Systematic Instruction of Early Math Skills

By

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To my parents, Janet and Chuck Hardy, for their endless love, support, and encouragement.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Mathematics is considered one of the core subject areas in schools in the United States for children of all ages (Cross, Woods, & Schweingruber, 2009). One reason for the increasing focus on mathematics is recognition by key stakeholders of the importance of mathematically competent adults in the global economy (Sarama & Clements, 2009). There is also research demonstrating the existence of disparities between math achievement in students from the United States compared to their counterparts in some countries in East Asia and Europe (Provasnik, Gonzales, & Miller, 2009). In a study of student achievement in 45 countries, U.S. fourth and eighth graders were consistently outperformed by students from Hong Kong, Japan, and Singapore (Provasnik et al., 2009). On some measures of math performance, U.S. students were outperformed by students from additional countries including Chinese Taipei, Kazakhstan, Russian Federation, England, Latvia, and Korea (Provasnik et al., 2009). In an analysis of math proficiency in Class of 2011 students, U.S. students performed thirty-second in a comparison of 65 countries (Peterson, Woessmann, Hanushek, & Lastra-Anadón, 2011).

Although math is generally considered a highly valued outcome for elementary and secondary students, there has traditionally been less of a focus on math skills during the preschool years (Cross et al., 2009). However, there has been an increased focus on teaching math skills to preschool children in recent years by government education agencies as well as professional organizations. Math is one of the five essential domains in the Head Start Child Development and Early Learning Framework (Office of Head Start, 2010). The National

Council of Teachers of Mathematics included standards for preschoolers in their book, *Principles and Standards for School Mathematics* (NCTM, 2000). The National Research Council, in a report on math in early childhood, recommended that “all early childhood programs should provide high-quality mathematics curricula and instruction” (Cross et al., 2009, p. 345).

Sarama and Clements (2009) identified several factors contributing to the increasing emphasis on mathematical learning in young children, including (a) changing theories about young children’s ability to understand and learn mathematical concepts, (b) research that indicates early knowledge is predictive of later achievement, and (c) evidence that there is a mismatch between children’s intuitive math knowledge and the instruction they receive in school (Sarama & Clements, 2009). Of particular relevance is the research that indicates early math skills predict later achievement. In several studies, early number sense was found to be correlated with achievement in later grades (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Jordan, Glutting, & Ramineni, 2010). In these three studies, math achievement was measured using portions of the Woodcock-Johnson III (Woodcock, McGrew, & Mather, 2001). Researchers found that early number sense was predictive of math achievement at the end of first grade (Jordan et al., 2007) and third grade (Jordan et al., 2009, 2010).

These findings have been replicated by other researchers. In an analysis of data from the Early Childhood Longitudinal Study, Kindergarten Class of 1998-9 (ECLS-K), Denton and West (2002) found a relationship between math skills in kindergarten and math skills in first grade. Children’s early math learning was measured using assessment items related to a range of math skills (e.g., number sense, geometry and spatial sense, data analysis, algebra, etc.). Additionally, in a meta-analysis of six longitudinal studies, including the ECLS-K data, Duncan and colleagues (2007) found that early math skills (e.g., number sense) were more predictive of later school achievement than early reading skills, early attention skills, and early social-emotional skills.

Further, there is research that demonstrates there are disparities in math achievement among children based on socioeconomic status (SES) and race. In their analysis of the ECLS-K data, Denton and West (2002) reported discrepancies in math performance between children from low-SES families and those from higher-SES families. Children from families with low SES scored a half-standard deviation below the national average on the standardized math score used in the ECLS-K study (Denton & West, 2002). Race-related disparities were also found. White and Asian children consistently outperformed black children in math achievement. White children entered kindergarten with a mean math score above the national average, but black children entered kindergarten with a mean below the national average (Denton & West, 2002).

In an analysis of the data from the Pre-Elementary Educational Longitudinal Study (PEELS), Markowitz and colleagues (2006) found differences in children's math knowledge based on socioeconomic status. On the Applied Problems subtest of the Woodcock-Johnson III, children from families with low SES scored more than one standard deviation below the mean, which was significantly below children from other income categories (Markowitz et al., 2006). On the Quantitative Concepts subtests of the Woodcock-Johnson III, children from families with low SES scored one standard deviation below the mean, which was significantly below children from the two highest SES groups (Markowitz et al., 2006). Additionally, on both subtests of the Woodcock-Johnson III, black and Latino children performed significantly lower than white children (Markowitz et al., 2006).

There is limited research about math knowledge in young children with disabilities. Not surprisingly, however, there is evidence of discrepancies in math skills between children with and without disabilities. In their analysis of the PEELS data, Markowitz and colleagues (2006) found there were differences between children with and without disabilities on several measures of math knowledge. These differences were greater for children with more significant disabilities. On the Quantitative Concepts subtest of the Woodcock-Johnson III, children with a developmental delay, learning disability, intellectual disability, or low-incidence disability performed more than one standard deviation below the mean (Markowitz et al., 2006). On the Applied Problems subtest, children with a learning disability or other health impairment performed one standard deviation below the mean (Markowitz et al., 2006). Children with a developmental delay, autism, or low incidence disability performed more than one standard deviation below the mean, and children with an intellectual disability performed more than two standard deviations below the mean (Markowitz et al., 2006).

Thus, there is evidence to suggest: (a) mathematics is a highly valued outcome for young children, (b) early math skills are related to later math achievement, and (c) there are significant disparities among young children on measures of math achievement.

Early Mathematics Skills and Instruction in Early Childhood Classrooms

Although there has been work dedicated to articulating the range of early math skills relevant to young children (e.g., Clements & Sarama, 2009; Cross et al., 2009, Office of Head Start, 2010; NCTM, 2000), there is little documentation of the type of math instruction that is provided in preschool classrooms, and the research that does exist points to a paucity of math instruction in preschool classrooms. In an analysis of 652 preschool programs in 11 states, Early and colleagues (2010) found that on average only 8% of the day included instruction in mathematics. Klibanoff and colleagues (2006) recorded, transcribed, and analyzed one hour of teacher talk in 26 preschool classrooms. They found wide variety in the number of instances of math talk by teachers (from 1 to 104, with a mean of 28.3 and a standard deviation of 24.2). Also, there was little variety in the type of talk. For instance, approximately half of the input was related to cardinality (i.e., naming the number in a set). Additionally, math input was measured broadly, such that any talk related to math concepts was counted, regardless of whether there was any instruction involved (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). In an unpublished doctoral dissertation, Ehrlich (2007) measured teacher math talk in preschool classrooms and found similar results, with a wide variety in the amount of teacher talk, but little variety in the type, with the majority related to cardinality. Rudd and colleagues (2008), in a study of six preschool classrooms, found that planned mathematical activities did not occur in the classrooms. In an analysis of the mathematical language used by the teachers in the classrooms, researchers found that the mathematical talk that did occur revolved around low-level thinking skills (such as labeling numbers, commenting on quantity, etc.) (Rudd, Lambert, Satterwhite, & Zaler, 2008). These studies, while limited in scope, document a lack of systematic math instruction in preschool classrooms.

The above research indicates that there is dearth of math instruction in preschool classrooms. Thus, it is necessary to consider how best to provide math instruction to preschoolers, including those who are typically developing, those with disabilities, and those who are at risk for school failure.

Response to Intervention: A Framework for Providing Effective Instruction to All Children

Response to intervention (RTI) is an instructional framework that is gaining prominence in the field of early education (e.g., Buysse et al., 2013; Greenwood et al., 2011; Pretti-Frontczak et al., 2007). The purpose of RTI is to identify children who need additional support and to provide that support in the classroom, with the goal of preventing the need for later referrals to special education (Carta & Greenwood, 2013). RTI consists of a system of tiered levels of intervention: (a) universal, which involves implementing a core curriculum that is sufficient for most students, (b) secondary, which involves using more targeted interventions for children who are not responsive to the core curriculum, and (c) tertiary, which involves individualized interventions for non-responders to the primary and secondary levels (NCRTI, 2010). Universal screening and ongoing progress monitoring are needed to assess whether children are responding to interventions and making progress (Methe & VanDerHeyden, 2013).

RTI is an emerging model in early childhood settings, and much of the existing research on RTI for young children is related to early literacy (Carta & Greenwood, 2013). However, there is evidence that the field has begun to apply an RTI framework to early math learning (e.g., Methe & VanDerHeyden, 2013). Methe and VanDerHeyden (2013) describe applying the RTI model to early mathematics using the following five steps: (1) conducting universal screening, (2) identifying and addressing class- or program-wide problems, (3) identifying individual learning problems, (4) providing tier two and three interventions, and (5) monitoring progress and outcomes.

Progress monitoring is an essential component of RTI frameworks (Odom & Fettig, 2013; Stecker, Fuchs, & Fuchs, 2008). One progress monitoring tool is IGDIs-Early Numeracy

(Hojnoski & Floyd, 2004). The following skills are measured using this tool: rote counting, quantity comparison, numeral identification, and counting. There are other existing measures for monitoring preschool children's progress in the following skills: numeral identification, counting, rote counting, classification, ordinality/number line, subitizing, comparison, and addition and subtraction (Clarke & Shinn, 2004; Foegen, Jiban, & Deno, 2007; Methe & VanDerHeyden, 2013; Methe, Hintze, & Floyd, 2008; VanDerHeyden, Broussard, & Cooley, 2006; VanDerHeyden et al., 2004). However, these measures have generally not been used to assess children's responses to different tiers of instruction. They are also limited in scope to primarily skills related to number sense. Additional measures are needed for the range of early math skills, including cardinality, seriation, patterning, shape identification and manipulation, directionality and spatial sense, measuring, and graphing.

Central to the application of the RTI model is the use of evidence-based teaching strategies. Core curricula, secondary interventions, and tertiary interventions must all be evidence-based. In a recent critical review of the literature, Hardy (2013a) analyzed the research around math instruction for preschoolers. There were three types of studies analyzed in the review: (a) curriculum studies, (b) instructional interventions studies, and (c) cognitive processes studies. The first category, curriculum studies, included studies that were designed to evaluate the efficacy of a specific math curriculum or to compare the efficacy of two different math curricula. In these studies, the curriculum served as the primary source of math instruction for all children in the classroom. The second category of studies, designated instructional interventions studies, included studies in which specific instructional procedures were used to teach math skills. The purpose of these studies was to examine the growth in children's math skills as a result of instructional interventions. The third category, cognitive processes studies, included studies in which the purpose was to evaluate children's math performance under different conditions at a single point in time. For example, in one study, children's performance on sorting was evaluated when the adult modeled sorting before asking the child to do so, compared to when the adult did not model sorting first. The purpose of these studies was not to impact child learning or evaluate an instructional procedure, but rather to garner information about the cognitive processes involved in completing math tasks.

The analysis of the curriculum studies informs what is known about the universal tier of instruction in the RTI framework. The analysis of the instructional interventions and cognitive processes studies informs what is known about the secondary/tertiary tiers of instruction in the RTI framework and provides information on designing interventions for children who do not respond to tier one.

Most of the participants in the curriculum studies (Arnold, Fisher, & Doctoroff, 2002; Clements & Sarama, 2007b; Clements & Sarama, 2008; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Klein, Starkey, Clements, Sarama, & Iyer, 2008; Sarama, Clements, Starkey, Klein, & Wakely, 2008; Starkey & Klein, 2000 study 1 and 2) were children from families with low SES in Head Start or state-funded preschool programs, and most of these studies either did not include children with disabilities or included relatively few children with disabilities. When they did include children with disabilities, researchers did not report analyses on this subgroup of children. The curriculum most often used was *Building Blocks* (Clements & Sarama, 2007a). Other curricula used in these studies included *Pre-K Mathematics Curriculum* (Klein, Starkey, & Ramirez, 2002) and researcher-created curricula. The length of the intervention in the curriculum studies ranged from six weeks to a year. The amount of math instruction varied widely and was typically not reported in sufficient detail to analyze dosage. In all of the studies, the results were strong, meaning that children in the intervention groups had greater improvements in math learning than children in the control groups. However, six of the eight curriculum studies (Arnold et al., 2002; Clements & Sarama, 2007b; Clements & Sarama, 2008; Sarama et al., 2008; Starkey & Klein, 2000 study 1 and 2) were coded as having weak methodological rigor, primarily because the authors did not include data on reliability and procedural fidelity.

The participants in the instructional and cognitive processes studies (Ciancio, Rojas, McMahon, & Pasnak, 2001; Clements, 1984; Curtis, Okamoto, & Weckbacher, 2009 study 1 and 2; Daugherty, Grisham-Brown, & Hemmeter, 2001; Holcombe, Wolery, & Werts, 1993; Kidd et al., 2012; McGivern et al., 2007; Murphy, Bates, & Anderson, 1984; Pasnak, Greene, Ferguson, & Levit, 2006; Ramani & Siegler, 2008; Ramani & Siegler, 2011; Ramani, Siegler, & Hitti, 2012 study 1 and 2; Siegler & Ramani, 2008; Siegler & Ramani, 2009; Vandermaas-Peeler, Boomgarden, Finn, & Pittard, 2012; Williamson, Jaswal, & Meltzoff, 2010 study 1 and 2) were from a range of racial and socioeconomic backgrounds, and the majority of them did not have reported disabilities. The skills targeted for instruction varied widely across studies, and multiple skills were taught in most of the studies. In the studies with moderate to strong results (Ciancio et al., 2001; Clements, 1984; Curtis et al., 2009 study 2; Daugherty et al., 2001; Holcombe et al., 1993; Kidd et al., 2012; McGivern et al., 2007; Pasnak et al., 2006; Ramani & Siegler, 2008; Ramani et al., 2012 study 1 and 2; Siegler & Ramani, 2008, Siegler & Ramani, 2009; Williamson et al., 2010 study 1 and 2), the skills taught most frequently were numeral identification, one-to-one correspondence, ordinality/number line, and sorting and classification.

The characteristics of instruction (e.g., instructional strategies, feedback strategies) in the instructional and cognitive processes studies were typically not clearly specified in the description of study procedures. Although this makes it difficult to determine which features would be associated with better child outcomes, the studies with moderate to strong results (Ciancio et al., 2001; Clements, 1984; Curtis et al., 2009 study 2; Daugherty et al., 2001; Holcombe et al., 1993; Kidd et al., 2012; McGivern et al., 2007; Pasnak et al., 2006; Ramani & Siegler, 2008; Ramani et al., 2012 study 1 and 2; Siegler & Ramani, 2008, Siegler & Ramani, 2009; Williamson et al., 2010 study 1 and 2) included the following characteristics: (a) instruction was provided individually or in small groups; (b) instruction was provided using work tasks or games; (c) instructional strategies included modeling, mands, prompts, and providing information; (d) feedback strategies included reinforcing children when correct and providing them with opportunities to demonstrate the correct answer after it was modeled; and (e) instruction lasted at least 12 sessions.

Together, the results of the curriculum studies, instructional intervention studies, and cognitive processes studies provide some evidence of effective practices that could be used at all tiers of an RTI framework for math. The analysis of these studies indicates that *Building Blocks* (Clements & Sarama, 2007a), *Pre-K Mathematics Curriculum* (Klein et al., 2002), and researcher-created curricula may all be efficacious for teaching early math skills to children from low-SES families. The analysis also resulted in the identification of several instructional and feedback strategies that could be used to inform the design of tier two and three interventions. However, several weaknesses of the studies were identified, including that: (a) the skills chosen for instruction were usually not based on children's needs, (b) the intervention characteristics were not specified with replicable precision, and (c) many of the studies had weak methodological rigor.

In the studies in this review, the selection of target skills was based on children's individual needs in only one study (Daugherty et al., 2001). Because there is a wide range of early math skills, it is important to identify strategies for determining which math skills should be highest priority for instruction. This might be accomplished by determining which skills are foundational or prerequisite skills for important math outcomes. Skills that are more highly predictive of later success in math should be a higher priority for instruction. For example, early math skills related to number sense are viewed as a foundational skill for later mathematics learning (Jordan et al., 2010; Methe & VanDerHeyden, 2013). Skills to target for instruction might also be determined by assessing individual children and determining which skills are most appropriate given their developmental needs.

In the studies in this review, the intervention procedures were described with replicable precision in only two studies (Daugherty et al., 2001; Holcombe et al., 1993). Both of these studies included only children with disabilities as participants, and the instructional strategy used was constant time delay. The feedback strategies used in both studies included reinforcing children for correct responses, but incorrect responses were addressed differently in each study. In one of the studies, incorrect responses were ignored (Holcombe et al., 1993), and in the other study, incorrect responses were followed by a prompt (Daugherty et al., 2001). In both studies, there were moderate to strong outcomes for children. Intervention characteristics must be precisely described in order to facilitate replication by other researchers and practitioners and also to allow for component analyses to determine which aspects of instruction are critical for teaching specific skills. These component analyses will allow for the further refinement of instructional practices.

Many of the studies in this review had weak methodological rigor. Specifically, only eight studies had at least moderate internal validity (Clements & Sarama, 2008; Clements et al., 2011; Daugherty et al., 2001; Holcombe et al., 1993; Klein et al., 2008; Murphy et al., 1984, Ramani et al., 2012 study 2; Williamson et al., 2001 study 1). Areas that were scored low across studies included the collection of reliability data and the collection of procedural fidelity data. Most of the studies had at least moderate external validity. However, the participants, intervention characteristics, and intervention dosage were often not adequately described.

Overall, the results of the literature review indicate that although a variety of curricula may be effective for tier one instruction, there is limited information about how to determine skills to target for tier two/three instruction, what instructional and feedback strategies can be effective for teaching these targeted skills, and whether instruction on targeted skills is generalized and maintained by children. Furthermore, the findings of much of the existing research are limited because of methodological issues.

Purpose of the Current Study

The purpose of the current study is to add to the research base on early math instruction by evaluating the efficacy of an intervention to teach early math skills to preschoolers. The study was designed to provide information on the efficacy of an instructional procedure that could be used as a tier two and/or three intervention for early math skills. The current study contributes to the literature in four additional ways. First, the skills targeted for instruction were based on each child's demonstrated deficits in math, which was not typically done in previous research. This was done through the use of a systematic assessment of each child's early math skills. Second, the activities, instructional strategies, and feedback strategies were based on the limited evidence base on intervention procedures used to teach math. For example, the intervention included modeling and prompting, and an error correction procedure was used to provide the child with feedback. Third, the intervention procedures and dosage were systematically implemented and described with specificity. Fourth, the study was designed to be methodologically rigorous, using an experimental single-subject design, regular collection of reliability and procedural fidelity data, visual analysis of graphed data, and a thorough description of participants.

The specific research questions were:

1. Is a systematic modeling and prompting procedure with error correction effective in helping preschoolers acquire discrete early math skills?
2. Is a systematic modeling and prompting procedure with error correction effective in helping preschoolers generalize discrete early math skills to other materials in the classroom?
3. Is a systematic modeling and prompting procedure with error correction effective in helping preschoolers maintain discrete early math skills?
4. Do teachers view instruction in early math skills, the systematic instructional procedure, and the effects of instruction in discrete early math skills as socially valid?

CHAPTER II

METHODS

Participants

Three children participated in this study. All three children were typically developing boys who were four years of age at the start of the study and were in the same classroom of their child care center. An overview of characteristics for each of the three participants is provided in Table 1.

Table 1. *Participant Characteristics*

	Jason	Orion	She'quan
Age	4 yr., 2 mo.	4 yr., 6 mo.	4 yr., 0 mo.
Sex	Male	Male	Male
Race	Black	Black	Black
Socio-economic status	Middle	Low	Low
Mullen Early Learning Composite			
Standard score	109	79	89
Percentile rank	72	8	22
Descriptive category	Average	Below average	Average
TEAM			
Raw score	16	6	7
T-score	-1.74	-20.52	-17.25
Scaled score	290	178	198
Competency score	48.26	29.48	32.75

Note: In a study of 360 children with an average age of 4.25 years, the mean T-Score was 44.42, with a standard deviation of 7.85.

Each child participant was assessed to confirm that he met the following inclusion criteria: (a) child has attended at least 80% of the previous 30 days of school, (b) child is 48 to 72 months of age, (c) child can maintain attention in adult-directed activities of 20 minutes in length with minimal supports, (d) child has deficits in at least five skill domains from the Math

Screening Instrument, and (e) child can count up to 5 items. Inclusion criteria a and b were assessed by having the teacher complete a child screening checklist. Criterion c was assessed based on teacher report and by researcher observation of behaviors during screening activities. Criteria d and e were evaluated by the researcher after completion of the researcher-developed Math Screening Instrument (see Appendix A; Hardy, 2013b). On the Math Screening Instrument, a range of early math skills was assessed: (a) counting and cardinality, (b) comparing quantities, (c) comparing sizes, (d) sorting and classification, (e) seriation, (f) patterning, (g) ordinality/number line, (h) shape manipulation, (i) numeral identification, (j) addition and subtraction, and (k) measuring.

The Mullen Scales of Early Learning (Mullen, 1995) was administered to provide a measure of each child's developmental status. A summary of the participants' early learning composite scores on the Mullen is presented in Table 1. Jason and She'quan's early learning composite standard score was average. Orion's early learning composite standard score was below average. The Tools for Early Assessment in Math (TEAM; Clements, Sarama, & Wolfe, 2011) was administered to assess each child's early math skills. A summary of the participants' TEAM scores is presented in Table 1. Orion and She'quan performed below average for their age, and Jason performed slightly above average.

Settings

All of the children were in the same classroom in a childcare center serving children from low-income backgrounds in a large southern city. All study activities, except for generalization sessions, occurred seated at a table in a conference room or staff break-room near the children's classroom. Generalization sessions occurred in the classroom, seated at a child-sized table or on the floor.

Measures

Classroom measures. Classrooms were assessed using two measures. The first measure was the Preschool Classroom Mathematics Inventory (PCMI; Frede, Weber, Hornbeck, Boyd, & Worth, 2006). The PCMI is a measure of the mathematics environment in early childhood classrooms. The PCMI is scored on a rating scale from one to five, with a score of one indicating minimal evidence of developmentally appropriate math instruction and a score of five indicating strong evidence of developmentally appropriate math instruction. The items on the PCMI are: (1) materials for counting, comparing, estimating, and recognizing number symbols; (2) materials for measuring and comparing amount: volume, weight, length, height, distance, and area; (3) materials for classifying and seriating; (4) materials for geometry and spatial positions/relations; (5) teachers encourage the use of one-to-one correspondence; (6) teachers encourage children to count and/or write numbers for a purpose; (7) teachers encourage children to estimate and compare numbers; (8) teachers encourage children's use of mathematical terminology and reflection on mathematical problems; (9) teachers encourage children to compare and measure: volume, weight, length, height, distance, and area; (10) teachers encourage children to classify and seriate; and (11) teachers encourage children's concepts of geometry and spatial positions/relations. The PCMI was completed by the researcher.

The second measure was a researcher-created Teacher Math Practices Questionnaire (see Appendix B). Because the PCMI was completely observational, the Teacher Math Practices Questionnaire was designed to ascertain the teacher's perspective on the math practices present in the classroom. The questionnaire was designed to determine what math curriculum and math practices were place in the children's classroom. The questionnaire included questions about the math curriculum or approach used by the teacher; the types of activities, materials, and strategies used to teach math skills to children; and the teacher's perspective on how children best learn math skills. The Teacher Math Practices Questionnaire was completed by the lead teacher in the classroom.

Child measures. Three child assessment measures were used. The first child measure was the Mullen Scales of Early Learning (Mullen, 1995), which was used to provide an overall estimate of each child's developmental status. The Mullen Scales of Early Learning is a standardized, norm-referenced measure with five scales: (1) gross motor, (2) visual reception, (3) fine motor, (4) receptive language, and (5) expressive language. Each child was assessed on each scale, excluding gross motor. The Mullen was administered by a project staff member who had been trained to reliably administer it as part of a research project funded by the Institute of Education Sciences.

The second child measure was the Tools for Early Assessment in Math (TEAM; Clements et al., 2011), which was used to provide an overall estimate of each child's math knowledge. The TEAM is a standardized, norm-referenced assessment of math skills for children from preschool to second grade. It contains two scales: number and shape. The TEAM was administered by the researcher.

On the TEAM, for ease of interpretability, raw scores are translated into T-scores, and grade equivalents are indicated. However, guidance is not given for comparing a child's performance to the typical performance of other children in the same age group. Thus, it is difficult to determine if a child's performance on the TEAM is above or below that of same-aged peers. However, in a study in which the TEAM was developed (Clements, Sarama, & Liu, 2008), 360 preschool children were assessed using the TEAM at two points in time. At the first time point, their average age was 4.25, roughly equivalent to the children in the current study. In the TEAM development study, the children were from 34 low-income and 12 mixed-income classrooms. The average T-score among these 360 children was 44.42, with a range of 20.25 to 69.08 and a standard deviation of 7.85. These scores serve as a comparison for the children in the current study.

The third measure was the Math Screening Instrument (see Appendix A; Hardy 2013b). The Math Screening Instrument is a researcher-designed measure of children's specific early math skills, developed for the purposes of this study. It was designed to be measure early math skills across all relevant domains, and thus provided more detailed information from multiple math domains than the TEAM. Items on the instrument were developed based on the literature on early childhood math development (e.g., Charlesworth & Lind, 2010; Clements & Sarama, 2009) and standards related to early math, including the Head Start Child Development and Early Learning Framework (Office of Head Start, 2010). Items on the Math Screening Instrument related to counting, quantity comparison, size comparison, sorting and classification, seriation, patterning, ordinality, comparing numerical magnitudes, shape identification, shape manipulation, numeral identification, addition, subtraction, and measurement. These categories are consistent with existing theory and research about early math skills.

The Math Screening Instrument was used to identify target skills for instruction. The Math Screening Instrument was administered to each child by the researcher over three sessions. During each session, the researcher administered each item by giving the child materials and a task direction (e.g., "Count the bears"). Each task was assessed in three probe trials (one per session). If the child demonstrated a task in one session, that task was not probed in the subsequent sessions (to ensure that only items the child did not know were in the pool of potential target skills). For some skills, there were multiple items designed to assess the child's performance of the skill. Often, these items were of increasing complexity (e.g., counting five items, then counting 10 items). If the child did not perform the easier task correctly, the child was not probed on the more difficult task for that skill. Only items that the child could not demonstrate in all three probe sessions were considered as potential target skills.

Materials

For probe sessions, a set of manipulatives was used. These manipulatives varied depending on the skills targeted for instruction (described in the procedures section below), but included different color and size foam shapes (sorting), colored blocks (counting and patterning), and pattern blocks with picture cards (shape manipulation).

For instructional sessions, two types of activities were used: manipulatives and art activities. At the beginning of each instructional session, the child was shown visuals for each of the two types of activities and was asked to choose the activity he wanted to play. Materials varied depending on the skills targeted for instruction. For the manipulatives activities, the materials were different color and size foam shapes (sorting), Unifix® cubes (patterning), small rubber animals (counting), and pattern blocks with picture cards (shape manipulation). For the art activities, the materials were play dough, play dough tools, and small rubber vehicles (counting); different size and color construction paper shapes, glue, and paper (sorting); markers and paper strips with repeating pictures (patterning); and colored paper shapes mimicking pattern blocks, glue, and picture cards (shape manipulation). Photographs of the manipulatives and art materials used in intervention sessions are presented in Appendix C.

Manipulatives used during probes differed from those that were used during instruction. However, there were some materials that remained constant. For example, for the target skill of shape manipulation, pattern blocks were used in both probe and intervention sessions. The picture cards, however, differed for the two session types. For the target skill of sorting, foam shapes were used in both session types. However, different colors and shapes were used depending on the session type (e.g., in the probe sessions, the child would sort orange, purple, and blue squares, circles, and diamonds; in the instructional sessions, the child would sort green, yellow, and red rectangles, ovals, and triangles). Photographs of the manipulatives used in probe sessions are presented in Appendix D.

For generalization sessions, materials differed from those used during probes and instruction. It was originally the intention to use solely materials from the children's classroom. However, due to the scarcity of materials available in the classroom relevant to each skill, it was necessary to use other materials. Materials used included colored plastic spools (counting), different size and shape puzzle pieces (sorting by shape and size), different color and shape beads (sorting by color and shape), different colored plastic stacking caps (patterning), and tangrams and construction paper figures (shape manipulation). The plastic spools used for counting were from the child's classroom; all other materials were provided by the researcher and were brought in and out of the classroom for generalization sessions. Photographs of the manipulatives used in generalization sessions are presented in Appendix E.

Response Definitions and Measurement

Data were collected live during sessions by the researcher using trial recording (Ayres & Gast, 2010). Sessions were approximately 10 to 20 minutes in length. There were four session types: probe, instructional, generalization, and maintenance.

In probe, generalization, and maintenance sessions, the child was given a task direction (e.g., “Put them together by shape”). There were three possible child responses: correct, error, and no response. A *correct response* was recorded if the child physically demonstrated or said the correct answer. A correct response was also scored if a child initially answered incorrectly and then spontaneously produced the correct response within five seconds. An *error* was recorded if the child physically demonstrated or said a wrong answer after the task direction. A *no response* was recorded if the child did not respond or said, “I don’t know.”

Correct and incorrect responses varied based on the skill. For each skill, correct and incorrect answers were operationalized prior to beginning data collection. For example, the skill of counting was defined as counting using one-to-one correspondence while pointing at or moving items and saying the corresponding number and ending on the correct number. An incorrect counting response was defined as the child guessing a number without using one-to-one correspondence, skipping items when counting, or counting some items more than once. Examples of correct and incorrect responses for each skill are provided in Table 2.

Table 2. *Definitions of Correct and Incorrect Responses for Each Skill*

Skill	Correct response	Incorrect response
Counting	Counts using 1:1 correspondence, pointing at or moving items and saying corresponding number. Must end with correct number.	Child guesses (correct or incorrect answer), counts without using 1:1 correspondence, skips some items, or counts some items more than once.
Sorting	Puts into separate piles based on attribute named. Must sort all items correctly.	Child spreads items out, puts in piles based on incorrect attribute, or only sorts some of the items correctly.
Patterning	Extends pattern at least 3 times correctly. Stops when done (rather than adding additional colors).	Extends pattern 1-2 times correctly only, does 1 time and then adds items incorrectly, or extends pattern 3 times correctly and then keeps adding items incorrectly.
Shape manipulation	Puts the shapes on top of the picture, filling the picture fully, with no extra shapes on picture.	Piles shapes on top of picture, puts shapes on picture but does not fill picture correctly, puts some shapes on picture correctly but does not complete picture fully, or puts extra shapes on picture.

In instructional sessions, there were two types of trials, demonstration and practice. In demonstration trials, the child was not directed to respond. However, it was possible that the child would independently imitate the researcher's demonstration of the skills. If the child imitated the researcher's verbal and/or physical behavior, an *imitated correct* was recorded. If the child followed the researcher's model with an attempt of the behavior that was incorrect (e.g., counted to nine instead of eight), an *imitated error* was recorded. In practice trials, a task direction (e.g., "Count how many spaces to move") was provided. There were three possible child responses: unprompted correct, prompted correct, and prompted error. An *unprompted correct* response was recorded if the child physically demonstrated or said the correct answer after the task direction. When the child did not respond to the task direction or responded incorrectly to the task direction, a prompt was provided. The child's response to the prompt was then coded as either a prompted correct or a prompted error. A *prompted correct* was recorded when a prompt was necessary and the child responded correctly to the prompt. A *prompted error* was recorded when a prompt was necessary and the child responded incorrectly or did not respond to the prompt. For each trial, only one child behavior was coded. Either the initial response was recorded if correct or the response to the prompt was recorded if a prompt was necessary. Therefore an incorrect response to the task direction was not recorded, as it was followed by a prompt for a correct response. In this case, only the response to the prompt was coded. In addition to these responses, spontaneous demonstrations of the skill were sometimes recorded (e.g., without task direction or prompt). If the child spontaneously and correctly demonstrated a skill, a *spontaneous correct* was recorded. If the child spontaneously and incorrectly demonstrated a skill, a *spontaneous error* was recorded. Data collection sheets for the four session types are presented in Appendices F, G, H, and I.

Interobserver Agreement

Interobserver agreement data were independently collected by project staff members. One of the staff members had a doctoral degree in early childhood education, and the other had a master's degree in early childhood special education. Both had prior experience collecting data as part of randomized control trials and/or single-subject studies. Project staff members were trained in practice sessions with non-study children prior to the beginning of data collection. Prior to collecting data, project staff members had to reach at least 90% agreement across three consecutive observations for each session type. During the study, interobserver agreement data were collected at least 30% of sessions for the two children who completed the study and 29.27% of sessions for the child who completed part of the study. Percentage agreement was calculated using the point-by-point agreement method. Total agreements were divided by the sum of agreements and disagreements and were then multiplied by 100 to derive a percentage (Repp, Deitz, Boles, Deitz, & Repp, 1976). Interobserver agreement was recorded on the IOA Calculation Form (see Appendix J).

Experimental Design

A multiple probe design (conditions) across skills, replicated across participants was used (Gast & Ledford, 2010). In multiple probe designs, baseline data are collected intermittently, with at least three sessions of consecutive baseline probes prior to the introduction of the independent variable. When baseline data are stable, the independent variable is introduced in a staggered fashion across the three skills. In multiple probe designs, experimental control is demonstrated by stable baseline data and the immediate change in the dependent variable after introduction of the independent variable (Gast & Ledford, 2010). Data in the other tiers should remain stable until the independent variable is introduced in each tier (Gast & Ledford, 2010). Through the staggered introduction of the independent variable, the following threats to internal validity are controlled for in multiple probe designs: maturation and history (Gast, 2010). Additional threats to internal validity are testing, instrumentation, procedural infidelity. Testing was addressed through the use of a probe design, rather than continuous baseline data collection, as well as measures to avoid session fatigue by participants. Instrumentation was addressed through the frequent collection of interobserver agreement data. Procedural fidelity was addressed through the frequent collection of data on the researcher's adherence to study procedures.

A multiple probe design is appropriate to use in this study because the dependent variable, specific math skills, are functionally independent, functionally similar, and non-reversible (Gast & Ledford, 2010). A multiple probe design is preferred to a multiple baseline design because a multiple probe design does not entail continuous baseline data collection (Horner & Baer, 1978).

Procedures

Child assessment. Prior to baseline, target skills were identified for each child. Children were first assessed using the Math Screening Instrument (see Appendix A). Children were next assessed using the Mullen Scales of Early Learning (Mullen, 1995) and the Tools for Early Assessment in Math (TEAM; Clements et al., 2011).

Target skill selection. Target skills were selected from a list of possible math skills that were assessed using the Math Screening Instrument (see Appendix A). Each participant's performance on the Math Screening Instrument is presented in Table 3. The unknown skills for each child were rank-ordered by difficulty by the researcher. The skills chosen were those considered developmentally "next" for each participant. It was originally planned to target skills related to number sense (e.g., counting, cardinality, numerical magnitude comparison, quantity comparison, ordinality, addition, and subtraction). However, each child demonstrated a need to learn more basic early math skills. Thus, the skills chosen for Jason and She'quan were sorting, patterning, and shape manipulation. The skills chosen for Orion were counting, sorting, and patterning.

Table 3. *Participants' Unknown Skills Based on the Math Screening Instrument*

Jason	Orion	She'quan
• Comparing quantities—less (large and small difference) • Sorting—size, shape, and function • Oddity—function • Seriation—ordering and inserting • Patterning • Ordinality—ordering and insertion • Comparing numerical magnitudes—more and less (large and small difference) • Shape identification—rectangle, triangle • Shape manipulation—triangle, filling picture • Addition • Subtraction • Measuring length • Measuring weight	• Counting to 10 • Comparing quantities—less (large and small difference) • Sorting—color, size, shape, function • Oddity—size, maybe function • Seriation—ordering and inserting • Patterning • Ordinality—ordering and insertion • Comparing numerical magnitudes—more and less (large and small difference) • Shape identification—all • Shape manipulation—rectangle, triangle, filling picture • Addition • Subtraction • Measuring length • Measuring weight	• Comparing quantities—less (large and small difference) • Sorting—color, size, shape, maybe function • Oddity—color, size, function • Seriation—ordering and inserting • Patterning • Ordinality—ordering and insertion • Shape identification—circle, square, rectangle, rhombus, oval • Shape manipulation—triangle, filling picture • Numeral ID—1-10 • Addition • Subtraction • Measuring length • Measuring weight

For each target skill identified for a child, two specific behaviors were taught. For example, all of the participants had patterning as a target skill. The two behaviors taught were completing AB patterns and completing ABB patterns. The specific behaviors taught were identified based on the child's performance on the Math Screening Instrument. A complete list of each participant's target skills and specific behaviors for each skill is presented in Table 4.

Table 4. *Participants' Target Skills and Specific Behaviors*

Skill	Jason		Orion		She'quan	
	Behavior 1	Behavior 2	Behavior 1	Behavior 2	Behavior 1	Behavior 2
1	Sorting by shape into 3 groups	Sorting by size into 3 groups	Counting to 8	Counting to 10	Sorting by shape into 3 groups	Sorting by size into 3 groups
2	Completing AB pattern by color	Completing ABB pattern by color	Sorting by color into 3 groups	Sorting by shape into 3 groups	Completing AB pattern by color	Completing ABB pattern by color
3	Making picture with 3 shapes	Making picture with 4 shapes	Completing AB pattern by color	Completing ABB pattern by color	Making picture with 3 shapes	Making picture with 4 shapes

Description of the children's classroom. After collecting child assessment data, two measures were used to describe the type of math activities and practices that occurred in the classroom. The first measure was a researcher-created Teacher Math Practices Questionnaire (see Appendix B). The second measure was the Preschool Classroom Mathematics Inventory (PCMI; Frede et al., 2006). Both measures are described in greater detail in the measures section above. The Teacher Math Practices Questionnaire was completed by the lead teacher in the children's classroom, and the PCMI was completed by the researcher.

Probe conditions. During the probe condition sessions, the researcher presented the task direction (e.g., "Count the cars") and then provided a five second response interval. The child's response was recorded. The researcher reinforced correct responses and ignored errors and no responses. It was planned that if the child requested for the adult to provide the correct answer, the researcher would say, "We'll learn that later." In the study, however, the children never requested the correct answer. The child received three probes per target behavior, for a total of six trials per skill and 18 trials per probe session. A probe condition occurred across all three tiers before intervention began in tier one. After data were stable and low, intervention in tier one began. Subsequent probe conditions occurred after the child reached criterion in each tier of intervention (three consecutive sessions at 100%). There were four probe conditions over the course of the study.

Intervention. A prompting procedure with two types of trials, demonstration and practice, were used in intervention sessions. In demonstration trials, the researcher provided a description of the task (e.g., “I’m going to put them together by size”) and provided information about how to complete the task (e.g., “To put them together by size, you look at each shape to see whether it is little, medium, or big. You put the little ones together, the medium size ones together, and then the big ones together”). The researcher then modeled completing the task, without delivering a task direction. After completing the task, the instructor provided a descriptive comment (e.g., “I put them together by size!”). See Figure 1 for a flowchart of demonstration trials. See Table 5 for the task direction and information provided for each skill. The child received two demonstration trials for each behavior, for a total of four demonstration trials per instructional session. Each session began with a demonstration trial, and the remaining demonstration trials were interspersed with practice trials so that approximately every third trial was a demonstration trial.

Figure 1. *Demonstration and Practice Trials Flowcharts*

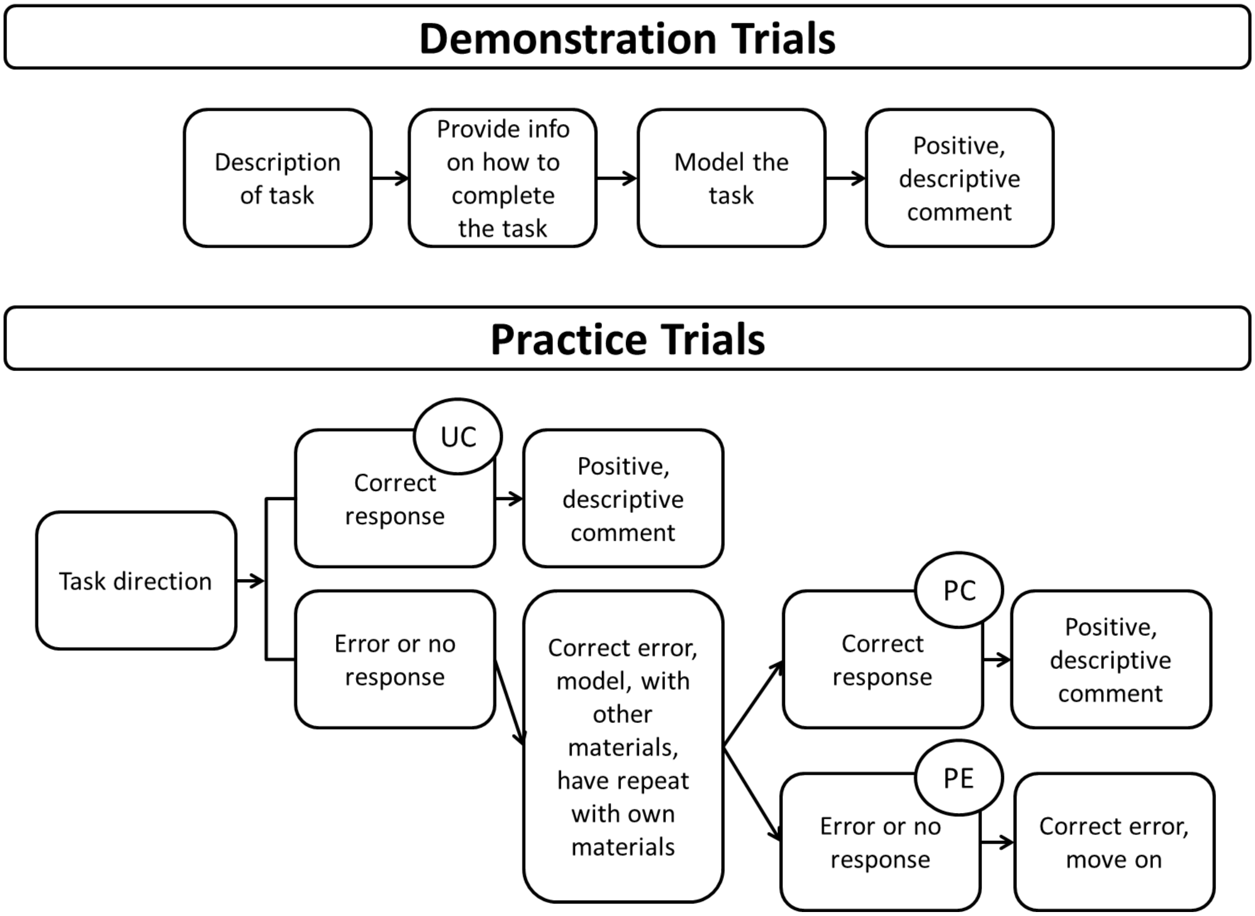


Table 5. *Task Direction and Information Provided for Each Skill*

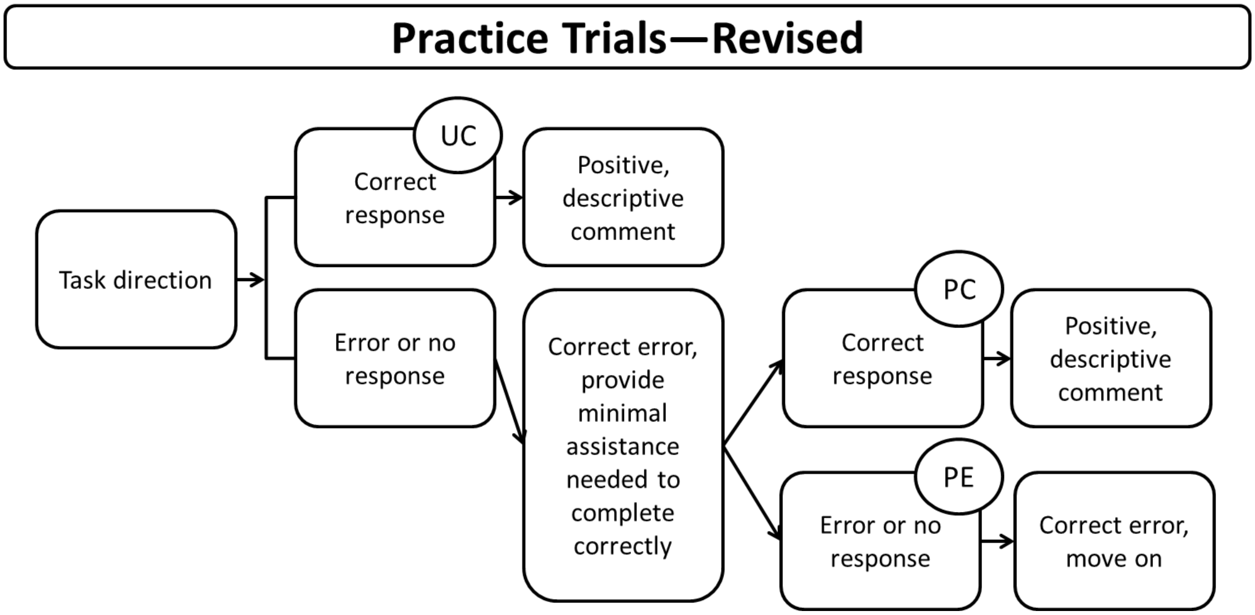
Skill	Task direction	Information provided
Counting	Count the X.	Put your finger on each one and move it as you count.
Sorting by color	Put them together by color.	To put them together by color, see which ones are the same color and put them together.
Sorting by shape	Put them together by shape.	To put them together by shape, see which ones are the same shape and put them together.
Sorting by size	Put them together by size.	To put them together by size, you look at each picture to see whether it is little, medium, or big. You put the little ones together, the medium size ones together, and then the big ones together.
Completing AB and ABB patterns by color	Keep going with the pattern.	Look at how the colors repeat and copy it. XYXY [substitute actual color names].
Making pictures with 3 or 4 shapes	Put the blocks together to make the picture.	Fit the blocks together like a puzzle so the whole picture is covered. Make sure the blocks stay in the lines.

In practice trials, the researcher provided the task direction (e.g., "Put them together by size"). The adult then provided a five-second response interval. If the child responded correctly, the researcher provided a positive, descriptive comment (e.g., "You did it—you put them together by size!"). This was recorded as an unprompted correct. If the child responded incorrectly, the researcher corrected the error and covered the child's materials if necessary (e.g., if the child incorrectly extended a pattern using Unifix® cubes, the Unifix® cubes were covered). The researcher then modeled the correct response using different materials. The researcher then uncovered the child's original materials (if they had to be covered) and reconfigured the materials so they were no longer an incorrect example but rather were arranged as they originally were when the trial began. The child was then directed to repeat the correct response using the original materials, with the researcher's model visible. If the child repeated the correct response, the researcher provided a positive, descriptive comment, and a prompted correct was recorded. If the child did not repeat the correct response, an error was recorded. See Figure 1 for a flowchart of practice trials. The child received eight practice trials per instructional session. Two behaviors were taught, with four trials per behavior. For example, for the skill of counting for Orion, counting to eight and ten were the behaviors. The child received four trials for counting to eight and four trials for counting to ten.

It also was possible that the child would imitate the researcher during the demonstration trials or spontaneously demonstrate the skill at any point the instructional sessions. If a correct spontaneous demonstration or correct imitation of the skill occurred, the researcher provided a positive, descriptive comment. If an incorrect spontaneous demonstration or imitation of the skill occurred, the researcher corrected the child and modeled the correct response. The child was not directed to repeat the correct response. If the child spontaneously demonstrated other math skills (i.e., the skills taught or to be taught in other tiers), the researcher ignored the child. Imitated corrects, imitated errors, spontaneous corrects, and spontaneous errors were recorded on the data collection form (see Appendix G).

Intervention modifications. The intervention was modified in two ways for all participants in the course of the study. First, the number of demonstration and practice trials presented to each child was decreased by half because the children were beginning to show signs of fatigue in the intervention sessions. Indications of fatigue included the child asking to go back to the classroom and exhibiting non-compliance to directions. When these behaviors began happening more frequently, a decision was made to modify the number of trials. This change happened in session 37 for Jason, session 12 for Orion, and session 32 for She'quan. The second modification was a change in the error correction procedure. Originally, the researcher corrected the child's error, modeled the correct response with other materials, and then directed the child to repeat the correct response with his own materials. This procedure sometimes led to the child being allowed to complete a task incorrectly and then repeat the incorrect response after it was modeled correctly. Thus, the procedure was modified so that the researcher corrected the error right after the child made it (rather than waiting till the child completed the incorrect response) and did the skill correctly with the child, providing the minimal physical and/or verbal assistance needed. For example, if the skill was patterning, and the child incorrectly completed the pattern, the researcher might say, "The pattern is one white cube, then two green cubes. Put a green cube next." Researcher judgment was used to decide when to interrupt the child's error. If the child appeared to be trying to correct his error, additional wait time was provided. However, if the child appeared unaware that he made an error or continued making the error, the researcher interrupted the response. This modification was instituted in tier two for each participant. See Figure 2 for a revised flowchart of practice trials.

Figure 2. *Revised Practice Trials Flowchart*



The intervention was modified in two additional ways for She'quan in tiers one and two. In tier one, She'quan had difficulty sorting by shape and size using the materials for those behaviors (small, medium, and large shapes). He did not appear to be attending to the relevant dimensions of the stimuli. Thus, the materials were modified to be pictures of different sized animals and different shapes that were all the same size. This was to help him attend to only size when asked to sort by size and only shape when asked to sort by shape. As he began to acquire the behaviors, the stimuli were gradually changed to the original materials for this skill.

In tier two, She'quan had difficulty learning both behaviors for the target skill at the same time. After three sessions with a significant percentage of initial errors (requiring a prompt), the researcher determined that the task needed to be simplified. Thus, the intervention was modified so that She'quan received instruction on one behavior, then on the second, and then on both together. These modifications are noted on She'quan's graph (see Figure 5).

Generalization. Generalization sessions occurred during probe and intervention conditions approximately every fifth session (once per week). Generalization to the classroom and other materials was measured. The primary researcher conducted the generalization sessions in the child's classroom, using other materials that were not used during intervention and classroom materials when possible. The researcher used the same task direction as in practice trials. The child received two probe trials per behavior, for a total of four probe trials per skill and 12 probe trials per session. The researcher reinforced correct responses and ignored errors and no responses. It was planned that if the child requested for the adult to provide the correct answer, the researcher would say, "We'll learn that later." In the study, however, the children never requested the correct answer.

Maintenance. Maintenance probe sessions were conducted approximately one month after the end of the last probe condition. Two sessions were conducted. The child received three probe trials per behavior, for a total of six probe trials per target skill and 18 probe trials per session. The researcher reinforced correct responses and ignored errors and no responses. In addition to these maintenance sessions conducted after the completion of all intervention, the probe conditions serve as a measure of maintenance of skills acquired in previous tiers.

Procedural Fidelity

Procedural fidelity data were independently collected using a direct, systematic recording system. A combination of checklists and trial recording was used to measure the researcher's behavior. Checklists were used to measure researcher behavior that occurred one time per session (e.g., researcher worked with the child in a one-on-one session), and trial recording was used to measure behavior that occurred multiple times in one session (e.g., providing prompt after task direction and response interval; providing positive, descriptive comment after each correct response). Procedural fidelity data were collected at least 30% of sessions for the two children who completed the study and 29.27% of sessions for the child who completed part of the study. Total occurrences was divided by the sum of occurrences and planned occurrences and was multiplied by 100 to derive a percentage (Gast, 2010). Examples of behaviors that were measured during probe condition sessions include: providing the task direction, providing a response interval, and praising a correct response. Examples of behaviors that were measured during instructional sessions include: providing the task direction, providing the prompt, and reinforcing a correct response. Procedural fidelity forms for each session type are presented in Appendices K, L, M, and N. The form for intervention sessions was revised when the instructional procedure was modified.

Social Validity

Social validity was measured by asking the teacher to complete a questionnaire with questions about the goals of the study, the procedures of the study, and the effects of the study (Wolf, 1978; see Appendices O and P). Because only one teacher was involved in the current study, it was not anonymous. The first set of questions was asked prior to the beginning of data collection and after data collection ended. The teacher was asked if she observed the child engaging in the skills targeted during the intervention. The second set of questions related to whether the teacher viewed early math instruction as important for preschoolers. The third set of questions related to the instructional procedures used in the study. The teacher was asked to view a video of the researcher implementing the intervention and respond to questions about whether the informant would be willing and able to implement the intervention.

CHAPTER III

RESULTS

Interobserver Agreement

Interobserver agreement data were collected during 32.76% of sessions for Jason, 29.27% of sessions for Orion, and 30.38% of sessions for She'quan. Data on the percentage of sessions in which IOA data were collected for each session type for each participant are presented in Table 6. The mean IOA percentages for Jason, Orion, and She'quan were 99.47%, 98.13%, and 99.13%, respectively. The mean and range IOA percentages for each session type for each participant are also presented in Table 6.

Table 6. *Mean and Range IOA Percentages and Percentage of Sessions in Which IOA Data Were Collected*

Session type		Across all participants	Jason	Orion	She'quan
Across all session types	Mean	99.03%	99.47%	98.13%	99.13%
	Range	87.50-100%	90-100%	87.50-100%	87.50-100%
	% sessions collected	31.25%	32.76%	29.27%	31.17%
Probe	Mean	100%	100%	100%	100%
	Range	n/a	n/a	n/a	n/a
	% sessions collected	26.67%	16.67%	16.67%	41.67%
Intervention	Mean	98.78%	99.23%	97.75%	99.11%
	Range	87.50-100%	90-100%	87.50-100%	87.50-100%
	% sessions collected	31.62%	37.14%	34.48%	26.42%
Generalization	Mean	99.07%	100%	100%	98.33%
	Range	91.67-100%	n/a	n/a	91.67-100%
	% sessions collected	33.33%	33.33%	16.67%	41.67%
Maintenance	Mean	100%	100%	n/a	n/a
	Range	n/a	n/a	n/a	n/a
	% sessions collected	50%	50%	n/a	n/a

Procedural Fidelity

Procedural fidelity data were collected during 32.76% of sessions for Jason, 29.27% of sessions for Orion, and 30.38% of sessions for She'quan. Data on the percentage of sessions in which procedural fidelity data were collected for each session type for each participant are presented in Table 7. The mean procedural fidelity percentages for Jason, Orion, and She'quan were 99.14%, 98.16%, and 99.48%, respectively. The mean and range procedural fidelity percentages for each session type for each participant are also presented in Table 7.

Table 7. *Mean and Range Procedural Fidelity Percentages and Percentage of Sessions in Which Procedural Fidelity Data Were Collected*

Session type		Across all participants	Jason	Orion	She'quan
Across all session types	Mean	99.07%	99.14%	98.16%	99.48%
	Range	88.24-100%	88.24-100%	90.91-100%	93.33-100%
	% sessions collected	31.25%	32.76%	29.27%	31.17%
Probe	Mean	99.43%	97.73%	100%	100%
	Range	88.24-100%	95.45-100%	n/a	n/a
	% sessions collected	26.67%	16.67%	16.67%	41.67%
Intervention	Mean	98.93%	99.10%	97.79%	99.58%
	Range	88.24-100%	88.24-100%	90.91-100%	94.12-100%
	% sessions collected	31.62%	37.14%	34.48%	26.42%
Generalization	Mean	99.26%	100%	100%	98.67%
	Range	93.33-100%	n/a	n/a	93.33-100%
	% sessions collected	33.33%	33.33%	16.67%	41.67%
Maintenance	Mean	100%	100%	n/a	n/a
	Range	n/a	n/a	n/a	n/a
	% sessions collected	50%	50%	n/a	n/a

Classroom Measures

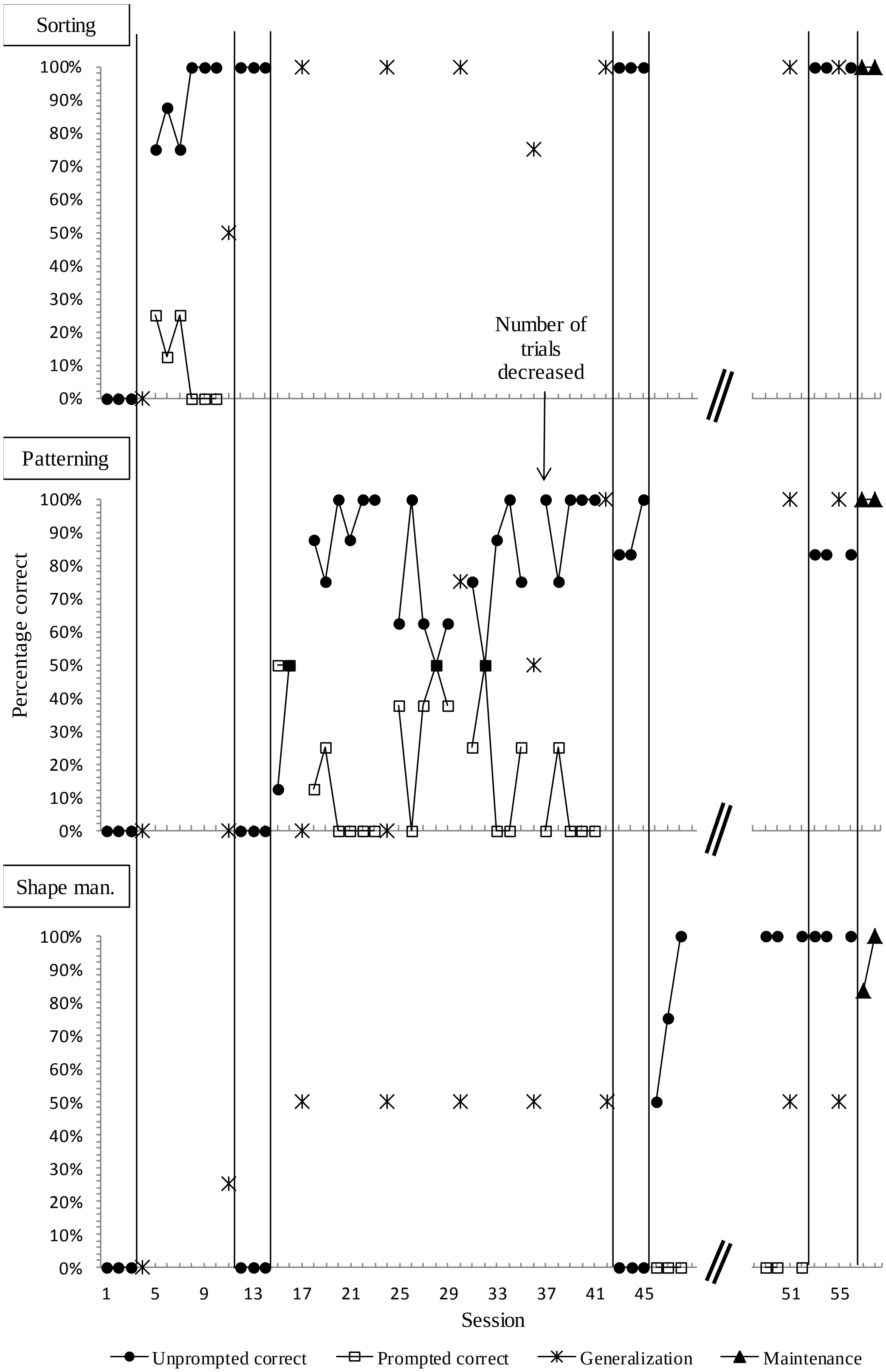
On the Teacher Math Practices Questionnaire, the teacher reported the use of the *Frog Street Pre-K Curriculum* (Schiller, Flor Ada, Campoy, & Mowry, 2010). She also noted the presence and use of a math center, manipulatives, and hands-on math activities. She emphasized the importance of making math fun. On the Preschool Classroom Mathematics Inventory, the classroom had a mean score of 1.36, which corresponds with minimal evidence of mathematics instruction. The range was one (minimal evidence) to three (some evidence). The highest score possible on this measure was five.

Jason

Acquisition of target behaviors. The data on Jason's acquisition of target behaviors are presented in Figure 3. Unprompted correct responses are represented with a closed circle and prompted corrects are represented with an open square. In the first probe condition, Jason demonstrated 0% of his target behaviors in all tiers of instruction. In the second probe condition, he demonstrated 0% of tier two and three behaviors, and in the third probe condition, he demonstrated 0% of tier three behaviors.

Jason rapidly acquired his target behaviors in tier one, reaching criterion in six sessions. In the second, third, and fourth probe conditions, he demonstrated 100% of tier one behaviors. Jason acquired his target behaviors in tier two more slowly, reaching criterion in 23 sessions. His data were variable throughout this condition before reaching criterion during the final three sessions. In the third probe condition, immediately following tier two instruction, he demonstrated an average of 88.89% of tier two behaviors. In the fourth probe condition, he demonstrated an average of 83.33% of tier two behaviors. Jason again rapidly acquired his target behaviors in tier three, reaching criterion in five sessions. In the fourth probe condition, immediately following tier three instruction, he demonstrated 100% of tier three behaviors.

Figure 3. *Jason's Acquisition, Generalization, and Maintenance of Target Behaviors*



Generalization of target behaviors. The data on Jason's generalization of target behaviors to untrained materials in the classroom are presented in Figure 3. Generalization probes are represented with an asterisk. In probe condition one, Jason demonstrated 0% of tier one target behaviors in generalization sessions. By the end of tier one, he demonstrated 50% of target behaviors in generalization sessions. In tier two, he demonstrated an average of 95% of tier one behaviors in generalization sessions, and, in tier three, he demonstrated 100% of tier one behaviors in generalization sessions.

Prior to intervention in tier two, Jason demonstrated 0% of tier two behaviors when measured in generalization sessions. His use of tier two behaviors in generalization sessions during tier two was variable, but reached 100% by the end of tier two. In tier three, he demonstrated 100% of tier two behaviors in generalization sessions.

In probe condition one, Jason demonstrated 0% of his tier three target behaviors in generalization sessions. Beginning at the end of tier one, he began to demonstrate some tier three behaviors in generalization sessions (prior to receiving instruction in tier three behaviors). He continued to demonstrate these behaviors with 50% accuracy in generalization sessions in tiers two and three. Jason never fully demonstrated tier three behaviors in generalization sessions.

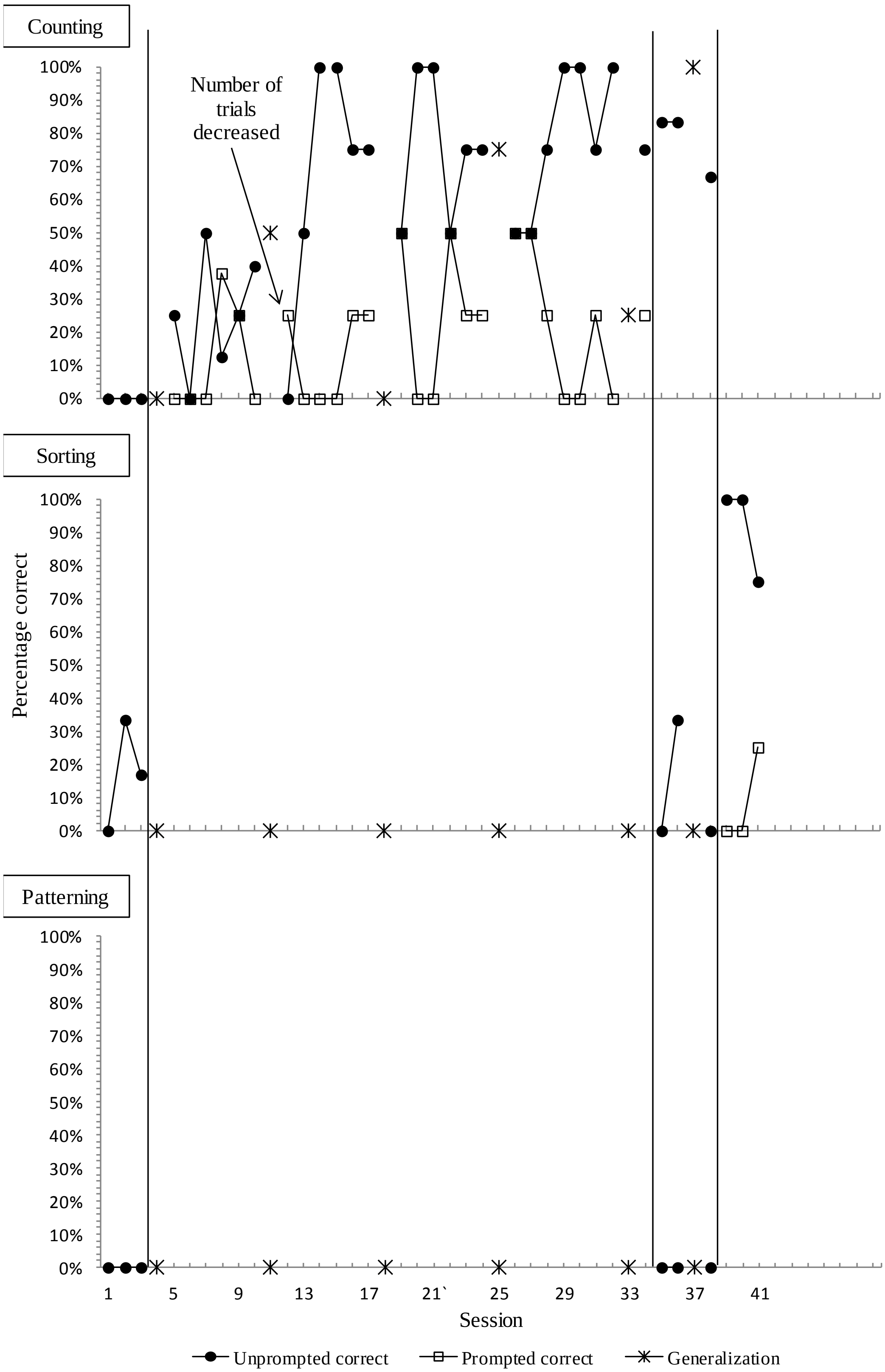
Maintenance of target behaviors. The data on Jason's maintenance of target behaviors are presented in Figure 3. Maintenance probes are represented with a closed triangle. Across two maintenance sessions one month after intervention ended, Jason demonstrated 100% of tier one and tier two behaviors and an average of 91.67% of tier three behaviors.

Orion

Acquisition of target behaviors. The data on Orion's acquisition of target behaviors are presented in Figure 4. Unprompted correct responses are represented with a closed circle and prompted corrects are represented with an open square. In the first probe condition, Orion demonstrated 0% of his target behaviors in tiers one and three and an average of 16.67% of tier two behaviors. In the second probe condition, he demonstrated an average of 11.11% of tier two behaviors and 0% of tier three behaviors. Orion began tier two right before winter break, but he did not return to the school after winter break. Thus, the data do not extend beyond initial tier two data collection.

Orion did not reach criterion levels of responding in tier one, which lasted 26 sessions. Though he did not attain three consecutive sessions at 100% accuracy, the decision was made to move to the next tier after he had a total of seven sessions at 100%. Overall, his tier one data had an ascending trend, but there was significant variability. For the first eight sessions, his responding never exceeded 50% unprompted correct. However, beginning at session 14, his level of responding ranged from 50-100% unprompted correct responses. It was during this period of time that he had seven sessions at 100%. In the second probe condition, immediately following intervention in tier two, he demonstrated 77.78% of the tier one behaviors. Orion was beginning to demonstrate acquisition of tier two behaviors when winter break occurred.

Figure 4. *Orion's Acquisition and Generalization of Target Behaviors*



Generalization of target behaviors. The data on Orion's generalization of target behaviors are presented in Figure 4. Generalization probes are represented with an asterisk. In probe condition one, Orion demonstrated 0% of tier one target behaviors in generalization sessions. During tier one intervention, he demonstrated tier one target behaviors variably in generalization sessions. However, in probe condition two, he demonstrated 100% of tier one target behaviors in generalization sessions. Prior to intervention in both tier two and three, Orion demonstrated 0% of tier two target behaviors in generalization sessions. Further data on generalization were not collected because the child withdrew from the study.

She'quan

Acquisition of target behaviors. The data on She'quan's acquisition of target behaviors are presented in Figure 5. Unprompted correct responses are represented with a closed circle and prompted corrects are represented with an open square. In the first probe condition, She'quan demonstrated 0% of his target behaviors in all tiers of instruction. In the second probe condition, he demonstrated 0% of tier two and three behaviors, and in the third probe condition, he demonstrated 0% of tier three behaviors.

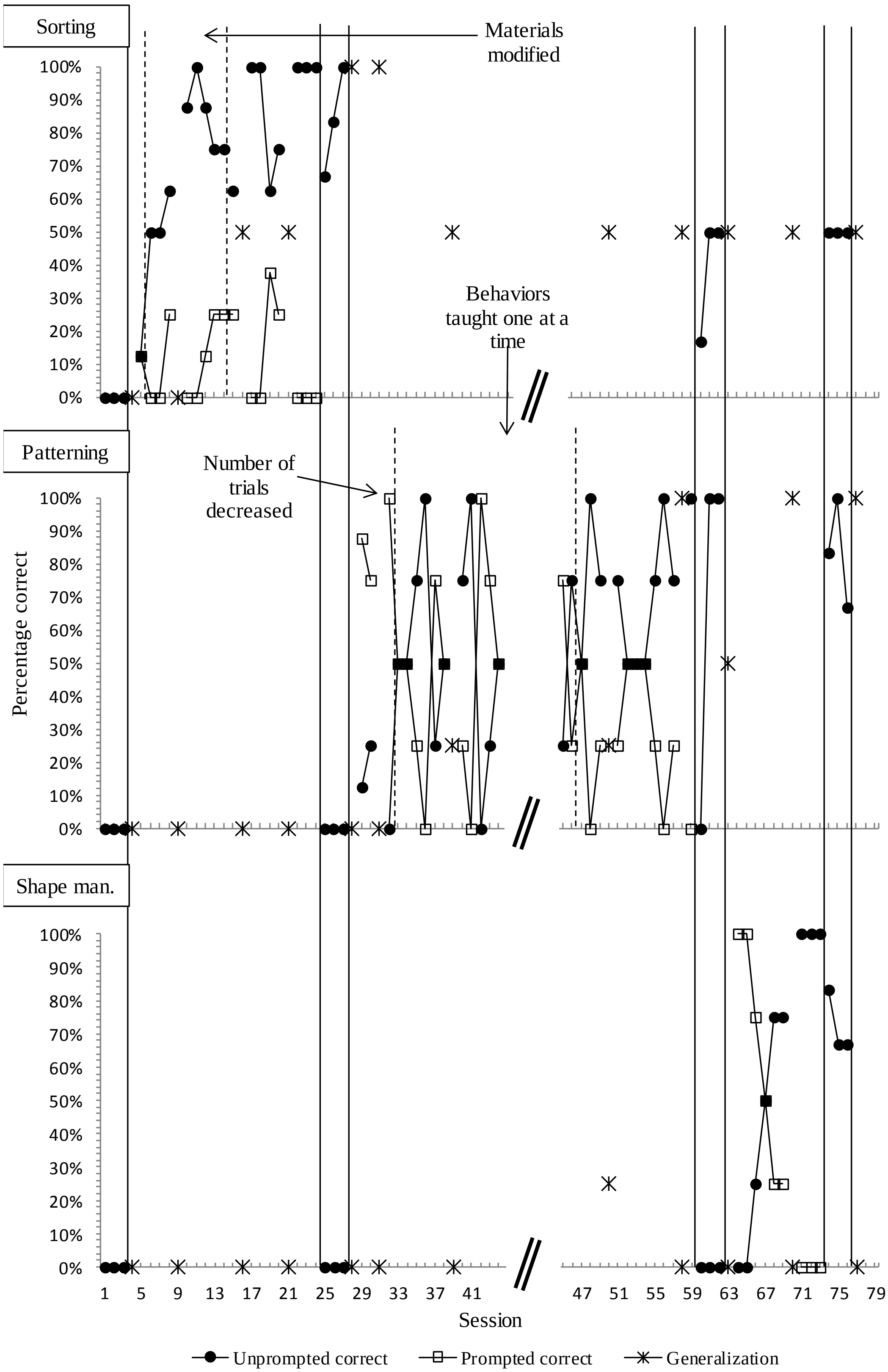
She'quan acquired his target behaviors in tier one, reaching criterion in 17 sessions. Overall, his tier one data had an ascending trend, but there was significant variability. At the beginning of the intervention, his data gradually accelerated to 100% unprompted correct responses in six sessions, and then they were variable before stabilizing at and reaching criterion at session 24. In the second probe condition, he demonstrated an average of 83.33% of tier one behaviors. In the third probe condition, he demonstrated an average of 38.89% of tier one behaviors. In the fourth probe condition, he demonstrated an average of 50% of tier one behaviors. She'quan's tier two data had an ascending trend but never reached criterion. There was significant variability in She'quan's tier two data. Although he had three sessions at 100% unprompted correct (not including two sessions at 100% when the modified materials were used), he did not have three consecutive sessions at 100%. The decision was made to move to the next tier when two of the last three sessions were at 100%. In the third probe condition, immediately following tier two instruction, he demonstrated an average of 66.67% of his tier two behaviors. In the fourth probe condition, he demonstrated an average of 83.33% of his tier two behaviors. She'quan acquired his target behaviors in tier three, reaching criterion in nine sessions. In the fourth probe condition, immediately following tier three instruction, he demonstrated 72.22% of tier three behaviors.

Generalization of target behaviors. The data on She'quan's generalization of target behaviors to untrained materials in the classroom are presented in Figure 5. Generalization probes are represented with an asterisk. In probe condition one, She'quan demonstrated 0% of tier one target behaviors in generalization sessions. By the end of tier one, he demonstrated 50% of target behaviors in generalization sessions. At the beginning of tier two, he demonstrated 100% of tier one behaviors in generalization sessions, but by the end of tier two, he only demonstrated 50% of tier one behaviors in generalization sessions. In tier three, he demonstrated 50% of tier one behaviors in generalization sessions.

Prior to intervention in tier two, She'quan demonstrated 0% of tier two behaviors when measured in generalization sessions. His use of tier two behaviors in generalization sessions in tier two was variable, but reached 100% by the end of tier two. She'quan's demonstration of tier two target behaviors in generalization sessions in tier three was again variable, but it reached 100% at the end of tier three.

In probe condition one, She’quan demonstrated 0% of his tier three target behaviors in generalization sessions. In tier two, he demonstrated an average of 6.25% of tier three target behaviors. In tier three, he demonstrated 0% of tier three target behaviors in generalization sessions. She’quan never demonstrated tier three behaviors in generalization sessions after intervention in tier three was complete.

Figure 5. *She'quan's Acquisition and Generalization of Target Behaviors*



Spontaneous and Imitated Demonstrations

In addition to the data collected in each instructional trial, data were also collected when the child spontaneously demonstrated a skill or imitated a skill. These spontaneous or imitative demonstrations of skills could be either correct or incorrect. The instances of spontaneous and imitated corrects and errors were recorded for each participant. The data are presented in Table 8. Two participants, Jason and She'quan, never had any spontaneous or imitated demonstrations of skills. Orion sometimes spontaneously or imitatively demonstrated skills, both correctly and incorrectly.

Table 8. *Spontaneous and Imitated Demonstrations for Each Participant*

	Jason			Orion			She'quan		
Tier	1	2	3	1	2	3	1	2	3
Spontaneous corrects	0	0	0	8	1	n/a	0	0	0
Spontaneous errors	0	0	0	5	0	n/a	0	0	0
Imitated corrects	0	0	0	2	0	n/a	0	0	0
Imitated errors	0	0	0	1	0	n/a	0	0	0

Dosage

Dosage was measured in two ways. The first was the number of trials in each intervention condition, and the second was the length of intervention sessions. The number of trials in each intervention condition for each participant is presented in Table 9. There was significant variability in the number of trials in each intervention condition both within and across participants. Jason had 48 trials to criterion in tier one (sorting), 164 in tier two (patterning), and 24 in tier three (shape manipulation). Orion had 128 trials to near-criterion in tier one (counting). She'quan had 128 trials to criterion in tier one (sorting), 116 trials to near-criterion in tier two (patterning), and 36 trials to criterion in tier three (shape manipulation). Overall, patterning appeared to require a larger number of trials than other skills, and shape manipulation was acquired more quickly. There was significant variability across participants in how many trials were required for the skill of sorting.

Table 9. *Number of Trials in Each Intervention Condition*

Skill	Jason	Orion	She'quan
Counting	n/a	128 (Tier 1)	n/a
Sorting	48 (Tier 1)	n/a	128 (Tier 1)
Patterning	164 (Tier 2)	n/a	116 (Tier 2)
Shape manipulation	24 (Tier 3)	n/a	36 (Tier 3)

The average length of intervention sessions in each tier for each participant is presented in Table 10. There was variability in the length of sessions for each participant, but on average, Jason's sessions were 14 minutes long, Orion's sessions were 18 minutes long, and She'quan's sessions were 12 minutes long. Across the all participants, the average length of sessions in tier one was 18 minutes, in tier two was 13 minutes, and in tier three was six minutes. Note that the number of trials in intervention sessions was decreased by half in session 37 for Jason (tier two), session 12 for Orion (tier one), and session 32 for She'quan (tier two). Thus, the decrease in length of session could be simply because the number of trials was decreased.

Table 10. *Length of Intervention Sessions in Minutes*

Tier		Across all participants	Jason	Orion	She'quan
Across all tiers	Mean	0:14	0:14	0:18	0:12
	Range	0:03-0:40	0:04-0:26	0:05-0:40	0:03-0:25
1	Mean	0:18	0:12	0:19	0:19
	Range	0:05-0:40	0:06-0:25	0:05-0:40	0:14-0:24
	Skill	n/a	Sorting	Counting	Sorting
2	Mean	0:13	0:17	0:08	0:11
	Range	0:05-0:26	0:05-0:26	0:06-0:13	0:05-0:25
	Skill	n/a	Patterning	Sorting	Patterning
3	Mean	0:06	0:06	n/a	0:06
	Range	0:03-0:11	0:04-0:10	n/a	0:03-0:11
	Skill	n/a	Shape manipulation	n/a	Shape manipulation

Social Validity

Social validity data were collected both prior to and after data collection. On both the pre- and post-questionnaires, the teacher indicated that she strongly agreed that math skills are important for children to know and that children need instruction to learn math skills. This did not change from pre-intervention to post-intervention. The teacher also responded to questions about how often she saw each target child engage in their target math skills, both prior to and after data collection. Those data are presented in Table 11. There was some change in the teacher's report of the frequency with which she observed each child demonstrating his target skills in the classroom, with a slightly higher frequency for two of Jason's skills and a slightly higher frequency for one of She'quan's skills, as reported on the post-questionnaire. The teacher also viewed a short video of the instructional procedure and answered questions related to the procedure. She strongly agreed that the instructional procedure seemed appropriate to use with preschool children, she could use the procedure in her classroom with individual children, and she could use the procedure in her classroom with small groups of children.

Table 11. *Teacher Report of Frequency of Skill Demonstration in the Classroom*

Tier	Jason		Orion		She'quan	
	Pre	Post	Pre	Post	Pre	Post
1	Sometimes	Sometimes	Never	n/a	Rarely	Rarely
2	Sometimes	Often	Rarely	n/a	Never	Rarely
3	Never	Often	Rarely	n/a	Never	Never

CHAPTER IV

DISCUSSION

The purpose of this study was to investigate whether a systematic instructional procedure was effective in helping children acquire, generalize, and maintain early math skills. One child (Jason) acquired all three of his target skills at criterion levels. One child (She'quan) acquired two of his target skills at criterion levels. She'quan did not reach criterion on the third skill (patterning). Although there was a clear effect, with three sessions at 100%, the sessions were not consecutive. One child (Orion) had seven sessions at 100% unprompted correct, but, again, they were not three consecutive sessions. He made progress toward acquiring a second, and then withdrew from school, and thus, the study. Therefore, across children, there were three demonstrations/replications (for Jason). Because She'quan did not reach criterion on all three of his skills, a functional relation cannot be established, although he made progress toward criterion levels of responding. Overall, the results would have been strengthened if Jason and She'quan had acquired their skills more rapidly and if She'quan had reached criterion on all three of his skills. The generalization results were mixed. Jason demonstrated generalization of two skills, Orion demonstrated generalization of one skill, and She'quan demonstrated generalization of one skill. Maintenance of skills was measured for one child (Jason). One month after the end of the last probe condition, Jason maintained all three skills at or near 100%. However, data on children's use of target skills in probe conditions following instruction on those skills is further evidence of maintenance. This occurred for six skills across the three participants.

Some of the results of this study require further examination, particularly around the variability in the data on efficiency of children's learning, the complexity of skills taught, and the use of demonstration trials. When evaluating the efficacy of an instructional procedure, it is necessary to consider the efficiency with which children learn. This was done in the current study by examining the number of trials to criterion. When the number of trials in each tier decreases with each subsequent tier, this often suggests that children might be "learning to learn." Although that pattern was present in the current study only for one participant, these data are difficult to interpret because of the variability in the data on number of trials to criterion. There were other possible reasons for this variability. First, over the course of the study, the number of trials per session was decreased due to session fatigue. The number of trials presented in intervention sessions was originally four demonstration trials and eight practice trials. The children seemed to experience session fatigue, which led to reducing the number of trials to two demonstration trials and four practice trials. This reduction in the number of trials appeared to prevent fatigue and was sufficient for children to acquire the behaviors taught. However, it is possible that presenting a greater number of trials per session would have resulted in more efficient acquisition. From the data, it is unclear how the change in number of trials per session affected the efficiency of learning.

The variation in the number of trials in intervention sessions, both within and across children, was also likely related to the complexity of the skill and the number of behaviors taught for each skill. Jason and She'quan had over 100 trials to criterion (or near criterion) for the skill of patterning, in marked contrast to the number of trials to criterion Jason had for both of his other skills and She'quan had for one of his other skills. These data suggest that patterning may have been a more difficult skill to learn. This could be because the skill was not developmentally "next" for the participants. For example, it's possible that the skill of duplicating patterns should have been taught before extending patterns. Duplicating patterns is a precursor to being able to extend patterns, and it is possible that Jason and She'quan did not have this prerequisite skill. This highlights the need for more sensitive measures of children's early math learning. Learning to pattern also may have been more difficult because multiple patterns were taught at the same time (AB and ABB). For some skills, the two behaviors taught were more similar and less complex (e.g., teaching Orion to count to 8 and 10). To address the issues with the difficulty of patterning and learning multiple behaviors for one skill at one time, a procedural modification was made. For the skill of patterning for She'quan, it was necessary to make a modification to the instructional procedure to focus on teaching one behavior, then the second behavior, and then both behaviors together. Learning both behaviors at once and being able to switch back and forth between the two during one session appeared to be too difficult for this participant, for this skill. Thus, when selecting the number and type of behaviors to target for instruction for a given skill, attention must be paid to the complexity of the skill and the task demands associated with learning multiple examples of the skill at the same time. It is possible that children learn a skill more deeply if they learn multiple variations (e.g., learning to complete AB and ABB patterns) and that this ameliorates concerns with how long it takes to acquire the skill. It might also be that for more complex skills, it is simply necessary to teach one behavior at a time, as was done for She'quan.

A final issue to consider when interpreting the results of this study relates to the use of demonstration trials in the instructional procedure. In the current study, the number of demonstration trials was held constant throughout the intervention condition rather than fading them as the children began to acquire the behaviors. In some instructional procedures, children are provided with a complete model of the correct behavior only in the first few sessions (e.g., constant time delay's zero second delay trials) or only after they fail to demonstrate the behavior (e.g., the system of least prompt's most intrusive prompt). In this instructional procedure, children were provided with some support (in the form of modeling) in every session, even as they began to demonstrate mastery of the behaviors. This may be more important when children are learning complex skills, such as the ones taught in this study (as compared to the type of skills typically taught with instructional procedures such as constant time delay). Further investigation of the use of demonstration trials throughout the intervention versus concentrated early in the instructional process relative to the type of outcomes being addressed is needed.

Contributions to the Literature

This study contributes to the literature on early math instruction for preschoolers in several ways. Evidence from this study indicates that children who are at risk and who have not been exposed to a tier one math curriculum can acquire, maintain, and, to some extent, generalize, early math skills. The target skills and how they were selected in the current study is different than other studies looking at instruction around math. Much of the previous research focused on only those math skills related to number sense, and the identification of target skills was often secondary to testing the instructional procedure. Previous research rarely included measures of maintenance and generalization of any early math skills. Further, much of the research on early math instruction has been limited by a number of methodological issues such as failure to collect reliability and procedural fidelity data and providing insufficient information about the intervention and dosage (Hardy, 2013a). Many of these methodological issues were addressed in the current study. Specifically, this study included: (a) a systematic instructional procedure, which was reported with replicable precision, (b) collection of reliability and procedural fidelity data, (c) collection of generalization and maintenance data, and (d) information on intervention dosage.

An additional contribution of this study is that the skills taught to children were based on the children's identified needs. The Math Screening Instrument, which was developed as part of this study, was used to provide a measure of children's skills across all domains of early math and included the assessment of: (a) counting, (b) comparing quantities, (c) comparing sizes, (d) sorting and classification, (e) seriation, (f) patterning, (g) ordinality/number line, (h) shape manipulation, (i) numeral identification, (j) addition and subtraction, and (k) measuring. Much of the research in early math instruction has focused primarily on number sense and has not been tailored to meet the needs of the children in the studies. This tool potentially provided a more sensitive approach for selecting skills to target for instruction.

Limitations of the Current Study

There were three main limitations to this study. The first is that the use of art activities possibly negatively impacted acquisition of target behaviors. Graphs of Jason's unprompted correct responses are presented in Figure 6. Sessions in which manipulatives were used are represented with a closed diamond, and sessions in which art activities were used are represented with an open diamond. Jason chose the art activity for six of his tier two sessions, and his tier two data were variable, with 23 sessions needed to reach criterion. He chose the art activity only once in both his tier one and three intervention sessions, and he acquired those behaviors more efficiently (six and five sessions to criterion, respectively). Jason also made more initial errors in sessions in which art activities were used, compared to sessions in which manipulatives were used. He had 40% prompted corrects and prompted errors in art activities, compared with 12.79% when manipulatives were used (see Table 12).

Figure 6. *Jason's Unprompted Correct Responses and Materials Used in Each Session*

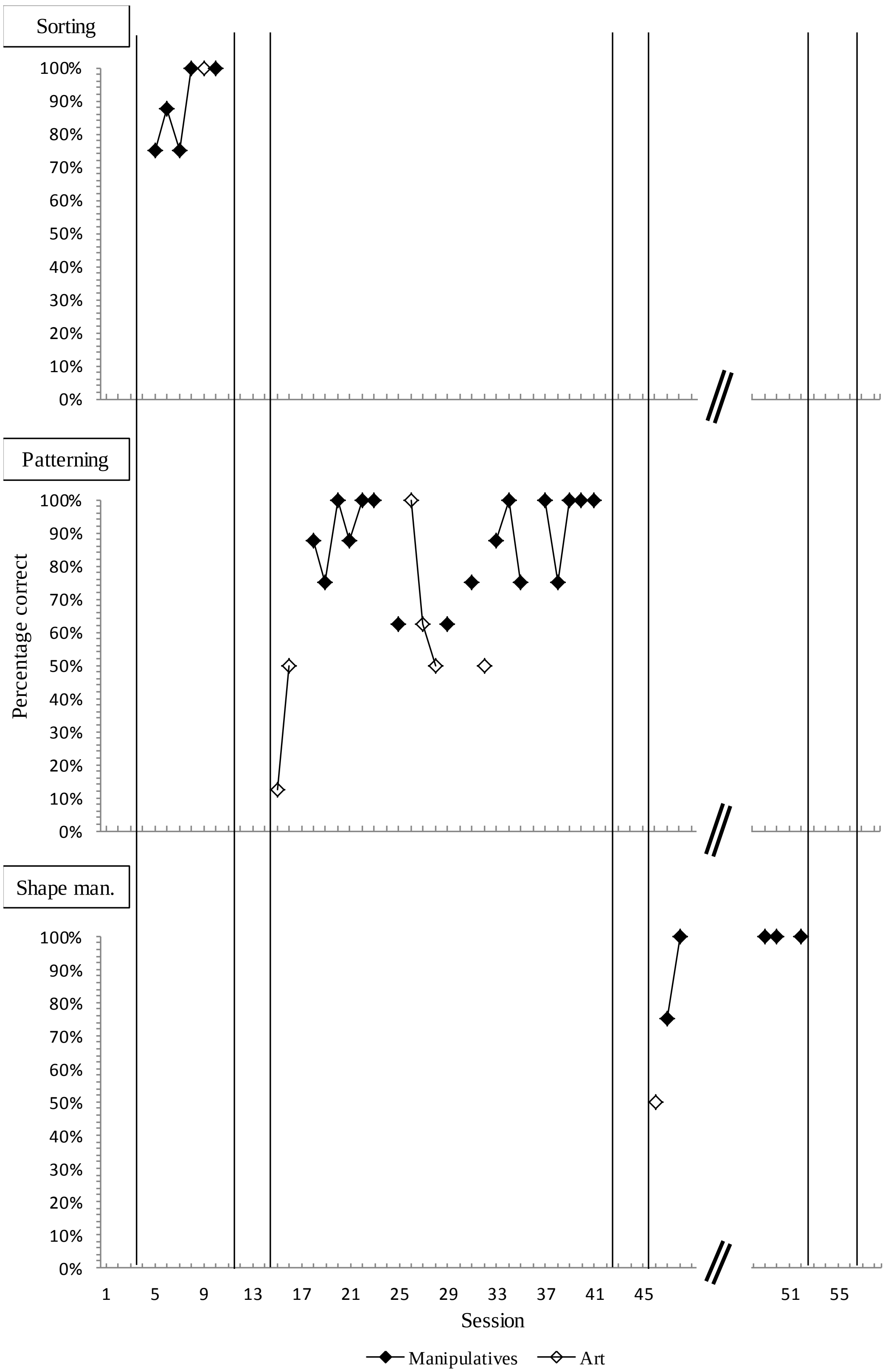
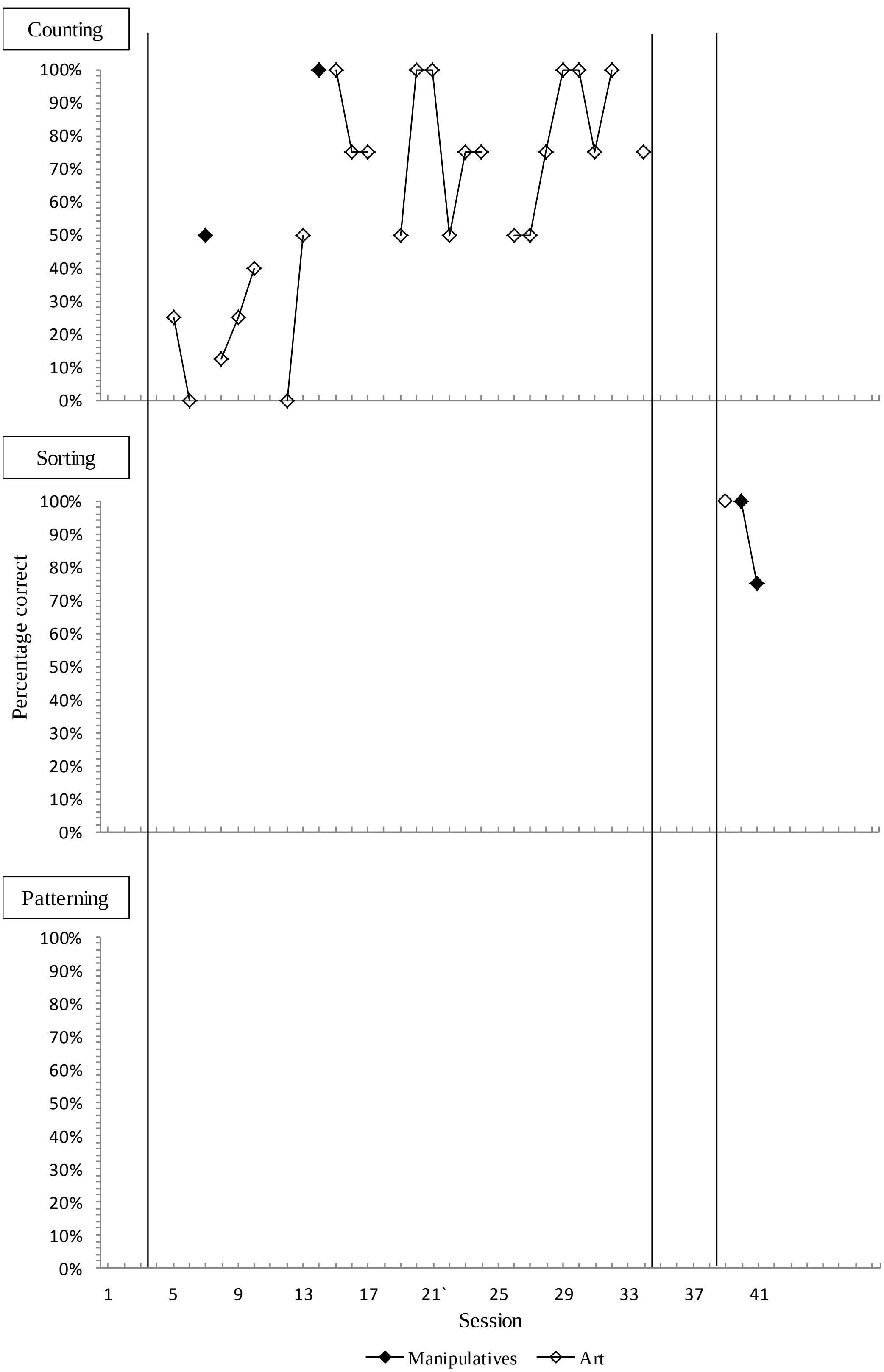


Table 12. *Comparison of Prompted Errors and Prompted Corrects in Sessions with Manipulatives versus Art Activities*

	Jason		Orion		She'quan	
	Man.	Art	Man.	Art	Man.	Art
Percentage prompted errors	0.00%	5.00%	20.00%	25.64%	4.10%	22.22%
Percentage prompted corrects	12.79%	35.00%	5.00%	17.95%	27.87%	36.11%
Percentage prompted errors and prompted corrects	12.79%	40.00%	25.00%	43.59%	31.97%	58.33%

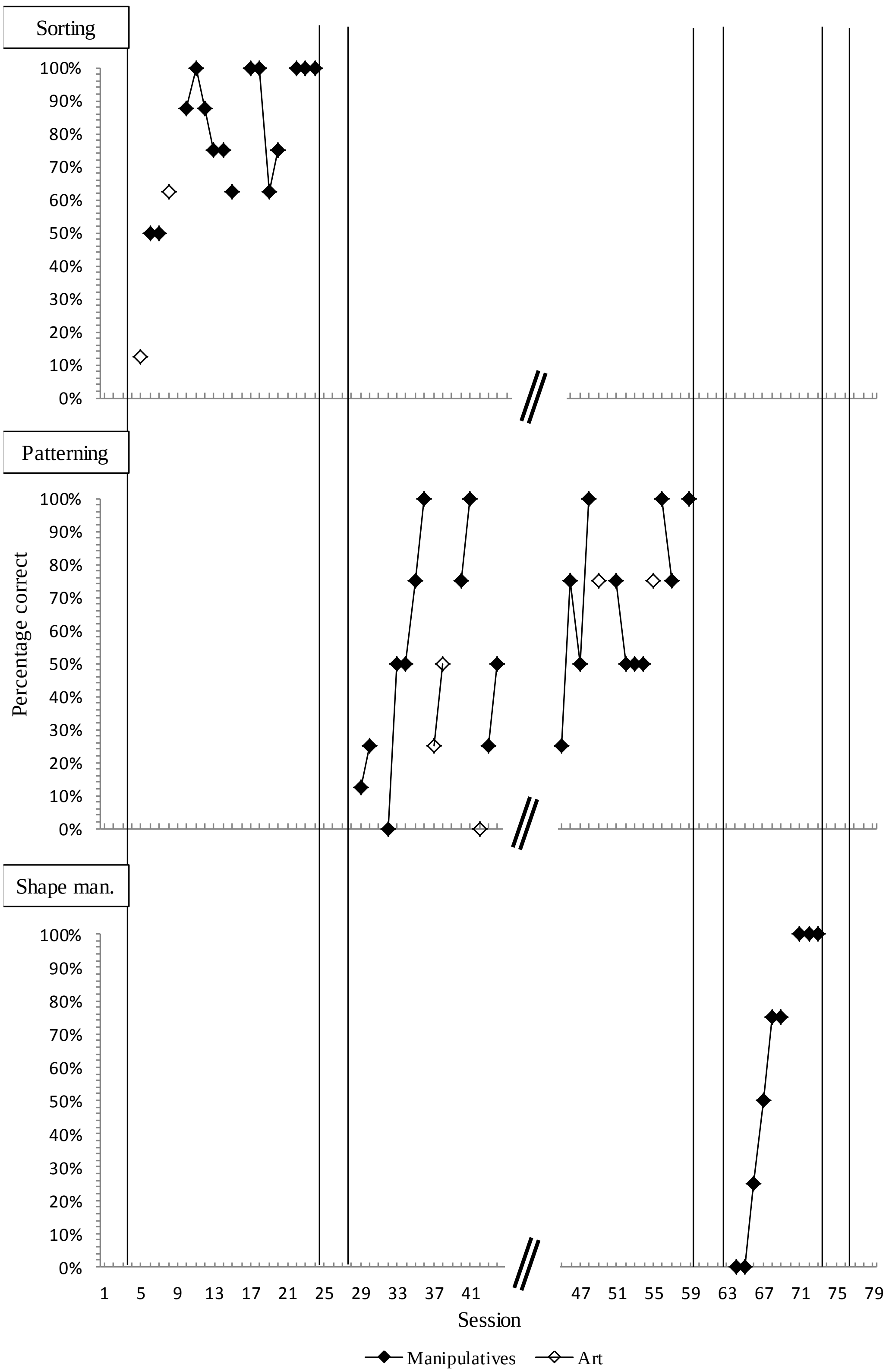
Graphs of Orion's unprompted correct responses are presented in Figure 7. Sessions in which manipulatives were used are represented with a closed diamond, and sessions in which art activities were used are represented with an open diamond. Orion chose the art activity for most of his tier one sessions, and his tier one data were variable, with 26 sessions needed to reach near-criterion. Orion had 43.59% prompted corrects and prompted errors in art activities, compared with 25.00% when manipulatives were used (see Table 12).

Figure 7. *Orion's Unprompted Correct Responses and Materials Used in Each Session*



Graphs of She'quan's unprompted correct responses are presented in Figure 8. Sessions in which manipulatives were used are represented with a closed diamond, and sessions in which art activities were used are represented with an open diamond. She'quan chose art activities in two of his tier one intervention sessions, and there was some variability to these data. He chose art activities in five of his tier two intervention sessions, and these data also were variable, with 27 sessions to near-criterion. She'quan did not choose art activities in any of his tier three intervention sessions, and there is little variability in these data. She'quan had 58.33% prompted corrects and prompted errors in art activities, compared with 31.97% when manipulatives were used (see Table 12).

Figure 8. *She'quan's Unprompted Correct Responses and Materials Used in Each Session*



Although it is clear that the participants had more variable responding when art activities were chosen and they tended to make more initial errors in art activities, it is difficult to determine why these activities had this effect on the children's learning. It is possible that the art activities led to more mistakes because the children were focused on the creative aspect of the activity, rather than attending to the task direction and math instruction. It is also possible that it was harder for children to correct mistakes made during the art activity (e.g., using the incorrect color marker when extending a pattern) or that the art component of the activity added additional cognitive demands to the activity, making it more difficult to complete (e.g., having to glue the shapes down after sorting them).

The second limitation relates to the error correction procedure used. The initial error correction procedure allowed children to repeat incorrect responses. For example, for the behavior of sorting by size, if the child performed it incorrectly, the adult modeled the behavior again and had the child repeat the behavior independently. Thus, the child had a second opportunity to perform the behavior incorrectly. As a result, the error correction procedure was revised. If the child sorted by size incorrectly, the adult physically and/or verbally prompted the child to perform the behavior correctly, rather than expecting the child to perform the behavior correctly independently. Essentially, only unprompted correct and prompted correct responses were possible (unless the child refused to perform a behavior). It is possible that children's acquisition of target behaviors would have been more efficient had the revised error correction procedure originally been used. This is because the revised procedure eliminated the possibility of the child making errors (without immediate help to correct the errors).

The third limitation relates to the generalization sessions. The goal was to use materials that were typically in the classroom to measure whether children could generalize the skills taught using researcher-provided materials to the existing classroom materials. However, that was not possible due to the lack of materials available in the classroom. Additionally, the generalization materials for the tier three skill (shape manipulation) for Jason and She'quan were problematic. Although it is possible that the children simply did not know the skill sufficiently well to generalize, another possible explanation relates to the stimuli used. One of the stimuli appeared to be too easy (i.e., the outline of the picture suggested very clearly which shapes should be used to fill the picture), leading to both participants demonstrating the behavior before intervention in that tier. The other stimulus appeared to be too difficult (i.e., the picture was very complex, and it was difficult to determine how to put the shapes together to form the picture), leading to both participants being unable to demonstrate the behavior in the generalization setting even after reaching criterion in that tier.

Implications for Research

One of the goals of this study was to better inform how to implement a response to intervention framework around early math skills. Specific implications for an RTI framework relate to: (a) the necessity for having a tier one curriculum in place, (b) the need for evidence-based tier two and three interventions, (c) the need for comprehensive and sensitive assessment tools, (d) the need for strategies for determining which skills to teach, and (e) the importance of professional development.

RTI models are based on the assumption that high quality tier one instruction is occurring in the classroom and that tier two and three interventions are for those children who do not respond to tier one interventions (Carta & Greenwood, 2013). Although the classroom used in the current study was NAEYC accredited, there was minimal evidence that tier one math instruction (based on the results of the Preschool Classroom Mathematics Inventory) was occurring. This is consistent with other research in this area that has found very little time in preschool classrooms is generally spent on math (e.g., Early et al., 2010; Ehrlich, 2007; Klibanoff et al., 2006; Rudd et al., 2008). Not surprisingly, each participant in the current study had significant deficits in early math skills. A recent review of the literature suggests that the presence of any math curriculum results in statistically significant differences in child outcomes when compared to "business as usual" (Hardy, 2013a).

Although there is research on effective early math curricula that could be conceived as tier one, little research has been done on interventions that could be used at tiers two and three. The instructional procedure in this study was examined as a possible tier two or tier three intervention. Although the instructional procedure was effective for two children, further research on the procedure is needed with respect to RTI. To more fully inform the implementation of tier two and three interventions, additional research is needed on key features of the instructional procedure, including investigating its use: (a) with differing numbers of trials, (b) with different populations of children, including children with disabilities, (c) with a wider variety of math skills, (d) in small groups, (e) with classroom teachers as the interventionists, and (f) when compared with other interventions. Additionally, the social validity of the procedure should be further measured with a larger number of teachers. Although only one teacher was involved in this study and thus rated the intervention for social validity, it would be useful to have a wider range of opinions about the social validity of the procedure.

To implement an RTI model, it is necessary to have measures that can be used to assess the full range of early math skills and that can be used as a progress monitoring tool. Formal assessment tools such as the TEAM (Clements et al., 2011), which was used in this study to provide an overall picture of each child's math functioning, are not sensitive enough to provide information about the specific math skills a child needs to learn and are not sensitive to changes in early math skills as a result of intervention. Existing progress monitoring tools, such as IGDIs-Early Numeracy (Hojnoski & Floyd, 2004), are limited to specific categories of math skills, such as number sense, and do not provide information on the range of math skills needed by young children. The Math Screening Instrument, used in this study, was designed to assess all domains of early math and to be sensitive to small changes. Psychometric studies are needed to further examine the possible use of this measure as a progress monitoring tool.

Additional research is also needed on how to identify which early math skills to teach. The Math Screening Instrument was used in this study to identify the skills the children did not know, but it does not provide information about how to identify the most appropriate skills to teach if a child has deficits across multiple domains of math. Learning trajectories (Clements & Sarama, 2009) is a framework for understanding how learning typically progresses within a particular type of math skill (e.g., counting, shape manipulation). However, there is no tool for determining in which order to teach math skills, if a child has unknown skills in multiple domains. For example, in this study, the order of instruction of skills was chosen based on researcher judgment (e.g., that sorting comes developmentally before patterning), but more scientific methods are needed. Further research is needed to determine if different domains of math skills need to be taught in a particular order, and, if so, what should be that order.

Findings from this study as well as previous work in this area that suggest very little math instruction occurs in preschool classrooms indicate a need for research on professional development. Professional development around early math skills must address three issues: (a) teachers' beliefs about math, (b) teachers' knowledge about early math skills, and (c) teachers' knowledge of and ability to use interventions for increasing children's early math skills. Teachers must believe math skills are important for young children in order to have "buy-in" for interventions in early math. There is evidence that teachers believe math is less important than other skill areas, such as social-emotional competence and literacy (Kowalski, Pretti-Frontczak, & Johnson, 2001; Lee & Ginsburg, 2007a). There is also evidence that preschool teachers believe math instruction should only occur if children are "ready" (Lee & Ginsburg, 2007b) and that teachers often underestimate children's capacity for math learning (Cross et al., 2009).

Teachers must also have an understanding of the range of early math skills relevant to young children. There is evidence to suggest that preschool teachers do not understand the full range of early math skills (Cross et al., 2009). Professional development around early math skills must include both content related to what skills to teach and pedagogical knowledge about how to teach. It is always necessary to investigate whether evidence-based instructional approaches can be translated into practice, and a necessary component of this is whether teachers and other classroom staff can implement the instructional procedure with fidelity and if this leads to positive child outcomes.

Implications for Practice

There are two implications for practice. The first is that the systematic instructional procedure, when implemented with fidelity, was effective in helping children acquire, generalize, and maintain early math skills. However, further evidence is needed to establish that the instructional procedure is efficient. Long periods of instruction were required for at least some skills for every child in this study. This could be because the skills require this amount of instruction, but it also could be because the instructional procedure is not effective for some math skills or for some children.

The instructional procedure is an example of an adaptive intervention. Adaptive interventions are those in which the intervention components and/or dosage can be modified to meet the needs of individuals (Collins, Murphy, & Bierman, 2004). Adaptive interventions have a number of advantages (Collins et al., 2004), including that they may be more efficient to use and more attractive to practitioners. Finally, the instructional procedure was viewed, by the classroom teacher in the current study, as appropriate and feasible for use in the classroom.

The second implication for practice relates to the Math Screening Instrument, which was developed by the researcher for the current study and was used to identify skills to target for instruction. This instrument was designed to be a measure of all domains of math skills relevant to young children. The instrument could be used in two ways by teachers. It could help them identify the type of skills that should be included in the tier one math curriculum, thus helping them plan tier one instruction. It also could be used by teachers to identify children's strengths and needs related to early math skills. It is the first such comprehensive tool of its kind. Although much work is needed on establishing the reliability and validity of the tool for progress monitoring, it has the potential to address a need in the field for comprehensive progress monitoring tools for early math skills for young children.

Conclusion

This study provides some evidence for the effectiveness of a systematic instructional procedure that can be used as part of an RTI framework for early math instruction. However, in order for an RTI framework to successfully be implemented by the field, much further research is needed. Additional research is needed on instructional procedures and their components, skills to target for instruction, tools for progress monitoring, and methods of professional development. Given the evidence regarding the power of measures of early math skills to predict later achievement and the disparities among young children in early math skills, this research is of paramount importance.

Appendix A

Math Screening Instrument

Child ID: __________________ Instructor: __________________

Session 1 date: __________________ Data collector: __________________

Session 2 date: __________________

Session 3 date: __________________

Directions: Present the materials and give the task direction. Mark C, E, or NR for each item.

Item	**Task direction**	**Trial 1**	**Trial 2**	**Trial 3**
Counting—5 objects	Count the X.			
Counting—10 objects	Count the X.			
Comparing quantities—large difference	Which has more?			
Comparing quantities—large difference	Which has less?			
Comparing quantities—small difference	Which has more?			
Comparing quantities—small difference	Which has less?			
Comparing sizes—bigger	Which is bigger?			
Comparing sizes—smaller	Which is smaller?			
Comparing sizes—longer	Which is longer?			
Comparing sizes—shorter	Which is shorter?			
Comparing sizes—heavier	Which is heavier?			
Comparing sizes—lighter	Which is lighter?			
Sorting—color, 3 groups of 3	Put all the X ones together.			
Sorting—color, 5 groups of 3	Put all the X ones together.			
Oddity—color, group of 3	Which one is different?			
Oddity—color, group of 6	Which one is different?			
Sorting—size, 3 groups of 3	Put all the X ones together.			

Sorting—size, 5 groups of 3	Put all the X ones together.			
Oddity—size, group of 3	Which one is different?			
Oddity—size, group of 6	Which one is different?			
Sorting—shape, 3 groups of 3	Put all the X ones together.			
Sorting—shape, 5 groups of 3	Put all the X ones together.			
Oddity—shape, group of 3	Which one is different?			
Oddity—shape, group of 6	Which one is different?			
Sorting—orientation, 3 groups of 3	Put all the X ones together.			
Sorting—orientation, 5 groups of 3	Put all the X ones together.			
Oddity—orientation, group of 3	Which one is different?			
Oddity—orientation, group of 6	Which one is different?			
Sorting—function, 3 groups of 3	Put all the X ones together.			
Sorting—function, 5 groups of 3	Put all the X ones together.			
Oddity—function, group of 3	Which one is different?			
Oddity—function, group of 6	Which one is different?			
Seriation—3 objects	Put them in order from smallest to largest.			
Seriation—3 objects	Put them in order from largest to smallest.			
Seriation—6 objects	Put them in order from smallest to largest.			
Seriation—6 objects	Put them in order from largest to smallest.			
Seriation—insertion, 3 objects	Put this one where it goes in order.			
Seriation—insertion, 6 objects	Put this one where it goes in order.			
Patterning—extending AB, color	Keep going with the pattern.			
Patterning—extending AB, shape	Keep going with the pattern.			
Patterning—extending ABC, color	Keep going with the pattern.			
Patterning—extending ABC, shape	Keep going with the pattern.			
Ordinality—1-5	Put the numbers in order.			
Ordinality—1-10	Put the numbers in order.			

Ordinality—insertion, 1-5	Put this number in the right spot.			
Ordinality—insertion, 1-10	Put this number in the right spot.			
Comparing numerical magnitudes—big difference	Which is bigger?			
Comparing numerical magnitudes—big difference	Which is smaller?			
Comparing numerical magnitudes—small difference	Which is bigger?			
Comparing numerical magnitudes—small difference	Which is smaller?			
Shape manipulation—making a square from 2 shapes	Put the shapes together to make a square.			
Shape manipulation—making a triangle from 2 shapes	Put the shapes together to make a triangle.			
Shape manipulation—picture with 3 shapes	Put the shapes together to make an X.			
Shape manipulation—picture with 6 shapes	Put the shapes together to make an X.			
Numeral identification—1	What number is this?			
Numeral identification—2	What number is this?			
Numeral identification—3	What number is this?			
Numeral identification—4	What number is this?			
Numeral identification—5	What number is this?			
Numeral identification—6	What number is this?			
Numeral identification—7	What number is this?			
Numeral identification—8	What number is this?			
Numeral identification—9	What number is this?			
Numeral identification—10	What number is this?			
Addition—adding 1	You have X and I give you 1. How many do you have?			
Addition—adding 2	You have X and I give you 2. How many do you have?			
Subtraction—taking away 1	You have X and I take away 1. How many do you have?			
Subtraction—taking away 2	You have X and I take away 2. How many do you have?			
Measuring length—small	How long is X?			
Measuring length—large	How long is X?			

Measuring weight—small	How much does X weigh?			
Measuring weight—large	How much does X weigh?			

Appendix B

Teacher Math Practices Questionnaire

Dear Teacher,

As part of your participation in this research study, you are being asked to share information about the practices you typically use to support children's math learning in your classroom. Your responses will be confidential, and will not be shared with anyone. Thank you for your participation!

1. What kind of math curriculum or approach, if any, do you use?

2. Please list everything you do that helps the children in your classroom learn math skills. What kinds of activities do you do, materials do you provide, and strategies do you use?

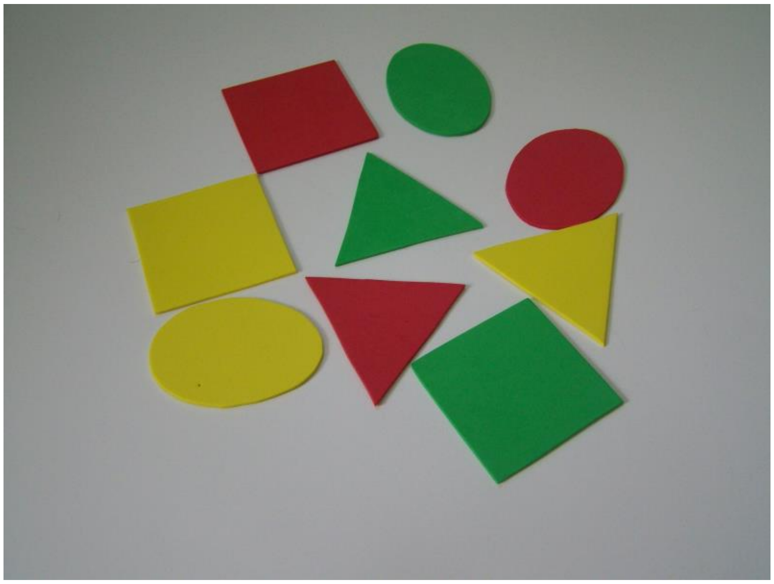
3. How do you think children learn math skills best?

Appendix C
Intervention Condition Materials

Counting, manipulatives activity:

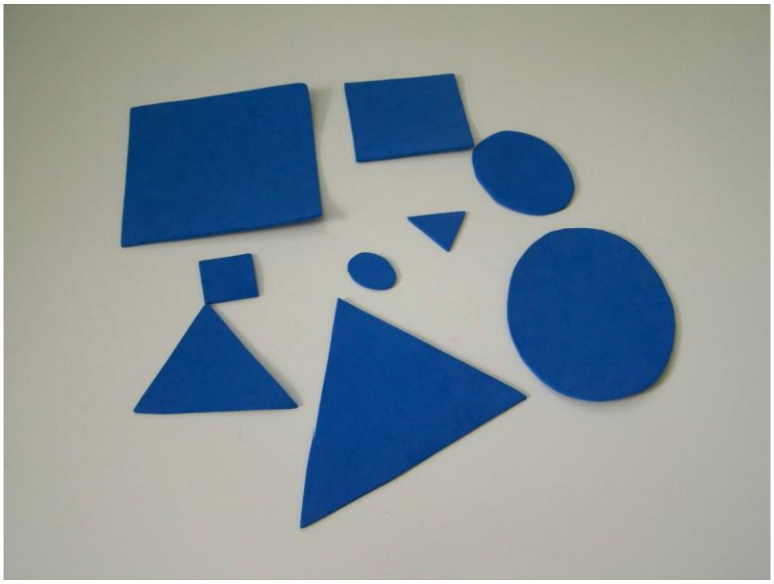
Counting, art activity:

Sorting by color and shape, manipulatives activity:

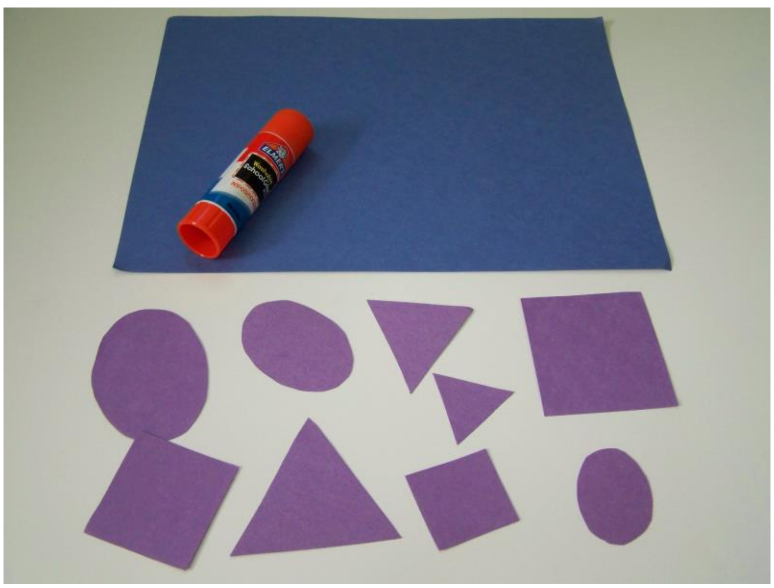


Sorting by color and shape, art activity:

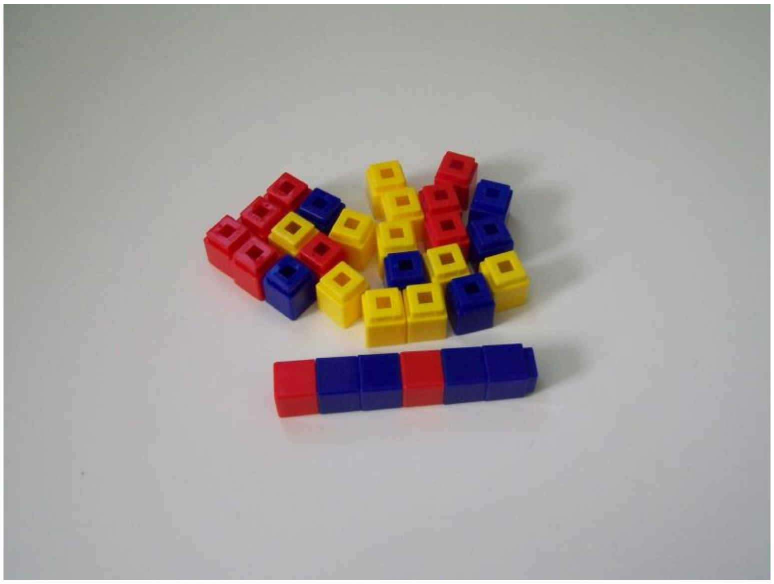
Sorting by shape and size, manipulatives activity:



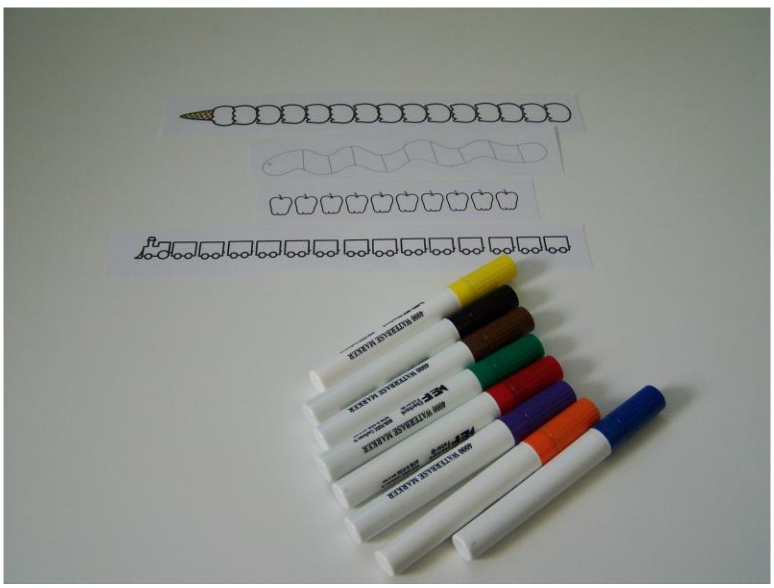
Sorting by shape and size, art activity:



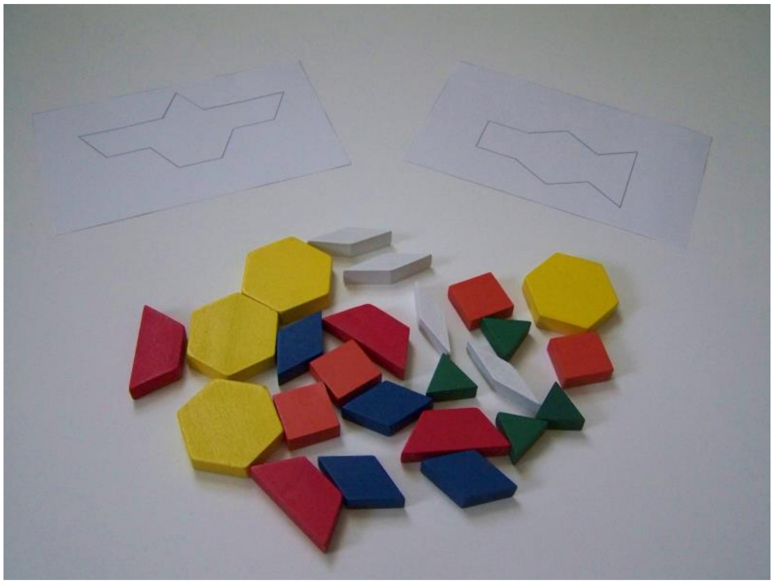
Patterning, manipulatives activity:



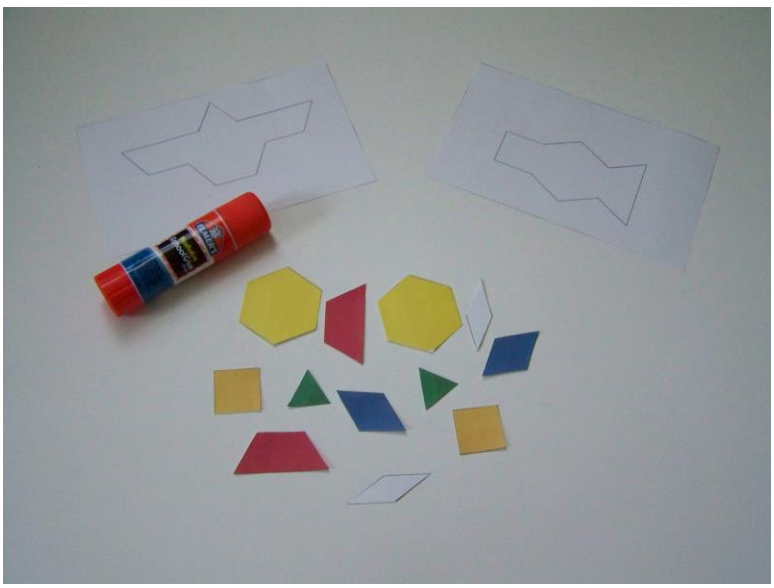
Patterning, art activity:



Shape manipulation, manipulatives activity:

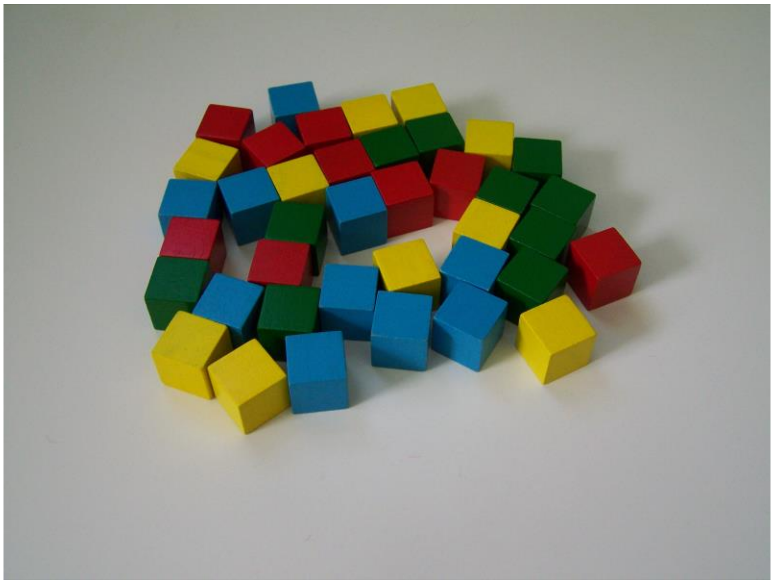


Shape manipulation, art activity:

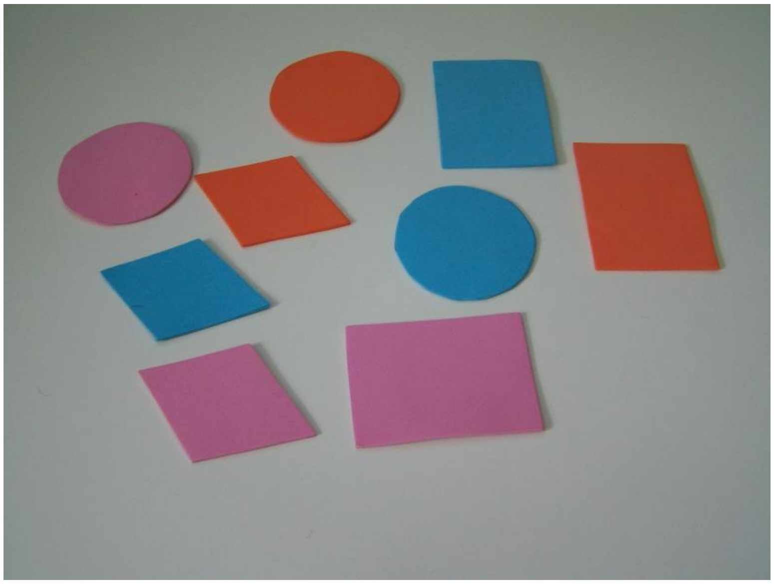


Appendix D
Probe Condition Materials

Counting and patterning:

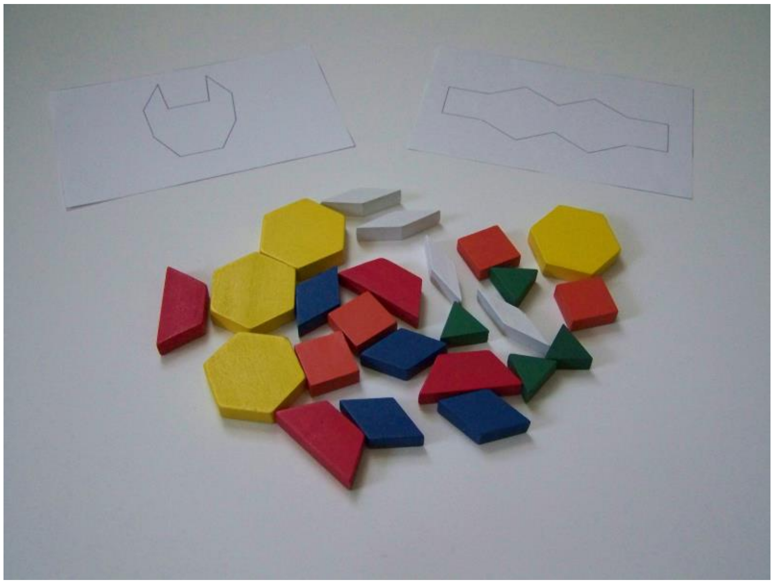


Sorting by color and shape:



Sorting by shape and size:

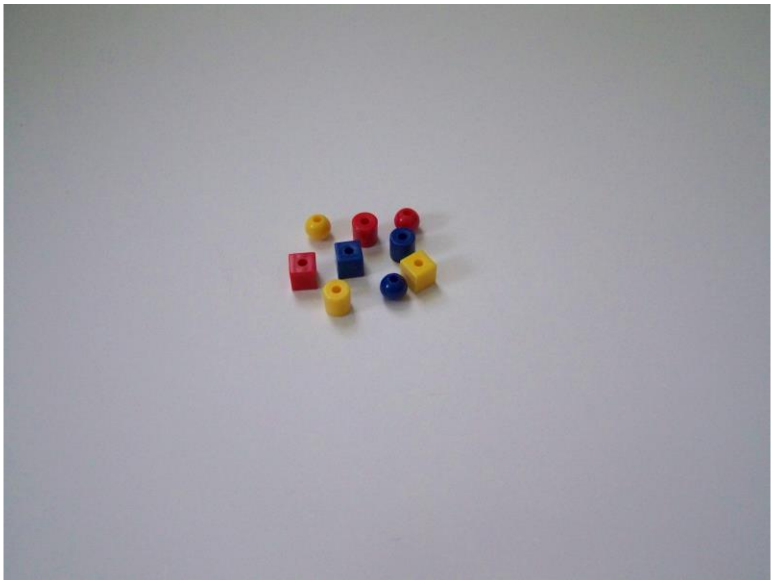
Shape manipulation:



Appendix E
Generalization Condition Materials

Counting:

Sorting by color and shape:



Sorting by shape and size:

Patterning:

Shape manipulation:

Appendix F
Probe Sessions Data Collection Form

Child ID: ____________________ Instructor: ____________________

Date: ____________________ Data collector: ____________________

Start time: ____________________ End time: ____________________

Reliability session: ☐ No ☐ Yes

Target:			
Beh. 1:			
Trial	C	E	NR
1			
2			
3			
Beh. 2:			
Trial	C	E	NR
1			
2			
3			
Tier one correct:	/6		

Target:			
Beh. 1:			
Trial	C	E	NR
1			
2			
3			
Beh. 2:			
Trial	C	E	NR
1			
2			
3			
Tier two correct:	/6		

Target:			
Beh. 1:			
Trial	C	E	NR
1			
2			
3			
Beh. 2:			
Trial	C	E	NR
1			
2			
3			
Tier three correct:	/6		

Appendix G

Instructional Sessions Data Collection Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Start time: ________________ End time: ________________

Reliability session: ☐ No ☐ Yes

Activity type:

☐ Work ☐ Art

Activity description: ____

Target:	

Beh. 1:			
Trial	UC	PC	PE
1			
2			
3			
4			

Beh. 2:			
Trial	UC	PC	PE
1			
2			
3			
4			

Total correct:	/8

Tally of imitated corrects: ____________________ Total imitated corrects: ______

Tally of imitated errors: ____________________ Total imitated errors: ______

Tally of spontaneous corrects: ____________________ Total spontaneous corrects: ______

Tally of spontaneous errors: ____________________ Total spontaneous errors: ______

Appendix H

Generalization Data Collection Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Start time: ________________ End time: ________________

Reliability session: ☐ No ☐ Yes

Materials description: ____

Target:			

Target:			

Target:			

Beh. 1:			
Trial	C	E	NR
1			
2			

Beh. 1:			
Trial	C	E	NR
1			
2			

Beh. 1:			
Trial	C	E	NR
1			
2			

Beh. 2:			
Trial	C	E	NR
1			
2			

Beh. 2:			
Trial	C	E	NR
1			
2			

Beh. 2:			
Trial	C	E	NR
1			
2			

Tier one correct:	/4

Tier two correct:	/4

Tier three correct:	/4

Appendix I

Maintenance Data Collection Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Start time: ________________ End time: ________________

Reliability session: ☐ No ☐ Yes

Target:	

Beh. 1:			
Trial	C	E	NR
1			
2			
3			

Beh. 2:			
Trial	C	E	NR
1			
2			
3			

Tier one correct:	/6

Target:	

Beh. 1:			
Trial	C	E	NR
1			
2			
3			

Beh. 2:			
Trial	C	E	NR
1			
2			
3			

Tier two correct:	/6

Target:	

Beh. 1:			
Trial	C	E	NR
1			
2			
3			

Beh. 2:			
Trial	C	E	NR
1			
2			
3			

Tier three correct:	/6

Appendix J

IOA Calculation Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector 1: ________________

Data collector 2: ________________

Type of session:

☐ Probe
☐ Instructional
☐ Generalization
☐ Maintenance

Formula:

$$\frac{Agreements}{Agreements + Disagreements} \times 100 = \%\ of\ Agreement$$

Point-by-point agreement:

Agreements = ______ Disagreements = ______ % of Agreement = ______

Appendix K

Probe Sessions Procedural Fidelity Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Instructions: Mark + if behavior occurred, 0 if it did not, and — if behavior was not applicable.

Worked with child one-on-one	
Conducted 6 trials for each target skill for a total of 18 trials	
Provided praise throughout session for behaviors unrelated to math skills	
Total present	/3

Trial	Skill/behavior	Child response			Instructor behavior				Mark 1 if trial correct:
		C	E	NR	Provided task direction	Waited 5 sec	Provided praise (C)	Ignored and moved on (E or NR)	
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
								Total trials correct:	/18

Total % correct:

Correct = ______ Total = ______ % correct = ______

Appendix L

Instructional Sessions Procedural Fidelity Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Instructions: Mark + if behavior occurred, 0 if it did not, and — if behavior was not applicable.

Worked with child one-on-one	
Offered child a choice of two activities	
Conducted 4 demonstration trials	
Conducted 8 practice trials	
Interspersed demonstration trials with practice trials	
Total present	/5

Demonstration trials:

Trial	Instructor behavior				
	Provided task description	Provided info on how to perform task	Modeled behavior	Provided positive, descriptive comment	Mark 1 if trial correct:
1					
2					
3					
4					
				Total trials correct:	/4

Practice trials:

Trial	Behavior	Child response			Instructor behavior							
		UC	PC	PE	Provided task direction	Waited 5 sec	Provided positive, descriptive comment (UC)	Corrected error	Did together with child	Provided positive, descriptive comment (PC)	Corrected error, moved on (PE)	Mark 1 if trial correct:
1												
2												
3												
4												
5												
6												
7												
8												
											Total trials correct:	/8

Spontaneous and imitative utterances:

Trial	Child behavior					Instructor behavior			
	SC	SE	SOS	IC	IE	Provided positive, descriptive comment (SC or IC)	Corrected error, modeled, had repeat (SE or IE)	Ignored (SOS)	Mark 1 if trial correct:
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
								Total trials correct:	/

Total % correct:

Correct = ______ Total = ______ % correct = ______

Appendix M

Generalization Sessions Procedural Fidelity Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Instructions: Mark + if behavior occurred, 0 if it did not, and — if behavior was not applicable.

Worked with child one-on-one	
Conducted 4 trials for each target skill for a total of 12 trials	
Provided praise throughout session for behaviors unrelated to math skills	
Total present	/3

Trial	Skill/behavior	Child response			Instructor behavior				Mark 1 if trial correct:
		C	E	NR	Provided task direction	Waited 5 sec	Provided positive, descriptive comment (C)	Ignored and moved on (E or NR)	
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
								Total trials correct:	/12

Total % correct:

Correct = ______ Total = ______ % correct = ______

Appendix N

Maintenance Sessions Procedural Fidelity Form

Child ID: ________________ Instructor: ________________

Date: ________________ Data collector: ________________

Instructions: Mark + if behavior occurred, 0 if it did not, and — if behavior was not applicable.

Worked with child one-on-one	
Conducted 6 trials for each target skill for a total of 18 trials	
Provided praise throughout session for behaviors unrelated to math skills	
Total present	/3

Trial	Skill/behavior	Child response			Instructor behavior				Mark 1 if trial correct:
		C	E	NR	Provided task direction	Waited 5 sec	Provided positive, descriptive comment (C)	Ignored and moved on (E or NR)	
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
								Total trials correct:	/18

Total % correct:

Correct = ______ Total = ______ % correct = ______

Appendix O

Social Validity Pre-Questionnaire

Name: ____________________________________ Date: ____________

Directions: Please mark an X in the column that best reflects your answer to each question.

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
1. Math skills are important for preschool children to know.					
2. Preschool children need instruction to learn math skills.					

Child ID: _________________	Often	Some-times	Not sure	Rarely	Never
3. Have you seen the child engaging in the following skill in your classroom: sorting objects by shape and size					
4. Have you seen the child engaging in the following skill in your classroom: making patterns					
5. Have you seen the child engaging in the following skill in your classroom: putting shapes together to make pictures					

Child ID: _________________	Often	Some-times	Not sure	Rarely	Never
6. Have you seen the child engaging in the following skill in your classroom: counting objects					
7. Have you seen the child engaging in the following skill in your classroom: sorting objects by color and shape					
8. Have you seen the child engaging in the following skill in your classroom: making patterns					

Child ID: ___________________	Often	Some-times	Not sure	Rarely	Never
9. Have you seen the child engaging in the following skill in your classroom: sorting objects by color and shape					
10. Have you seen the child engaging in the following skill in your classroom: making patterns					
11. Have you seen the child engaging in the following skill in your classroom: putting shapes together to make pictures					

Appendix P

Social Validity Post-Questionnaire

Name: ____________________________________ Date: ____________

Directions: Please mark an X in the column that best reflects your answer to each question.

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
12. Math skills are important for preschool children to know.					
13. Preschool children need instruction to learn math skills.					

Child ID: _________________	Often	Some-times	Not sure	Rarely	Never
14. Have you seen the child engaging in the following skill in your classroom: sorting objects by shape and size					
15. Have you seen the child engaging in the following skill in your classroom: making patterns					
16. Have you seen the child engaging in the following skill in your classroom: putting shapes together to make pictures					

Child ID: _________________	Often	Some-times	Not sure	Rarely	Never
17. Have you seen the child engaging in the following skill in your classroom: sorting objects by shape and size					
18. Have you seen the child engaging in the following skill in your classroom: making patterns					
19. Have you seen the child engaging in the following skill in your classroom: putting shapes together to make pictures					

Directions: Please watch the video I gave you. Then answer the following questions.

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
20. The instructional strategy used in this study seems appropriate to use with preschool children.					
21. I could use this instructional strategy in my classroom with individual children.					
22. I could use this instructional strategy in my classroom with small groups of children.					

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