# FAULT-TOLERANT ACTIVE VIBRATION CONTROL

By

Chakradhar Reddy Byreddy

Dissertation

Submitted to the Faculty of the

Graduate School of Vanderbilt University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in

Mechanical Engineering

August, 2006

Nashville, Tennessee

Approved:

Kenneth D. Frampton

Nilanjan Sarkar

Eric J. Barth

Michael Goldfarb

Gautam Biswas

To my beloved wife, Swapna, infinitely supportive

and

To my family, Sarath and Pushpa, truly caring

# ACKNOWLEGEMENTS

There are many people who have helped and motivated me during my Doctor of Philosophy program. My professors, family and friends have all helped in one way or another. I'm very much obliged to all of them for their constant support and encouragement.

In particular, my very special thanks goes to my advisor, Dr. Kenneth D. Frampton for his discerning guidance and also, for taking the time to understand me as an individual and motivating all the time. I would like to thank Dr. Nilanjan Sarkar, Dr. Eric Barth, Dr. Michael Goldfarb and Dr. Gautam Biswas for their time in reviewing the dissertation and also, for being part of my dissertation committee. I'm grateful to all those with whom I have had the pleasure to work during other related projects.

Nothing has been more important to me in the pursuit of this degree than the members of my family. I would like to thank my parents, brother and sister-in-law, whose love and guidance are with me in whatever I pursue. Most importantly, I wish to thank my loving and supportive wife, Swapna, for her endless support during my course of study and providing unending inspiration.

TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

CHAPTER I

INTRODUCTION AND SUMMARY


Introduction

Fault Detection and Isolation (FDI) has received considerable attention and widespread growth in recent years for various dynamic systems [1, 2, 3, 4]. The FDI of different failures play a vital role in maintaining the stability and reliability of the plant. Different applications of complex dynamic systems have emphasized the need for accurate and timely diagnosis of sensors and actuators that are part of these subsystems. Also, these applications demand stringent requirements on safety and reliability. Many researchers pursued the investigation in designing various FDI algorithms and techniques for real world applications [5, 6, 7].

The FDI problem is mainly concerned with the failure state of a dynamic system based on the residuals from comparative relationship amongst the system variables. One of the simplest ways to solve fault detection is by hardware redundancy. The major drawback of hardware redundancy is the extra cost associated with the addition of different elements. Other redundancies that exist in a dynamic system are in the form of analytical redundancies. Analytical redundancy [8] mainly refers to the dynamic relationship between dissimilar outputs. These dynamic relationships provide additional redundancy equations that produce residuals useful to FDI scheme. Finally, a third type of residual can be formed from self-test type redundancy relationships. In this

redundancy the dynamic relationship is strictly written in terms of a single instrument output.

There are numerous approaches to the design of FDI systems. They are classified as: a) Fault detection filter [9, 10, 11]; b) Innovation-based diagnosis systems [12]; c) Multiple model adaptive filters [1]; and, d) Analytical redundancy systems [8]. Design of detection filters and its application for vibration control is the main area of focus in this research work. Beard [9] developed the detection filter in 1971. This work was later expanded by Jones [10] in 1973, and then reformulated by several researchers.

Detection filters are designed so that output error residuals are produced with directional characteristics that can be associated with a set of known design fault directions. In this research work the main emphasis is on the Beard-Jones (BJ) filter. The BJ filter constitutes a technique for generating closed-loop residuals that have certain directional characteristics. The concepts used by Beard are based on an observer that is constructed so that the presence of faults in the system causes the direction of the residual vector in the output space to be in fixed in time (unidirectional). The concept of fault detectabilty is achieved if the output error residual is one-dimensional when any fault event occurs, and the closed-loop eigenvalues are arbitrarily assignable. In order to satisfy the above conditions, detection filter theory is based on the reference model approach used in observer theory and state estimation theory. Thus, a Beard-Jones detection filter (BJDF) is developed.

Beard detection filters mainly focus on the determination of a cyclic basis representation for the closed-loop system in which the output error has certain directional properties. Beard adopted the matrix algebra techniques for the calculation of the

detection gain matrix. Jones extended Beard's work by using linear operators and vector space techniques. Later, Massoumnia [13] used a geometric approach for the design of detection gain matrix. Both approaches and methodologies for designing the detection gain matrix are rather indirect and overly complicated, and are unfamiliar to the most engineers. In the mid 1980's White and Speyer [6, 14, 15] proposed a spectral technique for the design of detection gain matrix by assigning the eigenvalues and eigenvectors directly. Therefore, the detection filter needs to be constructed inside and outside the detection space simultaneously, which is a limitation of this method. In order to avoid this problem, Kim and Park [16, 17, 18] proposed a methodology based on the invariant zero approach.

An important component of any practical active control system is the transducers used for the implementation of control. Actuators are used to apply control signals to the system in order to change the system response. Sensors are required for measurement, which in turn can be used to estimate important system variables. These sensors provide necessary information to the controller to determine the performance of the system. These sensors and actuators provide the link between the controller and physical system to be controlled. In the presence of actuator or sensor failures the control performance of the system is widely affected.

The main motivation behind this work is to the use the idea of FDI for vibration control. In order to maintain optimal performance for active vibration control in the presence of failures, the concept of fault-tolerant control is achieved. Also, for maintaining the optimal performance due to failures, controller reconfiguration needs to be implemented. Since controller reconfiguration is a discrete phenomenon and, vibration

control is a continuous phenomenon, the interaction between the two is achieved by hybrid automata. The main focus of this work is to deal with the failure of actuators in active vibration control. In this research work a new technique is proposed for the design of the fault detection for high order systems. In addition to this a new methodology is developed for systems with feed-through dynamics.

Chapter II mainly deals with the study of parity space techniques and BJ filter. The various advantages and limitations of both FDI methods are studied in detail for fault-tolerant active vibration control. This chapter establishes the criteria for the desired FDI filter to be used for experimental studies. Chapter III focuses on the BJ filter for high order systems. A new design procedure that uses the concept of invariant zero method is developed for systems such as beams, plates and shells (high order systems). Also, a new methodology is developed for systems (from system identification) with feed-through dynamics for the design of BJ filter.

Chapter IV deals with the experimental verification of the proposed BJ filter design technique in discrete-time. Chapter V focuses on the experimental results of the fault-tolerant active vibration control for actuator failures. This experimental results shows that there is stability for the system under switching of various controllers for different fault case scenarios. Finally, chapter VI studies parametric failures of a system. In this chapter, parameter estimation method and BJ filter are used for these kinds of failures. Thus, simulations of fault-tolerant control are established with the presence of parametric failures.

Summary of Manuscript 1

Active vibration control plays a crucial role in improving the performance objective of physical systems. The main objective of active vibration control is to reduce the vibration of a physical system by automatic modification of the system's structural response. An active vibration control system can be modeled in many ways, but the most important components of such a system are the sensors (to detect the vibration), the electronic controller (to manipulate the signal from detector), and the actuators (component that influences the mechanical response of the system). Thus, failures in any of these components play a vital role in the performance of active vibration control. These component failures are referred as additive failures. The main objective in this section of proposed work is to study the fault-tolerant control with additive failures. The physical system studied in this work is a simply supported beam as shown in Figure 2-1.

The system considered in this work is a three input-three output system. Initially, only three structural modes are considered. This is because current parity relation and BJ filter design techniques only apply to lower order systems. Two different types of FDI filters are studied for the application of vibration control. The various advantages and drawbacks of these filters are studied in detail. The two types of FDI filters are: parity relations and BJ filter.

A constant gain output feedback compensator is designed for the vibration control for different fault case scenarios. Compensator design for the three fault case scenarios and one nominal case (no failure) have been designed is shown in Figure 2-2. The three fault case scenarios are: first actuator failure; second actuator failure; and, first and second (both) actuator failures. This research work implements the idea of multiple

switching of faults back and forth. The fault-tolerant control is the process of detecting failures (in this case actuator failures) and appropriately switching compensators to obtain optimal vibration control performance.

The process of fault-tolerant control is shown schematically in Figure 2-3 for parity space technique. In this diagram, there are multiple controllers and observers corresponding to different failures. Figure 2-5 explains the concept of threshold for switching for parity space techniques. Figure 2-6 shows controller reconfiguration for multiple fault case scenarios. Figure 2-7 shows the schematic representation for fault-tolerant control using BJ filter. In this diagram, there are multiple controllers and single BJ filter (mutual detectability) for different failure conditions. Figure 2-8 explains the concept of threshold for switching for BJ filter. Figure 2-9 shows controller reconfiguration for multiple fault case scenarios for BJ filter. Figures 2-10 and 2-11 shows the advantages and limitations of both the FDI techniques to be used for experimental studies. Finally, in this manuscript it is concluded that the BJ filter is more appropriate than the parity space technique for active vibration control.

<u>Summary of Manuscript 2</u>

In this manuscript, the BJ filter is designed for different cases of physical system. The cases depend on the system order, which in turn depends on the number of modes considered for the physical system. A new methodology for BJ filter design with feed through dynamics is also developed. Two types of plant models are used in this investigation: theoretical beam models and a model based on system identification of an

experimental setup. The fault case scenarios considered are the same as in the previous manuscript. And, the compensator design is based on constant gain output feedback.

Based on this experimentally obtained data, a state variable model was created based on ARX technique. One unique aspect of this model, and one common to many experimentally derived models, is that the feed-through matrix, $D$ in Equation (3-3), is non-zero. This necessitates the development of a new technique for designing FDI filters. Many systems that are encountered have feed-through dynamics (i.e. $D \neq 0$). However, there is as yet no way of dealing with this situation when designing BJ filters. This is because potential actuator failures have a direct effect on the output of the system (which in turn can be interpreted to be sensor failures). In order to avoid the confusion and isolate the actuator failure, a new BJ filter design method is presented for this particular case in this manuscript.

The open loop system model in the absence of failures is given in Equation (3-3). The traditional BJ detection filter in this case is assumed to be of the form shown in the Equation (3-7). Consider a system similar to Equation (3-3), but with a non-zero $D$ matrix. In addition to this consider a failure in actuator one. This particular case can be described in Equation (3-14).

In order to develop a BJ filter it is assumed that $\delta u_1$ behaves according to a first order dynamics shown in Equation (3-15). The value of $\alpha$ plays a pivotal role in the convergence of the continuous residual in fault-tolerant control. This is because, if the response of first order dynamics is faster than the actual system, then the rate of convergence of the continuous residual is fast. If the BJ theory fault is considered to be $\mu = \delta u_1$, then Equations (3-14) and (3-15) can be combined in to a new state-space form

shown in Equation (3-16).

Now, the usual BJ detection design outlined previously can be used on the appended system of Equation (3-16). It is possible that the appended system of Equation (3-16) might not be controllable even though the original system is controllable. The method described above deals with occurrence of multiple faults simultaneously at the same time. Also, if the system is not mutually detectable, it is possible to implement occurrence of faults in parallel.

For low order system, the detection gain matrix $L$ shown in Equation (3-7) is obtained by using the invariant zeros approach. In some situations it is not possible to design a stable closed-loop detection filter gain for higher order systems using invariant zeros approach. Thus, it is essential to present a new technique for higher order systems by using the idea behind the LQR design with eigenstructure assignment capability to minimize the quadratic cost function shown in Equation (3-20). This optimal gain matrix is then added to the original detection gain matrix (calculated using invariant zero approach) to obtain a closed-loop stable detection filter $L'$ shown in Equation (3-18).

In order to establish the capabilities of a fault-tolerant control system under various conditions, three different cases are considered and simulated. These cases are: a high order plant with full order BJ filter; a high order plant with truncated BJ filter; and, an experimentally obtained high order plant with a high order BJ filter. The necessary conditions and results for these cases are explained in detail using Figure (3-5), Figure (3-6), Figure (3-7) and Figure (3-8). It is demonstrated that the new BJ filter design technique can be used for the high order systems to achieve fault-tolerant control.

Summary of Manuscript 3

In this manuscript, BJ filter is designed for experimental model shown in Figure (4-1). The system identification of the model is established based on the schematic shown in Figure (4-2). Six sensors and six actuators are used for the system identification. The data is collected from the sensor and actuator uses Equation (4-1) for system identification and, from this Equation (4-2) is derived. The order of the system model in the experimental is chosen to be 36, and frequency responses of the all signal paths match the experimental results. Figure (4-3) and Figure (4-4) show the experimental and analytical frequency responses from the two signals. It is clearly established that the state space model derived from the experiment represents the beams dynamics.

Conversion of continuous BJ filter for high order systems (from manuscript 2) to discrete-time creates unstable poles. Therefore, it is necessary to design BJ filter in discrete-time. For low order system, the detection gain matrix $L$ shown in Equation (4-3) is obtained by using the invariant zeros approach. In some situations it is not possible to design a stable closed-loop detection filter gain for higher order systems using invariant zeros approach. Thus, it is essential to present a new technique for higher order systems by using the idea behind the LQR design with eigenstructure assignment capability to minimize the quadratic cost function shown in Equation (4-13)). This optimal gain matrix is then added to the original detection gain matrix (calculated using invariant zero approach) to obtain a closed-loop stable detection filter $L'$ shown in Equation (4-11).

Figure (4-5) shows the simulations of the experimental model in discrete-time with and without noise. The disturbance in the subplot in Figure (4-6) is used to create

9

vibrations in the experimental setup. Figure (4-6) shows the residuals of actuator one and actuator two failures at different times. The limitations of BJ filter for real-time analysis are explained in manuscript 3.

<div align="center">Summary of Manuscript 4</div>

In this manuscript, fault-tolerant control is conducted for vibration control of a simply supported beam shown in Figure (5-2). The system identification of the model is established based on the schematic shown in Figure (5-1).  Six sensors and six actuators are used for the system identification. The data is collected from the sensor and actuator to use Equation (5-1) for system identification. The order of the system model in the experimental is chosen to be 36, and frequency responses of the all signal paths match the experimental results. Figure (5-3) and Figure (5-4) show the experimental and analytical frequency responses from the two signals. It is clearly established that the state space model derived from the experiment represents the beams dynamics.

The schematic for fault-tolerant active vibration control is shown in Figure (5-5). The controller design for various fault case scenarios is shown in Figure (5-6). Figure (5-7) shows the time history of residual, fault states and sensor signal for actuator two failure. Figure (5-8) shows the continuous residuals for both actuators one and two failures. Figure (5-11) shows the control performance for different fault case scenarios. Finally, it is fault-tolerant active vibration control is conducted in real-time using the BJ filter. Also, from the experimental results it can be concluded that during switching of controllers, the physical system remains stable.

Summary of Manuscript 5

In current technologies, given the high reliability required of systems, the ability to detect a system fault at the earliest possible stage is of primary interest. Therefore, damage detection (parametric failures) is an important asset. The design of a diagnostic tool for this purpose requires that many issues be addressed. Most of previous literature on vibration based damage detection techniques lack in use for the concept of fault-tolerant control.

Even when a proposed FDI scheme is technically good for general failures, ease of use considerations remains a core issue in engineering practice. The coupling of the physical system makes the implementation of FDI and its results more complicated. As a consequence, fewer investigations have been done for parameter failures as compared with additive failures. Therefore, there is a great incentive for developing better methods to accomplish the concept of FDI for multiplicative faults. Furthermore this work can be extended to designing the appropriate controller for multivariable control with the presence of failures. The majority of work conducted on FDI over the past years is on the faults associated with sensors and actuators.

In this manuscript, parameter estimation is achieved for continuous multi-input multi output (MIMO) models with multiplicative faults. The advantage of the parameter estimation over other FDI methods is the ease of use for fault-tolerant control. The theory and development of Beard-Jones (BJ) filter for fault detection of parametric failures is studied in detail. BJ filter design is mainly implemented for sensor and actuator failures. Therefore, it is very important to study the pros and cons of both methods studied in this work for fault-tolerant control. For fault-tolerant control there is a transition from

continuous time to discrete event systems during the switching of the controllers. Therefore, it is also essential to use the idea of hybrid automata for this work. Thus, one of the critical issues is to integrate the FDI scheme, hybrid automata and active vibration control to achieve the objective of fault-tolerant control.

In this study the parametric failure considered to be the addition of mass to the beam. Other parametric failure that can be addressed is the change in the stiffness of the beam $K_P$ shown in Equation (6-2). This study features two types of fault case scenarios. The first one is where there is no change in the mass of the beam and, second one is the case where a significant amount of mass is added to the beam. It is very important to know the location of the additional mass, as it affects the mode shape of the beam. In this study the additional mass is located at half the length of the beam. The addition of mass of the beam can be expressed as shown in Equation (6-14). Similarly, mathematical representation of stiffness parameters can be implemented. In case of parameter estimation, the change in one of the parameter value is considered as the key to change in the physical system with the addition of mass.

The primary goal of this work is to implement two different FDI schemes for parametric failures. Simulations for the multiplicative failures have been conducted on a simply supported beam, which in turn demonstrate the ability of such a system to maintain performance and stability are shown in Figures (6-4) and (6-6). The primary contribution of this work is to study the limitations of both methods for multiplicative failures. Also, this work will help to develop robust FDI methods with the presence of both the additive and multiplicative failures.

<u>References</u>

[1] R. Patton and S. Wilcox, "Comparison of Two Techniques of IFD Based on a Nonlinear Stochastic Model of an Aircraft," Proc. 7th IFAC Symposium on Identification and System Parameter Estimation, York UK, 1986.

[2] A. Emami-Naeini et al., "Robust Detection and Isolation, and Accommodation for Sensor Failures," NASA Contract Report No. CR-174825, July 1986.

[3] J.C. Deckert, M.N. Desai, J.J. Deyst and A.S. Willsky, "F-8 DFBW Sensor Failure Identification using Analytic Redundancy," IEEE Transaction Automatic Control. Vol. AC-22, 975-803, 1977.

[4] P. Olin, J. Park and G. Rizzoni, "Failure Detection and Isolation for an Active Suspension System," VEL Report No. 390055-90-12, Vehicular Electronics Laboratory, The University of Michigan, Ann Harbor, December 1990.

[5] J. Gertler, "Fault detection and diagnosis in engineering systems," 1st Edition, Marcel Dekker, New York, 1998.

[6] R.K. Douglas and J.L. Speyer, "Robust fault detection filter," AIAA Journal of Guidance, Control and Dynamics, Vol. 19, 1996.

[7] J. Chen, J. Patton Ron and Zhang Hong-Yue, "Design of Unknown Input Observers and Robust Fault Detection Filters," International Journal of Control, Vol. 63, 1996.

[8] A.S.Wilisky and H.L. Jones, "A Generalized Likelihood Ratio Approach to the Detection and Estimation of Jumps in Linear Systems," IEEE transaction on Automatic Control, Vol. AC-21, 108-112, May 1976.

[9] R.V. Beard, "Failure Accommodation in Linear System through Self-Reconfiguration," Ph.D. Thesis, Massachusetts Institute of Technology, MA, 1971.

[10] H. Jones, "Failure Detection in Linear Systems," Report No. T-608, The Charles Stark Draper Laboratory, Cambridge, MA, 1973.

[11] R. Scattolini and N. Cattane, "Detection of sensor faults in a large flexible structure," IEEE Proceedings-Control Theory Applications, 145, 5, 1999.

[12] A.S.Wilisky et al., "Dynamic Model-Based Techniques for the Detection of Incident Freeways," IEEE transaction on Automatic Control, Vol. AC-25, 347-360, 1980.

[13] M. Massoumnia, "A Geometric Approach to the Synthesis of Failure Detection Filters," IEEE Transactions Automatic Control, 31, 839-846, 1986.

[14] J.E. White, "Detection Filter Design by Eigensystem Assignment," Ph.D. Dissertation, University of Texas, Austin, 1985.

[15] J.E. White and J. Speyer, "Detection Filter Design: Spectral Theory and Algorithms," IEEE Trans., AC-32, 593-603, 1987.

[16] Y. Kim and J. Park, "A Condition of the Eigenvalues of Detection Filters for Disturbance Attenuation: An Invariant Zero Approach," Conference on Decision and Control (CDC), Hawaii, USA, 2003.

[17] Y. Kim and J. Park, "On the Noise Response of Detection Filters: A Special Consideration on the Relation Between Detection Space and Completion Space," IEEE Proceedings – Control Theory and Applications, 150, 4, 443-447, 2003.

[18] Y. Kim and J. Park, "On the Approximation of Fault Directions for Mutual Detectability: An Invariant Zero Approach," IEEE Transactions on Automatic Control, 50, 6, 851-855, 2005.

CHAPTER II

MANUSCRIPT 1

FAULT-TOLERANT VIBRATION CONTROL FOR ADDITIVE FAULTS

Chakradhar Byreddy[1]      Kenneth D. Frampton[2]

Department of Mechanical Engineering

Vanderbilt University, Nashville TN 37235

*(Submitted to International Journal of Control)*

Abstract

The objective of this work is to detect failures in a vibration control system and adapt the control system in order to maintain optimal performance. Fault detection and isolation (FDI) filters, which are a subset of state observers specifically designed to detect and identify known types of system failures, are used to detect sensor and actuator malfunctions also called additive failures. The output of such filters is used, along with hybrid automata, to reconfigure feedback compensators in order to maintain closed loop objectives. Such reconfiguration allows the system to continue operating optimally under certain, pre-defined system failures. In this work, two types of FDI filters are compared:

[1]Graduate Research Assistant, Department of Mechanical Engineering, Vanderbilt University, 2301 Vanderbilt Place, Nashville, TN-37235. Email: chakradhar.r.byreddy@vanderbilt.edu, Phone: 1-615-322-1760, Fax: 1-615-343-6687
[2]Assistant Professor, Department of Mechanical Engineering, Vanderbilt University, 2301 Vanderbilt Place, Nashville, TN-37235. Email: ken.frampton@vanderbilt.edu, Phone: 1-615-322-2778, Fax: 1-615-343-6687

those based on parity relations and those based on Beard-Jones filter concepts. The primary contribution of this work is the integration of active control with fault detection and hybrid system management. There are several challenges inherent in this effort but the most important is the management of compensator switching. Since switching involves system discontinuity, therefore stability of the system is very difficult to guarantee. Simulation results demonstrate some of these challenges and the effectiveness of the two types of FDI filters in maintaining suitable closed loop performance in the presence of system malfunctions.

**Keywords**: Fault-Tolerant, Parity Relations, Beard-Jones and Additive Faults.

## Introduction

Most large-scale engineering systems are equipped with embedded systems for the purpose of monitoring and control. Hence, these kinds of systems require very high reliability. Therefore, it is essential that the components in such systems perform self-diagnosis in locating faults and failures. Such a self-diagnostic method helps in vastly increasing the system efficiency and reducing the costs associated with risk. In the past there has been a widespread growth in the sophisticated diagnostic procedures in various embedded systems. These embedded systems consist of sensors that measure system variables, a microprocessor controller that determines the methodology to reduce the error between the actual and desired output, and an actuator that executes the commands initiated by the controller to ascertain the desired performance.

One of the most active areas of research related to FDI is improving the implementation and design procedure using various methods. These methods, helped in

advent of more reliable and fault-tolerant applications. In this type of control, the most common form of faults is referred to as additive faults. These are typically failures of sensors and actuators. Also, there exist parametric faults such as changes in underlying system parameters. This work is limited to the identification of additive faults. The occurrence of any failure can cause the system to under-perform it's desired objective. For example, if an actuator fails then the controller designed to perform optimally with it will no longer perform optimally. Systems that can adapt to such unexpected faults would be of great advantage due to improved robustness and stability. Thus, fault-tolerant control is designed to enable an increase in efficiency of the whole system. Also, this helps in improving the life expectancy of the plant.

The objective of this research work is to implement the various methods to detect sensor and actuator failures, isolate the faults and adapt the feedback control in order to maintain closed loop performance in the presence of these faults. Many approaches have been proposed for the detection of additive faults such as actuator and sensor failures (Beard, 1971; Gertler, 1998). In this paper, FDI filters based on parity relations and Beard-Jones methodologies are studied. Also, studies are conducted on the pros and cons of these methodologies for fault-tolerant control. In this research work, pre-defined fault case scenarios are implemented and the closed loop control performance in the presence of the faults is studied. Furthermore, these pre-defined fault case scenarios can be extended depending on the requirement of the problem for achieving fault-tolerant control. It is established that the stability of the system while switching between the various controllers due to the presence of faults is critical.

Background

The primary contribution of this work is the integration of active control with fault detection and hybrid system management. Active vibration control of structures is one of the most important applications in the filed of smart structures. Currently, with the advent of smart structures, active vibration control is being applied to large-scale systems that are quite complex in nature. The control system objective is to reduce the sensitivity to disturbances and achieve active vibration control. Much work has been done on the development of various closed loop and adaptive algorithms. Complex multivariable control system architectures have been developed in order to handle complicated controller designs (Clark, Saunders, and Gibbs, 1998). The various adaptive structures paved the way for numerous new concepts associated with distributed and decentralized vibration control (Maroti et al., 2002). Most of this previous work pertains to the development of techniques related to feedback control and enhancing the performance capability of the vibration system. In addition, theories and issues associated with the implementation of distributed sensors and actuators in vibration control are developed in (Fuller, Elliot, and Nelson, 1997). FDI is a recently expanding research area. Today's FDI filters are applied to wide variety of fields such as aeronautics, health monitoring systems, etc. (Douglas and Speyer, 1996). Most of the works with FDI's are associated with the development of new techniques and methods to identify various faults. Different types of fault detection filters have been developed based on the requirement, which is the closed-loop control performance of the plant. Various concepts and development of FDI's have been extensively studied (Chen, Patton and Zhang, 1996).

A hybrid system is defined as a dynamical system with both discrete and continuous components. These kinds of systems are mostly employed where safety issues are high priority. Such hybrid systems can be modeled using hybrid automata. These logical dynamic systems are called finite-state machines, since they can only achieve a finite number of systems states. Also, systems that interact with the environment are modeled using hybrid automata. Systems that interact with the environment are highly volatile, hence called reactive systems. Reactive systems can capture system specifications using functional, behavioral and structural views. Thus, reactive systems can be modeled using finite-state machines. Hybrid systems use the concepts of hybrid control, which is a combination of feedback and adaptive control. Another reason for the use of hybrid control architectures is to have robust performance for narrow band disturbance rejection (Sievers and Von Flotow, 1992).

The contribution of this research work is the combined application of active vibration control, FDI's techniques and hybrid automata to achieve fault-tolerant control. Furthermore, current research work yields a better understanding of robustness and stability of such systems concerning hybrid switching. Since the functional demand of the system is quite complex, the stability of the system plays a vital role in the FDI's. In this paper, the stability associated with switching between the controllers and its impact on the closed-loop control performance of the system has been studied. Furthermore, studies are specifically conducted on parity relations and Beard-Jones filters that are more robust to disturbance rejection and stability during the switching of the appropriate compensators for fault-tolerant control.

<u>Mathematical Model of the System</u>

In order to implement the concept of fault-tolerant control, a multi-input multi-output (MIMO) system is modeled. The MIMO system under consideration is a simply supported beam (1-D) subjected to a random disturbance as shown in Figure 2-1. A state space model is developed for the beam including surface bonded by piezoceramic transducers (Hagwood, Chung, and Von Flotow, 1990), which act simultaneously as actuators and sensors. The model includes not only the dynamic coupling effects between the structure and actuators or sensors through piezoelectric effect, but also the structural dynamic effects introduced by mass and stiffness contributions of the piezoelectric transducers.
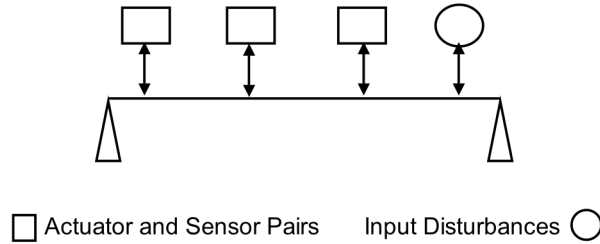


Figure 2-1: Simply supported beam with sensors and actuators

The solution assumes that the structural displacement can be expressed as the summation of the orthogonal function called modes (Meirovitch, 1990). The modes of a simply supported beam can be represented as a linear expansion of assumed modes and generalized coordinates in the following form

$$w(x,t) = \sum_{n=1}^{mod} \psi_n(x) q_n(t) \qquad (2\text{-}1)$$

where $w(x,t)$ is the beam displacement, $\psi_n(x)$ is the nth mode shape and $q_n(t)$ is the generalized coordinate of the nth mode. In general the number of assumed modes is limited to a finite number, and the importance of this result helps the design and analysis of controlled adaptive structures. Following the development of Hagwood et al., the equation of motion for the beam becomes (Hagwood, Chung, and Von Flotow, 1990)

$$[M_p + M_t]\{\ddot{q}(t)\} + [K_p + K_t]\{q(t)\} = [Q^d(t)] + [\Theta]\{v(t)\} \qquad (2\text{-}2)$$

where $M_p$ and $M_t$ are the mass matrices for the beam and piezoceramic transducers respectively, similarly $K_p$ and $K_t$ are the stiffness matrices for the beam and piezoceramic transducers respectively, $q(t)$ is a vector of generalized coordinates, $Q^d(t)$ is a vector of generalized disturbances, $\Theta$ is the electromechanical coupling matrix and $v(t)$ is the vector of control voltages applied to the transducers which act simultaneously as sensors and actuators (Vipperman, 1996).

The beam model of Equation (2-2) can be cast in the state-space forms as follows (Clark, Saunders, and Gibbs, 1998)

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad (2\text{-}3)$$

where $x$ is the state vector, $u$ is a vector of control and disturbance inputs associated with the actuators, and $y$ is the output vector associated with the sensors.

The model used for simulations consisted of three structural modes. Using such a small model was necessary in order to meet the observability requirements for the Beard-Jones filter design (discussed later). The use of a low order model for a vibrating beam means that there is the possibility for spillover in the closed loop design, and inaccuracy in the simulations. However, constant gain output feedback using collocated transduction

20

has been shown to avoid spillover problems (Clark, Saunders, and Gibbs, 1998). Furthermore, low-pass filtering of the sensor signals sent to the FDI filters would limit the effects of higher order modes on the detection process.

<p align="center">Output Feedback Compensator</p>

In this paper, constant gain output feedback compensators are used to minimize the beam vibration (Levine and Athans, 1970). This is a very simple form of control, however it has been demonstrated to be very effective in such applications (Clark, Saunders, and Gibbs, 1998). The type of control is not critical to the results shown here and the work could easily be extended for complex, dynamic compensators. Each possible actuator and sensor failure leads to different configurations of the MIMO system. When a particular actuator fails it is removed from the feedback loop and only the remaining actuators are used to reduce vibration. The compensator is designed based on pre-defined or prior knowledge of the faults that can occur in the system. The control law is implemented

$$u = -Ky \tag{2-4}$$

where $K$ is the constant feedback gain matrix, $u$ is the vector of control inputs and $y$ is the output vector containing the appropriate sensors. The feedback gain matrix can be found my minimizing the cost function (Levine and Athans, 1970) as shown in Equation (2-4)

$$J_i = \int_{t_0}^{t_\infty} (x^T \begin{bmatrix} C_i \\ [I \quad 0] \end{bmatrix}^T \begin{bmatrix} W_s & 0 \\ 0 & W_m \end{bmatrix} \begin{bmatrix} C_i \\ [I \quad 0] \end{bmatrix} x + Rv_i^2) dt \tag{2-5}$$

where $J$ is the cost function associated with each fault case scenario, $x$ is the system state vector, $C$ is the state-space system matrix from Equation (2-3), $W_s$ is the penalty associated with the sensor signals, $W_m$ is the penalty associated with the various modes, and finally R is the control effort penalty. The details associated with the implementation and calculation of the output feedback gain matrix can be found in (Levine and Athans, 1970).

The MIMO system considered in this work is a three input-three output system. Compensator design for the three fault case scenarios and one nominal case (no failure) have been designed. The three fault case scenarios are: 1. First actuator failure; 2. Second actuator failure; and, 3. First and second (both) actuator failures. The compensator performance is shown in Figure 2-2, which depicts the transfer function between the input and the sensor outputs for the various fault scenarios and for the uncontrolled transfer function. Note that the best attenuation is achieved when all the actuators are operational (no fault). As one would expect, when one actuator has failed the control performance is degraded and, when two actuators have failed the performance is the poorest. However, even when there are actuator failures the system reduces the amplitude, and system stability is still maintained. On the other hand, if no-fault controller were used with failed actuators then the performance would be suboptimal and, possibly, unstable.
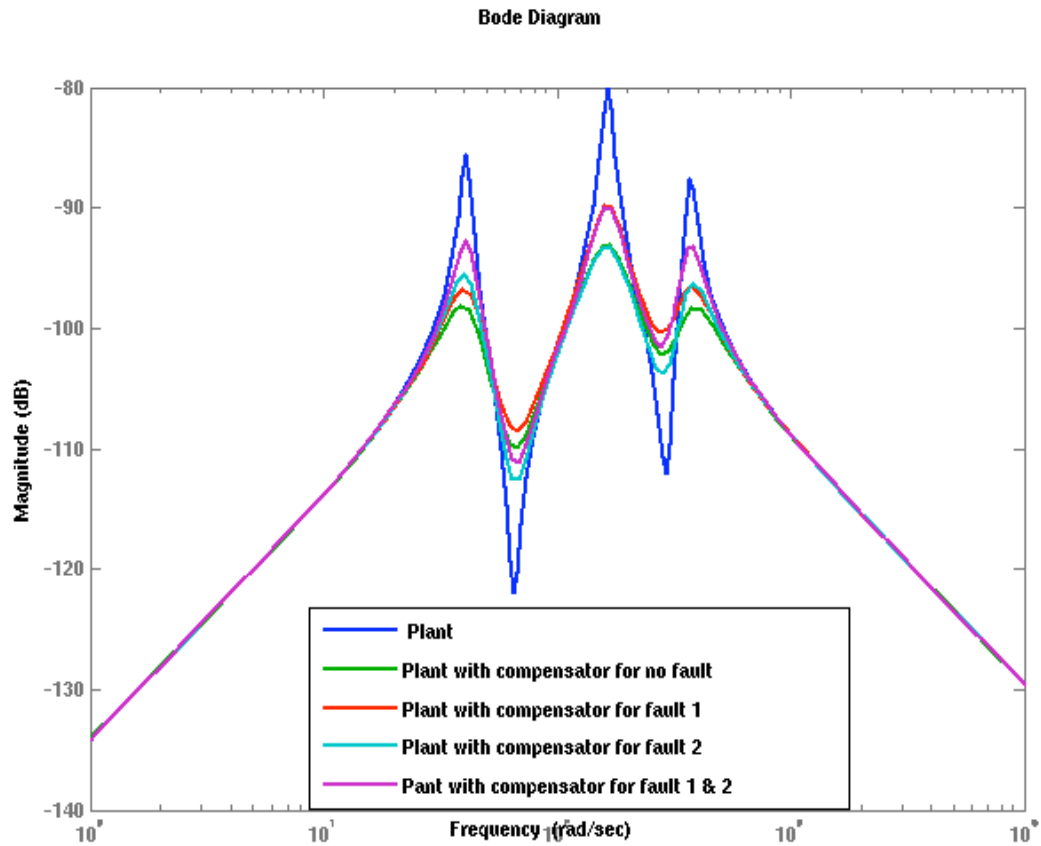
Figure 2-2: Transfer function for various fault case scenarios

## Hybrid Automata

In the current work, a framework related to the faults is constructed based on hybrid automata. A hybrid automaton is a modeling formalism of the system specifications and, it is also an analysis of algorithms dealing with the hybrid system. The hybrid automata are used because may affect the behavior of the system in a discontinuous manner as does switching from one controller to another. In hybrid automata, continuous states are converted to discrete states based on pre-defined rules. In this case the discrete states correspond to particular failure modes of the system. These

failure modes (not to be confused with structural modes) are also referred to as finite-states.

Hybrid automata consist of components like variables, control graphs, initial flow conditions, jump conditions, and events. The set of all these components constitute the existence of hybrid modeling. The manner in which hybrid automata are executed results in transition from continuous dynamic evolution to discrete changes in the system dynamics. This kind of modeling is required to facilitate the representation of abrupt failures (actuator and sensor failures) caused by external or exogenous actions and by switching controllers. In general discrete transitions can be triggered by internal system events (fault detection) or, external trigger events. These systems have been simulated using state-charts. State-charts allow multilevel decomposition, concurrency, encapsulation, and broadcast communication mechanisms. Thus, state-charts are modeled using Matlab/Simulink/Stateflow environment for visual modeling and simulation of hybrid systems that have continuous signals mixed with discrete and event driven dynamics. Various other software tools exist that are used to model hybrid systems (Hytech, 1997).

### Formulation of FDI Filters

The primary contribution of this work is to detect actuator faults and then to switch among pre-designed, optimal controllers to maintain (as well as possible) system performance and stability. Actuator faults are detected and isolated with Fault Detection and Isolation (FDI) filters based on either parity relations (Gertler, 1999) or Beard-Jones filters (Beard, 1971). In both methods, the measured behavior of the system is compared

with that of a system estimator referred to as a fault observer. Behavior deviations between the actual system and the observer indicate the presence of faults. These deviations are called primary residuals, which in turn are subjected to a linear transformation, to obtain the desired fault-detection and isolation properties. The linear transformation is termed a residual generator. The theory behind each approach is discussed below.

*Parity Relations*

For convenience (and consistency with Gertler, 1999), the plant is described here using transfer functions rather than the state-space relationships such that

$$y(t) = M(q)u(t) + S(q)p(t) \qquad (2\text{-}6)$$

where $M(q)$ and $S(q)$ are rational transfer function matrices. The vector $u(t)$ consists of control inputs and $p(t)$ is composed of additive faults. In detail, $M(q)$ is the transfer function between the control input and sensor outputs, $S(q)$ is the transfer function between the faults, $p(t)$, and the sensor outputs. The parity relation approach defines the residual generator as follows

$$r(t) = W(q)[y(t) - M(q)u(t)] \qquad (2\text{-}7)$$

where $r(t)$ is the vector of residuals, and $W(q)$ is called the residual generator. The residual generator operates on the error between the actual system output, y(t), and the predicted system output. The objective is to design $W(q)$ so that the residual vector has a desirable response to errors resulting from system faults. This goal is achieved by solving the equation

$$W(q)S(q) = Z(q) \qquad (2\text{-}8)$$

where $Z(q)$ is the desired residual generator dynamics. Solving Equation (2-8) for $W(q)$ means that the residual generator will equal the inverse of the fault transfer function times the desired dynamics of the residual generator. The desired response used in this investigation was a first order that is "slow" relative to the plant. Such a response would ensure that the fault generator would not respond to plant noise and that it would damp out oscillations in the residual preventing erratic switching. Further detail on the theory and implementation of parity relation can be found in (Gertler, 1997).

*Beard-Jones Filter*

The Beard-Jones detection filter approach has received significant attention among due to its unique properties. These filters are, essentially, traditional state observers whose free parameters are assigned such that the output (residual) vector has certain directional properties that can be associated with a set of faults (Beard, 1971). This detection filter is one of the most widely known diagnostic observers to be developed. The most significant aspect of the Beard-Jones approach is to identify the fault in a plant, which in turn is independent of the mode of the fault as long as the fault alters the plant to a certain degree.

The Beard-Jones detection filter has been modified for various problems. And, new approaches are developed to suit complex cases. Various methods were subsequently analyzed and interpreted as an eigenstructure assignment (Kim and Park, 1999 & 2003; Park, 1991; Park and Rizzoni, 1994; White, 1985 &1987). The open loop dynamic model in the absence of failures is given in Equation (2-3). The detection filter in this case is assumed to be of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + D(y - C\hat{x})$$ (2-9)

where $\hat{x}$ is the state estimate and $D$ is the detection gain. The state error is defined as

$$\varepsilon \overset{\Delta}{=} x - \hat{x}$$ (2-10)

Then D is chosen such that the output error

$$\bar{\varepsilon} = y - C\hat{x}$$ (2-11)

has restricted directional properties in the presence of a failure. Therefore, observer dynamics, in the absence of failures, becomes

$$\dot{\varepsilon} = G\varepsilon$$ (2-12)

where $G$ is defined as

$$G \overset{\Delta}{=} A - DC$$ (2-13)

The presence of an additive fault (i.e. actuator failure) in the system can be modeled by adding an additional input term to the system shown in Equation (2-3) to produce a new system equation

$$\dot{x} = Ax + Bu + f_i\mu_i$$ (2-14)

where $f_i$ is a nx1 design failure direction associated with the i$^{th}$ actuator failure and $\mu_i$ is time-varying scalar component of the fault which may be function of $x(t)$ or $u(t)$. The information regarding $\mu_i$ is essential to differentiate plant failures modeled by the same design failure direction. Thus, for actuator failures, the error is

$$\dot{\varepsilon} = G\varepsilon + f_i\mu_i$$
$$\varepsilon = C\varepsilon$$ (2-15)

Finally, the detection gain is determined using certain methodologies developed by

(Kim, 1999, 2003), in which $\bar{\varepsilon}$ is proportional to $Cf_i$ in response to failure corresponding

to that modeled by the direction $f_i$. Also, the theory behind the sensor failure is similar to

the actuator failure and, can be found in (Kim and Park, 1999 & 2003; Park, 1991; Park

and Rizzoni, 1994; Rizzoni and Min, 1989; White, 1985 &1987).


<p style="text-align:center">Fault-Tolerant Control</p>

In general, fault-tolerant control is the process of detecting failures (in this case

actuator failures) and appropriately switching compensators in order to maintain

performance. This process is shown schematically in Figure 2-3.  In both cases a

compensators are designed for each of the various fault case scenarios and also for the

nominal case (no fault). This set of compensators is called the controller library.

Similarly, fault observers were designed based on the closed loop system for each of the

fault case scenarios.  This set of observers is referred to as the observer library. The

hybrid automata selects the appropriate controller and observer from the libraries based

on the fault mode detected by the FDI's.

Results are presented in this section for fault-tolerant control implementations using

either parity relation or Beard-Jones based fault observers.  The system objective is to

minimize the vibration amplitude of a beam under the potential for actuator failures. The

results presented will demonstrate the effectiveness of each approach and identify the

relative strengths and weaknesses of the different methods.
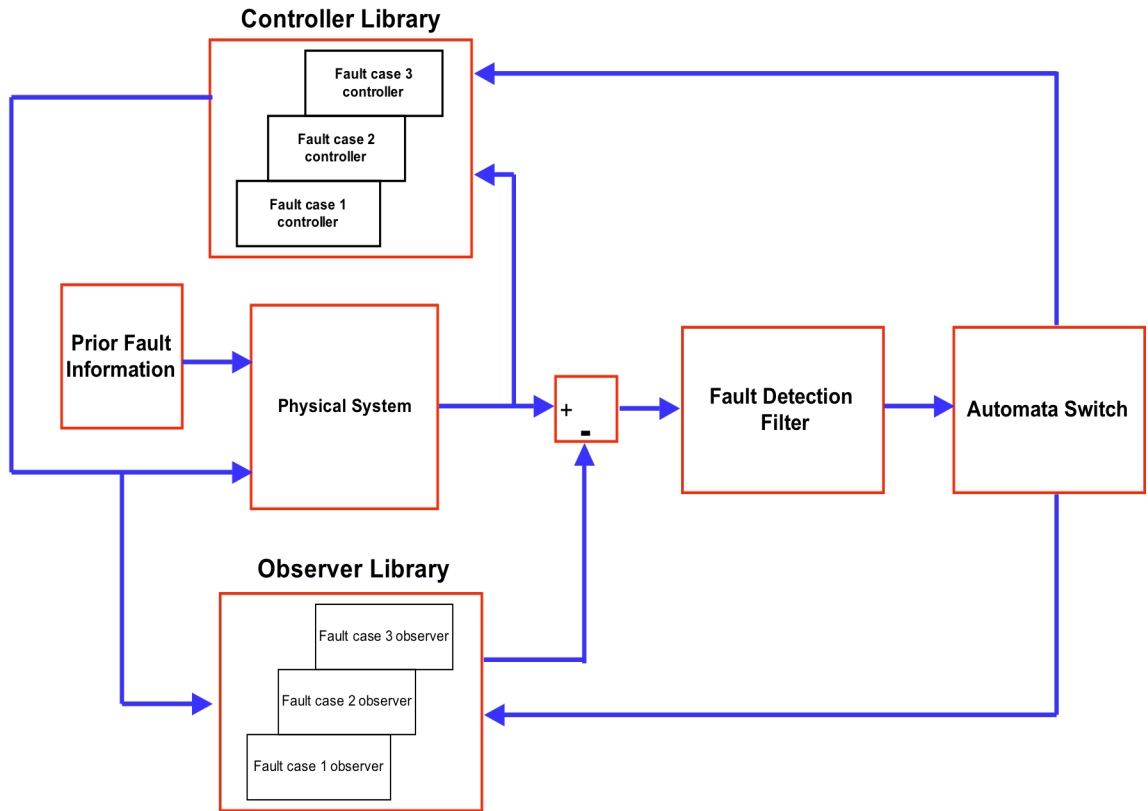
Figure 2-3: Hybrid model for parity FDI

*Fault-Tolerant Control with Parity Relations*

In this section results are shown for simulations of the hybrid system shown in Figure 2-3 and employing parity relation based FDI. Upon simulation start-up the system is operating under nominal conditions making use of a 3-input/ 3-output compensator. A fault is simulated by injecting a non-zero input into the system fault input (i.e. making $p(t)$ non-zero). While the approach can detect any form of $p(t)$, unit step functions are used for all faults in this work. Furthermore, while this approach can accommodate any combination of sensor and actuator failures, we have restricted this investigation to 3 possible fault case scenarios: fault case 1 corresponds to a fault in actuator 1; fault case 2 corresponds to fault in actuator 2; and fault case 3 is a fault in both actuators 1 and 2

simultaneously.
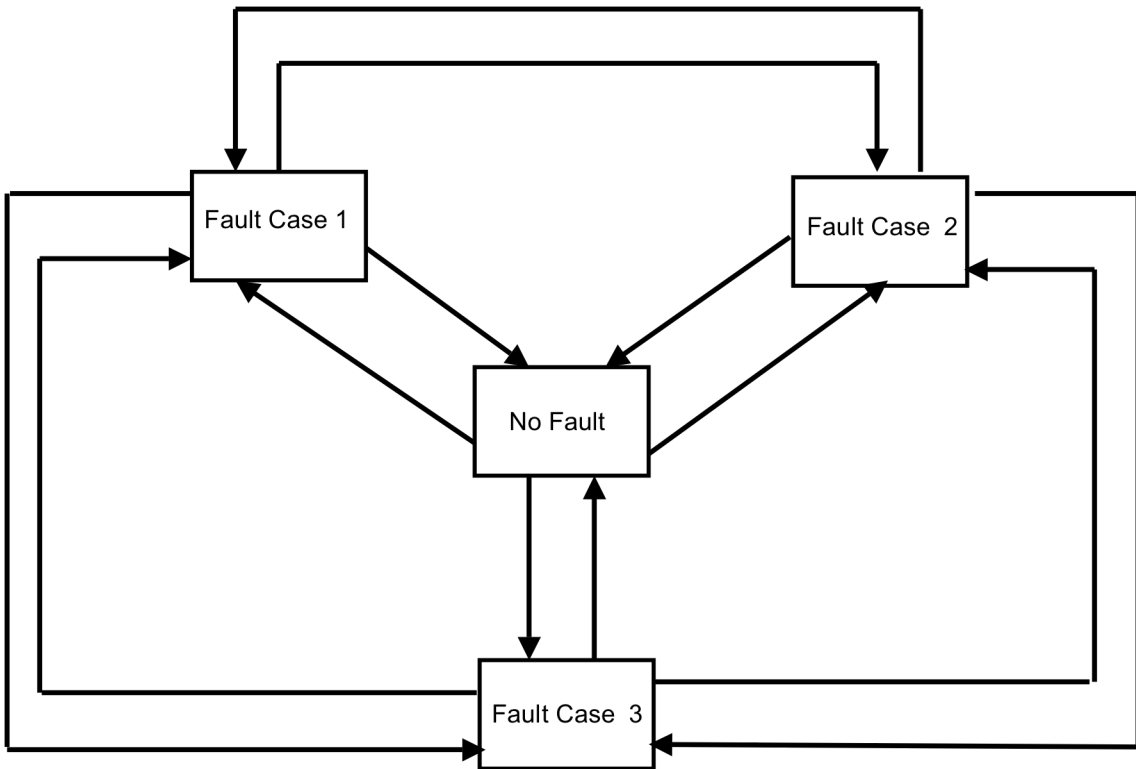


Figure 2-4: Failure Modes

When a fault is introduced the physical system response will deviate from the observer estimate. This deviation is detected by the FDI filter resulting in a non-zero residual output. The continuous residual output vector is then converted into a discrete residual vector. The discrete residual vector has 3 elements, one for each fault case. If the FDI residual is greater than a minimum threshold, indicating a fault, then the corresponding discrete residual vector element is set to 1. Otherwise it is set to zero. This discrete fault vector is then fed to the automata switch. The automata, shown in the state chart of Figure 2-4, switches based on the values in the discrete residual vector. There are four possible automata states; one for each fault case scenarios such that

$$
r = \begin{cases} [0\ 0\ 0] \text{ - No Fault} \\[12pt] [1\ 0\ 0] \text{ - Fault Case 1} \\[12pt] [0\ 1\ 0] \text{ - Fault Case 2} \\[12pt] [1\ 1\ 0] \text{ - Fault Case 3} \end{cases}
$$

(2-16)
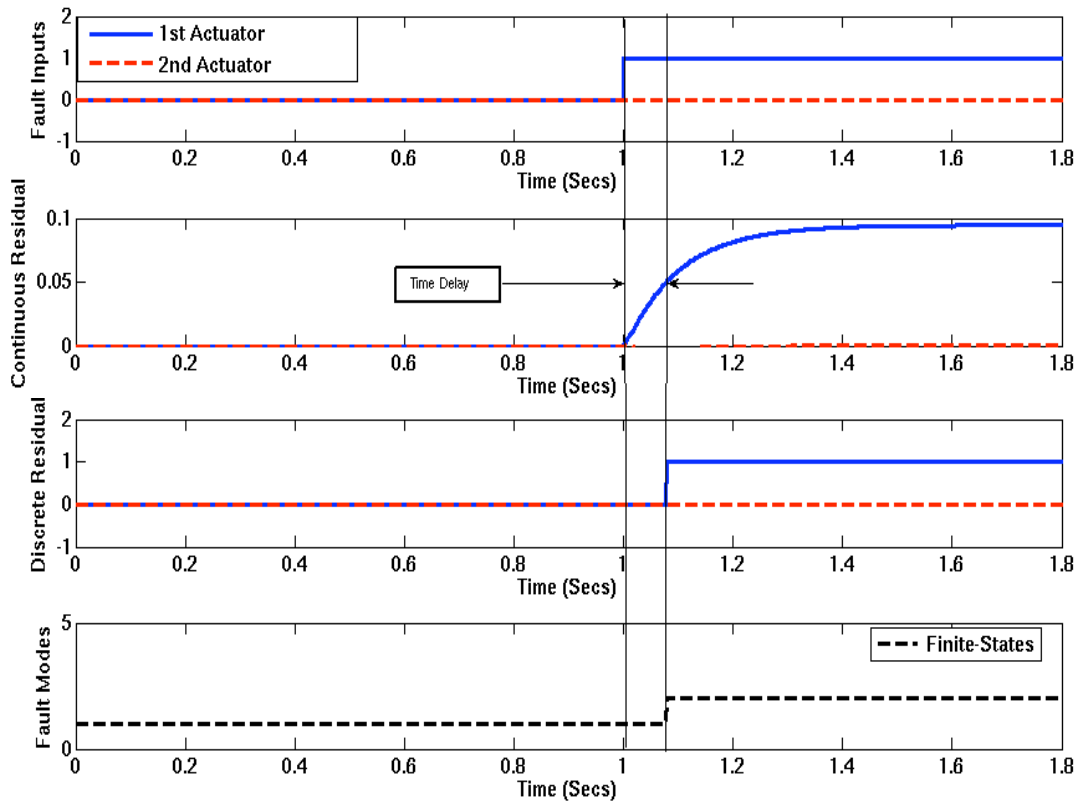


Figure 2-5: Time history explaining the concept of threshold

These transitions encompass all possible failure scenarios; i.e. going from nominal to failure case 1; going from failure case 1 to failure case 3; going from failure case 3 to failure case 2; etc. The results of a specific simulation are shown in Figure 2-5, which shows the time history of the fault, $p(t)$; the continuous residual vector, $r(t)$; the discrete residual vector; and finally the fault modes. The fault occurs at a time of one second in

31

the first actuator. The difference between the measured plant response and the observer output is input to the residual generator, which creates the continuous residual second plot of Figure 2-5. When the desired response reaches a threshold value, a fault is deemed to have occurred and the discrete residual vector is changed appropriately. This results in a change in the fault mode. Due to the use of a threshold in converting the residual from continuous to discrete, a time delay is introduced between the actual fault occurrences and switching of the finite-states. This time delay helps to ensure that system noise will not produce false positives in the fault detection process.

Note that when a switch occurs the system will experience some transient behavior. However this transient behavior is small in magnitude and brief in duration compared to the disturbance. Once steady state has been reached, the response of the system will be that predicted in Figure 2-2 for the appropriate fault mode. A result for a more complex fault scenario is shown in Figure 2-6. In this case all four fault modes occur at some time while faults in actuators 1 and 2 turn on and off. From the subplot 1 in Figure 6, actuator 1 and 2 have failed between zero and 10 seconds i.e. fault case scenario 3. Between 15 and 20 seconds, the second actuator malfunction has been rectified, which means that at that instant of time the finite-state is in fault case scenario 1. Between 25 and 30 seconds the first actuator malfunction has been rectified, which means that the finite-state is in the no fault case scenario. Note that the fault mode tracks the actual fault condition very well. Furthermore, it has been demonstrated (although not proven) that the system is stable when switching between fault case scenarios at a rate that is "slower" than the overall system dynamics. When fault occurrences are faster than the plant dynamics there is a potential for instability.
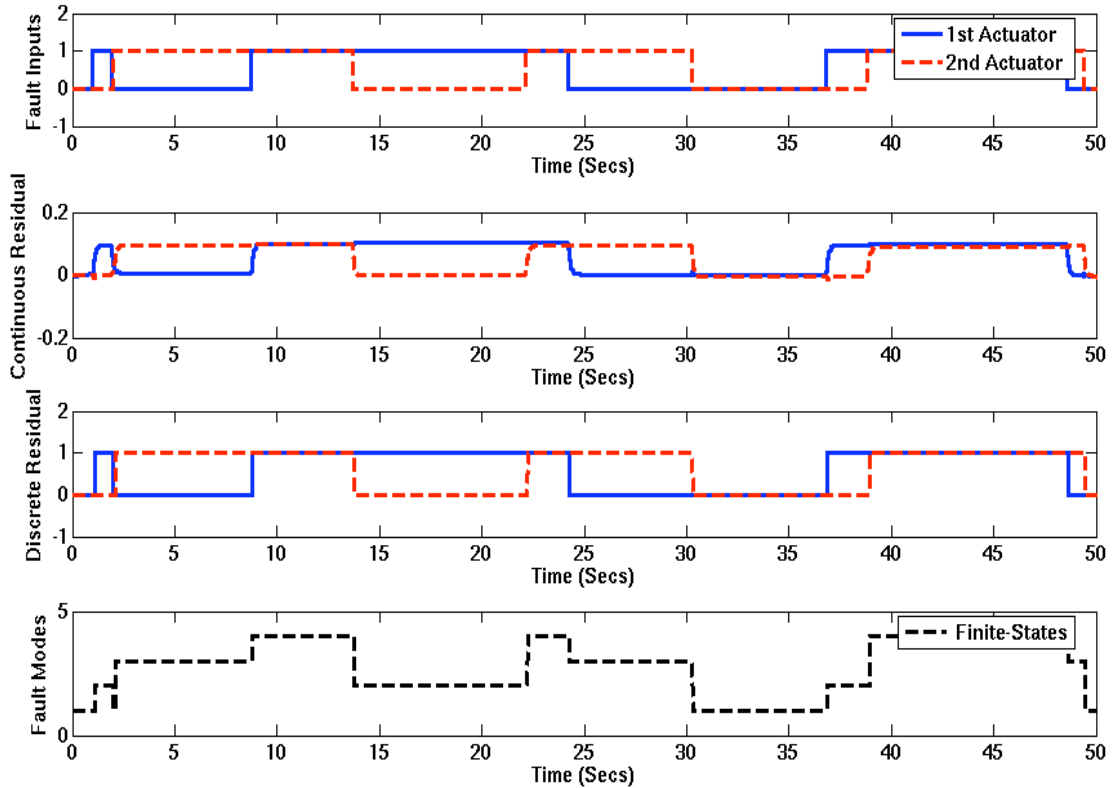
Figure 2-6: Time history of fault inputs, desired response, residual vector, and fault modes

*Fault-Tolerant Control with Beard-Jones Filters*

Beard-Jones filters are used in exactly the same manner as the parity filters of the previous section. One of the main features of the Beard-Jones filter is that it is possible to use a single filter to detect all possible faults. The implementation of the detection is based on various assumptions and condition, which can be found in (Kim, 1999, 2003; Park, 1991; Park and Rizzoni, 1994; White, 1985; White, 1987). The physical system used here satisfied all the conditions necessary and essential to design a Beard-Jones filter. In the current research work it was possible to design a single detection filter for all the possible fault case scenarios. Therefore, the fault tolerant control system depicted in

Figure 2-7 is identical to that of Figure 2-3, except that it does not require an observer library.
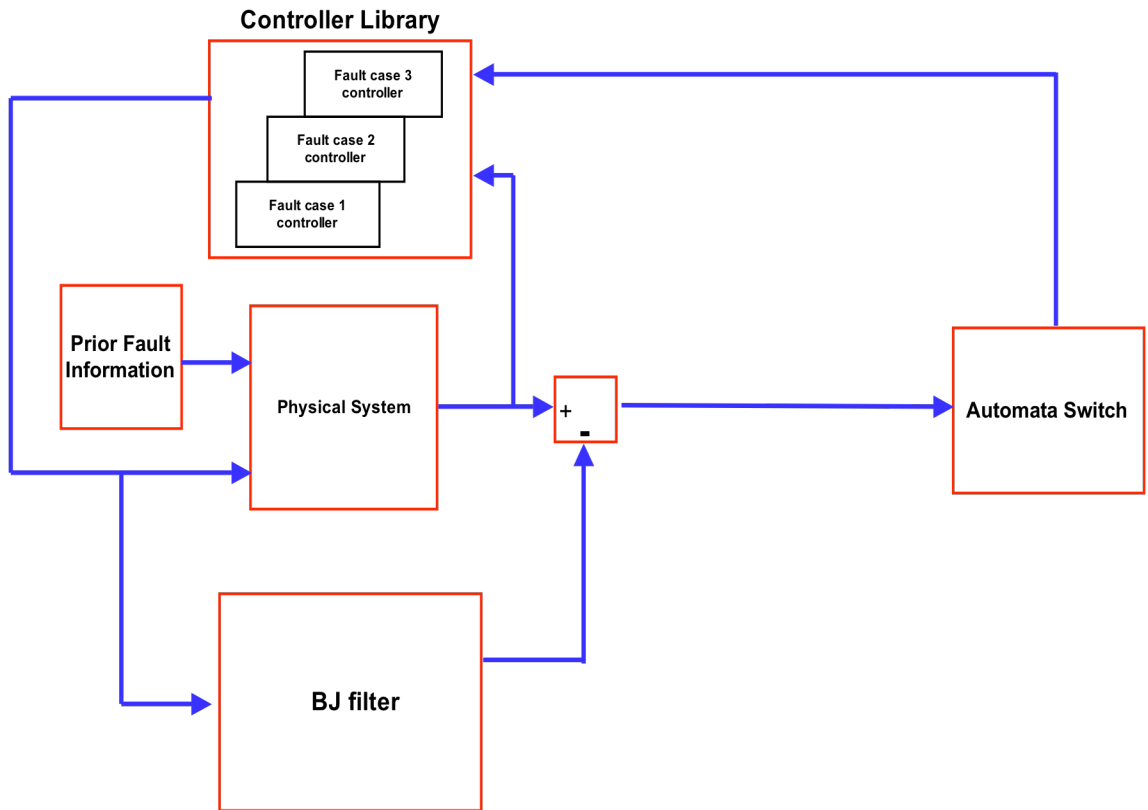


Figure 2-7: Hybrid model for Beard-Jones FDI

Again, upon simulation start-up the system is operating under nominal conditions. A fault input is simulated by introducing a non-zero input into the system as described in the above section. Similar to the parity approach, this approach can accommodate any combination of sensor and actuator failures, but is restricted to actuator failures here. The results of a specific simulation are shown in Figure 2-8, which shows the time history of the fault; the continuous residual vector; the discrete residual vector; and the fault mode. The fault occurs at a time of two seconds. The difference between the measured plant

response and the detection filter is input fault to the fault isolation to get the residual

vector shown in the second plot of Figure 2-8.
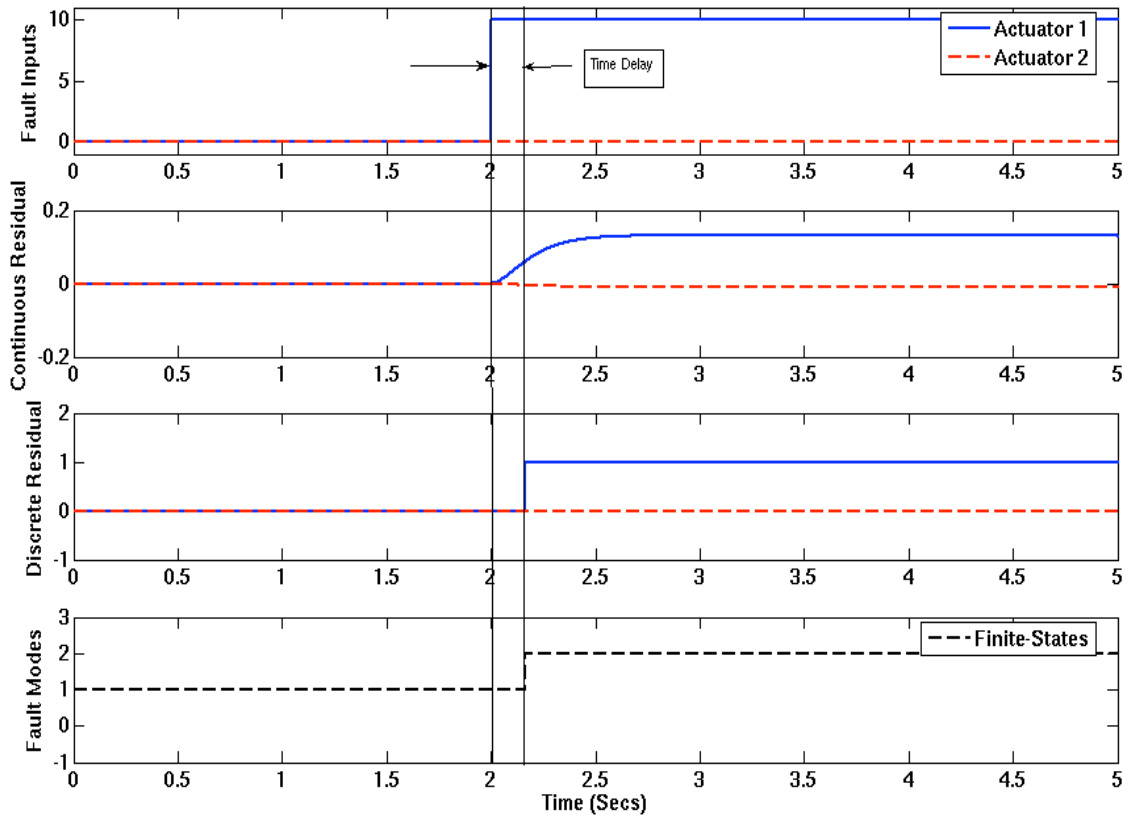


Figure 2-8: Time history explaining the concept of threshold

When the continuous residual reaches a threshold value a fault is deemed to have

occurred. Hence, there is a switch in the failure modes in the finite-state machine. The

response of the fault observer was chosen such that the residual dynamics were "slow"

compared to the system (similar to the parity relation case). Due to this "slow" behavior,

a time delay was created between the fault occurrences and switching of the finite-states

using the values of residual vector, as shown in the Figures 2-4 and 2-8. This time delay

helps to ensure that system noise will not produce false positives in the fault detection process.
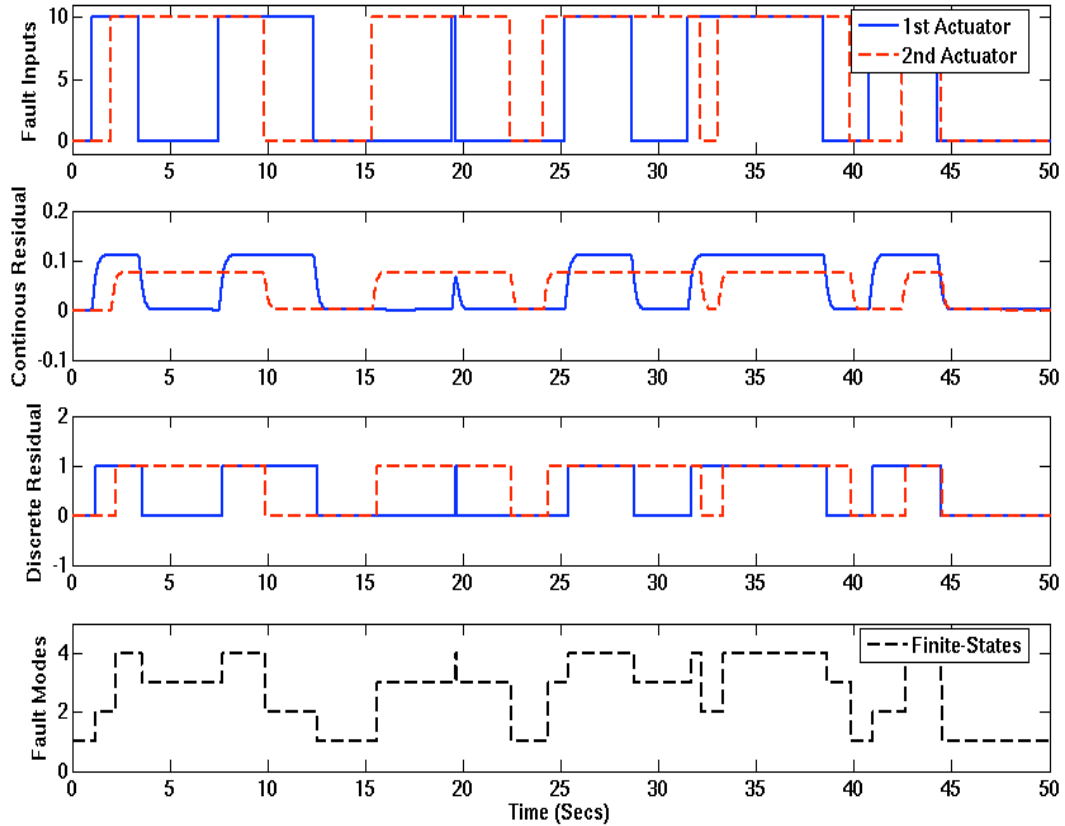


Figure 2-9: Time history of fault inputs, desired response, residual vector, and fault modes

The case were multiple faults occur at various times is shown in Figure 2-9. From the subplot 1 in Figure 2-6, actuator 1 and 2 have failed between zero and 10 seconds i.e. fault case scenario 3. At exactly 10 seconds, the first actuator malfunction has been rectified, which means that at that instant of time the finite-state is in fault case scenario 1. Between 18 and 20 seconds, the first actuator malfunction has been occurred and rectified, which means that the finite-state is in fault case scenario 3 and fault case

scenario 1. Thus, in this work it has been established that the system is stable from switching between fault case scenarios and, also from fault case scenarios to the nominal case.

Comparisons Under Non-Ideal Circumstances

One of the key features in this work is to compare the pros and cons of the two types of FDI filters used for controller switching. Previously discussed results have demonstrated that both filter types are good at isolating faults under ideal circumstances. However, their ability to perform under less than ideal circumstances has yet to be established. To this end, simulations were conducted under two cases: 1) when noise was added to the sensor signals prior to being provided to the FDI filters and 2) when the plant model deviates from the actual plant.

Figure 2-10 shows results from simulations, which included noise, added to the sensors prior to being supplied to the FDI filters.  In this case a fault was introduced to actuator 1 after 1 second and actuator 2 after 2 seconds.  Figure 10a shows the continuous residuals from Beard-Jones filters without noise and with noise having an RMS signal to noise ration of 99.5 percent. Note that even with very large noise levels the residual still tracks the faults reasonably well. This could be improved by "slowing" the response of the Beard-Jones filter thus reducing the high frequency noise transmission. Figure 10b shows the same results for the parity relation FDI filters.  Again, the filters continue to successfully track the fault with a high noise level. Thus, it can be concluded that both filters are robust to sensor noise.
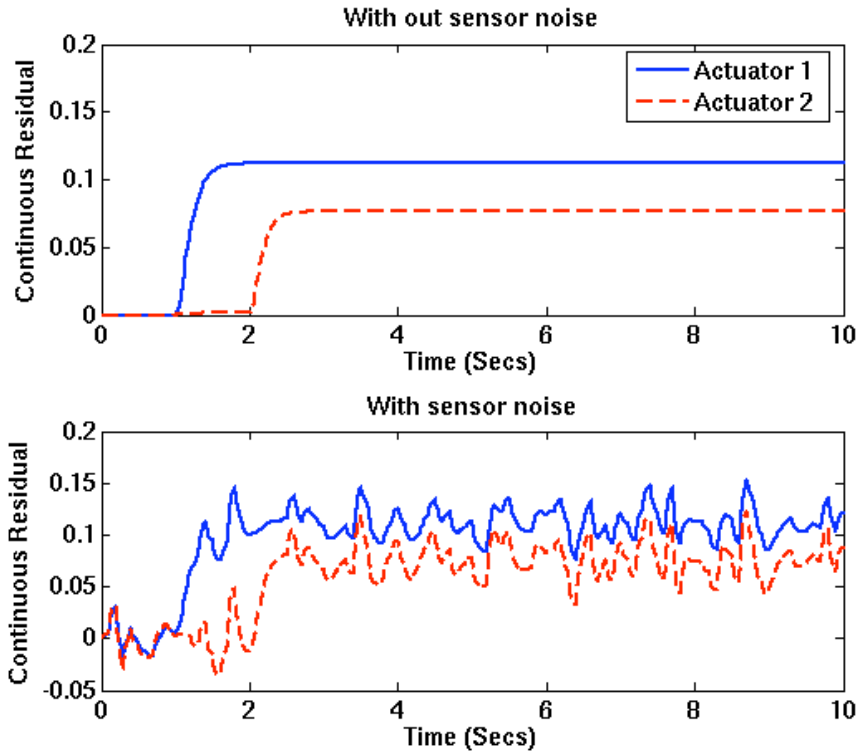
Figure 2-10:  a) Time history of continuous residual for FDI with Beard-Jones filter
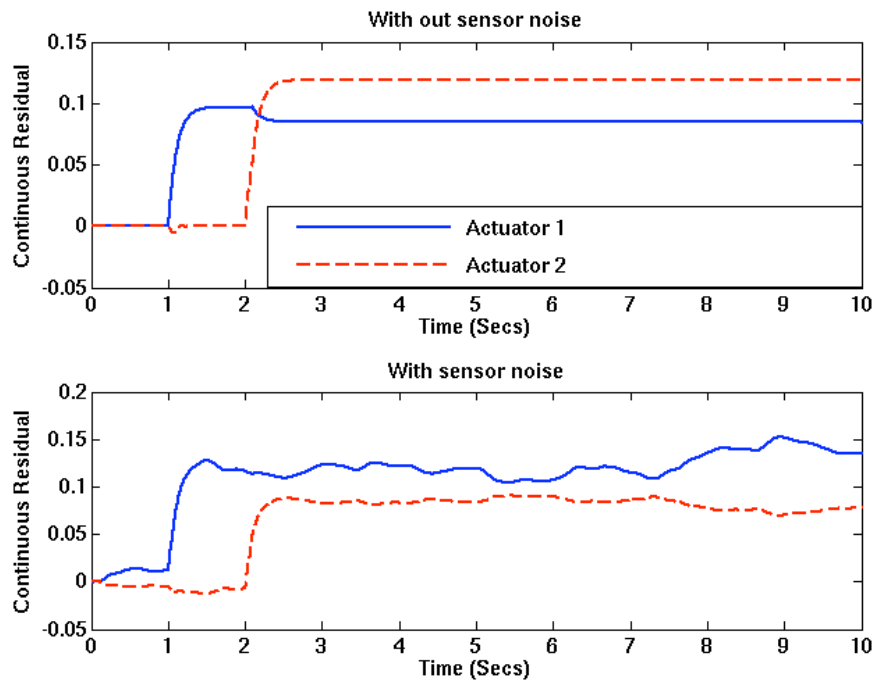


Figure 2-10:  b) Time history of continuous residual for FDI with parity relations
The second non-ideal situation is when the plant model does not exactly match the

actual plant. Simulations were run using the same FDI filters as previously used, but the natural frequency of one beam mode was changed by 50 percent (thus resulting in a change in one entry in the system A matrix). Results from these simulations are shown in Figure 2-11. In this case a failure in actuator 1 occurred after 1 second. Note that the parity relation filter simultaneously indicates a fault in actuator 2 shortly after the actual failure. However, the Beard-Jones filter shows only a slight deviation in the actuator 2 residual. Clearly, the parity relation based FDI filter is very sensitive to model uncertainty. Thus, FDI with parity is not robust to model mismatching than the Beard-Jones filter. These results are schematically shown in Figure 2-11.
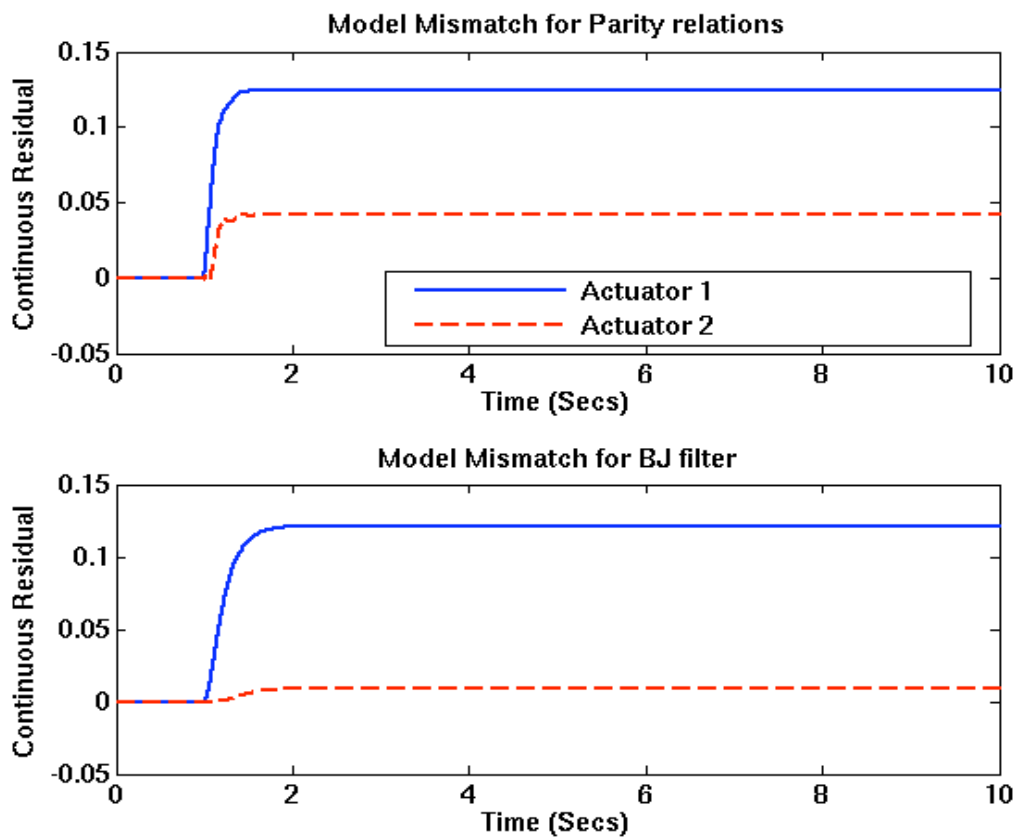


Figure 2-11: Time history of continuous residual for FDI for model mismatching

Stability of Fault-Tolerant Control

The issue of stability is critical when switching occurs between observers and controllers. It has been shown that the vibration of the physical system can be suppressed using different controllers under various fault case scenarios. But to understand the conditions under which the system might be unstable is an open problem (Lin and Antsaklis, 2005). The key area for the open problem is during the threshold period, just before switching. It has been concluded that for general cases of higher order systems, and more than two fault modes, the existence of a common quadratic Lyapunov function for a switched systems is still lacking. Additional methods (Liberzon, Hespanha, and Morse, 1999; Liberzon and Tempo, 2003; Lin and Antsaklis, 2004) are proposed to show the stability of switching for linear systems can be accomplished under certain conditions, which in turn remains an area of interest for many researchers.  However, the stability of a multiple-fault system cannot be proven.

Conclusions

The primary goal of this work is to implement a FDI scheme for fault-tolerant vibration control reconfiguration. Simulations have been conducted on a simply supported beam, which demonstrate the ability of such a system to maintain performance and stability. The primary contribution of this work is the integration of active control with fault detection and hybrid system management. Also, in this work the concepts implemented for switching the compensators have been studied. In this work various filter design criteria have been carefully examined in order to achieve the concept of fault-tolerant control. Furthermore, comparison of the FDI filters is conducted. This study

helped in choosing the appropriate filter for various experimental studies. Finally, the stability associated with switching from no fault case to fault case scenarios and back and forth has been studied.

## Acknowledgements

## References

[1] Beard, R.V. 1971. "Failure Accommodation in Linear System through Self-Reconfiguration," Ph.D. Thesis, Massachusetts Institute of Technology, MA.

[2] Chen J., Patton Ron J., and Zhang Hong-Yue. 1996. "Design of Unknown Input Observers and Robust Fault Detection Filters," International Journal of Control, 63.

[3] Clark, R.L., Saunders, W.R., and Gibbs, G.P. 1998. "Adaptive Structures: Dynamics & Control," 1st Edition, John Wiley & Sons Inc., New York.

[4] Douglas, R.K., and Speyer, J.L. 1996. "Robust Fault Detection Filter," AIAA Journal of Guidance, Control and Dynamics, 19.

[5] Fuller, C.R., Elliot, S.J., and Nelson, P.A. 1997. "Active Control of Vibration," 1st Edition, Academic Press, London.

[6] Gertler J. 1997. "Fault Detection and Isolation using Parity Relations," Control Engineering Practice, 5, 653-661.

[7] Gertler J. 1998. " Fault Detection and Diagnosis in Engineering Systems," 1st Edition, Marcel Dekker, New York.

[8] Gertler, J. 1999. "Residual Generation in Model-Based Fault Diagnosis," Control-Theory and Advanced Technology, 9.

[9] Hagwood N W., Chung W.H., and Von Flotow A. 1990. "Modeling of Piezoelectric Actuator Dynamics for Active Structural Control," Journal of Intelligent Material Systems and Structures, 1, 327-354.

[10] HYTECH, Homepage: http://embedded.eecs.berkeley.edu/research/hytech//

[11] Kim, Y., and Park, J. 1999. "An Analysis of Detection Spaces using Invariant Zeros," American Control Conference, San Diego, California, USA.

[12] Kim, Y., and Park, J. 2003. "A Condition of the Eigenvalues of Detection Filters for Disturbance Attenuation: An Invariant Zero Approach," Conference on Decision and Control (CDC), Hawaii, USA.

[13] Levine W., and Athans M. 1970. "On the Determination of the Optimal Constant Output Feedback Gains for Linear Multivariable Systems," IEEE Transaction of Automatic Control, 15, 44-80.

[14] Liberzon D., Hespanha J.P., and Morse A.S. 1999 "Stability of Switched Linear Systems: A Lie-Algebraic Condition," Systems and Control Letters, 37, 117-122.

[15] Liberzon D., and Tempo R. 2003 "Gradient Algorithm for Finding Common Lyapunov Function," IEEE Conference Decision Control, 4782-4787.

[16] Lin H., and Antsaklis P.J. 2004 "A Necessary and Sufficient Condition for Robust Asymptotic Stabilizability of Continuous-Time Uncertain Switched Linear Systems," IEEE Conference Decision Control.

[17] Lin H., and Antsaklis P.J. 2005 "Stability and Stabilizability of Switched Linear Systems: A Short Survey of Recent Results," International Symposium on Intelligent Control & Mediterranean Conference on Control and Automation.

[18] Maroti, M., Frampton K., Karsai G., Bartok S., and Ledeczi A. 2002" Experimental Platform for Studying Distributed Embedded Control Applications," Proc. Languages, Compilers, and Tool for Embedded Systems Conference, Berlin.

[19] Meirovitch, L. 1990. "Dynamics and Control of Structures," Wiley Interscience, New York.

[20] Park, J. 1991. "A Unified Theory of Fault Detection and Isolation in Dynamic Systems," Ph.D. Dissertation, The University of Michigan, Ann Arbor.

[21] Park, J., and Rizzoni, G. 1994. "An eigenstructure assignment algorithm for the design of fault detection filters," IEEE Trans., AC-39, 1521-1524.

[22] Rizzoni, G., and Min, P.S. 1989 "Detection of sensor failures in automotive engines," IEEE Trans., 38, 148-158.

[23] Sievers, L.A., and Von Flotow, A.H. 1992. "Comparison and Extensions of Control Methods for Narrowband Disturbance Rejection," ", IEEE Trans. Signal Processing, 40, 2377-2391.

[24] Vipperman, J.S. 1996. "Adaptive Piezoelectric Sensoriactuators for Active Structural Acoustic Control," Ph.D. Thesis, Duke University.

[25] White, J.E. 1985. "Detection Filter Design by Eigensystem Assignment," Ph.D. Dissertation, University of Texas, Austin.

[26] White, J.E., and Speyer, J. 1987. "Detection Filter Design: Spectral Theory and Algorithms," IEEE Trans., AC-32, 593-603.

CHAPTER III

MANUSCRIPT 2

FAULT-TOLERANT ACTIVE VIBRATION CONTROL FOR HIGHER ORDER
SYSTEMS

Chakradhar Byreddy[1]   Kenneth D. Frampton[2]   Yongmin Kim[3]   Tao Tao[4]

*(Submitted to Journal of Systems and Control Letters)*

Abstract

The objective of this work is to demonstrate a fault-tolerant vibration control

system applicable in higher order systems. System failures are detected and isolated by

Beard-Jones (BJ) filters. When such a failure is detected, the Fault Detection and

Isolation (FDI) filter output is supplied to a hybrid automaton that switches the system to

a new controller specifically designed for the faulty system condition. The closed loop

system, therefore, maintains optimal performance and stability under failure conditions.

The two most significant contributions of this work are: 1) the demonstration of a fault

[1]Graduate Research Assistant, Department of Mechanical Engineering, Vanderbilt
University, 2301 Vanderbilt Place, Nashville, TN-37235. Email:
chakradhar.r.byreddy@vanderbilt.edu, Phone: 1-615-322-1760, Fax: 1-615-343-6687.
[2]Assistant Professor, Department of Mechanical Engineering, Vanderbilt University,
2301 Vanderbilt Place, Nashville, TN-37235. Email: ken.frampton@vanderbilt.edu,
Phone: 1-615-322-2778, Fax: 1-615-343-6687.
[3]CTO, Nano-Tronix, 3FI, Younghan bldg., 1546-3, Seocho-Dong, Seocho-gu, Seoul 137-
872, South Korea. Email: kym@nano-tronix.com, Phone: +82-25819520, Fax: +82-
25819511.
[4]Graduate Research Assistant, Department of Mechanical Engineering, Vanderbilt
University, 2301 Vanderbilt Place, Nashville, TN-37235. Email: tao.tao@vanderbilt.edu,
Phone: 1-615-322-1760, Fax: 1-615-343-6687.

adaptive control system applicable to higher order systems, and 2) a new methodology for designing BJ filters for high order systems, and systems with feed-through dynamics (i.e. a non-zero **D** matrix). The capabilities of such a system are demonstrated through simulations based on analytical and experimentally obtained system models. The results provide a benchmark for the design of detection filters for use in fault-tolerant vibration control.

**Keywords:** Beard-Jones filter, Switching, Vibration control, Hybrid automata, System identification.

## Introduction

A key feature for long-term safety in critical systems is the implementation of Fault Detection and Isolation (FDI) for fault-tolerant control. Many of these systems are large-scale, so it is important that fault-tolerant control systems (and the associated FDI filters) be able to accommodate their higher-order dynamic models. One example of such an application is the control of vibrations in structural beams, plates and shells. Active vibration control has been studied for many years. The goal of active vibration control is to employ sensors and actuators (integrated into a structure) to reduce vibration amplitudes. Much work has been done on the development of feed-forward, feedback and adaptive algorithms [6]. Complex multivariable control system architectures have also been developed in order to handle complicated controller designs. These various methods have paved the way for numerous new concepts associated with distributed and decentralized vibration control [22]. While some work has been done in the detection of

44

faults in vibration control systems [26], no work has yet been done in fault-tolerant vibration control.

Most fault detection work to date has been concern with additive faults (i.e. actuator and sensor failures), and relatively low order systems [1,4,11]. These studies can be widely classified into four different categories [1,4,11,4,27]: 1. Algorithms based on kalman filters, 2. Parity space techniques, 3. Diagnostic observers and, 4. Parameter estimation methods. It has been previously established that BJ type diagnostic observers offer several advantages in vibration control applications [2,3]. One of these advantages is that BJ filters utilize subspace concepts to associate the residual with the system faults, thereby permitting simultaneous FDI [12,13,14,15].

However, two limitations to BJ filters limit their applicability to higher order systems. One limitation is that, when using currently available techniques, it is very difficult to obtain a BJ observer design that is stable for high order systems. The second limitation is that there are no existing design techniques that can accommodate a model with feed-through dynamics (i.e. a state space model with non-zero $\mathbf{D}$ matrix). These limitations are particularly challenging when the application is active vibration control. This is because the systems of interest (beams, plates or shells) require very large order models to ensure accuracy. Secondly, the models are frequently obtained by system identification or model order reduction (or both), which frequently result in feed-through dynamics.

Therefore, the primary objectives are to: 1) present new methods for designing BJ filters applicable for high order systems with feed-through dynamics and, 2) demonstrate the use of such filters in a fault-tolerant vibration control application. The system

45

considered here consists of a simply supported beam model, which is under closed loop vibration control. System actuators are allowed to fail, and this failure is detected by the BJ filter. The BJ filter output residual is sent to a hybrid automaton, which in turn determines the specific fault mode, and then switches to the appropriate feedback controller for the system condition.

The discussion begins with a description of the vibrating beam theoretical model, an alternative system model derived from an experimental platform, and the feedback control design. This is followed by a summary of BJ FDI theory along with a description of the new BJ design methods for feed-through dynamics and high order systems. Next, the simulation of the complete hybrid system is described. Finally, results that demonstrate the effectiveness of the BJ filters and the fault-tolerant control system are presented.

## System Modeling and Compensator Design

Two types of plant models are used in this investigation: theoretical beam models and a model based on system identification of an experimental setup. Both of these structural models are described in this section along with the feedback compensator design method.

### *Theoretical Model*

The plant under consideration is a simply supported beam subject to a random disturbance. The beam is equipped with three piezoelectric transducers that act simultaneously as actuators and sensors [30]. The modeling technique follows that

presented by Hagwood et al. [9], which uses a Rayleigh-Ritz technique to create the equations of motion for the full electromechanical system. This representation includes the mass and stiffness contributions of the piezoelectric transducers as well as the dynamics of the transducers themselves. The solution assumes that the structural displacement can be expressed as summation of orthogonal function of the following form

$$w(x,t) = \sum_{n=1}^{N} \psi_n(x)q_n(t) = \sum_{n=1}^{N} \sin(\frac{n\pi x}{a})q_n(t) \qquad (3\text{-}1)$$

where $w(x,t)$ is beam displacement, $\psi_n$ is n[th] mode shape, and $q_n(t)$ is the generalized coordinate of the nth mode. The result is a set of N coupled ordinary differential equations of the form

$$[\mathbf{M_p} + \mathbf{M_t}]\{\ddot{q}(t)\} + [\mathbf{K_p} + \mathbf{K_t}]\{q(t)\} = [\mathbf{Q^d}(t)] + [\Theta]\{\mathbf{u}(t)\} \qquad (3\text{-}2)$$

where $\mathbf{M_p}$ and $\mathbf{M_t}$ are the mass matrices for the beam and piezoceramic transducers respectively, similarly $\mathbf{K_p}$ and $\mathbf{K_t}$ are the stiffness matrices for the beam and piezoceramic transducers respectively, $\mathbf{q(t)}$ is a vector of generalized coordinates $q_n$, $\mathbf{Q^d(t)}$ is a vector of generalized disturbances, $\Theta$ is the electromechanical coupling matrix which relates the applied control voltages, $\mathbf{u(t)}$ to the modal equations. These equations can be cast in state variable form as follows [6]:

$$\dot{x} = \mathbf{Ax} + \mathbf{Bu}$$
$$y = \mathbf{Cx} + \mathbf{Du} \qquad (3\text{-}3)$$

47

where $\mathbf{x}$ is the state vector containing the generalized coordinates, $q_n(t)$ and their derivatives ($\mathbf{x} = \{\mathbf{q} \ \dot{\mathbf{q}}\}^T$), $\mathbf{u}$ is a vector of control and disturbance inputs, and $\mathbf{y}$ is the output from each piezoceramic transducer.

*Experimental Model*

It is common in the applications of control system and observer design, to base the design upon a model that is obtained through experimental system identification. Such a model is obtained here in order to demonstrate its unique aspects and for comparison with theoretical models. A simply supported beam experiment was constructed for observer and controls investigations, as shown in Figure 3-1. The beam is clamped at the both ends, with grooves machined near both ends to approximate simply supported boundary conditions. The material properties used to develop the beam model are those for 2024-T4 aluminum shown in Table 3-1.



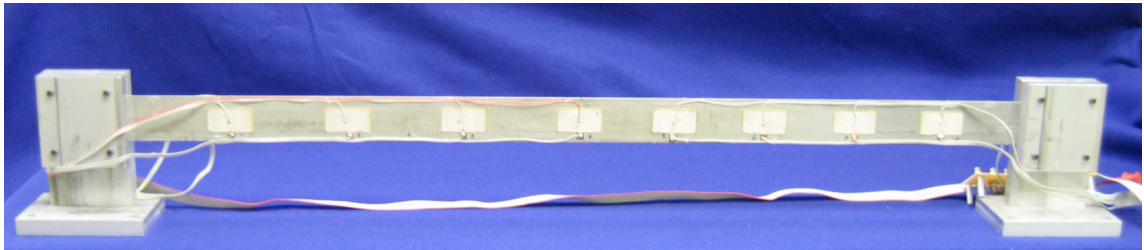Figure 3-1: Experimental set up for system identification

This work focuses on the vibrations from 0-600Hz, which includes the first nine modes. These nine natural frequencies have been theoretically predicted to be: 6.5, 26.1, 58.7, 104.3, 163.0, 234.7, 319.5, 417.3, and 528.2 Hz. System identification results discussed later show that the actual natural frequencies match the theoretical values very

well. The system was excited with broadband noise, and the input-output signals are collected. Band-limited white noise (0-600 Hz) was used as the disturbance to excite the beam, and the beam vibration was measured with PZT patches. The sensor signals were amplified and filtered with 4-pole Butterworth low-pass filters having a cut-off frequency of 600 Hz. The controller was implemented on a dSPACE DS1103 PPC board with AD/DA conversions. The control output was amplified by a 790A06 power amplifier from PZB Piezotronics, Inc. Further details on the set up, procedure and implementation can be found in [29].

Lead Zirconate Titanate (PZT) patches were used as sensors and actuators. Since PZT materials have direct and inverse piezoelectric effect when an external load is applied, an electric charge is produced at the surface of the material. Similarly, when a voltage is applied to the material, a strain is induced within. Sensor and actuator patches were attached on opposite sides of the beam, and at the same locations. A band-limited voltage was applied to PZT1 in Figure 3-1 (left most transducer) as the disturbance, whose coordinate along the beam is 0.11m. Four collocated pairs of PZT patches, PZT2, PZT3, PZT6 and PZT8 in Figure 3-1, were used as control transducers, and their coordinates are 0.25m, 0.39m, 0.75m and 0.98m respectively. The size of each PZT patch is 0.055m by 0.027m. In order to obtain the most accurate system model from which controllers were designed a system identification approach was used. When the vibration displacements of the beam are small, a linear model can reasonably represent the system. Four sensors and four actuators were used for the control system resulting in sixteen transfer functions from the inputs to outputs.

Table 3-1: Properties of the physical system

| Properties of the simply supported beam in SI units | |
|---|---|
| Plate thickness | 0.003175 m |
| Plate density | 2770 kg/m$^3$ |
| Plate length | 1.065 m |
| Plate width | 0.0508 m |
| Plate modulus | 73.1e9 Pa |
| Modal damping ratio | 0.05 |

In addition, the path between disturbance and sensors was also identified. Band-limited white noise was applied to each actuator and sensor data was collected from each sensor. This data was used to get the Auto-Regression with extra inputs model (ARX), shown in equation (3-4).

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_1 u(t-1) + \ldots + b_m u(t-m) \qquad (3-4)$$

A batch least squares solution was used to find the desired ARX parameters, and a multi-input multi-output (MIMO) state space model was then derived from the ARX model [21]. In order to choose an appropriate system order and obtain the optimal system model, the frequency responses of all signal paths were measured and system identification was performed. The final order of the identified model was chosen to be 36, since it provided the best fit with the lowest order. In Figures 3-2a and 3-2b, the transfer functions measured directly from system identification data are depicted with solid lines, and the transfer functions derived from the corresponding state-space model were shown

with dotted lines. As shown, the state space model represents the beam dynamics very well, in both magnitude and phase of the system response.
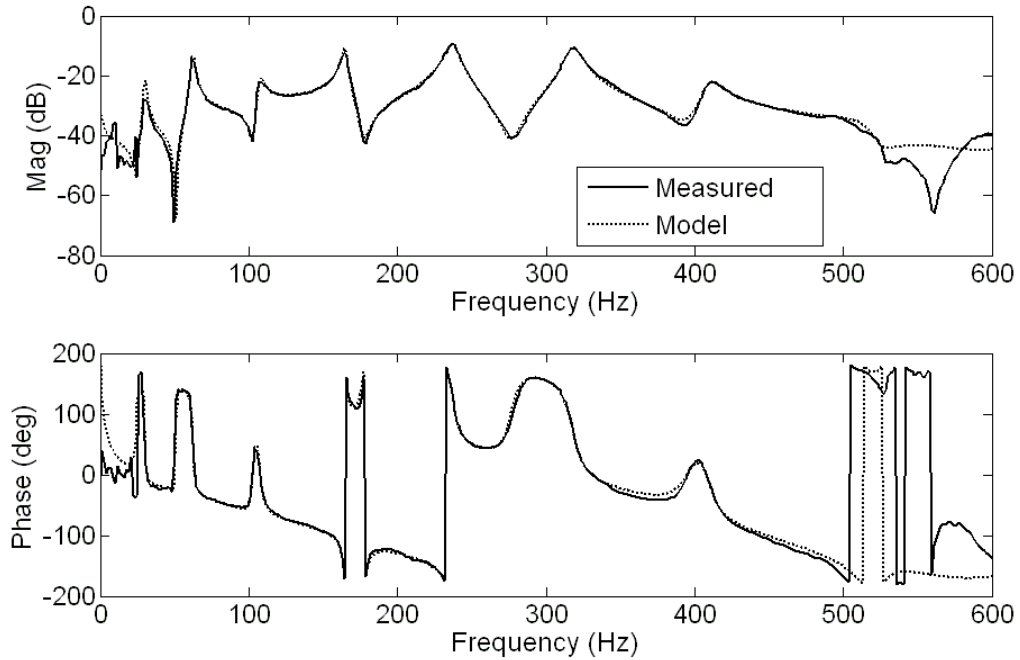


Figure 3-2a: Experimental and analytical frequency response from the disturbance to sensor 1

The discrepancy at the low frequency range (0-20 Hz) is due to the effect of environmental noise on the response measurements. All other system transfer functions compared similarly well. Furthermore, the frequency peaks of the identified model occur at 8.7, 29.3, 61.0, 105.9, 165.4, 235.8, 318.9, 413.3, and 515.4 Hz. These values compare very well with the theoretical values discussed earlier [29]. One unique aspect of this model, and one common to many experimentally derived models, is that the feed-through matrix, **D**, is non-zero. This necessitates the development of a new technique for designing FDI filters, as discussed later.
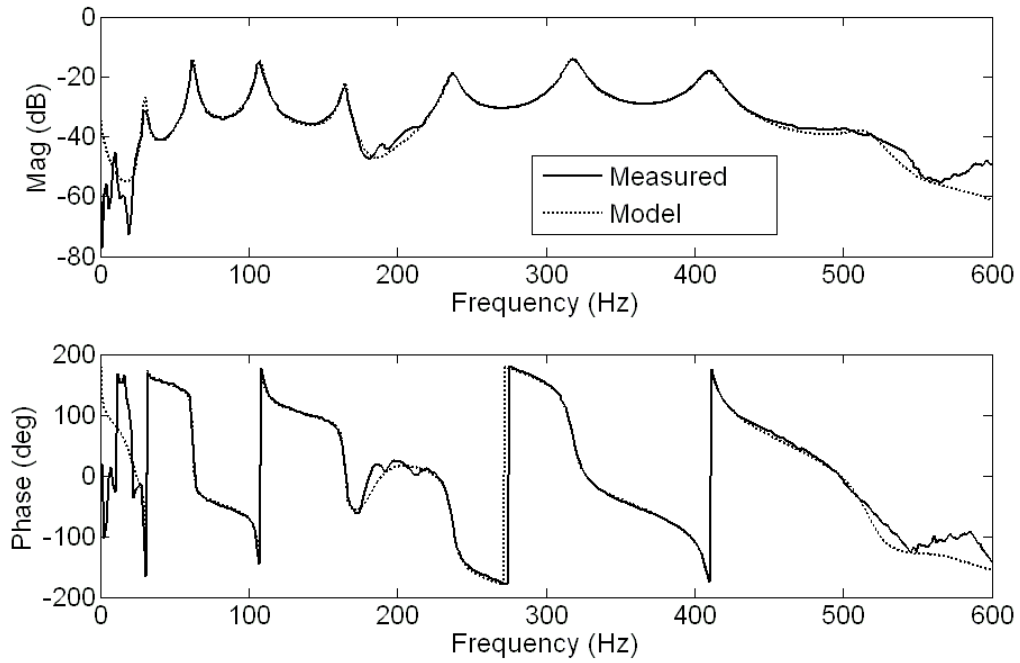
Figure 3-2a: Experimental & analytical frequency response from disturbance to sensor 4

*Compensator Design*

The control approach used in this study is constant gain output feedback. While this is a very simple form of control, it has been demonstrated to be very effective for vibration control [5]. Furthermore, the fault-tolerant system presented here could easily accommodate other, more sophisticated, compensators.

The control law implemented here is of the form

$$\mathbf{u} = -\mathbf{Ky} \tag{3-5}$$

where **K** is the feedback gain matrix, **u** is the control voltage and **y** is the output vector. The feedback gain matrix can be found by minimizing the cost function

$$J = \int_{t_0}^{t_\infty} \left[ \mathbf{x}^\mathbf{T} \mathbf{C}^\mathbf{T} \mathbf{W} \mathbf{C} \mathbf{x} + \mathbf{u}^\mathbf{T} \mathbf{R} \mathbf{u} \right] dt \tag{3-6}$$

52

where J is the cost, $\mathbf{x}$ is the system state vector, $\mathbf{C}$ is the system output matrix, $\mathbf{W}$ is the sensor signal penalty, and $\mathbf{R}$ is the control effort penalty [16]. The objective of each compensator is to minimize the sensor signal, which is related to vibration amplitude. This was accomplished by setting the performance penalty weight equal to the appropriate dimensional identity matrices multiplied by a constant. The constants are selected in order to achieve a control system performance that one could reasonably expect to obtain in a laboratory setting [5, 6, 30]. These values are held constant in all compensator designs.

In this work, each possible fault leads to a different system configuration. When a particular actuator fails, it is removed from the feedback loop and only the remaining actuators are used for closed loop control. Therefore, several controllers are designed based on each possible fault case scenario. These compensators are maintained in a controller library that is accessed whenever the system fault states changes. Further details and implementation methodology for this approach can be found in [5,16].

The four possible fault case scenarios are: 1) the nominal system with no fault, 2) first actuator failure, 3) second actuator failure and, 4) first and second actuator failures. The performance of each of these controllers under steady state conditions is shown in Figure 3, which depicts the transfer function between the disturbance input and first sensor output. The uncontrolled transfer function is also shown in Figure 3-3. The best vibration attenuation is achieved when all the actuators are operational (no fault). As one would expect, when one actuator failed the control performance is degraded and, when two actuators failed the performance is the poorest. Furthermore, it can be concluded from Figure 3-3 that a failure of the second actuator has more impact on

control performance than failure of the first actuator. This is because the second actuator was able to observe more structural modes than actuator one. However, even when there are actuator failures, system performance and system stability are still maintained. On the other hand, if the nominal controller continued to operate with failed actuators, the performance would be suboptimal and might be even unstable.
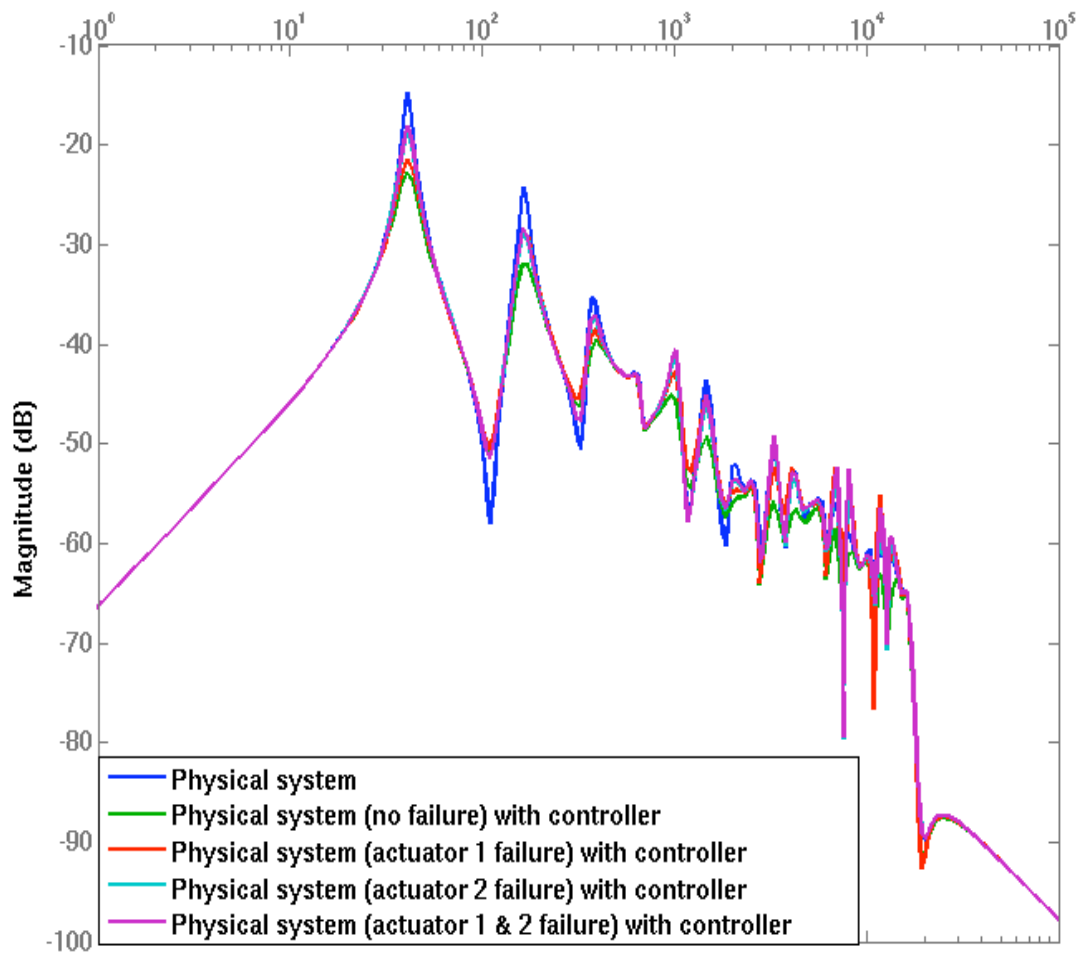


Figure 3-3: Transfer function for various fault case scenarios

BJ Detection Filter Theory and Design

BJ filters are a special case of the traditional Luyenberg observer. The difference is that for a BJ filter, the "free" parameters of the observer feedback matrix are selected in such a way that the output residual has specific directional properties when specific faults occur. Therefore, the residual can be monitored to both detect a fault, and isolate the specific fault, which has occurred. The basic theory of BJ filters is summarized in this section. This is followed by the development of two modifications to existing feedback matrix design techniques. The first modification enables BJ filters to be designed for systems with feed-through dynamics while the second modification presents a gain matrix design suitable for high order systems.

*The Traditional BJ Filter*

BJ detection filters are traditional observers designed in such that the output residual vector has specific directional properties that can be associated with specific faults [12,13,14,15,24,25,31,32]. Assuming we have a system model in the absence of failures, as given in Equation (3-3), the BJ filter is of the form

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \qquad (3\text{-}7)$$

where $\hat{\mathbf{x}}$ is the state estimate and $\mathbf{L}$ is the detection gain matrix. The state error is as

$$\varepsilon \overset{\Delta}{=} \mathbf{x} - \hat{\mathbf{x}} \qquad (3\text{-}8)$$

Now the observer gain matrix $\mathbf{L}$ is chosen in such a way that the output error

$$\bar{\varepsilon} = \mathbf{y} - \mathbf{C}\hat{\mathbf{x}} \qquad (3\text{-}9)$$

has restricted directional properties in the presence of a failure. Therefore, when there are no faults present, the closed loop dynamics become

$$\dot{\varepsilon} = \mathbf{G}\varepsilon \qquad (3\text{-}10)$$

where $\mathbf{G}$ is defined as

$$\mathbf{G} \overset{\Delta}{=} \mathbf{A} - \mathbf{LC} \qquad (3\text{-}11)$$

The presence of an additive fault, especially an actuator fault, can be modeled by adding a term to the open loop dynamic system shown in Equation (3-3) such that

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{f_i}\mu_i \qquad (3\text{-}12)$$

where $\mathbf{f_i}$ is a nx1 design failure direction associated with the $i^{th}$ actuator failure and $\mu_i$ is a time-varying scalar which may be function of x(t) or u(t). Thus, the system output error in the presence of faults becomes

$$\dot{\varepsilon} = \mathbf{G}\varepsilon + \mathbf{f_i}\mu_i$$
$$\bar{\varepsilon} = \mathbf{C}\varepsilon \qquad (3\text{-}13)$$

The detection gain is designed in such a way that the directionality of the residual, $\bar{\varepsilon}$, corresponds to specific faults. Design procedures are presented by Beard [1] and Jones [11], and more recently by Kim et al. [12,13,14,15]. In this case, $\bar{\varepsilon}$ is proportional to $\mathbf{Cf_i}$ in response to a failure corresponding to the direction $\mathbf{f_i}$. It should be noted that the sensor failure is similar to the actuator failure as can be found in [12,13,14,15,24,25,26,31,32].

*Design of BJ Filter with Feed-Through Dynamics*

It is not uncommon to encounter systems with feed-through dynamics (i.e. $\mathbf{D} \neq \mathbf{0}$), particularly when the working model results from model order reduction or system identification. However, there is as yet no way of dealing with this situation when designing BJ filters. This is because a potential actuator failure has a direct effect on the output of the system (which in turn can be interpreted to be a sensor failure). In order to

avoid the confusion and isolate the actuator failure, a new BJ filter design method is presented for this particular case.

Consider a system similar to Equation (3-3), but with a non-zero **D** matrix. In addition to this consider a failure in actuator one. This particular case can be described

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{b_1}\delta u_1$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{d_1}\delta u_1$$

(3-14)

where $\mathbf{b_1}$ and $\mathbf{d_1}$ are the first column vectors of **B** and **D** respectively, and $\delta u_1$ represents the deviation of the first input caused by the failure in the actuator one. In order to develop a BJ filter it is assumed that $\delta u_1$ behaves according to first order dynamics such that

$$\frac{d\delta u_1}{dt} = \alpha \delta u_1 + \eta$$

(3-15)

where $\alpha$ and $\eta$ are constants. If the BJ theory fault is considered to be $\mu = \delta u_1$, then Equations (3-14) and (3-15) can be combined in to a new state-space form

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b_1} \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mu \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C} & \mathbf{d_1} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mu \end{bmatrix} + \mathbf{Du}$$

(3-16)

Now, the usual BJ detection design outlined previously can be used on the appended system of Equation (3-16). It is possible that the appended system of Equation (3-16) may not meet all the requirements necessary to implement a BJ filter (i.e. observabilty and mutual detectability). In such a case other means must be employed to design or implement fault detection filters.

*Design of Detection Gain Matrix*

There are several gain selection methods available that work well for low order systems [1,11,12,13,14,15]. However, when the system order is large (greater than 10 or so), it is very difficult to use these methods and achieve a stable closed-loop detection filter.

In order to overcome this difficulty an unstable detection gain matrix is created using the invariant zero approach, and then modified to ensure a stable result. In most physical the detection orders are equal to one, which means that, for a given fault vector $\mathbf{f}$, the triplet $(\mathbf{A, f, C})$ has no invariant zero [12,13,14,15]. In this case, the invariant zero approach yields a detection gain matrix $\mathbf{L}$ that is given by

$$\mathbf{L} = (\mathbf{AF} - \mathbf{F\Delta})(\mathbf{CF})^* \tag{3-17}$$

where $\Delta$ is a diagonal matrix whose elements are given as the eigenvalues associated with the detection space of $\mathbf{F}$ and * indicates pseudo-inverse. Next, the result of equation (3-17) is modified to ensure a stable result according to the Equation (3-18)

$$\mathbf{L}' = \mathbf{L} + \mathbf{E}(\mathbf{I} - (\mathbf{CF})(\mathbf{CF})^* \tag{3-18}$$

If the observer gain $\mathbf{L}'$ is applied to error state Equation (3-10) then,

$$\mathbf{G} = (\mathbf{A} - \mathbf{L}'\mathbf{C}) = (\mathbf{A} - \mathbf{LC}) - \mathbf{E}(\mathbf{I} - (\mathbf{CF})(\mathbf{CF})^*)\mathbf{C} \tag{3-19}$$

Now, if we define $\mathbf{A_f} = \mathbf{A} - \mathbf{LC}$ and $\mathbf{C_f} = [\mathbf{I} - (\mathbf{CF})(\mathbf{CF})^*]\mathbf{C}$, then $\mathbf{G} = \mathbf{A_f} - \mathbf{EC_f}$ and the dual of $\mathbf{G}$ is $(\mathbf{A_f^T} - \mathbf{C_f^T E^T})$. One will note that the definition of $\mathbf{G}$ is equivalent to the LQR control problem of finding an $\mathbf{E}$ that stabilizes $(\mathbf{A_f, C_f})$ and minimizes

$$J = \sum \left[ \mathbf{z^T Q z} + \mathbf{m^T R m} + 2\mathbf{z^T N m} \right] \tag{3-20}$$

where $\mathbf{z}$ and $\mathbf{m}$ are the states and inputs associated with the new pair $(\mathbf{A_f, C_f})$. The weights $\mathbf{Q}$ and $\mathbf{R}$, can be altered to affect the closed-loop eigenvalues of the filter. It is

58

assumed $\mathbf{N} = \mathbf{0}$ in these calculations. Finally, once a suitable matrix $\mathbf{E}$ is obtained the modified BJ filter gain matrix $\mathbf{L'}$ can be obtained from Equation (3-18).

<u>Hybrid System Simulation</u>

Simulations of the fault-tolerant control system require a hybrid approach [10] due to the discrete switching between the compensators. This is accomplished by using a hybrid automaton to monitor the residual and execute controller switch. This methodology is shown in Figure 3-4. The hybrid model consists of the physical system or plant; a library of pre-designed controllers; the BJ filter; and the automata switch.
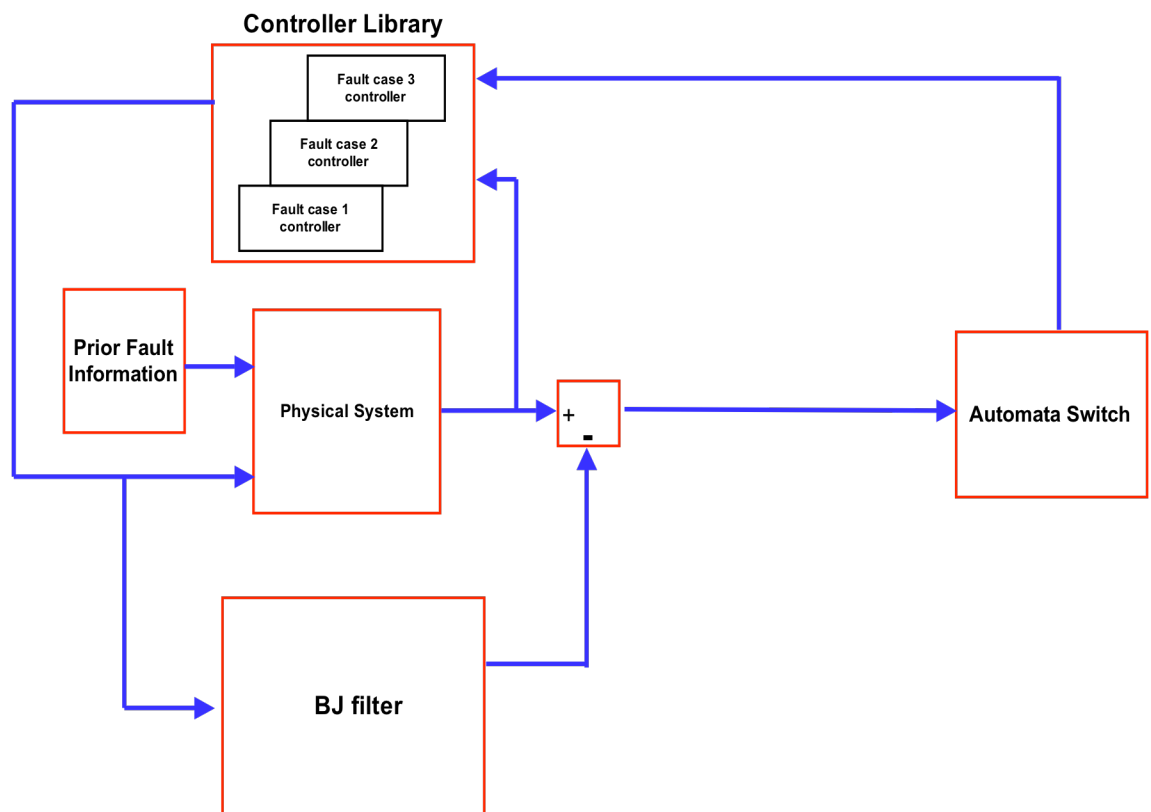


Figure 3-4: Hybrid model for Beard-Jones FDI

A simulation consists of starting the system in the nominal state. A fault is simulated by injecting a non-zero input into the system. Due to the presence of fault, the physical system response deviates from that predicted by the BJ filter, resulting in a non-zero residual output from the filter called the continuous residual. When the continuous residual exceeds a pre-set threshold values, the corresponding entry in the discrete residual vector is set to one. If the continuous residual value falls below the threshold, the discrete vector is set to zero. So, the discrete residual vector can take one of the following forms

$$r = \begin{cases} [0\ 0\ 0] \text{ - No Fault} \\ [1\ 0\ 0] \text{ - Fault Case 1} \\ [0\ 1\ 0] \text{ - Fault Case 2} \\ [1\ 1\ 0] \text{ - Fault Case 3} \end{cases}$$

(3-21)

This discrete residual vector is supplied to the automaton switch, which in turn selects the appropriate controller based on the fault case. The automaton is capable of detecting and switching between any of the four fault cases. One of the key features in the BJ filter is that it is able to detect multiple faults with a single filter; referred to as mutual detectability. Therefore, a single BJ filter is used to observe the system in any of the four fault modes. Not all systems are mutually detectable, however in systems where it is not; multiple BJ filters are designed and operated in parallel.

Results

In order to establish the capabilities of a fault-tolerant control system under various conditions, four different cases are considered and simulated, as described in the previous section. These cases are: 1) a high order plant with a full order BJ filter, 2) a

60

high-order plant with a truncated BJ filter, 3) a reduced-order plant with a reduced-order BJ filter, and 4) an experimentally obtained high order plant with a full order BJ filter. The necessary condition to study these cases is explained in detail in the following sections.

*High Order Plant with Full Order BJ Filter*

Many of the physical systems in active vibration control are continuous (i.e. beams, plates and shells) and, therefore require higher order models for more accurate representation. The first case considered here is modeled using 40 states (i.e. modes = 20 in Equation (3-1)). The BJ filter and feedback compensators are designed based on this $40^{th}$ order model. One of the main difficulties in designing the BJ filter of this order is to ensure that the large number of closed-loop eigenvalues lie the left-hand plane (LHP). This is achieved by using the LQR idea as described in previous section to design the detection filter gains.

Simulation results are shown in Figure 3-5 for fault-tolerant control of such a system. Figure 3-5 shows time histories of (a) the fault inputs, (b) the output from sensor one, (c) the continuous residual, and (d) the fault mode. Note that faults occur in actuator one 1 at 0.2 and actuator 2 at 0.4 seconds respectively. The continuous residual rises quickly to a steady state value when the fault occurs. Once the continuous residual crosses the pre-defined threshold a fault is deemed to have occurred and the fault mode (which began in mode one or no fault case) changes to mode two and the appropriate controller is selected from the controller library. When the controller switch occurs a transient is noted in the sensor but these quickly decay over time and the system

61

performance returns to normal. The results of Figure 3-5 demonstrate the ability of such a fault-tolerant control system to work well in a vibration control application. However, one key aspect of these systems is that they typically have dynamics, which are not included in the plant model. This case is considered in the following section.



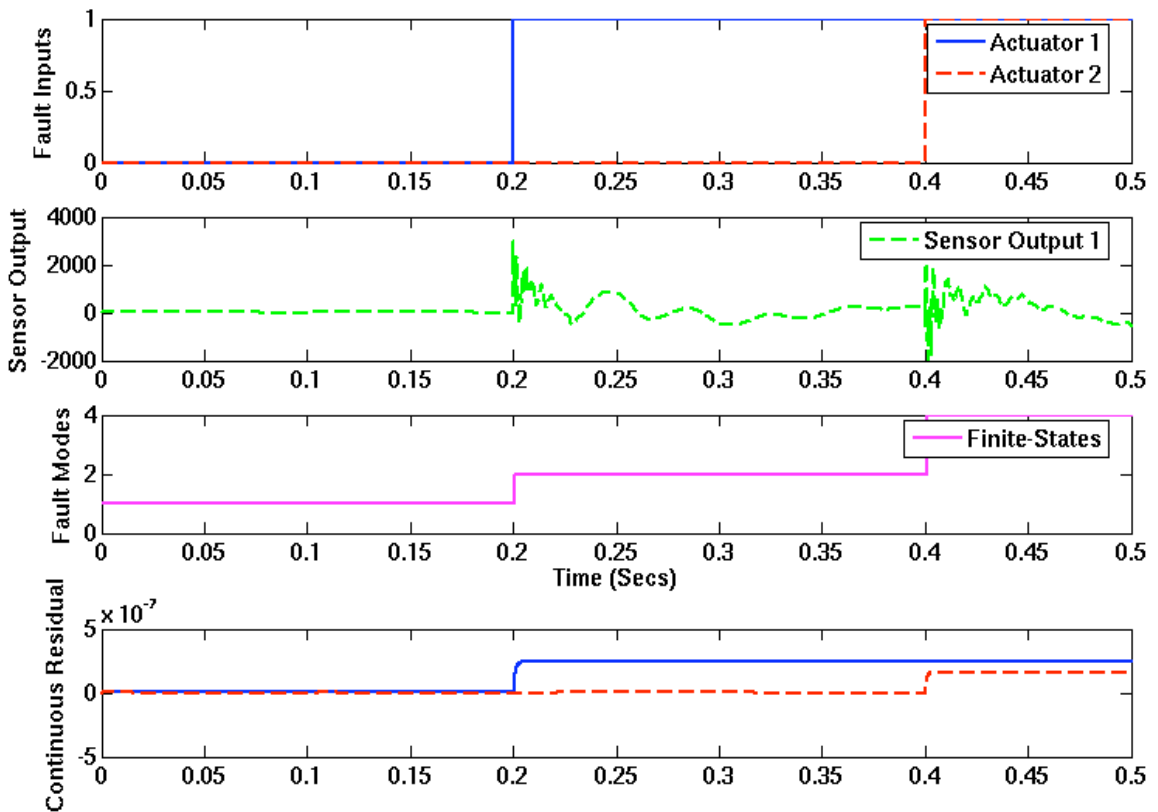Figure 3-5: Time history of fault inputs, desired response, fault modes, and continuous residual for 20 modes theoretical model

*High Order Plant with Truncated BJ Filter*

Since the continuous systems (such as beams, plates and shells) theoretically possess an infinite number of modes, it is common to develop observer and controllers based on the truncated system models. To demonstrate the effectiveness of BJ filters and

fault-tolerant control in this case a theoretical systems model was created with six states (i.e. modes = 3 in Equation (3-1)). A BJ filter and a controller library were designed based on this sixth order system. Furthermore, the BJ filter was designed using the invariant zeros approach described in above section. However, simulations were run using a 40 state (20 mode) beam model.

Results of this simulation are shown in Figure 3-6, which shows the time histories of the fault, the continuous residuals, the fault modes, and a sensor signal. Since the threshold between continuous and discrete residual is user defined, a time delay is expected between the occurrence of failures and switching of the fault modes. This time delay helps in ensuring that system output error will not produce false positives in the detection process. In this particular case, since the plant has higher order dynamics than the BJ filter, it results in false positives. But the time delay (threshold period is user defined) takes care of the output error associated with the higher order dynamics. Therefore, Figure 3-6 upholds the justification of multiple faults and switching of controllers back and forth for this particular physical system. But in some analysis the threshold value might not be able to eradicate the false positives, and detect faults that are not present as shown in Figure 3-7. One of the prime reasons to design a BJ filter with a $40^{th}$ order (the first case) is if the amplitudes of the false positives is grater than or equal to the threshold condition, then switching of the controllers occurs. Thus, even though there were no faults, the filter concurs that fault occurred. Therefore, it is important to study the first case in which the plant has $40^{th}$ order model and BJ filter design based on $40^{th}$ order.
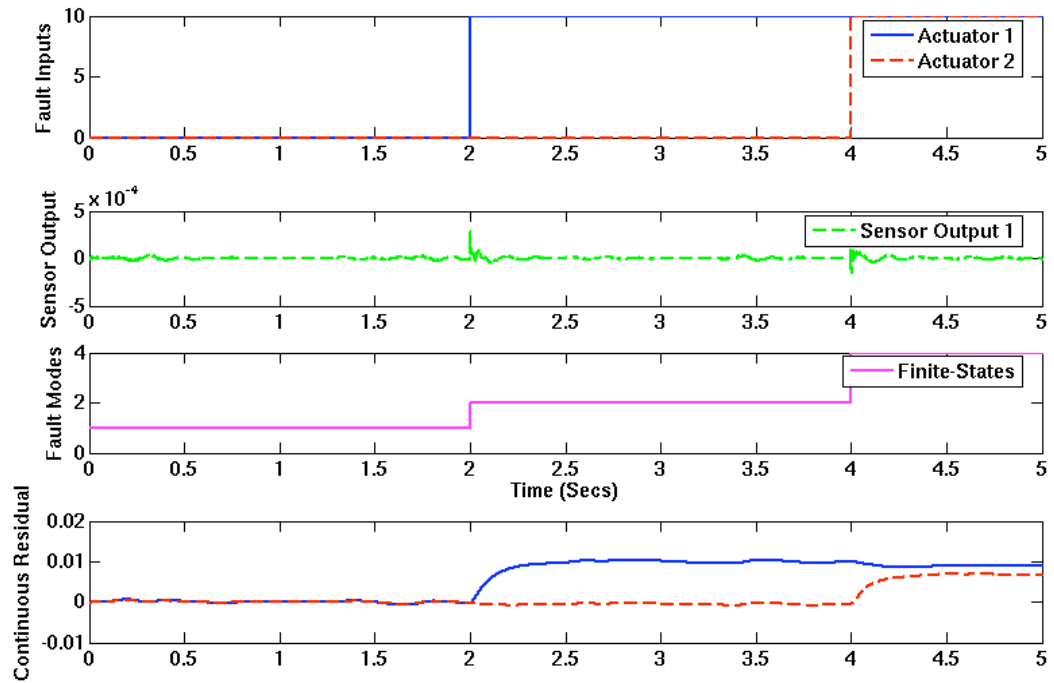
Figure 3-6: Time history of fault inputs, desired response, fault modes, and continuous residual for 20 modes theoretical model (BJ filter design based on three modes theoretical model)
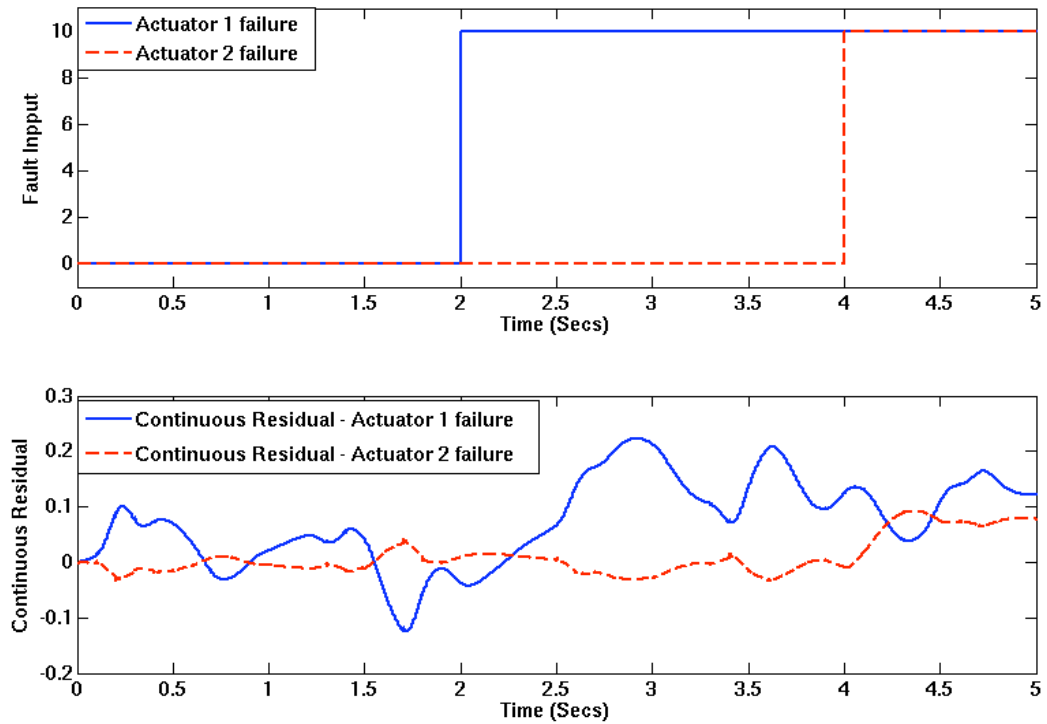


Figure 3-7: Time history explaining the limitation of 20 modes theoretical model (BJ filter design based on three modes theoretical model) due to the false positive

64

*Experimental High Order Plant with High Order BJ Filter*

Most of the systems that are modeled experimentally, frequently encounters with feed-through dynamics. This particularly is common when the system model is developed from system identification methods. A method for designing BJ filter is discussed in above section. The system model is based on the beam experiment shown in Figure 3-1.
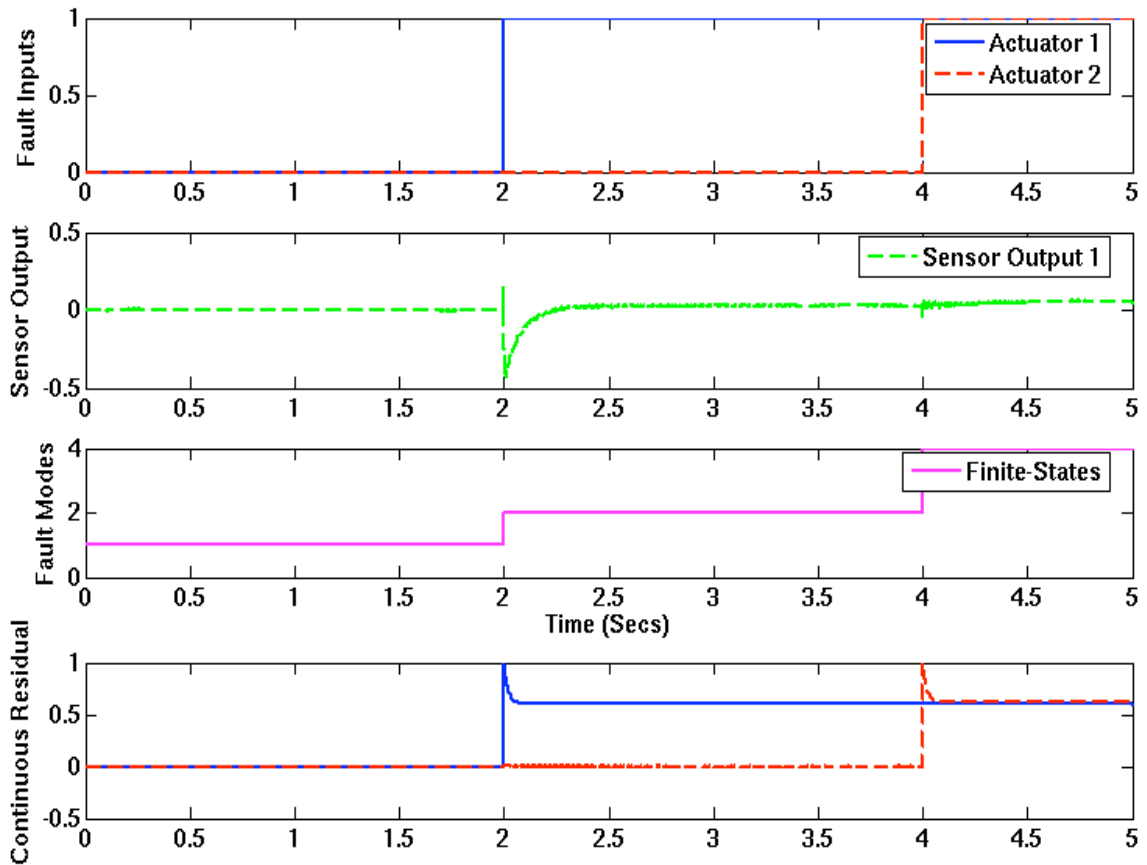


Figure 3-8: Time history of fault inputs, desired response, fault modes, and continuous residual for $36^{th}$ order system (experimental model) with $\mathbf{D} \neq \mathbf{0}$

The final model is a $36^{th}$ order, and the resulting BJ filter design is based on $37^{th}$ order. Also, the LQR method is used to design the BJ filter feedback matrix to ensure that the closed-loop poles are in the LHP, as described in above section. The results of the simulation are shown in Figure 3-8. This figure explains the concept of multiple faults

and switching of controllers back and forth for this particular physical system. The convergence of the continuous residual depends of the value of $\alpha$. The computational time for simulation is far less when compared to the first case. This case helps to design the BJ filter for experimental validation of FDI for active vibration control.

## Comments on Stability

When there is switching between various controllers, the core issue that needs to be dealt with is the importance of stability for the linear systems. Figure 3 clearly established the fact the system is stable if it is individually operational. To assess the various conditions under which the system might be unstable is a vast area of research and also, an open problem [19,20]. The key area for the open problem is the threshold period; how the system behaves. It's been concluded that for general cases of higher order systems and more than two fault modes, the existence of a common quadratic Lyapunov function for a switched systems is still lacking and an open problem. Additional methods [17,18,19] are proposed to show the stability of switching for linear systems (not higher order systems) can be accomplished under certain conditions, which in turn remains an area of interest for many researchers.

## Conclusions

The main objective of this work is to implement the concept of FDI for active vibration control. In this work, a new method is proposed to design a BJ filter for certain case of higher order systems. Simulations are conducted for various orders of the physical systems. Also, the methodology to design the filter for various order systems is studied in

66

detail. The primary contribution of this work is for fault-tolerant control using hybrid system management. This work was further extended on the experimental data associated with the system identification of the physical system.

References

[1] R.V. Beard, Failure Accommodation in Linear System through Self-Reconfiguration, Ph.D. Thesis, Massachusetts Institute of Technology, MA, 1971.
[2] C. Byreddy, K.D. Frampton, Simulations of Fault-Adaptive Vibration Control, ASME International Mechanical Engineering Congress and Exposition, Florida. 2005.
[3] C. Byreddy, K.D. Frampton, Fault-Tolerant Vibration Control for Additive Faults, Journal of Intelligent Materials and Structures, 2006. (Submitted for Publication)
[4] J. Chen, J. Patton Ron, Zhang Hong-Yue, Design of Unknown Input Observers and Robust Fault Detection Filters, International Journal of Control, 63, 1996.
[5] R.L. Clark, D.E. Cox, Multi-Variable Structural Acoustic Control with Static Compensation, Journal of Acoustic Society of America, 1997.
[6] R.L. Clark, W.R. Saunders, G.P. Gibbs, Adaptive Structures: Dynamics & Control, 1$^{st}$ Edition, John Wiley & Sons Inc., New York, 1998.
[7] R.K. Douglas, J.L. Speyer, Robust Fault Detection Filter, AIAA Journal of Guidance, Control and Dynamics, 19, 1996.
[8] C.R. Fuller, S.J. Elliot, P.A. Nelson, Active Control of Vibration, 1$^{st}$ Edition, Academic Press, London, 1997.
[9] N.W. Hagwood, W.H. Chung, A. Von Flotow, Modeling of Piezoelectric Actuator Dynamics for Active Structural Control, Journal of Intelligent Material Systems and Structures, 1, 327-354, 1990.
[10] HYTECH, Homepage: http://embedded.eecs.berkeley.edu/research/hytech//
[11] H. Jones, Failure Detection in Linear Systems, Report No. T-608, The Charles Stark Draper Laboratory, Cambridge, MA, 1973.
[12] Y. Kim, J. Park, An Analysis of Detection Spaces using Invariant Zeros, American Control Conference, San Diego, California, USA, 1999.

[13] Y. Kim, J. Park, A Condition of the Eigenvalues of Detection Filters for Disturbance Attenuation: An Invariant Zero Approach, Conference on Decision and Control (CDC), Hawaii, USA, 2003.

[14] Y. Kim, J. Park, On the Noise Response of Detection Filters: A Special Consideration on the Relation Between Detection Space and Completion Space, IEEE Proceedings – Control Theory and Applications, 150, 4, 443-447, 2003.

[15] Y. Kim, J. Park, On the Approximation of Fault Directions for Mutual Detectability: An Invariant Zero Approach, IEEE Transactions on Automatic Control, 50, 6, 851-855, 2005.

[16] W. Levine, M. Athans, On the Determination of the Optimal Constant Output Feedback Gains for Linear Multivariable Systems, IEEE Transaction of Automatic Control, 15, 44-80, 1970.

[17] D. Liberzon, J.P. Hespanha, A.S. Morse, Stability of Switched Linear Systems: A Lie-Algebraic Condition, Systems and Control Letters, 37, 117-122, 1999.

[18] D. Liberzon, R. Tempo, Gradient Algorithm for Finding Common Lyapunov Function, IEEE Conference Decision Control, 4782-4787, 2003.

[19] H. Lin, P.J. Antsaklis, A Necessary and Sufficient Condition for Robust Asymptotic Stabilizability of Continuous-Time Uncertain Switched Linear Systems, IEEE Conference Decision Control, 2004.

[20] H. Lin, P.J. Antsaklis, Stability and Stabilizability of Switched Linear Systems: A Short Survey of Recent Results, International Symposium on Intelligent Control & Mediterranean Conference on Control and Automation, 2005.

[21] L. Ljung, System Identification: Second Edition, Prentice Hall Information and System Sciences Series, New Jersey, 1999.

[22] M. Maroti, K.D. Frampton, G. Karsai, S. Bartok, A. Ledeczi, Experimental Platform for Studying Distributed Embedded Control Applications, Proc. Languages, Compilers, and Tool for Embedded Systems Conference, Berlin, 2002.

[23] L. Meirovitch, Dynamics and Control of Structures, Wiley Interscience, New York, 1990.

[24] J. Park, A Unified Theory of Fault Detection and Isolation in Dynamic Systems, Ph.D. Dissertation, The University of Michigan, Ann Arbor, 1991.

[25] J. Park, G. Rizzoni, An eigenstructure assignment algorithm for the design of fault detection filters, IEEE Trans., AC-39, 1521-1524, 1994.

[26] G. Rizzoni, P.S. Min, Detection of sensor failures in automotive engines, IEEE Trans., 38, 148-158, 1989.

[27] R. Scattolini, N. Cattane, Detection of sensor faults in a large flexible structure, IEEE Proceedings-Control Theory Applications, 145, 5, 1999.

[28] L.A. Sievers, A.H. Von Flotow, Comparison and Extensions of Control Methods for Narrowband Disturbance Rejection, IEEE Trans. Signal Processing, 40, 2377-2391, 1992.

[29] T. Tao, K.D. Frampton, Distributed Vibration Control with Sensor Networks, SPIE 13th Annual Symposium Smart Structures and Materials NDE for health Monitoring and Diagnostics, San Diego, California, 2006.

[30] J.S. Vipperman, Adaptive Piezoelectric Sensoriactuators for Active Structural Acoustic Control, Ph.D. Thesis, Duke University, 1996.

[31 J.E. White, Detection Filter Design by Eigensystem Assignment, Ph.D. Dissertation,

University of Texas, Austin, 1985.

[32] J.E. White, J. Speyer, Detection Filter Design: Spectral Theory and Algorithms, IEEE Trans., AC-32, 593-603, 1987.

CHAPTER IV

MANUSCRIPT 3

EXPERIMENTAL VERIFICATION OF BEARD-JONES FAULT DETECTION AND
ISOLATION FILTER FOR ACTUATOR FAILURES IN ACTIVE VIBRATION
CONTROL

Chakradhar Byreddy[1] Tao Tao[2]  Kenneth D. Frampton[3] Yongmin Kim[4]

*(Submitted to Journal of Smart Materials and Structures)*

Abstract

In this paper Fault Detection and Isolation (FDI) method is integrated with active
vibration control of a flexible structure. Beard-Jones (BJ) filter is chosen as the
appropriate detection method for the application of active vibration control with the
presence of multiple actuator failures. This paper investigates the difficulties in the
implementation and limitations of the BJ filter for vibration control of a simply supported

---

[1]Department of Mechanical Engineering, Vanderbilt University, 2301 Vanderbilt Place,
Nashville, TN-37235. Email: chakradhar.r.byreddy@vanderbilt.edu, Phone: 1-615-322-
1760, Fax: 1-615-343-6687.
[2]Graduate Research Assistant, Department of Mechanical Engineering, Vanderbilt
University, 2301 Vanderbilt Place, Nashville, TN-37235. Email: tao.tao@vanderbilt.edu,
Phone: 1-615-322-1760, Fax: 1-615-343-6687.
[3]Assistant Professor, Department of Mechanical Engineering, Vanderbilt University,
2301 Vanderbilt Place, Nashville, TN-37235. Email: ken.frampton@vanderbilt.edu,
Phone: 1-615-322-2778, Fax: 1-615-343-6687.
[4]CTO, Nano-Tronix, 3FI, Younghan bldg., 1546-3, Seocho-Dong, Seocho-gu, Seoul 137-
872, South Korea. Email: kym@nano-tronix.com, Phone: +82-25819520, Fax: +82-
25819511.

beam. Furthermore, this paper provides a new design technique for the design of the detection filter in discrete-time. Experimental results are presented to show the effectiveness and limitations of the BJ filter. In this experiment, the beam vibration control is conducted for the structural modes under 600Hz. In addition, the results demonstrate the capability to conduct experiments for achieving fault-tolerant active vibration control for additive faults (actuator or sensor failures).

## Introduction

An important criterion for the safety in dynamical systems is to incorporate the idea of Fault Detection and Isolation (FDI) in to the plant. Most of the development of FDI is inspired by large scale and complex systems that perform critical tasks. The idea behind FDI is to detect and isolate the kind of failure and compensate for the failure (fault-tolerant). Researchers developed new design techniques and concepts in the field of FDI in the early 1980s and mid 1990s [1,4,6]. These studies can be broadly classified in to different categories: a. Algorithms based on kalman filters b. Parity space techniques c. Diagnostic observers and, d. Parameter estimation methods. Advantages and limitations of various FDI methods mainly depend on the kind of the dynamical systems.  In area of active vibration control specifically for multiple actuator failures. parity space techniques and diagnostic observers are studied in detail [2,3]. Parity space techniques are limited to low order systems and corresponds to open-loop direction residual formation [3]. In addition to this, there is an instability factor associated with the inversion in the calculation of the residual using the parity space techniques. Therefore, diagnostic observer called the Beard-Jones (BJ) detection filter is used in this research work. The BJ

filter constitutes a technique for generating closed-loop residuals that have certain directional characteristics. The active vibration control on a simply supported beam is achieved by a closed-loop control system.

Beard [1] detection filter mainly focuses on the determination of a cyclic basis representation for the closed-loop system in which the output error has certain directional properties. Beard adopted the matrix algebra techniques for the calculation of the detection gain matrix. Jones [6] extended the Beard's work by using linear operators and vector space techniques. Later, Massoumnia [12] used geometric approach for the design of the detection gain matrix. Furthermore, Jones incorporated many failure directions for a single detection filter (multiple failures). Both the approaches and methodologies for designing the detection gain matrix are rather indirect and overly complicated, which are unfamiliar to the most engineers. In the mid 1980's White and Speyer [17,18] proposed a spectral technique for the design of detection gain matrix by assigning the eigenvalues and eigenvectors directly. Therefore, the detection filter needs to be constructed inside and outside the detection space simultaneously, which is a limitation of this method. In order to avoid this problem, Kim and Park [7,8,9,10] proposed a methodology based on the invariant zero approach.

The current invariant zero approach for the design of detection filter is mainly applicable to low order systems. This limitation is particularly challenging when the application is active vibration control. This is because the systems of interest (beams, plates and shells) require very large order models to ensure accuracy. Furthermore, the design of the detection gain matrix needs to be carried out in discrete-time for the real-time analysis. A new procedure for the design of detection gain matrix in discrete-time is

72

proposed for large order systems. Therefore, the primary interest of this research work is to: a) study the new design technique and, b) demonstrate experimentally the use of such filters in active vibration control application. The discussion begins with the description of the experimental setup and followed by the new design technique. Finally, the experimental results that demonstrate the effectiveness of this method for multiple failures in active vibration control are presented. This work later is applied experimentally to conduct fault-tolerant active vibration control.

## Experimental Setup

The instrumentation arrangement in our experimental setup is shown schematically in Figure 4-1. The physical system is a beam made of aluminum 2024-T4, and the physical parameters are listed in Table 4-1.

Table 4-1 Properties of the flexible structure

| Plate thickness | 0.003175 m |
|---|---|
| Plate density | 2770 kg/m$^3$ |
| Plate length | 1.065 m |
| Plate width | 0.0508 m |
| Plate modulus | 73.1e9 Pa |
| Modal damping ratio | 0.05 |

The beam is clamped at both ends, with grooves machined near both ends to approximate the simply supported boundary condition. System identification results in our experiments showed that the beam's dynamic responses are very close to a simply supported beam.
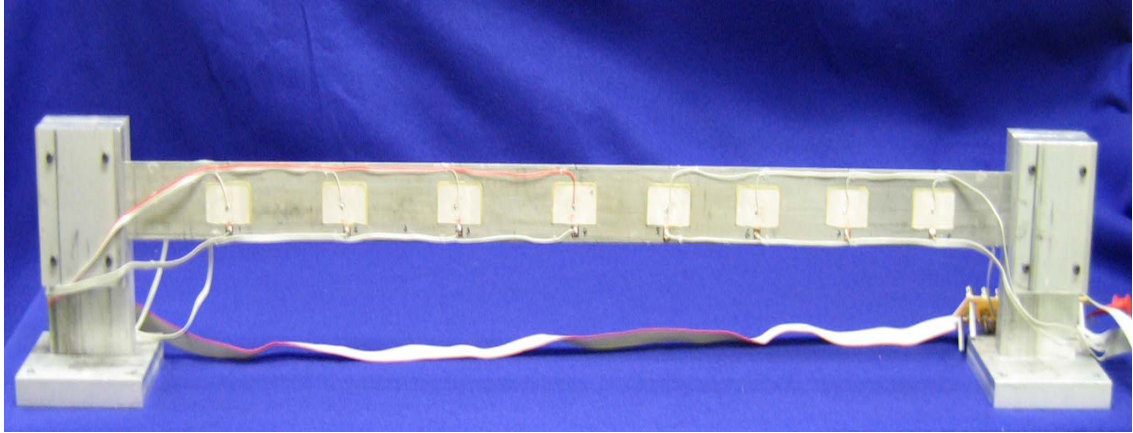
Figure 4-1. Experimental setup for FDI with multiple actuators/sensor pairs

Lead Zirconate Titanate (PZT) transducers are attached along the beam acting as sensors and actuators, shown in Figure 4-2. The size of each PZT patch is 0.055m by 0.027m, and patches are not evenly distributed along the beam. All sensors are on the same side of the beam, and all actuators are on the opposite side. The band-limited noise is applied to PZT1 (left most transducer, with the coordinate 0.11m) as the disturbance. Although the placements of the transducers will affect the control performance, the transducers were chosen randomly in our experiments for a general purpose.

The instrumentation arrangement is shown schematically in Figure 4-2. Band-limited white noise (0-600 Hz) is used as the disturbance to excite the beam, and the beam vibration is measured with PZT patches. The sensor signals are amplified and filtered with 4-pole Butterworth low-pass filters having a cut-off frequency of 600 Hz. The controller is implemented on a dSPACE DS1103 PPC board with AD/DA conversions. The control output is amplified by a 790A06 power amplifier from PZB Piezotronics, Inc.

<u>System Identification</u>

As specified previously, the system model obtained from theoretical derivation matches the experimental result pretty well. However, since the performance of the controller design depends much on the accuracy of the system model, the dynamics of the beam was obtained with system identification technique.
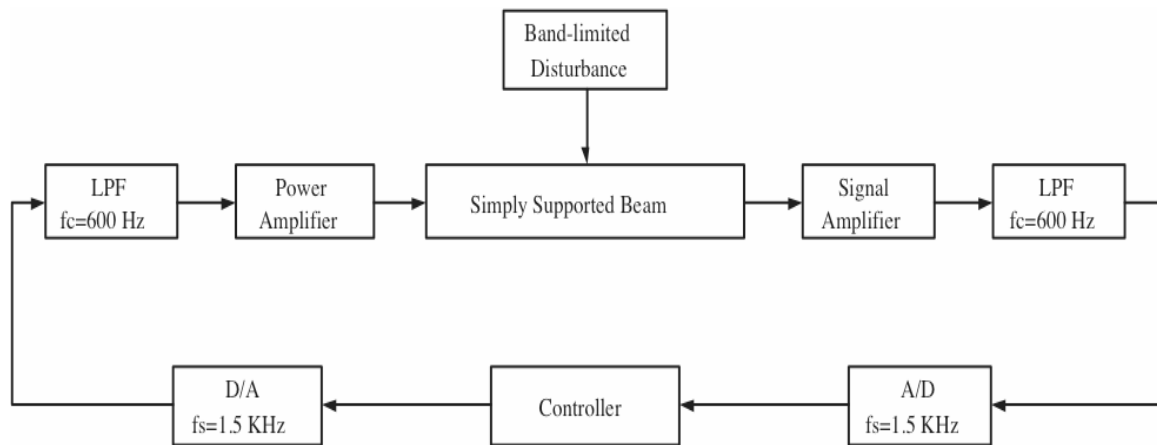


Figure 4-2. Schematic representation of the experimental setup for closed loop system

Six sensors and six actuators are used for the control system, which results in 36 transfer functions from the inputs to outputs. In addition, the path between disturbance and sensors is also identified. A band-limited white noise is applied to each actuator, and then all sensor and actuator data are collected to derive the Auto-Regression with extra inputs (ARX) [11] model:

$$y(t)+a_1y(t-1)+\ldots+a_ny(t-n)=b_1u(t-1)+\ldots+b_mu(t-m) \tag{4-1}$$

The ARX parameters are obtained using batch least squares solution, and then a multi-input multi-output (MIMO) state-space model is derived from the corresponding ARX model in the following form

$$x[n+1] = Ax[n] + Bu[n]$$
$$Y[n] = Cx[n] + Du[n]$$

(4-2)

The order of the system model in our experiment was chosen to be 36, and frequency responses of all signal paths match the experimental results really well. Experimental and analytical frequency responses of two signal paths are shown in figures 4-3 and 4-4. In figure 4-3, the solid lines represent the transfer function from the disturbance to sensor 1 measured directly from system identification data, while the dotted lines represent the derived state-space model with 36 states.
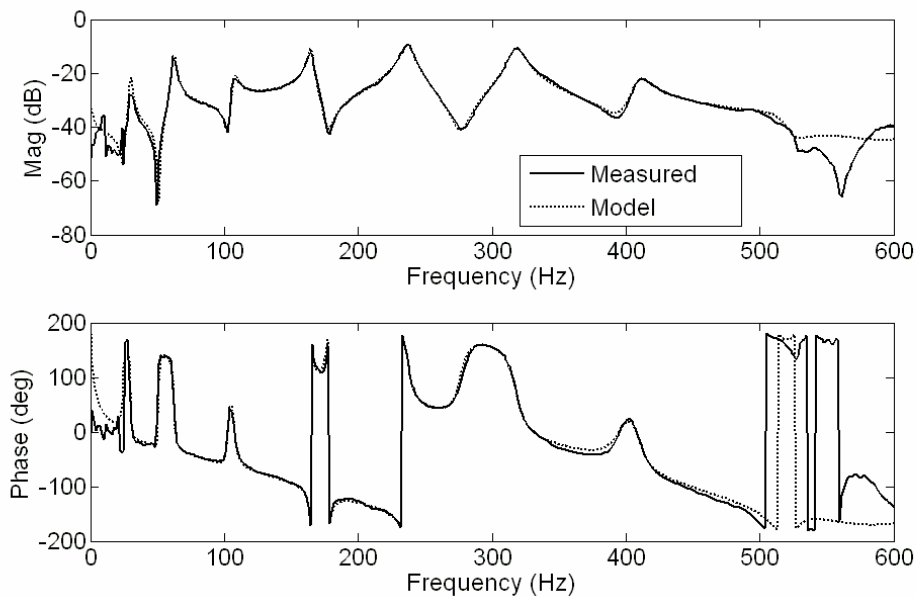


Figure 4-3. Experimental and analytical frequency responses from the disturbance to sensor 1

It is cleared shown that the state-space model represents the beam dynamics very well, in both magnitude and phase of the system response. Similarly, the transfer function from the disturbance to sensor 4 also shows good match between experimental and analytical responses, as in figure 4-4. All other signal paths have similar results. Another

important characteristic of the control system is also shown in figures 4-3 and 4-4, which is the unique modal sensitivity for different sensors. The modal sensitivity difference is because of the locations of different sensors, which serves as the criteria when implementing modal grouping strategy later.
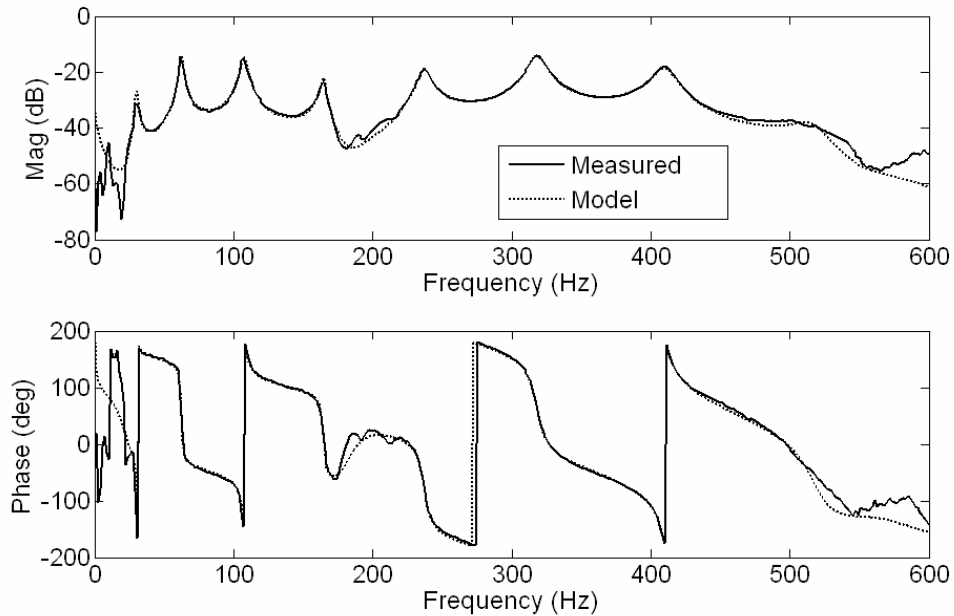


Figure 4-4. Experimental and analytical frequency responses from the disturbance to sensor 4

BJ filter Theory and Design

BJ filters are a special case of the traditional Luyenberg observer. The difference is that for a BJ filter, the "free" parameters of the observer feedback matrix are selected in such a way that the output residual has specific directional properties when specific faults occur. Therefore, the residual can be monitored to both detect a fault, and isolate the specific fault, which has occurred. The basic theory of BJ filters is summarized in this section. This is followed by the development of two modifications to existing feedback

matrix design techniques. The first modification enables BJ filters to be designed for systems with feed-through dynamics while the second modification presents a gain matrix design suitable for high order systems.

BJ detection filters are traditional observers designed in such that the output residual vector has specific directional properties that can be associated with specific faults [7,8,9,10,13,14,15,17,18]. Assuming we have a system model in the absence of failures, the BJ filter is of the form

$$\hat{\mathbf{x}}[\mathbf{n+1}] = \mathbf{A}\hat{\mathbf{x}}[\mathbf{n}] + \mathbf{Bu}[\mathbf{n}] + \mathbf{L}(\mathbf{y}[\mathbf{n}] - \mathbf{C}\hat{\mathbf{x}}[\mathbf{n}])$$
(4-3)

where $\hat{\mathbf{x}}$ is the state estimate and $\mathbf{L}$ is the detection gain matrix. The state error is as

$$\varepsilon[\mathbf{n}] \overset{\Delta}{=} \mathbf{x}[\mathbf{n}] - \hat{\mathbf{x}}[\mathbf{n}]$$
(4-4)

Now the observer gain matrix $\mathbf{L}$ is chosen in such a way that the output error

$$\bar{\varepsilon}[\mathbf{n}] = \mathbf{y}[\mathbf{n}] - \mathbf{C}\hat{\mathbf{x}}[\mathbf{n}]$$
(4-5)

has restricted directional properties in the presence of a failure. Therefore, when there are no faults present, the closed loop dynamics become

$$\varepsilon[\mathbf{n+1}] = \mathbf{G}\varepsilon[\mathbf{n}]$$
(4-6)

where $\mathbf{G}$ is defined as

$$\mathbf{G} \overset{\Delta}{=} \mathbf{A} - \mathbf{LC}$$
(4-7)

The presence of an additive fault, especially an actuator fault, can be modeled by adding a term to the open loop dynamic system shown in Equation (4-2) such that

$$\mathbf{x}[\mathbf{n+1}] = \mathbf{Ax}[\mathbf{n}] + \mathbf{Bu}[\mathbf{n}] + \mathbf{f_i}\mu_i[\mathbf{n}]$$
(4-8)

where $\mathbf{f_i}$ is a nx1 design failure direction associated with the i$^{th}$ actuator failure and $\mu_i$ is a time-varying scalar which may be function of $x(t)$ or $u(t)$. Thus, the system output error in the presence of faults becomes

$$\varepsilon[\mathbf{n}+1] = \mathbf{G}\varepsilon[\mathbf{n}] + \mathbf{f_i}\mu_i[\mathbf{n}]$$
$$\bar{\varepsilon}[\mathbf{n}] = \mathbf{C}\varepsilon[\mathbf{n}]$$

(4-9)

The detection gain is designed in such a way that the directionality of the residual, $\bar{\varepsilon}$, corresponds to specific faults. Design procedures are presented by Beard [1] and Jones [6], and more recently by Kim et al. [7,8,9,10]. In this case, $\bar{\varepsilon}$ is proportional to $\mathbf{Cf_i}$ in response to a failure corresponding to the direction $\mathbf{f_i}$. It should be noted that the sensor failure is similar to the actuator failure as can be found in [7,8,9,10,13,14,15,16].

There are several gain selection methods available that work well for low order systems [7,8,9,10,13,14,15,17,18]. However, when the system order is large (greater than 10 or so), it is very difficult to use these methods and achieve a stable closed-loop detection filter.

In order to overcome this difficulty an unstable detection gain matrix is created using the invariant zero approach, and then modified to ensure a stable result. In most physical the detection orders are equal to one, which means that, for a given fault vector $\mathbf{f}$, the triplet $(\mathbf{A, f, C})$ has no invariant zero [7,8,9,10]. In this case, the invariant zero approach yields a detection gain matrix $\mathbf{L}$ that is given by

$$\mathbf{L} = (\mathbf{AF} - \mathbf{F\Delta})(\mathbf{CF})^*$$

(4-10)

where $\Delta$ is a diagonal matrix whose elements are given as the eigenvalues associated with the detection space of $\mathbf{F}$ and * indicates pseudo-inverse. Next, the result of equation (10) is modified to ensure a stable result according to the equation (4-11)

$$L' = L + E(I - (CF)(CF)^*) \tag{4-11}$$

If the observer gain $L'$ is applied to error state equation then,

$$G = (A - L'C) = (A - LC) - E(I - (CF)(CF)^*)C \tag{4-12}$$

Now, if we define $A_f = A - LC$ and $C_f = [I - (CF)(CF)^*]C$, then $G = A_f - EC_f$ and the dual of $G$ is $(A_f^T - C_f^T E^T)$. One will note that the definition of $G$ is equivalent to the LQR control problem of finding an $E$ that stabilizes $(A_f, C_f)$ and minimizes

$$J = \sum_{n=1}^{\infty} \left[ z[n]^T Qz[n] + m[n]^T Rm[n] + 2z[n]^T Nm[n] \right] \tag{4-13}$$

where $z$ and $m$ are the states and inputs associated with the new pair $(A_f, C_f)$. The weights $Q$ and $R$, can be altered to affect the closed-loop eigenvalues of the filter. It is assumed $N = 0$ in these calculations. Finally, once a suitable matrix $E$ is obtained the modified BJ filter gain matrix $L'$ can be obtained from Equation (11).

## Results

In this section, the FDI implementation and its limitations are studied in detail to understand the significance of incorporating the FDI technique for active vibration control. The controller in this research work is based on H2 optimal theory. The H2 optimal control is effective and robust at attenuating the structural vibrations of the beam. The first part of the results section provides a platform for the designing the BJ detection filter in discrete-time. Simulations results are provided to show the effectiveness of the detection filter design technique. The remaining section deal with the experimental results.

The system identification is based on six inputs and six outputs system. But in this research work we consider only three inputs and three outputs system. The locations of the actuator-sensor pairs are at 0.255 m, 0.535 m and 0.865 m. In order to validate the detection filter design technique simulations results are shown in Figure 4-5.
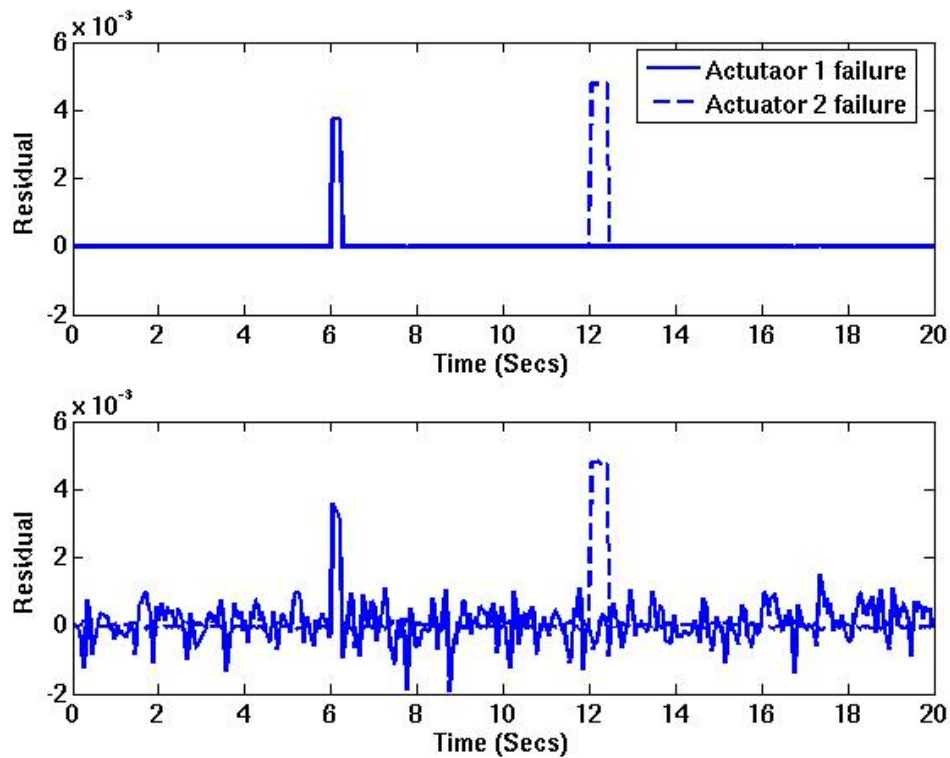


Figure 4-5. Simulation results for actuator one and two failures without and with sensor noise

In first subplot in Figure 4-1 at six seconds actuator one fails. Thus, we can see a residual output (indicated in solid line). Similarly in the first subplot in Figure 4-5 at 12 seconds actuator 2 fails. Therefore, we can see a residual output (indicated by the dotted line). In the second subplot in Figure 4-5 we can see the occurrence of actuator one and actuator two at six and 12 seconds respectively. But in this subplot, we can see lot of noise associated with the residual. This is due to the fact that a band-limited white noise

is added to the sensor output in the simulation. The significance of the sensor noise to the

residual output is explained in the experimental results.
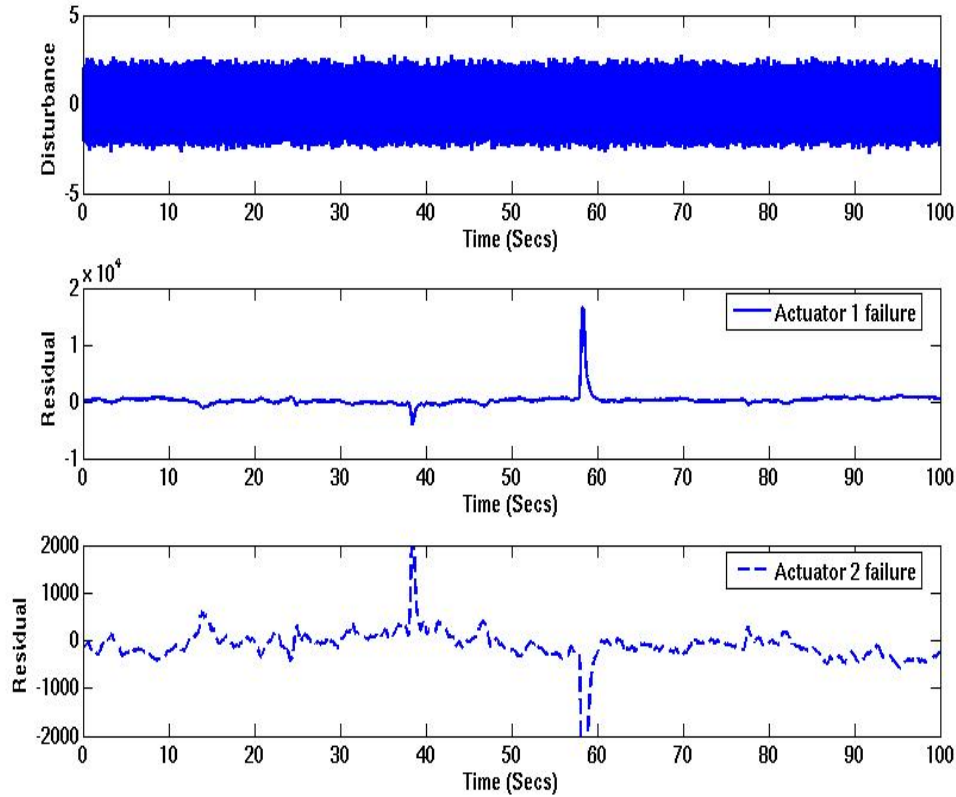


Figure 4-6. Experimental results for actuator one and two failures

Experimental results are shown in Figure 4-6. The actuator failures are simulated by

unplugging the control inputs from the dSPACE DS1103 PPC board. The first subplot in

Figure 4-6 is the input disturbance to the beam. This disturbance creates the vibration in

the beam. The second subplot in figure4-6 shows the failure of actuator one at around 58

seconds. The residual for actuator one is in the positive direction (indicated by solid line).

This indicates the isolation of the failure. At the same time, we can see a negative

residual in the third subplot in Figure 4-6. This residual does not appear in the simulation

results but appears in the experimental results. This is due to the fact that there is lot of sensor noise associated with the experiment. The signal to noise ratio is high in this case. The eignevalues are moved close the origin of the unit circle in discrete-time. This helps in reducing the bandwidth of the transfer function associated with the detection space and enhances the ability to filter noises. Also, at the same time, the response for the fault signal becomes large while the response speed becomes slow. Furthermore, the ratio between the gains associated with the detection filter and control inputs is order of 1000. Therefore, the noise is amplified by the detection filter gains. We have two kinds of independent spaces: detection and completion space in the invariant zero approach. The noise can be reduced by changing the detection space eigenvalues. In addition, the there is a model mismatch between the actual plant and the detection filter. All these reasons create a negative residual rather than zero residual for no fault. The third subplot in figure 6 shows the failure of actuator two at around 38 seconds. The residual for actuator two is in the positive direction (indicated by dotted line). At the same time, we can see a negative residual in the second subplot in Figure 4-6.

## Conclusions

The primary objective of this work is to design the BJ detection filter for high order systems in discrete time. Experimental results are conducted for multiple faults on the flexible structure. The significant contribution of this work is to implement the BJ filter in real-time application for actuator failures. This work can be further extended to achieve the concept of fault-tolerant active vibration control.

References

[1] R.V. Beard, Failure Accommodation in Linear System through Self-Reconfiguration, Ph.D. Thesis, Massachusetts Institute of Technology, MA, 1971.
[2] C. Byreddy, K.D. Frampton, Simulations of Fault-Adaptive Vibration Control, ASME International Mechanical Engineering Congress and Exposition, Florida. 2005.
[3] C. Byreddy, K.D. Frampton, Fault-Tolerant Vibration Control for Additive Faults, Journal of Intelligent Materials and Structures, 2006. (Submitted for Publication)
[4] J. Chen, J. Patton Ron, Zhang Hong-Yue, Design of Unknown Input Observers and Robust Fault Detection Filters, International Journal of Control, 63, 1996.
[5] R.K. Douglas, J.L. Speyer, Robust Fault Detection Filter, AIAA Journal of Guidance, Control and Dynamics, 19, 1996.
[6] H. Jones, Failure Detection in Linear Systems, Report No. T-608, The Charles Stark Draper Laboratory, Cambridge, MA, 1973.
[7] Y. Kim, J. Park, An Analysis of Detection Spaces using Invariant Zeros, American Control Conference, San Diego, California, USA, 1999.
[8] Y. Kim, J. Park, A Condition of the Eigenvalues of Detection Filters for Disturbance Attenuation: An Invariant Zero Approach, Conference on Decision and Control (CDC), Hawaii, USA, 2003.
[9] Y. Kim, J. Park, On the Noise Response of Detection Filters: A Special Consideration on the Relation Between Detection Space and Completion Space, IEEE Proceedings – Control Theory and Applications, 150, 4, 443-447, 2003.
[10] Y. Kim, J. Park, On the Approximation of Fault Directions for Mutual Detectability: An Invariant Zero Approach, IEEE Transactions on Automatic Control, 50, 6, 851-855, 2005.
[11] L. Ljung, System Identification: Second Edition, Prentice Hall Information and System Sciences Series, New Jersey, 1999.
[12] M. Massoumnia, A Geometric Approach to the Synthesis of Failure Detection Filters, IEEE Trans. Autom. Control, 31,839-846, 1986.
[13] J. Park, A Unified Theory of Fault Detection and Isolation in Dynamic Systems, Ph.D. Dissertation, The University of Michigan, Ann Arbor, 1991.
[14] J. Park, G. Rizzoni, An eigenstructure assignment algorithm for the design of fault detection filters, IEEE Trans., AC-39, 1521-1524, 1994.
[15] G. Rizzoni, P.S. Min, Detection of sensor failures in automotive engines, IEEE Trans., 38, 148-158, 1989.
[16] R. Scattolini, N. Cattane, Detection of sensor faults in a large flexible structure, IEEE Proceedings-Control Theory Applications, 145, 5, 1999.
[17] J.E. White, Detection Filter Design by Eigensystem Assignment, Ph.D. Dissertation, University of Texas, Austin, 1985.

[18] J.E. White, J. Speyer, Detection Filter Design: Spectral Theory and Algorithms, IEEE Trans., AC-32, 593-603, 1987.

CHAPTER V

MANUSCRIPT 4

EXPERIMENTS OF FAULT-TOLERANT ACTIVE VIBRATION CONTROL FOR A
SIMPLY SUPPORTED BEAM

Tao Tao, Chakradhar Byreddy, and Kenneth D.  Frampton

Department of Mechanical Engineering

Vanderbilt University, Nashville, TN 37235, USA

Abstract

In this paper a fault-tolerant active vibration control system is applied to a simply supported beam with high order.  System failures are detected and isolated by Beard-Jones (BJ) filters, and then a controller specifically designed for the faulty system is switched on, in order to maintain optimal control performance and stability under failure conditions.  The BJ filters are designed based on system identification model for a simply supported beam.  The controller library includes four controller designs which are used for different fault situations.  The performance of a fault adaptive control system applicable to higher order systems are demonstrated experimentally, and the result provide a benchmark for the design of detection filters for use in fault-tolerant vibration control.

**Keywords**: Beard-Jones filter, Vibration control, System Identification, Switching

## Introduction

Active Control has been used to reduce structural vibrations for many years[1,2,3,4], and the application of active vibration control has been extended to large-scale systems[5,6,7,8]. Many control algorithms, such as adaptive feedback and adaptive feedforward controls, have been developed for different situations[9,10]. Since sensors and actuators are normally involved in such active control systems, the implementation of Fault Detection and Isolation (FDI) for sensor or actuator failures have been investigated for long-term safety[11,12,13,14,15]. However, no work has been done in fault-tolerant vibration control, since the high order vibration system limits the application of traditional fault-tolerant strategies.

The traditional fault detection work can be widely classified into four categories: 1. Algorithm based on Kalman filters; 2. Parity space techniques; 3. Diagnostic Observers; 4. Parameter estimation methods. The BJ filters are based on diagnostic observers, and have been demonstrated previously to offer several advantages in vibration control applications[11,12,16]. One of these advantages is that BJ filters utilize subspace concepts to associate the residual with the system faults, thereby permitting simultaneous FDI.

As described before, traditional fault detection algorithms including BJ filters are limited to relatively low order systems, since it is very difficult to obtain a BJ observer design that is stable when the system has high orders. However, the vibration systems, such as beams and plates, require large order models to ensure accuracy[5,17,18], which makes it hard to implement a fault toleration vibration control. Another limitation is that there are no existing design techniques than can accommodate a model with feed-through

dynamics (i.e. a state space model with non-zero D matrix). The vibration system models are normally obtained with system identification techniques, which usually result in models with feed-through dynamics[19,20,21].

In this work, the performance of a fault-tolerant active vibration control system is demonstrated experimentally. The fault tolerant method in this paper is based on BJ filters, and applicable for high order systems with feed-through dynamics. A simply supported beam with three pairs of piezoelectric transducers acting as sensors and actuators is the active structure investigated. The work presented here begins with a description of the experimental platform, followed by system identification results. Then, the design of fault-tolerant BJ filters applicable for high order systems are presented. Finally, the performance of the BJ filters and the fault-tolerant control system is demonstrated.

## Experimental Setup

The instrumentation arrangement used in the experimental setup is shown schematically in figure 5-1. The simply supported beam is disturbed by a band-limited white noise (0 - 600Hz) , and sensor signals are amplified and filtered with four-pole Butterworth low-pass filters. Distributed controllers are implemented on a dSPACE DS1103 PPC board, and control signals are amplified by a 790A06 power amplifier from PZB Piezotronics, Inc.

The physical system is a beam made of aluminum 2024-T4, and the physical parameters are listed in table 5-1. The beam is clamped at both ends, with grooves machined near both ends to approximate the simply supported boundary condition[18].

System identification results have shown that the beam's dynamic response is very close theoretical predictions for a simply supported beam.
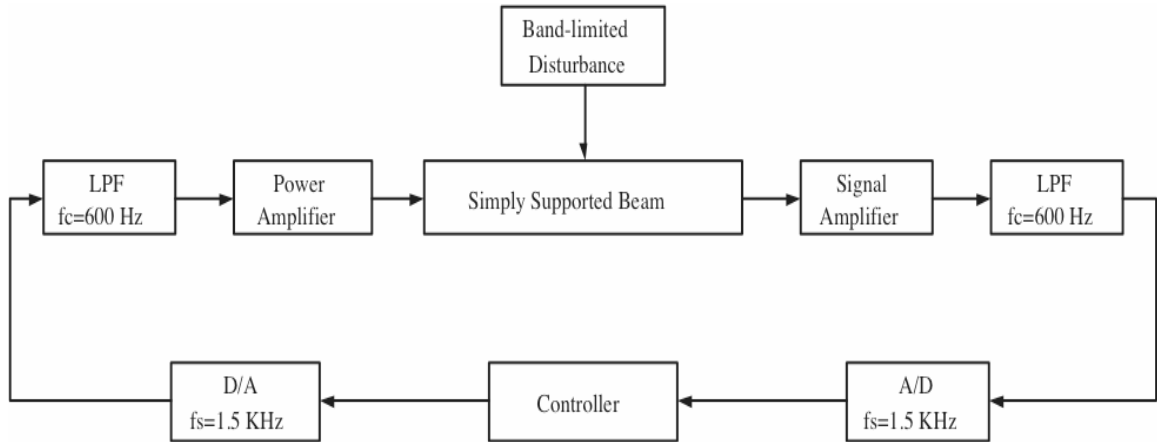


Figure 5-1. Block diagram of the experimental setup.

Table 5-1. Physical parameters of the experimental beam

| Physical parameters | Values |
| --- | --- |
| Density | 2700 (kg/m3) |
| Thickness | 0.0032 (m) |
| Length | 1.0650 (m) |
| Width | 0.0508 (m) |
| Young's Modulus | 73.1E9 (Pa) |

Lead Zirconate Titanate (PZT) transducers are attached along the beam acting as sensors and actuators, as shown in figure 5-2.  The size of each PZT patch is 0.055m by 0.027m, and patches are not evenly distributed along the beam.  All sensors are on the same side of the beam, and all actuators are on the opposite side.  The band-limited noise is applied to PZT1 (left most transducer, with the coordinate 0.11m) as the disturbance.

It is known that the transducer placements will affect control performance[19,22], and so the transducers were chosen to maximize sensitivity to the structural modes below 600 Hz. The three collocated pairs of transducers selected were: PZT2 (with a coordinate 0.26m) , PZT4 (with a coordinate 0.54m), and PZT7 (with a coordinate 0.87m) along the beam (as shown in figure 5-2).
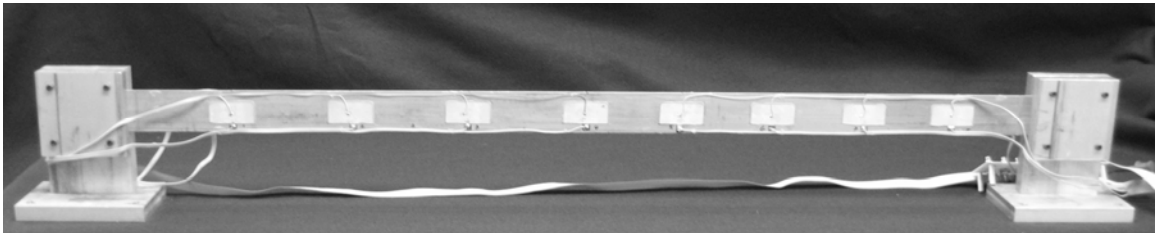


Figure 5-2. The experimental beam with multiple sensor/actuator pairs.

## System Identification

As specified previously, the system model obtained from theoretical derivation matches the experimental result well. However, since the control performance depends on the accuracy of the system model, the dynamics of the beam were obtained using experimental system identification.

Six sensors and six actuators were used for the control system, resulting in 36 transfer functions from the inputs to outputs. In addition, the path between disturbance and all sensors was also identified. A band-limited white noise was applied to each actuator, and then all sensor and actuator data were collected to derive the Auto-Regression with eXtra inputs (ARX) model[21]:

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_1 u(t-1) + \ldots + b_m u(t-m)$$

(5-1)

The ARX parameters were obtained using a batch least squares solution, and then a multi-input multi-output (MIMO) state-space model was derived from the corresponding ARX model.
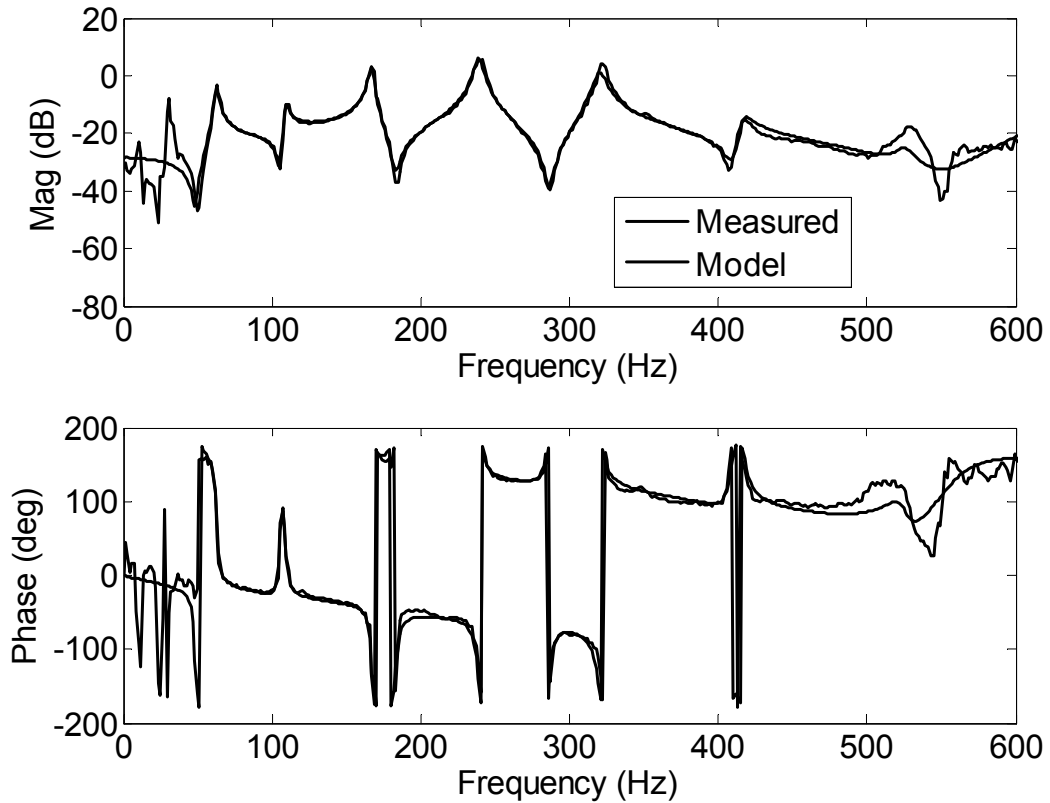


Figure 5-3. Experimental and analytical frequency responses from the disturbance to sensor 1.

The order of the system model in our experiment was chosen to be 36, and frequency responses of all signal paths match the experimental results very well. Experimental and analytical frequency responses of two signal paths are shown in figures 5-3 and 5-4. In figure 5-3, the solid lines represent the transfer function from the disturbance to sensor 1 measured directly from system identification data, while the

dotted lines represent the derived state-space model with 36 states. It is clearly shown that the state-space model represent the beam dynamics very well, in both magnitude and phase of the system response. Similarly, the transfer function from the disturbance to sensor 2, shown in figure 5-4, demonstrates a good match between experimental and analytical responses. The other signal paths have similar results.
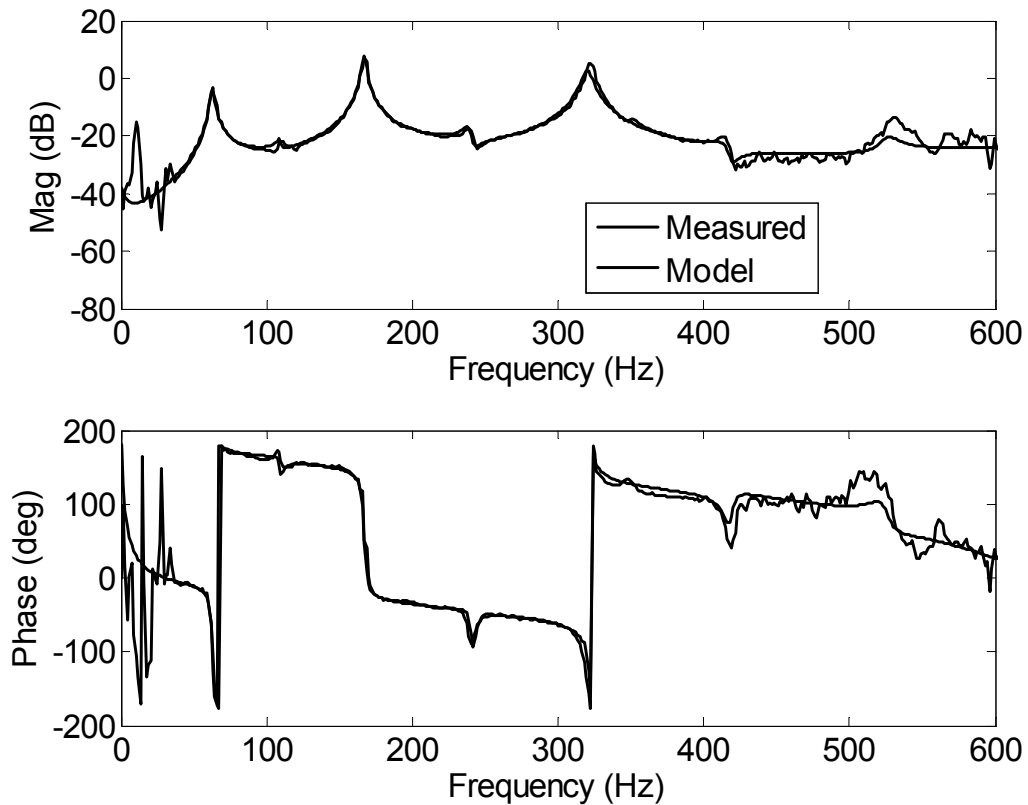


Figure 5-4. Experimental and analytical frequency responses from the disturbance to sensor 2.

Design and Theory of BJ Filter

BJ filters are a special case of the traditional Luyenberg observer. The difference is that for a BJ filter, the "free" parameters of the observer feedback matrix are selected in such a way that the output residual has specific directional properties when specific

faults occur. Therefore, the residual can be monitored to both detect a fault, and isolate the specific fault, which has occurred. The basic theory of BJ filters is summarized in this section. This is followed by the development of two modifications to existing feedback matrix design techniques. The first modification enables BJ filters to be designed for systems with feed-through dynamics while the second modification presents a gain matrix design suitable for high order systems.

*BJ Filter*

BJ detection filters are traditional observers designed in such that the output residual vector has specific directional properties that can be associated with specific faults[14,23,24,25,26,27]. The model for BJ FDI is shown in figure 5-5.
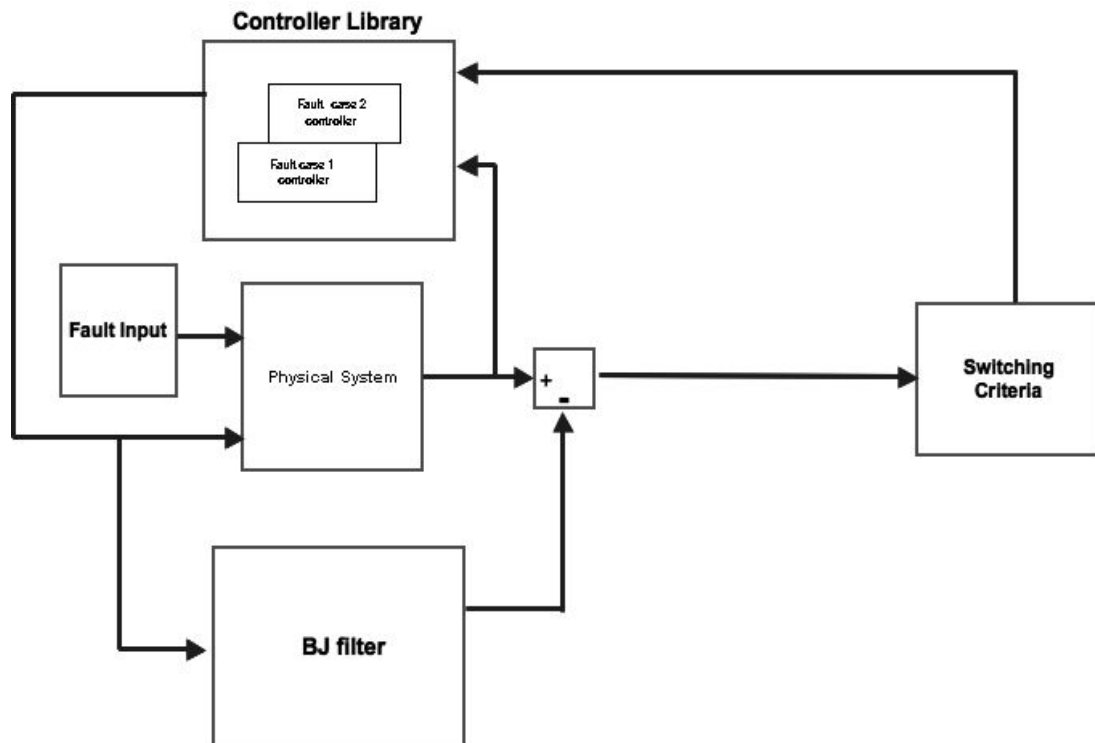


Figure 5-5. Basic model for Beard-Jones FDI.

The BJ filter is of the form

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$

(5-2)

where $\hat{\mathbf{x}}$ is the state estimate and $\mathbf{L}$ is the detection gain matrix. The state error is as

$$\varepsilon \overset{\Delta}{=} \mathbf{x} - \hat{\mathbf{x}}$$

(5-3)

Now the observer gain matrix $\mathbf{L}$ is chosen in such a way that the output error

$$\bar{\varepsilon} = \mathbf{y} - \mathbf{C}\hat{\mathbf{x}}$$

(5-4)

has restricted directional properties in the presence of a failure. Therefore, when there are no faults present, the closed loop dynamics become

$$\dot{\varepsilon} = \mathbf{G}\varepsilon$$

(5-5)

where $\mathbf{G}$ is defined as

$$\mathbf{G} \overset{\Delta}{=} \mathbf{A} - \mathbf{L}\mathbf{C}$$

(5-6)

The presence of an additive fault, especially an actuator fault, can be modeled by adding a term to the open loop dynamic system obtained from system identification

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{f_i}\mu_i$$

(5-7)

where $\mathbf{f_i}$ is a nx1 design failure direction associated with the ith actuator failure and $\mu_i$ is a time-varying scalar which may be function of $x(t)$ or $u(t)$. Thus, the system output error in the presence of faults becomes

$$\dot{\varepsilon} = \mathbf{G}\varepsilon + \mathbf{f_i}\mu_i$$
$$\bar{\varepsilon} = \mathbf{C}\varepsilon$$

(5-8)

The detection gain is designed in such a way that the directionality of the residual, $\bar{\varepsilon}$, corresponds to specific faults. Design procedures are presented by Beard and Jones,

and more recently by Kim et al.[14,23,24,25]. In this case, $\bar{\varepsilon}$ is proportional to $\mathbf{Cf_i}$ in response to a failure corresponding to the direction $\mathbf{f_i}$.

*Design of Detection Gain*

There are several gain selection methods available that work well for low order systems[11,13,15,28,29]. However, when the system order is large (greater than 10 or so), it is very difficult to use these methods and achieve a stable closed-loop detection filter.

In order to overcome this difficulty an unstable detection gain matrix is created using the invariant zero approach, and then modified to ensure a stable result. In most physical the detection orders are equal to one, which means that, for a given fault vector $\mathbf{f}$, the triplet (A, $\mathbf{f}$, C) has no invariant zero. In this case, the invariant zero approach yields a detection gain matrix $\mathbf{L}$ that is given by

$$\mathbf{L} = (\mathbf{AF} - \mathbf{F\Delta})(\mathbf{CF})^*$$

(5-12)

where $\Delta$ is a diagonal matrix whose elements are given as the eigenvalues associated with the detection space of $\mathbf{F}$ and * indicates pseudo-inverse. Next, the result of equation (5-12) is modified to ensure a stable result according to the equation (5-13)

$$\mathbf{L'} = \mathbf{L} + \mathbf{E(I} - (\mathbf{CF})(\mathbf{CF})^*$$

(5-13)

If the observer gain $\mathbf{L'}$ is applied to error state equation (5) then,

$$\mathbf{G} = (\mathbf{A} - \mathbf{L'C}) = (\mathbf{A} - \mathbf{LC}) - \mathbf{E(I} - (\mathbf{CF})(\mathbf{CF})^*)\mathbf{C}$$

(5-14)

Now, if we define $A_f = A - LC$ and $C_f = [I - (CF)(CF)^*]C$, then $G = A_f - EC_f$ and the

dual of $G$ is $(A_f^T - C_f^T E^T)$. One will note that the definition of $G$ is equivalent to the

LQR control problem of finding an $E$ that stabilizes $(A_f, C_f)$ and minimizes

$$J = \sum \left[ z^T Q z + m^T R m + 2 z^T N m \right]$$

(5-15)

where $z$ and $m$ are the states and inputs associated with the new pair $(A_f, C_f)$. The

weights $Q$ and $R$, can be altered to affect the closed-loop eigenvalues of the filter. It is

assumed $N = 0$ in these calculations. Finally, once a suitable matrix $E$ is obtained the

modified BJ filter gain matrix $L'$ can be obtained from equation (5-13).


*Continuous/Discrete Residuals and Fault Modes*

Where a fault is detected in the system, the physical system response deviates

from that predicted by the BJ filter, resulting in a non-zero residual output from the filter

called the continuous residual. When the continuous residual exceeds a pre-set threshold

values, the corresponding entry in the discrete residual vector is set to one. If the

continuous residual value falls below the threshold, the discrete vector is set to zero. This

discrete residual vector is supplied to select the appropriate controller based on the fault

case. One of the key features in the BJ filter is that it is able to detect multiple faults with

a single filter; referred to as mutual detectability. Therefore, a single BJ filter is used to

observe the system in any of the four fault modes. Not all systems are mutually

detectable, however in systems where it is not; multiple BJ filters are designed and

operated in parallel.

*Controller design*

The distributed controllers in this work are designed based on H2 optimal control theory[30,31]. The approach used here is no different from traditional H2 control theory. But the arrangement and implantation in a distributed manner is unique. Such H2 optimal control has been proven effective and robust at attenuating structural vibration in centralized strategy, and it is extended here to a distributed architecture.

The basic block diagram of H2 closed-loop system is shown in figure 5-6, where G is the generalized plant, K is desired controller, w is the exogenous input vector consisting of the disturbance and sensor noises, u is the control signal vector, and y is the plant output vector. In figure 5-5, z is the output to be minimized which consists of the filtered actuator signals, system states and plant outputs. The goal of H2 optimal control is to compute an internally stabilizing controller K, which minimizes the transfer function $\|T_{zw}\|_2$. Details concerning the calculation of the optimal controller K can be found in reference[30,31].
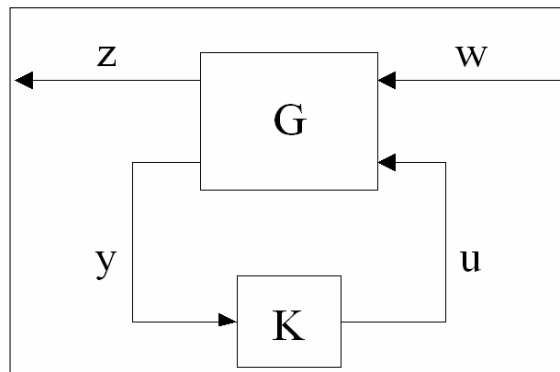


Figure 5-6. Basic H2 closed-loop system.

In the control library, there are four controllers: controller for no fault, controller for actuator 1 failure, controller for actuator 2 failure, and controller for actuator 1 & 2 failure. All four controllers were designed based on H2 optimal control strategy.

Experimental Results

The failure of an actuator was implemented by unplugging the BNC cable from the Digital Analog Converter (DAC) on the dSPACE connection panel. And the time history of continuous residual, discrete residual, finite state and the output of sensor 2 were presented in figure 5-7. It is shown that where there is a failure at actuator 2 around 38 seconds, the BJ filter detects the failure and the discrete residual is set to be 1 for actuator 2. The value of finite state is 3, which represents the failure of actuator 2 and switches the controller. Although the performance of the controller is a little worse after switch, the closed-loop system is stable and the transition between controller switch is negligible.
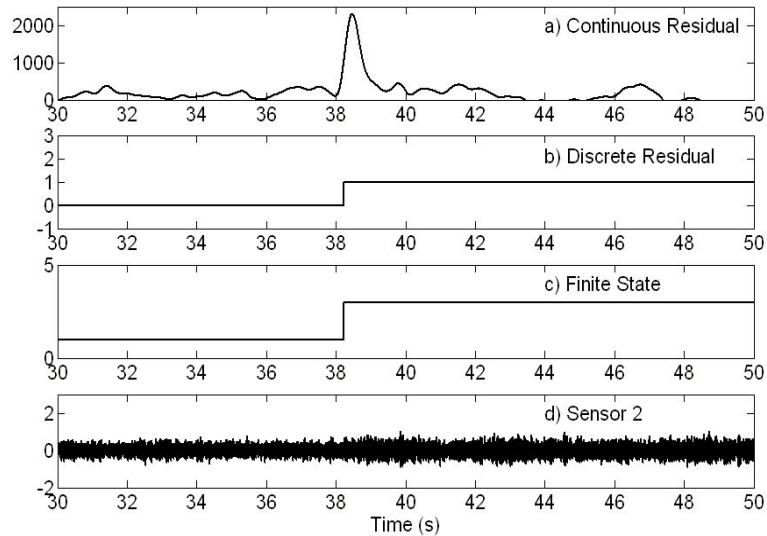


Figure 5-7. Time history of residuals and finite state for actuator 2 failure.

The results of the experiment with two actuator failures are shown in figures 5-8, 5-9 and 5-10.  The failure of actuator 2 happened around 40 seconds, and the failure of actuator 1 took place around 60 seconds.  The continuous residuals for both actuators are shown in figure 5-8, and the BJ filter in our system detected both failures well.  The discrete residuals and finite state for actuators 1 & 2 failures are shown in figure 5-9.  When actuator 2 failed around 40 seconds, the discrete residual for actuator 2 was changed to 1, and the finite state was set to be 3, which switched the controller to the specific one in the controller library.  Then, when actuator 1 failed around 60 seconds, the discrete residual for actuator 1 was changed to 1, and the finite state was set to be 4 and the controller was switched again.  The corresponding sensor signals are shown in figure 10.
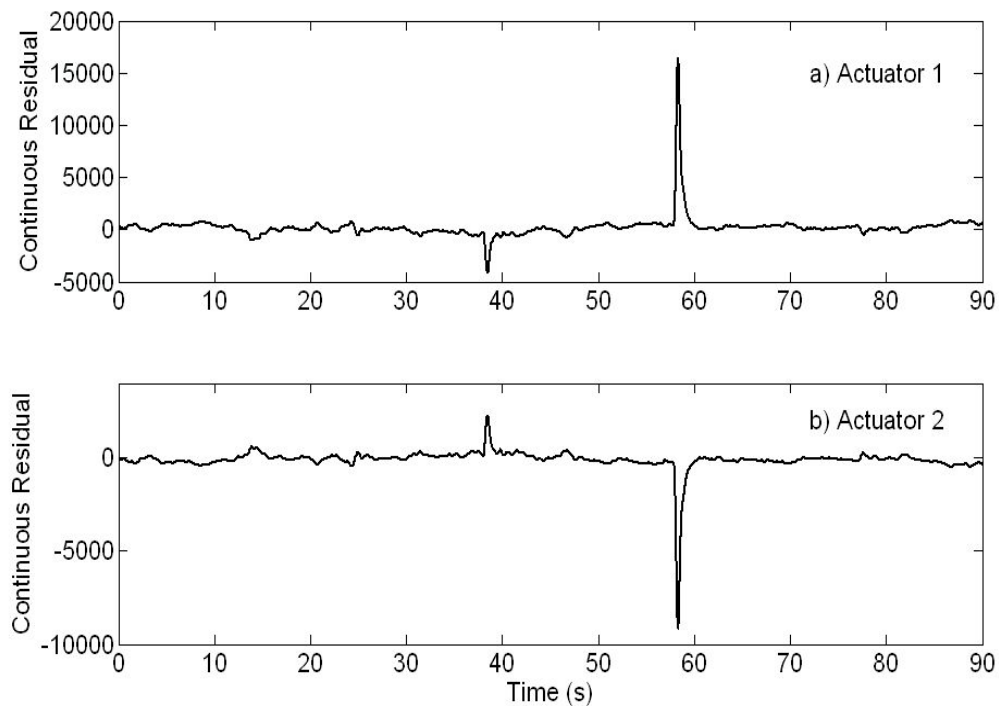


Figure 5-8. Continuous residuals for actuators 1 & 2 failures.
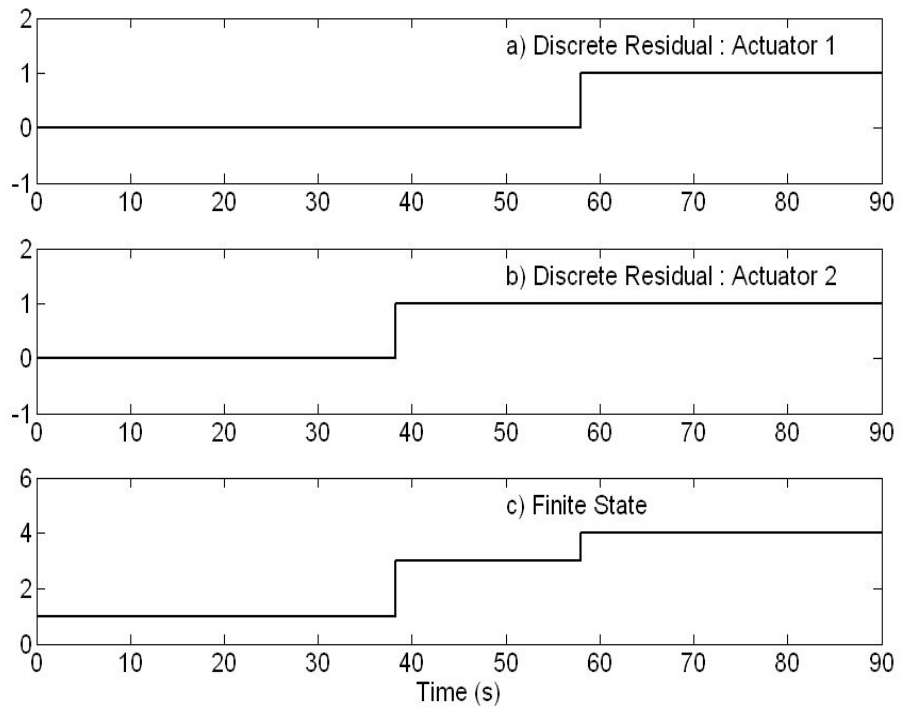
Figure 5-9. Discrete Residuals and finite state for actuators 1 & 2 failures
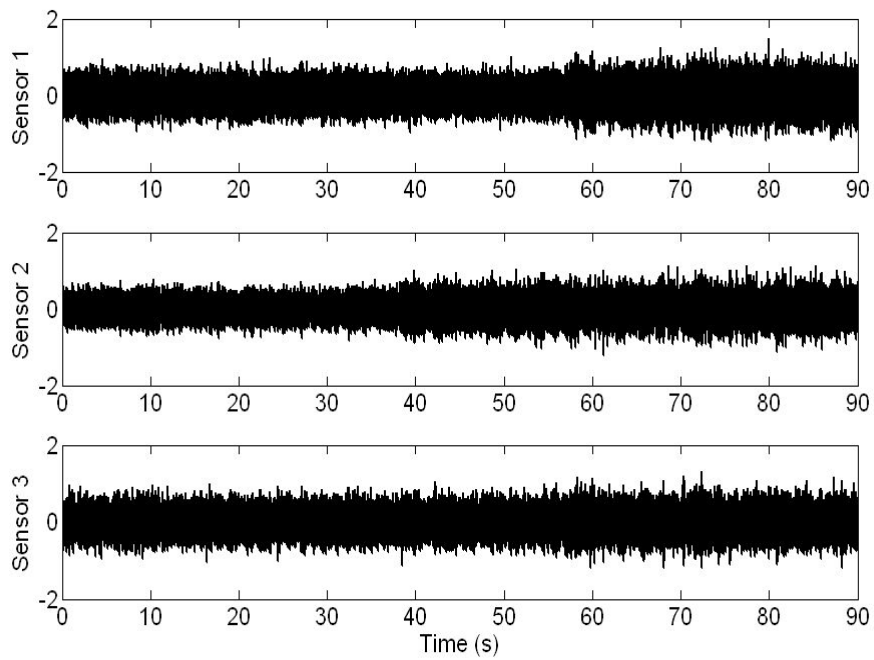


Figure 5-10. Sensor signals for actuator 1 & 2 failures.
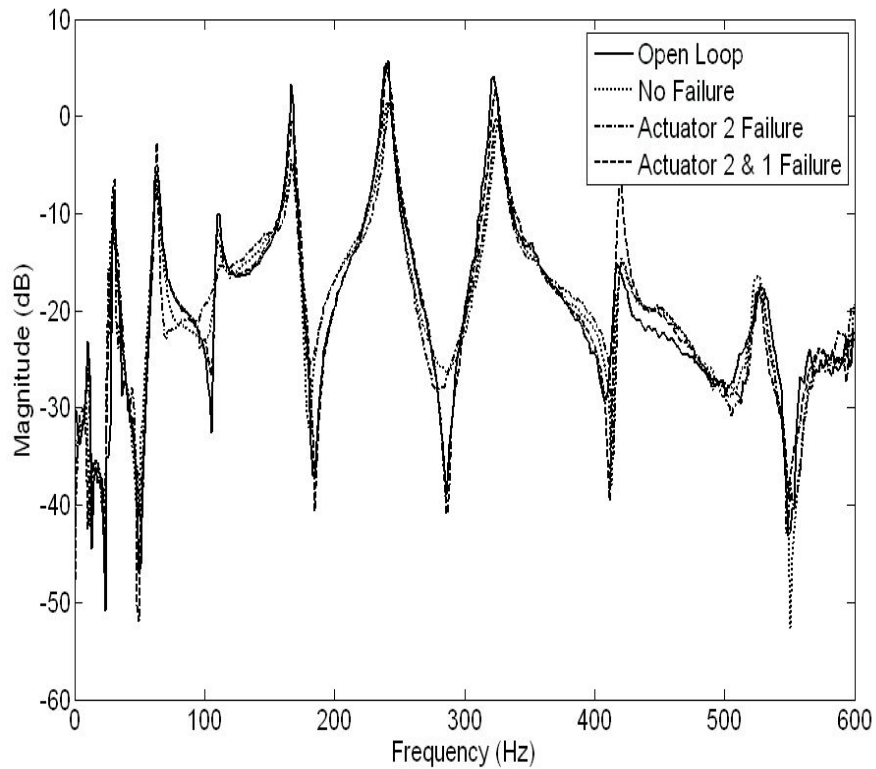
Figure 5-11. Transfer functions from the disturbance to sensor 1.

The transfer functions from the disturbance to sensor 1 in different fault situations are shown in figure 5-11. It is shown that the system with no actuator failures has the best control performance, but the closed-loop system in our system is stable and fault-tolerant, even the control performance is compromised.

## Conclusions

In this work the fault-tolerant active vibration control system is implemented experimentally. The method is demonstrated applicable high order systems, such as the vibrational system with 36 states.

101

References

[1] C. R. Fuller, "Experiments on reduction of aircraft interior noise using active control of fuselage vibration," *Journal of the Acoustical Society of America*, Vol. 78, No. S1, pp. S88, 1985.

[2] S. Burke and J. Hubbard, Jr., "Active vibration control of a simply supported beam using a spatially distributed actuator," *IEEE Control Systems Magazine*, Vol. 7, No. 4, pp. 25-30, 1987.

[3] W. T. Baumann, W. R. Saunders, and H. H. Robertshaw, "Active suppression of acoustic radiation from impulsively excited structures," *Journal of the Acoustical Society of America*, Vol. 90, No. 6, pp. 3202-3208, 1991.

[4] R. L. Clark and D. E. Cox, "Multi-variable structural acoustic control with static compensation," *Journal of the Acoustical Society of America*, Vol. 102, No. 5, pp. 2747-2756, 1997.

[5] O. N. Baumann, W. P. Engels, and S. J. Elliott, "A comparison of centralized and decentralized control for the reduction of kinetic energy and radiated sound power," *Proceedings of Active04*, Williamsburg, VA, 2004.

[6] M. J. Brennan, S. J. Elliott, and X. Huang, "A demonstration of active vibration isolation using decentralized velocity feedback control," *Smart Materials & Structures*, Vol. 15, No. 1, pp. N19-N22, 2006.

[7] K. D. Frampton, "Distributed group-based vibration control with a networked embedded system," *Smart Materials & Structures*, Vol. 14, No. 2, pp. 307-314, 2005.

[8] G. West-Vukovich, E. Davison, and P. Hughes, "The decentralized control of large flexible space structures," *IEEE Transactions on Automatic Control*, Vol. 29, No. 10, pp. 866-879, 1984.

[9] W. T. Baumann, "An adaptive feedback approach to structural vibration suppression," *Journal of Sound and Vibration*, Vol. 205, No. 1, pp. 121-133, 1997.

[10] S. C. Douglas, "Fast implementations of the filtered-X LMS and LMS algorithms for multichannel active noise control," *IEEE Transactions on Speech and Audio Processing*, Vol. 7, No. 4, pp. 454-465, 1999.

[11] H. Jones, "Failure detection in linear systems," The Charles Stark Draper Laboratory, Cambridge, MA Report No. T-608, 1973.

[12] R. V. Beard, "Failure accommodation in linear systems through self-reorganization," in *Aeronautics and Astronautics*. MA: Massachusetts Institute of Technology, 1971.

[13] J. Chen, R. J. Patton, and H. Y. Zhang, "Design of unknown input observers and robust fault detection filters," *International Journal of Control*, Vol. 63, No. 1, pp. 85-105, 1996.

[14] Y. Kim and J. Park, "An analysis of detection spaces using invariant zeros," *American Control Conference*, San Diego, CA, USA, 1999.

[15] R. Scattolini and N. Cattane, "Detection of sensor faults in a large flexible structure," *Iee Proceedings-Control Theory and Applications*, Vol. 146, No. 5, pp. 383-388, 1999.

[16] C. Byreddy and K. D. Frampton, "Simulations of fault-adaptive vibration control," *ASME International Mechanical Engineering Congress and Exposition*, Floria, 2005.

[17] W. J. Manning, A. R. Plummer, and M. C. Levesley, "Vibration control of a flexible beam with integrated actuators and sensors," *Smart Materials & Structures*, Vol. 9, No. 6, pp. 932-939, 2000.

[18] W. T. Thomson and M. D. Dahleh, *Theory of Vibration with Applications*. Upper Saddle River, NJ: Prentice Hall, 1998.

[19] R. L. Clark, W. R. Saunders, and G. P. Gibbs, *Adaptive Structures: Dynamics and Control*. New York: Wiley, 1998.

[20] J.-N. Juang, *Applied System Identification*. Englewood Cliffs, NJ: Prentice Hall, 1994.

[21] L. Ljung, *System Identification: Theory for the User*. Upper Saddle River, NJ: Prentice Hall, 1998.

[22] D. Halim and S. O. R. Moheimani, "An optimization approach to optimal placement of collocated piezoelectric actuators and sensors on a thin plate," *Mechatronics*, Vol. 13, No. 1, pp. 27-47, 2003.

[23] Y. Kim and J. Park, "A condition of the eigenvalues of detection filters for disturbance attenuation: an invariant zero approach," *Conference on Decision and Control (CDC)*, Hawaii, USA, 2003.

[24] Y. Kim and J. Park, "Noise response of detection filters: relation between detection space and completion space," *Iee Proceedings-Control Theory and Applications*, Vol. 150, No. 4, pp. 443-447, 2003.

[25] Y. Kim and J. Park, "On the approximation of fault directions for mutual detectabilty: An invariant zero approach," *Ieee Transactions on Automatic Control*, Vol. 50, No. 6, pp. 851-855, 2005.

[26] J. Park and G. Rizzoni, "An Eigenstructure Assignment Algorithm for the Design of Fault-Detection Filters," *Ieee Transactions on Automatic Control*, Vol. 39, No. 7, pp. 1521-1524, 1994.

[27] J. E. White and J. L. Speyer, "Detection Filter Design - Spectral Theory and Algorithms," *Ieee Transactions on Automatic Control*, Vol. 32, No. 7, pp. 593-603, 1987.

[28] R. K. Douglas and J. L. Speyer, "Robust fault detection filter design," *Journal of Guidance Control and Dynamics*, Vol. 19, No. 1, pp. 214-218, 1996.

[29] G. Rizzoni and P. S. Min, "Detection of Sensor Failures in Automotive Engines," *Ieee Transactions on Vehicular Technology*, Vol. 40, No. 2, pp. 487-500, 1991.

[30] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ: Prentice-Hall, 1995.

[31] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard H2 and H∞ control problems," *IEEE Transactions on Automatic Control,* Vol. 34, No. 8, pp. 831-847, 1989.

CHAPTER VI

MANUSCRIPT 5

THE USE OF HYBRID AUTOMATA FOR FAULT-TOLERANT VIBRATION
CONTROL FOR PARAMETRIC FAILURES

Chakradhar Byreddy[1]   Kenneth D. Frampton[2]   Kim Yongmin[3]

*(Submitted to SPIE 2006 Conference)*

Abstract

The purpose of this work is to make use of hybrid automata for vibration control
reconfiguration under system failures.  Fault detection and isolation (FDI) filters are used
to monitor an active vibration control system. When system failures occur (specifically
parametric faults) the FDI filters detect and identify the specific failure. In this work we
are specifically interested in parametric faults such as changes in system physical
parameters; however this approach works equally well with additive faults such as sensor
or actuator failures. The FDI filter output is used to drive a hybrid automaton, which

---

[1]Graduate Research Assistant, Department of Mechanical Engineering, Vanderbilt
University, 2301 Vanderbilt Place, Nashville, TN-37235. Email:
chakradhar.r.byreddy@vanderbilt.edu, Phone: 1-615-322-1760, Fax: 1-615-343-6687.
[2]Assistant Professor, Department of Mechanical Engineering, Vanderbilt University,
2301 Vanderbilt Place, Nashville, TN-37235. Email: ken.frampton@vanderbilt.edu,
Phone: 1-615-322-2778, Fax: 1-615-343-6687.
[3]CTO, Nano-Tronix, 3FI, Younghan bldg., 1546-3, Seocho-Dong, Seocho-gu, Seoul 137-
872, South Korea. Email: kym@nano-tronix.com, Phone: +82-25819520, Fax: +82-
25819511.

selects the appropriate controller and FDI filter from a library. The hybrid automata also implements switching between controllers and filter in order to maintain optimal performance under faulty operating conditions. The biggest challenge in developing this system is managing the switching and in maintaining stability during the discontinuous switches. Therefore, in addition to vibration control, the stability associated with switching compensators and FDI filters is studied. Furthermore, the performance of two types of FDI filters is compared: filters based on parameter estimation methods and so called "Beard-Jones" filters. Finally, these simulations help in understanding the use of hybrid automata for fault-tolerant control.

**Keywords:** Hybrid Automata, FDI, Parametric Faults and Fault-Tolerant Control

<div align="center">Introduction</div>

FDI is a key area of research for large-scale complex dynamic systems where reliability and safety is a high priority. Therefore, FDI is required for optimal performance in the presence of faults. Thus, identification of transfer function models with various fault case scenarios is required for tuning and design of controllers. For this purpose the physical system parameters are estimated. FDI issues associated with the multi-input multi-output (MIMO) systems are a focus of attention especially for parametric failures. Even when a proposed FDI scheme is technically great for general failures, ease of use consideration remains a core issue for engineering practice.

The coupling of the physical system makes the implementation of FDI and its results more complicated. As a consequence, very fewer investigations are done for parameter failures as compared with additive failures. Therefore, there is a vast area of

<div align="center">105</div>

incentive for developing better methods to accomplish the concept of FDI for multiplicative faults. Furthermore this work can be extended for designing the appropriate controller for multivariable control with the presence of failures. The majority of work conducted on FDI over the past years is on the faults associated with sensors and actuators[1,2,3,4].

Previous literature mainly concentrated on developing system identification algorithms for parametric estimation. This idea of system identification is used for the implementation of FDI for parametric failures. One such algorithm is a new recursive algorithm on model orders[5,6]. In this paper, the proposed algorithm worked well for a fault-tolerant control on a MIMO system. Also, simulation results and modeling scheme was very effective. Most of the algorithms developed in the field of the parameter estimation have been extensively developed based on the discrete time MIMO models for system identification[7].

In the work that is conducted currently, parameter estimation is done for continuous MIMO models for multiplicative faults rather than system identification. The advantage of the parameter estimation over other FDI methods is the use of this concept for fault-tolerant control. The theory and development of Beard-Jones (BJ) filter for fault detection of parametric failures is studied in detail. BJ filter design is mainly implemented for sensor and actuator failures[1]. Therefore, it is very important to study the pros and cons of both methods studied in this work for fault-tolerant control. For fault-tolerant control there is a transition from continuous time to discrete event systems during the switching of the controllers. Therefore, it is essential to use the idea of hybrid automata for this work. Thus, one of the critical issues is to integrate the FDI scheme, hybrid automata and

active vibration control to achieve the objective of fault-tolerant control.

## System Modeling

In order to implement the concept of FDI scheme for fault-tolerant control, a simple MIMO system is modeled. The MIMO system under consideration is a simply supported beam (1-D) subjected to a random disturbance as shown in Figure (6-1). The state space model is then developed for a simple beam with the surface bonded with a piezoceramic material[8]. The beam is equipped with numerous piezoelectric transducers that act simultaneously as sensors and actuators.
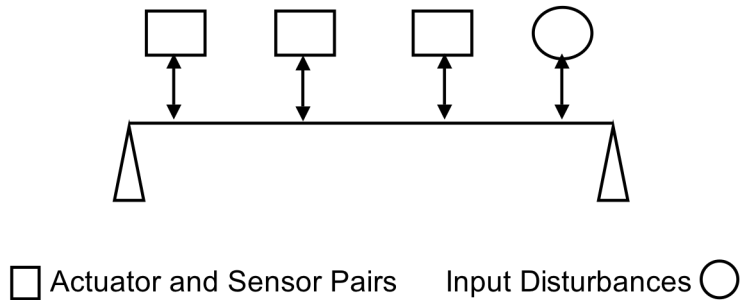


Figure 6-1: Simply supported beam with piezoelectric actuators and sensors

The beam models the effects of dynamic coupling between a structure and an electric network through the piezoelectric effect. The coupling for the electromechanical system is represented by mass and stiffness contributions of the piezoelectric transducers as well as the dynamics of the transducers itself. The solution assumes that the structural displacement can be expressed as a summation of the orthogonal functions (called modes). The modes of a simply supported beam can be represented as linear expansion of assumed modes and generalized coordinates in the following form

$$w(x,t) = \sum_{n=1}^{\text{mod}} \psi_n(x) q_n(t)$$

(6-1)

where $w(x,t)$ is the beam displacement, $\psi_n(x)$ is the n$^{\text{th}}$ mode shape and $q_n(t)$ is the generalized coordinate of the n$^{\text{th}}$ mode. In general, the expansion of the number of assumed modes is limited to finite number, and the importance of this result help the design and analysis of controlled adaptive structures.

$$[M_p + M_t]\{\ddot{q}(t)\} + [K_p + K_t]\{q(t)\} = [Q^d(t)] + [\Theta]\{v(t)\}$$

(6-2)

where $M_p$ and $M_t$ are the mass matrices for the beam and piezoceramic transducers respectively, similarly $K_p$ and $K_t$ are the stiffness matrices for the beam and piezoceramic transducers respectively, $q(t)$ is a vector of generalized coordinates, $Q^d(t)$ is a vector of generalized disturbances, $\Theta$ is the electromechanical coupling matrix and $v(t)$ is the vector of control voltages applied to the transducers which act simultaneously as sensors and actuators. The beam model can be cast in the state space form as follows

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(6-3)

where $x$ is the state vector containing the generalized coordinates, $q_n(t)$ and their derivatives, $u$ is a vector control and disturbance inputs, and $y$ is the output for each piezoceramic transducer and is proportional to the strain rate such that

$$x = \{\mathbf{q} \quad \dot{\mathbf{q}}\}^T$$

(6-4)

The transfer function of the beam can be written in mathematical form as

$$H(s) = \sum_{n=1}^{\text{mod}} \frac{\sigma_n(x_i)\sigma_n(x_j)}{s^2 + 2\xi\omega_n + \omega_n^2}$$

(6-5)

where $\omega_n$ is the natural frequency of the $n^{\text{th}}$ orthogonal coordinate, $\xi_n$ is the damping ratio of the $n^{\text{th}}$ orthogonal coordinate, $\sigma_n(x_i)$ is a function of the assumed modes, and $\sigma_n(x_j)$ is the $n^{\text{th}}$ entry of the vector $\sigma_n(x_i)$. In this paper, for MIMO system we considered three modes for the beam model. Therefore, the MIMO system in transfer function approach can be represented as follows

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} * \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

(6-6)

where $Y_i$ is the output at node $i$, $u_i$ is the input at node $i$ and, $H$ is the transfer function represented in the Equation (6-5).

## Theory

In this section the theory associated with parameter estimation and BJ filter design are explained in detail.

*Parameter Estimation*

The basic principle used for representing an unknown system and estimating the system parameters is a Recursive Least Squares (RLS) structure[9]. The relationship between output and input sequences of the system can be written in the mathematical form as

$$y(t) = \varphi_1(t)\theta_1 + \varphi_2(t)\theta_2 + \ldots\ldots\varphi_n(t)\theta_n = \varphi(t)\theta \tag{6-7}$$

where $y$ is the observed variables, $\theta_1, \theta_2, \theta_3 \ldots\ldots\theta_n$ are unknown parameters and $\varphi_1, \varphi_2, \varphi_3, \ldots\varphi_n$ are known functions that are dependent on r known variables. The variables $\varphi_i$ are called the regression variables, and the model in the Equation (6-7) is called the regression model. The continuous RLS estimation can be written as

$$\dot{\hat{\theta}} = P(t)\varphi(t)e(t)$$

$$e(t) = y(t) - \varphi^T(t)\hat{\theta}$$

$$\dot{P} = \alpha P(t) - P(t)\varphi(t)\varphi^T(t)P(t) \tag{6-8}$$

The parameters associated with the modes are $a_n = 2\xi\omega_n, a_{n+1} = \omega^2_n$. In this work, Equation (6-5) is used to define the various combinations of parameters as mentioned in Equation (6-7). An important observation is the numerator of the transfer function in Equation (6-5) is known (since it depends on the location of sensor and actuators).

*BJ filter design*

The BJ detection filter has been modified for various problems. And, new approaches are developed to make the problem more tractable for different complex cases. In addition to these, solutions to the problem are developed to understand the mutual interaction of physical and mathematical interpretation. Various methods are subsequently analyzed and interpreted as an eigenstructure assignment[10,11,12,13,14]. The open loop dynamic model in the absence of failures is given in Equation (6-3). In case of parametric failures the system can be represented in Equation (6-9)

$$\dot{x} = (A + \Delta A)x + Bu$$
$$y = Cx \tag{6-9}$$

This system can be written in the form of actuator failures as shown in Equation (6-10)

$$\dot{x} = Ax + Bu + \sum_i (\Delta a_i)x_i$$
$$y = Cx \tag{6-10}$$

where $(\Delta a_i)$ is $i$th column vector of $\Delta A$ and $x_i$ is the $i$th element of $x$. Therefore, $(\Delta a_i)$ is considered as the fault event vector since the above equation is similar to the actuator fault. Designing a fault detection filter for Equation (6-10), the information that comes from fault is shown in Equation (6-11)

$$\dot{e} = (A - KC)e + \sum_i (\Delta a_i)x_i$$
$$r = Ce \tag{6-11}$$

When $r \neq 0$, fault is deemed to occur. The details of BJ filter design can be found in references[15,16].

Design of Controller

In this paper, constant gain output feedback compensators are used to minimize the beam vibration. Each possible actuator and sensor failure leads to different configurations of the MIMO system. When a particular actuator fails it is removed from the feedback loop and only the remaining actuators are used to reduce vibration. The compensator is designed based on prior knowledge of the faults that can occur in the system. The control law implemented is

$$u = -Ky_i \tag{6-12}$$

111

where $K$ is the constant feedback gain matrix, $u$ is the local input and $y_i$ is the output associated with each sensor. The feedback gain matrix can be found my minimizing the cost function shown in Equation (6-13)

$$J_i = \int_{t_0}^{t_\infty} (x^T \begin{bmatrix} C_i \\ [I \quad 0] \end{bmatrix}^T \begin{bmatrix} W_s & 0 \\ 0 & W_m \end{bmatrix} \begin{bmatrix} C_i \\ [I \quad 0] \end{bmatrix} x + Rv_i^2)dt$$

(6-13)

where $J$ is the cost function associated with each fault case scenario, $x$ is the system state vector, $C$ is the state-space system matrix from Equation (6-3), $W_s$ is the penalty associated with the sensor signals, $W_m$ is the penalty associated with the various modes, and finally $R$ is the control effort penalty. The details associated with the implementation and calculation of the output feedback gain matrix can be found in reference[17].

The MIMO system considered in this work is a three input-three output system. Compensators for the two fault case scenarios (including the nominal case – no failure) have been designed. Since the most dominant modes are the first three modes, the compensator design considered in this current work includes only three modes. Figure 6-2 shows the physical system with control for different cases. From Figure 6-2, it can be concluded that there is a shift in mode frequencies with the addition of mass. Also, from the plot it can be inferred that there is no change in the second mode frequency (since the location of the additional mass is exactly at half the length of the beam)

<u>Results</u>

In this study the parametric failure is considered to be the addition of mass to the beam. Other parametric failure that can be extended for this work is change in the

stiffness of the beam $K_P$. This study features two types of fault case scenarios. The first one is the case where there is no change in the mass of the beam and, second one is the case where a significant amount of mass is added to the beam.
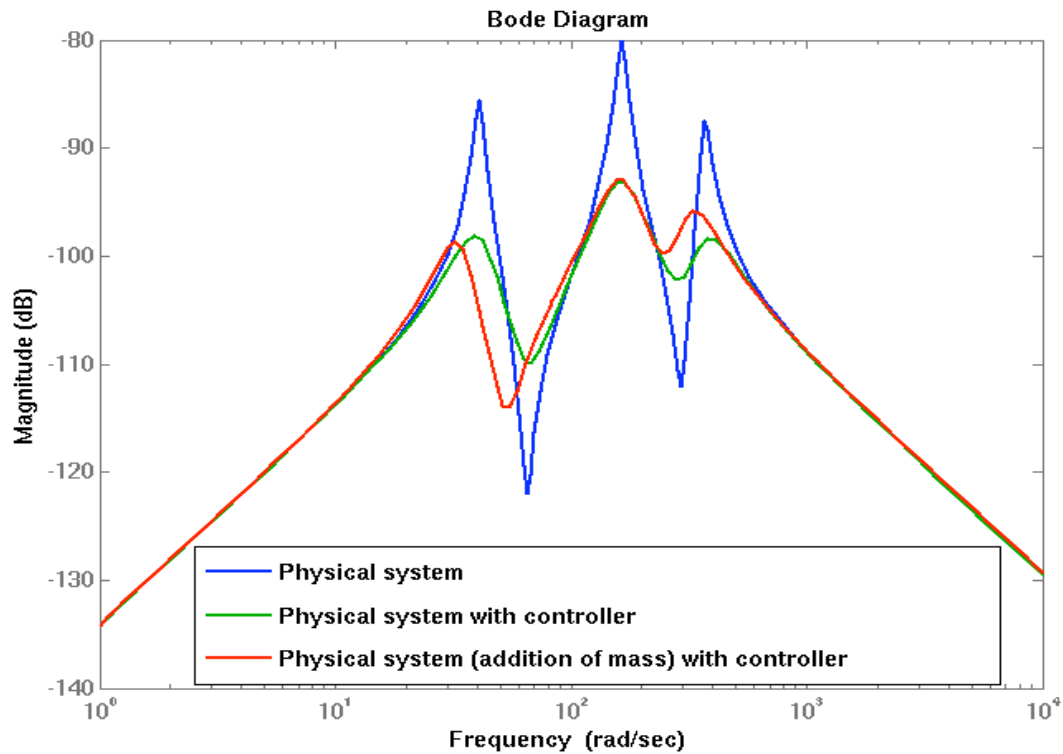


Figure 6-2: Vibration suppression of the physical system for different parametric failures

It is very important to know the location of the additional mass, as it affects the mode shape of the beam. In this study the additional mass is located at half the length of the beam. The details of the additional mass of the beam can be expressed as shown in Equation (6-14)

$$\Delta M = m * \begin{bmatrix} \phi_1\phi_1 & \phi_1\phi_2 & \phi_1\phi_3 \\ \phi_2\phi_1 & \phi_2\phi_2 & \phi_2\phi_3 \\ \phi_3\phi_1 & \phi_3\phi_2 & \phi_3\phi_3 \end{bmatrix}$$

(6-14)

where $\phi_i = \sin(\dfrac{n\pi z_i}{L})$, $z_i$ is the location of additional mass and $n$ is appropriate input-output combination. Similarly, mathematical representation of stiffness parameters can be implemented. In case of parameter estimation, the change in one of the parameter value is considered as the key to change in the physical system with the addition of mass. Since there are only two fault case scenarios, the total number of transitions is four ($2^2$). These transition conditions are modeled using a hybrid automaton. The hybrid model for this is shown in Figure 6-3. It is important to note that in the hybrid model for parameter estimation there is a controller library (there are only two different controllers for two fault case scenarios) and a system library.

Whenever there is a change in the system parameter, this change is represented in the discrete residual vector. This vector is the input to the hybrid automata. Based on the values of the discrete vector (transition conditions), the fault mode changes and thereby, reconfiguring to the appropriate controller. Figure 6-4 shows the discrete residual vector and fault modes with the occurrence of parametric failure. The switching in the controller takes place when the change in the parametric value reaches the threshold value. In this simulation, the change in the parametric fault occurred at a time less than the change in the fault mode from one (corresponds to nominal case) to two (addition of mass). The time to reach the convergence of the parameter value depends on $\alpha$ in the RLS algorithm, which in turn determines the threshold time for switching. In this simulation, the total number of parameters estimated is 20. But any one change in parameter

114

corresponds to change in the physical system. But it is not possible to dynamically determine the location and mass that's been added to the beam. Thus, we can conclude that it is very difficult to isolate the fault rather than detecting the fault.
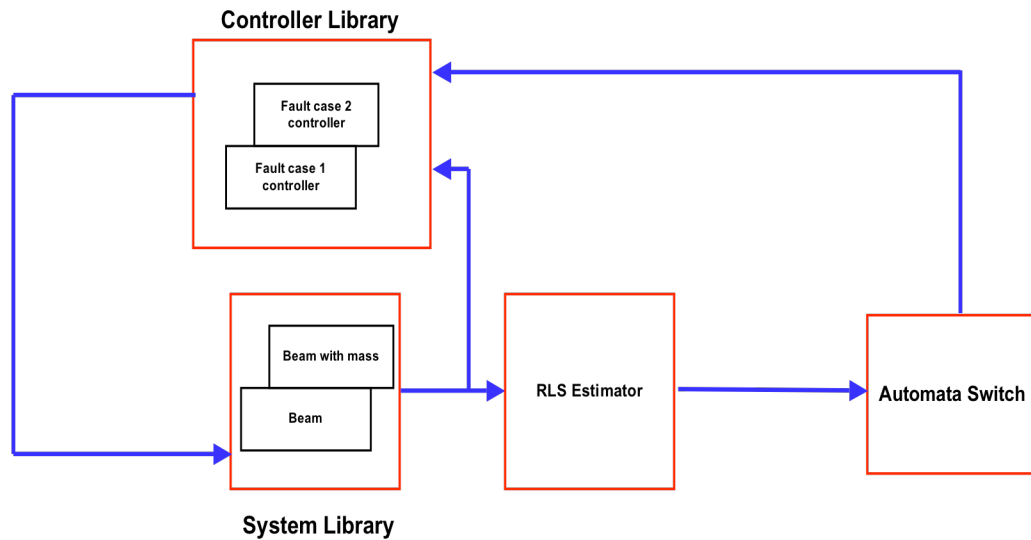


Figure 6-3: Hybrid model for parameter estimation

This method mainly helps in the experimental validation of the parametric failure for the simply supported beam. Also, in the RLS algorithm is important to note that this method is robust to sensor noise. The main disadvantage of this method is it is very difficult to implement for higher order vibrating systems. This is due to the fact that for higher order systems, there are many parameters that need to converge. Furthermore, the rate of convergence increases with increase in number of parameters. Due to this it is better to study BJ filter design for parametric failures for higher order systems. But there are several challenges associated with the design. The next paragraph thus deals with pros and cons of BJ filter design. Also, in the next section the difference in the hybrid model with two different methods is studied.

The hybrid model associated with the BJ filter design is shown in Figure 6-5. There

is a significant difference as compared to the parameter estimation method. In the BJ filter design, the control input and error output are input to the BJ filter. Also, there is no system library for the implementation of parametric failure. The prior fault information is the input to the beam, which changes the dynamics of the beam.
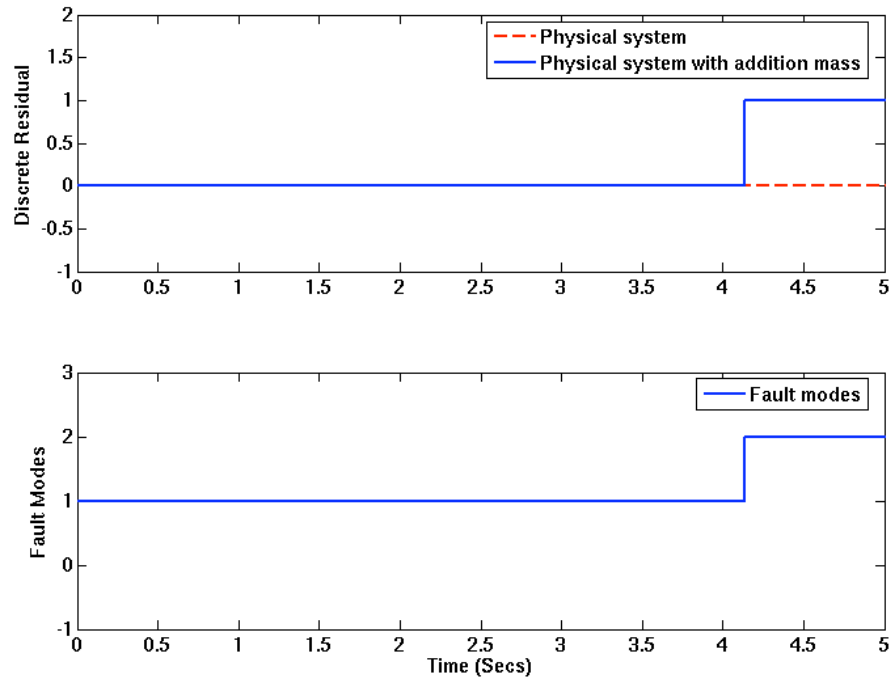


Figure 6-4: Time history of discrete residual and finite-states for parameter estimation method

Figure 6-6, shows the occurrence of the fault at exactly two seconds as a parametric failure (even though in the simulation process it is considered as an actuator failure). The failure occurs at two seconds but the switching of the controller takes place after some time called the threshold period. The threshold period is different for parameter estimation and BJ filter method. Since there are only two fault scenarios, there are only two finite-states.

There are several drawbacks to the BJ filter design for the parametric failures for

active vibration control. The most important is that there are many sources for $r \neq 0$, such as model uncertainties, actuator failure, sensor failure, parametric failure, noises etc. Thus whenever there is simultaneous presence of these faults, it is impossible to isolate the source of failure. Therefore, if we don't know if the failure occurred from the particular kind, the controller design might not be stable. This is due to the fact that the controller design might be for parametric failure, but in the system $r \neq 0$ is for actuator failure.
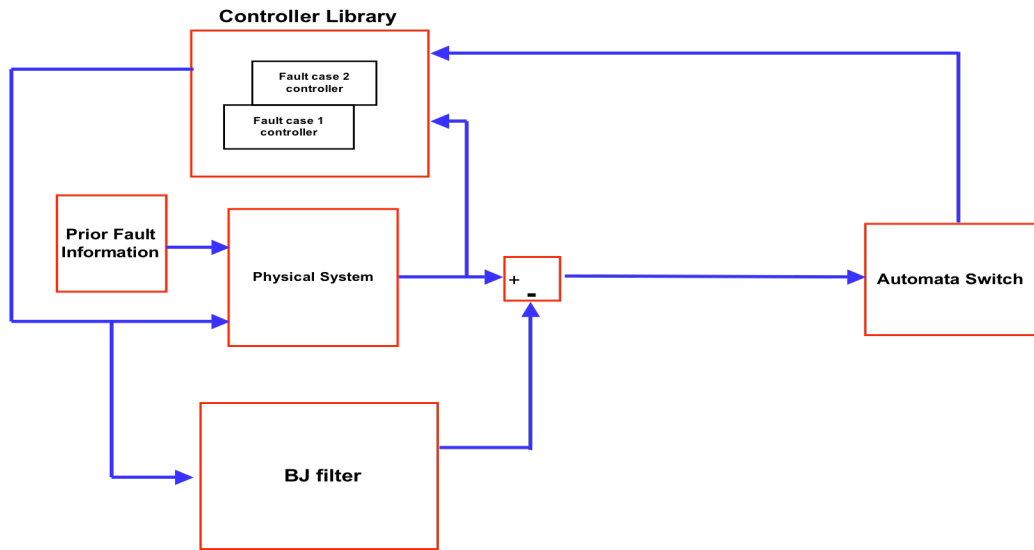


Figure 6-5: Hybrid model for BJ filter

## Conclusions

The primary goal of this work is to implement two different FDI schemes for parametric failures. Simulations for the multiplicative failures have been conducted on a simply supported beam, which in turn demonstrate the ability of such a system to maintain performance and stability. The primary contribution of this work is to study the limitations of the both the methods for multiplicative failures. Also, this work helps in future to develop robust FDI methods with the presence of both the additive and

multiplicative failures. The stability associated with vibration control during switching between the different physical systems is studied.
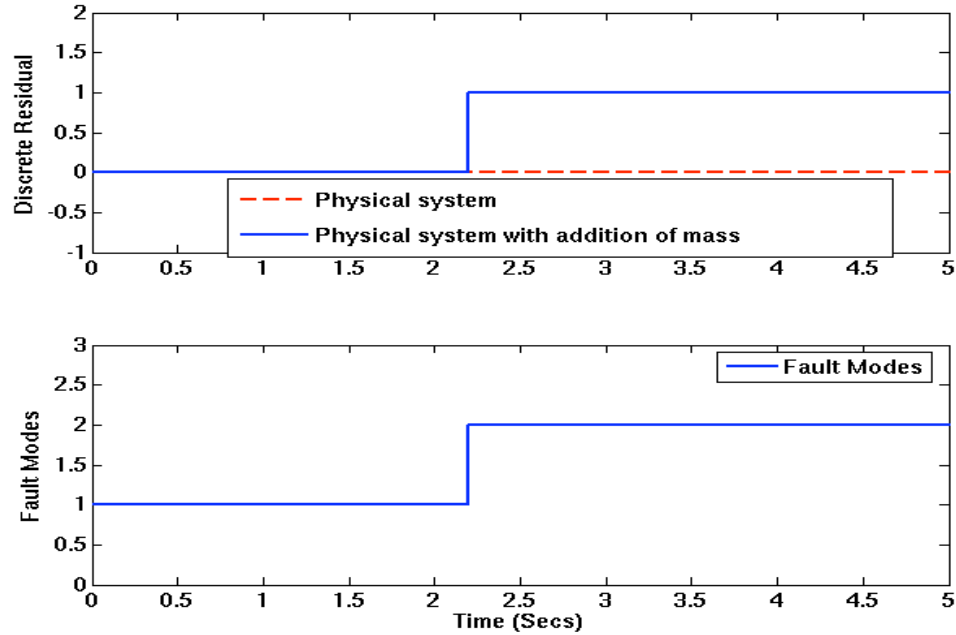


Figure 6-6: Time history of discrete residual and finite-states for BJ filter design

## Acknowledgements

## References

[1] Beard, R.V., "Failure accommodation in linear system through self-reconfiguration," Ph.D. *Dissertation*, Massachusetts Institute of Technology, MA, 1971.
[2] Gertler J., *Fault detection and diagnosis in engineering systems, 1$^{st}$ Edition*, Marcel Dekker, New York, 1998.
[3] Douglas, R.K. and Speyer, J.L., "Robust fault detection filter," AIAA Journal of

Guidance, Control and Dynamics, 19, 1996.

[4] Chen J., Patton Ron J. and Zhang Hong-Yue, "Design of unknown input observers and robust fault detection filters," International Journal of Control, 63, 1996.

[5] Guangjie, F. and Haiwei, M., "New recursive algorithm on model orders and its applications," Proceeding of the 4th world congress on intelligent and automation control, June10-14, 2002, Shangai, P.R. China.

[6] Ninness, B., Gomez, J-C. and Weller, S., "MIMO system identification using orthonormal basis functions," Proceedings of the 34th IEEE Conference on Decision and Control, 13-15 Dec. 1995, 1, 703 -708.

[7] Stoica, P. and Jansson, M.,"A transfer function approach to MIMO system identification," Proceedings of the 38th IEEE Conference on Decision and Control, 7-10 Dec 1999, 3, 2400-2405.

[8] Hagwood N W., Chung W.H. and Von Flotow A., "Modeling of piezoelectric actuator dynamics for active structural control," Journal of Intelligent Material Systems and Structures, 1, 1990, 327-354.

[9] Astrom K.J. and Wittenmark, B., *Adaptive control*, *2nd Edition*, Addison-Wesley, New York, 1995.

[10] White, J.E., "Detection filter design by eigensystem assignment," Ph.D. *Dissertation*, University of Texas, Austin, May, 1985.

[11] Park, J., "A unified theory of fault detection and isolation in dynamic systems," Ph.D. *Dissertation*, The University of Michigan, Ann Arbor, 1991.

[12] Park, J. and Rizzoni, G., "An eigenstructure assignment algorithm for the design of fault detection filters," IEEE Trans., 1994, AC-39, 1521-1524.

[13] White, J.E. and Speyer, J., " Detection filter design: spectral theory and algorithms," *IEEE Trans.*, 1987, AC-32, 593-603.

[14] Rizzoni, G. and Min, P.S., " Detection of sensor failures in automotive engines," IEEE Trans., 1989, 38, 148-158.

[15] Kim, Y. and Park, J., "An analysis of detection spaces using invariant zeros," American Control Conference, San Diego, California, USA, 1999.

[16] Kim, Y. and Park, J., "A condition of the eigenvalues of detection filters for disturbance attenuation: An invariant zero approach," Conference on Decision and Control (CDC), Hawaii, US, 2003.

[17] Levine W. and Athans M., "On the determination of the optimal constant output feedback gains for Linear Multivariable Systems," IEEE Transaction of Automatic Control, 15, 1970, 44-80.

CHAPTER VII

CONCLUSIONS


FDI techniques are studied in detail for the application of active vibration control. Two types of FDI filters are studied in this research work; BJ and parity relations. In this study a simply supported beam with sensors and actuators is chosen as the appropriate flexible structure for vibration control. It is concluded that parity space techniques are inadequate for high order systems such as beams, shells and plates. BJ filter is chosen as the appropriate filter for high order systems. FDI is incorporated with active vibration control to achieve fault-tolerant active vibration control.

The main focus in this work is ability to perform optimal vibration control performance with multiple actuator failures. In order to achieve this new methodology using the invariant zero approach is developed for designing the detection filter gains for high order systems. In addition to this, a new technique for the design of detection gains for systems with feed-through dynamics is developed. Simulation results provide adequate information about fault-tolerant active vibration control for multiple actuator failures. Also, a detection filter is designed for the experimental data (system identification) of a simply supported beam.

Experimental FDI is conducted on a simply supported beam. It is concluded that the BJ filter is robust to disturbance and less appropriate to high sensor noise. Experimental results validated the new design methodology for detection gain matrix for

high order systems. Also, experimental results for fault-tolerant control demonstrate that during switching of controllers the system is found to be stable. In addition to this, the results are adequate to conclude that there was significance optimal vibration control performance with the presence of multiple actuator failures.