# ESSAYS ON THE EFFECTS OF TRADE LIBERALIZATION AND OIL PRICE VOLATILITY IN OPEN ECONOMIES

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To my wonderful son, David, my joy and inspiration

To my grandmother, Umweni Ebowe, my unsurpassed role model

To my mother, Rosaline Idehen, and stepfather Joseph Idehen, amazingly supportive

and

To my beloved husband, Akin, infinitely dependable

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CHAPTER I

# ARE TARIFF REDUCTIONS ALWAYS PRO-COMPETITIVE: A CASE OF BETRAND COMPETITION IN DIFFERENTIATED PRODUCTS

## Introduction

Are reductions in trade costs a substitute for competition policy when firms compete in an international environment? Are the effects of trade liberalization[1] always pro-competitive and does it matter whether firms compete in quantities or prices? These are questions that various literatures have attempted to address. While some studies have assumed cost symmetries among competing firms (Rotemberg and Saloner 1989, Syropoulos 1992, Mendi, Moner-Colonques and Sempere-Monerris, 2006), and Bond and Syropoulos, 2008; others (Fung 1992) have considered cost asymmetries between firms in examining the effects of reduction in trade costs on collusion and the strategic interactions among firms. In examining these effects, these studies have assumed either product homogeneity or differentiation, single-market or multi-market contacts and price or quantity competition. While most of these literatures focused on interactions among firms in a single market setting, evidence suggests that international cartels usually involve multi-market interactions.

Davidson (1984) studied the effects of trade costs on collusive incentives of oligopolistic firms in a single market setting and found that trade costs make deviations more attractive to the home firm than the foreign firm because they introduce cost asymmetries, so that the domestic firm gets a higher market share than the foreign. In such a setting, Davidson showed that trade costs in the neighborhood of free trade make collusion more sustainable. Diverting away from the single-market

[1] Used interchangeably as a reduction in tariff or trade cost, although it can take on various other facets.

setting assumption and assuming multi-market interactions, Bond and Syropoulos (2008) examined the effect of trade liberalization on cartel profitability and sustainability in the presence of binding incentive constraints. They considered quantity competition in homogenous and differentiated products and in both cases, found that trade liberalization facilitates collusion in the neighborhood of free trade but makes collusion harder to sustain in the neighborhood of the prohibitive tariff. Thus, the effects of trade liberalization on collusion seem to depend on the initial levels of trade costs. In particular, trade liberalization is pro-collusive when trade costs are already low but pro-competitive when trade costs are high. Does this result also carry over to situations in which firms compete in prices? To examine this, I extend the model of Bond and Syropoulos (2005) in which firms compete in a multi-market setting with differentiated products but compete in prices instead of quantities.

My motivation for studying the case of price competition is that in reality, firms are more likely to choose prices and let the market determine the demand at such prices. Although firms can conceivably choose quantities especially when there are capacity constraints, if firms are looking to support collusion by harsher punishment payoffs, choosing prices would be better than choosing quantities, hence my focus on prices.

I find that the relationship between trade costs and the minimum discount factor for which the collusive outcome is sustainable depends, to a great extent, on the degree of product substitutability and the initial levels of trade costs— for sufficiently low degree of product substitutability and intermediate trade costs or extremely high degree of product substitutability, the minimum discount factor is monotonically increasing and in such cases, trade liberalization is pro-collusive. On the other hand, for intermediate degrees of product substitutability, the minimum discount factor is decreasing for low levels of trade costs and increasing for sufficiently high levels of trade costs. In such cases, trade liberalization can be either pro-competitive or pro-collusive depending on the initial levels of trade costs.

This result suggests that the effect of trade liberalization on collusion is dependent on the degree to which the traded goods are substitutable and the prevailing levels of trade costs—high or low. While reductions from low levels of trade cost may be pro-competitive, reductions from high levels may not.

Since trade liberalization may make collusion more difficult to sustain for some levels of trade cost, one would expect welfare enhancing effects of trade liberalization for such levels of trade costs. Thus, I considered the effect of a tariff reduction on social welfare for both the collusive outcome and the Nash equilibrium and find as expected that social welfare is unambiguously improved with a tariff reduction for both cases.

The effect of transport costs, as opposed to tariffs, on social welfare is also considered and I find that although tariff reductions are welfare enhancing, reductions in transport costs may not be, depending on the initial levels of transport cost. In particular, when transport costs are sufficiently high enough that trade would be eliminated, reductions in transport cost are welfare reducing but when transport costs are initially low enough that trade occurs, further reductions in transport cost are welfare enhancing. Thus, a reduction in trade cost has welfare enhancing effect only if trade occurred before the reduction in cost.

Bond and Syropoulos (2008) observed that trade liberalization will lead to an improvement in domestic welfare if and only if it results in a reduction in the domestic price. When trade occurs, the foreign firm charges a price that is increasing in the per-unit trade cost. Thus, a reduction in trade cost means a reduction in the export price. Also, the price charged by the domestic firm reduces as a result (strategic complementarity of prices). Although firms' profits from their home markets decrease, the increase in consumers' surplus and export revenues more than makes up for the decrease in home market profits so that welfare increases overall. Thus, a tariff reduction reduces the domestic price and ultimately leads to welfare enhancement.

## Price Competition and the Non-Cooperative Equilibrium

I consider a symmetric, two country Bertrand Oligopoly in differentiated products with a single firm in each country. Markets are assumed to be segmented so that consumers can only buy the goods from their domestic markets. Firms are symmetric in that they have the same zero marginal cost of production and each firm has a linear inverse demand function given by:

$p = A - q - \gamma x^*$ , $\bar{p} = A - x - \gamma q^*$, $p^* = A - q^* - \gamma x$, $\bar{p}^* = A - x^* - \gamma q$ where $(p, \bar{p})$ are prices of the domestic firm in the domestic and foreign markets respectively, $(p^*, \bar{p}^*)$ are prices of the foreign firm in the foreign and domestic markets respectively, $(q, x)$ are domestic output and export by the domestic firm respectively, $(q^*, x^*)$ are foreign output and export by the foreign firm respectively, $A$ is a market demand parameter and $\gamma$ is a measure of the degree of substitutability between domestic output and export with $0 < \gamma < 1$.

By inverting the inverse demand functions, I have $q = a - bp + d\bar{p}^*$ , $q^* = a - bp^* + d\bar{p}$ , $x = a - b\bar{p} + dp^*$, $x^* = a - b\bar{p}^* + dp$ where $a = A/(1+\gamma)$, $b = 1/(1-\gamma^2)$, $d = \gamma/(1-\gamma^2)$ and each of the above quantities have non-negativity constraints. In particular, given per unit trade cost $t$, the range of prices for which domestic output and export of the home firm are respectively positive satisfy:

$$p < p^{\max}(\overline{p}^*) \equiv A(1-\gamma) + \gamma \overline{p}^* \qquad \text{(i)}$$

$$\overline{p}^{\min}(p^*) \equiv \min\{t,\ -\tfrac{1}{\gamma}(A(1-\gamma) - p^*)\} < \overline{p} < \overline{p}^{\max}(p^*) \equiv A(1-\gamma) + \gamma p^* \qquad \text{(ii)}$$

By symmetry, the ranges of prices for the foreign firm satisfy:

$$p^* < p^{*\max}(\overline{p}) \equiv A(1-\gamma) + \gamma \overline{p} \qquad \text{(iii)}$$

$$\overline{p}^{*\min}(p) \equiv \min\{t,\ -\tfrac{1}{\gamma}(A(1-\gamma) - p)\} < \overline{p}^* < \overline{p}^{*\max}(p) \equiv A(1-\gamma) + \gamma p \qquad \text{(iv)}$$

The following proposition established the best responses of each firm to the other firm's choice of prices in the domestic and foreign market respectively.

**Proposition 1**

If firms do not play weakly dominated strategies, the best responses of the firms given $\gamma$ and $t$ are

$$\overline{p}^* = \begin{cases} t & \text{p} \le \dfrac{A(\gamma-1)+t}{\gamma} \\ \dfrac{A(1-\gamma)+t}{2} + \dfrac{\gamma p}{2} & \dfrac{A(\gamma-1)+t}{\gamma} < p < \dfrac{A(2-\gamma-\gamma^2)+\gamma t}{2-\gamma^2} \\ \dfrac{A(\gamma-1)+p}{\gamma} & \dfrac{A(2-\gamma-\gamma^2)+\gamma t}{2-\gamma^2} \le p < \dfrac{A(2-\gamma)+\gamma t}{2} \\ \dfrac{A+t}{2} & p \ge \dfrac{A(2-\gamma)+\gamma t}{2} \end{cases};$$

$$p = \begin{cases} 0 & \overline{\text{p}}^* \le \dfrac{A(\gamma-1)}{\gamma} \\ \dfrac{A(1-\gamma)}{2} + \dfrac{\gamma \overline{p}^*}{2} & \dfrac{A(\gamma-1)}{\gamma} < \overline{p}^* < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{A(\gamma-1)+\overline{p}^*}{\gamma} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \le \overline{p}^* < \dfrac{A(2-\gamma)}{2} \\ \dfrac{A}{2} & \overline{p}^* \ge \dfrac{A(2-\gamma)}{2} \end{cases}$$

Proof:

Global profits for the domestic and foreign firm are given respectively as:

$$\prod(p, p^*, \overline{p}, \overline{p}^*, t) = p(a - bp + d\overline{p}^*) + (\overline{p} - t)(a - b\overline{p} + dp^*)$$

$$\prod{}^*(p, p^*, \overline{p}, \overline{p}^*, t) = p^*(a - bp^* + d\overline{p}) + (\overline{p}^* - t)(a - b\overline{p}^* + dp)$$

Letting $\Pi_i$ denote partial derivative of global profit with respect to $i$, the first order conditions for the home firm's optimization problem are:

$$\prod{}_p = a + d\overline{p}^* - 2bp$$

$$\prod{}_{\overline{p}} = a + bt + dp^* - 2b\overline{p}$$

By symmetry, the first order conditions for the foreign firm's optimization problem are:

$$\prod{}_{p^*} = a + d\overline{p} - 2bp^*$$

$$\prod{}_{\overline{p}^*} = a + bt + dp - 2b\overline{p}^*$$

First, if a firm's rival sets a price such that the maximum price it can set as a function of its rival's price, would be less than the firm's marginal cost, the firm's best response would be to set at least its marginal cost and supply nothing. For instance, if the domestic firm sets a price p ≤ (A (γ-1) +t)/γ, then the foreign firm can choose to set its price at the level of trade cost or at any price greater than $\overline{p}^{*\max}(p)$ and any of these prices would be a best response. At these prices, export as well as the corresponding profit would be zero. Although the foreign firm can conceivably choose any price below t and supply zero export so that its profit are still zero, such a choice of price is eliminated under the assumption that firms do not play weakly dominated strategies. Prices below t are weakly dominated strategies because they yield the same zero profit as setting $\overline{p}^* = t$ as long as export is zero, but negative profits when the price is sufficiently low that export would be positive.

If a firm's rival sets an interior price for which two-way trade can occur, then the firm's best response would be to set a price which corresponds to positive output levels. Thus, the interior price pair for which trade occurs is derived by solving for the prices at which the above first order conditions equal zero.

If a firm chooses the highest price that just forces its rival out of the market, but which is less than the single firm monopoly price, the best response of its rival would be to choose a price which is at least the maximum price it can set in that market and supply nothing. The maximum price any firm can set in each market is the price at which its rival's output equals zero and this price is derived by equating the rival's demand function in that market to zero and solving for the price.

Finally, for any price choice of its rival which is at least as high as the monopoly price, the firm's best response would be to choose its monopoly price. □

The figure below illustrates the various sections of the reaction functions for the domestic and foreign firm with $p_2$ representing the price of the foreign firm and $p_1$ that of the domestic firm.[2]

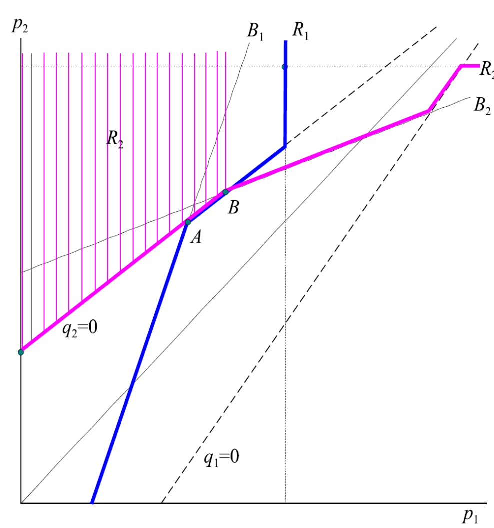



Fig. 1 Reaction Functions for the Domestic and Foreign Firm

[2] For zero exports, any price in the shaded region is also a best response by the foreign firm.

The firms' reaction functions are piecewise linear and piecewise differentiable; in particular, the slope of the reaction functions are less than unity in some trade cost ranges for which exports are positive [$\frac{\partial \text{ p}}{\partial \, \overline{\text{p}}^*} = \frac{\partial \, \overline{\text{p}}^*}{\partial \text{ p}} = \frac{\gamma}{2} < 1$] , greater than unity for higher trade cost ranges for which exports are zero [$\frac{\partial \text{ p}}{\partial \, \overline{\text{p}}^*} = \frac{\partial \, \overline{\text{p}}^*}{\partial \text{ p}} = \frac{1}{\gamma} > 1$] and flattens out at the monopoly price after that. Moreover, the foreign firm's reaction function is increasing in trade costs for positive exports, thereby allowing the domestic firm to increase its price since the firms' prices are strategic complements; thus, the resulting Nash equilibrium will be sensitive to the prevailing levels of trade costs.

Given the reaction functions of the firms, the following proposition establishes a unique Nash equilibrium in which no firm plays a weakly dominated strategy.

**<u>Proposition 2</u>**

There exist a unique Nash equilibrium in which,

$$p^N = p^{*N} = \begin{cases} \dfrac{A(1-\gamma)(2+\gamma)+\gamma t}{4-\gamma^2} & 0 \le \text{t} \le \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{A(\gamma-1)+\text{t}}{\gamma} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} < \text{t} \le \dfrac{A(2-\gamma)}{2} \\ \dfrac{A}{2} & \text{t} > \dfrac{A(2-\gamma)}{2} \end{cases}$$

$$\overline{p}^N = \overline{p}^{*N} = \begin{cases} \dfrac{A(1-\gamma)(2+\gamma)+2t}{4-\gamma^2} & 0 \le \text{t} \le \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \text{t} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} < \text{t} \le \dfrac{A(2-\gamma)}{2} \\ \dfrac{A+t}{2} & \text{t} > \dfrac{A(2-\gamma)}{2} \end{cases}$$

Proof:

Assuming that trade costs are such that the zero output lines for both firms are always above their best response lines so that the reaction functions are as given in Proposition 1, then it is easy to show that the equilibrium prices in the domestic market are as given above and derive directly from the intersection of the best response functions. □

**Proposition 3**

The corresponding Nash Equilibrium profits for the equilibrium price pair above are:

$$\Pi^N(t) = \Pi^{*N}(t) = \begin{cases} \dfrac{2A^2(\gamma^2+\gamma-2)^2 - 2At(\gamma^2+\gamma-2)^2 + t^2(\gamma^4-3\gamma^2+4)}{(\gamma^2-4)^2(1-\gamma^2)} & 0 \le t \le \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(A-t)[A(\gamma-1)+t]}{\gamma^2} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} < t < \dfrac{A(2-\gamma)}{2} \\ \dfrac{A^2}{4} & t \ge \dfrac{A(2-\gamma)}{2} \end{cases} \qquad (1)$$

Proof:

Derives directly from substitution of the equilibrium prices into the expression for global profits. □

The global Nash equilibrium profits are the same for both firms so that I can henceforth focus only on the competition in the home market without any loss of generality.

When trade costs are high so that exports are zero, the Nash equilibrium profit is strictly concave in t, increasing for $\frac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \le t < \frac{A(2-\gamma)}{2}$ and maximized at $t = \frac{A(2-\gamma)}{2}$. For the interior equilibrium, the Nash equilibrium profit is strictly convex in t; in particular, $\Pi_t^N > (<) 0$ as $t > (<) \frac{A(1-\gamma)^2(2+\gamma)^2}{\gamma^4-3\gamma^2+4}$ and is minimized at $t_{\min}^N = \frac{A(1-\gamma)^2(2+\gamma)^2}{\gamma^4-3\gamma^2+4}$ which is decreasing in the degree of substitutability, $\gamma$, and approaches A as $\gamma \to 0$ (goods become relatively more independent) and zero as $\gamma \to 1$

(goods become close substitutes). The prohibitive level of trade cost that drives the foreign firm out of the domestic market completely under the Nash equilibrium with positive exports is $\bar{t}^N = \frac{A(1-\gamma)(2+\gamma)}{2-\gamma^2}$, suggesting that the level of trade cost for which exports is prohibited in a homogenous products market in which firms have the same marginal cost and compete in prices is zero. It is also observed that $\Pi^N(\bar{t}^N) = \frac{A^2(1-\gamma^2)}{(2-\gamma^2)^2} < (>) \Pi^N(0) = \frac{2A^2(1-\gamma)^2(2+\gamma)^2}{(\gamma^2-4)^2(1-\gamma^2)}$ as $\gamma \to 0(1)$. This is a similarity between the Cournot and Bertrand competition. In the homogenous products case, a prohibitive trade cost yields the monopoly profit for the home firm because it completely drives the foreign firm out of the domestic market and makes the domestic firm a monopoly in the home market. As goods become differentiated, exports become important for firms' profits so that at the prohibitive trade cost, exports are zero and firms' profits are significantly reduced.

Thus, trade costs in the neighborhood of the prohibitive level of trade cost are a way to shield domestic firms from competition in a homogenous products market. However, the effect on profit of such prohibitive trade costs is different for the Bertrand case than for the Cournot. In the Bertrand case, the firm's profit continues to increase in the trade cost even after trade has been eliminated, but for the Cournot case, the firm's profit does not change.

## Tariffs Vs. Transport Costs—Effect on Social Welfare

Although it is irrelevant for firms whether trade costs take the form of tariff or transport costs since they enter the firms' profits in the same way, when considering welfare effects, it is important that the effects of tariffs be treated separately from the effects of trade costs.

When trade costs take the form of transport costs instead of tariffs, the effect on equilibrium prices remain the same as that of a tariff since firms perceive both costs to be an increase in the per unit cost of supplying output in the foreign market. Since equilibrium prices remain the same under both scenarios, equilibrium outputs remain the same as well as equilibrium profits. Thus, consumers' surplus and profits are unchanged in the social welfare function. However, transportation costs, as opposed to tariffs, do not generate any export revenues for the government so that social welfare is just the sum of consumers' surplus and profit only.

In that case, social welfare with transportation cost is given as:

$$V^N = \begin{cases} 1/2\ (q^2 + x^{*2}) + \gamma q x^* + pq + (\bar{p}^* - t)x^* & 0 \leq t < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ 1/2\ (q_0^2) + p_0 q_0^2 & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \leq t < \dfrac{A(2-\gamma)}{2} \end{cases}$$

where $q, q_0, p$, and $p_0$ are Nash equilibrium quantities and prices with subscripts denoting quantity (price) for the corner solution. Substituting these into the above formula yields the welfare expression:

$$V^N = \begin{cases} \dfrac{2(A^2 - At)(1-\gamma)(3-2\gamma)(2+\gamma)^2 + t^2(12 - 9\gamma^2 + 2\gamma^4)}{2(4-\gamma^2)^2(1-\gamma^2)} & 0 \leq t < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(A-t)[A(2\gamma-1)+t]}{2\gamma^2} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \leq t < \dfrac{A(2-\gamma)}{2} \end{cases}$$

For $0 \leq t < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2}$, there is an interior solution in which two-way trade occurs and welfare with transport costs is convex in t. For transport costs sufficiently low, exports are positive; a further decline in transport costs causes the profit of the home firm in the home market to decrease but the increase in consumer surplus and export profit from the foreign market more than compensates for the decline in profits in the home market so that welfare improves overall. But for sufficiently high tariffs in the neighborhood of $t = \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2}$, the home firm can not charge the monopoly price, so it charges a price below that and which depends positively on transport cost; a decrease in transport costs causes the price and profits of the home firm to decrease; consumer surplus and export profit increase but by less than the decline in home firm's profit so that welfare decreases overall. We know this because the derivative of the welfare function for when exports are zero is given by $\frac{A(1-\gamma)-t}{\gamma^2}$ which is negative for $t = \frac{A(2-\gamma-\gamma^2)}{2-\gamma^2}$. Thus, in the range of transport costs for the interior solution in which two-way trade occurs in equilibrium, welfare is U-shaped just as in the case of Brander and Krugman (1983) and minimized at $V_{\min}^N = \frac{A^2(9-4\gamma^2)}{2(12-9\gamma^2+2\gamma^4)}$. The level of trade cost at which this minimum is attained is:

$t_{\min}^v = \frac{A(1-\gamma)(2+\gamma)^2(3-2\gamma)}{12-9\gamma^2+2\gamma^4}$. It is easy to show that $V^N(t_{\min}^V) > V^N(\frac{A(2-\gamma)}{2})$, so that welfare attains its global minimum when trade costs are so high that monopoly prevails in each market.

When transport costs are sufficiently high such that trade is eliminated but the domestic firm can not charge its single market monopoly price, i.e. for

$\frac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \leq t < \frac{A(2-\gamma)}{2}$, the welfare function is decreasing in transport cost. So, welfare increases as transport costs decrease because the price of the home firm decreases and it's sale increases since $q = \frac{A-t}{\gamma}$ and $\partial q/\partial t < 0$. Thus, even though trade does not occur, reductions in trade costs unambiguously cause welfare to improve.

Brander and Krugman (1983) found that there are always welfare losses from reductions in transport cost when transport costs are high. Our result shows that this is true only for the interior solution in which two-way trade occurs. When transport costs are sufficiently high so that trade does not occur, reductions in transport costs lead to welfare improvement because although it leads to a decrease in profit of the domestic firm, the increase in consumer surplus, which arises from increased output, more than compensates for the loss in profit. Thus, a reduction in transport cost will have a pro-competitive effect as opposed to the result of Brander and Krugman (1983).

When trade costs take the form of tariffs, social welfare under the Nash equilibrium is the sum of global profits, consumers' surplus and tariff revenues, and is given by:

$$W^N = \begin{cases} \dfrac{(Q^N)^2}{2} + \prod^N + \mathrm{t\,x}^N & 0 \leq \mathrm{t} < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(Q_0^N)^2}{2} + \prod_0^N & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \leq \mathrm{t} < \dfrac{A(2-\gamma)}{2} \end{cases}$$

where $Q^N$, $\mathrm{Q}_0^N$, $\prod^N$, $\prod_0^N$, and $\mathrm{x}^N$ are Nash equilibrium quantities, profits and export under the interior and corner solutions respectively. Substituting these into the above formula yields the welfare expression:

$$W^N = \begin{cases} \dfrac{4A^2(1-\gamma)(2+\gamma)^2(2-\gamma^2) - 2At(1-\gamma)^2(2+\gamma)^3 - t^2(4+8\gamma-3\gamma^2-5\gamma^3)}{2(1-\gamma)(4+4\gamma-\gamma^2-\gamma^3)^2} & 0 \leq \mathrm{t} \leq \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(A-t)[A(2\gamma-1)+t]}{2\gamma^2} & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} < \mathrm{t} \leq \dfrac{A(2-\gamma)}{2} \end{cases}$$

Taking a piecewise derivative of the welfare function with tariffs yields:

$$W_t^N = \begin{cases} \dfrac{-2A(1-\gamma)^2(2+\gamma)^3 - 2t(4+8\gamma-3\gamma^2-5\gamma^3)}{2(1-\gamma)(4+4\gamma-\gamma^2-\gamma^3)^2} < 0 & 0 \leq \mathrm{t} < \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{A(1-\gamma)-t}{\gamma^2} < 0 & \dfrac{A(2-\gamma-\gamma^2)}{2-\gamma^2} \leq \mathrm{t} < \dfrac{A(2-\gamma)}{2} \end{cases}$$

Welfare with tariffs is concave and decreasing in tariffs for the entire range of tariffs specified and attains its maximum under free trade. Thus, tariff reductions are welfare enhancing for all tariff levels.

In the limit case of homogenous products, the welfare function is given by $(1/2)(A^2 - t^2)$ which is also decreasing in t and maximized under free trade; although free trade is best for welfare maximization, it is detrimental to firms whose profits are driven to zero under the Nash equilibrium.

It is also important to note that welfare under tariffs and transport costs are both decreasing in the degree of product substitutability so that the effects of reductions in trade costs would be more profound as goods become more substitutable.

Comparison of Welfare under Tariffs and Transport Costs

For zero trade costs, both welfare functions are equivalent and given for the differentiated products case as: $W^N(0) = V^N(0) = \dfrac{2A^2(2+\gamma)^2(2-\gamma^2)}{(4+4\gamma-\gamma^2-\gamma^3)^2}$. For the case of homogenous products, both welfare measures are equal to $A^2/2$.

For positive non-prohibitive tariffs for which two-way trade occurs, welfare with tariffs is higher than welfare with transport cost because , although tariffs just as transport costs, are costs to the firms, they also serve as revenue to the government when trade occurs but transport costs are a waste of resources that do not yield any additional government revenue. This wasteful effect of transport costs on welfare is more profound for distant substitutes since the exports markets become relatively more important for firms' profits and more resources end up being wasted as trade occurs.

For sufficiently high trade costs for which no trade occurs in equilibrium, both welfare measures are equal. Specifically, at the prohibitive tariff level, the following holds: $W^N(\bar{t}^N) = V^N(\bar{t}^N) = \dfrac{A^2\gamma(4-\gamma)}{8} < V^N(0) = W^N(0)$.

# CHAPTER II

# THE SUSTAINABILITY OF COLLUSION

## The Unconstrained Collusive Outcome

The collusive outcome involves both firms agreeing on a pair of prices $(p, \overline{p})$ for their domestic and foreign markets respectively in order to maximize global collusive profits; then, depending on the threshold level of tariff under this outcome, both firms decide whether to supply positive or zero exports.

The collusive price pair has to be such that the discounted profits from colluding in every period given those prices is no less than the sum of deviation profit—should any firm deviate from the collusive outcome—and the continuation punishment profit after deviation. Thus, the incentive compatibility constraint for a given trade cost $t \geq 0$ and discount factor $0 < \delta < 1$ is:

$$\Pi^C(p, \overline{p}, t) \geq (1-\delta)\Pi^D(p, \overline{p}, t) + \delta\, \Pi^P(t)$$

where $\Pi^C(p, \overline{p}, t)$, $\Pi^D(p, \overline{p}, t)$, and $\Pi^P(t)$ are the collusive (agreement), deviation and punishment profits respectively. We assume grim trigger strategies for the punishment phase so that the punishment payoff above would be the same as the Nash payoff. We also assume that renegotiation costs[3] are prohibitively high, so that once firms have successfully decided on the collusive outcome and punishment payoffs, they do not meet to renegotiate the punishment payoff in the event that a firm deviates. The Nash payoff remains the punishment payoff in every period after deviation.

[3] These include, but are not limited to, sanctions, fines and penalties that firms would face if caught negotiating prices since such negotiations are prohibited by law.

The threshold level of trade cost after which firms switch to zero exports is given as $\overline{t}^C = A(1-\gamma)$. This cutoff level of trade cost is less than that under the Nash equilibrium. However, as $\gamma \to 0$, the cutoff levels of trade costs under the Nash equilibrium and collusive outcome converge.

The following proposition establishes the collusive profit for the firms given this cutoff level of trade cost.

**Proposition 4**

The global collusive profit given the cutoff level of trade cost $\overline{t}^C = A(1-\gamma)$ is:

$$\Pi^C(t) = \begin{cases} \dfrac{A^2}{2(1+\gamma)} - \dfrac{At}{2(1+\gamma)} + \dfrac{t^2}{4(1-\gamma^2)}, & 0 \le t \le A(1-\gamma) \\ \dfrac{A^2}{4}, & t > A(1-\gamma) \end{cases} \tag{2}$$

Proof:

Given the collusive price pair $(p, \overline{p})$, global collusive profits are:

$$\Pi^C(p, \overline{p}, t) = \begin{cases} (a - bp + d\overline{p})p + (\overline{p} - t)(a - b\overline{p} + dp), & 0 \le t < A(1-\gamma) \\ (A-p)p & t \ge A(1-\gamma) \end{cases}$$

Solving the optimization problem yields the collusive prices

$$p^C = A/2 \text{ and } \overline{p}^C = \frac{A+t}{2}$$

Substituting these prices into the expressions for global collusive profits yield the profit functions for the given collusive prices. □

The collusive profit is decreasing in trade cost for $0 \le t < A(1-\gamma)$ and constant for trade costs equal to or exceeding the prohibitive level. It is also important to note that $\Pi^C(0) = \dfrac{A^2}{2(1+\gamma)} > \Pi^C(\overline{t}^C) = \dfrac{A^2}{4}$ so that the collusive profit in the neighborhood of free trade is higher than that which is obtainable when trade costs are prohibitive or the single market monopoly profits when exports are zero.

The intuition behind this result is that, to the extent that goods are differentiated, every market matters for firms' global profits which are maximized only under free trade. As trade costs become prohibitive, the maximum profit any firm can get is the monopoly profit from its domestic market since the rival firm is completely driven out of the market.

But when goods are homogenous, the global collusive profits are the same under free trade as is when trade costs are prohibitive. On the other hand, when goods are independent, firms can price independently without any need to collude as each firm becomes a localized monopoly in its own product and captures the monopoly profits in all markets.

## Deviation Profits

If a firm decides to deviate from the collusive outcome, it does so by lowering its price. The deviation payoffs are obtained by applying the reaction functions to the cartel prices derived in the previous section. The incentive to deviate is higher for close substitutes than for distant substitutes since the deviating firm will capture a larger market share when goods are closely substitutable than when they are not.

A deviating firm decides what its optimal deviation strategy should be in each market (deviation to an interior solution or deviation to a corner) given the level of trade cost and collusive price of its rival in that market. From the reaction function of the home firm in the home market, the threshold level of trade cost after which it deviates to a corner is $\bar{t}_0^{dh} = \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2}$ and for $0 \leq t < \bar{t}_0^{dh}$ deviation involves setting a price that allows positive export in the home market. On the other hand, for low levels of trade costs satisfying $0 \leq t < \frac{A(\gamma^2 - 2 + 2\gamma)}{2\gamma}$, deviation of the home firm in the foreign market involves setting a price to capture the entire market. Thus, the threshold level of trade cost after which the home firm deviates to an interior in the foreign market is $\bar{t}_i^{df} = \frac{A(\gamma^2 - 2 + 2\gamma)}{2\gamma}$. The corresponding deviation profits for the home and foreign markets respectively are:

$$\Pi_h^D(t) = \begin{cases} \frac{[A(2-\gamma)+\gamma t]^2}{16(1-\gamma^2)}, & 0 \leq t < \frac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \\ \frac{(A-t)[A(2\gamma-1)+t]}{4\gamma^2}, & \frac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \leq t < A(1-\gamma) \\ \frac{A^2}{4} & t \geq A(1-\gamma) \end{cases}$$

$$\Pi_f^D(t) = \begin{cases} \frac{A^2(2\gamma-1)-2A\gamma t}{4\gamma^2}, & 0 \leq t < \frac{A(\gamma^2-2+2\gamma)}{2\gamma} \\ \frac{[A(2-\gamma)-2t]^2}{16(1-\gamma^2)}, & \frac{A(\gamma^2-2+2\gamma)}{2\gamma} \leq t < \frac{A(2-\gamma)}{2} \\ 0 & t \geq \frac{A(2-\gamma)}{2} \end{cases} \tag{3}$$

The profit from deviating to an interior solution in the home market is strictly increasing in t whereas it is concave in t for deviation to a corner solution and maximized at $t = A(1-\gamma)$. The deviation profits in the foreign market are decreasing in the trade cost for the ranges specified for which the home firm deviates to a corner or an interior solution. The prohibitive level of trade cost that prevents deviation from the collusive outcome in the domestic market is $\bar{t}_h^D = A(1-\gamma)$ which is less than the prohibitive level of trade cost that prevents deviation in the export market $\bar{t}_f^D = \frac{A(2-\gamma)}{2}$.

For sufficiently high levels of $\gamma$, only a corner deviation is possible in the home market whereas in the foreign market, corner deviation occurs only for low values of $t$ but for high values of $t$, interior deviation will occur. On the other hand, for sufficiently low levels of $\gamma$, only an interior deviation is possible in the foreign market whereas in the home market, interior deviation occurs only for low values of $t$ but for high values of $t$, a corner deviation will occur. Thus, although the total deviation profits depends on the levels of $t$ and $\gamma$, in the neighborhood of free trade, the deviation profits corresponding to low and high levels of $\gamma$ respectively are

$$\Pi_{l\gamma}^{D}(t) = \frac{2A^2(2-\gamma)^2 - 2At(4-4\gamma+\gamma^2) + t^2(4+\gamma^2)}{16(1-\gamma^2)}$$

$$\Pi_{h\gamma}^{D}(t) = \frac{(2A^2 - 2At)(2\gamma-1) - t^2}{4\gamma^2}$$

The following proposition captures the results on deviation payoffs.

**<u>Proposition 5</u>**

The deviation payoffs in the domestic and export markets and corresponding deviation profits for low and high degree of substitutability between export and domestic output in the neighborhood of free trade are given by:

$$\Pi_{h}^{D}(t) = \begin{cases} \dfrac{[A(2-\gamma)+\gamma t]^2}{16(1-\gamma^2)}, & 0 \le t < \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(A-t)[A(2\gamma-1)+t]}{4\gamma^2}, & \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \le t < A(1-\gamma) \\ \dfrac{A^2}{4} & t \ge A(1-\gamma) \end{cases}$$

$$\Pi_{f}^{D}(t) = \begin{cases} \dfrac{A^2(2\gamma-1) - 2A\gamma t}{4\gamma^2}, & 0 \le t < \dfrac{A(\gamma^2-2+2\gamma)}{2\gamma} \\ \dfrac{[A(2-\gamma)-2t]^2}{16(1-\gamma^2)}, & \dfrac{A(\gamma^2-2+2\gamma)}{2\gamma} \le t < \dfrac{A(2-\gamma)}{2} \\ 0 & t \ge \dfrac{A(2-\gamma)}{2} \end{cases}$$

$$\Pi_{l\gamma}^{D}(t)=\frac{2A^2(2-\gamma)^2-2At(4-4\gamma+\gamma^2)+t^2(4+\gamma^2)}{16(1-\gamma^2)}$$

$$\Pi_{h\gamma}^{D}(t)=\frac{(2A^2-2At)(2\gamma-1)-t^2}{4\gamma^2}$$

Proof**:**

Already presented in the previous section. □

## Sustainability of the Collusive Outcome

I assume that both firms revert to the one-shot Bertrand Nash equilibrium forever after in case any firm deviates from the collusive outcome and under the assumption that both firms have the same discount factor $\delta\in(0,1)$, the incentive compatibility constraint for which the collusive outcome is sustainable in all periods of the repeated game is given as: $\Pi^C(p,\bar{p},t)\geq(1-\delta)\Pi^D(p,\bar{p},t)+\delta\Pi^N$ where the profit functions are as defined in the previous section. The minimum discount factor for which this holds is given by $\underline{\delta}(\gamma,t)=\frac{\Pi^C-\Pi^D}{\Pi^N-\Pi^D}$, which depends on the level of trade cost and the degree of product substitutability.

I find that there is a non-monotonic relationship between the minimum discount factor and trade cost which is strongly influenced by the degree of product substitutability. In the limit case of homogenous products, the corresponding Nash, collusive and deviation profits are given respectively by $\Pi_{\gamma=1}^{N}=t(A-t)$, $\Pi_{\gamma=1}^{C}=\frac{A^2}{4}$ and $\Pi_{\gamma=1}^{D}=\frac{A(A-t)}{2}$. The minimum discount factor for which collusion is sustainable is thus: $\underline{\delta}(1,t)=\frac{A}{2(A-t)}$.

This discount factor is increasing in trade costs, minimized under free trade at 0.5—a familiar textbook result for the minimum discount factor required for collusion to be sustainable in a homogenous goods duopoly market in which firms compete in prices—and maximized under the prohibitive level of trade cost.

This result suggests that collusion is easier to sustain for lower trade costs than for higher trade costs. This is because the punishment payoff is increasing in trade costs and maximized at the prohibitive level. As the punishment payoff increases, the incentive compatibility constraint becomes more difficult to satisfy and the possibility of a breakdown in the collusive outcome increases. Thus, reductions in trade costs make the collusive outcome more sustainable by increasing the collusive payoff and reducing the punishment payoff.

This result was also derived by Lommerud and Sørgard (2001), who found that when collusion is supported by grim trigger strategies, the short term gain from deviation is lower than the long term loss in continuation profits so that collusion becomes easier to sustain when trade is liberalized.

When goods are independent, the Nash, collusive and deviation profits are all equal so that a firm can price as a monopoly without any need for collusion.

Bond and Syropoulos (2008), in the case of quantity competition in homogenous products, found that the minimum discount factor for which the collusive outcome is sustainable is a decreasing function of $t$ for $t \in (0, \frac{A}{2}]$, so that reduction in trade costs in this range will make the collusive outcome less sustainable. They also find that there is a sense in which trade liberalization may facilitate the collusive outcome when trade costs are already low. They generate this latter result from the strict convexity of deviation profits for allocations of domestic and foreign outputs that satisfy the monopoly outcome at $t = 0$. At $t = 0$, the minimum discount factor will be attained by equalizing domestic and foreign output levels and this cross-hauling of goods facilitates collusion by leading to lower deviation profits.

Although this result derived for $t = 0$ and homogenous products are the same for the case of price competition and quantity competition, it is important to point out the discontinuity of the minimum discount factor at $t = 0$ for the case of Cournot competition whereas there is no discontinuity for the Bertrand competition. Moreover, the collusion enforcing effect of trade liberalization at $t = 0$ in the Cournot case arises only for equal market shares, marking a difference between the Cournot and Bertrand competition.

The result generated for the limiting case of the Bertrand competition for $t > 0$ is different from that of the Cournot and it is not surprising because one would generally expect the opposite of the Cournot result since whereas quantities are strategic substitutes in oligopolistic markets, prices are strategic complements. For strategic substitutes, a firm's best response function is decreasing in its rival's output choice whereas for strategic complements, it is increasing in its rival's price choice.

For Bertrand competition, when firms are symmetric with the same marginal cost of production, and trade costs are sufficiently low so that firms can export, the domestic firm has a competitive advantage over the foreign firm in the domestic market and can always slightly undercut the price of the foreign firm and capture a significantly large percentage of the market if not the entire market. For the Cournot case, a firm can hardly capture the entire market even when it deviates from the collusive outcome; moreover, the punishment payoff is never driven to zero.

However, for the case of differentiated products Bertrand competition, we will see that the result that trade liberalization makes the collusive outcome more difficult to sustain, as derived for the homogenous product Cournot competition, is obtained for some intermediate degrees of product substitutability and trade costs sufficiently low enough.

We will demonstrate that the minimum discount factor is non-monotonic in the trade cost and the degree of substitutability and to determine how trade liberalization affects the minimum discount factor, and hence collusion for $\gamma \in (0,1)$, the following inequality regarding the cut-off trade costs for which the firm switches between outcomes for the different "regimes" (Nash, collusive and deviation respectively) will be useful:

$$t_0^{dh} < t_m^c < t_0^n < t_m^n \tag{4}$$

where the superscripts denote the regimes and the subscripts denote the outcomes; for example $t_0^{dh}$ is the cutoff trade cost for which a deviating firm chooses a corner outcome in its home market, $t_0^n$ is the cutoff trade cost for which a firm switches from an interior to a corner under the Nash equilibrium and $t_m^n$ is the cutoff trade cost for which a firm finds choosing the monopoly outcome optimal under the Nash equilibrium. As is observable from the previous section, the levels of trade costs are given by:

$$t_0^{dh} = \frac{A(2-2\gamma-\gamma^2)}{2-\gamma^2},\ t_m^c = A(1-\gamma),\ t_0^n = \frac{A(2-\gamma-\gamma^2)}{2-\gamma^2},\ t_m^n = \frac{A(2-\gamma)}{2};$$

and it is straightforward to show that the inequality above actually holds. All four levels of trade costs are decreasing in $\gamma$. There is however a fifth level of trade cost for which the firm switches from a corner to an interior in the foreign market. This level of trade cost is given by $t_i^{df} = \dfrac{A(\gamma^2-2+2\gamma)}{2\gamma}$ and is increasing in $\gamma$. As a result, its position in the above inequality (4) will depend on the value of $\gamma$. The diagram below presents all five levels of trade costs as functions on $\gamma$ for a given value of A=1.

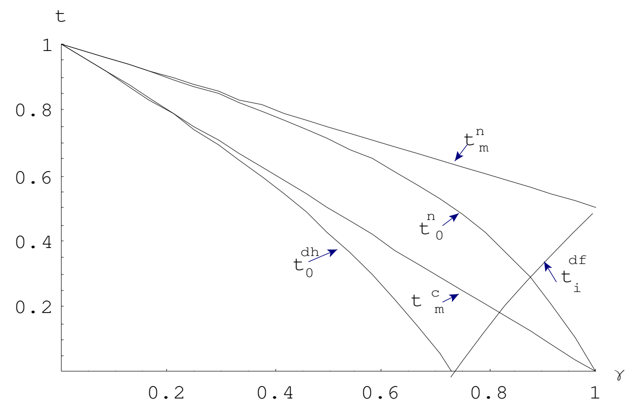



Fig. 2 Cutoff Tariffs as Functions of the Degree of Product Substitutability.

The values of $\gamma$ for which the trade cost functions intersect are given by: $t_i^{df} = t_0^{dh}$ at $\gamma^d = 0.73$, $t_i^{df} = t_m^c$ at $\gamma^c = 0.82$, $t_i^{df} = t_0^n$ at $\gamma^{n0} = 0.87$, $t_i^{df} = t_m^n$ at $\gamma^{nm} = 1$. These values will be useful for determination of the minimum discount factor in different ranges of $\gamma$. The ranges to be considered are: $\gamma \in \{(0,0.73),[0.73,0.82),[0.82,0.87),[0.87,1)\}$. We will take a value of $\gamma$ from each of the ranges specified above and illustrate the minimum discount factor for each value.

For $\gamma = 0$, there is no need for collusion since products are independent and each firm sets the monopoly price in each market even in the Nash equilibrium. Thus, consideration of the minimum discount factor breaks down.

For $\gamma \in (0,1)$, although the expressions for the minimum discount factors are complex and as such not presented in the paper, graphical representations of the minimum discount factor with respect to trade costs are presented below for specific values of $\gamma \geq 0.1$. Graphs are not presented for $\gamma \in \{0.3,0.4\}$ because the shape of the minimum discount factor is similar to that of $\gamma = 0.2$.

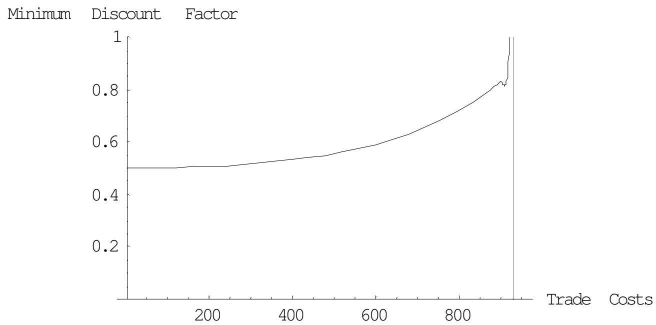



Fig. 3 Minimum Discount Factor and Trade Costs for $\gamma = 0.1$

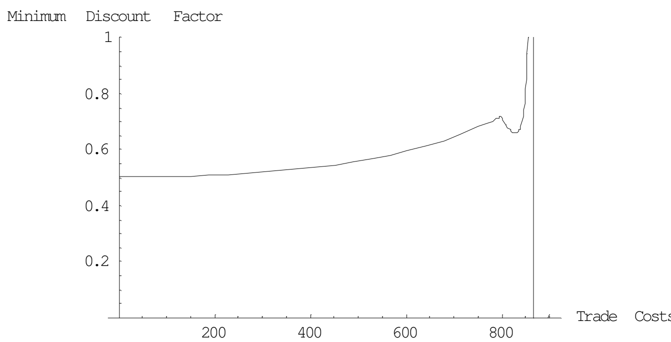



Fig. 4 Minimum Discount Factor and Trade Costs for $\gamma = 0.2$

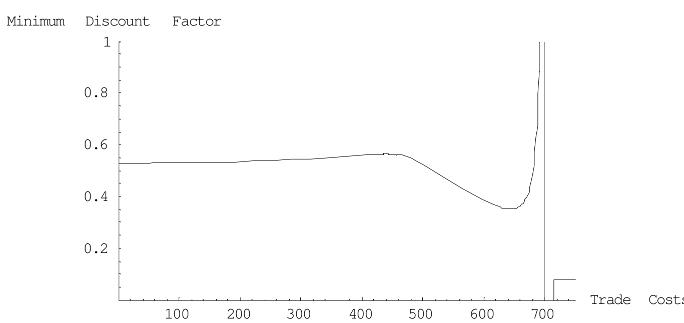



Fig. 5 Minimum Discount Factor and Trade Costs for $\gamma = 0.5$

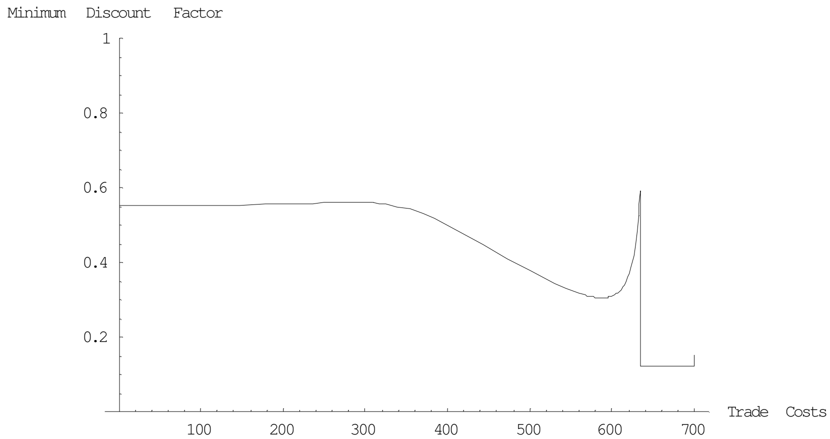



Fig. 6 Minimum Discount Factor and Trade Costs for $\gamma = 0.6$

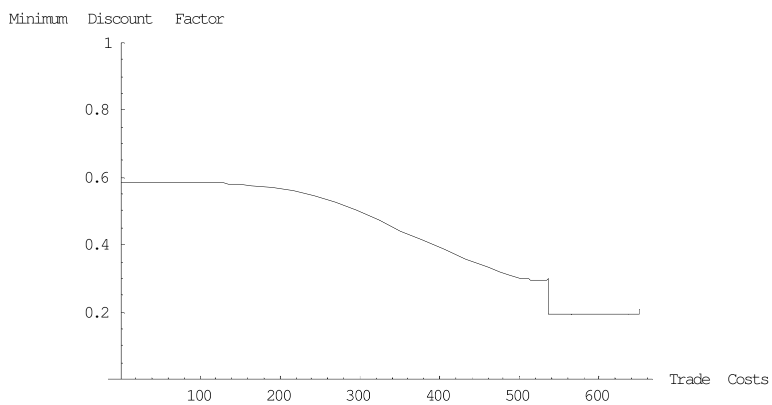



Fig. 7 Minimum Discount Factor and Trade Costs for $\gamma = 0.7$

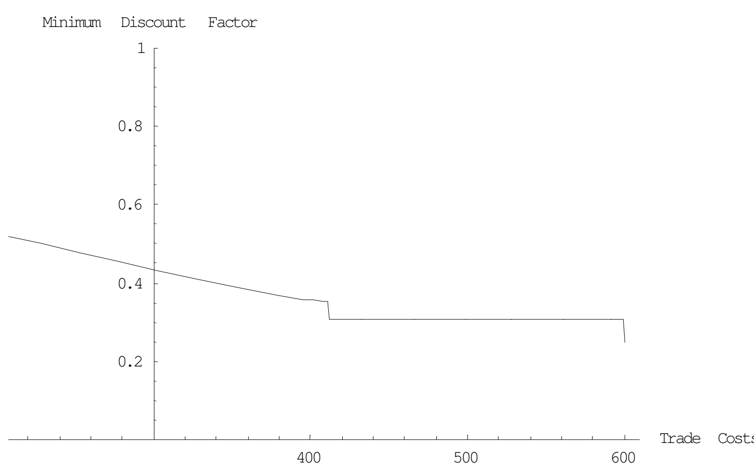



Fig. 8 Minimum Discount Factor and Trade Costs for $\gamma = 0.8$

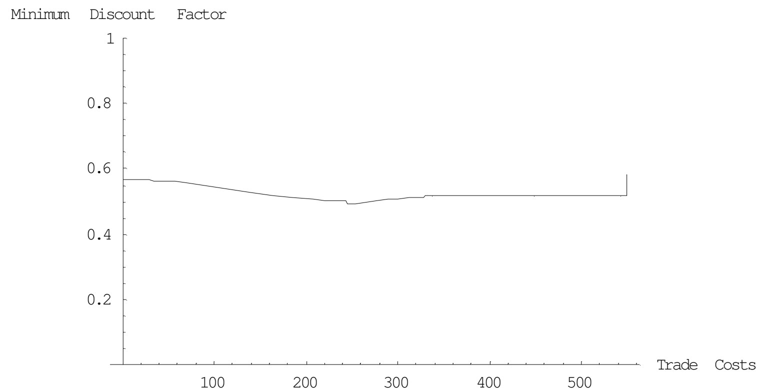



Fig. 9 Minimum Discount Factor and Trade Costs for $\gamma = 0.9$

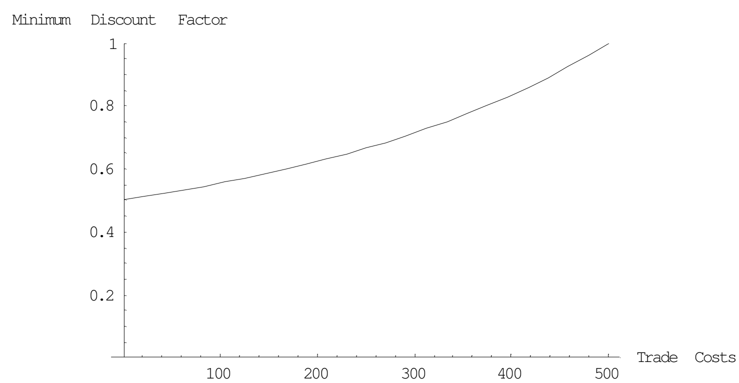



Fig. 10 Minimum Discount Factor and Trade Costs for $\gamma = 1$

Although the minimum discount factor exhibits a non-monotonic relationship with trade costs for $0 < \gamma < 1$, the following general conclusions can be made:

- For $\gamma < 0.5$, the minimum discount factor is increasing in trade costs for trade costs in the low to intermediate ranges. Thus, when products are distant substitutes and trade costs are not so high, trade liberalization will make the collusive outcome more sustainable just as in the case of homogenous products.
- For $\gamma < 0.6$, the minimum discount factor is convex in trade costs for sufficiently high trade costs especially in the neighborhood of the prohibitive level of trade

cost. In such a case, trade liberalization will have either a pro-competitive or pro-collusive effect depending on the initial level of trade cost.

- For $0.7 < \gamma \leq 0.9$, the minimum discount factor is decreasing in trade costs for sufficiently low levels of trade costs and especially in the neighborhood of free trade. Thus, in this range of $\gamma$ and trade costs, trade liberalization will make the collusive outcome more difficult to sustain and therefore have pro-competitive effects.

We have thus demonstrated clearly that the minimum discount factor may be decreasing or increasing with respect to trade costs and shown that the effect of trade liberalization on collusion and hence welfare is not so clear cut—it depends on the degree to which products are substitutable or differentiable and the prevailing levels of trade costs in the markets.

The case of homogenous products has already been treated in the literature (Lommerud and Sorgard (2001) for example) and touched on also in this paper and in that case, we saw that the minimum discount factor is an increasing function of trade costs. We also saw that this result is not only true for homogenous products but for all $\gamma < 0.5$ and sufficiently low trade costs.

<u>Limit Case of Homogenous Products and More Than 1 Firm in Each Country</u>

Suppose there are n>1 firms in each country. For $t = 0$, domestic firms have no cost advantages over foreign firms; the Nash equilibrium will involve all firms charging $p = 0$ and each supplying $q = A/2n$. This combination of price and output yields zero profit for each firm. Domestic output level will equals $A$ and welfare equals $A^2/2$.

For $t > 0$, the Nash equilibrium involves all foreign firms charging $\overline{p}^* = t$ in the domestic market and supplying zero exports while all domestic firms charge $p = 0$ and each supply $q = A/n$. Each firm still earns zero profit and welfare equals $A^2/2$.Social welfare in this case is higher compared to when there is one firm in each market. Regardless of the levels of trade costs, having more than one firm in each market is pro-competitive and welfare is higher because the increase in consumers' surplus as a result of increased output outweighs the decrease in firms' profits as they compete away their profits to zero.

Collusion may involve firms within and across countries. For trade costs equal to or beyond the prohibitive level $A/2$, collusion necessarily involves firms within a country (intra-national). In this case, each domestic firm charges the monopoly price $A/2$ and supplies $q = A/2n$. Each firm's profit is $A^2/4n$ which is decreasing in the number of firms. Welfare equals $3A^2/8$. Thus, welfare under collusion is unchanged when compared to the case of one firm in each country. The effect of more firms in each country is only a reduction in the collusive profit of each firm. The minimum discount factor for which the collusive outcome is sustainable is equal to $(n-1)/n$, which is increasing in n and highlights the sense in which increasing the number of firms in the domestic market can be pro-competitive when trade costs are sufficiently high.

For $t \in (0, A/2)$, intra-national collusion involves each of the foreign firms charging $\overline{p}^* = t$ and supplying zero exports whereas the domestic firms each set $p = t$ and supply $q = (A-t)/n$, thereby earning profits $t(A-t)/n$. As with the previous case, welfare is the same as when there is only one firm in each market,

$(A^2 - t^2)/2$, and the minimum discount factor is $(n-1)/n$, which is independent of trade costs.

On the other hand, international collusion involves all firms charging the monopoly price and each supplying $q = A/2n$ in its domestic market. This yields collusive profits of $A^2/4n$ which is higher than the profit level under intra-national collusion. The minimum discount factor for which the former is sustainable is $1 - \frac{A}{2n(A-t)}$ which is increasing in n, decreasing in t and maximized under free trade at $(2n-1)/2n$. Although the result still remains that increasing the number of firms in each country is pro-competitive in the neighborhood of $t = 0$, it is no longer the case that trade liberalization makes the collusive outcome more sustainable for $t > 0$ as is the case with one firm in each country. This is because the punishment pay-off for a deviating firm is zero and independent of the level of trade cost in its domestic market (same as in the foreign market) when there is more than one firm in each country whereas its punishment payoff in its domestic market is increasing in t for $t \in (0, A/2)$ and maximized at the prohibitive trade cost. Thus, a reduction in trade cost reduces the punishment payoff (or alternatively, increases the strength of the retaliatory payoff) for the case of one firm in each country. While a reduction in the trade cost has no effect on the punishment payoff when there is more than one firm in each country, the payoff from deviating in the export market is decreasing in t so that a reduction in trade cost increases this payoff and makes deviation more attractable.

Moreover, the minimum discount factor for international collusion is higher than what is necessary to sustain intra-national collusion so that although profits are higher under international collusion, intra-national collusion is more sustainable.

Thus, there is a trade-off between cartel sustainability and profitability when there is more than one firm in each country.

<u>Optimization Subject to Incentive Compatibility Constraint</u>

For a given $t \geq 0$, an agreement $(p, \overline{p})$ is sustainable if

$$Z(p, \overline{p}, t, \delta, \Pi^{N}(t)) = \Pi^{C}(p, \overline{p}, t) - (1-\delta)\Pi^{D}(p, \overline{p}, t) - \delta\ \Pi^{N}(t) \geq 0$$

where the collusive (agreement), deviation and Nash profits are given in previous sections.

The deviation profits depend on the degree of product substitutability and the levels of trade costs because these two determine whether the firm deviates to an interior or corner outcome in the domestic and foreign markets respectively. Specifically, for very low levels of $\gamma$ like $\gamma = 0.2$, only an interior deviation is possible in the foreign market whereas, in the domestic market, both interior and corner deviations are possible depending on the levels of trade costs. For low levels of the trade cost, as in the neighborhood of free trade, deviation in the domestic market is interior but for sufficiently high levels of trade cost, it is a corner.

On the other hand, for sufficiently high levels of $\gamma$ like $\gamma = 0.9$, only a corner deviation is possible in the domestic market whereas, in the foreign market, both interior and corner deviations are possible depending on the levels of trade costs. For low levels of the trade cost, as in the neighborhood of free trade, deviation in the foreign market is corner but for sufficiently high levels of trade cost, it is interior.

Thus, for sufficiently low levels of $\gamma$, we have the home and foreign market deviation profits given respectively as:

$$\Pi_h^D(\overline{p}) = \begin{cases} \dfrac{(a+d\ \overline{p})^2}{4b}, & 0 \le t < \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \\ \dfrac{(A-\overline{p})[A(\gamma-1)+\overline{p}]}{\gamma^2}, & \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \le t < A(1-\gamma) \end{cases}$$

$$\Pi_f^D(p,t) = \begin{cases} \dfrac{(a+dp-bt)^2}{4b}, & 0 \le t < \dfrac{A(2-\gamma)}{2} \end{cases}$$

For sufficiently high levels of $\gamma$, we have the home and foreign market deviation profits given respectively as:

$$\Pi_h^D(\overline{p}) = \begin{cases} \dfrac{(A-\overline{p})[A(\gamma-1)+\overline{p}]}{\gamma^2}, & 0 \le t < A(1-\gamma) \end{cases}$$

$$\Pi_f^D(p,t) = \begin{cases} \dfrac{(A-p)[A(\gamma-1)-\gamma t+p]}{\gamma^2}, & 0 \le t < \dfrac{A(\gamma^2-2+2\gamma)}{2\gamma} \\ \dfrac{(a+dp-bt)^2}{4b}, & \dfrac{A(\gamma^2-2+2\gamma)}{2\gamma} \le t < \dfrac{A(2-\gamma)}{2} \end{cases}$$

From the above, we can see that all outcomes are interior for very low levels of $\gamma$ and for trade costs in the range $0 \le t < \frac{A(2-2\gamma-\gamma^2)}{2-\gamma^2}$, so that total deviation profit is given as:

$$\Pi^D(p,\overline{p},t) = \begin{cases} \dfrac{(a+d\ \overline{p})^2+(a+d\ p-bt)^2}{4b}, & 0 \le t < \dfrac{A(2-2\gamma-\gamma^2)}{2-\gamma^2} \end{cases}$$

This deviation profit is decreasing in t and also strictly increasing and convex in $(p,\overline{p})$. But for sufficiently high levels of $\gamma$ and for trade costs in the range $0 \le t < A(1-\gamma)$, the Nash and collusive outcomes are interior whereas the deviation outcomes are both corners. In this case, total deviation profit is given as:

$$\Pi^D(p,\overline{p},t) = \begin{cases} \dfrac{(A-\overline{p})[A(\gamma-1)+\overline{p}]+(A-p)[A(\gamma-1)-\gamma t+p]}{\gamma^2}, & 0 \le t < A(1-\gamma) \end{cases}$$

which is decreasing in t and strictly concave in $(p,\overline{p})$.

For very low levels of $\gamma$ and $0 \leq t \leq \frac{A(2-2\gamma-\gamma^2)}{2-\gamma^2}$, the agreement profit is strictly concave in $(p, \overline{p})$, so that the constraint $Z(p, \overline{p}, t, \delta, \Pi^N(t)) = \Pi^C(p, \overline{p}, t) - (1-\delta)\Pi^D(p, \overline{p}, t) - \delta\ \Pi^N(t)$ is also concave in $(p, \overline{p})$ since the deviation profit is convex in $(p, \overline{p})$. For sufficiently high levels of $\gamma$ and $0 \leq t < A(1\text{-}\gamma)$, the agreement profit is strictly concave in $(p, \overline{p})$, but the deviation profit is concave in $(p, \overline{p})$ so that the constraint is the sum of a concave and convex set and as such, we cannot say much about the curvature of the constraint but we know at least that the constraint is a convex set because if we take any two points in the set, a linear combination of those two points also lie in the set.

For optimization subject to the incentive constraint, if we focus on the case of all interior outcomes and maximize the concave agreement profit subject to the concave constraint, we have the following:

$$\underset{p,\overline{p}}{Max}\ \mathrm{L} = \Pi^A(p, \overline{p}, t) + \lambda[\Pi^A(p, \overline{p}, t) - (1-\delta)\Pi^D(p, \overline{p}, t) - \delta\ \Pi^N(t)]$$

Denoting with subscript the derivative of the Langragian with respect to a variable, we have:

$$\mathrm{L}_p = (1+\lambda)\Pi^C{}_p - \lambda(1-\delta)\Pi^D{}_p = 0 \qquad (5)$$

$$\mathrm{L}_{\overline{p}} = (1+\lambda)\Pi^C{}_{\overline{p}} - \lambda(1-\delta)\Pi^D{}_{\overline{p}} = 0 \qquad (6)$$

$$\mathrm{L}_\lambda = \Pi^C - (1-\delta)\Pi^D - \delta\ \Pi^N \geq 0 \qquad (7)$$

$$\lambda \mathrm{L}_\lambda = \lambda(\Pi^C - (1-\delta)\Pi^D - \delta\ \Pi^N) = 0, \qquad \lambda \geq 0 \qquad (8)$$

When the incentive constraint is slack, i.e. holds with inequality, from (8), we have that $\lambda = 0$, and from (5) and (6), the solution to the constrained optimization problem is the same as that of the unconstrained problem; $p = \frac{A}{2}$ and $\overline{p} = \frac{A+t}{2}$, and since

$Z(p,\overline{p},t,\delta,\Pi^N(t))$ is concave in $(p,\overline{p})$, we know that $(p,\overline{p})$ is sustainable for any $\delta \geq \delta_{\min}$, where $\delta_{\min}$ the minimum discount factor consistent with low values of $t$ and $\gamma$ have already been computed.

For $\lambda > 0$, the incentive constraint binds and the monopoly outcome may no longer be sustainable as the collusive outcome as was the case when the incentive constraint was slack. From (5) and (6), we have that:

$$\frac{\lambda(1-\delta)}{1+\lambda} = \frac{\Pi^C{}_p}{\Pi^D{}_p} = \frac{\Pi^C{}_{\overline{p}}}{\Pi^D{}_{\overline{p}}} \Rightarrow \frac{a+2d\ \overline{p}-2bp-dt}{a+dp-bt} = \frac{a+2dp-2b\overline{p}+bt}{a+d\ \overline{p}} \qquad (9)$$

For $t=0$, this reduces to $\dfrac{a+2d\ \overline{p}-2bp}{a+dp} = \dfrac{a+2dp-2b\overline{p}}{a+d\ \overline{p}}$, so that $p=\overline{p}$. For interior optima, the first order condition is necessary but not sufficient. With only one binding constraint and two choice variables, the sign of the determinant of the 3x3 matrix of bordered second order conditions should be positive for $(p,\overline{p})$ to be an interior optimum.

From (9), we derive that the marginal rate of substitution between $p$ and $\overline{p}$ along the agreement and deviation profit curves must be equal; i.e.

$MRS_{p,\overline{p}} = \dfrac{\Pi_p^C}{\Pi_{\overline{p}}^C} = \dfrac{\Pi_p^D}{\Pi_{\overline{p}}^D}$. The derivatives of the deviation and agreement profits are given as: $\Pi^A{}_p = a - 2(b+d)p$ and $\Pi^D{}_p = \dfrac{d(a+dp)}{8b^2}$. From (9) and letting $1+\lambda=\theta$; $\lambda(1-\delta)=\beta$, we have

$p=\overline{p} = \dfrac{8ab^2\beta - \theta ad}{16b^2\beta(b+d)+\theta d^2} = \dfrac{A(1-\gamma)(8\beta-\gamma\theta+\gamma^3\theta)}{(1+\gamma)(16\beta+\gamma^2\theta-\gamma^3\theta}$, which when substituted into the marginal rate of substitution gives $MRS_{p,\overline{p}} = 1$.

We conjecture that $p = \overline{p}$ everywhere, and to test this, we need to verify that the sufficient condition holds. To do this, we check the determinant of the 3x3 matrix of bordered second order derivatives:

$$\begin{bmatrix} (1+\lambda)\Pi^C_{pp} - \lambda(1-\delta)\Pi^D_{pp} & (1+\lambda)\Pi^C_{p\overline{p}} - \lambda(1-\delta)\Pi^D_{p\overline{p}} & \Pi^C_p - (1-\delta)\Pi^D_p \\ (1+\lambda)\Pi^C_{\overline{p}p} - \lambda(1-\delta)\Pi^D_{\overline{p}p} & (1+\lambda)\Pi^C_{\overline{p}\overline{p}} - \lambda(1-\delta)\Pi^D_{\overline{p}\overline{p}} & \Pi^C_{\overline{p}} - (1-\delta)\Pi^D_{\overline{p}} \\ \Pi^C_p - (1-\delta)\Pi^D_p & \Pi^C_{\overline{p}} - (1-\delta)\Pi^D_{\overline{p}} & 0 \end{bmatrix}$$

Substituting $\Pi^C_p = a + d\ \overline{p} - 2bp$, $\Pi^D_p = \dfrac{d(a+dp)}{8b^2}$, $\Pi^C_{\overline{p}} = a + d\ p - 2b\overline{p}$,

$\Pi^D_{\overline{p}} = \dfrac{d(a+d\overline{p})}{8b^2}$, $\Pi^C_{pp} = \Pi^C_{\overline{p}\overline{p}} = -2b$, $\Pi^D_{pp} = \Pi^D_{\overline{p}\overline{p}} = \dfrac{d^2}{8b^2}$, and $\Pi^C_{p\overline{p}} = \Pi^C_{\overline{p}p} = 2d$,

$\Pi^D_{p\overline{p}} = \Pi^D_{\overline{p}p} = 0$ and the value $p = \overline{p}$ above into the above matrix gives a determinant

$$\frac{A^2 \gamma^2 (\beta(-1+\gamma)\gamma^2 - 16\theta)(\beta^2(-1+\gamma)(1+\gamma)(2+3\gamma) + 24\beta(-1+\theta) - 2(-1+\gamma)(1+\gamma)(-1+\theta)\theta)^2}{4(-1+\gamma)(1+\gamma)^4(-1+\theta)^2(-16\beta + (-1+\gamma)\gamma^2\theta)^2}$$

which is positive . Thus, the sufficient condition holds.

The above prices are less than the monopoly prices under free trade when the incentive constraint is slack. We know this because when the incentive constraint is slack and trade is free, then the collusive outcome is $p = \overline{p} = \dfrac{A}{2}$ but here the price pair we've derived for binding incentive constraint is less than the monopoly price since $\dfrac{1-\gamma}{1+\gamma} < 1$ and $\dfrac{8\beta - \gamma\theta + \gamma^3\theta}{16\beta + \gamma^2\theta - \gamma^3\theta} < \dfrac{1}{2}$. Thus, the maximum sustainable collusive profit for the case of binding incentive constraint under free trade is less than the monopoly profit. Since the deviation profits are increasing in the other firm's choice of monopoly prices, a reduction in prices below the monopoly price will reduce deviation incentives as is the case here. Thus, when trade costs are sufficiently low, as in the neighborhood of free trade, further reduction in trade costs reduces the deviation incentives and hence makes the collusive outcome more sustainable when the incentive constraint binds.

For $t > 0$, any price pair satisfying (9) is sustainable; we do not focus on determining the prices here. Our interest is on whether a reduction in the discount factor would cause trade to occur from a situation where trade did not initially occur. We know that since the constraint is monotone in $\delta$, there exists $\delta$ for which the constraint just binds. We also know that the pair of prices chosen by the cartel is increasing in $\delta$. The question that we then want to answer is: supposing we start from sufficiently high trade costs so that trade does not occur, will a decrease in $\delta$ which leads to a decrease in the collusive price cause firms to trade? To answer this question, we will consider how a decline in $\delta$ will affect the marginal rate of substitution along the agreement and deviation profits curve for the interior solution. If the marginal rates of substitution are equal, the firms will not trade; otherwise, trade will occur.

The marginal rate of substitution of the cartel profit for an interior solution is: $MRS^{A}_{p,\bar{p}} = \dfrac{a-2bp+2d\ \bar{p}-dt}{a-2b\bar{p}+2dp+bt}$ and that of the deviation profit is $MRS^{D}_{p,\bar{p}} = \dfrac{a+dp-bt}{a+d\ \bar{p}}$. If we evaluate these marginal rate of substitution at the boundary price $\bar{p}^{b} = t$, we get :

$$MRS^{A}_{p,\bar{p}} = \frac{A(1-\gamma)+\gamma t-2p}{A(1-\gamma)-t+2\gamma p} \text{ and } MRS^{D}_{p,\bar{p}} = \frac{A(1-\gamma)-t+\gamma p}{A(1-\gamma)+\gamma t}.$$

The former is increasing in p whereas the latter is decreasing in p. Since the marginal rates of substitution evaluated at the boundary are not equal, the firms will do better by trading. Thus, a reduction in the minimum discount factor will lead to trade.

When we compare these marginal rates of substitution along the interval $[p^N, p^c]$, where $p^c$ is the unconstrained domestic market price for the cartel, which is the same as the monopoly price, we get that: $MRS^A_{p,\bar{p}} \in [-\gamma, \quad \dfrac{A\gamma(2-\gamma-\gamma^2)-t\gamma(2-\gamma^2)}{A(\gamma-1)(2+\gamma)^2+t(4-3\gamma^2)}]$

and $MRS^D_{p,\bar{p}} \in [\dfrac{2A(2-\gamma-\gamma^2)-2t(2-\gamma^2)}{(A(1-\gamma)+\gamma t)(4-\gamma^2)}, \quad \dfrac{A(2-\gamma)-2t}{2A(1-\gamma)+2\gamma t}]$.

Finally, if we keep trade costs constant and reduce $\delta$ so that the collusive price reduces, we see that the marginal rate of substitution of the cartel profit along the boundary will increase whereas the marginal rate of substitution of the deviation profit along the boundary will decrease. In this fashion, further declines in $\delta$ which causes the above movements in the marginal rates of substitution will ensure that there exist a $p \in (p^N, p^c)$ for which the firms can't do better by trading, and this price will be the equilibrium price.

Welfare Effects of Tariff Reductions under the Collusive Outcome

When trade costs assume the form of tariffs, social welfare can be expressed as the sum of consumer surplus, global profits and tariff revenues when exports are positive. The expression is thus given by:

$W^C = \frac{1}{2}(Q^C)^2 + \Pi^C(t) + t\,x^C(t)$ where $Q^C = q^C + x^C$, $q^C = a - bp^C + d\bar{p}^C$, $x^C = a - b\bar{p}^C + dp^C$, $a = \frac{A}{1+\gamma}$,

$b = \frac{1}{1-\gamma^2}$, and $d = \frac{\gamma}{1-\gamma^2}$, $W^C$ is the national welfare function under collusion, $Q^C$ is the aggregate output level under collusion, $\Pi^C$ is the firm's profit under collusion, $q^C$ and $x^C$ are domestic firm's output and exports under collusion respectively. Recall from previous calculations that $\Pi^C(t) = \frac{2A^2(1-\gamma)-2At(1-\gamma)+t^2}{4(1-\gamma^2)}$, $q^C = \frac{A(1-\gamma)+t\gamma}{2(1-\gamma^2)}$, $x^C = \frac{A(1-\gamma)-t}{2(1-\gamma^2)}$ and

$Q^C = q^C + x^C = \dfrac{(2A-t)}{2(1+\gamma)}$. Thus, $W^C = \dfrac{1}{2}\left[\dfrac{(2A-t)}{2(1+\gamma)}\right]^2 + \dfrac{2A^2(1-\gamma)-2At(1-\gamma)+t^2}{4(1-\gamma^2)} + t\left[\dfrac{A(1-\gamma)-t}{2(1-\gamma^2)}\right]$

$$\Rightarrow W^C = \frac{4A^2(\gamma-1)(2+\gamma)-4At(\gamma-1)+t^2(1+3\gamma)}{8(1+\gamma)^2(\gamma-1)}$$

$$\Rightarrow \frac{\partial W^C}{\partial t} = \frac{4A(1-\gamma)+2t(3\gamma+1)}{8\,(\gamma-1)(1+\gamma)^2} < 0.$$

Thus, a tariff reduction unambiguously improves social welfare because it leads to an increase in aggregate output and therefore an increase in consumer surplus. Although there is a non-monotonic relationship between global collusive profits and tariffs when firms choose to export, the overall effect of a tariff reduction on social welfare is positive because of its effect on consumer surplus and tariff revenues. Again recall that Bond and Syropoulos (2008) noted that trade liberalization will lead to an improvement in domestic welfare if and only if it results in a reduction in the domestic price and that is the case here. The result obtained here is not surprising because the price charged by the domestic firm in the home market is independent of the tariff when firms collude but that of the foreign firm is an increasing function of the tariff. Thus, a tariff reduction ultimately reduces the relative price faced by domestic consumers and increases the purchasing power of their disposable income.

When firms collude under zero exports, tariff revenues are zero, so that the welfare function is the sum of consumer surplus and the single market monopoly profit. Thus, welfare is given as: $W^C = \frac{1}{2}(Q^C)^2 + \Pi^C$ where $Q^C = q^C = \frac{A}{2}$ and $\Pi^C = \frac{A^2}{4}$, implying that $W^C = \frac{3A^2}{8}$ a function that is independent of tariffs. Thus, trade liberalization has welfare-enhancing effects only when firms choose to export and then collude (collusive exporting).

## Conclusion

I study the effect of reductions in trade costs on the sustainability of collusion and social welfare in a duopoly market in which firms compete in prices and find that the relationship between trade costs and the minimum discount factor for which the collusive outcome is sustainable may be positive or negative, and this relationship depends, to a great extent, on the degree of product substitutability and the initial levels of trade costs.

For the limiting case of homogenous products, I find that a reduction in trade cost makes the collusive outcome more sustainable. This is because although trade liberalization makes deviation in the export market more attractive, it also makes punishment after deviation harsher. In the case of price competition, the loss in future profits as a result of the harsher punishment is more than the one-period gain from deviation. Hence the collusion-enhancing effect of trade liberalization as is found here.[4]

I also find that the optimal cartel agreements when trade is free and the incentive constraint binds involves firms charging a price less than the monopoly price in each market in order to facilitate collusion.

I also considered the effect of tariffs and transport costs on social welfare and find that reductions in tariffs unambiguously lead to improvements in social welfare, whereas reductions in transport costs may or may not lead to welfare improvements. In particular, when exports are positive in equilibrium, welfare is U-shaped; for sufficiently low levels of transport costs, further reductions in transport costs are welfare- enhancing but when transport costs are sufficiently high, reductions in transport costs are welfare-reducing.

[4] The opposite result is found for the case of quantity competition because in that case, firms engage in cross-hauling of identical products even under the punishment phase, whereas in the case of price competition, no cross-hauling of identical products occur under collusion or punishment.

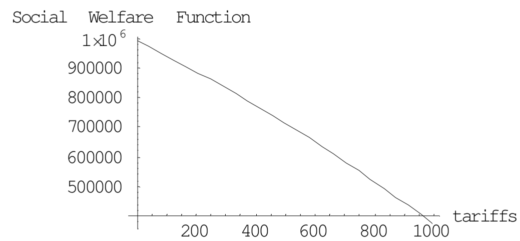
When transport costs are prohibitively high enough to warrant zero exports in equilibrium but the domestic firm prices lower than the monopoly price, welfare is a declining function of transport costs so that reductions in transport costs are welfare-enhancing. Moreover, welfare under free trade (zero tariffs and transport costs) is found to be higher compared to welfare under autarky (prohibitive tariffs).

These results show that, for welfare considerations, it matters whether trade costs assume the form of tariffs or transport costs. In particular, neither form of trade cost is a substitute for the other when considering trade policy. Trade liberalization, whether it takes the form of tariff reductions or reductions in transport costs, would be welfare enhancing only to the extent that the economy-wide gains from trade (if it occurs) outweighs the losses; or the gains to consumers from increased output outweighs the losses to firms from decreased profits if no trade occurs.

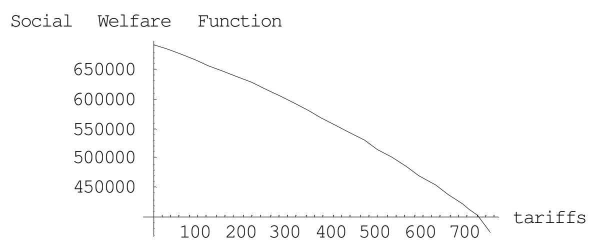
APPENDIX A

Simulation of the Welfare Function with Tariffs for A=1000 and $t \in [0, A(2-\gamma)/2]$.

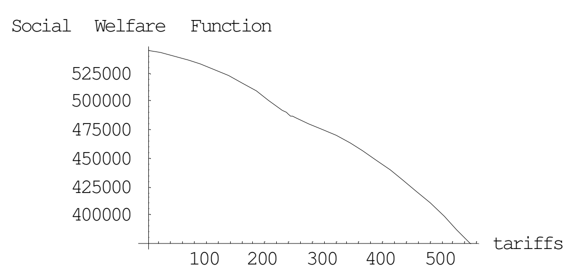
$\gamma = 0.01$




$\gamma = 0.5$

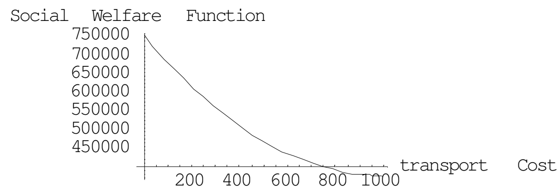



$\gamma = 0.9$

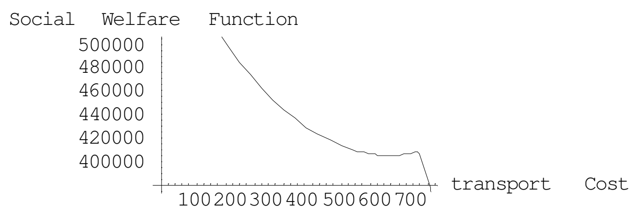


Simulation of the Welfare Function with Transport Costs for A=1000 and $t \in [0, A(2-\gamma)/2]$.

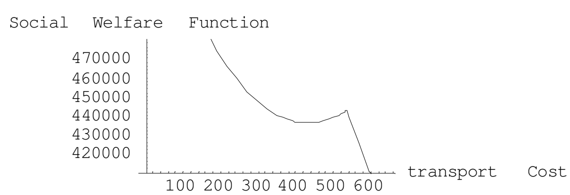
$\gamma = 0.01$



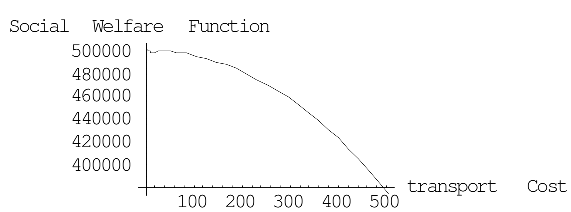

$\gamma = 0.5$




$\gamma = 0.7$




$\gamma = 0.99$



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## CHAPTER III

## THE EFFECT OF OIL PRICE VOLATILITY ON THE TERMS OF TRADE AND PRIVATE CONSUMPTION: A CASE STUDY OF NIGERIA

### Introduction

It has been observed by many authors[5] that changes in the world price of primary exported commodities can have significant effect on economic activity and the terms of trade of exporting countries. Fluctuations in the terms of trade may be because countries export different basket of goods than they import; but may also be due to monopolistic tendencies (pricing-to-market) and other factors that cause deviations from the Law of One Price. These authors demonstrate that changes in the terms of trade of developing countries are mostly due to fluctuations in the world price of a single primary commodity which they export. Moreover, fluctuations in consumption patterns can be linked to fluctuations in the terms-of-trade.

This paper studies the effects of oil (Nigeria's primary export, comprising between 90-95% of merchandise exports over the sample period of my analysis) price volatility on Nigeria's terms of trade and private consumption by considering a model of a small open economy in which a representative agent maximizes the expected discounted present value of utility—a function of consumption and leisure—subject to his resource constraints. The level of exports is endogenized and assumed to be a function of labor input alone. The reason for this assumption is because Nigeria, though an exporter of crude oil, imports gasoline for consumption. An implication of

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Cuddington and Urzúa, 1989; Bleaney and Greenaway, 1993; Backus and Crucini, 2000; Baxter and Kouparitsas, 2000; Bidarkota and Crucini; 2000 etc looked at the causes of fluctuations in the net barter terms of trade in recent years and most of them concluded that most of the variations in the net barter terms of trade can be attributed to variations in the price of oil. Moreover, most of the volatility in the terms of trade of developing countries are attributable to volatility in the prices of the primary commodities which they export.

this assumption, however, is that capital use will be captured by labor productivity, leading to a positive correlation between the two variables so that results would have to be interpreted with caution.

I will study the effect of oil price volatility on the representative agent's consumption pattern by addressing the following questions:

1. How volatile and persistent are Nigeria's terms of trade?
2. What is the relationship between the terms of trade and world price of oil?
3. How does volatility in the terms of trade affect the agent's consumption?

In addressing the above questions, I will use multivariate linear systems method to compute population moments and impulse response of consumption to exogenous terms of trade shocks and carry out stochastic simulations of the linear optimal control problem associated with the model. The agent's consumption pattern is expected to fluctuate with terms of trade shock. In particular, for the case of Nigeria, since oil exports constitutes about 95% of total export, a positive terms-of-trade shock is expected to have a positive effect on oil export earnings and thus output. On the other hand, oil is also an input in other sectors of the economy so that a positive terms-of-trade shock would have a negative effect on output. Thus, the overall effect of a positive terms-of-trade shock on output would depend on which effect dominates. If the positive effect dominates, then a positive terms of trade shock should lead to an increase in consumption but if the negative effect dominates, then one is not likely to observe such an increase in consumption.

## Variance Decomposition of Output

Before proceeding to a description of the model, I will carry out a variance decomposition of output. Variance decomposition is a different method of depicting system dynamics and it decomposes variations in an endogenous variable into the component shocks to the endogenous variables in a Vector Auto Regressions (VAR) model. The variance decomposition gives information about the relative importance of each random innovation to the variables in the VAR. All variables in the VAR are log-transformations of the original variables. The procedure is as follows:

Consider a VAR of order p, $y_t = \beta x_t + A_1 y_{t-1} + ..... + A_p y_{t-p} + \varepsilon_t$ where $y_t$ is a vector of k endogenous variables, $x_t$ is a vector of g exogenous variables comprising, in this case, of the relative price of oil and quantity of oil produced, and $\varepsilon_t$ represents innovations at time t with variance $\Omega$. If the VAR is invertible, the moving average (MA) representation is given by:

$$\bar{y}_t = A_1 \bar{y}_{t-1} + ..... + A_p \bar{y}_{t-p} + \varepsilon_t = (I - A_1 L - ..... - A_p L^p)^{-1} \varepsilon_t =$$

$$(I + \Psi_1 L + ..... + \Psi_p L^p)^{-1} \varepsilon_t$$

where $\bar{y}$ is the residual from regressing output on the relative price of oil and quantity of oil. The VAR coefficients, A, and the MA coefficients $\Psi$ must satisfy

$$(I - A_1 L - ..... - A_p L^p)\,(I + \Psi_1 L + ..... + \Psi_p L^p) = \mathrm{I}$$

$$(I + c_1 L + ..... + c_p L^p) = I \text{ where } c_1 = c_2 = ...... = 0.$$

Thus, $\Psi_1 = A_1$, $\Psi_2 = A_1 \Psi_1 + A_2$, ......., $\Psi_s = A_1 \Psi_{s-1} + A_2 \Psi_{s-2} + ...... + A_p \Psi_{s-p}$. The s-period ahead forecast error from the VAR is given by $\varepsilon_{t+s} + \Psi_1 \varepsilon_{t+s-1} + ... + \Psi_{s-1} \varepsilon_{t+1}$, which has a mean squared error $\Omega + \Psi_1 \Omega \Psi_1^{'} + ..... + \Psi_{s-1} \Omega \Psi_{s-1}^{'} =$

$$PP^{'} + \Psi_1 PP^{'} \Psi_1^{'} + .... + \Psi_{s-1} PP^{'} \Psi_{s-1}^{'} = \sum_{j=1}^{k} \left(p_j p_j^{'} + \Psi_1 p_j p_j^{'} \Psi_1^{'} + .... + \Psi_{s-1} p_j p_j^{'} \Psi_{s-1}^{'}\right)$$

where $p_j$ is the $jth$ column of P, a $k$ x $k$ lower triangular matrix with the standard deviations of the orthogonalized innovations along the main diagonal such that $PP^{'} = \Omega$.

The variance decomposition of output for a 10-period forecast horizon is displayed below in tabular form for two different orderings of the relative price of oil and productivity. All the variables in the model are log-transformations of the original variables. The ordering of the variables is given at the bottom of each table. The column S.E. is the forecast error of output for each forecast horizon. The remaining columns give the percentage of the variance due to each innovation such that each row adds up to 100. Since output comes first in the VAR ordering, the only source of the one period ahead variation is its own innovation so that the first number is 100 percent.

Over the 10-period horizon, the oil price index is shown to contribute a significantly greater percentage of the variance in output compared to productivity, regardless of the ordering of the variables—an average of about 24%. Since the ordering of variables does not matter for the variance decomposition, shocks to productivity and the relative price of oil can be perceived as independent.

TABLE 1

VARIANCE DECOMPOSITION OF OUTPUT

| Period | S.E. | GDP | POIL | LABORPROD |
|---|---|---|---|---|
| 1 | 0.030000 | 100.0000 | 0.000000 | 0.000000 |
| 2 | 0.040442 | 95.04201 | 4.945196 | 0.012792 |
| 3 | 0.045302 | 89.34683 | 10.62640 | 0.026774 |
| 4 | 0.048609 | 83.28443 | 16.55643 | 0.159140 |
| 5 | 0.051554 | 77.17922 | 22.47242 | 0.348364 |
| 6 | 0.054467 | 71.33420 | 28.11369 | 0.552109 |
| 7 | 0.057466 | 65.87156 | 33.38511 | 0.743336 |
| 8 | 0.060601 | 60.83358 | 38.25523 | 0.911197 |
| 9 | 0.063895 | 56.22073 | 42.72626 | 1.053008 |
| 10 | 0.067363 | 52.01414 | 46.81618 | 1.169683 |

Ordering: GDP LABORPROD POIL

| Period | S.E. | GDP | POIL | LABORPROD |
|---|---|---|---|---|
| 1 | 0.030000 | 100.0000 | 0.000000 | 0.000000 |
| 2 | 0.040442 | 95.04201 | 4.945290 | 0.012698 |
| 3 | 0.045302 | 89.34683 | 10.49099 | 0.162181 |
| 4 | 0.048609 | 83.28443 | 16.16402 | 0.551558 |
| 5 | 0.051554 | 77.17922 | 21.79094 | 1.029844 |
| 6 | 0.054467 | 71.33420 | 27.14488 | 1.520915 |
| 7 | 0.057466 | 65.87156 | 32.14820 | 1.980236 |
| 8 | 0.060601 | 60.83358 | 36.77611 | 2.390319 |
| 9 | 0.063895 | 56.22073 | 41.03218 | 2.747087 |
| 10 | 0.067363 | 52.01414 | 44.93345 | 3.052414 |

Ordering: GDP POIL LABORPROD

## Model Description

### **Preferences**

The representative agent has preferences $U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$ , $0 < \beta < 1$,

where $C_t$ is the amount of the manufactured consumption good imported in exchange for oil at time $t$, $L_t$ is leisure at time $t$ and $u(C_t, L_t)$ is assumed strictly increasing in its arguments, concave, twice continuously differentiable and $C_t$, $L_t$ are assumed to be always interior. The functional form assumed for the utility function is: $u(C_t, L_t) = \log(C_t) + \log(L_t)$.

**Production Function**

For the representative agent, output of oil at time $t$ is given by $Y_t = A_t N_t^{\alpha}$, where $A_t$ is productivity parameter at time $t$ and $N_t$ is labor input at time $t$ with $N_t > 0$.The production function is assumed to be concave and twice continuously differentiable, which implies that labor's share of output $\alpha$ satisfies $0 < \alpha \leq 1$.

**Resource Constraint**

In each period, the representative agent faces two resources constraints:

*) Total amount of time allocated to labor and leisure cannot exceed unity; $N_t + L_t \leq 1$. Since the utility function and output are increasing in $L_t$ and $N_t$ respectively, we can assume that the constraint holds with equality.

**) The value of consumption at any time $t$ cannot exceed the sum of disposable income and net change in financial wealth; $C_t \leq (1-\tau_t) q_t Y_t + B_t R_t - B_{t+1}$ where the price of the consumption good has been normalized to unity, $q_t$ is the price of oil in terms of the consumption good, $\tau_t$ is the income tax rate, $B_t$ is the stock of bonds brought into time $t$, $R_t = 1 + r_t$ is the gross rate of return on bonds at time $t$, $r_t$ being the real interest rate and $B_{t+1}$ is the stock of bond chosen at time $t$ for time $t+1$.

***) The exogenous variables in the model are labor productivity, the real gross rate of return on one-period bonds, government consumption $G_t$, and the terms of trade $q_t$.

Another variable not in the model but included in the solution algorithm for completeness and conformity with the programs to be used is lump-sum transfers $T_t$. Lump-sum transfers are set to zero because the government rarely engages in such activities as payments of social security, unemployment compensation and other transfer payments to citizens. In fact, retirees sometimes are not paid their pension allowances for years at a time. To this end, the government is modeled as collecting income taxes but not making any transfer payments. Thus, the government's budget constraint without debt financing is $G_t \le \tau_t Y_t$, where $G_t$ is government expenditures on schools, roads, national security etc and $\tau_t Y_t$ is revenue from income taxes at time $t$. For the rest of the analysis, the government is assumed to follow a balanced budget path in which $G_t = \tau_t Y_t$ for all $t$.

## Optimization Problem

The representative agent maximizes the expected discounted present value of utility subject to his resource constraints in * and **. The langragian for the optimization is:

$$L = \underset{\{C_t, N_t\}}{Max} \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \log(1 - N_t)] + \sum_{t=0}^{\infty} \beta^t \lambda_t [(1 - \tau_t) A_t q_t N_t^{\alpha} + B_t R_t - B_{t+1} - C_t]$$

where $\lambda_t$ is the shadow price of consumption and leisure and resource constraint multiplier at time $t$ and $\beta$ is the rate of time preference. For ease of notation, let $(1 - \tau_t) = \Omega_t$, then the first order conditions with respect to $C_t$, $N_t$, $B_{t+1}$ and $\lambda_t$ respectively are:

$$1/C_t = \lambda_t \qquad (10)$$

$$1/(1 - N_t) = \alpha \lambda_t \Omega_t A_t q_t N_t^{\alpha - 1} \qquad (11)$$

$$E_t\{\beta \lambda_{t+1} R_{t+1}\} = \lambda_t \qquad (12)$$

$$\Omega_t A_t q_t N_t^{\alpha} + B_t R_t - B_{t+1} - C_t = 0 \qquad (13)$$

for all $t = 0,1,2,....,\infty$ and the transversality condition is $\lim_{t\to\infty} \lambda_t B_{t+1} = 0$. The above first order conditions can be interpreted in terms of equal marginal benefit and marginal cost. For example, the expression on the left hand side of condition (11) is the marginal utility from increasing leisure by one unit whereas that on the right hand side is the marginal cost, which is the value of the marginal product of labor (the foregone alternative) multiplied by the marginal utility of consumption that the extra labor input would have generated. Also, expression (12) equates the marginal benefit from investing in one unit of bond today (which is the discounted marginal utility of consumption from the gross bond earnings) to the marginal cost (which is the marginal utility from consuming today instead of investing). Expression (13) is the intertemporal budget constraint.

## Near Steady State Dynamics

In the steady state, $A_t = A$, $C_t = C$, $N_t = N$, $q_t = q$, $\lambda_{t+1} = \lambda_t = \lambda$, $B_{t+1} = B_t = B$, $R_{t+1} = R_t = R$ implying that $\beta = 1/R$. Since labor hours is restricted to be between zero and one, then the only feasible growth rate of hour per capita is zero. From the resource constraint, we have that consumption and output must grow at the same rate in the steady state but labor hours does not grow in the steady state since it is constrained to be between zero and one. Moreover, we have $s_c = \Omega + s_b(\frac{1-\beta}{\beta})$, where $s_c$ and $s_b$ are consumption and bond share of output respectively.

Linearization of (10)-(13) around the steady state levels ($C$, $N$, $B$ and $\lambda$) yields expressions for the percentage deviations from steady state levels, denoted by a circumflex (^).

$$-\hat{C}_t = \hat{\lambda}_t \qquad (14)$$

$$[\frac{N}{1-N} + (1-\alpha)]\hat{N}_t = \hat{\lambda}_t + \hat{A}_t + \hat{q}_t + \hat{\Omega}_t \qquad (15)$$

$$-\hat{\lambda}_{t+1} + \hat{\lambda}_t = \hat{R}_{t+1} \qquad (16)$$

$$\hat{B}_{t+1} - \frac{\hat{B}_t}{\beta} = -s_c\hat{C}_t + \alpha\Omega\hat{N}_t + \Omega\hat{A}_t + \frac{s_b}{\beta}\hat{R}_t + \Omega\hat{\Omega}_t + \Omega\hat{q}_t \qquad (17)$$

Moreover at base prices, we have:

$$\hat{Y}_t = \hat{A}_t + \alpha\hat{N}_t \qquad (18)$$

$$\hat{Y}_t - \hat{N}_t = \hat{A}_t + (\alpha - 1)\hat{N}_t \qquad (19)$$

The last two expressions give approximate solutions for output and labor productivity. A detailed description of the solution algorithm for generating moments and impulse responses is provided in Appendix B.

## Data Description

### Trends in Variables and Growth Rates

Data for this paper were extracted from different sources including the Energy Information Administration website, World Bank's World Development Indicators and International Monetary Fund's International Financial Statistics. For some variables, data are available from 1960 to 2007 whereas others are only available from 1980 to 2006. As a result, the sample period for all variables is taken to be 1980 to 2006, which becomes a limitation of the model's estimation. The series were either first-differenced or second-differenced to ensure stationarity before being used in the analysis. The line graphs below show trends in the log-levels and ratios of the variables as well as their growth rates over the period under study.

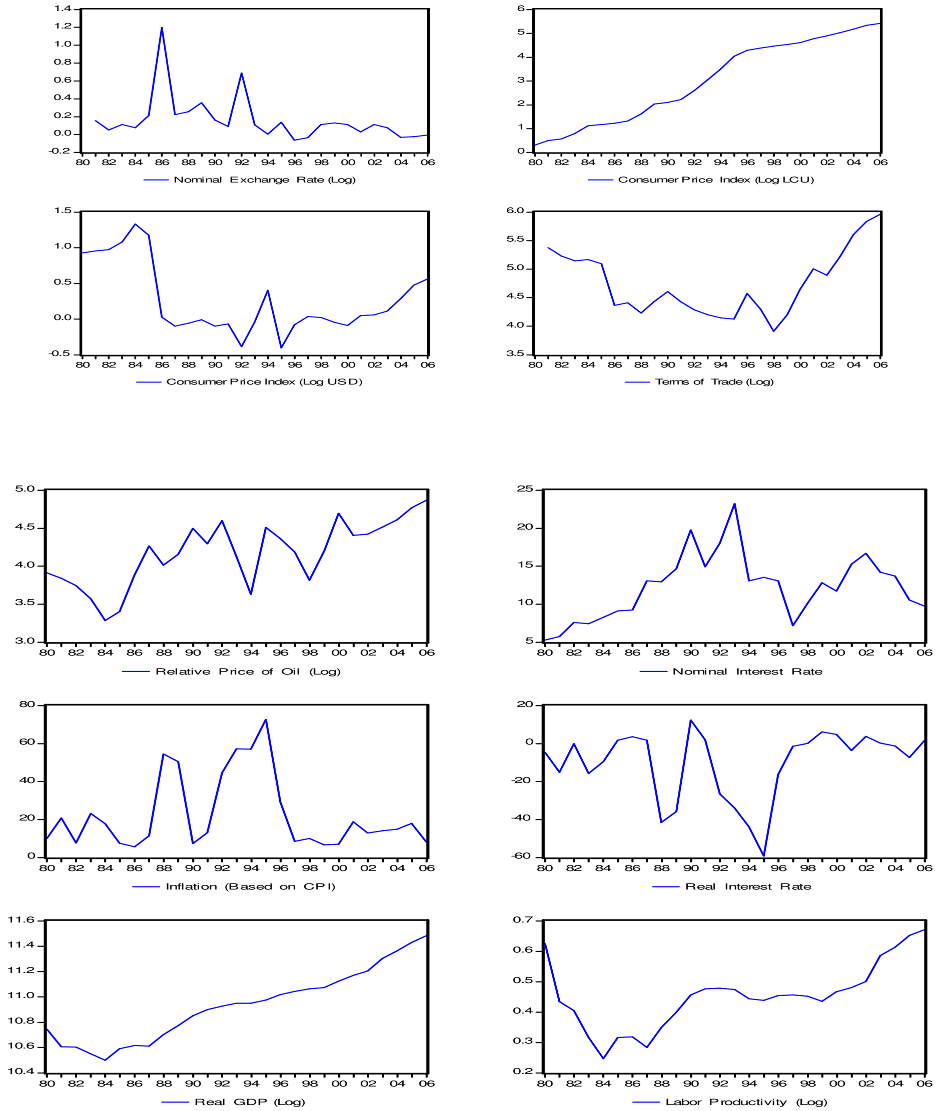


Fig. 11 Trend in Levels of Variables

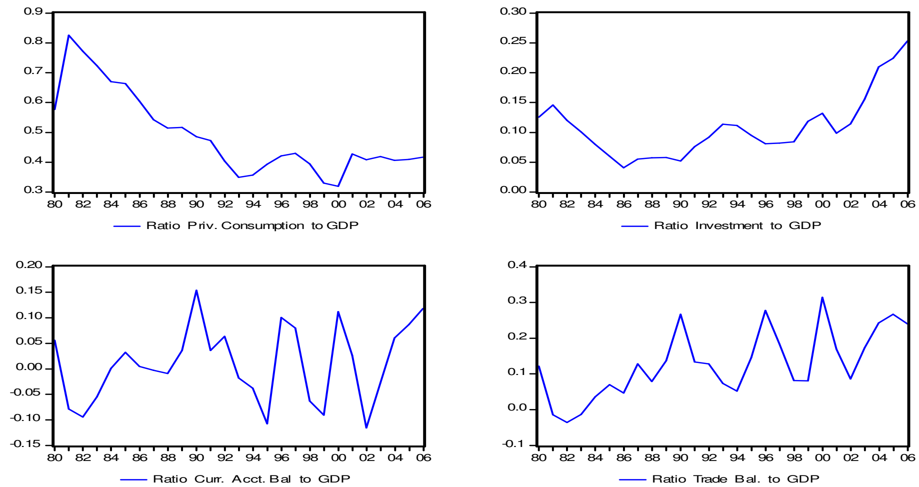


Fig. 12 Trend in Ratios of Variables

As is evident from the graph of the nominal exchange rate above, in the early 1980's, the Naira (Nigeria's currency) had a high value in terms of the US dollar because the country had discovered oil a decade before and oil was in high demand; the agriculture sector was doing well and generating exports for the nation, although the exports generated by agriculture had declined compared to the period before the oil boom. At the same time, negative real interest rates prevailed ex-ante in the financial sector but the effect on the Naira wasn't so obvious because the agriculture and oil sector more than made-up for the decline in investments that result from negative real interest rates.

By the late 80's to mid 90's, the agriculture sector which relied heavily on government subsidies and incentives had been almost completely abandoned in favor of oil making the export base even more concentrated given that oil already contributed about 90% of merchandise exports at the time. At the same time, inflation based on consumer price index was up and rising and by the late 80's to mid 90's it had reached an average of 65%. Thus, in spite of the comparatively high nominal interest rates, negative real interest rates prevailed in the economy.

Although Nigeria is a small open economy, its financial market is underdeveloped and not fully integrated into the world economy an as such, the real interest rates differ from that of the world economy. In integrated financial markets, domestic investors have the ability to purchase foreign assets and foreign investors have the ability to purchase domestic assets. Countries that are fully integrated into world financial markets should have identical expected rates of return for identical assets regardless of location but this was certainly not the case for Nigeria. As a result, the country became vulnerable and its investment share of real GDP declined consistently over the late 1980s and early 1990s. This led to a decline in the demand for the Naira in international currencies market by investors and ultimately a collapse of the nominal exchange rate. The collapse of the value of the Naira in international currencies market in the 1990's was the result of a combination of forces including a thin export base, weak domestic financial market and high inflationary conditions.

Another expected trend that is noticeable from figure 1 is that of the relative price of oil. Oil prices have increased dramatically over the past few years; the consumer price index (converted to US dollar so that both the numerator and denominator of the index are denominated in the same currency)has also been rising over the past few years but not by as much as the surge in oil prices. Thus, the resulting ratio of both indexes (the relative price of oil) has shown an upward trend. Similar to this trend is that of the terms of trade which has been increasing since the late 1990s.

Labor productivity and investment share of GDP—a measure of capital—also appear to have a similar trend according to the figure. As was mentioned earlier, although capital is not included in the model, it almost certainly still plays a role in the model's dynamics through its effect on labor productivity.

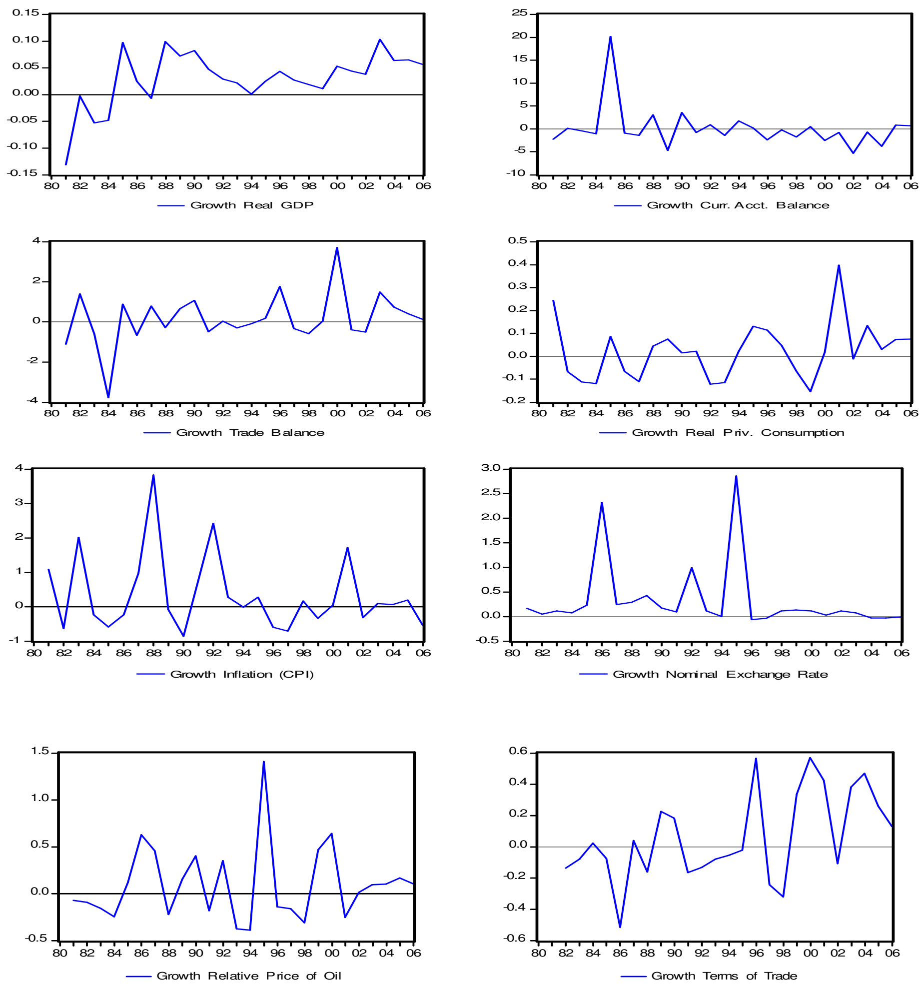


Fig. 13 Trend in Growth Rates of Variables

## Business Cycle Statistics (Cyclicality, Volatility and Persistence)

Three business cycle statistics are considered: cyclicality, volatility and persistence. The cyclicality of a variable is determined by the correlation of that variable with real GDP whereas volatility and persistence are measured by the standard deviation and first order autoregressive coefficients respectively. Augmented Dickey Fuller Unit Root tests were carried out to test for the stationarity of all variables and it was found that all the variables are integrated of order 1 (I (1)), except real GDP and gross investment which were integrated of order 2 (I (2)) at the five percent level of significance. Thus, all I (1) series were first-differenced and both I (2) second-differenced in order to render the series stationary so that the statistics are time independent. Table 2 below shows these statistics for variables considered in the model.

The statistics show that the nominal exchange rate is countercyclical while the current account balance, terms of trade, relative price of oil, labor productivity, employment, investment, private consumption and the trade balance are all procyclical. Labor productivity, the most procyclical of all the series, is the least volatile while current account balance is the most volatile, followed by real investment, nominal exchange rate, the terms of trade, the trade balance, the relative price of oil, employment and private consumption.

TABLE 2

BUSINESS CYCLE STATISTICS

| | CORRELATION WITH REAL GDP | STANDARD DEVIATION | FIRST ORDER AUTO REGRESSION COEFFICIENT |
|---|---|---|---|
| CURRENT ACCOUNT BALANCE | 0.214 | 0.876 | 0.080 |
| LABOR PRODUCTIVITY | 0.969 | 0.109 | -0.314 |
| EMPLOYMENT | 0.414 | 0.203 | -0.539 |
| NOMINAL EXCHANGE RATE | -0.116 | 0.669 | -0.122 |
| REAL GROSS INVESTMENT | 0.560 | 0.677 | -0.120 |
| REAL PRIVATE CONSUMPTION | 0.219 | 0.189 | 0.123 |
| TRADE BALANCE | 0.214 | 0.474 | 0.080 |
| TERMS OF TRADE | 0.047 | 0.566 | 0.46 |
| RELATIVE PRICE OF OIL | 0.260 | 0.422 | -0.73 |

The terms of trade, ratio of export price index to import price index, is found to exhibit more volatility than the relative price of oil, ratio of oil price index to domestic CPI. Although crude oil is the country's major export commodity, the country imports gasoline because of its low refining capacity. Thus, the price of oil is captured in both the export price index and import price index.

The effect of oil prices would be more dominant in the export price index than the import price index since oil constitutes the bulk of exports whereas machinery, transport equipments and manufactures whose prices are relatively less volatile constitute the bulk of import—about 68%. If the covariance between the export price index and import price index is negligible, given that the prices of the other imported commodities do not vary much, then volatility in the terms of trade would be mostly due to volatility in world oil prices. Also, there are other factors other than crude oil prices that influence the terms of trade, like tariffs, exchange rates etc, so that the volatility of the terms trade will also depend on the volatility of all these other factors.

With regard to persistence, the relative price of oil is the most persistent whereas the current account balance and trade balance are the least persistent; meaning that shocks to the relative price of oil will take longer to die out compared to shocks to any of the other variables.

## Model Estimation and Results

Ordinary Least Squares Regressions are run to determine the relationship, if any, between the oil price index relative to the domestic CPI and the terms of trade and then the terms of trade and growth rate of private consumption expenditure.

Modified versions of the R.G. King (1987) Matlab programs EC475#2 and EC475#3 are employed in the computation of population moments and impulse responses.

Parameters of the original programs are calibrated to the data for Nigeria. Data for real returns on bonds is not available and computation of the real interest rate using the nominal returns on three month deposits and inflation based on consumer price index led to a negative average real interest rate over the period under study. To side step this problem, an output function depending on labor and capital was estimated for the economy and the estimated value of marginal product of capital was taken to be the real interest rate, a value of 2 percent. Values of parameters used in the computation of population moments and impulse response are given in table 3 below.

TABLE 3

DESCRIPTION OF VARIABLES AND VALUES OF PARAMETERS

| Variable | Description | Value |
|---|---|---|
| nbar | Steady State Level of Hours | 0.2 |
| alpha | Labor's Share of Output | 0.9 |
| gammax | Growth Rate of Labor Augmenting Technical Change | 1 |
| r | Real Interest Rate | 0.02 |
| beta | Time Preference Parameter | 0.98 |
| capomega | One Minus the Tax Percentage | 0.8 |
| sb | Bond's Share of Output | 0.1 |
| sT | Transfer's Share of Output | 0 |
| sc | Consumption Share of Output | 0.802 |
| avrelpoil | Average Relative Price of Oil over Sample Period Normalized to One | 1 |
| Rho | Diagonal Transition Matrix of Forcing Variables | 0.9 |

Table 4 below shows the estimated coefficients of the OLS regressions with Heteroscedasticity-Autocorrelation Consistent (HAC) standard errors and co-variances. Dependent variables are in columns and the explanatory variable, the relative price of oil, in row, with p-values in parentheses. These estimates are generated so as to determine whether variations in the terms of trade can be explained by variations in world oil prices—the second question of this paper, and also to determine the supply response of oil to changes in the relative price of oil. Variables were log-transformed before they were used in the regressions. The two regressions that were run included:

$$\frac{p_t^x}{p_t^m} = \chi + \delta_1(\frac{p_t^o}{cpi_t}) + v_t \qquad (20)$$

$$O_t = \vartheta + \phi_1(\frac{p_t^o}{cpi_t}) + \varepsilon_t \qquad (21)$$

where $\frac{p_t^x}{p_t^m}$, $\frac{p_t^o}{cpi_t}$, and $O_t$ represent the terms of trade, relative price of oil, and oil export at time t respectively. The variables at the end of the equations are the error terms which capture the effects of other potential explanatory variables not included in the models.

The terms of trade is modeled as a function of the relative price of oil. All else equal, an increase in the relative price of oil, since oil is a major component of exports, should lead to an increase in the export price index; also, since gasoline is imported, an increase in the relative price of oil should lead to an increase in the import price index. If the covariance between the export price index and import price index is negligible, then one would expect variations in the terms of trade to be mostly explained by variations in world oil prices.

Finally, oil exports are modeled as determined by relative oil prices alone in order to determine the country's supply response of oil to changes in world oil prices.

These regressions were used because the sole objective was to determine the relationship between the variables of interest and whether the relationship between them is significant. Results of the regressions analyses show a significantly positive relationship between the terms of trade and the relative price of oil; also variations in the terms of trade are shown to be explained mostly by variations in the relative price of oil—a high 83% as shown by an $R^2$ value of 0.83. This is not surprising since machinery, transportation equipment and manufactures whose prices are relatively less volatile, constitute about 68% of imports, so that much of the variations come from the relative price of oil.

There is also a significantly positive supply response of oil to the relative price of oil. Thus, with increasing world oil prices, revenues from oil exports would increase, leading to an increase in GDP.

TABLE 4

RESULTS OF REGRESSIONS ANALYSES

| | Terms of Trade | Oil Export |
|---|---|---|
| Relative Price of Oil | 0.425<br>(0.001)* | 0.48<br>(0.000)* |

* Denotes significance at the 5% level.

The table below, derived from running the aforementioned PC-Matlab programs, shows the coefficients of the decision rules and hence the linkage between the co-state variable (shadow price of consumption), controls (private consumption and labor input) and flow variables (wages and output); and the state variable (bonds) and exogenous variables (productivity, real interest rate, tax rate and the terms of trade).

The results show that an increase in productivity has the same effect on consumption and employment as an increase in the terms of trade or an income tax reduction. This could be because the persistence parameters were set to be the same for all shocks since I am unable to test for differences in persistence across variables given the limited time span of the data.

TABLE 5

SIMULATION RESULTS

| | Bonds (B) | Productivity (A) | Real Interest Rate (R) | Terms of Trade (q) | Capomega ( $\Omega$ ) |
|---|---|---|---|---|---|
| Shadow Price of Consumption (λ) | -0.0070 | -0.1666 | 7.4941 | -0.1666 | -0.1666 |
| Private Consumption (c) | 0.0070 | 0.1666 | -7.4941 | 0.1666 | 0.1666 |
| Labor Input (n) | -0.0200 | 2.3813 | 21.4116 | 2.3813 | 2.3813 |
| Wage (w) | 0.0000 | 0.0000 | 0.9000 | 0.0000 | 0.0000 |
| Output (y) | -0.0180 | 3.1432 | 19.2704 | 2.1432 | 2.1432 |

Another reason for productivity, terms of trade and income tax shocks having the same effect on consumption and employment in the model is because they are isomorphic due to the set up of the model, i.e. their effect of a percentage change on consumption and effort are of identical form in the model. For example, from equation 6, a percentage change in either productivity, terms of trade or income tax will lead to a change in effort of magnitude $[\frac{N}{1-N}+(1-\alpha)]^{-1}\equiv 1/0.35$ percentage, given the models parameters. Thus, a modification of the model might lead to differences in the effects of all three shocks.

A reduction in income taxes by one percent will result in an increase in employment by 2.4%, leading to an overall increase in output of 2.1% and a jump in private consumption to a new higher steady state level by 0.17%.

Although the effects of an increase in productivity are similar to those of a tax reduction, the former has a larger effect on output because an increase in productivity generates additional increase in output leading to more than a ‘one-to-one’ effect whereas a decrease in income tax leads to an increase in effort since the incentive to work increases, but the effect on output of the increase in effort is less than 'one-to-one' since labor's share of output is less than unity.

Focusing on the effects of changes in the terms of trade, the main objective of this paper, the result that private consumption responds positively to a shock in the terms of trade by jumping to a new higher steady state arises due to the constancy of the real interest rate combined with the permanent income hypothesis behavior of consumption; changes that are perceived transitory such as a temporary shock in the terms of trade would have no effect on private consumption over time.

The real interest rate and private consumption exhibit a negative relationship. The negative response of private consumption to real interest rate as shown in this model arises due to the reinforcement of the substitution effect by the income effect. An increase in the real interest rate is an increase in the opportunity cost of current consumption leading to a negative substitution effect; an increase in the real interest rate also leads to an increase in the consumer's value of wealth which should lead to an increase in private consumption; also for an underdeveloped and highly indebted country like Nigeria, the net-indebtedness is negative leading to a negative wealth effect. Thus, the overall effect of real interest rate on private consumption is negative.

The figures below show impulse response of private consumption, effort and output to a unit shock in the terms of trade and the real interest rate. Similar to the simulation results, private consumption is shown to jump to a new higher level with a terms-of-trade shock but does not change over time. Also, labor input decreases over time with a terms of trade shock, whereas output increases over time.

An increase in a country's export price index relative to its import price index leads to increased export and thus, increased output. Since output equals income in equilibrium, income increases; as income increases, leisure increases so that labor input decreases over time. As with the simulation results, the effect of a unit shock in productivity and taxes are the same as with a terms-of-trade shock and as such not included here.

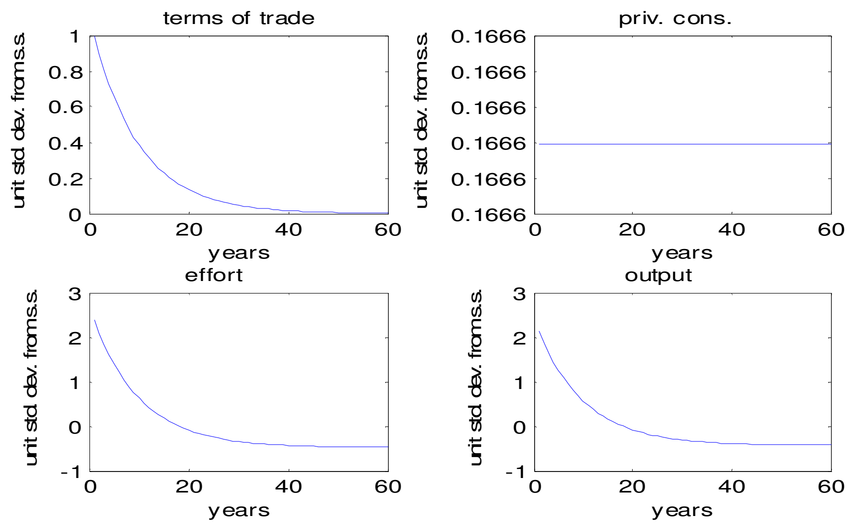


Fig. 14. Effect of a Unit Terms of Trade Shock

On the other hand, a unit increase in the steady state level of bonds cause a jump in the levels of private consumption, effort or output over time; private consumption jumps to a new higher steady state level and remains there, whereas effort and output jump to new lower steady state levels. An increase in the steady state level of bonds leads to an increase in wealth for the representative consumer. As a result, private consumption jumps up, effort down and output down since output depends on effort. Since the variables do not show any trend over time with respect to a unit increase in bond, the impulse response graphs are not presented below.

Finally, a shock to the interest rate leads to an increase in private consumption over time because it increases the value of wealth over time. As the value of wealth increases, the representative agent consumes more leisure and reduces effort, leading to a decline in output over time. This indicates that there is a role to be played by the financial market in affecting growth in private consumption over time.

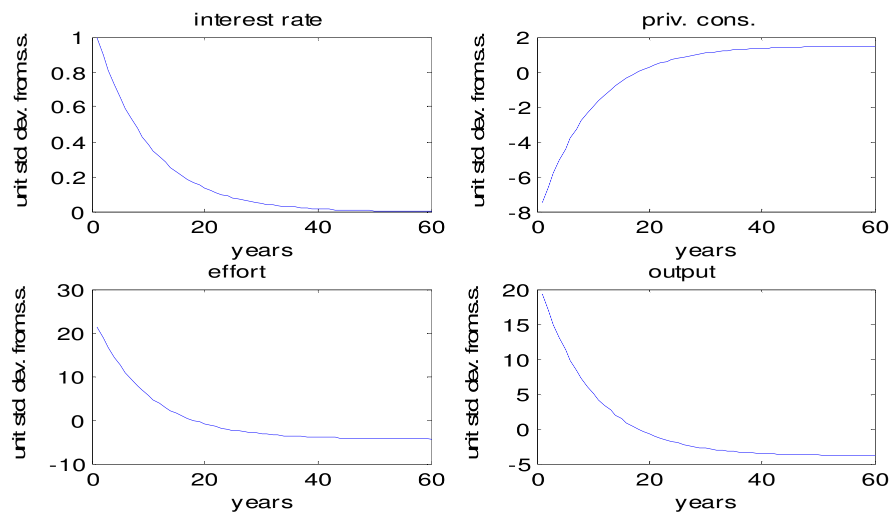


Fig. 15. Effect of a Unit Interest Rate Shock

## Conclusion

I studied the effect of oil price volatility on a representative agent's consumption pattern and Nigeria's terms of trade. A small open economy rational expectations model was considered. Ordinary Least Squares regressions were estimated to determine the percentage variation in the terms of trade that can be explained by variations in the relative price of oil. I find that variability in the terms of trade can be mostly explained by variability in world oil prices.

For a small open economy with a concentrated export base and well diversified import base, it is expected that increases in the price of its principal export and thus its export price index be reflected in its terms of trade. In the case of Nigeria, oil though a major export, is also an import so that fluctuations in oil prices are captured by both the export price index and import price index. If the export price index and import price index do not vary much together, then volatility in oil prices would lead to an even greater volatility in the terms of trade.

Computations of moments and impulse responses were also carried out using a linear approximations approach and solution algorithm introduced by R.G. King (1987). I find that real private consumption jumps to a new higher steady state level with a terms-of-trade shock but does not change over time.

I also find that real private consumption improves over time with a real interest rate shock and that this effect is stronger compared to that of productivity and the terms of trade. Thus, in order for real interest rate shocks to have positive wealth effects and lead to improvements in private consumption in the long run, policies that strengthen the financial markets and lead to debt reduction should be emphasized.

APPENDIX B

Solution Algorithm

Expressions (14) and (15) relate the controls $\hat{C}_t$ and $\hat{N}_t$ to the controlled state and co-state variables, $\hat{B}_t$ and $\hat{\lambda}_t$, as well as the exogenous variables $\hat{A}_t$, $\hat{R}_t$, $\hat{O}_t$ , $\hat{T}_t$ , $\hat{q}_t$ and $\hat{\Omega}_t$ . These expressions may be written as:

$$M_{cc}\begin{bmatrix}\hat{C}_t\\ \hat{N}_t\end{bmatrix} = M_{cs}\begin{bmatrix}\hat{B}_t\\ \hat{\lambda}_t\end{bmatrix} + M_{ce}\begin{bmatrix}\hat{A}_t\\ \hat{R}_t\\ \hat{O}_t\\ \hat{T}_t\\ \hat{q}_t\\ \hat{\Omega}_t\end{bmatrix} \tag{22}$$

where the $M_{cc}$ and $M_{cs}$ are 2x2 matrices as there are two control variables and two state, co-state variables and the $M_{ce}$ is a 2x6 matrix as there are six exogenous variables in the model. $M_{cc}$ relates controls to controls, $M_{cs}$ relates controls to states and $M_{ce}$ relates controls to exogenous variables. Thus, we have that

$$M_{cc} = \begin{bmatrix}-1 & 0\\ 0 & \frac{N}{1-N}+(1-\alpha)\end{bmatrix},\ M_{cs} = \begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix} \text{ and } M_{ce} = \begin{bmatrix}0 & 0\,0\,0\,0\,0\\ 1 & 0\ \ 0\,0\,1\,1\end{bmatrix}$$

Expressions (16) and (17) relate variations in controlled state and co-state to variations in controls and exogenous variables. These expressions may be written as:

$$M_{ss}(B)\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix} = M_{sc}(B)\begin{bmatrix}\hat{C}_{t+1}\\ \hat{N}_{t+1}\end{bmatrix} + M_{se}(B)\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix} \tag{23}$$

where $M_{ss}(B)$, $M_{sc}(B)$ and $M_{se}(B)$ are matrix polynomials in the backshift operator B at most of power 1. The notations are the same as above, so that $M_{ss}$ relates state to state variables, etc. The above can be rewritten as

$$M_{ss0}\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}+M_{ss1}\begin{bmatrix}\hat{B}_{t}\\ \hat{\lambda}_{t}\end{bmatrix}=M_{sc0}\begin{bmatrix}\hat{C}_{t+1}\\ \hat{N}_{t+1}\end{bmatrix}+M_{sc1}\begin{bmatrix}\hat{C}_{t}\\ \hat{N}_{t}\end{bmatrix}+M_{se0}\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix}+M_{se1}\begin{bmatrix}\hat{A}_{t}\\ \hat{R}_{t}\\ \hat{O}_{t}\\ \hat{T}_{t}\\ \hat{q}_{t}\\ \hat{\Omega}_{t}\end{bmatrix} \tag{24}$$

So that

$$\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}=-[M_{ss0}]^{-1}M_{ss1}\begin{bmatrix}\hat{B}_{t}\\ \hat{\lambda}_{t}\end{bmatrix}+[M_{ss0}]^{-1}M_{sc0}\begin{bmatrix}\hat{C}_{t+1}\\ \hat{N}_{t+1}\end{bmatrix}+[M_{ss0}]^{-1}M_{sc1}\begin{bmatrix}\hat{C}_{t}\\ \hat{N}_{t}\end{bmatrix}+[M_{ss0}]^{-1}M_{se0}\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix}+[M_{ss0}]^{-1}M_{se1}\begin{bmatrix}\hat{A}_{t}\\ \hat{R}_{t}\\ \hat{O}_{t}\\ \hat{T}_{t}\\ \hat{q}_{t}\\ \hat{\Omega}_{t}\end{bmatrix} \tag{25}$$

Since $M_{cc}$ is a square matrix, $M_{cc}$ is invertible; thus, from expression (22), we have that

$$\begin{bmatrix}\hat{C}_{t}\\ \hat{N}_{t}\end{bmatrix}=[M_{cc}]^{-1}M_{cs}\begin{bmatrix}\hat{B}_{t}\\ \hat{\lambda}_{t}\end{bmatrix}+[M_{cc}]^{-1}M_{ce}\begin{bmatrix}\hat{A}_{t}\\ \hat{R}_{t}\\ \hat{O}_{t}\\ \hat{T}_{t}\\ \hat{q}_{t}\\ \hat{\Omega}_{t}\end{bmatrix} \tag{26}$$

$$\begin{bmatrix}\hat{C}_{t+1}\\ \hat{N}_{t+1}\end{bmatrix}=[M_{cc}]^{-1}M_{cs}\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}+[M_{cc}]^{-1}M_{ce}\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix} \tag{27}$$

But from (16) and (17), we derive the matrices:

$$M_{ss0}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ M_{ss1}=\begin{bmatrix}0 & -1\\ \frac{-1}{\beta} & 0\end{bmatrix},\ M_{sc0}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ M_{sc1}=\begin{bmatrix}0 & 0\\ -s_c & \alpha\Omega\end{bmatrix},\ M_{se0}=\begin{bmatrix}0 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

and $M_{se1}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ \Omega & \frac{s_b}{\beta} & 0 & 0 & \Omega & \Omega\end{bmatrix}$

Since $M_{sc0}$ is a zero matrix, expression (14) reduces to

$$\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}=-[M_{ss0}]^{-1}M_{ss1}\begin{bmatrix}\hat{B}_t\\ \hat{\lambda}_t\end{bmatrix}+[M_{ss0}]^{-1}M_{sc1}\begin{bmatrix}\hat{C}_t\\ \hat{N}_t\end{bmatrix}+[M_{ss0}]^{-1}M_{se0}\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix}+[M_{ss0}]^{-1}M_{se1}\begin{bmatrix}\hat{A}_t\\ \hat{R}_t\\ \hat{O}_t\\ \hat{T}_t\\ \hat{q}_t\\ \hat{\Omega}_t\end{bmatrix}$$

and substitution of expression (26) into the above yields

$$\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}=-[M_{ss0}]^{-1}[M_{ss1}-M_{sc1}M_{cc}{}^{-1}M_{cs}]\begin{bmatrix}\hat{B}_t\\ \hat{\lambda}_t\end{bmatrix}+[M_{ss0}]^{-1}M_{se0}\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix}+[M_{ss0}]^{-1}[M_{se1}+M_{sc1}M_{cc}{}^{-1}M_{ce}]\begin{bmatrix}\hat{A}_t\\ \hat{R}_t\\ \hat{O}_t\\ \hat{T}_t\\ \hat{q}_t\\ \hat{\Omega}_t\end{bmatrix} \quad (28)$$

Let $W=-[M_{ss0}]^{-1}[M_{ss1}-M_{sc1}M_{cc}{}^{-1}M_{cs}]$, $X=[M_{ss0}]^{-1}M_{seo}$ and

$Z=[M_{ss0}]^{-1}[M_{se1}+M_{sc1}M_{cc}{}^{-1}M_{ce}]$, then expression (28) can be written as

$$\begin{bmatrix}\hat{B}_{t+1}\\ \hat{\lambda}_{t+1}\end{bmatrix}=W\begin{bmatrix}\hat{B}_t\\ \hat{\lambda}_t\end{bmatrix}+X\left[\begin{bmatrix}\hat{A}_{t+1}\\ \hat{R}_{t+1}\\ \hat{O}_{t+1}\\ \hat{T}_{t+1}\\ \hat{q}_{t+1}\\ \hat{\Omega}_{t+1}\end{bmatrix}\right]+Z\begin{bmatrix}\hat{A}_t\\ \hat{R}_t\\ \hat{O}_t\\ \hat{T}_t\\ \hat{q}_t\\ \hat{\Omega}_t\end{bmatrix}$$

The matrix $W$ is referred to as the State Co-state transition matrix and given the matrices above, we have that

$W=\begin{bmatrix}\frac{1}{\beta} & \frac{\Omega}{1-\alpha(1-N)}+(\frac{1-\beta}{\beta})s_b\\ 0 & 1\end{bmatrix}$, $X=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0 & 0\end{bmatrix}$ and

$$Z=\begin{bmatrix}\frac{\Omega}{1-\alpha(1-N)} & \frac{s_b}{\beta} & 0 & 0 & \frac{\Omega}{1-\alpha(1-N)} & \frac{\Omega}{1-\alpha(1-N)}\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

The eigenvalues of W are $(1, \frac{1}{\beta})'$ and the corresponding Eigen vectors are

$$\begin{bmatrix} \dfrac{\dfrac{\Omega}{1-\alpha(1-N)}+(\dfrac{1-\beta}{\beta})s_b}{(\dfrac{1-\beta}{\beta})} \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Define $V$ as the matrix of Eigenvectors of $W$ and $\mu$ a diagonal matrix with the Eigen values of $W$ arranged in ascending absolute value, then

$$V = \begin{bmatrix} \dfrac{\dfrac{\Omega}{1-\alpha(1-N)}+(\dfrac{1-\beta}{\beta})s_b}{(\dfrac{1-\beta}{\beta})} & 1 \\ 1 & 0 \end{bmatrix} \text{ and } \mu = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{\beta} \end{bmatrix}$$

The solution to the difference equation at date $t$ is given by

$$\begin{bmatrix} \hat{B}_t \\ \hat{\lambda}_t \end{bmatrix} = V\mu^t V^{-1} \begin{bmatrix} \hat{B}_0 \\ \hat{\lambda}_0 \end{bmatrix} + \sum_{h=0}^{t} V\mu^h V^{-1} X \begin{bmatrix} \hat{A}_{t-h+1} \\ \hat{R}_{t-h+1} \\ \hat{O}_{t-h+1} \\ \hat{T}_{t-h+1} \\ \hat{q}_{t-h+1} \\ \hat{\Omega}_{t-h+1} \end{bmatrix} + \sum_{h=0}^{t} V\mu^h V^{-1} Z \begin{bmatrix} \hat{A}_{t-h} \\ \hat{R}_{t-h} \\ \hat{O}_{t-h} \\ \hat{T}_{t-h} \\ \hat{q}_{t-h} \\ \hat{\Omega}_{t-h} \end{bmatrix}$$

In combination with the law of motion for the exogenous variables, the expressions above form a linear system which consists of the state ($\hat{B}$), co-state ($\hat{\lambda}$) and the exogenous variables to be used for the computation of (i) impulse responses; (ii) population moments; and (iii) stochastic simulations. With a single bond stock and first order autoregressive processes for the exogenous variables, the linear system that expresses the optimal evolution of the state variable (B) and the exogenous state variables A, R, q, and $\Omega$ is:

$$s_{t+1} = \begin{bmatrix} \hat{B}_{t+1} \\ \hat{A}_{t+1} \\ \hat{R}_{t+1} \\ \hat{O}_{t+1} \\ \hat{T}_{t+1} \\ \hat{q}_{t+1} \\ \hat{\Omega}_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_1 & \pi_{BA} & \pi_{BR} & \pi_{BO} & \pi_{BT} & \pi_{Bq} & \pi_{B\Omega} \\ 0 & \rho_{AA} & \rho_{AR} & \rho_{AO} & \rho_{AT} & \rho_{Aq} & \rho_{A\Omega} \\ 0 & \rho_{RA} & \rho_{RR} & \rho_{RO} & \rho_{RT} & \rho_{Rq} & \rho_{R\Omega} \\ 0 & \rho_{OA} & \rho_{OR} & \rho_{OO} & \rho_{OT} & \rho_{Oq} & \rho_{O\Omega} \\ 0 & \rho_{TA} & \rho_{TR} & \rho_{TO} & \rho_{TT} & \rho_{Tq} & \rho_{T\Omega} \\ 0 & \rho_{qA} & \rho_{qR} & \rho_{qO} & \rho_{qT} & \rho_{qq} & \rho_{q\Omega} \\ 0 & \rho_{\Omega A} & \rho_{\Omega R} & \rho_{\Omega O} & \rho_{\Omega T} & \rho_{\Omega q} & \rho_{\Omega\Omega} \end{bmatrix} \begin{bmatrix} \hat{B}_t \\ \hat{A}_t \\ \hat{R}_t \\ \hat{O}_t \\ \hat{T}_t \\ \hat{q}_t \\ \hat{\Omega}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{A,t+1} \\ \varepsilon_{R,t+1} \\ \varepsilon_{O,t+1} \\ \varepsilon_{T,t+1} \\ \varepsilon_{q,t+1} \\ \varepsilon_{\Omega,t+1} \end{bmatrix} \equiv Ms_t + \varepsilon_t$$

where $\varepsilon_A$, $\varepsilon_R$, $\varepsilon_O$, $\varepsilon_T$, $\varepsilon_q$ and $\varepsilon_\Omega$ are shocks to the system which are serially uncorrelated but may be contemporaneously correlated. The $\rho_{ij}$ coefficients, $i, \mathrm{j} = \{A, \mathrm{R}, \mathrm{O}, \mathrm{T}, \mathrm{q}, \Omega\}$, govern the model's exogenous dynamics and under the assumption that the shocks to the system are temporary, the exogenous process for $\hat{A}$, $\hat{R}$, $\hat{O}$, $\hat{T}$, $\hat{q}$ and $\hat{\Omega}$ is stationary. This paper adopts the R.G. King (1987) Matlab programs EC475#2 and EC475#3 for computation of impulse responses and population moments, making slight adjustments to accommodate parameters of my model where necessary.

The interest of this paper is in the productivity and the terms of trade shocks. Given the limited data used for this analysis, differences in persistence parameters could not be estimated. Also, the results of the variance decomposition of output coupled with the very low values of the off-diagonal elements of the covariance matrix of the terms of trade and productivity—0.0002, suggests that the shocks are independent. Thus, the diagonal elements of the M system matrix above are assumed to be equal and the off-diagonal elements are set to zero, so that M becomes an upper triangular matrix.

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