ESSAYS ON LIQUIDITY AND TRADING ACTIVITY

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CHAPTER I

IS HEAVY TRADING GOOD OR BAD FOR PRICE DISCOVERY? EVIDENCE FROM OPTIONS

1. Introduction

A certain 'normal' level of trading volume occurs in financial markets due to investors' changing liquidity or portfolio needs. However, the system is frequently perturbed by abnormal trading shocks; and how these shocks affect price discovery depends on the source of such trading. If investors trade due to changes in their private information sets, trading will lead to more efficient prices. On the other hand, if the shocks are triggered by cognitive biases, as suggested by studies in behavioral finance, trading will push prices away from their true equilibrium value.

In this paper, I use options to examine whether market prices on average are more or less informative on high volume days, that is, whether shocks in option volume are caused by informed investors or psychological factors. Because the spot and option markets are tightly related, I study the effect of cross-volume shocks as well, where cross-volume shocks are defined as abnormal trading volume in the spot market. In addition, under the information arrival hypothesis, I examine whether the data are consistent with option trading on volatility related information.

Many prior studies provide arguments on whether volume helps or hurts prices.[1] On the one hand, in Easley, Keifer, and O'Hara (1997a,b), unusually high volume indicates the presence of informed traders and leads to more informative prices as new information is disseminated through the trades. Similarly, trading activity can reflect differences of opinion among market participants. Heavy trading aids in price discovery through the aggregation of investors'

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[1]A related empirical study is Barclay and Hendershott (2003), who find that prices in after-hour trading, when trading volume is low, appear to be less accurate than transaction prices during the trading day.

heterogeneous valuation models and/or interpretation of the common information set (Brandt and Kavajecz (2003)). Cao and Ou-Yang (2003) provide a theoretical model in which option volume increases in both information arrival and the dispersion of its interpretation. Moreover, Admati and Pfleiderer (1988) argue that heavy liquidity trading can further price discovery as well, in the sense that periods of concentrated liquidity trades attract informed investors to the markets. In equilibrium, with endogenous information acquisition, prices are more efficient when trading is intense. More generally, volume may proxy for the rate of information flow (Copeland (1976)), which in turn implies a monotonic positive relation between volume and the efficiency of the price. This interpretation is commonly adopted in empirical studies (for instance, Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), Lamoureux and Lastrapes (1990), and Andersen (1996) for the Mixture of Distribution Hypothesis).

On the other hand, heavy trading may hurt price discovery. In economies with irrational investors, volume shocks may result from cognitive biases. That abnormal volume can reflect behavioral sources is readily illustrated by the case of Massmutual Corporate Investors (MCI) and MCI Communications (MCIC), two unrelated companies with similar ticker symbols. Rashes (2001) documents MCI's abnormal volume reactions to MCIC news announcements, along with other commonly known cases of ticker symbol confusion. More generally, Huddart et al. (2005) find that 'psychological factors are pervasive and strong enough to be an important determinant of equity volume,' and that spikes in trading volume reflect the psychological effect on investors' trading decisions of previous price levels, such as 52-week highs and lows. Also, many academic studies argue that investors are overconfident. Overconfident investors tend to trade too much and, in a time-series context, volume reflects variations in investor sentiment (Odean (1998, 1999), Barber and Odean (2000, 2001, 2002), and Benos (1998)). Even sophisticated traders can display behavioral biases. Dow and Gorton (1997) propose a model in which one of the reasons why managers' trade is to show their employers that they are working hard. Coval and Shumway (2005) examine the trading behavior of Chicago Board of Trade proprietary traders and document important irrationalities driven by loss-aversion. Moreover, some studies suggest that rational investors may find it profitable in the short run to trade on the same side of the market irrational traders do, if convergence is expected to be slow (Brunnermeier and Nagel (2004)).[2]

That there are irrational episodes in every market is hardly disputed in the literature. What is disputed is whether individual irrationalities can aggregate into market wide anomalies.[3] Critics argue that these episodes are random and are eventually priced out of the market, so that their effect on price behavior is negligible (Ross (2005) and Fama (2005)).

In this paper, I contribute to this debate by examining abnormal volume, and its relation to market efficiency. To perform a test of market efficiency, I focus on the option market, where both implied and ex post realized volatilities are available; therefore, a natural measure arises for the market's pricing error. While the relation between implied and realized volatility has been widely studied, the model-free implied volatility utilized in this paper directly allows for a pricing error interpretation. The model-free implied volatility is calculated without specific assumptions on the market's true option pricing model (Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000)). Therefore, it provides a direct test of market efficiency since it is free from the joint hypothesis problem (Jiang and Tian (2005)). In contrast, previous studies are unable to use the relation to gauge market efficiency using traditional parametric implied volatility measures. While I concentrate on the model-free implied volatility, I also use Black-Scholes (1973) implied volatilities calculated from at-the-money options, where model biases are minimized.

To detect periods of intense trading, I adjust the volume series for time-series characteristics and calendar day variation, and orthogonalize spot and option volume. I define abnormal trading as residual volume arising from the adjustment procedure (Marsh and Wagner

[2] Although traditionally many argued that irrational investors will not survive in the market, recent papers believe that survival is possible (De Long, Shleifer, Summers, and Waldmann (1991)). In addition, Kogan, Ross, Wang, and Westerfield (2005) find that irrational traders can affect prices significantly even if trading decreases their wealth over time.

[3] Through social contagion, such as groupthink or herding.

(2004)) and use it to assign each day in the sample period to a high or a low volume state. I also consider well-documented econometric biases associated with implied volatility tests, such as telescoping maturity and errors-in-variables, and employ several methods to calculate realized volatility including range-based measures that are shown to perform better than other estimators, for instance, the absolute return (Alizadeh, Brandt, and Diebold (2000)).

The results indicate that the option market is more efficient on days when option volume is abnormally high. A similar, but weaker, result is found for spot volume. That the pricing error is smaller in high option volume states is consistent with the notion that on average, abnormal trading is not indicative of shocks to irrational investors' demands but rather, more likely to be due to changes in investors' information set or in their interpretation of it. While occasional behavioral trading shocks are likely to take place in any market, the results suggest that, on average, they do not play an important role. As a robustness check, I find that the results are invariant to two alternative weighting schemes for option volume. The rationale behind the weighting schemes is to place more weight on the option contracts that informed traders are likely to prefer. In particular, I use (inverse) option bid/ask spread-weighted and vega-weighted option volume. The later weighting scheme investigates volatility related information trading in the option markets.

As suggested by Jiang and Tian (2005), interpreting the results I take account of the non-zero volatility risk premium and use a constant adjustment. I allow the constant adjustment to differ across the volume states and find significant differences. This implies that the volatility risk premium depends on trading volume as well. However, since the volatility risk premium is not observable, the constant adjustment or the proxies suggested by Chernov (2002) provide an inferior modeling framework. To further eliminate potential econometric biases, I adopt the Kalman filter to estimate the system. As a side result, the Kalman filter provides an estimated time-series of the volatility risk premium.

Also, I disaggregate option volume into two components: the change in open interest and a net component, or the difference between total volume and the change in open interest. This is based on Day and Lewis (2004) who use the decomposition to approximate the speculative and hedging components of derivative volume. I examine whether abnormal shocks to speculative and/or hedging demands help or hurt price discovery in the option markets. Since the decomposition into speculative and hedging components is based on open interest, the results will be suggestive of the types of trades that affect the number of contracts outstanding. When option trading is decomposed into speculative and hedging activities, the findings indicate that the presence of speculators enhances price discovery while for hedging demand, no significant relation is found. In addition, put related hedging demands are harmful for price discovery, which is consistent with Bollen and Whaley (2004), who argue that portfolio insurance demands cause temporary price pressures when no natural counterparties arise for the trade.

The current study is related to two papers in the literature. First, Mayhew and Stivers (2003) examine the time-series forecasting performance of Black-Scholes (B-S) implied volatility. In their cross section of stocks, they observe that only for stocks with the very largest option trading volume does implied volatility show significant forecasting ability. This result is important, and is consistent with the notion that securities that receive more investor attention have more information efficiency in pricing. In the cross-section, this constitutes a friction-based argument. In contrast, I investigate a second role for volume; in particular, whether periodic abnormal trading activity reflects information arrival as suggested by previous studies. Second, Donaldson and Kamstra (2004) show that the B-S implied volatility outperforms the volatility forecasts produced by ARCH models when NYSE volume is high. This latter result may naturally arise since the Black-Scholes model has friction biases. For instance, Longstaff (1995) finds that the discrepancy between the B-S implied volatility and realized volatility decreases when market conditions are closer to the assumptions underlying the B-S model; that is, when transaction costs are lower and liquidity (measured by trading activity and other proxies) is higher. Though the Donaldson and Kamstra result is also consistent with the conjecture that spot trading enhances option market efficiency, they are unable to draw conclusions about market efficiency due to the joint hypothesis problem. In contrast, my paper focuses on market efficiency. Also, while examining the role of spot volume for option prices is very interesting, option volume has a more important role for answering the questions raised in my paper. This is because spot volume only has an indirect influence on option prices (through its effect on the spot price), while option volume has a direct effect.

Interestingly, this would not be true, if options were truly redundant. This is because with redundancy, the spot-volume-effect is the only effect one could expect, as options could be priced even if no trades occurred in the option market. Therefore, the results of my paper have implications not only for the role of volume for market efficiency, but also, for the redundancy argument.

The paper is organized as follows. Section 2 provides an analytic framework for examining the relation between volatility forecasts and volume. Section 3 discusses the data sources and volume adjustments. Section 4 provides the estimation results and discusses the robustness analyses. Section 5 concludes.

2. Analytic Framework

In this section, I introduce a framework for examining the efficiency of price discovery. The section explains the difficulties of measuring pricing errors and provides a link between the current approach and prior literature on implied and realized volatility. It also discusses how volume states are incorporated in the tests.

2.1. Tests for the Rationality of the Market's Forecast

Price discovery is commonly defined as the process by which information is impounded in the market prices, or, alternatively, the process by which observed market prices approach the true underlying value. Therefore, efficiency is usually interpreted in terms of pricing errors, i.e., by how close market prices are to the underlying true asset price. The underlying true price is generally not observable, however. For instance, since stocks do not expire, the only measurement on the underlying latent price process is the market price; hence, it is not clear how to capture the pricing error. For options, there are two price measures (expressed in units of volatility). First, volatility forecasts are implied in the option's price. Second, an ex post measure is also available: after expiry, realized volatility can be estimated for the relevant time period. Hence, the pricing efficiency can be assessed by comparing the implied and realized volatilities.[4] With two signals on the true underlying process, options provide a unique opportunity to assess market efficiency. When the market price is observable and an unbiased or consistent estimate of realized volatility is employed, the pricing error representation is analogous to the argument underlying the non-parametric testing problem of comparing model implied moments to the empirical densities (for instance, Ait-Sahalia (1996)). In that case, since the nonparametric/empirical measure is always consistent, the bias in the assumed model can be determined by the distance between the empirical and the model implied densities.

A more rigorous discussion of this idea is related to Canina and Figlewki (1993). In particular, when the econometrician knows the market's true pricing model, by inverting the pricing model, the market's expected future volatility is directly observable (no uncertainty):

$$IV_t = E_{M,t}(s_{t+1}), \tag{1}$$

where IV_t is the volatility embedded in current option prices maturing at $t+1$, s_{t+1} is the true volatility from t to $t+1$. The right-hand-side denotes the market's expectation of next period's

[4]This assumes that the volatility risk premium is zero. When the volatility risk premium is not zero, all expectations must be taken with respect to the risk neutral probability measure, thus the implied volatility will be larger than the realized volatility by the volatility risk premium (see, for instance, Chernov (2002)). This case will also be considered in this paper.

volatility. IV_t, the inverse price, represents the market price in units of implied volatility. Based on the observation that, by definition the underlying volatility can be decomposed into its expected value at time *t* and a random component orthogonal to the time *t* information set,

$$\sigma_{t+1} = E_t(\sigma_{t+1}|\Phi) + \eta_{t+1} \qquad \text{with} \qquad E_t(\eta_{t+1}|\Phi)=0, \tag{2}$$

the rationality of the market's forecast can be examined in the following linear regression framework:

$$\sigma_{t+1} = \alpha + \beta \cdot IV_t(\Phi) + \varepsilon_{t+1} \tag{3}$$

where Φ indicates the set of all currently available information and ε_{t+1} is the regression residual. In equation (3) σ_{t+1}, the true underlying volatility, is replaced by an estimate of realized volatility, which when unbiased/consistent, does not change the validity of the test. The market's expectation is rational if $\alpha = 0$ and $\beta = 1$.

2.2. Black-Scholes Implied Volatility

The framework above assumes that the econometrician knows the option pricing model Therefore, equation (3) provides a direct test of market efficiency. In reality, the option pricing model is not known; hence in prior studies equation (3) is described as a joint hypothesis test, rather than a test of market efficiency.

Many studies employ the Black and Scholes (B-S) (1973) implied volatility (IV^{B-S}) as the embedded option price volatility. However, if the market's pricing model is not the B-S, inverting the option prices via the B-S model will not provide the market's true expectation (i.e.,

(1) will not hold). Indeed, many argue that it is inconsistent to derive volatility expectations from a constant volatility model (Britten-Jones and Neuberger (2000)). When we know that the B-S implied volatility is not the market's true volatility expectation, it is not straightforward to expect that the B-S implied volatility and the true volatility (or realized volatility) are linearly related. Therefore, one must question whether equation (3) provides a suitable framework for testing in this case.

Interestingly, Bandi and Perron (2003) show that an approximate linear relation holds regardless of the validity of the B-S option pricing model. For instance, when volatility is stochastic but the volatility risk premium and the correlation between price and volatility are zero, Hull and White (1987) show that the option price equals the expected value of the Black-Scholes (1987) price evaluated at the average integrated volatility. That is,

$$O_{HW,t-1}(\sigma_{\tau} = \sqrt{V_{\tau}}) \approx E_{t-1}[O_{B-S,t-1}(\sigma_{\tau} = \sqrt{\overline{V}_{t,\tau}})] \qquad (4)$$

where σ, τ, and V are the standard deviation, time to maturity, and underlying volatility, respectively and

$$\overline{V}_{t,\tau} = \frac{1}{\tau}\int_{t}^{t+\tau} V_s \, ds \, . \qquad (5)$$

Moreover, the Black-Scholes formula is shown to be nearly linear in volatility for at-the-money options (Feinstein (1988)). Therefore,

$$E_{t-1}(O^{atm}{}_{B-S,t-1}(\sigma_{\tau} = \sqrt{\overline{V}_{t,\tau}})) \approx O^{atm}{}_{B-S,t-1}(\sigma_{\tau} = E_{t-1}(\sqrt{\overline{V}_{t,\tau}})) \qquad (6)$$

That is,

$$E_{t-1}[\sqrt{\bar{V}_{t,t}}\ ] \approx \sigma^{atm}{}_{B\text{-}S,t}. \qquad (7)$$

where $\sigma^{atm}{}_{B\text{-}S,t}$ is the at-the-money Black-Scholes volatility and the expectation is taken with respect to the risk-neutral probability measure. This implies that when the price dynamics can be described by Hull and White (1987)-type stochastic volatility models, the Black-Scholes implied volatility of at-the-money options is approximately the average volatility over the life of the option. Hence, the relation between realized and implied volatility is approximately linear and equation (3) is still valid but, as it was mentioned above, has to be interpreted in the joint hypothesis framework. As Poteshman (2000) points it out, this approximation hinges on B-S being linear in volatility for at-the-money options. Therefore, squaring volatility or taking a logarithmic transformation invalidates the approach due to Jensen's inequality. In addition to Hull and White (1987)-type stochastic volatility models, Jones (2001) argues that the approximate linear relation between realized volatility and the Black-Scholes implied volatility remains valid when the correlation between price and volatility is non-zero (i.e., there is a leverage effect).

Many studies estimate the forecasting power of implied volatility for subsequent realized volatility. These studies rely on the linearity results above. For example, Day and Lewis (1992), Lamoureux and Lastrapes (1993), Canina and Figlewski (1993), Figlewski (1997), Christensen and Prabhala (1998), Blair et al. (2001), and Chernov (2002) study the information content of implied volatility and compare it to other forecasts of future volatility, such as the ones obtained from Autoregressive Conditional Heteroskedasticity (ARCH) -type models. Evidence provided by these studies is mixed, for instance, Canina and Figlewski (1993) question the success of implied volatility to improve forecasts based on historical price information. However, Christensen and Prabhala (1998) attribute this finding to econometric problems. Day and Lewis

(1993) find that implied volatilities are informative but biased. Blair et al. (2001) report that implied volatility provides the majority of relevant information on future realized volatility in-sample. Out-of-sample, the implied volatility forecast is the most accurate forecast available. Overall, a recent survey by Poon and Granger (2003) concludes that implied volatility provides a superior forecast of future volatility.

Two main analyses are applied in the literature to examine the information content and performance of implied volatility. Both of these rely on the linearity results discussed above. First, studies augment the ARCH-type systems with implied volatility in the variance equation. In this framework, the coefficient of the implied volatility term is interpreted as the incremental information content of option prices that is not included in the historical price process (for instance, Day and Lewis (1992). A likelihood ratio test is then performed to compare the original model based solely on past information with the augmented specification. Second, the following, so called encompassing regressions are estimated:

$$s_{realized,t} = b_0 + b_1 s_{implied,t-1} + b_2 s_{historical,t} + e_t. \qquad (8)$$

where s denotes volatility (for instance, Chernov (2002) and Poteshman (2000)). Equation (8) is based on equation (3). First, implied volatility is efficient, if $b_2 = 0$. Moreover, as in equation (3), unbiasedness requires $(b_0, b_1)=(0,1)$. Most studies mentioned above generally find that estimates of the b_0 coefficient are significantly different from zero, while the estimates of b_1 are less than one. Thus, they conclude that implied volatility provides a biased forecast. Evidence on b_2 is mixed.

2.3. Volatility Risk Premium

Chernov (2002) and Bandi and Perron (2003) illustrate that the approximate linear relation between realized and B-S implied volatility is preserved even when the volatility risk premium is not zero and/or the underlying asset price has jumps, for instance. However, in these settings, the additional risk premiums must be incorporated in the analyses. For instance, when the volatility risk premium is priced, the expectation in equation (7) is taken with respect to the risk neutral probability measure, Q, (when this risk is not priced, the risk-neutral and the objective probability measures coincide):

$$E^{Q}_{t-1}[\ \sqrt{\overline{V}_{t,t}}\ ] \approx \sigma^{atm}_{B\text{-}S,t}. \tag{9}$$

Bandi and Perron (2003) show that eqation (9) implies the following:

$$E_{t-1}[\ \sqrt{\overline{V}_{t,t}}\ ] + RP_t \approx \sigma^{atm}_{B\text{-}S,t}. \tag{10}$$

where the expectation on the left-hand-side is taken with respect to the objective probability measure and *RP* represents the volatility risk premium.

The important implication of this result is that, although the linear relation remains valid, equation (3) no longer provides valid inference. This is because omitting the volatility risk premium (or the jump risk premium) causes β to be downward biased due to the omitted variable problem, while α will capture the expected price of volatility risk and thus, will not be zero even when implied volatility is unbiased. The valid test in this case is described by the following regression model with the null hypothesis that H_0: $(\alpha,\beta) = (0,1)$:

$$\sigma_{t+1} = \alpha + \beta \cdot IV_t(F) + \gamma RP_t + \varepsilon_{t+1} \tag{11}$$

Chernov (2002) and Poteshman (2000) find that the forecasting bias is largely eliminated when they account for non-zero volatility risk premium. On the other hand, Neely (2003) finds that permitting a non-zero price for volatility risk does not influence the forecasting performance of implied volatility. However, these results have to be interpreted with caution since *RP* is not observable, and using an arbitrary specification/proxy may introduce additional biases (Bandi and Perron (2003)).

The results discussed in this subsection provide a motivation for modeling realized volatility as a linear function of the Black-Scholes implied volatility. These results indicate that at-the-money B-S implied volatility remains approximately unbiased under fairly general conditions when the B-S assumptions are not satisfied. Therefore, I use B-S implied volatilities calculated from at-the-money options in this study, however, as in most previous studies, the joint hypothesis concern cannot be completely eliminated. In contrast, I also use model-free implied volatility, which is free from the joint hypothesis problem. In a recent paper, Jiang and Tian (2005) suggest a direct test for the information efficiency of S&P 500 index options by utilizing a model-free implied volatility measure. The next subsections introduce the model-free measure and describe the tests employed in this study.

2.4. Model-free Implied Volatility

The model-free implied volatility presents an alternative way of measuring volatility embedded in the price of options and provides the expected sum of squared returns under the risk-neutral measure. However, unlike traditional option-implied volatilities that are derived from specific assumptions on the market's true pricing model (for instance, Black and Scholes (1973) and Heston (1993)), the model-free implied volatility is calculated from option prices directly, without any particular assumption on the underlying price process (Carr and Madan (1998), Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000)). The model-free measure of implied volatility (IV^{MF}), which provides the risk-neutral expectation of the integrated volatility is given by the following:

$$IV_{t,t+t}^{MF} = E_0^Q[\int_{T_1}^{T_2} (\frac{dS_t}{S_t})^2] = 2(\int_0^\infty \frac{C(t+t,K) - C(t,K)}{K^2} dK) \qquad (12)$$

where S_t is the underlying asset, r_t is the risk-free rate, $C(t, K)$ denotes the European call price maturing at time t with strike price K. Due to risk-neutrality, the presence of a non-zero volatility risk premium discussed in the previous subsections becomes an important issue for model-free implied volatilities as well. The result stated in equation (12) is derived by Britten-Jones and Neuberger (2000) under the assumption that a continuum of strike prices and maturities is available in the market, and that the price process has no jumps. However, Jiang and Tian (2005) extend the result and show that (12) can be accurately approximated using a finite, empirically realistic number of options and suggest that it remains valid for stochastic processes with jumps (see also Bollerslev et al. (2005)).[5]

The model-free implied volatility is becoming increasingly popular for both academics and practitioners. For instance, the Chicago Board of Option Exchange (CBOE) has recently adopted the model-free implied volatility approach to calculate its implied volatility index (VIX). The new approach is based on a discrete approximation to equation (12).

[5]It is important to re-emphasize the difference between the B-S and the model-free measures. The model-free measure is an empirical measure; therefore, it is always consistent. In contrast, the B-S implied volatility is the market's expected volatility filtered through the B-S model. If the B-S model is not the true pricing model, the filtering distorts the true expectations; hence the joint hypothesis problem.

2.5. Realized Volatility Measures

To assess the impact of trading volume on option price discovery, an ex post proxy of the underlying volatility process, i.e., the estimated realized volatility, is required. Furthermore, the estimated realized volatility must provide a good approximation to the true underlying price. This imposes the requirement that the chosen estimator satisfy given statistical properties, such as consistency or unbiasedness.

There are various proposed methods for calculating realized volatility. The traditional method is based on close-to-close squared returns (r_t^2). The annualized value of the *n*-day volatility is given by the following[6]:

$$s_{realized} = \sqrt{\frac{252}{n}\sum_{t=1}^{n} r_t^2} \tag{13}$$

Since the mean is assumed to be well-approximated by 0 at the daily frequency, the divisor is *n* rather than *n-1* since no degrees of freedom are lost to estimate the mean.

Parkinson (1980) proposes a measure based on the daily high and low values:

$$s_{realized} = \sqrt{\frac{252}{n}\sum_{t=1}^{n} \frac{1}{4\ln 2}(\ln H_t - \ln L_t)^2} \ , \tag{14}$$

where H_t and L_t are high and low values respectively. Andersen (2000) argues that high frequency (intra-day) data are better able to approximate the underlying continuous process. Surprisingly, Andersen and Bollerslev (1998) show that the daily range provides similar

[6]It is important to note that, since taking the square-root is a concave transformation, based on Jensen's inequality, the realized volatility estimators are upward biased for the true volatility.

information content to intra-day returns sampled at the four-hour frequency. Brandt and Jones (2002) and Chernov (2002) motivate the performance of the range by assuming that returns are described by a constant volatility Brownian motion: volatility is constant during the day but is allowed to change daily. Though the range suffers from the discretization bias, Brandt and Jones (2002) argue that for the S&P 500 index this bias affects the daily range negligibly due to the liquidity of the index. As a robustness check, this paper employs both measures of realized volatility in the empirical tests.

2.6. Estimation Method

This paper uses both the model-free and the B-S implied volatilities and builds on the predictive regression framework described in equation (3). In addition, I address the issue of a non-zero volatility risk premium described in equation (11). The models are modified however, to accommodate the idea of volume states. That is, the bias in implied volatility as measured by the joint behavior of a and b in the equations is allowed to vary based on whether the market price (i.e., implied volatility) is formed on high or low option/stock volume days. The type of asymmetry will be suggestive of the cause: implied volatility is expected to be less biased in high, than in low, volume states when abnormal trading shocks are triggered by information arrival On the other hand, if trading shocks on average are due to changes in investors' sentiment, implied volatility will be more biased in high volume states.

Volume states are defined by abnormal trading in the option and stock markets, respectively. Since the stock and option markets are intimately related, spot volume may play an important role in price discovery in the option market. This is because heightened spot volume will affect the spot discovery process, which in turn affects the option's price. Section 3 below explains how the volume series are adjusted to represent surprise/abnormal values. This is generally required in the literature in order to improve the statistical properties of these series and remove the deterministic variation caused, for instance, by calendar day effects. In addition, surprise volume provides an intuitive way to pinpoint high/low volume periods. For instance, Marsh and Wagner (2004) use surprise volume as a proxy for information flow and incorporate asymmetric volume regimes in the conditional volatility equation in their GARCH model. Donaldson and Kamstra (2004) also incorporate switching volume states.

To account for the regime structure implied by the own- and cross-volume effects, I estimate predictive regressions of the following form:

$$\sigma^2_{realized,t} = \alpha_1 I_{\{V_{t-1}>0\}} + \alpha_2 I_{\{V_{t-1}\le 0\}} + \beta_1 \sigma^2_{implied,t-1} I_{\{V_{t-1}>0\}} + \beta_2 \sigma^2_{implied,t-1} I_{\{V_{t-1}\le 0\}} + \varepsilon_t \qquad (15)$$

where V_t represents adjusted (abnormal) volume (spot and/or option). $I_{V>\alpha}$ is an indicator variable that takes the value of 1 when volume is larger than a predetermined threshold level α, and 0 otherwise. Since the adjusted volume series reflect surprise or abnormal volume, α is usually set to zero in the empirical implementation of the tests. However, I also examine various tail percentiles defined on abnormal volume. More specifically, using the abnormal volume series, for each day I create an indicator variable that equals 1 if the current abnormal volume exceeds a pre-specified percentile of abnormal volume in the previous 60 days. Under this setting, α changes over time. In equation (15), both the intercept and the slope coefficients are allowed to change between high volume and low volume states. Therefore, this framework is equivalent to a Seemingly Unrelated Regression (SUR) structure. The specification allows for an asymmetric constant adjustment when the volatility risk premium is not zero.

Since model-free implied volatility provides the risk-neutral expected sum of squared returns, if volatility risk is priced, equation (15) has to be adjusted in the spirit of equation (11). In addition, subsection E questions whether a constant adjustment provides an econometrically sound way to interpret the volatility risk premium in equation (11). As before, difficulties arise since the volatility risk premium is not observable. Therefore, I employ the Kalman filter to estimate the following system:

$$s^2_{realized,t} = \sum_{r=1}^{2} a_r I_r + \sum_{r=1}^{2} b_r s^2_{implied,t-1} I_r + \sum_{r=1}^{2} g_r RP_t I_r + e_t$$
$$RP_t = w + fRP_{t-1} + d' X_t + h_t \tag{16}$$

where, as in equation (11), *RP* represents the volatility risk premium, *X* represents the vector of independent variables that drive the risk premium, *r* indicates the realized volume state, and I_r is an indicator variable that takes the value of 1 when volume state *r* occurs and 0 otherwise. The Markov structure is suggested by Bollerslev et al. (2005), who also find that the *X* vector contains macro-finance variables, such as market volatility, the price earnings ratio of the market, credit spread, industrial production, the producer price index, and non-farm employment.

The *t* subscript of realized volatility stands for the *t*-th time interval for which the realized volatility is estimated. I examine daily data as well as a sample based on 30 calendar day non-overlapping windows. The window length represents a trade-off between the sample size used in the estimation and the biases imposed by using sampling windows that are shorter than the forecast window. For instance, the 30 calendar day interval matches the horizon embedded in the implied volatility estimates employed in this study, however, it significantly reduces the available sample size. The shorter windows cause an overlap in the error term, which can be reasonably corrected by Hansen's (1982) GMM method (Canina and Figlewski (1993), Poteshman (2000)). A more serious problem is that even under the assumption of mean reversion, the volatility process may have substantial temporary deviations from its mean. The use of time periods that are short relative to the forecast window of implied volatility increases the likelihood of observing some of these deviations; hence, estimates of realized volatility based on these observations will not provide an accurate measure of average volatility until expiry. For the shorter horizon, the square-root-of-time rule is used to bring the realized and implied volatility to a common time unit.

Equations (15) and (16) separate implied volatilities based on whether they are obtained in high volume or low volume regimes. I also estimate similar models based on weighted option volume as well as on the speculative and hedging components of derivative trades.

3. Data Description and Volume adjustments

3.1. Data and Descriptive Statistics

Data used in this paper contain trading volume, open, close, high, and low values of the S&P 500 (SPX) index; option data for the corresponding SPX options, and the corresponding VIX implied volatility index of the CBOE. Data on the S&P 500 are obtained from *Yahoo! Finance*. Data on SPX option trading activity are obtained from two sources: the CBOE and *OptionMetrics*, and contain daily observations on volume, open interest, as well as volume and open interest for Long-term Equity Anticipation Securities (LEAPS) puts and calls (i.e., long-dated put and call options) for the time period of January 2, 1996-June 30, 2004. Also, data on option vegas, B-S implied volatilities, and bid/ask spreads are from *OptionMetrics*. The sample period contains 2139 daily observations. Additional data are obtained to account for the effect of macroeconomic news announcements and changes in the composition of the index.[7]

For the implied volatility measures I use two sources. The B-S implied volatility is calculated from the implied volatilities reported in *OptionMetrics*. I follow the method suggested by Ni, Pan, and Poteshman (2005) and calculate the B-S based market price of equity volatility by

[7]Information on announcement dates for the consumer price index, producer price index, the employment situation, and productivity is obtained from the website of the Bureau of Labor Statistics. Federal Open Market Committee meeting dates are available from the website of the Federal Reserve Bank. Index additions and deletions are listed by Bloomberg Data Services.

averaging the implied volatilities of an at-the-money put and an at-the-money call to form a straddle. I choose these options by requiring that they have the same expiration date and strike price, restricting time to expiration between 5 and 50 days and the strike to closing index price between 0.8 and 1.20, and choosing the pair that is most at-the-money. If there is more than one pair that satisfies these criteria, I choose the one with the shortest time to expiration.

Following Bollerslev et al. (2005), I use the CBOE's VIX index to measure model-free implied volatility. The VIX ('new VIX') implied volatility index of the CBOE is calculated using SPX option prices and expresses the market's expectation of 30 calendar day volatility.[8] The new VIX is calculated using the model-free implied volatility formula based on the theoretical results of Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000). Under the new methodology, the VIX calculations utilize a wide range of strike prices, rather than only at-the-money options; hence the resultant implied volatility uses information contained in option prices across the volatility skew. The correlation between the B-S implied volatility calculated from the straddle's price and the VIX for the sample period is 0.89.

Figure 1.1 graphs the closing values of the VIX and the S&P 500 indices. For the full sample, the correlation between the implied volatility index and the S&P 500 is 0.13. However, in sub-samples significant differences arise. In the first part of the sample period, the VIX and SPX series are strongly positively correlated (for instance, for the period January, 1996 to December, 1997, the correlation coefficient is 0.75), while in more recent years the correlation is significantly negative (for instance, for the sample period of January, 1999 to December, 2001, the correlation coefficient is -0.60). Consistent with the leverage effect, the correlation between the change in VIX and change in the S&P 500 index return is significantly negative (-0.77).

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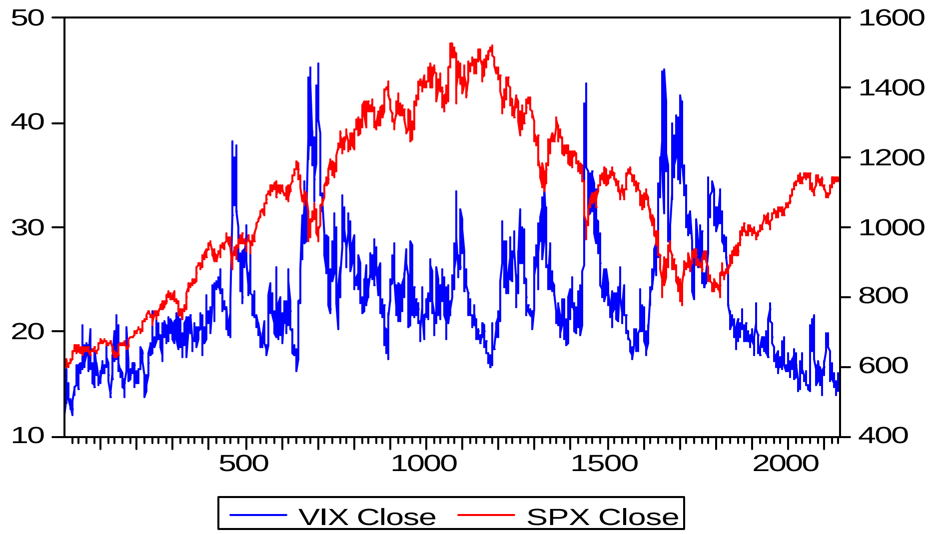
[8] The CBOE adopted a new methodology for calculating implied volatilities on September 22, 2003. Prior to the change in methodology, VIX represented the implied volatility index of S&P 100 index (OEX) options. Since September 22, 2003, VIX refers to the implied volatility index of SPX options and the old VIX has been renamed to VXO. The CBOE provides a full price history for the new VIX from 1986 and continues reporting the old VIX implied volatility as VXO. The CBOE adopted the new VIX calculations in order to better conform to industry practices: to employ a pricing method that is more likely to be the actual pricing method the market uses to value SPX derivatives.

Figure 1.1. Closing Values for the VIX implied volatility and the S&P 500 indices



The figure depicts daily closing values for the CBOE's VIX implied volatility index and the S&P 500 index on a dual crossing scale in which the vertical axis on the left side provides the values of the VIX and the right vertical axis provides values of the S&P 500 index. The time period is January, 1996 to June, 2004.

Table 1.1 reports summary statistics for the S&P 500 index and for the CBOE's VIX implied volatility index. Panel A describes the distribution of the high, low, and closing values of both indices as well as the total trading volume of the S&P 500 (SPX) index, as expressed in units of number of shares traded, and the total trading volume of SPX options, given by the number of option contracts (including put and call volume in both SPX and SPX LEAPS options).

Panel B provides a breakdown of the total SPX option volume into call, put, LEAPS call, and LEAPS put options. In addition, a similar breakdown is reported for open interest. The table confirms that put volume for S&P 500 index options is significantly higher than call volume for both the short-term and the long-term options categories: average daily call volume excluding LEAPS is 41,685.04 contracts, while the average daily put volume equals 61,199.80 contracts. For LEAPS, the average daily put and call volumes are 977.88 and 2,754.38 contracts, respectively. Similarly, the average daily put open interest is 1,158,575.00 for short-term options and 479,543.10 for LEAPS, in contrast, the average call open interest is 868,359.70 for short-term, and 286,146.8 for long-term options. S&P 500 index puts are used by institutional investors to provide portfolio insurance, which explains the high volume of puts in the sample.

Table 1.1 Descriptive Statistics for the S&P 500 and VIX indices

The table reports summary statistics for the S&P 500 index and for the CBOE's VIX implied volatility index. Panel A describes the distribution of the high, low, and closing values of both indices as well as the total trading volume of the S&P 500 (SPX) index as expressed in the number of shares traded and the total trading volume of SPX options given by the number of option contracts (including put and call volume in both SPX and SPX LEAPS options). Panel B provides a breakdown of total SPX option volume into call, put, LEAPS call, and LEAPS put options. In addition, a similar breakdown is reported for open interest. The time period is January 2, 1996 to June 30, 2004.

Panel A

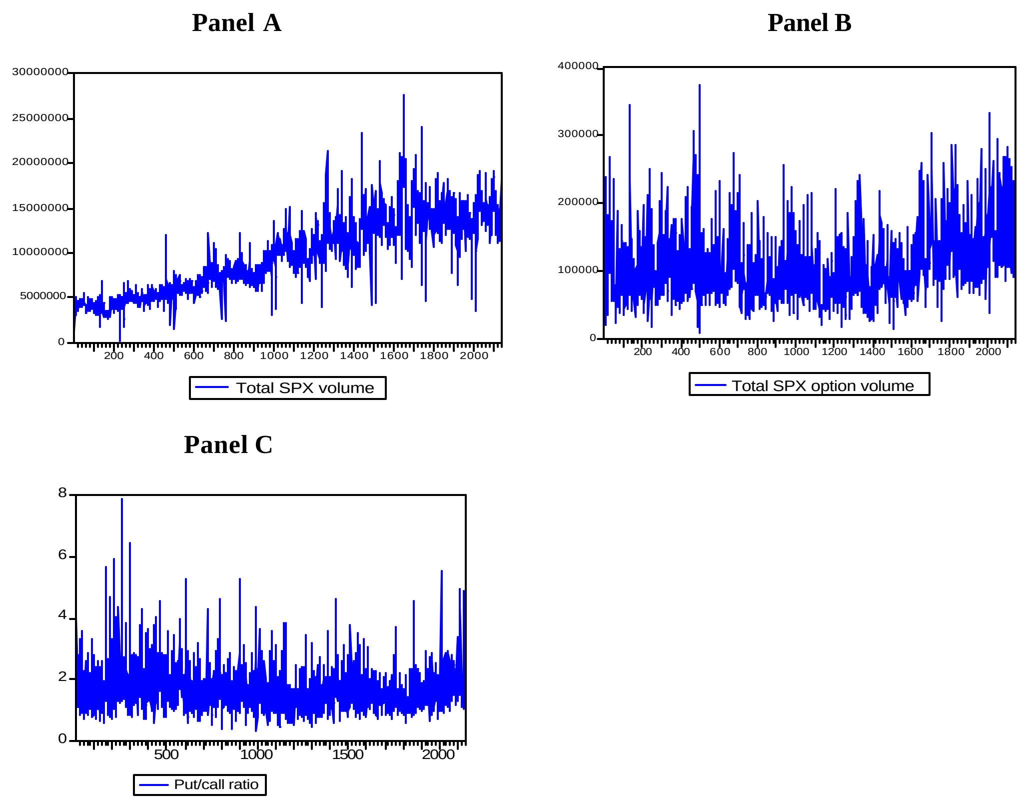
	Index Values						Volume	
	SPX Close	SPX High	SPX Low	VIX Close	VIX High	VIX Low	SPX	SPX Option
Mean	1,069.99	1,077.83	1,061.09	22.98	23.77	22.42	9,665,774	106,624
Median	1,089.45	1,096.95	1,079.98	21.83	22.58	21.32	9,386,000	98,866
Maximum	1,527.46	1,552.87	1,518.46	45.74	49.53	45.58	27,755,600	375,840
Minimum	598.48	602.71	597.29	12.00	12.29	11.11	149,900	7,656
Std.Dev.	235.88	238.05	233.83	5.82	6.13	5.64	4,197,066	47,871
Skewness	-0.09	-0.10	-0.09	0.99	1.06	0.96	0.37	1.11
Kurtosis	2.16	2.17	2.16	4.04	4.29	3.94	2.49	4.86
Jarque-Bera	65.25	64.20	66.08	445.90	548.74	406.88	71.19	751.96
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
N	2,139	2,139	2,139	2,137	2,137	2,137	2,139	2,136

Panel B

	Volume				Open Interest			
	SPX Call	SPX Put	SPX Leap Call	SPX Leap Put	SPX Call	SPX Put	SPX Leap Call	SPX Leap Put
Mean	41,685	61,199	978	2,754	868,360	1,158,575	286,147	479,543
Median	37,181	56,059	59	1,586.5	841,081	1,097,825	206,894.5	511,261.5
Maximum	167,199	219,324	39,841	77,231	1,602,823	2,823,283	797,954	708,225
Minimum	2,837	4,587	0	5	21,917	32,336	7,775	4,627
Std.Dev.	22,252.76	28,985	2,770.58	4,394.06	222,968.40	339,173	241,324.50	141,212.00
Skewness	1.39	1.25	5.51	7.60	0.47	1.24	0.54	-0.76
Kurtosis	5.97	5.42	45.20	95.48	2.96	5.14	1.85	3.14
Jarque-Bera	1,474.84	1,077.72	169,258.40	781,791.30	78.54	949.97	220.26	205.41
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
N	2,138	2,138	2,136	2,136	2,129	2,130	2,128	2,128

Panel A, B, and C of Figure 1.2 display daily volume for the S&P 500 index, the corresponding daily total index option volume, and the put/call ratio for options on the index, respectively.

Figure 1.2. Daily Volume and Put/Call Ratio



Panel A, B, and C of the figure display daily volume for the S&P 500 index, daily total S&P 500 index option volume, and the put/call ratio for the index options, respectively. Spot volume is expressed in the number of shares, while option volume is represented by the number of contracts. The time period depicted in the figure is January, 1996 to June, 2004.

Spot volume is expressed in number of shares, while option volume is represented by the number of contracts. The correlation between the S&P 500 index volume and the total SPX option volume is 0.36. The figure indicates that spot volume is non-stationary. This issue will be revisited below. In addition, option volume displays time-series dependence. The first and second order autocorrelation coefficients are 0.56 and 0.44, respectively, for total option volume on the SPX, and 0.91 and 0.87, respectively for SPX spot volume.

Table 1.2 describes the distributional characteristics of the daily hedging and speculative components of total, put, and call volume of the S&P 500 options. The hedging and speculative components are defined based on the intuition that day-to-day changes in open interest are likely driven by hedging-related trades (Day and Lewis (2004)). Therefore, volume generated by hedgers is proxied by the absolute change in the relevant open interest series (total, put, or call open interest) and the speculative component is expressed as the difference between the volume series and the corresponding hedging volume estimate.

Table 1.2. The Speculative and Hedging Components of Option Volume

The table describes the distributional characteristics of the daily hedging and speculative components of total, put, and call volume of the S&P 500 options. Volume generated by hedgers is proxied by the absolute change in the relevant open interest series (total, put, or call open interest) and the speculative component is expressed as the difference between the relevant volume series and the corresponding hedging volume estimate. For some days in the sample, the change in open interest is larger than total trading volume. These observations are coded as missing and are not included in the calculations in this table. The time-period for which the daily summary statistics are calculated is January, 1996 to June, 2004.

	total		Puts		Calls	
	Speculative	Hedging	Speculative	Hedging	Speculative	Hedging
Mean	75,202	29,887	45,178	17,255	30,862	12,508
Median	67,506	26,589.5	40,259	14,999	26,753	10,474
Maximum	347,531	139,916	194,817	88,669	157,524	139,741
Minimum	806	0	67	0	27	0
Std.Dev.	42,950.25	17,816.79	26,860.48	11,156.09	20,465.01	9,263.55
Skewness	1.17	1.31	1.27	1.46	1.44	2.65
Kurtosis	5.31	5.99	5.79	6.86	6.27	24.25
Jarque-Bera	877.21	1,286.39	1,153.32	1,898.17	1,508.06	37,990.96
Probability	0.000	0.000	0.000	0.000	0.000	0.000
N	1,946	1,946	1,941	1,941	1,901	1,901

These proxies have obvious weaknesses and are likely to misclassify some traders. Therefore, they will bias the tests against finding any significant relation. However, important groups, such as portfolio insurance demands, will be unambiguously classified in the appropriate category. Portfolio insurance trades are associated with net buying pressure (Bollen and Whaley

(2004)) since the market maker is required to absorb these trades. As a result, changes in these demands will always lead to changes in open interest.

For some days in the sample (not only days surrounding expiration days), the change in open interest is larger than total trading volume. These observations are coded as missing and are not included in the calculations in this table. The average daily estimated speculative component is larger than the hedging component for both calls and puts. The difference in means is statistically significant at the 1 percent level in all cases. One interesting question is whether calls or puts have a larger portion of speculative trading based on the proxy employed here. To answer this question, I calculate the proportion of total put (call) volume provided by the estimated speculative put (call) volume for each day in the sample. Due to the missing observations generated above, the sample for which both call and put speculative trading estimates are available contains 1,875 days. The average share of the speculative trading proxy is 0.70 and 0.69 for put and call volume respectively. On 982 (893) days, the share of speculative trading is larger for put (call) volume than call (put) volume.

3.2. Volume Adjustments

As mentioned above, spot volume is not stationary, while option volume displays a significant amount of time-series dependence. Figure 2 implies that option volume may have a quadratic trend. Also, deterministic variation in volume due to calendar day effects has been extensively documented in previous studies. Calendar day effects include days-of-the-week, holiday, and expiration date effects. These characteristics induce biases in empirical work, and are especially problematic in this study. For instance, without the adjustment, the trading days surrounding exchange holidays may fall in the low volume state. The volume series are commonly adjusted in the literature (see, Gallant et al. (1992), Campbell et al. (1993), Marsh and Wagner (2004) for examples of adjustment techniques). In addition, macroeconomic news announcement days and days when a change occurs in the index composition may affect trading volume in both the stock and the option markets.

The adjustments employed in this paper are based on Marsh and Wagner (2004). In addition to addressing the econometric problems above, adjusting the volume series also provides an intuitive way to define periods of heightened trading activity, which is a crucial point in this paper. This is because the adjustments detailed below create residual series, which can be interpreted as abnormal volume: the difference between realized trading activity and forecast trading volume, where the forecast is based on time-series dynamics and calendar characteristics.

First, consistent with the volume literature, a logarithmic transformation is applied to both the option and the spot volume series to improve the distributional properties (for instance, to reduce the problem of skewness). The transformation also improves the stationarity of spot volume. Following Marsh and Wagner, in order to eliminate the stochastic trend in spot volume, the Hodrick and Prescott (1997) filter[9] (HP filter) is applied to logarithmic spot volume. Thus, the two sets of variables to be adjusted for time-series dependence and calendar day effects are 1) the difference between the HP filtered and the actual logarithmic spot volume series and 2) the logarithm of the option volume series. The adjustment models are determined via the standard Box/Jenkins approach and estimated via ordinary least squares (OLS). The sample size is relatively large; hence, OLS is expected to perform well for the autoregressions. For both the logarithm of option volume and the de-trended spot volume, autoregressive terms of orders 1 and 2 are selected based on the Akaike and Schwarz information criteria. The adjusting regressions include day-of-the-week, holiday, macroeconomic announcement, expiration indicator variables, and contemporaneous and lagged cross-volume terms in addition to the autoregressive terms. The holiday indicator takes the value of 1 on trading days immediately preceding as well as following an exchange holiday. The logarithmic option volume is also adjusted for a quadratic

[9] Several values are tested for the smoothing parameter between $5 \cdot 10^6$ and 10^7. The results are not driven by the choice of the smoothing values.

trend, which proves to be an important determinant in the regression results. The adjusted R^2 of the regression for log SPX option volume is 0.56 with a Durbin-Watson statistic of 2.15. For the de-trended SPX spot volume series, the corresponding R^2 is 0.53 with Durbin-Watson equaling 1.90. For both series, the calendar day effects (day-of-the-week, holiday, expiration days), as well as the macroeconomic announcement days, are significant determinants of the deterministic variation.

The speculative and hedging volume components are adjusted in a similar manner. I use contemporaneous and lagged option volume in the adjusting regressions to capture relative rather than absolute measures of these components. One interesting result emerges from these adjustment regressions: macroeconomic announcement day indicators do not affect relative hedging volume, but significantly positively affect the speculative component (this is also true in levels (absolute) terms). This observation lends additional support to using the speculative and hedging proxies and is also consistent with the argument that directional informed trading[10] in index products may manifest itself in the speed with which certain traders react to public information, rather than trading on private signals (Schlag and Stoll (2004)).

4. Does Volume Improve Volatility Prediction?

In this section, I estimate the state-dependent predictive regressions described in Section 2. Subsections *4.1* and *4.2* focus on model-free implied volatility, while subsection *4.3* uses B-S implied volatility.

I estimate equations (15) and (16) using daily and monthly non-overlapping windows. Using non-overlapping intervals avoids the spurious correlation in realized volatility since successive realized volatility observations do not share returns used in calculating the previous period's value. However, at the daily frequency, a telescoping maturity problem remains since the overlap is not eliminated from the implied volatility measure. To see this, implied volatilities

[10] Un-directional, or volatility related trading will be discussed later in this paper.

on two consecutive dates are likely to be highly correlated since they represent the volatility expectation for two 30 calendar day windows that share 28 of the relevant calendar days. Poteshman (2000) argues that the telescoping maturity problem can be mitigated by using the method of Hansen (1982). On the other hand, telescoping maturity is fully addressed in the monthly sample. To calculate the 30 calendar day (i.e., monthly) realized volatility, I determine the actual calendar days that bracket a 30 day window and calculate the average squared return based on all transaction day values that fall in the given calendar day interval.[11] I start with January 2, 1996, the first date in my sample (day 0). I match up the implied volatility on day 0 with the following 30 calendar day realized volatility; then I proceed by matching up the implied volatility on day 30 with the following 30 calendar day volatility.

4.1. Model-free Implied Volatility and Abnormal Option Volume

Panel A of Table 1.3 reports the estimation results for the traditional (symmetric) predictive regression model (i.e., equation (3)) in which the only explanatory variable is the model-free implied volatility of SPX options using daily and monthly samples. While the number of observations utilized in the daily sample is 2,136, the sample size drops significantly in the monthly series, to 104 data points. The probability reported in parentheses under the coefficient estimates corresponds to the Wald (c^2)-test for the null hypotheses that $a = 0, b=1$, respectively. The c^2 probability corresponding to the null hypothesis that $(a,b) = (0,1)$, is reported in column three. Probability values less than 0.1 reflect that the null hypothesis can be rejected at a statistically significant level, while probability values exceeding 0.1 reflect that the null hypothesis cannot be rejected. The Wald-test is based on standard errors calculated from robust procedures[12]. The coefficient and significance estimates are in line with results reported in the literature. Poteshman (2000) provides a summary of the results from volatility forecasting

[11] A similar procedure is used for the other volatility measures.
[12] The method of Hansen (1982) is adopted.

regressions for a number of previous studies. Jiang and Tian (2005) estimate similar regressions using model-free implied volatility. The adjusted R^2 increases when I use the monthly sample, from 0.148 for daily to 0.351 for 30 calendar day observations.

Table 1.3. Results from the Predicting Regressions without Volume Censoring

Predicting regressions of the following form are estimated for the time period January, 1996-June, 2004:

$$s^{2}_{realized,t} = a + b\, s^{2}_{implied,t-1} + e_t$$

This model provides a regression of realized volatility on implied volatility and an intercept term and was discussed in equation (3). Probabilities corresponding to the Wald(χ^2)-test that the intercepts are zero and the slope is one, respectively, are reported in parentheses. A high probability value indicates inability to reject the relevant hypothesis. Panel B estimates the regression model using the instrumental variable approach. Reported are results when the instrumental variable is the lagged implied volatility as suggested by Fleming (1998). A high χ^2 probability value indicates that the relevant null hypothesis (i.e., α=0, β=1, or (α,β)=(0,1)) cannot be rejected (I also use * to highlight these cases).

Panel A	α (α=0)	β (β=1)	(α, β)=(0,1)
Daily sample	**-0.342**	**0.863**	>0.001
N=2136	(0.003)	(0.054)	
Adj. R^2=0.15			
Monthly sample	**-0.029***	**0.818***	>0.001
N=104	(0.849)	(0.129)	
PANEL B			
Daily sample	**-0.331**	**0.855**	>0.001
N=2133	(0.000)	(0.038)	
Monthly sample	**-0.037***	**0.822***	>0.001
N=103	(0.821)	(0.149)	

For both the daily and the monthly frequencies, the null hypothesis that *(a,b) = (0,1)* is rejected at high levels of significance. In the past, many studies interpreted this result as evidence that implied volatility is a biased estimator of expected volatility. However, since implied volatility is on average greater than subsequent realized volatility, the finding is consistent with the idea that volatility risk is priced. Therefore, implied volatility may exceed future realized volatility due to investors' risk aversion (Chernov (2002), Bandi and Perron (2003)) and the estimation results should be interpreted in the spirit of equation (8) with the intercept representing the expected volatility risk premium. The slope coefficient (b) is not significantly different from one for the monthly sample. Hence, with a constant adjustment related to the intercept, as suggested by Jiang and Tian (2005), the model-free implied volatility may be considered an unbiased estimator.

Panel B of Table 1.3 repeats the same analyses employing an instrumental variable (IV) approach. The IV approach is proposed by Christensen and Prabhala (1998) to address the error-in-variable problem that arises when the implied volatility is estimated with measurement errors. As in Christensen and Prabhala (1998), I employ two-stage least squares regressions with lagged implied volatilities and lagged realized volatilities as instruments (Chernov (2002)). The results do not change significantly in the IV procedure. This is consistent with Jiang and Tian (2005) who find essentially no measurement error in model-free implied volatility. In contrast, they also estimate standard OLS and IV regressions for Black-Scholes implied volatilities and document significant improvements in the performance once the error-in-variable problem is corrected. This suggests that when the model-free implied volatility is employed, accurate statistical inferences obtain without the instrumental variable estimation framework.

Panel A of Table 1.4 reports the results of the asymmetric regressions of the form of equation (15). First, I use the abnormal option volume series defined in Section 3 to determine high volume states. More specifically, Panel A provides estimated coefficients for the case when the indicator variable for the high volume state takes the value of one on days when abnormal option volume is positive and zero otherwise. The time subscript on abnormal volume and implied volatility are the same: abnormal volume and implied volatility are measured contemporaneously.

Table 1.4. Results from the Predicting Regressions with Option Volume Censoring Using Model-free Implied Volatility

Predicting regressions of the following form are estimated for the time period January, 1996-June, 2004:

$$s^{2}_{realized,t} = a_1 I_{\{V_{t-1}\in A\}} + a_2 I_{\{V_{t-1}\notin A\}} + b_1 s^{2}_{implied,t-1} I_{\{V_{t-1}\in A\}} + b_2 s^{2}_{implied,t-1} I_{\{V_{t-1}\notin A\}} + e_t$$

This model provides a regression of realized volatility on implied volatility and an intercept term. However, both the intercept and the slope coefficients are allowed to vary across high and low volume regimes. This is expressed by the multiplicative indicator term, $I_{\{V_{t-1}\in A\}}$, which takes the value of 1 in Panel A, when abnormal option volume is positive and 0 otherwise. In Panel B, volume characteristics are grouped in three distinct events. The first event contains observations for which previous day's abnormal option volume was non-positive. The second event occurs when previous day's abnormal option volume was greater than zero but less than the 70th percentile of the past 60 days. Event three indicates previous day abnormal option volume greater than the 70th percentile of the past 60 days. The abnormal volume series is based on the residual series adopted from volume filtering models which account for time-series characteristics, calendar day variation, macroeconomic announcement days, and expiration days in volume. Probabilities associated with the Wald (χ^2) coefficient tests that the intercepts are zero and the slope is one, respectively, are reported in parentheses. A high χ^2 probability value indicates that the relevant null hypothesis (i.e., α=0, β=1, or (α,β)=(0,1)) cannot be rejected (I also use * to highlight these cases).

Panel A	α_1 (α_1=0)	β_1 (β_1=1)	(α_1, β_1)=(0,1)	α_2 (α_2=0)	β_2 (β_2=1)	(α_1, β_1)=(0,1)
Daily sample	**-0.377**	**0.886***	<0.001	**-0.295**	**0.832**	<0.001
N=2136	(0.002)	(0.201)		(0.015)	(0.063)	
Adj R^2=0.147						
Monthly sample	**-0.270***	**0.985***	<0.001	**0.402***	**0.533**	<0.001
N=104	(0.138)	(0.914)		(0.115)	(0.015)	
Adj R^2=0.363						

Panel B	α_1 (α_1=0)	β_1 (β_1=1)	α_2 (α_2=0)	β_2 (β_2=1)	α_3 (α_3=0)	β_3 (β_3=1)
Daily sample	**-0.323**	**0.849***	**-0.375**	**0.854***	**-0.454**	**0.952***
N=2067	(0.011)	(0.106)	(0.028)	(0.246)	(0.005)	(0.691)
Adj R^2=0.148						
Monthly sample	**0.410***	**0.528**	**-0.308***	**0.989***	**-0.237***	**0.974***
N=99	(0.158)	(0.026)	(0.203)	(0.950)	(0.399)	(0.906)
Adj R^2=0.341						

The table reports that for both the daily and monthly samples, implied volatility performs better when abnormal option volume is high. The estimated slope coefficients are 0.886 and 0.832 for the high and low volume regimes, respectively, in the daily sample. In the monthly sample, the contrast between the high volume and the low volume slope coefficients is especially large, on option volume days when abnormal volume is positive, the estimated slope coefficient is 0.985, while on negative abnormal volume days the estimated slope coefficient is only 0.533. The Wald coefficient test indicates that the slope coefficients are insignificantly different from 1 in both cases on heavy option volume days and are less than 1 otherwise. The joint hypothesis that $(a_i , b_i) = (0,1)$ is strongly rejected in all cases. In addition, the intercepts vary significantly depending on the volume regime, which (loosely interpreted) indicates that the expected volatility risk premium differs across the volume states. In particular, the coefficients are more negative in high volume states, which indicates that a higher risk premium is demanded during these periods.

Panel B of Table 1.4 refines the definition of high volume states. Instead of basing the indicator variable on whether abnormal option volume on the day when implied volatility is determined is positive or negative, I create an indicator based on given percentiles of abnormal option volume in the past 60 days. Panel B disaggregates abnormal option volume into low, moderately high, and very high states. In particular, the low, moderately high, and very high states correspond to the events that abnormal option volume is less than 0, more than zero but less than the 70th percentile of the past 60 day abnormal volume, and more than the 70th percentile of the past 60 day abnormal volume, respectively. The aim is similar to that of Panel A, however, Panel B provides an improvement in that the right tail is not compared to the rest of the distribution, which aggregates very high and very low volume levels. The general conclusions that arise from the two panels support the idea that on average, high volume states are likely to indicate trading due to changes in the information environment rather than changes to investor biases. The result remains robust to the definition of heavy trading. In addition, the results from the decomposition of high volume states into moderately high and very high, as well as the analysis of the left tail provide further support to the information argument.

In additional analyses not reported in the paper, I use classification schemes in which the indicator variable takes the value of 1 when abnormal option volume is larger than the 90th percentile of the past 60 day abnormal option volume, and zero otherwise. In some ways, this measure may be better able to capture irrationalities or fads in the market and provides a robustness check for Panel A. For symmetry, I also consider abnormally low volume by looking at whether abnormal volume is less than the 10th percentile of the previous 60 days (in this case, the indicator variable takes the value of 1, and 0 otherwise). For instance, under the information arrival hypothesis, when information acquisition is endogenous and volume is abnormally low, informed trades may have fewer incentives to engage in information collection or to trade on their information. On the other hand, the irrationality argument has weak positive or no implications for the left tail of abnormal volume. For the monthly sample, the 10- and 90-percentile rules create very small sub-samples for which the indicator variable is 1; hence any statistical inference is likely to be invalid. For daily observations, the slope coefficient corresponding to abnormal option volume days, which are higher than the 90th percentile of the previous 60 days are not significantly different from 1, while the low volume day slope is significantly less than 1. When the indicator variable is based on the 10th percentile, the null hypotheses that each of the slopes are 1 cannot be rejected, and the coefficient estimates are 0.788 and 0.891 for the observations less than and greater than the 10th percentile, respectively.

4.2. Model-free Implied Volatility and Spot Volume

Table 1.5 reports a similar exercise using spot volume. In Panel A, the results are weaker than those reported in Table 4. While for daily observations, the slope coefficient on abnormally high volume days is significantly higher than on low volume days (0.978 and 0.719, respectively), for the monthly sample, the coefficients are nearly identical and the hypotheses that each of these slope coefficients equals 1 cannot be rejected. For Panel D, the abnormally high volume categories are associated with slope coefficients that are statistically indistinguishable from 1 and are significantly different from the low volume slope coefficients at the daily frequency. At the monthly frequency, the results are more ambiguous, which may result from the low number of 1's in the samples.

Table 1.5. Results from the Predicting Regressions with Spot Volume Censoring Using Model-free Implied Volatility

Predicting regressions of the following form are estimated for the time period January, 1996-June, 2004:

$$\sigma^2_{realized,t} = \alpha_1 I_{\{V_{t-1} \in A\}} + \alpha_2 I_{\{V_{t-1} \notin A\}} + \beta_1 \sigma^2_{implied,t-1} I_{\{V_{t-1} \in A\}} + \beta_2 \sigma^2_{implied,t-1} I_{\{V_{t-1} \notin A\}} + \varepsilon_t$$

This model provides a regression of realized volatility on implied volatility and an intercept term. Both the intercept and the slope coefficients are allowed to vary across high and low volume regimes. This is expressed by the multiplicative indicator term, $I_{\{V_{t-1} \in A\}}$, which takes the value of 1 in Panel A, when abnormal spot volume is positive and 0 otherwise. In Panel B, volume characteristics are grouped in three distinct events. The first event contains observations for which previous day's abnormal spot volume was non-positive. The second event occurs when previous day's abnormal spot volume was greater than zero but less than the 70th percentile of the past 60 days. Event three indicates previous day abnormal spot volume greater than the 70th percentile of the past 60 days. The abnormal volume series is based on the residual series adopted from volume filtering models which account for time-series characteristics, calendar day variation, macroeconomic announcement days, and expiration days in volume. Probabilities associated with the Wald (χ^2) coefficient tests that the intercepts are zero and the slope is one, respectively, are reported in parentheses. A high χ^2 probability value indicates that the relevant null hypothesis (i.e., α=0, β=1, or (α,β)=(0,1)) cannot be rejected (I also use * to highlight these cases).

Panel A	α_1 (α_1=0)	β_1 (β_1=1)	(α_1, β_1)=(0,1)	α_2 (α_2=0)	β_2 (β_2=1)	(α_1, β_1)=(0,1)
Daily sample	**-0.495**	**0.978***	<0.001	**-0.153***	**0.719**	<0.001
N=2132	(0.000)	(0.826)		(0.131)	(0.000)	
Adj R^2=1.151						
Monthly sample	**-0.023***	**0.817***	<0.001	**-0.051***	**0.830***	<0.001
N=100	(0.926)	(0.327)		(0.798)	(0.279)	
Adj R^2=0.336						

Panel B	α_1 (α_1=0)	β_1 (β_1=1)	α_2 (α_2=0)	β_2 (β_2=1)	α_3 (α_3=0)	β_3 (β_3=1)
Daily sample	**-0.173**	**0.730**	**-0.472**	**0.937***	**-0.578**	**1.036***
N=2068	(0.099)	(0.000)	(0.016)	(0.668)	(0.001)	(0.773)
Adj R^2=0.150						
Monthly sample	**-0.027***	**0.816***	**-0.374***	**0.970***	**0.082***	**0.796***
N=100	(0.895)	(0.248)	(0.128)	(0.865)	(0.879)	(0.619)
Adj R^2=0.334						

4.3. Black-Scholes Implied Volatility and Abnormal Option Volume

Up to this point, the analyses rely on the model-free implied volatility measure in order to directly test the impact of volume on price discovery and to avoid the joint hypothesis problem. In this sub-section, I report the results using the Black-Scholes (B-S) measure. Mayhew and Stivers (2003) study the time-series forecasting performance of Black-Scholes implied volatility. In their cross section of stocks, they observe that only for stocks with the very largest option trading volume does implied volatility show significant forecasting ability. Similarly, Donaldson and Kamstra (2004) show that the B-S implied volatility outperforms the volatility forecasts produced by ARCH models when NYSE volume is high. These results naturally arise since the Black-Scholes model has strong friction biases. For instance, Longstaff (1995) finds that the discrepancy between the B-S implied volatility and realized volatility decreases when market conditions are closer to the assumptions underlying the B-S model; that is, when transaction costs are lower and liquidity (measured by trading activity and other proxies) is higher. The current study extends these results by showing that model-free implied volatility improves with option and, to a lesser extent, spot volume; therefore, beyond the friction biases, the improvement documented in the B-S model can be due to changes in the efficiency of the price as well.

I adopt the methods proposed by Ni, Pan, and Poteshman (2005) and use the B-S implied volatility of at-the-money straddles using implied volatility information from *OptionMetrics*. Table 1.6 reports the asymmetric regression model employed in Panel A of Tables 4 and 5. Panel A shows that implied volatility performs better when option volume is high in both the daily and in the monthly samples, while the slope coefficient is significantly different from 1 during low volume regimes. The difference in the slopes is especially large in the monthly sample, in high volume states the slope is 0.95, while the corresponding slope in low volume states is 0.65. As in the case of model-free implied volatility, the results are less pronounced when the conditioning variable is abnormal spot volume. In particular, at the daily frequency, the high volume day slope coefficient is statistically indifferent from one, while the hypothesis that the low volume day slope coefficient is 1 is rejected. However, at the monthly frequency no significant differences occur across high and low volume slopes.

Table 1.6. Results from the Predicting Regressions with Option and Spot Volume Censoring Using Black-Scholes Implied Volatility

Predicting regressions of the following form are estimated for the time period January, 1996-June, 2004:

$$s^2_{realized,t} = a_1 I_{\{V_{t-1}\in A\}} + a_2 I_{\{V_{t-1}\notin A\}} + b_1 s^2_{implied,t-1} I_{\{V_{t-1}\in A\}} + b_2 s^2_{implied,t-1} I_{\{V_{t-1}\notin A\}} + e_t$$

This model provides a regression of realized volatility on B-S implied volatility and an intercept term. However, both the intercept and the slope coefficients are allowed to vary across high and low volume regimes. This is expressed by the multiplicative indicator term, $I_{\{V_{t-1}\in A\}}$, which takes the value of 1 in Panel A, when abnormal option volume is positive and 0 otherwise. In Panel B, the indicator variable takes the value of 1 when abnormal spot volume is positive, and 0 otherwise. The abnormal volume series is based on the residual series adopted from volume filtering models which account for time-series characteristics, calendar day variation, macroeconomic announcement days, and expiration days in volume. Probabilities associated with the Wald (χ^2) coefficient tests that the intercepts are zero and the slope is one, respectively, are reported in parentheses. A high χ^2 probability value indicates that the relevant null hypothesis (i.e., α=0, β=1, or (α,β)=(0,1)) cannot be rejected (I also use * to highlight these cases).

	α_1	β_1		α_2	β_2	
Panel A	(α_1=0)	(β_1=1)	(α_1, β_1)=(0,1)	(α_2=0)	(β_2=1)	(α_1, β_1)=(0,1)
Daily sample	**-0.409**	**0.868***	<0.001	**-0.246**	**0.783**	<0.001
N=2130	(0.000)	(0.112)		(0.000)	(0.029)	
Adj R^2=0.160						
Monthly sample	**-0.264***	**0.951***	<0.001	**0.179***	**0.653**	<0.001
N=101	(0.129)	(0.653)		(0.371)	(0.025)	
Adj R^2=0.369						
Panel B						
Daily sample	**-0.437**	**0.902***	<0.001	**-0.159***	**0.705**	<0.001
N=2127	(0.000)	(0.215)		(0.190)	(0.001)	
Adj R^2=0.161						
Monthly sample	**-0.010***	**0.790***	<0.001	**-0.043***	**0.793***	<0.001
N=101	(0.961)	(0.152)		(0.819)	(0.120)	
Adj R^2=0.358						

5. Robustness Analyses

5.1. Serial Correlation in Volume and Volatility

The results reported above are consistent with the information hypothesis. However, it is important to address the concern that introducing volume states in the analysis may result in spurious findings. Previous studies have found that on average, implied volatility is higher than subsequent realized volatility. This has been documented for both the B-S as well as model-free measures. Therefore, based on the well-established positive contemporaneous correlation between volume and volatility, a possible spurious relation may arise. In particular, high volume today (either spot or option) implies high volatility today, and if volatility is persistent, it also implies high volatility tomorrow. If realized volatility is indeed higher tomorrow following a high volume day, the discrepancy between implied and realized volatility may shrink based on this mean effect.[13] To eliminate this concern, I split the sample into high and low option (and spot) volume days and investigate the distributions of realized and implied volatilities in the subsamples. Tests of the equality of the means and the medians is used to check whether realized (and implied) volatility increases following high volume days. The tests indicate that the mean (median) realized volatility following a high option volume day is 0.90 (0.70) and the mean realized volatility following a low option volume day is 0.92 (0.66) and the null hypothesis of equal means (medians) cannot be rejected. Model-free implied volatility is marginally significantly higher (10% level) on high option volume days with a t-test p-value of 0.094. Thus the narrowing of the implied volatility – realized volatility spread is not likely to be driving the results.

For spot volume, the robustness check is less conclusive about a potentially spurious relation. The mean realized volatility is significantly higher (the null hypothesis of equality is

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[13] Although it may be misleading to simply look at the difference between implied and realized volatility since volatility risk may be priced.

rejected at the 5% level) following high volume days: 0.87 and 0.95 for low and high spot volume, respectively. Similarly, the mean implied volatility is significantly higher on high spot volume days: 1.42 and 1.47 for low and high spot volume days, respectively. However, medians across the subsamples do not reflect significant differences for realized volatility and the conclusions of the median tests for implied volatility differ across the various median tests applied.

5.2. Weighted Option Volume

That the pricing error is smaller in high option volume states is consistent with the notion that on average, abnormal trading is not indicative of shocks to irrational investors' demands but rather, more likely to be due to changes in investors' information set or in their interpretation of it. As an additional robustness check, in this sub-section I examine whether the results are invariant to two alternative weighting schemes[14] for option volume. In particular, I use (inverse) bid/ask spread-weighted and vega-weighted option volume. The rationale behind spread-weighting is that options with different strike prices and time-to-maturity have different liquidity characteristics, which are an important decision variable for the informed trader. Moreover, different option classes have different sensitivities to volatility; hence, informed traders with volatility signals will be more likely to trade contracts with a high vega. While spread weighting is consistent with both directional and volatility related information trading, vega-weighting focuses solely on the role of volatility related information trading in the option markets.

OptionMetrics reports the vega and the closing best bid and ask prices across all exchanges for each contract. Each day, I weight each option contract's volume by its vega and the inverse spread, respectively, then aggregate the weighted contract volumes to arrive at an

[14] I also check whether abnormal volume derived from daily aggregated option volume multiplied by the volatility of implied volatility exhibits similar results. Volatility is an alternative proxy for information arrival. Therefore the contemporaneous volatility of implied volatility, similarly to volume, reveals the level of informed trading. Results are robust to this alternative "weighting"-scheme.

aggregate option volume measure. As before, I adjust the newly created volume series and define volume regimes based on the residual series arising from the adjustment procedure. Table 1.7 reveals that the results remain robust to both weighting schemes described in this sub-section.

Table 1.7. Results from the Predicting Regressions with Weighted Option Volume Censoring Using Model-free Implied Volatility

Predicting regressions of the following form are estimated for the time period January, 1996-June, 2004:

$$s^2_{realized,t} = a_1 I_{\{V_{t-1}\in A\}} + a_2 I_{\{V_{t-1}\notin A\}} + b_1 s^2_{implied,t-1} I_{\{V_{t-1}\in A\}} + b_2 s^2_{implied,t-1} I_{\{V_{t-1}\notin A\}} + e_t$$

This model provides a regression of realized volatility on implied volatility and an intercept term. However, both the intercept and the slope coefficients are allowed to vary across high and low volume regimes. This is expressed by the multiplicative indicator term, $I_{\{V_{t-1}\in A\}}$, which takes the value of 1 in Panel A when the abnormal inverse spread-weighted option volume is positive and 0 otherwise. Panel B reports similar results for vega-weighted option volume. In Panel C and D, abnormal spread- and vega-weighted volume regimes are defined, respectively, based on whether appropriate abnormal volume is less than the 0, between 0 and the 70th, and more than the 70th percentile of the last 60 days' abnormal spread-weighted and vega-weighted volume, respectively. The abnormal volume series is based on the residual series adopted from volume filtering models which account for time-series characteristics, calendar day variation, macroeconomic announcement days, and expiration days in volume. Probabilities associated with the Wald (χ^2) coefficient tests that the intercepts are zero and the slope is one, respectively, are reported in parentheses. A high χ^2 probability value indicates that the relevant null hypothesis (i.e., α=0, β=1, or (α,β)=(0,1)) cannot be rejected (I also use * to highlight these cases).

Panel A

Daily sample N=2131 Adj R^2=0.153	**-0.519** (0.000)	**1.002*** (0.987)	<0.001	**-0.131*** (0.235)	**0.694** (0.000)	<0.001
Monthly sample N=103 Adj R^2=0.340	**-0.072*** (0.766)	**0.865*** (0.484)	<0.001	**-0.016*** (0.940)	**0.788*** (0.186)	<0.001

Panel B

Daily sample N=2131 Adj R^2=0.149	**-0.460** (0.000)	**0.956*** (0.574)	<0.001	**-0.223*** (0.106)	**0.767** (0.025)	<0.001
Monthly sample N=103 Adj R^2=0.342	**-0.111*** (0.550)	**0.891*** (0.453)	<0.001	**0.072*** (0.787)	**0.729*** (0.181)	<0.001

Table 1.7, continued

Panel C	α_1 (α_1=0)	β_1 (β_1=1)	α_2 (α_2=0)	β_2 (β_2=1)	α_3 (α_3=0)	β_3 (β_3=1)
Daily sample	**-0.143***	**0.702**	**-0.335***	**0.835***	**-0.753**	**1.177***
N=2074	(0.214)	(0.001)	(0.132)	(0.285)	(0.000)	(0.167)
Adj R^2=0.156						
Monthly sample	**-0.036***	**0.799***	**-0.100**	**0.782***	**-0.385***	**1.164***
N=100	(0.783)	(0.223)	(0.000)	(0.358)	(0.305)	(0.592)
Adj R^2=0.393						

Panel D	α_1 (α_1=0)	β_1 (β_1=1)	α_2 (α_2=0)	β_2 (β_2=1)	α_3 (α_3=0)	β_3 (β_3=1)
Daily sample	**-0.245**	**0.783**	**-0.314**	**0.866***	**-0.621**	**1.050***
N=2074	(0.096)	(0.048)	(0.080)	(0.310)	(0.000)	(0.620)
Adj R^2=0.150						
Monthly sample	**0.030***	**0.756***	**-0.194***	**0.967***	**-0.030***	**0.818***
N=101	(0.921)	(0.274)	(0.447)	(0.868)	(0.911)	(0.386)
Adj R^2=0.376						

In Panel A, the high and low spread-weigted volume betas are 1.002 and 0.694, respectively at the daily frequency. The hypothesis that the low volume beta is 1 is rejected by the Wald-test. At the monthly frequency, the corresponding betas are 0.865 and 0.788, respectively, although the Wald-test cannot reject that they are different from 1. For vega-weighting, the daily (monthly) betas are 0.956 (0.891) and 0.767 (0.729), respectively.

The time-series of states based on the vega-weighted and inverse spread-weighted rules, respectively, are not identical (although vega is highest for at-the-money contracts, which are often the most liquid as well). A four regime test (not reported) indicates that irregardless of whether the vega-weighted volume is abnormally high or not, the difference between low and high spread-weighted days is always statistically large. On the other hand, holding spread-weighting constant, vega-weighting improves between high and low regimes, however, when spread-weighting is high, the improvement is insignificant. This is consistent with the contention that liquidity is more important for the informed trader than the volatility sensitivity of the contract.

5.3. Speculative and Hedging Components

I disaggregate option volume into two components: the change in open interest and a net component, or the difference between total volume and the change in open interest. This is based on Day and Lewis (2004) who use the decomposition to approximate the speculative and hedging components of derivative volume. Day and Lewis argue that the speculative component of option volume is likely to capture information motivated trades; hence, it provides a more direct indicator of periods of heightened informed trading. Also, a high speculative component can be interpreted in terms of large differences in opinions in the market. In this case, trading will improve efficiency by aggregating the heterogeneous valuation models and beliefs (Brandt and Kavajecz (2003)). Under the information and differences of opinion interpretations, the accuracy of the option price is more strongly related to days when the speculative trading component is high. On the other hand, speculative demand may reflect irrational trading activity caused by overconfident investors' guesses or sudden fads in trading caused by a shock to investor sentiment. This would imply that price discovery is inversely related to speculative trading demands.

The hedging motivated component is not likely to indicate information motivated trades (Day and Lewis (2004)). In addition, based on Bollen and Whaley (2004), it may capture portfolio insurance demands, which cause temporary price pressures when no natural counterparties arise for the trade. Therefore, when the demand of hedgers is high, price discovery may actually be adversely affected. This can be especially true for puts as the market maker is the only party providing liquidity in some of the moneyness categories of these contracts.

When option volume is decomposed into speculative and hedging components based on day-to-day changes in open interest, the indicator variables are based on whether the relative components expressed are abnormally high. I examine several scenarios. The results for the case when abnormal total trading volume is interacted with the abnormal hedging and speculative ratios reveal that the pricing error is the smallest when both option volume and the speculative component are high: the estimated slope coefficient is 1.04. When trading volume is high and it is accompanied by an unusually high hedging ratio, the slope is estimated to decrease by -0.196, although the estimate is not statistically significant. Put related hedging demand significantly reduces the slope coefficient.

5.4. Model Errors when Volume is Low

One important concern in the test of the own- and cross-volume effects is that a specific pricing model may have biases, which can also depend on market liquidity. For instance, the Black and Scholes (1973) model is likely to perform better when transaction costs are small (Longstaff (1995)). Therefore, if trading activity proxies for liquidity, an improvement in the informativeness of implied volatility in high volume states (i.e., the own- and cross-volume effects) may be due to a decrease in the bias of the assumed pricing model (since the model assumptions are closer to the true market conditions) rather than to enhanced price discovery. It is also important to consider how model-free implied volatility may be affected by the discretization or truncation biases and whether these biases are more pronounced in low volume states.

To address the concern that I am merely capturing time variation in the B-S pricing model's bias due to time variation in frictions, I examine various measures of spot and option market liquidity on high and low option and spot market days. In addition, to address the behavior of model-free implied volatility, I examine the number of non-trading contracts across high and low volume regimes in both puts and calls. In addition, I look at the range determined by the highest and the lowest exercise price traded in all contracts, in puts and calls separately, and in the two nearest contracts. Jiang and Tian (2005) derive the sensitivity of model-free volatility to discretization and truncation and find that the available strike prices and the number of actual trading contracts in the market provide a model-free implied volatility measure that is very close to the theoretical value derived under the assumption of a continuum of strikes.

6. Conclusion

This paper examines whether the accuracy of the option's price is affected by the intensity of trading in the spot and option markets. In particular, I examine market efficiency on abnormally high volume days. The research question is motivated by numerous theoretical studies, which suggest that the effect of abnormal volume depends on why investors trade. For instance, if transactions occur due to information arrival, trading will improve market prices. On the other hand, if abnormal trading reflects shocks to irrational traders' demands, prices will be adversely affected. I empirically test the role of trading volume in the index option market by examining the relation between implied and realized volatilities across the different volume regimes.

This study provides several contributions to the existing literature. First, the tests implemented in the paper are based on two implied volatility measures. The Black-Scholes implied volatility is used since it is approximately unbiased under quite general conditions, however, the joint hypothesis problem cannot be completely eliminated. On the other hand, model-free implied volatility provides a direct test of market efficiency since it is derived without any assumptions on the underlying option pricing model or equivalently, on the underlying price process. Hence, unlike previous parametric implied volatilities, such as the Black-Scholes implied volatility, it avoids the joint hypothesis problem. Second, I improve the estimation methods by directly incorporating the latent feature of the volatility risk premium via the Kalman filter. Third, I provide evidence on the role of volatility related information trading in the option market. Fourth, I further disaggregate option volume into hedging and speculative trades and examine how abnormal speculative or hedging demands affect price discovery. The latter test provides implications on what types of trades result in changes in open interest.

The findings suggest that option prices are more efficient on high option volume days. This implies that on average, abnormal trading is not indicative of shocks to irrational investors' demands but rather, more likely to be due to changes in investors' information set or in their interpretation of it. However, evidence on the cross-volume effect (i.e., the role of spot volume) is somewhat ambiguous. While in some specifications spot volume appears to enhance option price discovery, the results are not robust: the role of spot volume becomes insignificant in alternative tests. This result is not surprising for the model-free measure since it may be driven by the fact that model-free implied volatility indicates volatility exposure that is neutralized to the stock price by construction. Thus any improvement in stock price efficiency will only be reflected in the realized volatility measure. However, a similarly inconclusive relation is found for B-S implied volatility. Together these results may indicate that directional informed trading in the index markets is not likely since private information on market-wide movements are difficult to obtain. Instead, information may pertain to changes in market volatility; hence option trading has a stronger informational role than spot trading for indices.

The information hypothesis is further supported by two alternative weighting schemes. Vega-weighting is especially interesting since it provides evidence consistent with volatility related informed demand in the option markets. Moreover, when trading volume is decomposed into speculative and hedging components, a high option volume associated with unusually large amounts of speculative trading enhances the option's price discovery. For hedging, no such relation is found. In addition, the speculative component of puts affects price discovery in a negative manner. This is consistent with portfolio insurance demands providing a net buying pressure, for which inventory premium is required.

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CHAPTER II

OPTION VOLUME AND THE PRICE DYNAMICS OF INDIVIDUAL STOCKS: A LINK TO THE INFORMATION SHARE OF OPTIONS

1. Introduction

It is well-established in the literature that contemporaneous stock volatility and (surprise) stock volume are significantly positively related. The Mixture of Distribution Hypothesis (MDH) suggests that the correlation arises from informed trading in the stock market, as information arrival is reflected in both stock volume and volatility.[15] In this paper, I extend the volume-volatility literature and document that stock volatility is positively related to surprise option volume as well. I also provide suggestive, though not conclusive, evidence that this cross-market relation is also driven by the mixture of distribution argument. The relevance of the MDH for options comes from a number of recent studies.[16] These studies find that the option market is an alternative channel through which information is incorporated into the underlying stock's price.

To establish the relation between option volume and spot volatility, I estimate individual time-series models for the 1,280 stocks in my sample using daily data between 1996 and 2004. The regressions include stock volume, along with option volume, to control for the traditional volume-volatility results. The time-series findings reveal that the cross-market volume-volatility correlation is not only positive, but for the majority of the stocks in my sample, it is significantly so. Unlike for surprise stock volume however, the statistical significance fails to be uniform.

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[15] See, for instance Copeland (1976), Ross (1989), Admati and Pfleiderer (1988), Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), Lamoureux and Lastrapes (1990), and Andersen (1996).

[16] While early empirical evidence on where informed traders initiate their trades is largely inconclusive (see, for instance, Anthony (1988), Stephan and Whaley (1990), Chan, Chung, and Johnson (1993), and Vijh (1990)), more recently, Chan, Chung, and Fong (2002) find that in the option market, information is included in quote revisions. Amin and Lee (1997) and Cao, Chen, and Griffin (2002) report evidence on information related option trading preceding earning announcements and takeover announcements, respectively. Pan and Poteshman (2004) show that option trading volume contains information for future stock prices. On the other hand, Stoll and Schlag (2004) conclude that price discovery in the German DAX is induced by futures trading rather than option trading. Chakravarty, Gulen, and Mayhew (2004) refer to SEC litigation cases of illegal insider option trading as direct evidence on the informational role of options.

I present the following two pieces of suggestive evidence in favor of an MDH-type explanation of the cross-market ties. First, if the positive correlation is indeed driven by information arrival in the option market, the strength of the correlation between option volume and stock volatility should vary from stock to stock in my sample. In particular, it should vary with firm and market characteristics that describe 1) the degree of asymmetric information surrounding the security and 2) whether the corresponding option market is attractive for the informed trader. The latter issue is important because adverse market characteristics, such as illiquidity can reduce the benefit of down-side protection, lower costs, and the inherent leverage (Black (1976)) in options. For instance, when the market for the underlying security is more liquid than those for the corresponding derivatives, informed traders are likely to choose the stock market itself (Easley, O'Hara, and Srinivas (1998)). Consistent with the tradeoff, Chakravarty et al. (2004) find evidence that informed trading in options varies across stocks.

To test this heterogeneity argument, I perform a cross-sectional analysis of the estimated volume-volatility correlations from the individual time-series. I use various measures to proxy for the degree of asymmetric information surrounding a given security (an extensive list is provided in Kelly (2005)), such as analyst coverage and institutional holdings. Similarly, I use relative spreads in the option market, the relative size of the option market, relative spread volatility, and the estimated informational share of options from Chakravarty et al. (2004) to proxy for how attractive the option market is for the informed trader. The results show that the estimated impact of suprise option volume is positively related to the relative size of the option market, and negatively related to the ratio of option to stock spreads and the number of analysts following the firm. Moreover, when the cross-sectional model includes a proxy for the information share of options, the model adjusted R-squared is very high (about 48%, though the R-squared is not comparable across the different models due to differences in the subsamples). The cross-sectional relations are robust for both the NYSE/AMEX and the Nasdaq subsamples.

The second piece of suggestive evidence relates to the stock's spread. If informed traders migrate between the stock and option markets, when informed traders choose the option market, the spot market makers' adverse selection costs decrease. Therefore, I examine the role of abnormal option volume for stock spreads using the individual time-series in my sample. I find that on average, spreads are negatively related to surprise option trading, which is consistent with the decreasing adverse selection costs argument.

Finally, in addition to the volume-volatility literature, this paper is also relevant for the debate on whether option trading influences the price process of the underlying stock. While previous studies in this area use an *indirect* test,[17] and concentrate on option listing using the pre- and post-listing periods in an event study, this paper provides a *direct* test by focusing on the effect of option volume on the volatility of individual stocks over time.[18] Moreover, the cross-sectional analyses in this paper could also be motivated by Grossman (1988), the central theoretical argument underlying the listing studies. Grossman suggests that the effect of option trading on the volatility of the underlying stock will depend on whether the option market can attract informed traders.

The plan of the paper is as follows. A brief analytical framework is presented in Section 2. Section 3 discusses the data sources and examines the characteristics of individual option and spot volume. Section 4 investigates the contemporaneous relation between option volume and return volatility using the individual time-series. The section also tests which firm, stock, and

[17] Evidence from the previous studies is also inconclusive. While some of the studies find that the volatility of the average stock declines after its option is introduced (see, for instance, Nathan Associates (1974), CBOE (1975), Nabar and Park (1988), Conrad (1989), Detemple and Jorion (1990), and Rao, Tripathy, Dukes (1991) using US data), others argue that volatility does not change in the post listing period or that parallel volatility changes occur in a non-optioned matching sample, thus the volatility effect is spurious and reflects a market-wide phenomenon. See, for instance Lamoureux and Panikkath (1994), Long, Schinski, and Officer (1994), Niendorf and Peterson (1997), Bollen (1998), and St. Pierre (1998) using US data. One paper, Wei, Poon, and Zee (1997), even reports a destabilizing result for OTC stocks. Detemple and Jorion (1990) suggest that the market-wide volatility change need not imply that the listing has no effect, since the introduction of a given option may help complete the market for all securities. However, the observation that the volatility effect flips signs across time (Bollen (1998)) challenges the strength of this argument or of an overall conclusion to the empirical results.

[18] Bessembinder and Seguin (1992) adopt this approach in a study of the effect of index futures listing on return volatility.

option market characteristics help explain the cross-sectional variation in the estimated dynamic coefficients. Section 5 discusses the robustness analyses. Section 6 concludes.

2. Analytic Framework and Empirical Methods

Following the traditional volume-volatility literature, I concentrate on surprise, rather than total volume. There are several reasons for this. For instance, from an empirical standpoint, surprise volume has more desirable statistical properties. Moreover, surprise volume is a more intuitive measure of informed trading/information arrival than total volume (Bessembinder and Seguin (1992) and Marsh and Wagner (2005)).

Hence, I first decompose option volume into an expected and an unexpected component. Then I examine how unexpected/surprise option volume affects spot return volatility for each individual stock (while also controlling for surprise stock volume), and how and why these effects vary in the cross-section of stocks in my sample. The decomposition of the individual daily option volume series is as follows.

$$ov_{t+1} = E_t(ov_{t+1}|F \cup sv_{t+1}) + h_{t+1} \qquad \text{with}$$

$$E_t(h_{t+1}|F \cup sv_{t+1})=0 \text{ and } \{cd_t, sv_t, sv_{t-1,\ldots,}ov_t, ov_{t-1,}\ldots\} \in F \qquad (1)$$

where *ov* and *sv* are option and spot volume, respectively, and F denotes the time *t* information set, which includes past option and spot volume observations, and calendar day effects (*cd*).[19]

[19] Alternatively, by definition, one could decompose the option and spot volume series based on the time t information set and a random component, which is orthogonal to the information set.

$$sv_{t+1} = E_t(sv_{t+1}|F) + n_{t+1}$$

$$ov_{t+1} = E_t(ov_{t+1}|F) + x_{t+1}$$

with $E_t(n_{t+1}|F)=0,\; E_t(x_{t+1}|F)=0,$ and $\{sv_t, sv_{t-1,\ldots,}ov_t, ov_{t-1,}\ldots\} \in F,$

Examples of calendar day effects include day of the week, holiday, and expiration day indicators. F is augmented by contemporaneous spot volume to accommodate the argument that options are derivative instruments. As a robustness analysis, I also include measures of total stock and option market volume in the decomposition based on Lo and Wang (2000) and Tkac (1999), who suggest that an adjustment for market-wide trading is required when partitioning volume into normal and abnormal trading. The subscript of the expectation operator indicates that the expectation is taken with respect to the information set available at time *t*. To resolve the time conflict, sv_{t+1} can be thought of as expected spot volume, which will be replaced by the realized volume measure in the empirical analyses. The unique component of option volume in this setup is h_{t+1}, which cannot be predicted based on information on past volume behavior or current spot trading activity. I use a vector autoregression framework with spot and option volume equations to estimate the expected and unexpected volume components.

To estimate the dynamic relation between surprise option volume and return volatility, I employ two methods. First, I use the GARCH framework. The results reported in this paper are based on a modified GARCH(1,1) specification, which has generally been preferred in the literature due to its parsimony and ability to adequately fit daily return series. Other specifications, such as the EGARCH and GARCH-M models, are also considered as robustness checks. The volume augmented GARCH(1,1) model is given by following standard system of equations:

$$
\begin{aligned}
r_t^i &= m_t^i + u_t^i \\
u_t^i \mid \Omega_{t-1} &\sim N(0, h_t^i) \qquad\qquad (2)\\
h_t^i &= a^i + b_1^i (u_{t-1}^i)^2 + b_2^i h_{t-1}^i + \sum_{k=1}^{K} g_k^i TA_{k,t}^i
\end{aligned}
$$

then examine the co-movements in n and x. The unique component of option volume is the component of x independent of n. In addition to obtaining the unique component, both methods provide information on volume betas, or the contemporaneous relation between option and spot volume.

where r_t^i represents the return on stock i, while m_t^i and W_{t-1} are the (possibly time-varying) mean of the return and the conditioning information set respectively. u_t^i represents the conditionally normally distributed but heteroskedastic error term with h_t^i denoting its time-varying variance, and the $TA^i_{k,t}$'s are measures of trading activity. I first estimate (2) with unexpected spot volume as the only trading activity variable, then I add option volume as an additional explanatory variable. This nested setup allows for measuring the incremental contribution contained in the surprise option volume series.

Several previous studies have employed models of the form of (2) with spot volume as the trading activity variable in the literature using the Mixture of Distributions Hypothesis. See, for instance, Marsh and Wagner (2004) for a recent application. Under the MDH explanation of the volume-volatility relation, volume is weakly exogenous in the sense of Engle, Hendry, and Richard (1983) in the GARCH setup.

The mean of the return series, m_t^i, is often described by a constant in the GARCH(1,1) estimations. However, in the cross-sectional analyses presented below one concern is that the error terms of the mean equation may not be independent across the stocks. As a result, cross-sectional estimates utilizing the coefficients from the time-series models may have biased standard errors and thus, these results are difficult to interpret. To address this possibility, I estimate the individual GARCH models by including a market proxy in the return equations. The market proxy is represented by the value-weighted CRSP index and is aimed at capturing missing common factors, which are a likely cause of cross-sectional dependence in the error terms (see, for instance, Jorion (1990)). In addition, to be consistent with the second methodology described below, I also include calendar day variables and lagged terms in the mean equation.

The second method follows Bessembinder and Seguin (1992) and estimates the following system of return and volatility equations in sequential/recursive estimation steps.

$$r_t^i = a_r^i + b_{r,1}^i r_{m,t} + \sum_{j=1}^{J_r} b_{r,j+1}^i r_{t-j}^i + \sum_{k=1}^{4} g_{r,k}^i D_{k,t} + d_r OCD_t + \sum_{j=1}^{J_r} f_{r,j}^i \hat{s}_{i,t-j}^i + u_t^i$$

$$\hat{s}_t^i = a_s^i + \sum_{j=1}^{J_{sd}} f_{s,j}^i \hat{s}_{t-j}^i + \sum_{k=1}^{4} g_{s,k}^i D_{k,t} + \sum_{j=1}^{J_{sd}} w_{s,j}^i \hat{u}_{t-j}^i + \sum_{k=1}^{K_{sd}} b_{s,k}^i TA_{k,t}^i + e_t^i \qquad (3)$$

$$\hat{s}_t^i = |\hat{u}_t^i| \sqrt{p/2}$$

where

r_t^i	spot return at time t
$r_{m,t}$	return on the market proxy at time t
D_{jt} *(j=1,..4)*	day of the week dummies, Wednesday is excluded
OCD_t	other calendar day variables, such as holiday and expiration day indicators
s_t^i	standard deviation of spot returns (estimated)
$TA_{k,t}^i$	spot and option trading activity variables

Model (2) is based on Schwert (1990) and provides an unbiased estimate of return volatility. I add the market proxy to the return equation to account for possible cross-sectional dependence in the error term. I estimate two models here as well. First, I estimate the system with only spot volume as the trading activity variable included in the variance equation. Next I add option volume.

3. Data and Volume Adjustments

Data on option volume for individual stocks are obtained from OptionMetrics. I use daily observations for the period January 2, 1996 – December 31, 2004. The OptionMetrics database contains detailed information on each option contract of a given stock. I use information on volume, daily best closing bid and best closing ask prices, and open interest.

Daily option volume and open interest are calculated by summing information across the individual option contracts. In particular, for each stock, option volume is obtained by summing across the dollar volume of the contracts written on the stock and traded on a given day. Since there is only one spot 'contract' for each stock, while there are multiple corresponding option contracts trading at different prices, I use dollar volume to make the spot and option volumes comparable. Moreover, dollar contract volume for derivatives is similar to an absolute delta weighted volume measure due to the close association between an option's delta and its premium. This addresses nonlinear relations between the spot and derivatives markets due to hedging.

I calculate option spread for the cross-sectional analyses by taking the trade weighted average of the various contracts' relative spread given by the ratio of the difference between the best ask and the best bid prices and their respective midpoints. I also calculate other option market liquidity measures based on the daily number of listed contracts and the ratio of non-zero volume contracts to the total number of available contracts. These are represented as the time-series means of the available series of daily contracts and daily proportion of non-zero contracts, respectively, in the cross-sectional analyses. I also use the daily time-series of total open interest and option volume to calculate measures of the option market's size and trading intensity, respectively, for the cross-sectional analyses. Daily changes in open interest provide a measure for the hedging demand component of derivative trades as in Day and Lewis (2004).

The source of individual stock information is the Center for Research in Security Prices (CRSP). I use stock cusip numbers to match CRSP and OptionMetrics. Although Nasdaq volume data are biased by the problem of double counting, not all tests employed in the paper are affected by this issue. Therefore, Nasdaq stocks are examined as well. The impact of double counting is discussed below. As with options, I calculate the daily dollar volume for each security. When a stock switches between Nasdaq and the NYSE (or AMEX) during the sample period, the sub-periods are analyzed separately as if they represented two distinct securities. To control for abnormal trading activity surrounding distribution events, such as stock splits and stock dividends, I eliminate all distribution days from the sample. For the cross-sectional analyses, I use information on the time-series of spot volume, shares outstanding, and return to calculate trading intensity, market capitalization, and return volatility, respectively.

In order to maintain reasonable statistical power and assure that the option market for a given stock is active, I require that a stock has at least 15 trading days with option volume of at least 50 contracts in a given month. I simultaneously require non-zero or non-missing spot volume, and non-missing stock return as well. Though these later requirements are generally satisfied when the former holds, this screen does find days when CRSP has no information on the stock. Once I eliminate all option months that do not satisfy these criteria, I further require at least 10 adjacent months of daily data for each security. This requirement is important since I will estimate autoregressive models via OLS, which are generally biased, but the bias becomes negligible when the time-series are sufficiently long. Since the sample period allows for daily observations for a maximum of 108 months for each stock, as a result of my data filter, some stocks disappear from the sample and then reappear. I also require at least 10 months of adjacent data for stocks that switch between Nasdaq and NYSE or AMEX in each subperiod determined by their listing location. The filter results in a final sample of 1,280 stocks with 1,623 daily time-series of at least 10 months length. I estimate each of the 1,623 time-series separately and then calculate the length-weighted average of the coefficient for stocks for which multiple time periods are available but no exchange switch occurs.

Measures of spot market liquidity for both the time-series and the cross-sectional analyses are extracted from the Market Microstructure database (MMD) of Vanderbilt University. The last day for which observations are available is August 31, 2004. Matching between the CRSP and the Market Microstructure Database is based on cusip information. The MMD utilizes the Lee-Ready (1991) algorithm and aggregates transaction level information from the TAQ dataset into daily averages. The aggregation employs standard data restrictions to insure the validity of the recorded trades and quotes. The source of analyst coverage is I/B/E/S, while information on institutional holdings is obtained from Thomson Financial's CDA/Spectrum.

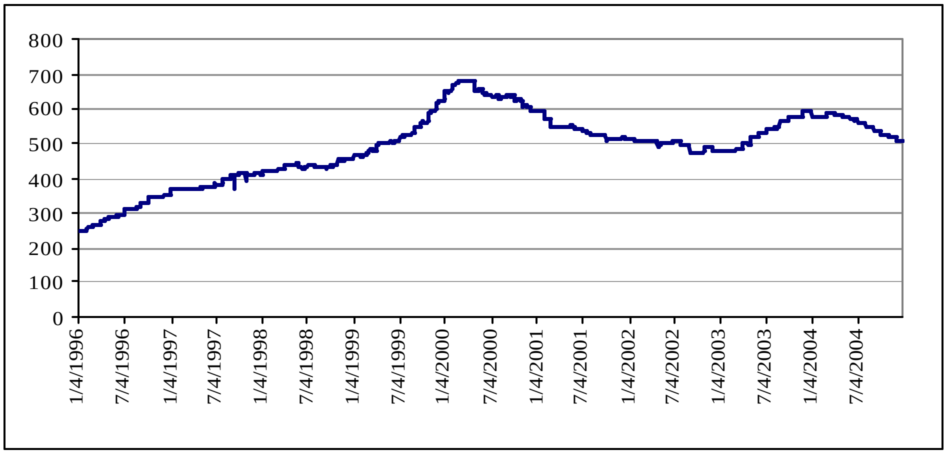
I use several sources to collect information on single stock futures. First, OneChicago provides detailed current and historic information on single stock futures listed on ONE. Also, Option Clearing Corporation reports daily open interest data for all trading single stock futures from March 17, 2003 and volume information for a sliding window of the last two years. Information on single stock futures from NQLX is from FutureSource[20].

After matching option volume information with other variables utilized in the analyses, 1,623 individual time-series remain. The average number of observations contained in a time-series is 685. The maximum length is 2,265, while the minimum is 188 daily observations. The average stock has 868 available data points. Figure 2.1 graphs the number of stocks represented in the sample, the average equity option, and the corresponding stock volume series for each day in the sample period. Option volume is multiplied by 100 to reflect the number of shares of stock underlying an option contract. Both option and spot volume are expressed in millions of dollars. Summary statistics pertaining to the cross-section of the average daily values of the individual series are presented in Table 2.1. The distribution of the mean spot and option volume series is highly skewed. The average (standard deviation of) mean dollar option volume is $8,169.93 (24,240.98), while the largest mean option volume equals $471,304.14 (Yahoo Inc. with 1,840 data points) and the smallest is $250.46 (Cytogen Corp. with 291 data points).

[20] I thank Yingmei Cheng for providing information on the listing dates of NQLX futures.

Figure 2.1. Sample Characteristics

Panel A



Panel B

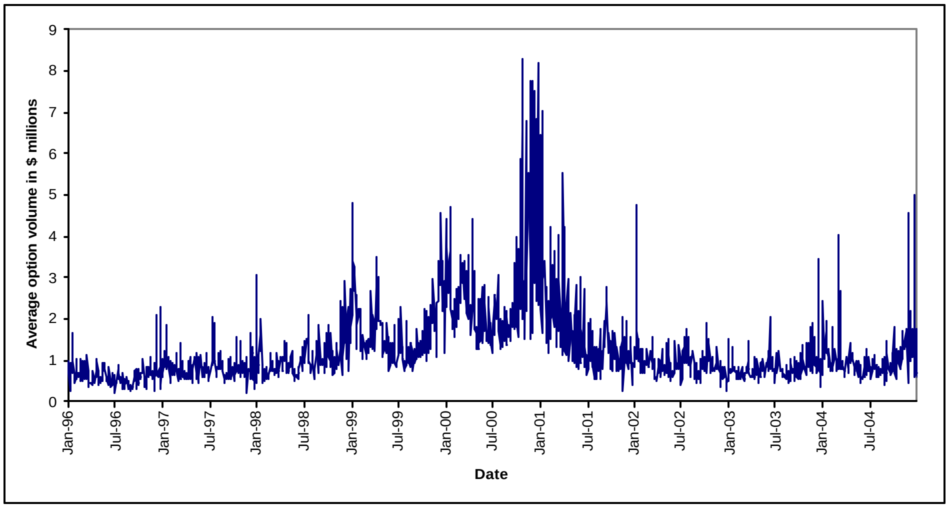
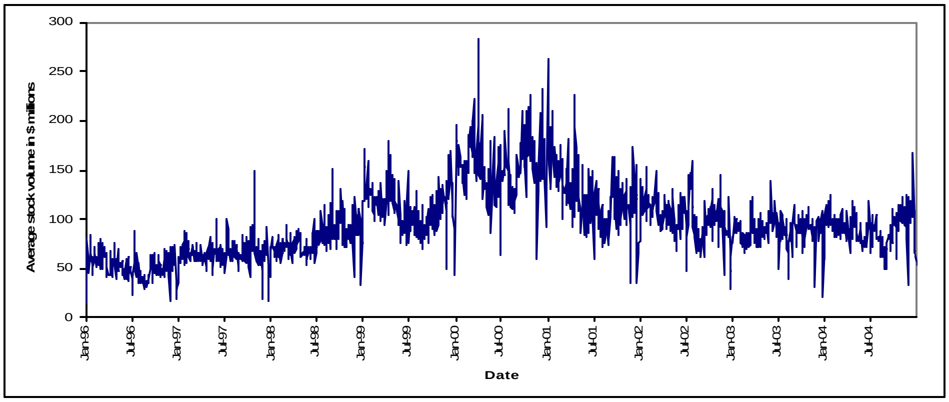


Figure 2.1, continued

Panel C



Panel A of Figure 1 reports the daily number of securities included in the final sample. The final sample consists of 1,280 individual stocks that have at least 15 days of at least 50 option contracts traded in a given month with at least 10 adjacent months satisfying this criterion. Panel B and C illustrate the behavior of aggregated option and spot volume, respectively, the unit of measurement for spot volume is dollar millions. Option volume is multiplied by 100 to reflect the number of shares corresponding to an option contract, and it is also expressed in dollar millions.

For spot volume, the average (standard deviation of) mean daily dollar volume is $66.46 million (119.62 million). The largest mean dollar volume is $1,867.00 million (Microsoft Corp. with 2,265 observations) and the smallest is $1.15 million (Immtech International Inc. with 228 time-series observations). The summary statistics are also described by three market capitalization groups, small, medium, and large.

Table 2.1. Summary Statistics

The sample includes 1,280 stocks traded on the NYSE, AMEX, or NASDAQ between January, 1996 and December, 2004. The descriptive statistics summarize the cross-sectional distribution of the 1,280 mean daily volume and return observations by market capitalization groups, where the means are calculated for each individual stock from the available time-series. The option and spot volume series are represented by dollar volume. Spot volume is in dollar millions.

	Option Volume	Spot Volume	Spot Return
Entire Sample			
Mean	8,169.93	66.45	0.0005
Median	2,644.52	33.28	0.0007
Standard Deviation	24,240.98	119.62	0.0023
Kurtosis	147.06	84.7	5.62
Skewness	10.35	7.6	-1.23
Minimum	250.46	1.15	-0.0137
Maximum	471,304.14	1,867.00	0.0101
Observations	1,280	1,280	1,280
Market Capitalization: Small (mean market capitalization = $606.92 m)			
Mean	2,495.57	16.62	0.0001
Median	1,421.80	12.93	0.0003
Std. Dev.	7,844.82	23.33	0.0027
Kurtosis	365.24	268.28	1.47
Skewness	18.46	14.77	-0.49
Minimum	250.46	1.15	-0.0108
Maximum	158,055.98	445.20	0.0072
Observations	427	427	427
Market Capitalization: Medium (mean market capitalization= $2,462.47 m)			
Mean	4,753.73	43.81	0.0004
Median	2,745.29	33.25	0.0009
Std. Dev.	5,812.85	32.40	0.0027
Kurtosis	17.24	8.64	5.46
Skewness	3.55	2.47	-1.55
Minimum	440.59	4.92	-0.0137
Maximum	51,057.04	246.11	0.0101
Observations	426	426	426
Market Capitalization: Large (mean market capitalization = $22,963.71 m)			
Mean	17,252.48	138.89	0.0008
Median	6,264.49	86.73	0.0008
Std. Dev.	39,273.05	181.96	0.0011
Kurtosis	56.09	37.17	16.50
Skewness	6.49	5.22	0.58
Minimum	459.88	9.83	-0.0059
Maximum	471,304.14	1867.00	0.0096
Observations	427	427	427

3.1 Properties of Volume: Expected and Unexpected components

This section partitions option volume into an expected and an unexpected component, and, simultaneously, investigates the joint dynamics of option and spot volume. The first subsection explores the univariate time-series properties of the volume series. The rest of the section uses a vector autoregressive framework for the joint dynamics.

3.1.1. Univariate Analyses of Volume

To gain some insight into the dynamic behavior of the volume series, I estimate univariate models that control for lagged values of the dependent variable and calendar day regularities. The use of calendar day indicators is motivated in Table 2.2, which reports descriptive statistics for the daily sum (i.e., aggregate volume) of the 1,623 option, spot, dollar option, and dollar spot volume series conditional on calendar day characteristics.

Table 2.2. Calendar Day Effects in Option Volume

The table reports univariate results for the calendar day effects in the aggregate option and spot volume series. The following calendar days are considered: 1) days of the week, 2) holidays, and 3) expiration days. The holiday indicator variable takes the value of 1 on the last trading day prior to a holiday as well as on the first trading day following it. The triple witching subsample contains trading volume on the third Friday of each quarter, while the '3rd Friday' subsample contains trading volume on the third Friday of each month. In Panel A, spot volume is in millions of shares. Panel B reports summary statistics on option volume in millions of contracts. Panel C and D report similar results for option and spot dollar volume, respectively.

Panel A. Aggregate Option Volume

	Total	Mon	Tue	Wed	Thu	Fri
Mean	1.63	1.49	1.64	1.68	1.67	1.69
Minimum	0.17	0.17	0.26	0.30	0.32	0.21
Maximum	5.77	3.23	4.08	4.66	4.10	5.77
N	2,265	431	465	463	454	452
	Holiday	Non-Holiday	Triple Witching	Non-Triple Witching	3rd Friday	Not a 3rd Fri
Mean	1.41	1.65	2.28	1.62	2.39	1.60
Minimum	0.21	0.17	0.74	0.17	0.69	0.17
Maximum	5.11	5.77	3.89	5.77	5.11	5.77
N	156	2,109	36	2,229	108	2,157

Table 2.2, continued

Panel B. Aggregate Spot Volume

	Total	Mon	Tue	Wed	Thu	Fri
Mean	1,388.84	1,277.03	1,388.89	1,459.35	1,445.11	1,366.65
Minimum	87.41	87.41	153.08	261.30	240.84	136.61
Maximum	3,775.12	3,360.79	3,446.42	3,775.12	3,455.04	3,568.78
N	2,265	431	465	463	454	452
	Holiday	Non-Holiday	Triple Witching	Non-Triple Witching	3rd Friday	Not a 3rd Fri
Mean	1,159.61	1,405.80	1,686.32	1,384.04	1,495.45	1,383.50
Minimum	136.61	87.41	409.36	87.41	288.11	87.41
Maximum	3,106.21	3,775.12	3,568.78	3,775.12	3,568.78	3,775.12
N	156	2,109	36	2,229	108	2,157

Panel C. Aggregate Dollar Option Volume

	Total	Mon	Tue	Wed	Thu	Fri
Mean	6.51	5.60	7.67	6.61	6.30	6.30
Minimum	0.59	0.59	0.88	0.78	0.96	0.62
Maximum	51.77	33.78	51.77	26.79	28.50	31.22
N	2,265	431	465	463	454	452
	Holiday	Non-Holiday	Triple Witching	Non-Triple Witching	3rd Friday	Not a 3rd Fri
Mean	6.60	6.51	8.20	6.49	8.66	6.41
Minimum	0.62	0.59	1.93	0.59	1.93	0.59
Maximum	48.14	51.77	28.38	51.77	31.22	51.77
N	156	2,109	36	2,229	108	2,157

Panel D. Aggregate Dollar Spot Volume

	Total	Mon	Tue	Wed	Thu	Fri
Mean	50,115.10	46,063.19	50,197.50	52,612.35	52,142.53	49,299.59
Minimum	3,823.04	3,823.04	6,568.54	11,195.75	10,820.13	5,874.53
Maximum	1.92E+05	1.40E+05	1.92E+05	1.56E+05	1.51E+05	1.38E+05
N	2,265	431	465	463	454	452
	Holiday	Non-Holiday	Triple Witching	Non-Triple Witching	3rd Friday	Not a 3rd Fri
Mean	41,766.66	50,732.63	60,481.22	49,947.68	54,211.55	49,910.00
Minimum	5,874.53	3,823.04	18,171.06	3,823.04	12,227.57	3,823.04
Maximum	1.09E+05	1.92E+05	1.38E+05	1.92E+05	1.38E+05	1.92E+05
N	156	2,109	36	2,229	108	2,157

The estimated means reveal the well-documented inverted U-shape pattern for spot volume during the week, with the highest trading activity occurring on Wednesday. For option volume, Monday displays the lowest activity, while the other days of the week show similar activity levels (for contract volume, Tuesday appears to be the highest). The results for Thursday and Friday may be inflated by the expiration day effects due to the univariate nature of these tests. A holiday effect is also captured in the table. The holiday sub-sample includes option and spot volume observations on the last trading day prior to a holiday as well as on the first trading day following it. As stock volume, option volume appears to be slower on trading days immediately surrounding exchange holidays. Tests of the equality of the means and medians confirm the significance of these results. In addition, average trading volume is significantly higher on triple witching days and on the third Friday of each month. This is consistent with Stoll and Whaley (1986, 1987, 1990), who analyze the price and volume effects during the expiration hour. Table 2.3 reports the average, minimum, and maximum autocorrelation coefficients up to order 6 and the correlation coefficients between the contemporaneous and lagged dollar option and spot volume series.

Table 2.3. Correlation and Autocorrelation Coefficients

Table 3 describes the average, minimum, and maximum of the 1,623 correlation and autocorrelation coefficients calculated for each of the available time series.

		Dollar option volume			Dollar spot volume		
		Average	Minimum	Maximum	Average	Minimum	Maximum
Dollar option volume	lag 0	1	1	1	0.517	-0.030	0.986
	lag 1	0.325	-0.033	0.829	0.325	-0.038	0.855
	lag 2	0.227	-0.056	0.744	0.232	-0.106	0.739
	lag 3	0.193	-0.107	0.711	0.198	-0.102	0.729
	lag 4	0.170	-0.098	0.645	0.179	-0.111	0.701
	lag 5	0.159	-0.079	0.754	0.166	-0.133	0.729
	lag 6	0.144	-0.118	0.654	0.154	-0.198	0.745
Dollar spot volume	lag 1	0.300	-0.091	0.836	0.531	0.101	0.894
	lag 2	0.219	-0.092	0.722	0.380	0.001	0.853
	lag 3	0.187	-0.090	0.735	0.326	-0.055	0.843
	lag 4	0.166	-0.106	0.682	0.297	-0.050	0.838
	lag 5	0.156	-0.136	0.674	0.278	-0.062	0.829
	lag 6	0.146	-0.135	0.689	0.259	-0.071	0.818

Consistent with the volume literature, I take the logarithm of dollar option and spot volume to improve the distributional properties of these series (for instance, to reduce the problem of skewness). I add an arbitrary small number, 0.0001, before I take the logarithm in order to avoid generating missing values when volume is zero. The logarithmic transformation reduces the average skewness of the time-series of dollar option (spot) volume from 6.83 (4.59) to -0.0002 (-0.88). In the univariate analyses, I follow the standard approach in the literature and I estimate a fifth[21] order autoregressive model via OLS which also includes day of the week, holiday, and expiration day indicator variables for the logarithmic dollar spot and option volume series separately using each of the 1,623 available time-series. These regressions also adjust for potential trends[22] in the individual series. Summary statistics of the coefficient estimates from the univariate time-series model are given in Panel A and Panel B in Table 2.4 for the spot and option volume regressions, respectively. The table reports the average coefficient estimates for the 1,623 time-series, the proportion of these coefficients that are significantly different from zero at least at the 10% two-tailed level, and the standard error weighted average coefficients. t-statistics are based on simple OLS standard errors. The univariate models reveal that both volume series are persistent: the first (second) order autoregressive term is significant in 99.75% (49.97%) and 96.73% (69.50%) of the individual series for spot and option volume, respectively. The holiday indicator is significant in 61.43% of the spot equations, and in only 29.08% of the option equations. The expiration day variable is a significant determinant of spot volume in 16.7% of the cases, while it is a significant determinant of option volume in 51.39% of the time-series. The mean of the cross-sectional distribution of the adjusted R^2's is 51.58% and 32.45% for the spot and option equations, respectively, which indicates that the models provide more explanatory power for spot volume.

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[21] Also estimate autoregressive models of order 2, 10, and 22. The results are robust to these alternatives.
[22] Lo and Wang (2000) find that volume time-series are very sensitive to the choice of the detrending method. I will also examine percentage changes in volume (in addition to (log) volume levels) in this section.

Table 2.4. Results from the time-series models that estimate the expected and unexpected components of the volume series

Panel A and B report summary statistics on the coefficient estimates from time-series models that model the dynamics of spot and option volume, respectively. These models are estimated separately. The dependent variable is the logarithm of dollar spot volume in Panel A, and the logarithm of dollar option volume in Panel B. The explanatory variables include autoregressive terms up to order five, day of the week indicators, a holiday indicator that takes a value of one immediately preceding and following an exchange holiday, indicator variables for the third Friday of each quarter and for the third Friday of each month, and the 'Daytill' variable that measures the number of trading days to expiration day. Column two in the table reports the average coefficient estimates across the 1,623 estimated time-series, column three reports the proportion of the estimated coefficients that are significantly different from zero at least at the 10% two-tailed level. Column four reports the standard error weighted average coefficient estimates. Panel C reports the results from the VAR system that estimates the spot and option volume equations simultaneously and includes cross-volume terms in the option equation.

Panel A. Spot Volume			
		Coefficients	
	Average	% Significant	SE weighted avg
Intercept	0.1006	56.25%	0.0473
Mon	-0.1060	39.49%	-0.1092
Tue	0.0221	13.80%	0.0156
Thu	-0.0287	14.91%	-0.0299
Fri	-0.1370	43.25%	-0.1275
Holiday	-0.2550	61.43%	-0.2076
Third Fri	0.0688	16.70%	0.0701
Triple Witching	0.1455	22.61%	0.1645
Trend	-0.0004	67.16%	0.00004
lag 1	0.4764	99.75%	0.4824
lag 2	0.0759	49.97%	0.0871
lag 3	0.0669	42.76%	0.0731
lag 4	0.0375	28.34%	0.0481
lag 5	0.0507	39.80%	0.0664

Panel B. Option Volume			
		Coefficients	
	Average	% Significant	SE weighted avg
Intercept	0.1322	48.12%	0.0982
Mon	-0.0256	16.02%	-0.0388
Tue	0.0073	10.60%	0.0153
Thu	-0.0082	9.00%	-0.0162
Fri	-0.0977	26.49%	-0.0993
Holiday	-0.1377	29.08%	-0.1213
Third Fri	0.3909	51.39%	0.3405
Triple Witching	-0.0463	9.06%	-0.0416
Daytill	-0.0067	37.95%	-0.0078
Trend	-0.0004	62.11%	0.00004
lag 1	0.2878	96.73%	0.3078
lag 2	0.1116	69.50%	0.1243
lag 3	0.0749	51.88%	0.0848
lag 4	0.0566	41.16%	0.0680
lag 5	0.0557	43.07%	0.0727

Table 2.4, continued

Panel C. Option volume equation from VAR

		Coefficients		
		Average	% Significant	SE weighted avg
Option volume	Intercept	0.0633	16.51%	0.0548
	lag 1	0.2255	77.57%	0.2097
	lag 2	0.0990	52.37%	0.1053
	lag 3	0.0687	39.56%	0.0771
	lag 4	0.0532	31.48%	0.0613
	lag 5	0.0505	31.42%	0.0606
Spot volume	lag 0	0.0864	44.05%	0.4610
	lag 1	0.1306	33.83%	-0.0290
	lag 2	-0.0083	21.44%	-0.0362
	lag 3	-0.0026	18.18%	0.0279
	lag 4	-0.0041	16.21%	-0.0226
	lag 5	0.0132	16.08%	-0.0164
Calendar day Variables	Mon	-0.0020	16.82%	0.0248
	Tue	0.0094	10.91%	0.0207
	Thu	-0.0118	9.18%	-0.0062
	Fri	-0.1202	17.68%	-0.0500
	Holiday	-0.1005	17.87%	-0.0099
	3rd Fri	0.4494	44.73%	0.3635
	Quarter End	-0.0719	12.75%	-0.1098
	Daytill	-0.0054	31.98%	-0.0067

3.1.2. Multivariate Analyses of Volume: Unique Component

Table 2.3 reports an average contemporaneous correlation of 0.51 between spot and option volume. Furthermore, R^2's indicate that a considerably high portion of the variation in the option equations is not explained by time-series characteristics and calendar day effects. The univariate approach ignores the connection between the option and spot series, which may contribute to the magnitude of the adjusted R^2's reported above. This subsection presents a bivariate analysis of the volume series. As in the univariate case, I use logarithmic dollar spot and option volume. The bivariate system is specified as a structural VAR in which spot volume is affected by its own lagged values and calendar day effects, while option volume is driven by contemporaneous spot volume, lagged option and spot volume, and calendar day variables.[23] I estimate the structural VAR via three stage least squares. Three stage least squares is equivalent to a seemingly unrelated regression (SUR) for simultaneous systems; and thus, it allows the spot and option volume error terms to be contemporaneously correlated. I standardize the log dollar spot and option volume series prior to estimating the VAR system. This is important for the cross-sectional analyses below.

Panel C of Table 2.4 above show summary statistics of the coefficient estimates from the VAR model for the option volume equations. The table reports the average coefficient estimates for the 1,623 time-series, the proportion of these coefficients that are significantly different from zero at least at the 10% two-tailed level, and the standard error weighted average coefficients. The univariate analysis above finds a mean (median) adjusted R^2 of 32.45% (29.10%) for option volume. In comparison, the mean (median) adjusted R^2 is 44.77% (43.64%) once contemporaneous and lagged spot volume enter the option equation, and the mean adjusted R^2 reported above is 51.58% for the spot volume equation. Though the R^2's increase, a significant portion of option volume remains unexplained. This points toward a significant unique component in option volume.

Panel A and B of Table 2.5 report the average and the maximum autocorrelations for lags 1, 2, and 3 for the standardized unexpected volume series based on the univariate and the VAR estimations, respectively. Both panels indicate that the standardized surprise option and spot volume series are no longer persistent. In Panel A, the univariate models retain a high correlation between option and spot volume.

[23] In addition to the asymmetric structural model described above, I also consider using a symmetric structural VAR in which both equations include a contemporaneous cross-market volume term and own- and cross-market lagged values (i.e., I allow contemporaneous and lagged option volume to enter the spot volume equation). Among the estimated parameters, the coefficients corresponding to the contemporaneous spot volume terms in the option equations are especially sensitive to the structural specification. The resulting estimates are somewhat troubling: in only 17% of the cases do they indicate a significant relation between option and spot volume, part of which is significantly negative. Because of this instability, I do not adopt the symmetric VAR.

Table 2.5. Correlation and autocorrelation coefficients for surprise option and spot volume

The table provides the correlation and autocorrelation coefficients for the surprise option and spot volume series. In Panel A, the surprise volume series are obtained from univariate time-series regressions that include lagged values of the dependent variables (log dollar spot and option volume, respectively) and calendar day indicator variables. In Panel B, the surprise volume series are obtained from a VAR model that jointly estimates the spot and option volume equations and includes contemporaneous and lagged cross-volume terms for options.

Panel A. Standardized residuals from the univariate time-series models

	Spot volume	Option volume	Spot vol (-1)	Option vol (-1)	Spot vol (-2)	Option vol (-2)	Spot vol (-3)
Spot volume	1						
Option volume	0.4103	1					
Spot vol (-1)	0.0008	0.0949	1				
Option vol (-1)	0.0460	-0.0069	0.4109	1			
Spot vol (-2)	-0.0069	0.0277	0.0006	0.0950	1		
Option vol (-2)	0.0053	-0.0110	0.0460	-0.0070	0.4113	1	
Spot vol (-3)	-0.0188	0.0061	-0.0070	0.0276	0.0003	0.0949	1
Option vol (-3)	-0.0054	-0.0179	0.0054	-0.0109	0.0459	-0.0068	0.4118

Panel B. Residuals from the VAR model of standardized log dollar spot and option volume

	Spot volume	Option volume	Spot vol (-1)	Option vol (-1)	Spot vol (-2)	Option vol (-2)	Spot vol (-3)
Spot volume	1						
Option volume	-1.1E-15	1					
Spot vol (-1)	-0.0009	0.0025	1				
Option vol (-1)	0.0049	-0.0061	0.0013	1			
Spot vol (-2)	-0.0069	0.0002	-0.0012	0.0025	1		
Option vol (-2)	0.0019	-0.0087	0.0051	-0.0063	0.0021	1	
Spot vol (-3)	-0.0170	-0.0033	-0.0070	4.8E-05	-0.0015	0.0023	1
Option vol (-3)	-0.0009	-0.0133	0.0019	-0.0086	0.0052	-0.0062	0.0031

That spot and option trading must be related is a straightforward observation. Yet, what this relation is and how it varies from stock to stock have not been discussed in the literature. To summarize the results of this section, though stock trading is an important determinant of option volume, on average, a large portion of option volume remains unexplained after accounting for hedging effects and time-series characteristics. On average, option volume has a significant unique component, rather than mostly representing derived demand. In the cross-section of the stocks in the sample, the spot-option relations as well as the explanatory power of the models display considerable variation.

4. Does Option Volume Affect Spot Volatility?

This section describes the empirical result for the option volume-spot volatility relation. Models (2) and (3) are estimated for each of the 1,623 time-series in the sample. The significance of the coefficients for the GARCH framework is based on heteroskedasticity consistent quasi-maximum likelihood (QML) covariances and standard errors using the methods proposed by Bollerslev and Wooldridge (1992). Standard errors for the system described in (3) are based on simple OLS estimates. The second subsection reports the results of the cross-sectional analyses.

4.1 Estimation Results

Table 2.6 aggregates the individual coefficient and standard error estimates for the augmented GARCH(1,1) with unexpected spot volume included in the variance equation. The surprise spot volume is obtained from the VAR model above. The table details the average and median parameter values and standard errors. The average parameters are based on equal-weighting as well as standard error weighting. For standard error weighting, the weights are given by one over the standard error of the estimated coefficient.

Table 2.6. Maximum Likelihood Estimate of the GARCH(1,1) Model with Spot Volume

The table presents the mean and the median parameter estimates and their associated p-values based on the maximum likelihood estimate of the 1,623 individual GARCH(1,1) models of the following form:

$$r_t^i = m_t^i + u_t^i$$
$$u_t^i \mid \Omega_{t-1} \sim N(0, h_t^i)$$
$$h_t^i = a^i + b_1^i (u_{t-1}^i)^2 + b_2^i h_{t-1}^i + b_3^i UV_{S,t}^i$$

where $UV_{S,t}^i$ is unexpected spot volume, estimated via the VAR system. The significance of the coefficients is based on Bollerslev-Wooldridge (1992) heteroskedasticity consistent standard errors. For the standard error weighted estimates, weights are given by one divided by the estimated standard error. The bottom section of the panels reports the percent of positive coefficients as well as the percent of significant values among the positive and the negative estimates.

	b_1		b_2		b_3	
	Estimates	**z-stat**	**Estimates**	**z-stat**	**Estimates**	**z-stat**
Equal weighted						
mean	-0.0203	-0.5837	0.0812	2.3234	0.0259	11.6486
median	-0.0165	-0.4961	0.0841	1.8838	0.0210	10.8312
Standard error weighted						
	Estimates		**Estimates**		**Estimates**	
mean	-0.0194		0.0923		0.0166	
median	-0.0154		0.0974		0.0124	
% +	36.11		84.04		99.69	
% + and sig.	5.42		53.91		99.57	
% - and sig.	20.27		0.99		0.25	

This weighting scheme assigns a more important role to the coefficients that are more accurate and attenuates the insignificant coefficients to zero. Also reported are the proportion of the sample of 1,623 coefficient estimates that are positive, the proportion of the coefficients that are positive and significant, and the corresponding value for the negative and significant coefficient estimates.

The table is consistent with existing evidence that the average magnitude and significance of the ARCH and GARCH coefficients decreases substantially when contemporaneous spot volume is included in the variance equation. In addition, as reported in previous studies, for the majority of the stocks, surprise spot volume is positively related to spot volatility. 99.69% of stocks display positive unexpected spot volume coefficient estimates, 99.57% are positive and significant at least on the 10% level.[24]

Table 2.7 reports similar results for the unexpected spot and option volume augmented GARCH(1,1) model. The table uncovers surprisingly strong results. In particular, the coefficient estimates corresponding to unexpected option volume is positive for 95.19% of the sample. In addition, 60.69% are significantly positive, while only 0.12% of the coefficients are negative and significant at least at the two-tailed 10% level. This closely compares to, but is weaker than the results obtained for spot volume. More specifically, when both volume series are included in the variance equation, 99.63% of the surprise stock volume coefficients are positive. 98.77% are significantly positive and 0.25% are significantly negative.[25]

The significance of the estimated surprise option volume coefficients for the majority of the stocks implies that option volume has incremental information for spot volatility. The contribution is examined via the likelihood ratio (LR) test in more detail. Table 2.8 reports the results. When the spot volume augmented model (restricted model) is compared to the spot and option volume augmented model (unrestricted model), the likelihood ratio test rejects the null hypothesis that components of option volume do not contain additional information 83.40% of the time (at least, at the 0.1 level). The average likelihood ratio test statistic is 6.14, which provides significance at the 0.025 level based on the $\chi^2(1)$ critical value. The results of the LR test confirm that for the majority of the stocks, option volume adds value to the estimation of return volatility.

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[24] In additional analyses not reported here, I also include the expected volume component along with surprise volume and find that for the majority of the stocks, expected spot volume has a positive, albeit insignificant effect. Furthermore, surprise volume is much more important for volatility than expected volume. The estimated coefficients on unexpected spot volume are approximately ten times higher than those on expected volume. These results are consistent with previously reported results in the literature (see, for instance, Bessembinder and Sequin (1992)).

[25] When both expected option volume and expected spot volume are included in the analyses, I find that they have no significant effect on the volatility of the average stock. The coefficients on the corresponding unexpected components are approximately ten times of those on the expected components. Unlike for spot volume, the average effect of expected option volume is negative, however, the average effect is not significant.

Table 2.7. Maximum Likelihood Estimate of the GARCH(1,1) Model with Spot and Option Volume

The table presents the mean and the median parameter estimates and their associated p-values based on the maximum likelihood estimate of the 1,623 individual GARCH(1,1) models of the following form:

$$r_t^i = m_t^i + u_t^i$$
$$u_t^i \mid \Omega_{t-1} \sim N(0, h_t^i)$$
$$h_t^i = a^i + b_1^i (u_{t-1}^i)^2 + b_2^i h_{t-1}^i + b_3^i UV_{S,t}^i + b_4^i UV_{O,t}^i$$

The significance is based on Bollerslev-Wooldridge (1992) heteroskedasticity consistent standard errors. $UV_{j,t}^i$ represents the unexpected volume series, with j=O(S) option (spot) volume, obtained from the VAR system. For the significance weighted estimates, weights are given by one minus the associated z-value. The bottom section of the table reports the proportion of positive coefficients as well as the proportion of significant positive values and the significant negative estimates.

	b_3		b_4	
	Estimate	**z-stat**	**Estimate**	**z-stat**
Equal weighted				
mean	0.0260	7.6982	0.0045	2.2439
median	0.0200	6.9965	0.0034	2.0120
Significance weighted				
	Estimate		**Estimate**	
mean	0.0156		0.0030	
median	0.0116		0.0022	
% +	99.63		95.19	
% + and sig.	98.77		60.69	
% - and sig.	0.25		0.12	

Table 2.8. Likelihood Ratio Test Results

Models 1-2 are based on model (2). Model 1 is represented by the spot volume augmented standard GARCH(1,1) specification and its variance equation is given by the following:

$$h_t^i = a^i + b_1^i (u_{t-1}^i)^2 + b_2^i h_{t-1}^i + b_3^i UV_{S,t}^i \qquad \text{(Model 1)}$$

Model 2 augments the variance equation with both spot and option volume:

$$h_t^i = a^i + b_1^i (u_{t-1}^i)^2 + b_2^i h_{t-1}^i + b_3^i UV_{S,t}^i + b_4^i UV_{O,t}^i \qquad \text{(Model 2).}$$

The mean test statistic represents the cross-sectional average of the 1,6230 likelihood ratio tests performed for the individual stocks. The table also reports the distribution of the test statistic, the critical values, and the portion of the values that are significant at least at the 10% level.

		Mean Test Statistic	Critical Value at 0.1	Critical Value at 0.05
Unrestricted	Model 2			
Restricted	Model 1			
Distribution of Test Stat.	$\chi^2(1)$	6.142	2.71	3.84
% exceeding critical value at 0.1	83.41			

Table 2.9 reports summary statistics for the estimation results for model (3). The table only reports results for the case when both the spot and option volume series are included as trading variables in the variance equation. Panel A uses the unexpected components estimated from the univariate time-series models. In Panel B, unexpected volume components in the variance equation are based on the VAR estimates.

Under the null hypothesis of no relation between option volume and spot volatility, 5 percent of the stocks are expected to display significant positive, and 5 percent are expected to exhibit significant negative coefficients, at the 10 percent two-tailed level. Therefore, the hypothesis that option trading has no effect on return volatility can be rejected for surprise option volume. The results indicate significant heterogeneity across the stocks; therefore, conclusions based on the average security are misleading. The next section provides a cross-sectional characterization of how and why the time-series results vary from stock to stock in the sample. It is important to note that the magnitude of the estimated coefficients can be meaningfully compared in the cross-section because all volume variables are standardized in the individual time-series analyses.

Table 2.9. Results from Model (3)

The table reports summary statistics for the 1,623 coefficient estimates for the trading activity variables in the variance equation of Model (3). Model (3) is given by the following set of equations that are estimated sequentially.

$$r_t^i = a_r^i + b_{r,1}^i r_{m,t} + \sum_{k=2}^{K_r+1} b_{r,k}^i r_{t-k}^i + \sum_{j=1}^{4} g_{r,j}^i D_{jt} + d_r OCD_t + \sum_{j=1}^{J_r} f_{r,j}^i \hat{s}_{t-j}^i + u_t$$

$$\hat{s}_t^i = a_s^i + \sum_{j=1}^{J_{sd}} f_{s,j}^i \hat{s}_{t-j}^i + \sum_{j=1}^{4} g_{s,j}^i D_{jt} + \sum_{k=1}^{K_{sd}} w_{s,k}^i \hat{u}_{t-k}^i + \sum_{l=1}^{L_{sd}} b_{s,l}^i TA_{lt}^i + e_t$$

$$\hat{s}_t = |\hat{u}_t| \sqrt{p / 2}$$

where

r_t	spot return at time t
$r_{m,t}$	return on the market proxy at time t
D_{jt} $(j=1,..4)$	day of the week dummies, Wednesday is excluded
OCD_t	other calendar day variables, such as holiday and expiration day indicators
s_t	standard deviation of spot returns (estimated)
$TA_{l,t}$	spot and option trading activity variables, in particular, surprise spot and option volume.

Panel A reports estimates based on unexpected volume obtained from univariate regression models on volume on its own lagged values and calendar day variables. In Panel B, the unexpected volume components are obtained from a VAR model, which estimates the option and spot volume equations simultaneously and includes a contemporaneous cross and lagged cross terms in the option volume equation.

Panel A. Based on expected and unexpected volume from the univariate model.

	Avg Coefficient	% positive	% positive and significant	Avg t-stat
Unexpected option vol	0.0043	96.80%	69.56%	2.7500
Unexpected spot vol	0.0145	99.45%	99.14%	8.9956

Panel B. Based on expected and unexpected volume from the VAR model.

	Avg Coefficient	% positive	% positive and significant	Avg t-stat
Unexpected option vol	0.0034	95.26%	61.68%	2.2911
Unexpected spot vol	0.0141	99.57%	99.32%	9.4972

4.2 Cross-Sectional Analyses of the Option Volume – Spot Volatility Relation

The results in Section 4.1 reveal a generally positive association between option volume and stock volatility. This represents an extension to the volume-volatility literature. In the rest of this paper, I examine whether this cross-market correlation is also consistent with the MDH (i.e., information arrival in the option market). In this section, I investigate whether the cross-sectional variation is related to the degree of asymmetric information surrounding the security and the probability of informed trading in the option market, by studying how different proxies for these conditions are related to the cross-section of the time-series estimates. In Section 4.3, I look at the effect of surprise volume on stock spreads.

4.2.1. Explanatory variables

I use several measures to depict the relevant firm and market characteristics. For instance, a commonly used proxy of information asymmetry is analyst coverage. Analyst coverage indicates attention surrounding the firm. When more analysts follow a firm, the prices are likely to be more information efficient. Therefore, the opportunity for informed trading is reduced in both markets. The attention measure is also expected to alter the relative role of the markets for the informed trader based on the theoretical results in Easley, O'Hara, and Srinivas (1998). Easley et al. show that the existence of a separating equilibrium, or in other words, informed trading in both markets, depends on the proportion of investors who are informed. The greater the proportion, the more likely it is that a separating equilibrium obtains. My second proxy for the degree of information asymmetry or attention is institutional holdings.

To measure how attractive the option market is for informed traders (i.e., the probability of informed trading), I use a number of relative liquidity measures. For instance, I follow Chakravarty et al. (2004) and use the ratio of option spread to spot spread to measure relative relations. This is based on the argument that the spread is an explicit cost, which also proxies for liquidity in the market, or the ability of an informed trader to hide so that prices do not immediately reflect her order flow. Fleming et al. (1996) find that low trading costs foster price discovery.

Similarly, I also use the ratio of the size of the option market to the size of the stock market. This is also related to informed traders' ability to hide in the market. Though smaller firms may provide more opportunities for new information acquisition, these opportunities may not translate into profitable trades when trading is thin. Ho and Michaely (1988) argue that in equilibrium traders may rationally choose to learn less about small firms.

It is important to note however, that previous literature points towards a dual role for both size and spread. First, larger firms/markets typically receive more investor attention and are more likely to be held by institutional investors who, through economies of scale in information collection, are assumed to be better informed. This implies that large firms have greater information efficiency in pricing and thus, provide fewer opportunities for informed trades.

Similarly, a higher spread indicates higher adverse selection costs as the market maker fears the presence of informed traders (Llorente et al. (2002) and Lee, Mucklow, and Ready (1993)); while this in turn implies that these stocks must be more suitable for information acquisition. Chakravarty et al. (2004) report that the option's information share is negatively related to the ratio of option to spot effective spreads, which suggests that the importance of trading costs dominates the market maker's response to asymmetric information. In addition, with multiple contracts and market makers in the option market, the adverse selection cost may be less visible in an aggregate (across contracts and markets) option spread measure.

Furthermore, I use relative volume in the option market. Since private information is incorporated into prices via the trading process, Chakravarty et al. (2004) argue that the information share of the option market is also related to the ratio of volumes. I also consider the estimated informational share of options in Chakravarty et al. for the relevant sub-sample of stocks, the relative volatility of option spread, and the volatility of analyst coverage as explanatory variables in the cross-section. All measures are represented by their time-series means (and standard deviations for the volatility measures) calculated for the sample period of each individual stock. The explanatory variables are summarized in Appendix 2.1.

4.2.2. Results

I estimate the cross-sectional regressions separately for the Nasdaq and NYSE/AMEX subsamples. The results are provided in Table 2.10.

Panel A of Table 2.10 uses the sample of NYSE/AMEX stocks and estimates 6 cross-sectional specifications. Because not all regressors are available for each stock, the sample size differs across the specifications. This also means that the goodness of fit statistic is not comparable across the columns. To compensate for this discrepancy, I reestimate model 2 in column 1 using only observations which are also available in the column 4 estimation. Similarly, I use the model in column 4 and reestimate it (in column 7), in order to match the sample in column 8. Panel B reports similar results for the Nasdaq sample, with the exception of Model 8, which cannot be estimated for this sample. This is because the CGM information share variable is only available for 60 stocks, none of which trades on Nasdaq. The results are estimated separately for the different exhanges because volume on Nasdaq suffers from the problem of double counting. Because of the difference in volume measures, the estimated coefficients from the time-series analyses may have different characteristics across the Nasdaq and non-Nasdaq firms. Moreover, microstructure differences may also bias cross-sectional results when the two subsamples are pooled together.

The results in Panel A show that the most significant determinants of how surprise option volume affects return volatility are the relative size and illiquidity of the option market, and analyst coverage. As predicted, relative size is positively, while relative liquidity and analyst coverage are negatively related to the impact of options. These variables describe the relative information environment of options, including the degree of information asymmetry surrounding the stock and the relative ease with which informed traders could hide in the option market.

Table 2.10. Cross-sectional Analysis of the Option Volume-Spot Volatility Coefficients

The table reports results on the cross-sectional analyses of the estimated option volume-spot volatility coefficients. Panel A uses NYSE/AMEX stocks, while in Panel B, the corresponding results for Nasdaq are reported. Relative size is the size of the option market relative to firm size. Relative spread is the ratio of option to stock spread. Analyst coverage indicates the average number of analysts following the security. CGM information share is the information share of the option market as reported in Chakravarty, Gulen, and Mayhew (2004). The CGM information share variable is only available for 60 stocks in the sample. Volatility of analyst coverage is the volatility of the number of analysts following the firm. Institutional holding is the percentage share of institutional ownership. T-statistics are reported in parentheses.

Panel A

	Cross-sectional model - NYSE/AMEX							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Intercept	0.564	0.836	0.783	0.756	0.821	0.904	1.281	1.863
	(11.221)	(13.667)	(13.057)	(12.586)	(10.676)	(10.931)	(4.999)	(4.768)
Relative size	0.052	0.067	0.059	0.056	0.032	0.052	0.082	0.077
	(5.500)	(7.034)	(6.487)	(6.144)	(2.632)	(4.377)	(3.950)	(3.832)
Relative spread		-0.110	-0.110	-0.110	-0.103	-0.101	-0.086	-0.098
		(-7.677)	(-7.919)	(-8.181)	(-6.704)	(-5.919)	(-2.644)	(-3.053)
Analyst coverage				-0.005	-0.075	-0.076	-0.183	-0.174
				(-3.203)	(-3.225)	(-3.263)	(-2.101)	(-2.055)
Vol. of analyst c.					0.015			
					(1.464)			
Institutional holding						0.041		
						(1.062)		
CGM info share								0.349
								(2.028)
Adj R^2	0.038	0.111	0.125	0.138	0.165	0.169	0.360	0.400
N	739	707	**585**	**585**	346	315	**45**	**45**

Table 2.10, continued

Panel B

	Cross-sectional model - Nasdaq					
	1	**2**	**3**	**5**	**6**	**7**
Intercept	0.361	0.872	0.683	0.670	0.532	0.670
	(4.661)	(9.046)	(6.406)	(6.249)	(4.440)	(4.755)
Relative size	0.048	0.016	0.002	0.005	-0.028	0.010
	(2.767)	(2.868)	(2.089)	(2.245)	(-1.127)	(0.427)
Relative spread		-0.180	-0.142	-0.143	-0.138	-0.139
		(-8.628)	(-6.606)	(-6.643)	(-5.852)	(-5.766)
Analyst coverage				-0.005	-0.073	-0.006
				(-1.327)	(-2.407)	(-1.405)
Volatility of analyst coverage					0.071	
					(4.384)	
Institutional holding						-0.031
						(-0.749)
CGM info share	-	-		-	-	-
N	884	802	**532**	**532**	422	
Adj. R-squared	0.007	0.089	0.085	0.086	0.134	

Also, there is a surprisingly strong association between the information share of options, as estimated by Chakravarty et al. (2004), and the option volume-spot volatility coefficient. In particular, the adjusted R-squared of the regression is 40% once the information share is included in the analyses. Unfortunately, the information share variable is only available for a very small subsample of 60 stocks, and not available for Nasdaq firms. With additional missing values for explanatory variables, the sample size drops to 45 in this regression. The volatility of analyst coverage only has a marginally significant positive effect, while institutional holdings are not significantly related to the impact of options on spot volatility. Panel B reports similar results.

4.3 Option Volume and Stock Spreads

The previous section offered some evidence that the cross-market volume-volatility relation is driven by the mixture of distribution argument. This section provides further support.

However, as it was mentioned in the Introduction, these sections should be viewed as suggestive, rather than conclusive.

An implication of informed trading in the option market (and that of the resulting MDH) is that the unexpected component of option volume is contemporaneously negatively related to stock spreads. This is because the informed trader's choice to trade in the option market, rather than in the stock market, reduces the adverse selection costs of the spot market maker.

To test this argument, I estimate individual time-series regressions with the relative effective stock spread[26] as the dependent variable. I take the individual time series of surprise option volume obtained from the VAR, and match them to daily information on stock spreads. Because data restrictions and the resulting time-series above are based on volume and return information only, I re-run my data filters with requiring non-missing values for the spread with 10 consecutive months of available data, as before. I then adopt a multivariate regression framework for each individual stock.

For the majority of the stocks, the daily time-series of the relative effective spread is persistent, and in about 10% of the cases, the augmented Dickey-Fuller test cannot reject the existence of a unit root. Where the unit root test is not rejected, I difference the spread series. I also check to see whether the behavior of the time-series is related to the length of the individual samples, but find no evidence.[27]

I rely on two sources to build a model of the spread over time. First, Chordia et al. (2005) find that spread at the daily frequency displays day-of-the-week and holiday effects, and is affected by stock volatility, stock volume, returns, and tick size changes.[28] Second, I also consider established models of the spread from cross-sectional analyses. In particular, I modify the Chordia et al. model to more carefully proxy for daily changes in inventory control and

[26] The effective spread is given by the daily average trade weighted effective spread.
[27] This is important because non-stationarity cannot be resolved by shortening the sample window.
[28] The first tick size change is June 24, 1997 to sixteenth. Decimalization was implemented between August, 2000 and January, 2001.

adverse selection costs for a given security. Hence, I use stock volatility multiplied by the average holding horizon for the given stock (estimated by the total trading time for the day divided by the number of transactions). This resembles the measure proposed by Stoll (1978). For order processing costs, I use inverse trading volume. Finally, in this specification, surprise option volume enters as a multiplicative term, since it is incorporated in the time varying coefficients of the inventory cost/adverse selection variable. This is a more direct test of whether the adverse selection component of the spread decreases when surprise option volume is high. The results are reported in Table 2.11.

The impact of surprise option volume is non-positive for the majority of the sample. In terms of statistical significance, approximately 37% of the option volume coefficients are significantly negative; the majority however is indistinguishable from 0. The model has obvious weaknesses, but overall, the results suggest that option trading can have an important impact on the stock's spread. Moreover, the estimated option volume – spot volatility coefficients from Section 4.1 above are strongly negatively related to the estimated impact of surprise option volume on stock spreads, with the correlation coefficient equaling -0.68. The average R-squared of these regressions is 0.35, while the median is 0.34.

5. Robustness Results

5.1 Changes in Volume Decomposition

I use alternative methods to arrive at the expected and unexpected components of option volume. These methods are described below. When I use these alternative definitions, the cross-sectional distribution of the time-series coefficients is qualitatively unchanged.

Table 2.11. Stock Spreads and Option Volume

The table summarizes the results from the individual time-series regressions in which the dependent variable is the relative effective spread (at the daily frequency). The model is described by the following equation:

$$Effectivespread_t = b_0+(b_1+b_2\cdot optionvolume_t)\cdot adverseselection_t+b_3\cdot orderprocessing_t + \sum_i b_i days\text{-}of\text{-}the\text{-}week_t + b_8\cdot holiday_t + b_9\cdot sixteenth_t + b_{10}\cdot decimals_t + time\text{-}series\ adjustments + e_t$$

Option volume is represented by the unexpected component of option volume from the VAR above. For adverse selection costs, I use stock volatility multiplied by the average holding horizon for the given stock (estimated by the total trading time for the day divided by the number of transactions). This resembles the measure proposed by Stoll (1978). For order processing costs, I use inverse trading volume. For each explanatory variable, the table indicates the percent of coefficient estimates that are positive (in the first column) and negative (second column). In parentheses, the first column contains the percent of estimates that are positive and significant, while the second column shows the percent of negative and significant values.

	+ (+ and sig.)	- (- and sig.)
Adverse Selection (A.S.)	94.77%	5.23%
	(73.59%)	(0.24%)
A.S.xSurprise OV	22.95%	77.05%
	(7.81%)	(37.46%)
Order processing	28.99%	71.01%
	(7.97%)	(41.87%)
Mon	46.14%	53.86%
	(5.72%)	(9.66%)
Tue	46.86%	53.14%
	(8.29%)	(11.03)
Thu	51.77%	48.23%
	(7.41%)	(6.44%)
Fri	49.44%	50.56%
	(4.35%)	(7.57%)
Holiday	40.18%	59.82%
	(3.70%)	(3.54%)
Tick1	19.32%	80.68%
	(9.58%)	(71.58%)
Tick2	6.28%	93.72%
	(1.85%)	(86.39%)

5.1.1. Percentage Change in Volume

Since volume time-series are very sensitive to the choice of the detrending method (Lo and Wang (2000)), I also estimate the joint dynamics of percentage changes in spot and option volume. Measuring volume in percentage changes avoids the problem of volume growth over time.

When spot and option volume are measured in percentage changes, the explanatory power of the univariate models drop significantly. The average adjusted R^2 in the spot equation is 5.68%, while the 99th (75th) percentile is 17.51% (8.20%). For the univariate option equation, the mean adjusted R^2 is 1.02%, while the 99th (75th) percentile is 17.41% (1.99%). When contemporaneous and lagged percent change in spot volume is included in the option equation, the average adjusted R^2 increases to 8.87%, with a 99th (75th) percentile of 62.01% (12.45%). This indicates that while spot volume is an important determinant of option trading, variation in option trading intensity remains largely unexplained.

5.1.2. The Role of Market-Wide Trading

Tkac (1999) and Lo and Wang (2000) uncover a factor structure in spot volume and argue that an adjustment to market volume is required when filtering the individual volume series. I use relative market capitalization weighted total volume for the stocks and options in my sample to proxy for market-wide trading activity. As before, I use an asymmetric model and only include the stock market volume variable in the individual spot equations, while I include both the option and the spot market proxies in the option volume equations.

The mean (median) adjusted R^2 in the spot equation increases to 53.71% (53.07%) once market-wide volume is added as an explanatory variable. In 83.49% of the sample, market volume is a significant (positive) determinant of individual stock volume at least at the 10% level. Interestingly, for the option equations, market-wide trading activity does not play an important role. Stock market volume has a significant impact on option trading only in 18.24% of the sample. The corresponding proportion for market-wide option volume is 0.00%. This lack of importance is also reflected in the adjusted R^2's, which decrease slightly when the market volume variables are included in the analyses. The mean (median) adjusted R^2 in this case is 44.03% (42.99%).

5.2 Changes in Model Specifications

For robustness, first, I estimate several time-series models to determine the expected and unexpected volume components. These include the symmetric and asymmetric VAR models discussed above. Second, I re-estimate the system with a contemporaneous and lagged equity returns in both equations for the following reasons. Firstly, option volume may reflect adjustments to hedge positions due to changes in the underlying stock price. This would imply that stock price changes (i.e., volatility) are naturally accompanied by option trading. By including return information in the option volume equations, I address the concern that surprise volume captures this hedging response. Secondly, stock volume may also reflect program trading induced by recent performance. Feedback traders increase spot volume in response to changes in the stock price. Just as in the case of hedgers in the option market, larger price changes may trip more trade indicators and are associated with more spot trading. The inclusion of return information does not alter the results.

Third, to address the issue that traders hedge more in volatile periods, that is, the hedging induced relation between option and spot volume is time-varying, I re-estimate the VAR models with time-varying parameters for the spot volume terms in the option equation. I allow these parameters to be functions of lagged return volatility. Alternatively, I use the model proposed in Bessembinder and Seguin (1993), which sequentially estimates the dynamics of return, trading activity, and volatility. In the sequential estimation steps, trading activity is regressed on past volatility, time-series terms, and calendar day variables. The residual trading activity from this regression is then used in the volatility equation.

Fourth, I extend Model 1 to include a GARCH-M specification, which is similar in spirit to Model 2, and an EGARCH structure, which allows for an asymmetric response in volatility.

Fifth, in the cross-sectional analyses, I include an explanatory variable that measures how heavily the coefficient estimates are based on the stock market boom period. This is because cross-market ties may differ across the tech boom period and during the early years of 2000. Also, as robustness check, I re-estimate the cross sectional tests on a restricted sample which only includes stocks with the most liquid option markets.

Finally, it is important to note that non-synchronicity in market closings is not likely to be a problem in the analyses. This issue was a serious concern in earlier studies, however, exchanges adopted changes in their hours of operation and during most of the sample period considered here, stock exchanges close at 4:00 p.m. E.T. while equity option trading ends at 4:02 p.m. E.T. For instance, the CBOE changed its closing time from 4:10 p.m. to 4:02 p.m. on June 26, 1997. Index options remain open until 4:15 p.m. but those are not included in the analyses. Though market closings are nearly simultaneous, non-synchronicity arises when one of the markets is stale, for instance, when trading stops early in the day, while the other market remains active until the closings. Llorente et al. (2002) use the last quote's midpoint of the day to address this issue based on the argument that market makers posting the quotes are likely to incorporate all available information to avoid offering the market a free option. I also use a sub-sample of the most liquid stock and option markets to address this concern.

6. Conclusion

This paper extends the volume-volatility literature by documenting a positive correlation between surprise option volume and stock volatility. I also find that the relation is statistically significant for the majority of the stocks.

In the stock market, the commonly accepted explanation for the volume-volatility relation is the Mixture of Distribution Hypothesis (MDH). The MDH suggests that both surprise stock volume and stock volatility are driven by the underlying directing process of information arrival; hence, they are contemporaneously correlated. Since many recent papers find evidence of informed trading in the option market, it is reasonable to conjecture that the option volume-stock volatility relation may also be driven by information arrival. The paper provides two pieces of suggestive, though not conclusive, evidence that this may indeed be the case.

First, in cross-sectional tests, I find that the volume-volatility relations are closely related to the information environment surrounding the individual securities, as well as to the relative liquidity of the option market. In other words, the latter indicates that the time-series results are strongest for stocks for which conditions in the option market allow informed traders to hide so that their order flow does not immediately reveal their private information.

Second, the analysis of stock spreads finds that they are non-positively related to surprise option volume, and in many cases the association is significantly negative. This result is consistent with the hypothesis that, when informed traders choose the option market, rather than the stock market (as reflected in the surprise component of option volume), the stock market maker's adverse selection costs decrease.

Appendix 2.1

Table 2.12. Variable Description

Variables	Explanation
Analyst coverage	A proxy for attention/asymmetric information. Obtained from IBES
Institutional Holdings	A proxy for attention/asymmetric information. Obtained from Thomson Financial
Volatility of analyst coverage	A proxy for uncertainty/asymmetric information. Obtained from IBES
Relative spread	A proxy for relative liquidity/how attractive the option market is for informed trading. Represented by the ratio of the spread in the option market (trade weighted across the contracts) and the spread in the stock market (trade weighted, based on TAQ). Obtained from Optionmetrics and the Market Microstructure Database
Relative spread volatilities	A proxy for changes in relative liquidity/uncertainty. Represented by the ratio of the standard deviation of the spread in the option market (trade weighted across the contracts) and the spread in the stock market (trade weighted, based on TAQ). Obtained from Optionmetrics and the Market Microstructure Database
CGM information share	A proxy for the amount of informed trading in the option market. Represented by the lower bound of the information share estimate from Chakravarty et al. (2004) for the relevant subsample of stocks.
Relative size of the option market	A proxy for the suitability of the option market for informed trade/relative liquidity/attention. Represented by the dollar open interest divided by the market capitalization of the stock. Obtained from the Market Microstructure Database and CRSP

For both the relative size and the relative spread measure, alternative definitions were also tested (such as the ratio of total volumes and the ratio of option spread to effective stock spread, respectively). Also the CGM information share was alternatively defined as the upper bound, as well as the average of the upper and lower bounds. These alternative definitions do not affect the results reported in Table 2.10 above.

Appendix 2.2

Single stock futures resumed trading on November 11, 2002. Since the equity futures market provides a new venue for traders of the underlying stock, the listing of single stock futures may (should) alter the dynamic relation between the spot and the option market. To estimate the effect of single stock listings, I collect information on stocks with single stock futures that are listed prior to March, 2004. This allows for a post listing sample length of at least ten months. When a stock is listed on both ONE and NQLX, I use the earlier listing day. I eliminate stocks for which not enough pre- and post-listing data points are available using the ten month criteria above for both sub-periods. The remaining sample contains 99 stocks. The inquiry is closely related to Bessembinder and Seguin (1992), who study the introduction of index futures on the dynamic relation between index volatilities and trading volume. I follow Bessembinder and Seguin when estimating the effect of single stock futures on the option volume – stock volatility relation and allow the option and spot volume slope coefficients to differ across the pre- and post-listing periods in the variance equation**.** Table 2.11 reports the results. The table indicates weak evidence that the listing of single stock futures decreases the association between spot volume and volatility as well as option volume and volatility. The effect is least ambiguous for the spot volume coefficients, which are significantly lower for 46.46% of the stocks after the introduction of the corresponding single stock futures. However, in 13.13% of the cases, the spot volume coefficient increases in the post-listing period. The corresponding results are 29.29% and 7.07% for option volume, respectively. The weak effect is surprising since equity futures provide new opportunities for both informed traders and hedgers. This may imply that during the first two years of its launch, the equity futures market was not fully integrated into the joint market environment and the observed futures transactions mostly reflect the support activities of the futures market makers. However, it is also consistent with the view that the single stock futures market has not been a success for the exchanges in general. In addition, the listing date may coincide with other events, which cause an unrelated regime switch for the securities so that tests of this nature have to be interpreted with caution.

Table 2.13. The Effect of Single Stock Futures Listings

The table reports the estimation results from a modified Model (3), which incorporates the listing dates of individual single stock futures for a sample 99 stocks. In particular, I follow Bessembinder and Seguin (1992) and allow the option and spot volume slope coefficients to differ across the pre- and post-listing periods in the variance equation.

	Mean	%Significant	
		+	-
Unexpected Option Volume	0.007	96.97%	
Unexpected Option Volume*$I_{SSF\ listed}$	-0.002	7.07%	29.29%
Unexpected Spot Volume	0.008	100%	
Unexpected Spot Volume*$I_{SSF\ listed}$	-0.001	13.13%	46.46%

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CHAPTER III

LIQUIDITY AND THE RISK-RETURN TRADEOFF

1. Introduction

The aim of this paper is to reexamine the intertemporal relation between risk and return. An ongoing debate in asset pricing focuses on this empirical tradeoff (see, most recently, Pastor et al. (2006)). Traditional theoretical pricing models imply that the market risk premium and the conditional market variance are positively correlated; however, empirical studies fail to establish the hypothesized results. Since the market portfolio occupies a central role in finance theory, with this lack of empirical support, the puzzle remains an important research question.

Early empirical findings suggest positive and negative risk-return relations with equal frequency.[29] More recently, many studies argue that the empirical inconsistency may arise from the mismeasurement of the conditional mean (Pastor et al. (2006) and Brandt and Kang (2004)), or the conditional variance (Harvey (2001), Brandt and Kang (2004), Bali and Peng (2004), and Ghysels, Santa-Clara, and Valkanov (2005)). Yet others believe that single-factor tests are misspecified, and use two-factor models to account for investors' hedging demands due to changes in the investment opportunity set. Hence; they include an additional, 'omitted' variable in the specification along with the market's variance. The additional variable represents the covariance of the state variable with the market return (see, for instance, Scruggs (1998) and Guo and Whitelaw (2005)).

The contribution of this paper is to consider the role of liquidity. If investors' true pricing model incorporates liquidity risk, tests of the risk–return tradeoff must also account for it.

[29] For instance, see Campbell and Hentschel (1992) and French, Schwert, and Stambaugh (1987) for evidence on a positive, Pagan and Hong (1991), Breen, Glosten, and Jagannathan (1989), Glosten, Jagannathan, and Runkle (1993), and Nelson (1991) for negative, and Chan, Karolyi, and Stulz (1992) and Baillie and DeGennaro (1990) for an insignificant risk-return tradeoff.

The proposed empirical tests are not a mere change of the state variable in the ICAPM sense. Hence; they differ from empirical models derived from two-factor models, such as Scruggs (1998). This is because liquidity risk in my tests originates from the liquidity adjusted CAPM of Acharya and Pedersen (2005). The liquidity adjusted CAPM divides systematic risk into market risk (conditional variance) and liquidity risk components. Interestingly, in the liquidity risk component, the volatility of liquidity matters as well, not just its covariance with the market return. This is different from Merton's (1973) ICAPM equilibrium equation, where the sole influence of the state variable is through its covariance with the market return. Furthermore, the choice of the underlying theoretical model (and the subsequent empirical specification) is not arbitrary. Due to the strong, overlapping relation between liquidity and transaction costs, the traditional solution to the ICAPM does not exist with liquidity as a state variable, and thus, it cannot be utilized as the underlying model of the empirical tests.[30] Although my tests are solely driven by this consideration, an additional advantage when compared to the ICAPM-based two factor risk-return approach is that the two factor models have frequently been criticized for the ad hoc choice of the state variables.

The study of liquidity in asset pricing is relatively new. Results on commonality in liquidity (Chordia et al. (2001), Huberman and Halka (2001), and Hasbrouck and Seppi (2001)) or the existence of a systematic liquidity factor (Pastor and Stambaugh (2003) and Sadka (2003)) provide support for the role of liquidity in pricing. Moreover, Acharya and Pedersen (2005)'s liquidity adjusted CAPM implies a role for liquidity in the risk-return tradeoff. Unlike in the empirical part of Acharya and Pedersen, but as in other studies of the behavior of risk and return, the aim here is to explore the time-series connection between expected returns and market, as well as liquidity risk. Thus, the paper is silent on whether the risk is priced cross-sectionally, although undoubtedly the issues are closely related.

Two main specifications are employed in the study. First, I examine the relation between

[30] I'd like to thank Anthony Lynch for an insightful discussion on the issue.

daily net-of-illiquidity-costs excess market returns and the total systematic risk, approximated by the variance of net-of-illiquidity-costs excess returns. Based on arguments identical to those that apply to the traditional CAPM, the liquidity adjusted CAPM implies a positive relation between expected net returns and conditional net return variance in the time-series context. I use the generalized autoregressive conditional heteroskedasticity (GARCH) framework with daily observations between January 2, 1973 and August 31, 2004 in the empirical tests, and confirm the positive relation.

The second specification explores the individual components of systematic risk. I decompose total systematic risk (or net-of-illiquidity-costs return variance) into the variance of gross returns, the covariance between gross returns and illiquidity costs, and the variance of illiquidity costs. I use univariate, as well as multivariate GARCH models to estimate this later specification. Since the univariate model specifies the dynamics of excess return (but not that of liquidity), it only allows for estimating the conditional return variance within the system, but not the liquidity risk components. Therefore, I estimate the liquidity risk components (i.e., the volatility of liquidity and the covariance between return and liquidity) prior to running the GARCH filters. I use daily data and calculate these quantities at the monthly frequency. As a result, the univariate GARCH tests are based on monthly data from January, 1973 to August, 2004. Since conditional asset pricing results pertain to *ex ante* variances and covariances, I use lagged liquidity variance and covariance measures, as has been suggested in the literature.

In the multivariate system, I model the dynamics of excess return and liquidity jointly, and as a result, ex ante values of their variances and the covariance are estimated within the system, and are passed through as latent. Since I do not have to calculate the variances and the covariance prior to estimating the GARCH model, multivariate tests are based on daily data. In both the univariate and multivariate tests, results confirm ties between liquidity risk and expected returns in the intertemporal setting. Excess market return is positively related to conditional liquidity volatility, and negatively related to the covariance between returns and liquidity.

Furthermore, in all cases, the positive relation between conditional return variance and excess return is restored once liquidity is included in the tests.

To confirm the main results of the paper, I employ the following robustness checks. First, I examine the sensitivity of the tests to changes in the calculation of liquidity, outliers in return, and adjustments for the investors' investment horizon. In addition, I use two different methods for calculating the monthly liquidity risk components for the univariate GARCH tests. I also examine whether the results differ across the different tick size regimes.

Finally, I investigate the possibility that liquidity risk proxies for idiosyncratic risk. Idiosyncratic volatility has been in the center of research recently with emphases on explaining observed time-series patterns (Campbell et al. (2001)) and determining whether idiosyncratic volatility matters for asset prices. The questions whether and to what extent idiosyncratic volatility and liquidity interact with each other have also been raised in recent studies. For instance, a new finding suggests that idiosyncratic volatility may dominate liquidity risk in cross-sectional tests (Spiegel and Wang (2005)). A theoretical link between liquidity and idiosyncratic volatility, through the market maker's inventory, has been suggested in Ho and Stoll (1980). Another possible connection results if illiquidity causes sub-optimal portfolio diversification, leaving the investors exposed to idiosyncratic risk. I use residuals from the Fama-French model with momentum to calculate the average idiosyncratic risk in the market.

The plan of the paper is as follows. Section 2 reviews the relevant literature. Section 3 describes the modeling framework. Section 4 introduces the data and calls attention to some of the empirical difficulties and restrictions. Section 5 and Section 6 provide the main results of the risk-return tests and the robustness analyses, respectively. Section 7 concludes.

2. Review of the literature

2.1 Risk-return tradeoff

Relevant theoretical models begin with Sharpe (1964) and Litner (1965). These papers directly relate the change in the price of the asset to its own variance or to the covariance between its return and the return on the market portfolio. Hence; for the market portfolio, they imply a simple positive linear relation between expected excess returns and the conditional variance. Merton's (1973) ICAPM implies a partial positive relation, since in this model, in addition to its conditional variance, the conditional excess return is also a linear function of its covariance with the investment opportunity set. Merton (1980) argues that under certain conditions, the covariance term (the hedge component) is negligible.

Harvey (1989) tests a version of the CAPM that allows for time-varying expected returns and time-varying conditional variances, and finds a positive contemporaneous relation between risk and return. A positive tradeoff is also supported in Campbell and Hentschel (1992) and French, Schwert, and Stambaugh (1987). Baillie and DeGennaro (1990) estimate a GARCH-M model and find that the risk-return relation is not significant.

Glosten, Jagannathan, and Runkle (1993) use a GARCH-M model that allows for seasonal patterns in volatility, differences in the impact of positive and negative innovations to returns, and allows nominal interest rates to predict the conditional variance. The paper reports a negative relation, but argues that the result does not invalidate the static theoretical models. This is because the cross-sectional prediction that risk adjusted return is equalized across securities does not necessarily imply an intertemporal relation. The risk-return tradeoff may not be positive if, for instance, the compensation investors require for risk is different in different time periods,[31]

[31] The GJR argument is that the risk adjusted returns can be equal across assets in each period, yet the market variance in period 1 is larger than in period 2 while the required return is smaller in period 1 than in period 2. Thus, investors require less compensation in period 1 to bare market risk. However, one problem with this argument is that this implies that 'everything stays the same' does not hold. That is, it must be

or if investors save more when the future is more uncertain. Pagan and Hong (1991), Breen, Glosten, and Jagannathan (1989), and Nelson (1991) also arrive at a negative coefficient.

Scruggs (1998) believes that empirical tests based on the conditional single-factor model may suffer from the omitted variable bias, if the covariance term in the ICAPM is not negligible. He suggests that the omitted variable bias could explain why empirical studies provide inconsistent results for the intertemporal relation. Scruggs uses long-term government bond returns as the second factor (or state variable) to proxy for the investment opportunity set. The inclusion of this second factor restores the positive (and significant) relation between the conditional market variance and the market risk premium. Guo (2003) argues however, that long-term government bond returns is an ad hoc state variable. In addition, Scruggs and Glabadanidis (2000) find that the original results are sensitive to the specification.

Goyal and Santa-Clara (2003) revisit the risk-return tradeoff by examining the role of idiosyncratic risk. This is implied by, for instance, Levy (1978), Malkiel and Xu (2001), and Huberman (2001), who suggest that investors may hold undiversified portfolios. Goyal and Santa-Clara find that idiosyncratic, rather than systematic risk matters, and uncover a positive, significant relation between the market's excess return and average stock variance. However, Wei and Zhang (2005) argue that the results are driven by the market's behavior in the 1990s.

Many studies argue that the inconsistency in this literature arises from the latent feature of the expected return and the market variance. Pastor et al. (2006) suggest using implied cost of capital for the conditional expected return. Harvey (2001) and Ghysels, Santa-Clara, and Valkanov (2005) concentrate on measuring the conditional variance. Bali and Peng (2004) use intraday data for estimating daily variances. Brandt and Kang (2004) propose a latent VAR, in which both the return and the variance series are left unobserved.

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that the state of the world between the two periods is different. Therefore a simple test of the relation between the market risk premium and the market variance is not correct due to an omitted variable problem since the other risk sources or state variables are not accounted for under this setting.

This paper reexamines the intertemporal relation between risk and return by incorporating the idea of liquidity risk. It provides a connection to single factor models, since the only 'state variable' is the net-of-illiquidity-cost return variance. Although the multifactor structure is not adopted, Scruggs's (1998) argument on the omitted variable bias remains relevant in tests where the total systematic risk is decomposed into market risk and liquidity risk. In these instances, the relation between gross excess return and the conditional return variance is a partial relation. I also address the latent feature of the conditional variance and covariance measures.

2.2 Liquidity premium and liquidity risk premium

When the cost of transacting is different across securities, the expected returns are different as well. This is a simple consequence of the discounted present value argument.[32] Therefore, a liquidity premium exists for illiquid assets even if liquidity has no impact on the risk of holding the assets (i.e., has no liquidity *risk* premium). Constantinides (1986) shows that transaction costs have a second order effect on investor utility. The result reflects the intuition that investors will trade less frequently if transaction costs are introduced. Other studies, such as Heaton and Lucas (1996), Aiyagari and Gertler (1991), Vayanos (1998) and Vayanos and Vila (1999) also conclude that investors should reduce their trading in the illiquid assets and require a small liquidity premium.[33]

While the transaction cost argument predicts that investors only require a small liquidity premium, empirical studies find significant differences in expected return across portfolios with

[32] Illiquid assets generate higher pre-transaction cost expected returns because the price of the illiquid asset is equivalent to the price of the liquid asset adjusted for the present discounted value of the transaction costs.

[33] Another possible attack on the importance of liquidity in asset pricing may come from the argument that investors holding liquid and illiquid assets simultaneously could avoid liquidating illiquid securities during a wealth shock. The issue is closely related to the frequency of trading in the illiquid asset mentioned above. However, this strategy seems viable only with small and short-lived wealth shocks. In addition, changes in the relative prices of the assets in the investor's portfolio may call for a specific portfolio rebalancing scheme that is different from the liquid-first illiquid-last method. Similarly, liquidating illiquid securities earlier to maintain a buffer of cash or liquid assets may not result in an optimal rebalancing rule. In addition, Duffie and Ziegler (2001) argue that while the strategy appears to be effective, it will fail when asset returns and bid-ask spreads are fat-tailed.

differing liquidity. For instance, Brennan and Subrahmanyam (1996) report a 6.6% annual spread in excess returns across the low-lambda (low price impact) and the high-lambda portfolios. In an early study connecting asset pricing and market microstructure, Amihud and Mendelson (1986) find that required returns are positively and concavely related to relative spreads. In addition, they identify a clientele effect which states that in equilibrium assets with larger spreads are held by investors with longer investment horizons. Chordia, Roll, and Subrahmanyam (2001), Huberman and Halka (2001) and Hasbrouck and Seppi (2001) find commonalities in liquidity. The presence of commonalities offers a natural connection to asset pricing. Pastor and Stambaugh (2003) explore whether market-wide liquidity is a state variable in pricing financial assets, and report that stocks whose return sensitivity to signed volume is higher have significantly higher expected returns.

To eliminate the inconsistency between theory and empirical findings, Lynch and Tan (2005) investigate the role of immediacy costs under assumptions of return predictability, wealth shocks, and stochastic transaction costs. Simulations show that these modeling assumptions can generate a liquidity risk premium which is significantly higher than the liquidity premium implied by the transaction cost literature. Huang (2003) assumes that investors face surprise liquidity shocks and they are constrained from borrowing against future income. As a result, the required liquidity risk premium is dependent on the expected length of the investment horizon as well as on the random nature of the horizon (uncertainty).

Stochastic liquidity is not necessary nor is it sufficient for liquidity risk premium to exist. For instance, liquidity risk premium can arise even when illiquidity costs are constant. Vayanos (2003) proposes an equilibrium model with stochastic volatility, assets with differing liquidity, and constant transaction costs. The model generates a flight to liquidity phenomenon and, consequently, the liquidity risk premium varies over time. Under turbulent market conditions, since investors prefer liquid securities, the price of liquid assets is bid up while the required return on illiquid assets that would induce fund managers to hold the security increases significantly.

Market microstructure studies weigh in on the role of liquidity in asset pricing as well. Stoll (2000) mentions that the order processing and inventory components of immediacy costs must be priced as they represent real economic costs: real resources are used to provide the market making services. However, the question whether the adverse selection component influences an asset's price is less clear since this only represents a wealth redistribution, thus no real costs are incurred. Similarly, O'Hara (2003) acknowledges that liquidity should affect asset prices since it is a tax-like cost borne by investors. However, whether liquidity can affect the risk of holding an asset is not obvious. O'Hara argues that for liquidity to affect the risk of holding an asset, that is, to induce a liquidity risk premium it would "have to be time varying, or at least be systematic in some sense (p. 1339)." O'Hara believes that the price discovery process may affect the risk of holding an asset. The question becomes whether the uninformed can diversify away the risk of losing against an informed trader.

In order to formally incorporate liquidity in asset pricing models, Jacoby et al. (2000) and Acharya and Pedersen (2005) derive liquidity adjusted CAPM's. The liquidity adjusted CAPM implies a testing framework for the risk-return tradeoff with liquidity risk. In this paper, I rely on the Acharya and Pedersen model to design the empirical tests. The Acharya and Pedersen model is described in more detail in the next section.

3. Modeling framework

The paper explores the role of liquidity risk in the intertemporal risk-return context, motivated by the Acharya and Pedersen (2005) model. In this section, I describe the Acharya and Pedersen model in detail and highlight the main results. The empirical design in this paper is based on these main results and is discussed in Section 2.3.

3.1 Theoretical predictions

Acharya and Pedersen (AP) (2005) use the assumptions of the single period CAPM to derive the liquidity based pricing model. Liquidity risk in their paper is generated by uncertainty about the illiquidity cost. Under this framework, illiquidity cost (C_t^i), defined as the per share cost of *selling* the security, is an exogenous stochastic process, described by a first order autoregressive model. P_t^i is the price process of the ith security and c_t^i represents the relative illiquidity cost:

$$c_t^i = \frac{C_t^i}{P_{t-1}^i}. \tag{1}$$

The AP model derives the following CAPM result for the net-of-illiquidity-cost returns:

$$E_{t-1}(r_t^i - r^f - c_t^i) = E_{t-1}(r_t^M - r^f - c_t^M)\frac{\mathrm{cov}_{t-1}(r_t^i - c_t^i, r_t^M - c_t^M)}{\mathrm{var}_{t-1}(r_t^M - c_t^M)} \tag{2'}$$

or

$$E_{t-1}(r_t^i - r^f - c_t^i) = l_{t-1}\mathrm{cov}_{t-1}(r_t^i - c_t^i, r_t^M - c_t^M) \tag{2}$$

where r_t^i (r_t^M) and c_t^i (c_t^M) are the return and relative illiquidity cost of asset i (the market portfolio) at time t respectively, r^f represents the risk-free rate, and l_{t-1} describes the market price of risk:

$$l_{t-1} = \frac{E_{t-1}(r_t^M - c_t^M) - r^f}{\mathrm{var}_{t-1}(r_t^M - c_t^M)}. \tag{3}$$

In addition to the standard CAPM prediction for *net* (of illiquidity costs) returns, the AP model is represented by the following equation in terms of *gross* excess returns:

$$E_{t-1}(r_t^i - r^f) = E_{t-1}(c_t^i) + l_{t-1}\mathrm{cov}_{t-1}(r_t^i, r_t^M) + l_{t-1}\mathrm{cov}_{t-1}(c_t^i, c_t^M) - l_{t-1}\mathrm{cov}_{t-1}(r_t^i, c_t^M) - l_{t-1}\mathrm{cov}_{t-1}(c_t^i, r_t^M) \tag{4}$$

This is the conditional liquidity adjusted CAPM. Three important results arise from (4), which could be viewed as forms of liquidity risk. First, investors demand higher returns on assets that are illiquid when market liquidity is low. Second, investors are willing to pay more (accept lower returns) on assets that are liquid when the market is down. Third, investors' demand is greater for securities that have higher returns when the market is illiquid. The study also finds support for persistence in illiquidity. Thus, unexpected shocks to illiquidity raise the expected future illiquidity and thus, raise expected returns by lowering contemporaneous prices (and returns).

For the market portfolio, liqudity risk collapses into the variance of market illiquidity and the covariance of illiquidity and returns. The model maintains the CAPM framework in that investors' marginal utility is only dependent on wealth, but here, wealth is determined by the net of transaction cost returns on the wealth portfolio, that is, *net* market returns play the role of the single state variable. Thus, the crucial argument of the paper is that true systematic risk comes from net returns rather than from gross returns.

Similarly to the traditional CAPM, the liquidity adjusted model implies a positive relationship between net expected returns and the variance of net returns intertemporally. When the net return variance is decomposed into a traditional market risk component and liquidity risk, as in the four beta representation above, the intertemporal relation between the conditional excess return and the individual components is a partial relation. Empirical tests of the resulting risk-return tradeoff are discussed in Section 2.3 below.

3.2 ICAPM and the role of liquidity in asset pricing

One might think of the two-factor model of Merton (1973) as an alternative theoretical framework for incorporating liquidity in asset pricing tests. In the CAPM, investment opportunities can change over time but the investors' utility is fully determined by wealth and the derived utility of wealth is state independent. The two-factor model introduces state dependence in the marginal utility of wealth. Many studies adopt the ICAPM equilibrium condition[34] with numerous alternative candidates for the state variable. Recent papers often label liquidity as a state variable (for instance, Pastor and Stambaugh (2003)).

One difficulty in using liquidity as a state variable in the ICAPM is that the derivation of the ICAPM does not account for the possibility of trading costs. Nor does it incorporate information on the investor's trading horizon or which assets in the portfolio are traded in order to meet consumption needs and, at the same time, optimize between liquidation strategies by considering the liquidity characteristics of the alternative assets at a given point in time. In other words, the ICAPM applies for arbitrary state variables with the assumption of no frictions. Unfortunately, when liquidity is chosen as a state variable, it explicitly introduces friction in the modeling framework since the notion of illiquidity is inconsistent with a frictionless world. As a result, the equilibrium equation cannot be considered valid in this setting. Because of this conceptual difficulty and the lack of theoretical support in the form of multi-period liquidity adjusted asset pricing, the ICAPM framework is not adopted in this study.

In comparison, Acharya and Pedersen (2005) explicitly model the effect of stochastic trading costs on asset prices. The next subsection introduces the empirical models based on the AP framework.

3.3 Empirical specifications

In this paper, the empirical tests of the risk-return tradeoff with liquidity rely on equations (2) and (4) above. Since a large literature argues that the conditional second moment of

[34]The equilibrium equation of the conditional Intertemporal Capital Asset Pricing Model (ICAPM) is given by the following:

$$E_{t-1}(r_{M,t}) = (\frac{-J_{WW}W}{J_W})s^2_{M,t} + (\frac{-J_{WX}}{J_W})s_{MX,t},$$

where J(W(t), X(t), t) is the derived utility of wealth function, W(t) is wealth, and X(t) describes the state. It implies an intertemporal relation between the market risk premium ($E_{t-1}(r_{M,t})$) and the conditional variance ($s^2_{M,t}$) as well as the conditional covariance between the returns and the state variable that drives changes in the investment opportunity set ($s_{MX,t}$).

asset returns follows a GARCH-type process,[35] I estimate the empirical counterparts of (2) and (4) as GARCH models. Along with a univariate model, which specifies the dynamics of return volatility, I also use a multivariate model. The multivariate model allows me to specify the dynamics of each conditional second moment, that is, the dynamics of liquidity volatility and the covariance between return and liquidity as well. This is motivated by Watanabe (2006), who argues that liquidity also follows a GARCH-type process.

For simplicity, in this section, the empirical models are specified as GARCH(1,1). In the empirical analyses, lag-length is determined by the Akaike information criterion, and the tests also accommodate asymmetries in volatility response to positive and negative shocks. These are discussed in Section 4 below.

3.3.1 Univariate GARCH models

Applying equation (2) above to the market portfolio, I use the following univariate system for net-of-illiquidity-cost market returns.

$$\begin{aligned} r_{M,t}^{N} &= b_0 + b_1 h_t^{N} + u_t \\ u_t \mid \Omega_{t-1} &\sim N(0, h_t^{N}) \\ h_t^{N} &= a + b_1 u_{t-1}^2 + b_2 h_{t-1}^{N} \end{aligned} \tag{5}$$

where $r_{M,t}^{N}$ is net market return (i.e., the market return minus the risk free rate and the illiquidity cost) and h_t^N represents its conditional variance.

For gross returns, as in equation (4) above, the conditional net second moment is disaggregated into three components:

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[35] Bollerslev, Chou, and Kroner (1992) provide a survey of the role of ARCH/GARCH-type models in finance.

$$r_{M,t} = b_0 + b_1 h_t + b_2 s^2_{L,t} + b_3 s_{ML,t} + b_4 c_{M,t} + u_t$$
$$u_t \mid \Omega_{t-1} \sim N(0, h_t) \qquad (6)$$
$$h_t = a + b_1 u^2_{t-1} + b_2 h_{t-1}$$

where $r_{M,t}$ is the excess return on the market, h_t represents the conditional market variance, $s^2_{L,t}$ is the conditional market liquidity variance, $s_{ML,t}$ is the conditional covariance between market return and liquidity, and $c_{M,t}$ is the expected liquidity cost (this was subtracted from the returns in (5)).

The risk-return tradeoff refers to the relation between expected returns and *ex ante* measures of risk (such as ex ante variance). In (5), the GARCH framework provides an ex ante measure of the net market return variance. In (6), market volatility is the only risk measure that passes through the GARCH framework as latent. The conditional variance of liquidity and the covariance between liquidity and return have to be estimated from the sample prior to running the GARCH model. To estimate these quantities, I use daily observations to arrive at the monthly frequency. Monthly frequency is chosen in order to increase the signal to noise ratio. First, I estimate the realized values of these risk measures based on the following formula first proposed by French et al. (1987) and shown for the return series:

$$s^2_{MT} = \sum_{t=1}^{T} r^2_{Mt} + 2\sum_{t=2}^{T} r_{Mt} r_{Mt-1} \qquad (7)$$

where $r_{M,t}$ represents the market return on day *t* in month *T*. I use non-overlapping daily values to calculate the monthly quantities. Equation (7) captures the autocorrelation induced by non-syncronous trading through the second term in the equation. The variance of liquidity and the covariance term are calculated similarly, except the formula is adjusted for the non-zero mean.

As a robustness check, I also calculate these variance and covariance values using the built-in variance and covariance functions in SAS. Realized variances and covariances include both an ex ante and unexpected components. To proxy for the ex ante values, I follow Goyal and Santa-Clara (2003), and use lagged values.

3.3.2 The multivariate GARCH model

The multivariate GARCH framework provides an alternative method for estimating ex ante risk measures. It circumvents the initial estimation stage by passing all variance and covariance terms through as latent. The multivariate model is specified as follows.

$$\begin{pmatrix} r_{M,t} \\ c_{M,t} \end{pmatrix} = \underline{a} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} s^{2}_{M,t} \\ s_{Mc,t} \\ s^{2}_{c,t} \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} x_1 + \begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix}$$

$$\begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix} | \Omega_{t-1} \sim N(\underline{0}, H_t = \begin{pmatrix} s^{2}_{M,t} & s_{Mc,t} \\ s_{Mc,t} & s^{2}_{c,t} \end{pmatrix}) \tag{8}$$

$$\begin{pmatrix} s^{2}_{M,t} \\ s_{Mc,t} \\ s^{2}_{c,t} \end{pmatrix} = C'C + B' \begin{pmatrix} e^{2}_{1,t-1} \\ e_{1,t-1}e_{2,t-1} \\ e^{2}_{2,t-1} \end{pmatrix} B + D' \begin{pmatrix} s^{2}_{M,t-1} \\ s_{Mc,t-1} \\ s^{2}_{c,t-1} \end{pmatrix} D$$

where $r_{M,t}$ is the excess return on the market, $c_{M,t}$ is a given measure of market liquidity, $s^{2}_{M,t}$ and $s^{2}_{c,t}$ represent the conditional market return variance and liquidity variance respectively, and $s_{Mc,t}$ stands for the conditional market return and liquidity covariance. The conditioning information set at time *t-1* is denoted by Ω_{t-1}. The model is estimated based on the restricted

BEKK[36] of Engle and Kroner (1995), with diagonal coefficient matrices B and D, and a lower triangular C coefficient matrix. The first equation of (8) describes the mean equation, that is, the joint dynamics of return and liquidity. The second equation characterizes the distribution of the errors. Finally, the third equation represents the intertemporal behavior of the variance-covariance matrix (vech representation). By construction, the positivity of H_t is automatically guaranteed if H_0 ³ 0 in the restricted BEKK model.

Unlike (5) and (6), (8) captures the dynamics of liquidity as well. Acharya and Pedersen (2005) advocate the idea of persistence in liquidity: illiquidity costs in their model follow an AR(1) process. Thus, x_1 can be set to $c_{M,t-1}$. Watanabe (2006) proposes a theoretical model cast in a multi-security trading framework in a Kyle (1985)-type centralized market with time-varying and conditionally heteroskedastic liquidity. Watanabe conjectures that liquidity follows a GARCH-type process based on the empirical characteristics he observes. Time-variation in the second moment of liquidity is also supported by the descriptive section of the paper discussed below. In addition, the literature offers some insight into the relationship between volatility and liquidity.

4. Measures of liquidity and data sources

4.1 Liquidity measures

I use Amihud's (2002) ILLIQ as the measure of liquidity in this paper, as in Acharya and Pedersen (2005). ILLIQ is often used in asset pricing tests because it requires only return and volume data, therefore; it allows for a large sample size. ILLIQ is first calculated for each individual security in the market portfolio, and is given by the following:

[36] An early version of Engle and Kroner (1995) was written by Baba, Engle, Kraft and Kroner. This explains the model's acronym (BEKK).

$$ILLIQ_t^i = \frac{1}{days_t^i} \sum_{d=1}^{days_t^i} \frac{| r_{td}^i |}{V_{td}^i} \tag{9}$$

where the superscript i indexes the ith security, t represents month t, d is day of the month, r_{td}^i and V_{td}^i are the return and the dollar volume (in millions) of security i on day d in month t, respectively. For tests based on daily observations, I simply calculate the ratio of absolute return and dollar volume for each security. To arrive at the value- and equal-weighted market liquidity figures, I take value- and equal-weighted averages of the individual stock measures, respectively. I report results for both value- and equal-weighting for the market portfolio. However, it is often argued that equal-weighting is more meaningful for liquidity studies. This is because in a pure stock-based market portfolio proxy, some of the most illiquid assets of the true market portfolio, such as private equity, real estate, etc., are ignored. Equal-weighting provides some compensation for this (see, for instance, Chordia et al. (2000) and Acharya and Pedersen (2005)).

As discussed in AP, ILLIQ classifies a given security as illiquid, if its price moves significantly in response to small volume. Moreover, ILLIQ is shown to be positively related to microstructure data based liquidity measures such as various spread measures or Kyle's (1985) lambda (see, for instance, Amihud (2002)). Since ILLIQ is not in the desired units of measurement, as it is expressed in "percent per dollar spent" (theoretical results in Acharya and Pedersen are based on "dollar cost per dollar spent"), AP suggest normalizing ILLIQ to address inflation (scaling by the ratio of the market capitalization of the market portfolio at the end of month *t-1* and at the end of the first month of the sample period), and to transform the scaled ILLIQ so that its mean and standard deviation is approximately the same as those of the effective half spread reported in Chalmers and Kadlec (1998). AP also caps the illiquidity measure at 30%. I follow their suggested normalization. In addition, I also calculate simple winsorized values of

ILLIQ from (9) above. In the tables below, the AP adjusted and the winsorized liquidity measures are represented by 'ILLIQ(AP)' and 'ILLIQ(w),' respectively. Unfortunately, the liquidity adjusted CAPM is not general enough to accommodate the various other dimensions of liquidity (such as depth).[37]

4.2 Data sources

One important shortcoming in studying liquidity in an asset pricing context is the tradeoff between using a good proxy for the market portfolio and recognizing that the microstructure of the NYSE and Nasdaq are significantly different. In this paper, I limit my sample to NYSE stocks. This eliminates all concerns about microstructure differences, but increases the likelihood that my market proxy is not close to the true market portfolio.[38] This approach is commonly adopted in the literature however. For instance, Glosten, Jagannathan, and Runkle (1993) also work with NYSE stocks to test the risk-return tradeoff for the market.

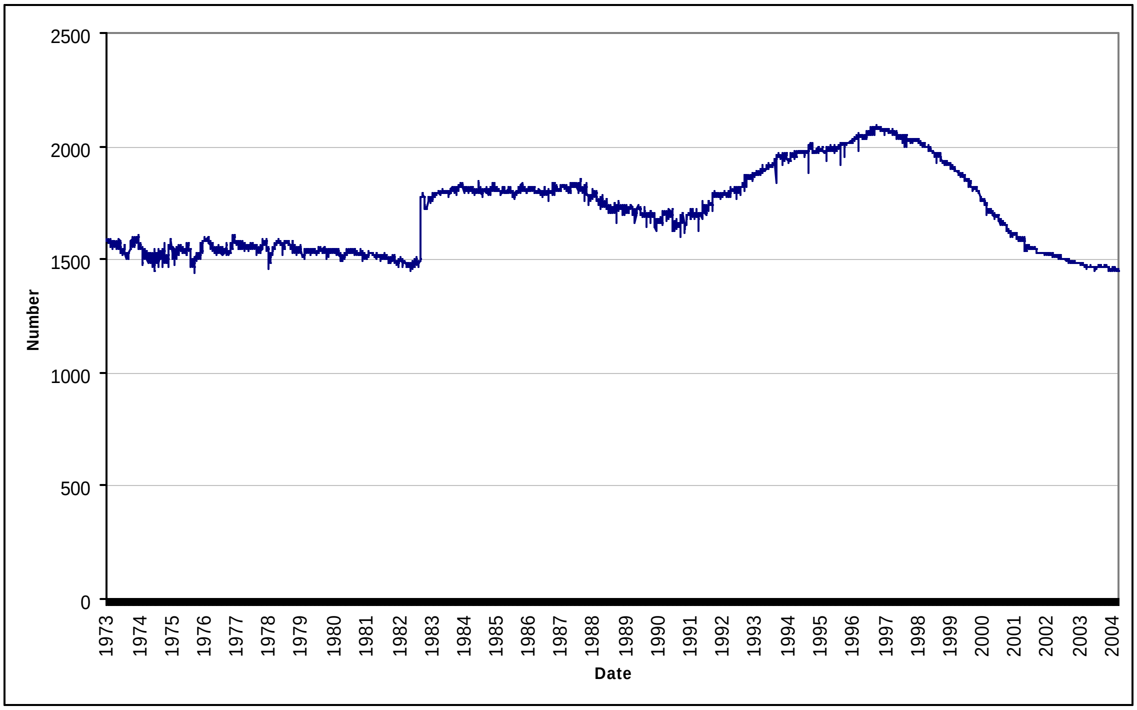
To construct ILLIQ, I use daily data on all NYSE stocks from the Center for Research in Security Prices (CRSP) from January 1, 1973 to August 31, 2004. The sample period provides 7995 daily and 380 monthly observations. I eliminate ADR's, units, etc.: only observations with share codes 10 and 11 are retained. I further require that the stock price is below \$999 and that it is higher than \$3. Finally, only stocks for which at least 15 observations are available in a given month are included in the market portfolio in that month. Figure 3.1 graphs the number of stocks in the market portfolio over time. To calculate access returns, I use the risk-free interest rate reported with the Fama-French factors in Wharton Research and Data Services (WRDS). This is

[37] Though market liquidity is not directly observable, measurements along its various dimensions are readily available. Liquidity is generally viewed as the ability to trade large numbers of shares in a short period of time with minimal price impact. Kyle (1985) identifies the following characteristics of liquidity: 1) tightness (reflected in the bid-ask spread); 2) depth; and 3) resiliency (the speed of return to equilibrium). In the literature, several proxies are constructed based on the price, time, and size/quantity dimensions.

[38] Return on the NYSE portfolio in my sample is highly correlated with the CRSP market return (NYSE/AMEX/Nasdaq). The correlation is 0.932 for the equally weighted, and 0.979 for the value weighted portfolios for 1973-2004 at the daily frequency.

the one month Treasury Bill rate.

Figure 3.1. Number of firms in the market portfolio



The figure shows the daily number of sample firms used in calculating the returns of the market portfolio from January 1, 1973 to August 31, 2004. The sample is restricted to NYSE stocks with share codes of 10 or 11, and requires at least 15 daily observations for a firm to be included in a given month. Stocks with a share price of less than $3 and $1000 or above are also excluded.

Two major reductions in the minimum tick size occurred recently in the markets. As a first step towards decimalization, NYSE switched to quoting stock prices in sixteenths of a dollar on June 24, 1997. The full decimalization was implemented in various steps on the NYSE between August 28, 2000 and January 29, 2001. Several studies have discussed the effect of tick size reductions on liquidity measures and other microstructure variables. For instance, Goldstein and Kavajecz (1998) find that spreads and depths decrease as a result of smaller tick sizes. Depths decrease as well since smaller tick sizes may discourage market makers and investors from providing liquidity. For instance, limit orders become easier to be “picked up” by informed traders. Furfine (2003) reports a similar finding after the decimalization. In addition, he argues that the price impact of trades has declined. Smaller spreads along with smaller depths indicate an ambiguous change in liquidity as a result of decimalization. Changes in the minimum requirements may introduce regimes in the time series of the liquidity variables utilized in this paper.

4.3 Investment horizon

A crucial issue in this study is the relation between the arbitrarily chosen holding period and investors' true holding horizon. For instance, net returns, the source of systematic risk in the liquidity adjusted CAPM, depend on this relation. When a monthly buy-and-hold strategy is assumed, it is straightforward to calculate the gross returns from the daily return series. However, most investors will not incur daily illiquidity costs, in fact, they may not trade in the given month at all. As a result, the liquidity adjustment must depend on the frequency at which they trade, thus their true holding horizon.

Acharya and Pedersen (2005) arrive at net returns by subtracting the product of illiquidity costs and turnover in the chosen holding period from the gross returns (for a sell transaction). To illustrate, if daily data are available for returns and illiquidity cost, and the assumed holding period is a month, gross returns are the product of daily returns (based on simple returns) within the month, turnover is the total volume in the month divided by the average total number of shares outstanding, and the illiquidity cost is calculated as a daily average (not aggregated for the month). The adjustment is based on the idea of the amortized spread proposed by Chalmers and Kadlec (1998). The amortized spread measures the cost of the spread over the investors' holding horizon, and takes the following form:

$$AS_T = \frac{\sum_{t=1}^{T} | P_t - M_t | \cdot V_t}{P_T SO_T}, \qquad (10)$$

where P_t and M_t are the transaction price and the price midpoint at time t respectively, V_t represents the volume, and SO_T measures the number of shares outstanding.

Unfortunately, the turnover adjusted ILLIQ only accounts for the average realized holding horizon of all investors. Thus it may not provide an accurate representation of the marginal investor. Nor does it capture some of the important implications of the stochastic horizon characterizing real-world portfolio holdings. However, despite the weaknesses, it represents the most sensible way of adjusting for the holding horizon in the analyses.

5. Results

5.1 Descriptive results

I calculate daily and monthly value- and equal-weighted return and liquidity series for the sample. Market return is expressed as a percentage, and is scaled by a 100. Descriptive statistics of the equal- (EWRET) and value-weighted return (VWRET) series are reported in Table 3.1. For comparison, the table also provides information on the CRSP value- and equal-weighted market indices (NYSE/AMEX/Nasdaq) for the sample period. In Panel B of the table, correlation coefficients based on daily observations reveal that the equal- and value-weighted return series from the NYSE sample used in this study are highly correlated with the corresponding CRSP market returns. The table reports descriptive statistics for four subperiods based on decades. For the entire sample period (1973-2004), the value-weighted returns display a correlation coefficient of 0.979, while the correlation equals 0.932 for the equal-weighted observations. This illustrates that my sample may be as good a proxy for the market portfolio as the corresponding CRSP indices. The table also provides information on total trading volume and shares outstanding.

Table 3.1. Descriptive statistics of the equal- and value-weighted return series

EWRET and VWRET denote the equal- and value-weighted market returns respectively. For comparison, the table also provides information on the CRSP equal- and value-weighted return series (CRSP-EW and CRSP-VW) for the sample period. Panel B describes correlation coefficients between the equal- and value-weighted return indices and the market return of CRSP.

Panel A	**Mean**	**Std.Dev**	**Minimum**	**Maximum**
1970's (1973-1979)				
EWRET	0.0610	0.0221	-4.4330	5.2584
VWRET	0.0442	0.0214	-3.5441	4.5965
CRSP-EW	0.0686	0.0167	-3.9940	3.7960
CRSP-VW	0.0198	0.0206	-3.4980	4.2640
Total Vol	22802944	229115	6973000	86454800
Total MktCap (in thousands)	22182715	66433	18150839	27968784
1980's (1980-1989)				
EWRET	0.0846	0.0174	-14.5039	9.4717
VWRET	0.0970	0.0198	-17.4886	9.0501
CRSP-EW	0.0674	0.0140	-10.3910	6.9310
CRSP-VW	0.0646	0.0190	-17.1350	8.6620
Total Vol	129980377	1350700	18001400	661990637
Total MktCap (in thousands)	49096594	275319	28010505	72837856
1990's (1990-1999)				
EWRET	0.0710	0.0135	-5.4972	3.2796
VWRET	0.0938	0.0162	-6.3988	4.8918
CRSP-EW	0.1275	0.0119	-5.4320	2.8000
CRSP-VW	0.0671	0.0163	-6.5950	4.8330
Total Vol	421534009	4653288	68579979	1433590115
Total MktCap (in thousands)	130152329	907696	72361338	233141633
after 2000 (2000-2004)				
EWRET	0.0665	0.0314	-5.0795	5.1046
VWRET	0.0556	0.0340	-5.2365	5.6876
CRSP-EW	0.0820	0.0297	-6.3530	4.8380
CRSP-VW	-0.0069	0.0385	-6.6280	5.3160
Total Vol	1316879209	8081317	360189702	2734195348
Total MktCap (in thousands)	268155388	402286	232725478	290857583

Table 3.1, continued

Panel B	Correlation
1970's (1973-1979)	
EWRET/CRSP-EW	0.9669
VWRET/CRSP-VW	0.9982
1980's (1980-1989)	
EWRET/CRSP-EW	0.9564
VWRET/CRSP-VW	0.9964
1990's (1990-1999)	
EWRET/CRSP-EW	0.9370
VWRET/CRSP-VW	0.9886
after 2000 (2000-2004)	
EWRET/CRSP-EW	0.8839
VWRET/CRSP-VW	0.9427

Table 3.2 shows descriptive statistics for market ILLIQ. As mentioned above, I make two adjustments to ILLIQ in equation (9). First, I use the normalization suggested by Acharya and Pedersen (2005). This adjusted variable is ILLIQ(AP) in the tables. Second, I winsorize the measure, as suggested in Amihud (2002), by setting all values smaller than the 1^{st} percentile or larger then the 99^{th} percentile of the ILLIQ distribution to the 1^{st} percentile and the 99^{th} percentile, respectively. I use ILLIQ(w) to refer to the winsorized measure. In the time series analyses below, the liquidity variables are adjusted by turnover so that, similarly to the amortized spread of Chalmers and Kadlec (1998), they represent appropriate illiquidity costs for the chosen investment horizon. Panel A of Table 3.2 provides descriptive statistics for the full sample, subperiods used in Table 3.1, and subperiods defined by the tick size regimes. I also investigate whether ILLIQ has decreased as a result of tick size changes. The 'z-score' column indicates the outcomes of tests for the equality of means. The table shows that both ILLIQ measures experienced a significant decrease (at the 1% level) in their means when compared to the previous tick size period, with the exception of the equal-weighted ILLIQ(AP) between the first and the second tick size regimes.

Table 3.2. Descriptive statistics for the liquidity measures

Panel A reports descriptive statistics for the scaled (ILLIQ(AP)) and winsorized (ILLIQ(w)) ILLIQ measures for the full sample (January 2, 1973-August 31, 2004), subsamples by decade, and subsamples by tick size regimes. The tick size regimes correspond to the following time periods: 01/01/73-06/04/97, 06/05/97-01/29/01 (sixteens), and 01/30/01-08/31/04 (decimals). Panel B shows the correlation coefficient between the following liquidity measures for the market portfolio: 1) ILLIQ (winsorized (ILLIQ(w)) and scaled (ILLIQ(AP)); 2) quoted relative spread (QRSPR); 3) relative effective spread (ERSPR); and 4) Roll's measure. Since the later three require intraday observations, this table is based on the sample period January 1, 1993 to August 31, 2004. In Panel B, all liquidity measures are based on value-weighting of the individual stock measures.

Panel A

	Mean	Std.Dev	Minimum	Maximum
Full sample (1973-2004)				
ILLIQ(AP) – equal-weighted	0.4615	0.0009	0.2996	0.9427
ILLIQ(w) – equal-weighted	0.3111	0.0046	0.0138	3.8145
ILLIQ(AP) – value-weighted	0.2633	0.0007	0.2522	0.3089
ILLIQ(w) – value-weighted	0.0277	0.0004	0.0006	0.3391
1970's (1973-1979)				
ILLIQ(AP) – equal-weighted	0.4860	0.0025	0.3140	0.9427
ILLIQ(w) – equal-weighted	0.8743	0.0132	0.1785	3.8145
ILLIQ(AP) – value-weighted	0.2702	0.0001	0.2574	0.3089
ILLIQ(w) – value-weighted	0.0820	0.0012	0.0226	0.3391
1980's (1980-1989)				
ILLIQ(AP) – equal-weighted	0.4120	0.0012	0.2969	0.6504
ILLIQ(w) – equal-weighted	0.2253	0.0020	0.0744	0.7231
ILLIQ(AP) – value-weighted	0.2624	0.0006	0.2552	0.2787
ILLIQ(w) – value-weighted	0.0217	0.0002	0.0060	0.0761
1990's (1990-1999)				
ILLIQ(AP) – equal-weighted	0.4913	0.0013	0.3502	0.7769
ILLIQ(w) – equal-weighted	0.1253	0.0016	0.0277	0.5483
ILLIQ(AP) – value-weighted	0.2626	0.0007	0.2558	0.2878
ILLIQ(w) – value-weighted	0.0078	0.0009	0.0015	0.0324
After 2000 (2000-2004)				
ILLIQ(AP) – equal-weighted	0.4668	0.0026	0.3201	0.8190
ILLIQ(w) – equal-weighted	0.0472	0.0006	0.0138	0.1271
ILLIQ(AP) – value-weighted	0.2564	0.0006	0.2522	0.2681
ILLIQ(w) – value-weighted	0.0017	0.0001	0.0006	0.0047

Table 3.2, continued

	Mean	z-test	Std.Dev	Minimum	Maximum
01/02/73-06/04/97					
ILLIQ(AP) – equal-weighted	0.4593		0.0011	0.2996	0.9427
ILLIQ(w) – equal-weighted	0.3885		0.0055	0.0380	3.8145
ILLIQ(AP) – value-weighted	0.2650		0.0007	0.2552	0.3089
ILLIQ(w) – value-weighted	0.0353		0.0005	0.0028	0.3391
06/05/97-01/29/01					
ILLIQ(AP) – equal-weighted	0.5066		0.0026	0.3502	0.8190
ILLIQ(w) – equal-weighted	0.0570	***	0.0005	0.0277	0.1271
ILLIQ(AP) – value-weighted	0.2593	***	0.0006	0.2555	0.2878
ILLIQ(w) – value-weighted	0.0025	***	0.0002	0.0014	0.0068
01/30/01-08/31/04					
ILLIQ(AP) – equal-weighted	0.4308	***	0.0021	0.3201	0.6622
ILLIQ(w) – equal-weighted	0.0402	***	0.0005	0.0138	0.0996
ILLIQ(AP) – value-weighted	0.2556	***	0.0006	0.2522	0.2654
ILLIQ(w) – value-weighted	0.0015	***	0.0001	0.0006	0.0045

Panel B

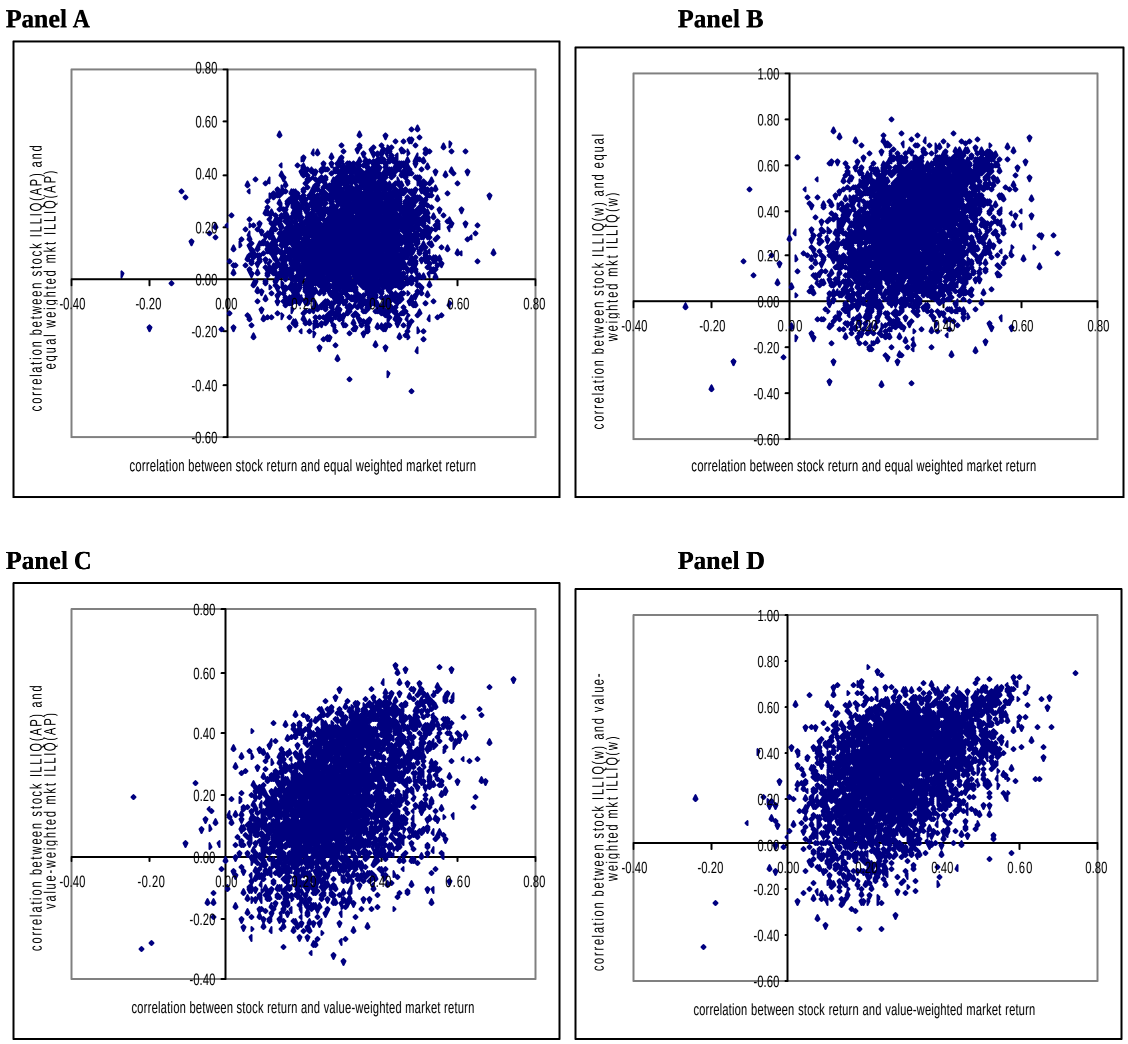
	ILLIQ(w)	ILLIQ(AP)	QRSPR	ERSPR	ROLL
ILLIQ(w)	1				
ILLIQ(AP)	0.6395	1			
QRSPR	0.6226	0.2857	1		
ERSPR	0.4734	0.2247	0.7986	1	
ROLL	-0.0278	-0.0576	-0.0042	0.1256	1

Panel B provides correlation coefficients between the ILLIQ measures and three intraday data based market liquidity measures. These three measures are 1) relative spread, 2) relative effective spread, and 3) Roll's measure. They are calculated using the Market Microstructure Database of Vanderbilt University, which aggregates intraday values from TAQ into daily quantities. Since the TAQ-based liquidity measures are only available from 1993, the correlation coefficients are calculated based on daily observations for 1993-2004. All liquidity measures in this panel represent equal-weighted averages of the corresponding liquidity measures of individual stocks. The correlation table indicates that the winsorized ILLIQ is more closely related to the spread measures than normalized ILLIQ. For instance, the correlation coefficient between ILLIQ(w) and the quoted relative spread is 0.62, while the corresponding correlation for ILLIQ(AP) is only 0.28. The panel illustrates that transforming liquidity measures in empirical analyses can have a significant impact on their behavior.

Finally, although this paper studies the aggregate market portfolio, Figure 3.2 offers cross-sectional information on the relation between return and liquidity. The figure graphs the correlation coefficients between each individual security's return and market return (horizontal axis) against their liquidity correlation with measures of aggregate market liquidity (vertical axis). For each stock in each panel of the figure, I calculate one return and one liquidity correlation coefficient using the entire sample. The number of observations used in calculating a given correlation coefficient varies by security, since the number of observations differs across the individual firms. For instance, Panel A of Figure 2 graphs the relation between a stock's return correlation with the return on the equal-weighted market portfolio (horizontal axis) and the stock's liquidity correlation (using ILLIQ(AP)) with the equal-weighted market ILLIQ(AP) measure (vertical axis) for all stocks in the sample. Panel B provides corresponding results using ILLIQ(w). Panels C and D use value-weighted market return and liquidity indices.

The figure is consistent with the finding that the majority of the individual stocks are positively correlated with the market's return. However, this is not entirely true for liquidity. While the largest portion of the scatter plot is in the first quadrant in all four panels, a significant number of data points fall in the fourth quadrant indicating individual stocks that are positively correlated with the market return but whose liquidity is negatively related to market liquidity. A priori, the graphs provide ambiguous insights in the role of liquidity risk. Stocks in the first quadrant appear especially vulnerable. If a market shock is combined with a market liquidity shock, these stocks and any of their portfolios are significantly affected. On the other hand, presence of fourth quadrant observations may imply that systematic liquidity risk is smaller, or easier to diversify.

Figure 3.2. Cross-sectional correlation coefficients



For each stock in the sample, I calculate correlation coefficients between its 1) return and the return on the market's return, 2) its liquidity and the market liquidity proxy. The figure shows the return correlation against the liquidity correlation. Market variables are equally weighted averages.

Graphs analyzing the cross-sectional distribution of the correlations for each tick size regime separately display a similar pattern (not reported in the paper). Since these figures do not reveal significant differences across regimes, they do not foreshadow crucial shifts in the potential role of liquidity risk across the sub-periods.

5.2 Model analyses

5.2.1 Univariate GARCH for the net-of-illiquidity-cost return series

To mitigate the impact of October, 1987, I winsorize the market return series at the 1st and 99th percentiles. In order to estimate (5), I create a daily net return series by subtracting the risk free rate and the estimated illiquidity costs from the daily return. The estimated illiquidity cost is given by ILLIQ (results for both ILLIQ(AP) and ILLIQ(w) are reported in the tables) adjusted to the investors' holding horizon. Panel A of Table 3.3 shows the quasi-maximum likelihood estimates of the univariate symmetric GARCH-M model for the net return series (Model (5) above). By construction, the GARCH framework uses an *ex ante* measure of the variance. For comparison to the previous literature, I also estimate the model without the liquidity adjustment (i.e., for gross excess returns, $j=G$ in the table). This is comparable to models estimated in previous risk-return analyses (see, for instance, French et al. (1987)). The estimates utilize the Bollerslev and Wooldrige (1992) robust standard errors and covariance. The Akaike information criterion is used to determine the optimal lag length for past variance and squared residuals in the variance equation.

Interestingly, the results indicate significant positive GARCH effects in the mean equation for both the liquidity adjusted, and the unadjusted models; that is, in both cases confirming the hypothesized positive risk-return relation. This seems to suggest that there is a risk-return tradeoff for the market portfolio even without the liquidity adjustment. Moreover, the simple linear relation between the excess return and the conditional market variance is a proportional relation, as the intercepts are not significantly different from zero. In both cases, volatility is very persistent, with the lagged volatility coefficient, β_2, around 0.93.

Table 3.3. Univariate generalized autoregressive conditional heteroskedasticity-in-mean (GARCH-M) results for net and gross returns

The table reports results for univariate GARCH-M models for daily net-of-illiquidity cost excess returns and for gross excess returns, based on the sample period of January 2, 1973 to August 31, 2004. Net-of-illiquidity cost excess return (r_t^N) is given by $r_{M,t}$-rf_t-$c_{M,t}$, where $r_{M,t}$ is the return on the market portfolio on day t, rf_t is the risk free rate, given by the one-month Treasury rate, and $c_{M,t}$ is the expected liquidity cost, given by $ILLIQ_{t-1}$ multiplied by $turnover_{t-1}$ (to adjust for investors' holding horizon). Gross excess return (r_t^G) is given by $r_{M,t}$-rf_t. Thus, models using the gross return do not adjust for liquidity, as in previous studies. Two models are estimated. For both, optimal lag length is selected based on the Akaike information criterion. The symmetric GARCH-M model is given by the following:

$$r_t^j = b_0 + b_1 h_t^j + u_t \qquad \text{with } j{=}N \text{ or } G$$

$$h_t^j = a + b_1 u_{t-1}^2 + b_2 h_{t-1}^j$$

The asymmetric component GARCH-M differs in its variance equation, in which mean reversion is allowed to a varying level m_t. d_t=1 indicates a negative shock.

$$h_t^j - m_t = q_1(u_{t-1}^2 - m_{t-1}) + q_2 d_{t-1}(u_{t-1}^2 - m_{t-1}) + q_3(h_{t-1}^j - m_{t-1}) \qquad \text{with } j{=}N \text{ or } G$$

$$m_t = w + r(m_{t-1} - w) + f(u_{t-1}^2 - h_{t-1}^j)$$

In parentheses are Bollerslev and Wooldridge (1992) standard errors.

	b_0	**b_1**	a_1	b_1	b_2	b_3	b_4
j=N	-0.0571	0.0301	0.0043	0.1741	1.2652	-0.1377	-0.3082
AP/Equal-w	(0.0117)	(0.0216)	(0.0013)	(0.0193)	(0.1151)	(0.0184)	(0.1058)
j=N	-0.0388	0.0635	0.0074	0.0535	0.9373		
AP/Value-w	(0.0164)	(0.0239)	(0.0015)	(0.0055)	(0.0062)		
j=N	0.0179	0.0430	0.0004	0.1396	1.7098	-0.1346	-0.7156
w/Equal-w	(0.0120)	(0.0220)	(0.0002)	(0.0145)	(0.0381)	(0.0138)	(0.0367)
j=N	0.0142	0.0763	0.0075	0.0539	0.9367		
w/Value-w	(0.0164)	(0.0240)	(0.0015)	(0.0055)	(0.0062)		
j=G	0.0576	0.0476	0.0003	0.1325	1.7291	-0.1289	-0.7332
Equal-w	(0.0123)	(0.0228)	(0.0001)	(0.0140)	(0.0331)	(0.0136)	(0.0322)
j=G	0.0186	0.0752	0.0071	0.0528	0.9383		
Value-w	(0.0164)	(0.0240)	(0.0015)	(0.0054)	(0.0061)		

Table 3.3, continued

Panel B - Asymmetric component GARCH-M

	b_0	b_1	q_1	q_2	q_3	w	r	f
j=N	-0.0717	0.0471	0.0600	0.0568	0.7536	0.5446	0.9948	0.0290
AP/Equal-w	(0.0122)	(0.0232)	(0.0183)	(0.0222)	(0.0369)	(0.0785)	(0.0018)	(0.0057)
j=N	-0.0348	0.0570	-0.0143	0.0802	0.9145	0.6366	0.9949	0.0268
AP/Value-w	(0.0005)	(0.0125)	(0.0120)	(0.0145)	(0.0184)	(0.0825)	(0.0016)	(0.0053)
j=N	0.0137	0.0418	0.0465	0.0780	0.5972	0.5712	0.9900	0.0498
w/Equal-w	(0.0120)	(0.0226)	(0.0223)	(0.0285)	(0.0792)	(0.0719)	(0.0026)	(0.0069)
j=N	0.0132	0.0778	-0.0247	0.0849	0.9181	0.6714	0.9935	0.0327
w/Value-w	(0.0148)	(0.0148)	(0.0127)	(0.0150)	(0.0195)	(0.0783)	(0.0019)	(0.0060)
j=G	0.0768	0.0124	0.0651	0.0574	0.7245	0.5297	0.9940	0.0300
Equal-w	(0.0123)	(0.0235)	(0.0196)	(0.0234)	(0.0436)	(0.0710)	(0.0019)	(0.0057)
j=G	0.0752	-0.0200	-0.0227	0.0827	0.9144	0.6570	0.9937	0.0324
Value-w	(0.0242)	(0.0160)	(0.0127)	(0.0153)	(0.0208)	(0.0809)	(0.0019)	(0.0060)

One problem with the above specification is that it does not allow for an asymmetric volatility response to positive and negative return innovations. Extensive empirical evidence points to the presence of an asymmetric response, which implies that the symmetric GARCH models may be misspecified. A popular model for accommodating the asymmetries is EGARCH (see Nelson (1991)). Ghysels et al. (2005) argue however, that the EGARCH model is not an appropriate model in the risk-return context because it constrains the persistence of the positive and negative shocks to be the same. Using a mixed data sampling approach (MIDAS), Ghysels et al. find that the asymmetry in the persistence of the impact on volatility is more important than the asymmetry in the impact itself. While negative shocks have a larger initial effect than positive shocks, their effect is temporary. In contrast, the effect of positive shocks is very persistent. Therefore, I use the two-component GARCH model of Engle and Lee (1999) to accommodate the asymmetries in both the volatility response, and its persistence to negative and positive return shocks, as suggested by Ghysels et al. The two-component GARCH is shown to behave similarly to their MIDAS estimator.

Panel B of Table 3.3 reports the asymmetric GARCH results for the net return series. Interestingly, the liquidity adjusted model remains robust to the new specification; however, the unadjusted model no longer shows a positive risk-return relation (b_1 coefficient in the table). For the equal- and value-weighted samples, the risk-return tradeoff is positive-insignificant and negative-insignificant, respectively. This is consistent with the traditional risk-return studies that find that the simple relation between excess return and the conditional market variance is very sensitive to the specification. For the liquidity adjusted models, the risk-return relation is positive and significant. The intercepts (b_0) are only indistinguishable from zero for the winsorized illiquidity measure (ILLIQ(w)), however. q_2 is significant and positive in all cases. This is the parameter that guides both the asymmetric response and its persistence. The positive coefficient indicates that the volatility response to negative return innovations is both larger and mean-reverts faster, as suggested by Ghysels et al. (2005). Overall, the results are consistent with the theoretical predictions in Acharya and Pedersen (2005) for the net-of-illiquidity-cost return series.

5.2.2 Univariate GARCH for the gross return series

The tests reported in Table 3.3 explore the relation between net market returns and total systematic risk, approximated by the variance of net returns. In this subsection, the net-of-illiquidity-costs return variance is decomposed into the variance of gross returns, the covariance between the gross return and the illiquidity cost, and the variance of the illiquidity cost. The decomposition follows equation (4) above, and allows for studying the individual role of each component. For instance, based on this decomposition, the relation between expected returns and the conditional variance of the market's return is a partial relation. This illustrates that when (4) is the true model, tests modeling the risk-return relation as a simple relation between excess return and conditional volatility are biased due to the omitted variable problem. As before, I use the GARCH framework to estimate the risk-return relation as in (6), and also use the asymmetric model discussed above.

Before estimating the GARCH models, I calculate sample equivalents for realized liquidity variance and return-liquidity covariance. Therefore, tests in this section are based on monthly observations. The estimated liquidity variance series are highly skewed in all cases. Therefore, I use a logarithmic transformation. Moreover, the turnover adjusted ILLIQ(AP) measure (or ILLIQ(w)), a regressor in (6), is not stationary. Therefore, in the analyses, I use its first difference.

Table 3.4 summarizes the results. All standard errors are based on the Bollerslev-Wooldridge (1992) adjustment. In all cases, the partial relation between excess return and the variance of the excess return is positive and significant. Moreover, the variance of liquidity is positively, while the return-liquidity covariance is negatively related to excess returns. The role of the liquidity variance appears to be especially strong. Expected illiquidity costs however, are not significantly related to the excess return. The relation is difficult to interpret as the theoretical model requires the level of liquidity, while the empirical implementation contains change in liquidity. The intercepts are significantly different from zero in the mean equation, which may reflect that they are capturing some of the liquidity-cost effects. As before, θ_2 is consistent with a significantly asymmetric response to return shocks in the variance equation.

One shortcoming of model (6) is that liquidity variance and the return-liquidity covariance have to be estimated from the sample. Since asset pricing provides predictions for conditional values, the realized values estimated from the sample cannot be directly used in the analyses. In Table 3.4, I use lagged values of the estimated realized liquidity variance and covariance to proxy for ex ante measures. Alternatively, French et al. (1987) disaggregate the estimated market variance into ex ante and unexpected components based on ARIMA models. Unfortunately, this method is less successful for the return-liquidity covariance, since the ARIMA model provides a poor forecast. Therefore, I do not adopt the French et al. method.[39] Furthermore, previous studies suggest that estimates of second moments are sensitive to the data frequency used in the estimation. In the next section, I circumvent these problems by using a multivariate framework. The multivariate model provides a convenient framework for estimating ex ante risk measures within the system. Hence, in the multivariate model, not only the variance of the market return, but also the variance of liquidity and the covariance between return and liquidity are passed through as latent.

Table 3.4. Univariate generalized autoregressive conditional heteroskedasticity-in-mean (GARCH-M) results for *gross* returns with liquidity risk

This table follows model (6) and estimates a univariate GARCH-M model for the excess return on the market portfolio, based on the sample period of January 2, 1973 to August 31, 2004. The excess return (r_t) is given by $r_{M,t}$-rf_t, where $r_{M,t}$ is the return on the market portfolio on day t, rf_t is the risk free rate, given by the one-month Treasury rate. As in (6), systematic risk is decomposed into market variance (which is passed through as a latent variable), and liquidity risk. Liquidity risk includes the variance of liquidity and the covariance between market illiquidity and returns. The liquidity risk components are sample estimates based on return volatility estimates in French et al. (1987), and are calculated prior to the estimation of the GARCH-M model. Two models are estimated. For both, optimal lag length is selected based on the Akaike information criterion. The symmetric GARCH-M model is given by the following:

$$r_t = b_0 + b_1 h_t + b_2 s^2_{L,t} + b_3 s_{ML,t} + b_4 c_{M,t} + u_t$$
$$h_t = a + b_1 u^2_{t-1} + b_2 h_{t-1} + b_3 u^2_{t-2} + b_4 h_{t-2}$$

The asymmetric component GARCH-M differs in its variance equation, in which mean reversion is allowed to a varying level m_t. d_t=1 indicates a negative shock.

$$h_t - m_t = q_1(u^2_{t-1} - m_{t-1}) + q_2 d_{t-1}(u^2_{t-1} - m_{t-1}) + q_3(h_{t-1} - m_{t-1})$$
$$m_t = w + r(m_{t-1} - w) + f(u^2_{t-1} - h_{t-1})$$

In parentheses are Bollerslev and Wooldridge (1992) standard errors.

[39] The results are sensitive to using lagged values vs. French et al. (1987).

Table 3.4, continued

Panel A - Symmetric GARCH-M

	b_0		**b_1**		**b_3**		**b_4**		**b_5**	a_1		b_1		b_2	
AP	1.9132		0.0417	**	0.4526	*	-0.3719	*	32.3621	1.8637	*	0.0773	**	0.8488	***
Equal-w	(1.4141)		(0.0161)		(0.2627)		(0.1762)		(39.7862)	(1.0727)		(0.0347)		(0.0640)	
AP	-0.4540		0.1887	*	0.1852		-13.7368	**	34.3629	2.4624	*	0.0549	*	0.8114	***
Value-w	(2.8621)		(0.1107)		(0.1948)		(4.3932)		(84.8575)	(1.4881)		(0.0317)		(0.0965)	
w	5.0886	***	0.0800	**	0.4530	***	-0.1967		48.826	0.1881		-0.0209	***	1.0106	***
Equal-w	(1.1808)		(0.0320)		(0.1472)		(0.2934)		(97.6021)	(0.1305)		(0.0008)		(0.0051)	
w	0.6510		0.1337	*	0.1905		12.4013		43.8366	2.0792	*	0.0643	*	0.8144	***
Value-w	(2.4249)		(0.0748)		(0.1918)		(11.4698)		(84.2894)	(1.2472)		(0.0329)		(0.0889)	

Panel B - Asymmetric component GARCH-M

	b_0		**b_1**		**b_3**		**b_4**		**b_5**	q_1		q_2		q_3	
AP	2.7442	***	0.0162	***	0.5359	**	-0.6281		5.3557	-0.2793	***	0.1346	**	0.9396	***
Equal-w	(0.9665)		(0.0033)		(0.2532)		(0.6395)		(35.9234)	(0.0587)		(0.0558)		(0.1341)	
AP	0.8192	***	0.0739	***	0.1035	**	-0.4007		41.9349	-0.3686	***	0.2054	***	0.9626	***
Value-w	(0.1288)		(0.0197)		(0.0344)		(4.9771)		(58.0885)	(0.0668)		(0.0406)		(0.1150)	
w	3.3152	***	0.0478	***	0.2076	**	-0.4884	*	-26.1560	0.0750	*	-0.0790		-0.9757	***
Equal-w	(0.4529)		(0.0106)		(0.0975)		(0.2915)		(87.1835)	(0.0398)		(0.0508)		(0.0221)	
w	-1.7119	***	0.1170	***	0.0638	**	-1.5682		-729.193	-0.4888	**	0.1822	***	1.1196	***
Value-w	(0.1424)		(0.0047)		(0.0298)		(2.0027)		(1216.2990)	(0.1950)		(0.0417)		(0.2246)	

5.2.3 Multivariate GARCH model

In this section, I estimate a multivariate GARCH-M model to test the risk-return tradeoff. As it was mentioned before, this is motivated by Watanabe (2003), who argues that liquidity follows GARCH-type dynamics as well. Table 3.5 reports results for (8) with the restrictions that B and D are diagonal coefficient matrices, and C is a lower triangular coefficient matrix (BEKK). These restrictions mean that both the variance of return and liquidity follow a univariate-type process, where each is driven by its own lagged value and lagged innovations from the corresponding mean equation. The covariance is driven by the lagged cross-product of return and liquidity volatility, and the lagged cross-product of the corresponding innovations from the mean equations.

Table 3.5. Multivariate generalized autoregressive conditional heteroskedasticity-in-mean (GARCH-M) results

This table follows model (8) and estimates a multivariate GARCH-M model for the excess return and the liquidity of the market portfolio, based on the sample period of January 2, 1973 to August 31, 2004. The excess return (r_t) is given by $r_{M,t}$-rf_t, where $r_{M,t}$ is the return on the market portfolio on day t, rf_t is the risk free rate, given by the one-month Treasury rate. With the appropriate parameter restrictions on (8), multivariate GARCH-M model takes the following form:

$$r_{m,t} = a_0 + a_{11} s^2_{M,t} + a_{12} s_{cM,t} + a_{13} s^2_{c,t} + f_1 c_{M,t} + e_{1,t}$$

$$c_{M,t} = a_1 + a_{21} s^2_{M,t} + f_2 c_{M,t-1} + e_{2,t}$$

with second moment dynamics as follows:

$$s^2_{M,t} = c_1^2 + d_1 s^2_{M,t-1} + b_1 e^2_{1,t-1}$$

$$s_{cM,t} = c_1 c_2 + d_1 d_2 s_{cM,t-1} + b_1 b_2 e_{1,t-1} e_{2,t-1}$$

$$s^2_{c,t} = c_2^2 + c_3^2 + d_2^2 s^2_{c,t-1} + b_2^2 e^2_{2,t-1}$$

where $r_{M,t}$ is the excess return on the market, $c_{M,t}$ is a given measure of market liquidity, $s^2_{M,t}$ and $s^2_{c,t}$ represent the conditional market return variance and liquidity variance respectively, and $s_{cM,t}$ stands for the conditional market return and liquidity covariance. The model is estimated based on the restricted BEKK of Engle and Kroner (1995), with diagonal coefficient matrices B and D, and a lower triangular C coefficient matrix in (8).

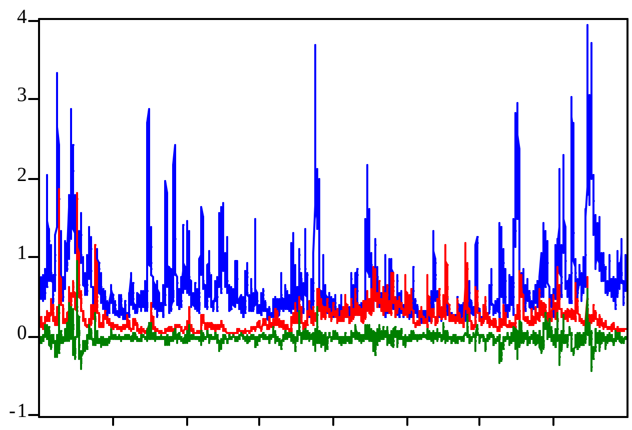
Table 3.5, continued

	AP/EW		W/EW		AP/VW		W/VW	
a_0	0.0100 (0.0069)		0.0305 (0.0000)	***	0.0145 (0.0059)	**	0.0231 (0.0172)	
a_{11}	0.0680 (0.0226)	***	0.0867 (0.0130)	***	0.0828 (0.0169)	***	0.0855 (0.0277)	***
a_{12}	14.8073 (3.9472)	***	0.7600 (0.3865)	**	5.9716 (0.0495)	***	-20.5438 (48.4077)	
a_{13}	-0.5006 (0.2066)	**	-1.6819 (0.4260)	***	27.9420 (17.0261)		-22.8900 (6.1173)	***
f_1	31.4504 (10.2170)	***	43.5066 (69.7086)		-6.1177 (39.4502)		8.0286 (871.3938)	
a_1	-0.0006 (0.0006)		0.0006 (0.0003)	*	0.0002 (0.0000)	***	0.00004 (0.00002)	**
a_{21}	0.0023 (0.0008)	***	-0.0004 (0.0003)		-0.00004 (0.00004)		-0.00003 (0.00002)	*
f_2	-20.8259 (1.7693)	***	-91.2015 (2.8358)	***	0.6299 (0.1510)	***	-99.3459 (2.6011)	***
c_1	0.1137 (0.0051)	***	0.1011 (0.0048)	***	0.0654 (0.0049)	***	0.0669 (0.0050)	***
d_1	0.9527 (0.0026)	***	0.9594 (0.0021)	***	0.9814 (0.0013)	***	0.9812 (0.0012)	***
b_1	0.2690 (0.0076)	***	0.2538 (0.0065)	***	0.1777 (0.0061)	***	0.1780 (0.0057)	***
c_2	0.0031 (0.0003)	***	0.0008 (0.0000)	***	0.0011 (0.0000)	***	0.00004 (0.00000)	***
c_3	-0.0003 (0.0002)	*	-0.0001 (0.0000)		-0.0009 (0.0001)	***	-0.00009 (0.00005)	**
d_2	0.9680 (0.0013)	***	0.9590 (0.0009)	***	0.7011 (0.0085)	***	0.9589 (0.0006)	***
b_2	0.2524 (0.0054)	***	0.3045 (0.0040)	***	0.5820 (0.0103)	***	0.3127 (0.0031)	***

In Table 3.5, I allow all second moments to enter the return equation (as in (4)). In addition, I also include the first difference of the turnover adjusted illiquidity costs. This is the same as in the previous section, with the exception that all second-moment regressors are estimated within the system. The only second moment that enters the liquidity equation is return volatility (I model the first difference of illiquidity costs due to non-stationarity).

The table indicates that excess return is significantly positively related to its conditional variance (a_{11}) and to the variance of liquidity (a_{12}), and significantly negatively related to the covariance between return and liquidity (a_{13}). As in the previous sections, the results are somewhat weaker for the value-weighted portfolio. This is consistent with the idea value-weighting overweights liquid assets, and as a result, any role liquidity may play for asset prices is harder to detect (see, for instance, Chordia et al. (2000) and Acharya and Pedersen (2005)). Return variance does not appear to be a significant driver of liquidity (a_{21}). It is positive and significant for the equally-weighted portfolio using ILLIQ(AP); however, it is not significant using ILLIQ(w). Figure 3.3 shows the estimated variances for return and liquidity, and the return-liquidity covariance.

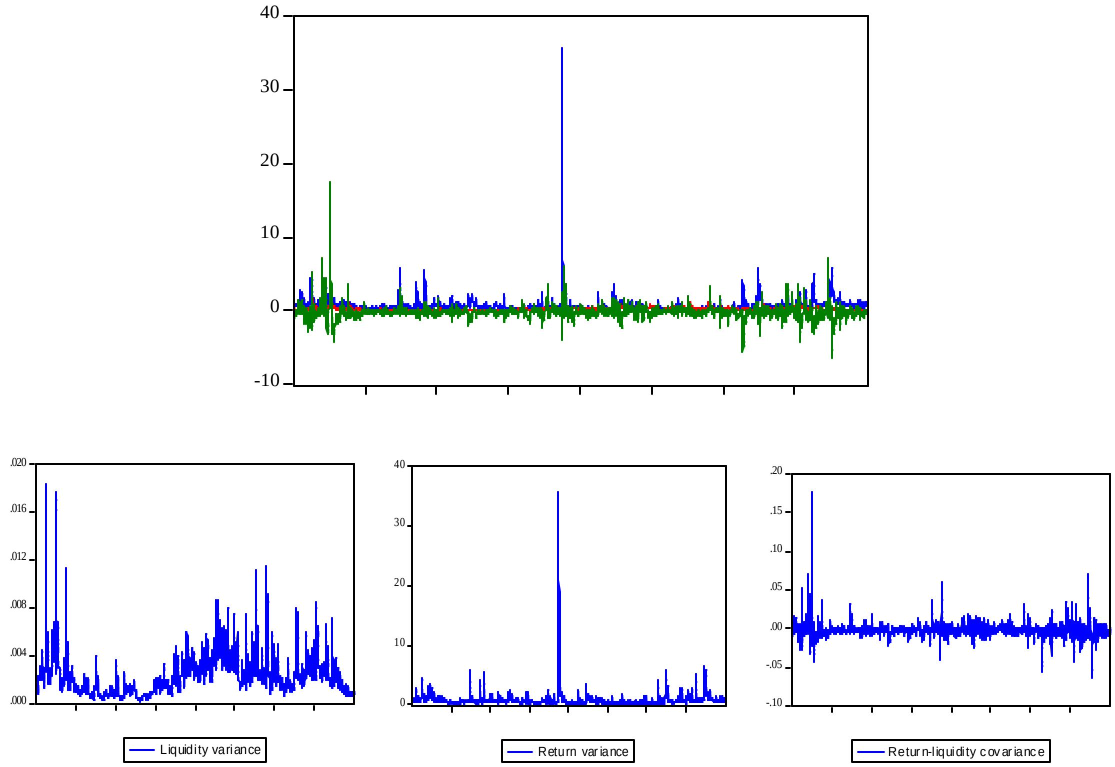
Figure 3.3. Variance and covariance estimates from the multivariate GARCH-M model



The figure shows the (scaled) estimated variances and the covariance from the multivariate GARCH model in the first column of Table 5. The graph in blue is the return variance series. The red line denotes the liquidity variance series (x100). The green line shows the covariance between return and liquidity (x10).

Each of the three series appears to be conditionally heteroskedastic, indicating that the GARCH framework is appropriate for modeling liquidity as well. The results in this section (as in the previous sections) utilize winsorized market returns to address the problems with extreme events, such as that of October, 1987. Figure 3.4 shows the estimated variances and the covariance series using the original (not winsorized) return series.

Figure 3.4. Variance and covariance estimates from the multivariate GARCH-M model (the return series are not winsorized)



This figure shows the estimated variances and the return-liquidity covariance for a model that corresponds to the first column of Table 5, except it utilizes excess returns that are not winsorized (unlike in Figure 3).

6. Robustness Analyses

6.1 Idiosyncratic volatility and liquidity

In this section, I investigate the possibility that liquidity risk proxies for idiosyncratic risk. Spiegel and Wang (2005) find that idiosyncratic volatility dominates liquidity risk in cross-sectional tests. A theoretical link between liquidity and idiosyncratic volatility, through the market maker's inventory, has been suggested in Ho and Stoll (1980). Another possible connection results if illiquidity causes sub-optimal portfolio diversification, leaving the investors exposed to idiosyncratic risk.

Many studies investigate whether idiosyncratic volatility matters for asset prices. For instance, Goyal and Santa-Clara (2003) use the following simple time-series regression to test the role of idiosyncratic volatility.

$$r_{M,t} = a + X_t^{'} b + e_t$$

where $r_{M,t}$ is the excess return on the market and the X_t vector includes a proxy for the conditional variance of the market portfolio and a measure of average stock variance. I combine their model with the model estimated in section 4.2.2 to examine the relation between idiosyncratic volatility and liquidity risk. I use several methods to estimate idiosyncratic risk. For instance, I use the average stock variance, as suggested by Goyal and Santa-Clara, and also, use the residuals from the Fama-French 3 factor model with momentum. Table 3.6 shows the results.

While including idiosyncratic volatility weakens the previous results, both the conditional return variance and the variance of liquidity remain positive and significant. The results become very sensitive to changes in the liquidity measure: idiosyncratic volatility (b_5) changes sign and significance across the four models reported in Table 3.6. The results are consistent with the finding that liquidity risk does not proxy for idiosyncratic volatility, and more specifically, idiosyncratic volatility does not matter in the risk-return trade-off.

Table 3.6. Liquidity risk and idiosyncratic volatility

I augment the model estimated in Table 4 with idiosyncratic volatility. Idiosyncratic volatility is estimated monthly from the residuals from the Fama-French model with momentum. The sample contains monthly observations from January, 1973 to August, 2004. Results are reported for the asymmetric GARCH-M with the following form:

$$r_t = b_0 + b_1 h_t + b_2 s^2_{L,t} + b_3 s_{ML,t} + b_4 c_{M,t} + b_5 v^i_t + u_t$$
$$h_t - m_t = q_1(u^2_{t-1} - m_{t-1}) + q_2 d_{t-1}(u^2_{t-1} - m_{t-1}) + q_3(h_{t-1} - m_{t-1})$$
$$m_t = w + r(m_{t-1} - w) + f(u^2_{t-1} - h_{t-1})$$

	AP/EW		W/EW		AP/VW		W/VW	
b_0	-0.4261		1.4224	***	1.3278		-1.9426	
	(0.7953)		(0.4327)		(1.8490)		(1.27449)	
b_1	0.0214	***	0.0056		0.0730	**	0.1234	***
	(0.0003)		(0.0059)		(0.0298)		(0.0384)	
b_2	-1.2034	***	-15.0429		-3.5809		-1.2067	
	(0.0768)		(20.2045)		(7.5317)		(2.1107)	
b_3	21.4221	***	0.1628	***	0.1474	*	-0.0866	
	(7.3254)		(0.0413)		(0.0804)		(0.0688)	
b_4	15.1857		111.3812		1.4306	*	0.0801	
	(27.1423)		(94.8170)		(0.8057)		(0.0838)	
b_5	-14.6069		1.1698	**	0.1618		-0.3175	
	(66.0569)		(0.7317)		(0.3665)		(0.5563)	
q_1	-0.2471	***	-0.3204	***	-0.3442	***	-0.3917	*
	(0.0724)		(0.0937)		(0.0394)		(0.2001)	
q_2	0.1079		0.1300	**	0.2054	***	0.1795	***
	(0.0908)		(0.0523)		(0.0547)		(0.0471)	
q_3	0.8815	***	1.0011	***	0.9388	***	1.0102	***
	(0.1569)		(0.1731)		(0.0967)		(0.2435)	
w	28.1429	***	30.2122	**	17.3788	***	17.3240	***
	(10.4724)		(12.1378)		(2.6418)		(2.7881)	
r	0.9254	***	0.9355	***	0.8954	***	0.8278	***
	(0.0477)		(0.0499)		(0.0467)		(0.0647)	
f	0.1718	**	0.2053	*	0.2043	***	0.3175	
	(0.0682)		(0.1053)		(0.0606)		(0.2088)	

6.2 Shortcomings of the liquidity measure

In this paper, I use ILLIQ, a measure first developed by Amihud (2002), as the measure of illiquidity. One advantage of ILLIQ for asset pricing is that it can be calculated for long time horizons, as it only requires return and volume information. The Acharya and Pedersen (2005) model also uses ILLIQ, which motivates the use of the measure in this paper. There are many caveats concerning ILLIQ however. For instance, it is based on total volume, as opposed to signed volume. More importantly, it may be dominated by its denominator, daily share volume. This is especially problematic because volume has many undesirable statistical properties over long horizons.

To investigate this issue, I calculate equal- and value-weighted market inverse volume measures for the entire sample, as well as for subsamples presented in Table 3.1 and 3.2. I find that the concern is valid, for the entire sample, the correlation coefficient between ILLIQ(w) and inverse volume is approximately 95.64% for the equal-weighted measures, and varies between 76.83% and 97.54% across the subsamples.

7. Conclusion

The study of liquidity in asset pricing is relatively new. Recent theoretical papers attempt to formalize the impact of liquidity on security prices by solving traditional asset pricing models under the assumption that trading costs are stochastic. This framework is somewhat limited in that only the price dimension of liquidity can enter pricing relations. To the extent that the price dimension represents other aspects of liquidity (for instance, time dimension as in Demsetz (1968)), this limitation may not be serious.

The new liquidity adjusted models also suggest a new direction for the risk-return tradeoff literature. If investors care about liquidity risk, the traditional risk-return tests are misspecified. The paper looks at this possibility by studying the intertemporal relation between the market risk premium and liquidity risk. In particular, I investigate the liquidity adjusted CAPM in which the source of systematic risk is given by the variation in net-of-illiquidity cost returns. For the market portfolio, the systematic risk is divided into the traditional market risk and a liquidity risk component. Ex ante measures of liquidity risk appear to be significant drivers of the market risk premium and obey the directional predictions of the theoretical models. In addition, the partial relation between expected returns and the conditional market variance is restored once the model adjusts for liquidity. These results have two important implications. First, they are consistent with the idea that investors consider liquidity risk to be important in addition to market risk in the risk-return tradeoff. Second, they extend the implications of the static liquidity adjusted CAPM in the dynamic context.

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