students are

able to judge both when it [is] appropriate to make a mathematical contribution and what [constitutes] an acceptable contribution. This [requires], among other things, that the students [can] judge what [counts] as a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical solution (Cobb, Stephan, McCain, Gravemeijer, 2001).

These skills are difficult for students to grasp and will require a teacher to deliberately work with them throughout the year.

There are many components in creating a productive lesson. All of these aspects were considered and referenced in the creation of the linear measurement lessons. The objectives from each section, the selection of the problem, the introduction of the problem to the class, the exploratory activity, and the final discussion, were used to form the following lessons.

**Lesson Plan One**

Objective

Students will be able to measure by correctly starting at the beginning of the object and use the measuring tool (their foot) without leaving gaps between each use.

Materials

Masking tape

Paper and pencils for each student

Introduction

Tell the students that today we have entered The Kingdom of Kai. The Kingdom of Kai is ruled by a very kind king named King Kai. However, today, King Kai has declared that he would like everything in his kingdom measured. Pause and ask the students if they would be willing to help King Kai by measuring objects around the room. Inform them that King Kai would like everything measured by feet. Ask for suggestions on how one would measure with feet. After students have had an opportunity to make numerous suggestions tell them that the suggestion of walking while measuring heel to toe will probably be the best for this situation. Explain to the students that their job is to measure multiple objects in the room (give examples such as the rug, the bookshelf, the desks, etc) using the heel to toe method. They are to then record the found measurements for the objects measured.

Exploratory Activity

Allow students 25-30 minutes to measure various objects around the room. During this time the teacher may walk around to monitor students and make sure everyone remains on task. Ideally, students are using this time to explore measuring while recording their results. The teacher’s job is to note the subtle differences in students’ measuring techniques. These differences should be highlighted later in the discussion.

Discussion

Begin the discussion by asking a volunteer to demonstrate the way he/she measured. If a student proclaims he/she measured differently ask that student to also demonstrate. Allow all different methods to be presented; if a distinct method is observed during the exploratory activity, but not volunteered during the discussion, ask the student to demonstrate. Once all methods have been presented allow students to comment on the differences. A typical difference that arises is the discrepancy between whether one should or should not “count” the first foot placed down. When this issue arises, ask a student with each technique to pace the distance of an object. As a student places his/her foot down, use a strip of masking tape to mark the distance covered. Encourage the students to participate in a discussion regarding the importance of covering the entire space measured with tape. Another discrepancy that may occur is whether or not students are leaving space or gaps between each foot. The tape can once again be used to engage the students in a discussion regarding this issue.

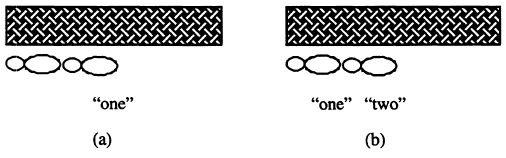
**Lesson One Explanation and Justification**

Placement of Lesson

This lesson will serve as the introduction to linear measurement. Before students are able to discuss more advanced properties of measurement they must first be able to employ successful techniques for measuring.

Student Thinking and Teaching Methods

There will likely be two differences in student thinking that should be addressed in discussion. Firstly, the concept of leaving spaces or gaps between each foot is one that should be discussed. Though this may seem obvious to an individual proficient in linear measurement this can be a difficult concept for a student exploring during an introduction to linear measurement. Students will often struggle with the distinction of measuring “as signifying the last pace (or amount of space it defined) rather than the space covered by the entire sequence of paces” (Cobb et al., 2001). If a student is struggling with the concept that measuring is “the space covered by the entire sequence of paces” the idea that spaces between steps is a problem which will not be inherent to him/her. An excellent method to address this concept is to have the teacher use the masking tape to illustrate the distance covered. Using masking tape will “support a conceptual discussion of these two different methods” (Stephan, et al., 2001). The goal of this discussion is not to inform the students of the “right way” but rather to allow them “an opportunity to reorganize their understanding about what it means to measure” (Stephan et al., 2001).

The second concept that often proves difficult for students is whether or not to “count” the first step while measuring distance. Picture (a) demonstrates the method when a student does not count the first step. Picture (b) is the method when students do “count” the first step. The pictures are from a 2001 study done by Stephan, Cobb, Gravemeijer, and Estes. 

In order to initiate the discussion concerning this discrepancy, ask students to “compare and contrast these two methods of measuring during the subsequent whole-class discussion” (Stephan et al., 2001). To illustrate the differences in a “conceptual discussion of these two methods, the teacher [should place] a piece of masking tape at the beginning and end of each pace” (Stephan et al., 2001). For students arguing for method (a), “measuring [is] not about covering the space defined by the [object]; rather, the goal of measuring [seems] to be to count, in some manner, the number of paces it [takes] to reach the end of the carpet” (Stephan et al., 2001). The masking tape illustrates this problem well in a manner that allows students access and therefore the opportunity to discuss the differences.

Ideally, at the conclusion of the lesson students will all be able to employ the most efficient method for measuring linear distances. The masking tape will help to illustrate differences and therefore lead students in a discussion, which will allow their thinking and measuring techniques to advance.

Assessment

The assessment for this lesson is formative and informal. It is important that the teacher monitor student thinking during the exploratory activity. The teacher should be looking for any method the students are employing to measure. Any differences may be highlighted during the whole class discussion. It is recommended that the teacher use a clipboard to note the differences in addition to the students using the different methods. The student names will be important during the discussion as well. Ideally, all students will volunteer their methods; however, if a method is needed to initiate the discussion concerning proper measuring it is the teacher’s responsibility to call on a student using this method if he/she does not volunteer. In this particular discussion the teacher should ensure that students have volunteered three methods of measuring: a correct method, a method that does not involve counting the first pace, and a method where there are gaps between each pace. If one of the incorrect measurement methods is not used by students it does not need to be introduced into the discussion. If all three methods are utilized by students in the exploratory activity it is important to include each method because these are the mathematical concepts for which the lesson is focused.

Assessment should also be done during the class discussion. It is important that changes and advances in student thinking are noted. The objective of the lesson is to have all students employing the correct method to measure linear distance with an understanding of why this method is best. In order to ensure the second component of the objective, student understanding, the teacher must monitor and evaluate the thinking of each student. The discussion should not conclude or change topics until all students have demonstrated an understanding of the importance of these two aspects of linear measurement: not leaving gaps, and “counting” the first step.

**Lesson Plan Two**

Objective

Students will be able to explain the practical use and advantages of having a standard unit of measure.

Materials

Masking tape

Paper and pencils for all students

A cutout of King Kai’s footprint

Introduction

Inform students that today King Kai has asked the class to report back to him on the exact measurement of a few objects. Relay that King Kai would like to know the distance of the desks, the cabinets, and a few other objects specific to the classroom. It is best to have seven or eight objects for students to measure. Remind the students to carefully record the results from each object so they are able to report back to the king. Have students work in pairs to measure the assigned objects.

Exploratory Activity Part I

Allow students 20-25 minutes to measure the various objects around the room. During this time the teacher may walk around to monitor students and make sure everyone remains on task. Students should use this time to measure the given objects and record their results. The teacher’s job is to note the differences in students’ results. These differences should be highlighted later in the discussion.

Discussion Part I

Begin by asking volunteers to come to the front of the room and record their results regarding the number of feet it took to measure the various objects. Each amount should be recorded on the board for all students to see. Continue with this process until many students, at least five or six, have shared their solutions.

Ideally, a student comments on the different answers concerning the amount of feet required to measure the same object. Depending on time, allow eight or nine students to volunteer answers before bringing the discrepancy to the students’ attention. If the students do not introduce this issue, the teacher may measure the object with his/her foot, considering it will be significantly larger than the students’ feet. Once the problem is highlighted, ask students for various hypotheses as to why this may have occurred. Record each suggestion on the board. After many solutions have been volunteered have two students explore the idea that the difference in foot size may be the problem. Select two students, ideally with dramatically different foot sizes, to measure one of the objects while the other students observe. Use the masking tape to mark the length of the first foot step. The purpose of the masking tape is to highlight the difference in length of the feet. Allow the students to discuss and debate this role. Traditionally, students will determine that a standard foot is needed. If this is not suggested by the students the teacher should suggest the use of a standard unit and have the students eventually measure with it to see the practical advantages. The students should be able to articulate the issue of having different lengths of measuring devices as well as the advantages of a standard unit. It may be helpful for students to list real examples and uses of the standard unit in order to illustrate these concepts. Once the standard unit has been suggested, reveal that the class is actually in possession of numerous copies of King Kai’s footprint. Allow the class to break up into pairs again this time measuring with the King’s footprint.

Exploratory Activity Part II

Allow students 10-15 minutes to measure some of the objects with King Kai’s footprint. The teacher should monitor to make sure students remain on task and record their results.

Discussion Part II

Gather students and once again allow them to share their results. Note that this time it takes the same amount of feet to cover the distance.

**Lesson Two Explanation and Justification**

Placement of Lesson

This lesson will follow the lesson on techniques for linear measurement. The purpose of this lesson is to communicate to students the importance of a standard unit of measurement. In order to be able to engage in further discussions on the qualities of measurement students must first develop an understanding of the necessity and efficiency of using a standard unit.

Student Thinking and Teaching Methods

Ideally, students will not struggle to measure the various objects around the room considering the techniques for linear measurement were addressed in the previous lesson. The students should notice discrepancies in results once the answers are displayed on the board. Traditionally, “eventually, students [discuss] the idea that their [feet are] different sizes, since some students [have] smaller feet than others (Stephan et al., 2001). If students are struggling to reach this conclusion, marking the distance of two students’ feet while pacing can spark discussion. The visual representation should provide students with the cue to discuss the difference in feet sizes.

Once it is accepted by the students that it is an issue that all students have different size feet, a solution can be offered. As discussed earlier, it is important that all students understand the issue/problem before the solution is offered in order to aid in the understanding of the importance of the solution. Students should understand that it would be impossible to communicate the size or length of an object without a standard unit. The impracticality of this inability to communicate as well as the ill effects this would have in our real life should be discussed.

Traditionally, “the need for a standard [foot is] suggested by the children, (and) the teacher [is able to give] each pair of students a standard [foot]” (Stephan et al., 2001). If students request the presence of King Kai it is acceptable to explain that “the king [is] receiving too many requests to measure things in his kingdom and could not be everywhere at once” (Cobb et al., 2001). Once again, the students do not always need to present the solution to the problem; however, they must agree that there is a problem and that the suggested solution is valid.

Assessment

Assessment for this lesson is formative and informal. The teacher should monitor student methods for measuring during the exploratory activities. If a student is struggling to measure correctly it is appropriate to engage the student in conversation. During this conversation, the teacher may remind the student of the problems and eventual discoveries of the previous lesson. During the initial class discussion it is important for the teacher to observe student understanding. In order for the standard foot to be important and acceptable to students they must first all understand the issue with everyone measuring with his/her own foot. The discussion should not be changed or progressed until the teacher is assured that every student determines this is problematic. Once the solution of the standard foot is presented the teacher must once again observe student comprehension to assess that every student understands the role and importance of the foot.

During the second exploratory activity, the teacher should once again monitor that all students are correctly measuring. If any problems arise it is once again appropriate to remind students of the problems and solutions of the previous lesson. Finally, all students must agree that using the standard unit of measurement is the solution to the earlier problem of various foot sizes. It is the teacher’s role to assess that all students demonstrate this comprehension. The whole class discussion should continue until this is observed.

**Lesson Plan Three**

Objective

Students will use and article the advantages of the “footstrip.”

Materials

Butcher paper

Pencils

Scissors

Introduction

Start by informing the students that today King Kai has ordered the class to measure the length of the playground (if the weather does not permit students to go outside then try to find another large distance such as the gym). Request that the students think about what they have measured so far in comparison to the playground in order to think of any differences. List the differences on the board. Once a student suggests that the playground is much larger, highlight this difference. Ask the students to brainstorm ways they could make the larger distance more manageable. Allow students to make multiple suggestions. Request that they use the cutout of King Kai’s foot to draw five of his feet on a strip of paper; provide scissors to cut excess paper. Ask the students to measure the playground with their new strip.

Exploratory Activity

Students should spend their time individually measuring the length of the playground with their strip for 20-25 minutes. Do not correct students if the placement of the King’s feet on the strip is odd or incorrect. The teacher should walk around taking note of student thinking. It is important to note the way students place the feet on the strip as well as the methods students employ when using the strip.

Discussion

Begin by asking students to show how they placed King Kai’s feet on the strip. Compare various methods used in the class. Have each student explain the rationale behind placing the feet the way they did. If the students do not initiate the topic ask them to reflect on how they measured when they were only using one foot and attempt to draw connections between that experience and drawing the five feet. Use this connection to illustrate to students the importance of placing the feet heel to toe on the footstrip.

Once this important concept has been explained and understood by students, ask the students to think about any advantages of measuring with this footstrip instead of an individual foot. Write down all student suggestions. By the conclusion of the conversion, among other concepts, students should understand that it is easier and more efficient to measure long distances with the footstrip instead of the individual King Kai’s foot. In order to help illustrate this, have one student measure an object with the individual foot and another measure with the footstrip. Have students explain the positives and potential negatives of the footstrip.

**Lesson Three Explanation and Justification**

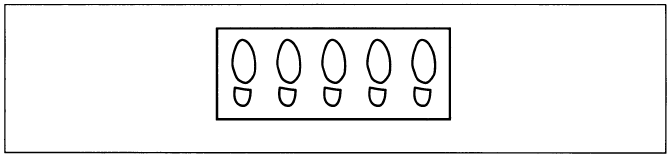
Placement of Lesson

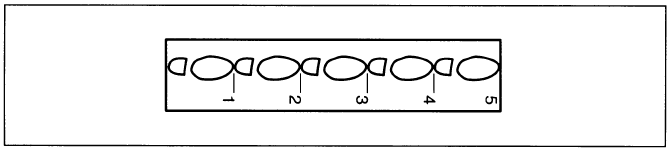
This lesson will follow the lesson regarding the importance of a standard unit. In order to be able to investigate the advantages of a multiple unit measuring tool students must first understand the advantages of a standard unit. This lesson will help students to further explore and understand the importance and efficiency of this characteristic of measurement.

Student Thinking and Teaching Methods

At the conclusion of the lesson, students must understand the placement of feet heel to toe on the footstrip as well as the efficiency of using the footstrip. During the introduction of the lesson students need to understand that the playground is much larger than the objects they have previously measured. This understanding will allow students to eventually understand the efficiency of the footstrip.

Ideally, the students will suggest placing multiple feet together; however, the teacher may inform the students of the footstrip. It is acceptable for “the teacher [to ask] students to draw five feet on a strip of paper and then [ask] how the king might use such a strip” (Stephan, Cobb & Gravemeijer, 2003). It is ideal if the students suggested the concept of the footstrip but it is more important that they eventually understand the use and advantages of the footstrip.

There may be multiple techniques for creating the footstrip employed during the exploratory activity of the lesson. In a 2003 study, Michelle Stephan, Paul Cobb, and Koeno Gravemeijer identified two common methods often used by students. The methods are the following: 



During discussion it is important to highlight both methods. In order to illustrate to students the correct way to measure, engage students in a conversation about the techniques used to measure when they only had one foot. The teacher’s goal is to help students draw connections between measuring with one foot and drawing feet on the footstrip.

The second main concept that needs to be addressed in the discussion is the efficiency of using a footstrip for larger items. According to current research, “students [realize] that solving problems [will] now be faster, indicating that some of them [anticipate] counting their individual paces more efficiently, that is, they [will] count faster by five paces than by single paces” (Stephan et al., 2003). This suggests that most students will be able to understand the efficiency of the footstrip without further explanation. If students are not able to initially understand the benefits of the footstrip, have one student measure an object with the single foot while another measures with the footstrip. Ideally, the object will be quite long so students are able to see the hassle of measuring in a particularly bothersome situation. Allow the students to discuss this concept. Guide students in making a conclusion which supports the footstrip for its efficiency.

Assessment

The assessment is formative and informal. The teacher should monitor student thinking during the exploratory activity. The student thinking during this activity will determine the course of the discussion. A teacher must monitor how students are placing the feet on the footstrip. Every method employed during the activity, ideally, should be addressed during the discussion. It is not necessary to correct inaccurate methods for measurement during the activity due to the fact that it will be addressed in discussion. By the conclusion of the discussion it is important for students to understand the reason one creates a footstrip by placing the feet heal to toe. The discussion should not conclude until this concept is clear to all students.

The other main concept to be assessed during discussion is the efficiency of the footstrip. Student should not only understand how to construct a footstrip but the importance of doing so. This assessment will be done entirely during the discussion. It is vital that all students understand this by the conclusions of the discussion.

**Lesson Plan Four**

Objective

Students will be able to articulate the use of the phrase and measurement concept “part of” in terms of the standard unit.

Introduction

Tell the students that today King Kai has ordered everyone to again measure the length of the playground with the footstrip (if the weather does not permit students to go outside then try to find another large distance such as the gym). This time it is incredibly important that all the results are recorded so students are able to send the results back to the King.

Exploratory Activity

Have students measure the distance of the playground with the footstrip. Allow them to work in pairs for 20-25 minutes. The instructor should walk around and monitor student measuring techniques. It is appropriate to engage a student in discussion if they are not using the footstrip correctly. Remind them of the previous discussion regarding the proper techniques for measuring using the footstrip. The instructor should also note how students handle the last pace required to measure the playground in an instance when a proficient measurer would use a fraction to represent this distance. These different techniques need to be highlighted during the discussion.

Discussion

Begin the discussion by asking students to share their results. When there are discrepancies ask the students to hypothesis why this may have occurred. If a student does not identify the last pace as being problematic specifically ask the students how they handled it. It may be necessary to demonstrate a scenario to students where the last pace does not fit evenly. Ask different students to share their methods. Continue this process until all distinct methods have been demonstrated; if a method is not volunteered, ask a student to demonstrate.

Once all the methods are demonstrated engage the students in a discussion about the best method of handling the last pace. Guide the students to realize that it is not accurate to either not count the last step or to count the entire step. Invite students to make suggestions of a solution considering that these two methods are not accurate. If students do not suggest it, ask them if it would be possible to just say “part of” for the last pace. Have a student demonstrate how it would work to write the number of paces and “part of.” Discuss any issues or concerns students have regarding this solution. By the conclusion of the discussion, students should understand that writing “part of” is an acceptable technique for handling the last step.

**Lesson Four Explanation and Justification**

Placement of Lesson

This lesson should be the final lesson of the unit. It will address how to handle the last pace. In order for students to be able to understand how to handle this issue they must be able to measure accurately and understand the benefits of a standard unit. It is also important that all students are using the standard unit so the results can be compared in the discussion. Preferably, students will be able to use the footstrip in order to exaggerate the issue with the last pace. It is easier for students to attempt to manipulate one standard foot in order to not address the problem with the last pace. For this reason it is best to place this lesson after the lesson introducing the footstrip.

Student Thinking and Teaching Methods

Students will, most likely, utilize many techniques to handle the last pace. There are three main techniques which research has found typical in classrooms. The simplest method is to simply have the foot extend beyond the object being measured (Stephan et al., 2003). These students “[seem] to simply count the number of paces [they need] to take to reach the end of the wall” with no concern that the foot extends beyond the object (Stephan et al., 2003). With these students one should focus on the end of the object relative to the end of their footstrip.

A second method used by students, is to simply insist that “an item [cannot] be measured if a person’s last foot [does] not fit exactly within the physical boundaries of the item” (Stephan et al., 2003). Engaging these students in a conversation brainstorming techniques to handle this situation can help to illustrate that a solution is possible.

The last common method used by students involves becoming creative with the placement of feet. The students using this method “simply [turn] their feet sideways so that the last foot [fits] exactly within the end of the rug, and they [count] the whole foot as ‘one’” (Stephan et al., 2003). These students should be engaged in a conversation concerning the proper techniques for measurement. A reminder and review of the first lesson would be beneficial for these students.

For all of the students, using each method, there is an overall misconception. In all of these cases students are not connecting measuring with “covering the amount of distance” (Stephan et al., 2003). This goal of covering distance through measuring should be highlighted to students in a discussion. The class should able to talk clearly about the goal and purpose of measuring. In the context of this discussion the various methods can be highlighted and explored. Through discussion and class exploration, where one student demonstrates each method and explains for the group, the class should be able to draw conclusion about why these various methods are not suitable methods of measuring. Students should understand that measuring is a way to “cover distance” and the counting the entire last foot or not counting the last foot at all are not meeting this goal of measuring. This understanding should help illustrate to students the benefits of using the “part of” solution to handle the last pace.

Assessment

The assessment for this lesson is formative and informal. The teacher should assess students for their understanding of the “part of” portion. Allow students to use any technique they determine is best during the exploratory activity. It is important to note the methods used to address this issue so they can be highlighted during the discussion. Each one of the methods should be discussed; if a method is not volunteered, prompt the student to demonstrate. By the conclusion of the lesson, all students should understand why the “part of” solution is more accurate than not counting the last pace or counting the “part of” pace as a full pace. Students should also understand which situations to use the “part of” pace. The discussion should not conclude until the students understand all of these aspects.

**Lesson Components**

These lessons address four distinct components to constructing a strong lesson: *learners and learning, learning environment, curriculum and instructional strategies*, and *assessment*. The lessons were formed on research done on introducing linear measurement in the primary grades. The c*urriculum and instructional strategies* were shaped in response to the success and issues that arose during research on introducing linear measurement in primary grades. The instructional strategies used to form the larger goals of the unit were also based on research done on techniques for successfully teaching mathematics in primary grades. These goals were broader than linear measurement and therefore draw from more diverse research.

*Learners and Learning* is central to all aspects of the four lessons. Each lesson is formed by taking careful consideration of the previous knowledge of students at the beginning of the lesson. The lessons are also created around research regarding the typical struggles of students. These struggles are acknowledged and the lessons are adjusted accordingly. The lessons are designed so these student misconceptions become apparent because they are in direct conflict with the exploratory activity in the lesson. Once these misconceptions are apparent, the discussions help students to resolve these issues. The goal is to provide students with an opportunity to encounter misconceptions, discuss issues, and form correct concepts. This process is also designed based on research done on successful teaching practices.

The *assessment* component of each lesson is addressed explicitly in the “Explanation and Justification” section following the lesson. The assessments are all formative and informal. The important aspect of the assessment in these lessons is not to give students grades. The important aspect is to gather information regarding student understanding. The assessment is done in two components in order to create a strong lesson. The first assessment is done during the exploratory activity. This assessment allows the teacher to gather information regarding students’ methods. This information will guide the discussion. The second opportunity for assessment is during the discussion. The discussion allows the teacher to observe and analyze the level of comprehension for each student. This information is vital. The objective of each lesson is to impart understanding on the students. In order to ensure that the objectives are being met during each lesson the teacher must constantly assess the students during the discussion. The specific aspects to be assessed are detailed during the “Explanation and Justification” section following each lesson.

Lastly, *learning environment* is addressed in all four lessons. An overview of an ideal learning environment is detailed in the initial “Lesson Goals and Criteria” section. These lesson goals and criteria were formed from current research on ideal mathematical practices in the primary grades. This learning environment can extend beyond these four lessons because they are not formed to be specific to linear measurement. Environments for specific lessons are outlined in the “Lesson Plans” as well as the “Lesson Explanation and Justification.” These specific learning environments are formed to help students achieve the unique lesson objective. Once again, these choices were based on current research on introducing linear measurement in the primary grades.

References

Ball, D. (1991) What’s All This Talk about “Discourse?”. *Arithmetic Teacher*, 39 (3), 44-48.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. W. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28*, 258-277.

Cobb, P., Gravemeijer, K., & Stephan, M. (2003). Coordinating Social and Individual Analysis: Learning as participation in Mathematical Practices. *Journal for Research in Mathematics Education*. 12, 68-103.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in Classroom Mathematical Practices. *The Journal of the Learning Sciences*, 10, 113-163.

Henningsen, M., & Stein, M. K. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics Education, 28*, 524-549.

Jackson, k. & Cobb, P. (2010). *Refining a Vision of Ambitious Mathematics Instruction to Address Issues of Equity.* Unpublished paper presented at National Council of Teachers of Mathematics Research Pre-Session, San Diego & American Educational Research Association, Denver.

Lappan, G. (1993). Implementing the Professional standards for teaching mathematics: What do we have and where do we go from here?. *Arithmetic Teacher,* 40(9), 524-26.

Russell, S. J. (2000). Developing Computational Fluency with Whole Numbers in the Elementary Grades. In Ferrucci, Beverly J. and Heid, M. Kathleen (Eds). The New England Math Journal, 32, (2), 40-54.

Stephan, M., Cobb, P., Gravemeijer, K., & Estes, B. (2001). The role of tools in supporting students' development of measuring conceptions. In A. Cuoco (ed.), 2001 *Yearbook of the National Council of Teachers of Mathematics* (pp. 63-­76)*.* Reston, VA: National Council of Teachers of Mathematics

Thompson, A., Philipp, R. Thompson, P., Boyd, B. (1994). Calculation and Conceptual Orientations in Teaching Mathematics. In A. Coxford (Ed.), *1994 Yearbook*

*of the NCTM* (pp. 79-92). Reston, VA: NCTM.

Whitenack, J., Knipping, N., Novinger, S., & Underwood, G. (2001) Second Graders Circumvent Addition and Subtraction Difficulties. *Teaching Children Mathematics*, 8, 228-232.