

An Analysis of the Random Walk Hypothesis based on Stock Prices, Dividends, and Earnings

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## Abstract

This paper explores the stationarity of price movements, dividend yields, and earnings yields for stock market indices and individual stocks within the broader context of the random walk hypothesis. In general, for a stock's price to follow a random walk, its future price must be unforecastable based on all currently available information in the stock market, including its price history. If a stock price is stationary in a given time period, its statistical process does not change over that time, meaning that the series has a deterministic trend, which could even be flat. This investigation tests for stationarity in the time series of prices and dividend yields of the Dow Jones Industrial Average (DJIA), the S&P 500 Index, and their underlying component stocks based on the results of univariate and panel unit root tests. I also test for the stationarity of earnings yields for the components of the DJIA. I find more evidence against the null hypothesis of a unit root for DJIA and its underlying stock prices than I do for the S&P 500 index and its component stocks. Dividend yields do not behave in a stationary fashion for the underlying components of the DJIA and S&P 500. Interestingly, earnings yields for the DJIA do exhibit more stationary-like behavior than the dividend yields for the DJIA and S&P 500, suggesting that earnings data have some predictability for stock prices.

## **1. Introduction**

It is no secret that in recent history names such as Warren Buffett and Bernard Madoff have risen to household status as a result of their influences on the economy and the way in which they have altered public perception of investing in the stock market. The persistent growth of fame of professional investment tycoons relies on the fact that the general public sees investment as a quick and efficient way to make money. Burton G. Malkiel defines investing as “a method of purchasing assets to gain profit in the form of reasonably predictable income (dividends, interest, or rentals) and/or appreciation over the long term<sup>1</sup>.” In order for investors to feel as if they are investing their money “wisely,” many attempt to make informed decisions by evaluating index performance, company and/or fund performance, general political and economic trends, and recommendations from trusted investment professionals, among other important factors. However, even upon acting on “informed” decisions, the typical investor will never see profit gains remotely similar to those enjoyed by Warren Buffett. This may lead one to

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<sup>1</sup> Malkiel (2007)

believe that, comparatively, Warren Buffett is smarter, better-informed, luckier, or better at predicting the future when it comes to investments. However, whether or not any of the above affirmations are true, like everyone else, even Buffett makes mistakes and loses money. This therefore begs the question: Can stock market movements really be predicted?

This investigation seeks to explore what is commonly known as the random walk hypothesis. As defined by Andrew W. Lo and A. Craig MacKinlay, the random walk hypothesis states that “in an informational efficient market—not to be confused with an allocationally or Pareto-efficient market—price changes must be unforecastable if they are properly anticipated i.e., if they fully incorporate the expectations and information of all market participants<sup>2</sup>.” Phrased alternatively, the random walk hypothesis asserts that “the history of stock price movements contains no useful information that will enable an investor consistently to outperform a buy-and-hold strategy in managing a portfolio<sup>3</sup>.” Finally, one may state the random walk hypothesis as:

$$p_t = \mu + p_{t-1} + \varepsilon_t \quad (1)$$

where  $p_t$  is the natural logarithm of a stock-price index  $P_t$  at time  $t$ ,  $p_{t-1}$  is the natural logarithm of a stock-price index  $P_{t-1}$  at time  $t-1$ ,  $\mu$  is the expected price change or drift, and  $\varepsilon_t$  should be independent and identically distributed (henceforth i.i.d.) random variables or a strict white noise.

If the hypothesis that stocks follow a random walk is entirely true, why is it that professional money managers and derivative analysts are some of the most highly-paid, highly-sought-out professionals in the world? Especially in the context of the economic crisis we face today, brought on fundamentally by the failure of some of the most highly-esteemed global

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<sup>2</sup> Lo and MacKinley (2002)

<sup>3</sup> Malkiel (2007)

financial institutions to accurately predict both consumer and market behavior, many investors have been struggling to determine the best way to maintain the value of their investments in the short-run and enjoy an increase in their value in the long-run. As a result, overall distrust in financial economists and investment professionals has grown significantly. This paper seeks to lay to rest some of this confusion and determine whether predictability indeed exists in some of the markets that unfailingly confound global investors.

This investigation explores this hypothesis and the various opposing theories that draw on fundamental and technical stock market analysis to predict stock market fluctuations. In doing so, the hope is to be able to answer the following: Are there significant dependencies evident in the movement of the stock market that prove that the stock market is not in fact a random walk? Ultimately, one will be led to either accept or reject the random walk hypothesis based on the results of univariate and panel unit root tests applied to overall indices, individual stocks, and panels of multiple stocks of which the utilized indices are comprised.

This investigation tests for stationarity in the time series of prices and dividend yields of the Dow Jones Industrial Average (DJIA), the S&P 500 Index, and their underlying component stocks based on the results of panel and univariate unit root tests. Additionally, I test for the stationarity of earnings yields for the thirty components of the DJIA. I find that prices of the DJIA and its underlying components behave in a more stationary manner than do the prices of the S&P 500 and its underlying components. Dividend yields behave in an equally non-stationary fashion for the underlying components of both the S&P 500 and DJIA. Furthermore, earnings yields for the DJIA prove to exhibit more stationarity than the dividend yields for the DJIA and S&P 500, signifying that earnings data have some predictability for stock prices.

## 2. Background

The exploration of the random walk hypothesis dates back to 1900 when a Random Walk model of market price was introduced by French mathematician Louis Bachelier in his study of the Brownian motion, i.e. the random movement of particles<sup>4</sup>. This initial study and its subsequent implications for the proven randomness of the stock market has since spawned ongoing debate regarding whether stock movements are completely random, semi-random, or decidedly forecastable. The opposing sides of this debate, both supported by innumerable investigations and proofs, can be explained generally on one side by Malkiel in A Random Walk Down Wall Street and on the other by Lo and MacKinlay in A Non-Random Walk Down Wall Street.

According to Malkiel, “short-run changes in stock prices cannot be predicted<sup>5</sup>.” As a result, it is his belief that instead of investing with a money-manager that aims to invest one’s money in stocks and funds that will “beat the market,” i.e. generate higher returns than the underlying index, one will experience higher long-run returns by following a buy-and-hold strategy of all of the underlying stocks in a given index. Rather than presenting conclusions drawn from regressions of actual economic data, Malkiel theoretically and empirically analyzes the prevailing methods used in technical and fundamental analyses and conjures doubt in their abilities to predict stock price movement. Malkiel’s famous experiment in which he asks his students to generate a stock chart of a fictitious asset initially selling at \$50 whose movements are based on the flip of a coin shows that while the price of a stock may appear to follow predictable cycles, “the ‘cycles’ in the stock charts are no more true cycles than the runs of luck

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<sup>4</sup> Nakamura and Small (2007)

<sup>5</sup> Malkiel (2007)

or misfortune of the ordinary gambler<sup>6</sup>.” Therefore, if the random walk hypothesis reigns as the truth, there is no way to accurately predict stock price movements.

This argument supporting the random walk hypothesis is supported by many economic scholars and financial economists. In their analysis of annual U.S. historical data of measures of interest rates, money, output, spending, and prices dating as far back as 1860 and ending in 1970, economists Charles R. Nelson and Charles I. Plosser fueled support for the random walk hypothesis. Using the Dickey-Fuller unit root test, Nelson and Plosser failed to reject the null hypothesis that macroeconomic time series are non-stationary variables without a tendency to return to a trend line. Hence, their investigation concludes that “macroeconomic models that focus on monetary disturbances as a source of purely transitory fluctuations may never be successful in explaining a large fraction of output variation<sup>7</sup>.” In a more recent study of stock market, exchange rate, and commodity market data, Tomomichi Nakamura and Michael Small find that the utilized financial data “whose first differences are independently distributed random variables or time-varying random variables” follow a random walk<sup>8</sup>. Unlike Nelson and Plosser, Nakamura and Small apply the small-surrogate method, which does not depend on data distribution, first to the original data and next to the first difference data.

In stark contrast to the aforementioned studies, it is a belief shared by many that “financial markets *are* predictable to some degree, but far from being a symptom of inefficiency or irrationality, predictability is the oil that lubricates the gears of capitalism<sup>9</sup>.” In a thorough investigation of U.S. market indices in first a general historical sense and second over specified periods of time, Lo and MacKinlay find the random walk hypothesis to be false. They attribute

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<sup>6</sup> Malkiel (2007)

<sup>7</sup> Nelson and Plosser (1982)

<sup>8</sup> Nakamura and Small (2007) specifically use data from Standard & Poor’s 500, Nikkei225, the British Pound/US dollar and Japanese Yen/US dollar exchange rates, gold prices, and crude oil prices.

<sup>9</sup> Lo and MacKinlay (2002)

the success experienced by Random Walk scholars to their tendency to confuse the random walk hypothesis with the closely-related Efficient Markets Hypothesis<sup>10</sup>. In summary, non-believers of the random walk hypothesis have found that while complete price change predictability may not be possible, the opposite, total randomness in price movement, fails to be true as well.

Referring directly to Lo and MacKinlay's research, a study of asset price memory in the short- and long-term in 44 economies, including both emerging and industrialized nations, finds that "markets with a poor Sharpe ratio are more likely to reject the random walk than better performing markets<sup>11</sup>." In other words, markets yielding lower returns for a given risk level are more likely to be predictable compared to markets yielding higher returns. While this study does not comprehensively disprove the random walk hypothesis, it casts doubt on the truth and universality of its assertions. Similarly, in an investigation of a dataset including 249 macroeconomic variables in the G7 countries, the evidence in favor of the random walk hypothesis appears weaker than in previous investigations of the same data<sup>12</sup>. Using a series of unit root tests, Yunus Aksoy and Miguel A. Leon-Ledesma draw stronger conclusions against unit roots in nominal and real asset prices. The absence of unit roots implies that the series are stationary, leading to the conclusion of the probability of asset price predictability and therefore weaker support for the random walk hypothesis.

In addition to the aforementioned conclusions drawn with regard to large, developed markets, the random walk hypothesis has been found false using data from smaller, lower-volume markets. Oktay Tas and Salim Dursunoglu use Dickey-Fuller unit root tests to test for

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<sup>10</sup> Lo and MacKinlay (2002) further detail that "unforecastable prices need not imply a well-functioning financial market with rational investors, and forecastable prices need not imply to opposite."

<sup>11</sup> Eitelman and Vitanza (2008)

<sup>12</sup> Aksoy and Leon-Ledesma (2008)



weak form market efficiency in the Istanbul Stock Exchange<sup>13</sup>. After obtaining consistent results from their series of tests, Tas and Dursunoglu conclusively reject the random walk hypothesis for the ISE. Equally comprehensive conclusions were drawn in a study of 11 African stock markets<sup>14</sup>. Performing joint variance ratio tests on the African stock market data from January 2000 to September 2006, Graham Smith was able to reject the random walk hypothesis in all 11 markets. In conclusion, just as many investigations have failed to reject the random walk hypothesis using a variety of test statistics and citing data from diverse markets across the globe, many other investigations have concluded the exact opposite.

The inconsistency of conclusions and lack of significant progress made in the quest for agreement over the truth or falsehood of the random walk hypothesis has led me to believe that although many investigations relating to this topic have been and are currently being conducted on wide ranges of financial data, the present results are inadequate. Many of the aforementioned studies rely on unit root tests (with slight variances) to determine whether the utilized macroeconomic and/or financial data follow a random walk. In my investigation, I plan to take this method one step further by implementing both unit root and panel unit root tests on the indices and underlying stocks in the Dow Jones Industrial Average, representative of the United States stock market, the FTSE 100 Index, comprising 100 companies listed on the London Stock Exchange, and the Nikkei 225 Index, a comprehensive index for the Tokyo Stock Exchange.

The need to test for unit roots when investigating the random walk hypothesis arises from the fact that “if a variable contains a unit root then it is non-stationary<sup>15</sup>.” It is necessary to determine whether the data variables being tested are non-stationary because testing non-

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<sup>13</sup> Tas and Dursunoglu (2005)

<sup>14</sup> Smith (2008) used stock market data from Botswana, Côte d’Ivoire, Egypt, Ghana, Kenya, Mauritius, Morocco, Nigeria, South Africa, Tunisia and Zimbabwe.

<sup>15</sup> Harris (1995)

stationary variables may lead to spurious regression results that could imply a significant economic relationship when in fact one may not exist. Thus, in general terms, if a unit root test proves that the data have a unit root, it is implied that the data are non-stationary and thus follow a random walk. Conversely, if the results show that the data do not have a unit root, one would reject the null hypothesis that the data follow a random walk.

The panel unit root test, a recently developed adaptation of the standard unit root test, is meant to increase the power of the unit root test by increasing the number of the samples being tested. This can be done, for example, by increasing the amount of time series and cross-sectional data used in the investigation. Economists and research analysts have increasingly turned to panel unit root tests to rectify discrepancies consistent in testing for unit roots in data spanning diverse economic variables, regions, and periods of time. For instance, Yoosoon Chang presents the “nonlinear IV methodology” which he finds “resolves the inferential difficulties in testing for unit roots arising from the intrinsic heterogeneities and cross-dependencies of panel models<sup>16</sup>.” Chang uses the usual IV estimation in the Augmented Dickey-Fuller regression to test for unit roots in a panel setting of various cross-sectional data, acknowledging and attempting to resolve the issues that tend to arise due to cross-sectional dependencies.

In this particular investigation, the null hypothesis is that the time series has a unit root and is thus non-stationary against the alternative. If results find that the data have a unit root, one will be led to fail to reject the null hypothesis and accept the implication that the data are non-stationary and may very well follow a random walk. On the other hand, if unit roots are absent in the data, one will be led to reject the null hypothesis in favor of the conclusion that the data are stationary and likely do not follow a random walk. Both univariate unit root tests and panel unit root tests will be necessary in this investigation. Whether a unit root is present will be

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<sup>16</sup> Chang (2003)

determined for each index as a whole and each underlying individual stock in the indices using a univariate unit root test. A panel unit root test will be utilized to determine if the panel of underlying stocks comprising each index has a unit root.

### **3. Identifying the Presence of Unit Roots in Global Market Indices**

In conducting the aforementioned unit root tests for the purpose of analyzing the stationarity of prices for overall market indices as well as their underlying components, the logarithm of closing prices will be used in order to normalize the data. Also, the time series will be in units of months. In order to determine the appropriate number of lags to be included in the tests in order to account for short-run dynamics affecting the data, the number of lags that minimizes the Akaike Information Criterion (AIC) will be implemented. The AIC is a commonly used measure of goodness of fit of a statistical model. The AIC compares several models, the best of which has the lowest AIC. Finally, while the data for the assorted indices and stocks date back to various dates, all data used end on October 1, 2009.

This investigation takes a top-down approach by first testing generally for unit roots in broad market indices from various countries. Specifically, closing price data for the following stock market indices are used: the S&P 500, the Dow Jones Industrial Average, the NASDAQ, the FTSE 100, the Deutscher Aktien Index (DAX), the CAC 40, the Nikkei 225, the Hang Seng, and the Straits Times. Using these data, a regression of the monthly closing level of each index on time is conducted using the Augmented Dickey-Fuller (ADF) test for unit roots. The ADF test is modeled as follows:

$$\Delta p_t = \alpha + (\beta - 1) p_{t-1} + \varepsilon_t \quad (2)$$

where the null hypothesis is that  $\beta - 1 = 0$ , i.e. the variable contains a unit root and the alternative hypothesis is  $\beta - 1 < 0$ , i.e. the variable does not contain a unit root. The results from these tests can be seen in Table 3.1.

*Table 3.1: Augmented Dickey-Fuller Test Results for Global Market Indices*

| Index         | Lags | Test Statistic | 1% critical value | 5% critical value | 10% critical value | Beginning Date |
|---------------|------|----------------|-------------------|-------------------|--------------------|----------------|
| S&P 500       | 7    | -2.057         | -3.96             | -3.41             | -3.12              | January 1950   |
| DJIA          | 6    | -3.89          | -3.96             | -3.41             | -3.12              | October 1928   |
| Nasdaq        | 2    | -2.238         | -3.96             | -3.41             | -3.12              | February 1971  |
| FTSE 100      | 1    | -2.003         | -3.988            | -3.428            | -3.13              | April 1984     |
| DAX           | 1    | -1.633         | -3.998            | -3.433            | -3.133             | November 1990  |
| CAC 40        | 1    | -1.501         | -3.995            | -3.432            | -3.132             | March 1990     |
| Nikkei 225    | 1    | -2.783         | -3.988            | -3.428            | -3.13              | January 1984   |
| Hang Seng     | 1    | -2.421         | -3.989            | -3.429            | -3.13              | December 1986  |
| Straits Times | 3    | -3.119         | -3.99             | -3.43             | -3.13              | December 1987  |

As shown in the above table, for all of the stock market indices investigated exclusive of the Dow Jones Industrial Average, one fails to reject the null hypothesis at the 10% critical value level. This means that for these indices, the data do appear to have a unit root. Possessing a unit root implies non-stationarity, leading to the preliminary conclusion that these indices do behave in a random-walk-like fashion. However, this test says little to nothing about the stationarity of the indices' underlying components or the stationarity of the panel of underlying components. Additionally, note that randomness is not completely analogous to non-stationarity.

The DJIA is the lone index for which one can reject the null hypothesis at the 5% critical value level. This means that according to the data collected and confirmable at the 5% level, the DJIA time series does not possess a unit root. Therefore, it follows that it is likely that the closing values of the DJIA do have a stationary relationship over the time period for which the data are collected. While it would be presumptuous to automatically conclude from this test that the movements of the DJIA over time do not follow a random walk, this presents an interesting

conclusion that merits further investigation. Reasons that could potentially lead to spurious conclusions include the relatively smaller size of the DJIA in terms of components compared to many of the other indices, the specific dataset used in my particular investigation, and the imperfect correlation between stationarity and randomness. However, compared to the other sets of data for which the presence of unit roots has been tested, the dataset for the DJIA is the most extensive, dating back to 1928.

#### **4. The Stationarity of Stock Prices**

##### *4.1. The Dow Jones Industrial Average*

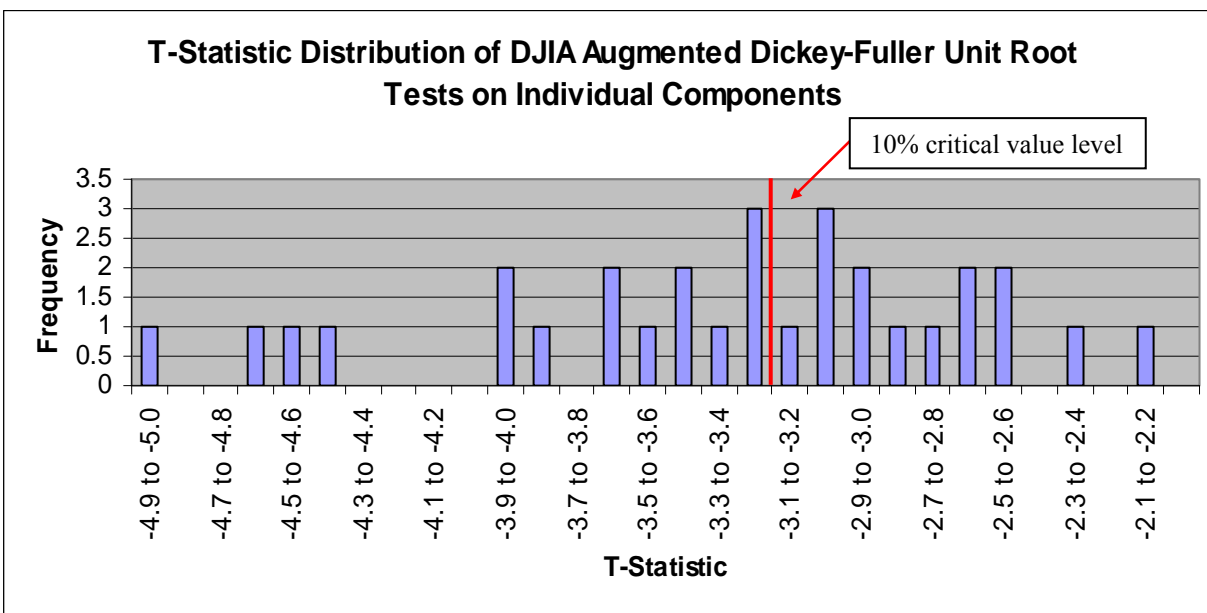
The next step in this investigation involved performing Augmented Dickey-Fuller tests on each of the thirty components of the DJIA. These tests were performed on datasets of varying length according to data availability. The results vary greatly in terms of ability or failure to reject the null hypothesis that the time series possess unit roots. While one is able to reject the null hypothesis with considerable confidence for certain stocks, e.g. CAT (Caterpillar Inc.), JNJ (Johnson & Johnson), PG (Procter & Gamble Co.), and UTX (United Technologies Corp.), one unequivocally fails to reject the null hypothesis at all critical value levels for many of the other stocks tested.

The explanations for the extreme variation in outcomes can come from many sources. As mentioned previously with regards to testing for unit roots in global market indices, discrepancies can arise from the particular dataset utilized as many of the time series extend back to distinct years. Additionally, because these data belong to time series for stocks of individual companies rather than comprehensive indices, movements in the pattern of closing prices can arise from company-specific events or histories. For example, Caterpillar Inc. engaged in many acquisitions during the span of the dataset used, likely affecting the stationarity of its stock's

prices during certain periods of time, altering the viability of the conclusion to reject the null hypothesis that the series has a unit root.

However, when the aggregate conclusions from the unit root tests are depicted graphically rather than numerically, a compelling story emerges. In order for the null hypothesis to be rejected at least at the 10% critical value level, the test statistic from the ADF tests must be at least less than -3.12 and at most less than -3.151.

*Figure 4.1.1*



The results from each of the individual ADF unit root tests are compiled in Figure 4.1.1. Each test statistic is sorted into its appropriate interval; the intervals are sorted in increments of 0.1. The red line in the depiction above marks the -3.12 to -3.151 range, below which the test statistics from each ADF test must fall in order to reject the null hypothesis at the 10% critical value level. As can be seen pictorially, the majority of components of the DJIA can be shown to not possess a unit root at varying levels of confidence. To be specific, between 53 and 57% of the test results show a rejection of the null hypothesis of the ADF test at least at the 10% level.

Thus, for many of the components of the DJIA, one can conclude that the time series do not have unit roots and may very well exhibit stationary relationships.

Though Figure 4.1.1 above essentially depicts a panel unit root test, it is necessary to consider the numerical results of various panel unit root tests for the panel of stocks comprising the DJIA. For completeness, the results from three separate panel unit root tests are considered. Table A.1 in Appendix A shows the results for three panel unit root tests: the Im-Pesaran-Shin unit root test, the Breitung unit root test, and the Levin-Lin-Chu unit root test. Each of these tests was conducted on a panel of the thirty stocks that comprise the DJIA. The dataset extends back to June, 2001, a date chosen so that all thirty components could be included in the regression based on the individual stock data collected. A total of seven lags were incorporated into each test, as seven was the maximum lag length required for the ADF unit root tests on the individual stocks. One is able to reject the null hypothesis of the Im-Pesaran-Shin unit root test that states that all of the panels have unit roots. This is consistent with the previous test results which have shown that the DJIA index does not have a unit root and that the majority of underlying components of the DJIA do not have unit roots. One is furthermore able to reject the null hypotheses of the Breitung and Levin-Lin-Chu unit root tests which also state that all series contain unit roots. Again, this result is consistent with previous test results that showed variability in the ability to reject the null hypothesis of the unit root test.

It is important to note that one of the assumptions inherent in ADF unit root tests is that the time series exhibit stability. This is likely an inaccurate assumption however for many stocks and indices whose time series contain structural breaks. With regards to this, the issue is wherein the strength of the panel unit root test lies. Panel unit root tests are able to test individual panels for the presence of unit roots using smaller cross-sections of time series data than are possible for

individual ADF unit root tests. Therefore, in order to take advantage of the strength of the panel unit root test versus the individual ADF unit root test, it is helpful to examine its results over short time periods, i.e. multiple year-long periods instead of one multi-year period. The results from these tests can be viewed in tables A.2 through A.9 in Appendix A.

There are several conclusions to be drawn from the panel unit-root test results for the periods 2001 to 2002, 2002 to 2003, 2003 to 2004, 2004 to 2005, 2005 to 2006, 2006 to 2007, 2007 to 2008, and 2008 to 2009 for the components of the DJIA. For seven out of eight of these time periods, namely the time periods 2002 to 2003 through 2008 to 2009, chronologically, the Im-Pesaran-Shin, Breitung, and Levin-Lin-Chu unit-root tests agree and lead to rejection of the null hypothesis in favor of the conclusion that not all panels possess unit roots. This result agrees with the results found when conducting panel unit-root tests for the span of eight years from 2001 to 2009 in one test. There is a discrepancy, though, in the test results for the period beginning in 2001 and ending in 2002. The Im-Pesaran-Shin and Breitung unit root tests fail to reject the null hypothesis, while the Levin-Lin-Chu unit root test rejects the null hypothesis. This variability in results shows that it is more likely than not that for the twelve month-period beginning in 2001 and 2002, the closing prices of the thirty components of the DJIA are non-stationary. It is possible that there are short-run dynamics affecting the stationarity of these stocks during this twelve-month period that is detected in a panel unit root test focusing only on that period. This proves that it is in fact worthwhile to take advantage of the unique ability of the panel unit root test to test for the presence of unit roots in a large number of panels over a small number of time periods as the results yielded are distinct.



#### 4.2. The S&P 500 Index

Following the same process as the investigation of the Dow, ADF unit root tests were performed individually on each of the 500 components of the S&P 500 index. As was true with the components of the Dow, the results of the ADF tests vary to an even larger degree due to the much greater number of components underlying the index and hence the greater variability between company histories. However, it is important to note the overall trend of the results found from these tests.

Figure 4.2.1

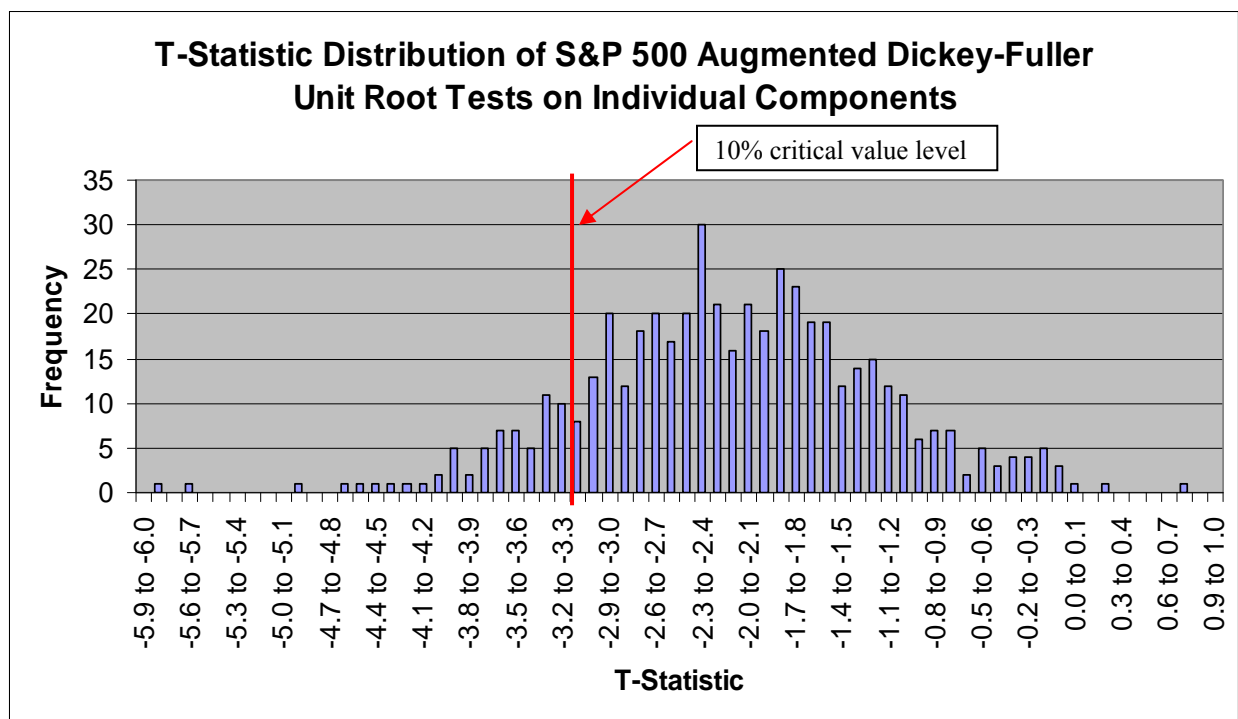


Figure 4.2.1 above depicts the overall trend of the individual ADF test results in aggregate, compiled in the same fashion as the previously shown graph of the ADF test results for the DJIA. The red line approximates the -3.12 to -3.24 range, the comprehensive range below which the null hypothesis of the ADF unit root test can be rejected at least at the 10% critical value level. In contrast to the results obtained from the data used for the components of the

DJIA, a minority of the components of the S&P 500 were shown to reject the null hypothesis at least at the 10% level. Specifically, only between 10 and 14.5% of the time series do not contain unit roots when evaluated at the 10% level. This represents a small minority compared to the aggregate results from the DJIA tests. Since the significance level is 10%, one would expect about 10% of the sample of tests to exhibit rejections of the null hypothesis. Thus, the fact that 10-14.5% of the unit root tests were able to reject the null hypothesis remains in line with our expectations, leaving little evidence against the null hypothesis for the S&P 500. As is also true for the DJIA, this general conclusion is consistent with the ADF unit root test of the S&P 500 index, for which the null hypothesis fails to reject. However, it is important to note that 10-14.5% rejection of the null hypothesis is a significant minority.

While this aggregate pictorial representation serves as a preliminary test for unit roots in the panel of stocks underlying the S&P 500 index, it is necessary to implement and analyze results from statistical panel unit root tests. Featured in table B.1 in Appendix B are tabulated results of the Im-Pesaran-Shin, Breitung, and Levin-Lin-Chu panel unit root tests. Each test was conducted on 490 panels out of the 500 stocks in the S&P 500 using data as far back as November, 2006. Twelve lags were used in conducting the Breitung and Levin-Lin Chu panel unit root tests while eight lags were incorporated into the Im-Pesaran-Shin panel unit root test. This difference in lags is due to the fact that the Im-Pesaran-Shin unit root test is impossible to conduct using more than eight lags. The ten stocks were omitted from these tests due to data restrictions<sup>17</sup>.

One is able to reject the null hypotheses of the Im-Pesaran-Shin, Breitung, and Levin-Lin-Chu panel unit root tests, which signifies that not all panels possess unit roots. This is consistent with the results from the ADF unit root tests on the individual components of the

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<sup>17</sup> The ten omitted stocks did not have enough available monthly price data in order to conduct the unit root tests.

index, which showed that between 10 and 14.5% of the stocks underlying the S&P 500 do not contain unit roots. The results from the ADF unit root test of the S&P 500 index as a whole show that as a collective, the 500 stocks within the S&P 500 do not move in a stationary fashion. But, when examined as individual panels, one is able to find certain components of the index that indeed move in a stationary manner. This overarching difference in results between the DJIA and S&P 500 may be caused by the higher level of company-related synchronicity and smaller number of component stocks in the DJIA when compared to the S&P 500.

In order to rectify the problem of assumed stability by the unit root test, it is necessary to consider the results of panel unit roots for the stocks in the S&P 500 index when segmented into year-long periods. The results of these panel unit root tests can be seen in tables B.2 through B.4 in Appendix B.

Tables B.2 through B.4 show the results from Im-Pesaran-Shin, Breitung, and Levin-Lin-Chu unit root tests on 490 stocks in the S&P 500 for the periods 2006 to 2007, 2007 to 2008, and 2008 to 2009. For only one period, beginning in 2008 and ending in 2009, did all three panel unit root tests agree. For this particular time series, the null hypothesis was uniformly rejected. On the other hand, for the period beginning in 2006 and ending in 2007, only the Im-Pesaran-Shin and Levin-Lin-Chu unit root tests rejected the null hypothesis. Furthermore, for the year between 2007 and 2008, the sole unit root test rejecting the null hypothesis is the Levin-Lin-Chu test.

This disparity in results from the panel unit root tests causes distrust in the results of the three-year-period panel unit root tests illustrated in table B.1 which consistently reject the null hypothesis. Overall, in comparison with the DJIA, the components underlying the S&P 500 seem to display far less stationarity in their price movements.

## 5. The Stationarity of Dividend Yields

While the results from the unit root tests on time series of index and stock prices provide some initial insight into their potential stationarity, it is necessary to explore other market indicators in order to strengthen our understanding of stationary relationships present in the stock market. Based on John Cochrane's studies in "Permanent and Transitory Components of GNP and Stock Prices," this investigation continues with analysis of the stationarity of the relationship between dividends and stock prices for the components of the DJIA and the S&P 500. Cochrane finds that "shocks to prices holding dividends constant are almost entirely transitory"<sup>18</sup>. I do this by exploring the mean-reverting effect that dividends have been shown to have on prices by testing for the presence of unit roots in dividend-price ratios, i.e. dividend yields.

Although preliminary results lead to the conclusion that price movements of the DJIA are potentially stationary while those of the S&P 500 are most likely non-stationary, many believe in the probable non-stationarity of stock prices in general. Additionally, as a historically fundamental measure of overall company profitability, which undeniably changes with universal economic/market shocks as well as company-specific occurrences, in general, dividends are believed to move in a non-stationary manner. Therefore, by the nature of statistical processes, it is my hypothesis that dividend yields will exhibit stationary behavior.

In order to test for stationarity in the dividend yields of the underlying components of the DJIA and the S&P 500, the ADF unit root test is further employed. Rather than using monthly time series data as were used for prices, half-yearly data are used for dividend yields. These tests regress the logarithm of the ratio of the sum of the dividends during the given half-yearly period and the closing price at the beginning of the half-yearly period, as a percentage, on time. For

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<sup>18</sup> Cochrane (1994)

example, for data from the first half of 2009, one would consider the following:

$$\log((\Sigma(D_{\text{jan1-jun30}, 2009})/P_{\text{jan1}, 2009}) * 100) \quad (3)$$

where  $(\Sigma(D_{\text{jan1-jun30}, 2009})$  represents the sum of the dividends paid during the period January 1, 2009 to June 30, 2009 and  $P_{\text{jan1}, 2009}$  represents the closing price on January 1, 2009. [In cases where there are zero dividends, 0.01 is substituted for the dividend-price ratio.] As with the index and price data, dividend data are obtained from the Compustat and CRSP databases as well as Yahoo Finance. Furthermore, lags are incorporated in the datasets based on minimizing the AIC and time series length varies based on data availability and company history. Unlike in the investigation of price stationarity, panel unit root tests are not conducted on dividend yields. This is due to the fact that the length of the majority of the dividend datasets is insufficient to accurately test for unit roots in a large panel of stocks.

### *5.1. The Dow Jones Industrial Average*

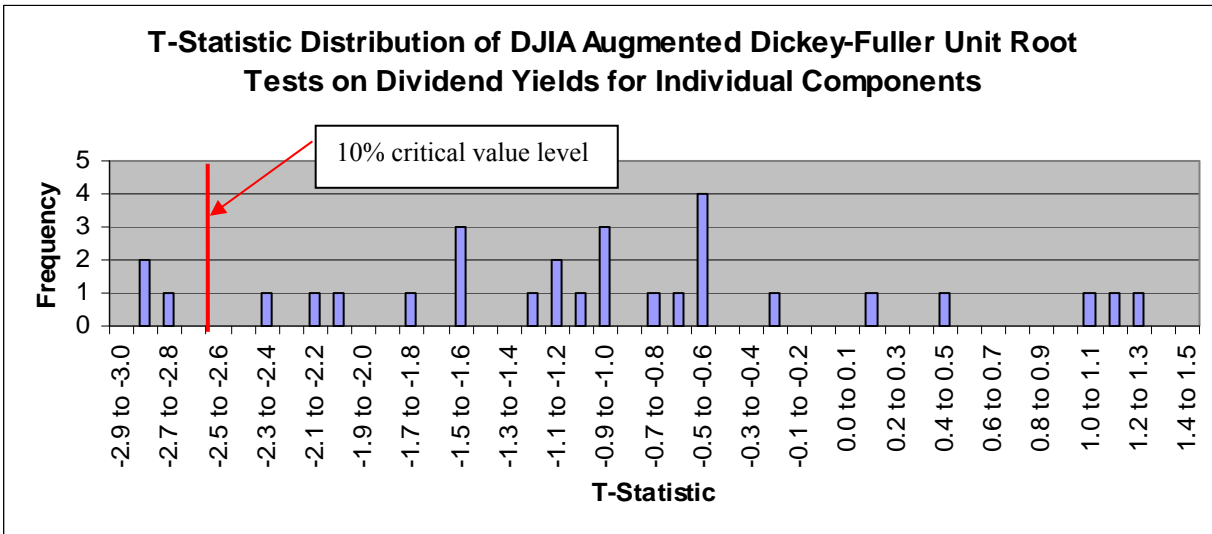
Due to the variability in results from the ADF unit root tests on dividend yields of the underlying components of the DJIA, it is helpful to view the aggregate results pictorially in Figure 5.1.1, which depicts the distribution of test statistics from the twenty-nine ADF tests grouped into increments of 0.1<sup>19</sup>. In order to reject the null hypothesis at at least the 10% critical value level, the test statistic must fall within or below the range -2.583 to -2.63, approximated by the red line Figure 5.1.1. As can be concluded visually from the graph, a minority of dividend yields are shown to reject the null hypothesis that the time series possesses a unit root. In particular, the dividend yields of three companies, or approximately 10.34% of the companies evaluated, do not have unit roots and thus exhibit stationary behavior. This result proves to be a

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<sup>19</sup> Insufficient data available for testing stationarity of dividend yields for Cisco Systems, Inc. (CSCO).

rejection of the initial hypothesis of this investigation, as it appears that overall, dividend yields do not behave in a stationary manner for the DJIA.

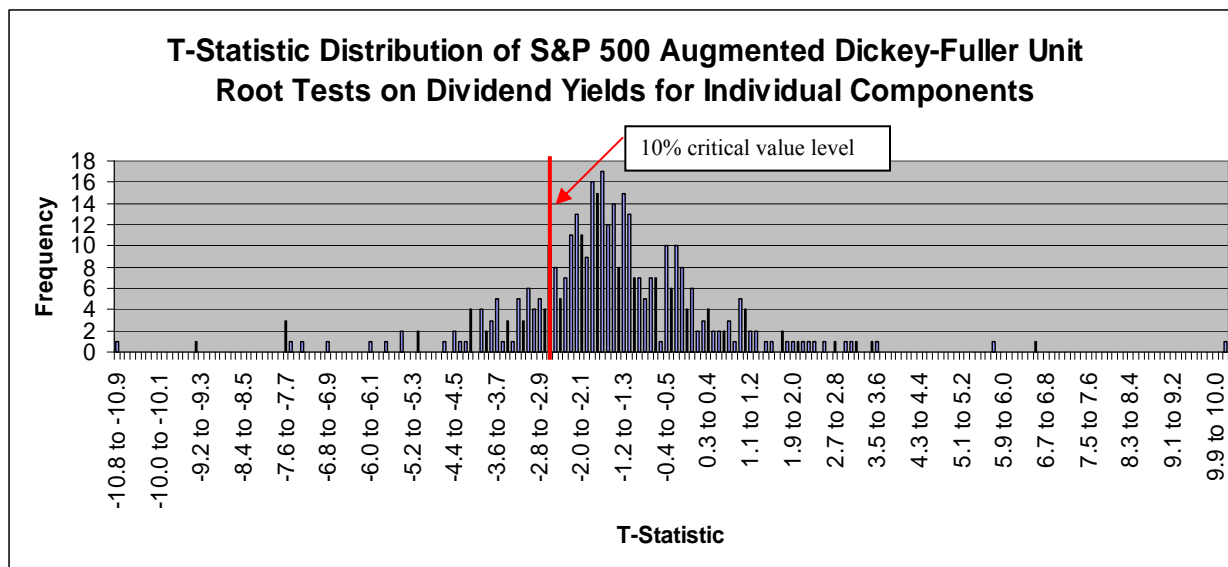
Figure 5.1.1



## 5.2. The S&P 500 Index

Due to data restrictions, the results from ADF tests on dividend yields of 382 of the 500 companies comprising the S&P 500 Index are depicted in the same fashion as Figure 5.1.1 in Figure 5.2.1 below. The red line approximates the -2.583 to -2.63 range, within or below which the test statistic must fall in order to reject the null hypothesis that the time series has a unit root at a critical value level of at least 10%. Between 20.57 and 22.66% of the companies evaluated are shown to reject the null hypothesis at at least the 10% critical value. Thus, I find more evidence against the null hypothesis of a unit root for dividend yield data for the S&P 500 and its underlying components than I do for prices. However, the difference in the proportion of companies comprising the S&P 500 for which dividend yield data reject the null hypothesis compared to price data is not dramatic.

Figure 5.2.1



## 6. The Stationarity of Earnings Yields

With the knowledge that, in general, it cannot be concluded that dividend yields behave in a stationary manner, one is led to speculate about the stationarity of earnings yields, another widely studied measure of a company's well-being. A company's earnings yield is measured by its earnings to price ratio. Unlike dividends which are not paid by every company, earnings per share are reported for all stocks in the market. Therefore, when considering time series of earnings yields in comparison to time series of dividend yields, considerably more data are available and consistent, making for more robust regressions.

Once again, the ADF unit root test is utilized to analyze the stationarity of earnings yields for the stocks comprising the DJIA. Quarterly time series data for the thirty stocks within the DJIA are used in this part of the investigation from the Compustat database. Note that lags are once more incorporated according to minimization of the AIC. All earnings data span the period of time from 1997 to 2009. Due to the occasional presence of negative earnings to price ratios, the logarithm of one plus the reported earnings per share divided by closing price is regressed on

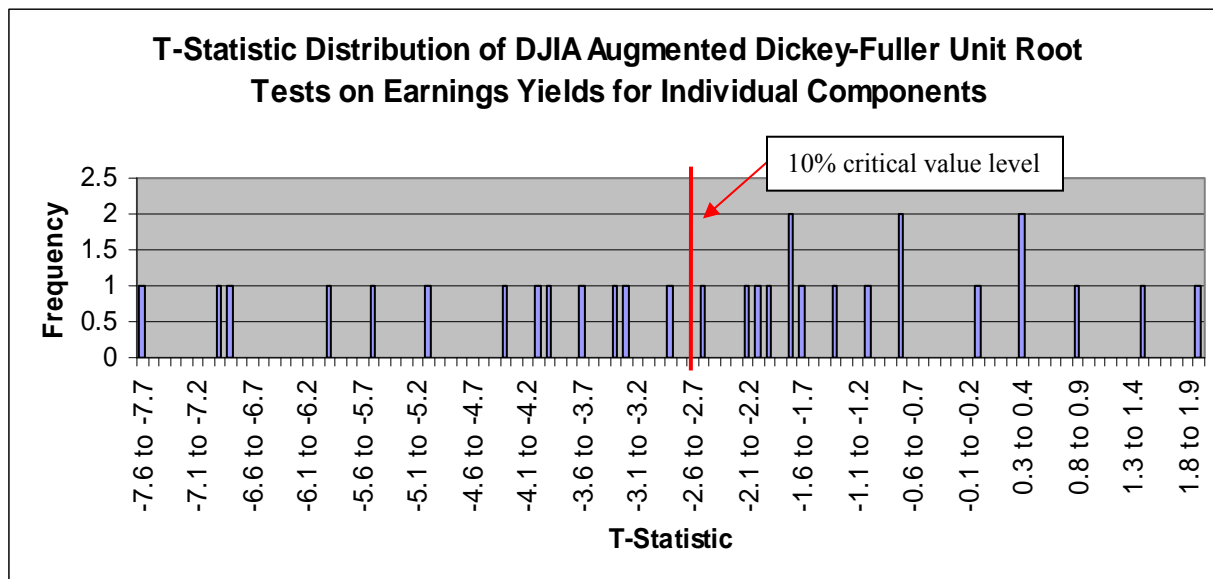
time. For example, if considering the first quarter of 2009, one would regress the following variable on time:

$$\log(1+E_{1Q2009}/P_{1Q2009}) \quad (4)$$

where  $E_{1Q2009}$  represents the company's reported earnings per share for the first quarter of 2009 and  $P_{1Q2009}$  represents the company's closing price for the first quarter of 2009. In the interest of comparing the results of the ADF tests on earnings yields with those on dividend yields, panel unit root tests are not conducted.

Figure 6.1 below depicts the results of the unit root tests on earnings yields for the components of the DJIA. The graph organizes the obtained test statistics into intervals of 0.1. The red line approximates the -2.604 to -2.63 range, within or below which the test statistic must fall in order to lead to rejection of the null hypothesis that the time series possesses a unit root at a critical value level of at least 10%.

Figure 6.1



The results show that for the DJIA, a much higher percentage of companies' earnings yields compared to dividend yields behave in a stationary manner. Specifically, just over 43% of



the stocks comprising the DJIA reject the null hypothesis of the ADF unit root test at least at the 10% critical value level. Hence, almost half of the companies' earnings yields do not have unit roots and are stationary.

## 7. Conclusion

It can be concluded that in general, the price movements of the Dow Jones Industrial Average are more stationary than those of the S&P 500 Index in recent history. This difference in behavior is likely due in large part to the comparatively high level of company synchronicity in the components underlying the DJIA. While exhibiting stationarity and not following a random walk are not completely analogous, one can be led to conjecture that the DJIA, as a more stationary index, does not follow a random walk, while the S&P 500, as a generally non-stationary index, does follow a random walk.

Interpretation of the results of the ADF unit root tests on dividend yields for the companies underlying the DJIA and S&P 500 does not lead to the conclusion that dividend yields are stationary, contrary to my preliminary hypothesis and earlier studies using older data. Only about 10% of companies comprising the DJIA were shown to reject the null hypothesis of the unit root test and hence not possess unit roots for dividend-price ratios. Additionally, only 5-10% more of the companies comprising the S&P 500 were shown to reject the null hypothesis of the unit root test for dividend yield data when compared to price data. The increase in the prevalence of growth stocks during the time period which my datasets generally span is likely to be a main cause of the suspected non-stationarity of dividend yields seen in the results above. A growth stock is defined as stock of a company whose earnings are expected to increase at a rate that is above average relative to the market. Instead of paying dividends to investors, growth stocks typically prefer to reinvest retained earnings in projects that contribute to the company's

growth and profitability. Therefore, although much focus has historically been on the fundamental relationship between dividends and prices as is the case in Cochrane's study, the results of this investigation suggest a shift away from consideration of dividend yields when analyzing stationary relationships in financial data from the past several decades.

Finally, based on the results from the ADF tests on earnings yields for the components of the DJIA, one can conclude that earnings yields are likely to be a more stationary process than dividend yields. Although this segment of the investigation was conducted solely on the companies within the DJIA, judging from the relatively similar percentages of rejection of the null hypothesis of the ADF unit root test for dividend yields for the DJIA and S&P 500 when compared with percentages of rejection of the null hypothesis of the ADF test for prices, one can suppose that such would be the case for earnings yields on the S&P 500 as well. As a result, due to the considerably higher percentage of stationary earnings yields, it may be more meaningful for financial data analysts to shift their focus away from dividend yields and toward price-earnings ratios when considering datasets from primarily the 1990's and 2000's. In order to firmly grasp the full weight of this assertion, an investigation of the stationarity of earnings yields for the companies comprising the S&P 500 would be an useful addition to this study.

## Appendices

### Appendix A: DJIA Test Results

*Table A.1: Panel Unit Root Test Results for the Dow Jones Industrial Average Index*

|                   | Im-Pesaran-Shin unit-root test | Breitung unit-root test    | Levin-Lin-Chu unit-root test |
|-------------------|--------------------------------|----------------------------|------------------------------|
| Test Statistic    | -1.9857                        | -2.6072                    | -13.0174                     |
| P-value           | 0.0235                         | 0.0046                     | 0.9785                       |
| Number of Lags    | 7                              | 7                          | 7                            |
| Number of Panels  | 30                             | 30                         | 30                           |
| Number of Periods | 101                            | 101                        | 101                          |
| Date range        | June, 2001 – October, 2009     | June, 2001 – October, 2009 | June, 2001 – October, 2009   |

*Tables A.2 through A.9: Panel Unit Root Test Results for the Dow Jones Industrial Average Index Using Year-Long Periods*

*Table A.2*

|                   | Im-Pesaran-Shin unit-root test | Breitung unit-root test       | Levin-Lin-Chu unit-root test  |
|-------------------|--------------------------------|-------------------------------|-------------------------------|
| Test Statistic    | 0.8735                         | 5.1950                        | -6.4201                       |
| P-value           | 0.8088                         | 1.000                         | 0.0889                        |
| Number of Panels  | 30                             | 30                            | 30                            |
| Number of Periods | 12                             | 12                            | 12                            |
| Date range        | October, 2001 – October, 2002  | October, 2001 – October, 2002 | October, 2001 – October, 2002 |

*Table A.3*

|                   | Im-Pesaran-Shin unit-root test | Breitung unit-root test       | Levin-Lin-Chu unit-root test  |
|-------------------|--------------------------------|-------------------------------|-------------------------------|
| Test Statistic    | -3.3779                        | -2.4488                       | -18.9756                      |
| P-value           | 0.0004                         | 0.0072                        | 0.0000                        |
| Number of Panels  | 30                             | 30                            | 30                            |
| Number of Periods | 12                             | 12                            | 12                            |
| Date range        | October, 2002 – October, 2003  | October, 2002 – October, 2003 | October, 2002 – October, 2003 |

Table A.4

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -2.5074                          | -0.0862                          | -17.7716                         |
| P-value           | 0.0061                           | 0.4657                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2003 –<br>October, 2004 | October, 2003 –<br>October, 2004 | October, 2003 –<br>October, 2004 |

Table A.5

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -4.0177                          | -1.8131                          | -14.1721                         |
| P-value           | 0.0000                           | 0.0349                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2004 –<br>October, 2005 | October, 2004 –<br>October, 2005 | October, 2004 –<br>October, 2005 |

Table A.6

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -3.1339                          | -0.4747                          | -14.8349                         |
| P-value           | 0.0009                           | 0.3175                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2005 –<br>October, 2006 | October, 2005 –<br>October, 2006 | October, 2005 –<br>October, 2006 |

Table A.7

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -2.7607                          | -0.7988                          | -16.9369                         |
| P-value           | 0.0029                           | 0.2122                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2006 –<br>October, 2007 | October, 2006 –<br>October, 2007 | October, 2006 –<br>October, 2007 |

Table A.8

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -4.4820                          | -2.9990                          | -15.2137                         |
| P-value           | 0.0000                           | 0.0014                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2007 –<br>October, 2008 | October, 2007 –<br>October, 2008 | October, 2007 –<br>October, 2008 |

Table A.9

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -2.2692                          | -0.1677                          | -13.1494                         |
| P-value           | 0.0116                           | 0.4334                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2008 –<br>October, 2009 | October, 2008 –<br>October, 2009 | October, 2008 –<br>October, 2009 |

*Appendix B: S&P 500 Test Results*

Table B.1: Panel Unit Root Test Results for the S&amp;P 500 Index

|                   | Im-Pesaran-Shin unit-root test    | Breitung unit-root test           | Levin-Lin-Chu unit-root test      |
|-------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Test Statistic    | -4.3582                           | -7.4933                           | -46.5339                          |
| P-value           | 0.0000                            | 0.0000                            | 1.0000                            |
| Number of Lags    | 8                                 | 12                                | 12                                |
| Number of Panels  | 490                               | 490                               | 490                               |
| Number of Periods | 36                                | 36                                | 36                                |
| Date range        | November, 2006 –<br>October, 2009 | November, 2006 –<br>October, 2009 | November, 2006 –<br>October, 2009 |

*Tables B.2 through B.4: Panel Unit Root Test Results for the S&P 500 Index Using Year-Long Periods*

*Table B.2*

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -9.4851                          | 4.4550                           | -58.4647                         |
| P-value           | 0.0000                           | 1.0000                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2006 –<br>October, 2007 | October, 2006 –<br>October, 2007 | October, 2006 –<br>October, 2007 |

*Table B.3*

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | 6.5529                           | 26.7396                          | -43.4758                         |
| P-value           | 1.0000                           | 1.0000                           | 0.0005                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2007 –<br>October, 2008 | October, 2007 –<br>October, 2008 | October, 2007 –<br>October, 2008 |

*Table B.4*

|                   | Im-Pesaran-Shin unit-root test   | Breitung unit-root test          | Levin-Lin-Chu unit-root test     |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| Test Statistic    | -16.9440                         | -11.3458                         | -99.3088                         |
| P-value           | 0.0000                           | 0.0000                           | 0.0000                           |
| Number of Panels  | 30                               | 30                               | 30                               |
| Number of Periods | 12                               | 12                               | 12                               |
| Date range        | October, 2008 –<br>October, 2009 | October, 2008 –<br>October, 2009 | October, 2008 –<br>October, 2009 |

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