# PROPAGATION THROUGH ENDOGENOUS INVESTMENT-SPECIFIC TECHNOLOGICAL CHANGE

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### Abstract

Many real business cycle models lack a significant propagation mechanism. Consequently most of the serial correlation in output is inherited from the serial correlation in the exogenous shocks. A simple model is presented to show there need not be any relationship between the serial correlation of the exogenous shocks, and that of output. This is accomplished by incorporating the well-documented fact that research spending has generated changes in the real price of capital.

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# 1    Introduction

It is often asserted that one problem with many real business cycle models is that there is only a minimal propagation mechanism for exogenous shocks (see, Cogley and Nason [1]). Serial correlation in output is largely generated by the exogenous technological shocks, and the model itself does not produce this correlation. Absent a theory of why the exogenous shocks are highly serially correlated, the model itself sheds no light on the issue of why output is highly serially correlated.

In this paper a very simple model is described in which *iid* exogenous shocks are translated into a process for aggregate output which has an arbitrarily high level of serial correlation. This feature is present in a model that does not rely on the existence of a high value for capital's share of output, externalities, imperfect competition, habit-formation type preferences, or any other of the many usual deviations from the standard competitive framework.[1]

The model here is a very simple parameterized version of a business cycle model, which delivers closed-form decision rules. However, it also borrows from a related literature. Greenwood Hercowitz and Krusell [2] make the compelling argument that it is falling real prices for new investment goods that accounts for most of the observed growth, with relatively little being left over to be explained by other factors, such as total factor productivity. Greenwood Hercowitz and Krusell [3] then utilize this reasoning to look at certain related business cycle issues. However, in neither of these papers do they explicitly model the mechanism by which the real price of capital falls. In this paper the changing real price of capital is driven by *endogenous* research spending. Increased research spending in one period lowers the cost of producing capital in a subsequent period. It is shown here that this important feature is can also be an additional source of propagation for exogenous technological shocks.

There is an apparent tension in many existing models because in order to generate high serial correlation in aggregate output, with low serial correlation in the exogenous shocks, requires a "capital share" which is too high to be consistent with measures of capital income. The findings of this paper show that there is no such tension or incompatibility between having a low value for capital's share, and a high persistence in output, even in the presence of *iid* exogenous shocks.

# 2    A Simple Model

Consider a model populated by identical representative agents with preferences written as follows

$$E\left[\sum_{t=1}^{\infty}\beta^t\log(c_t)\right],\tag{1}$$

where $c_t$ is the level of consumption in period $t$. The technology for the economy is standard, with output ($y_t$) being produced from capital alone. The resource constraint for the economy is written as follows:

$$c_t + x_t + q_t k_{t+1} = A_t k_t^\alpha \equiv y_t.\tag{2}$$

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[1] There are few other papers that address the isues documented by Cogley and Nason [1]. Perli and Sakellaris [5] investigate this topic by constructing a two-sector model with human capital. They find that an important feature is the elasticity of substitution between unskilled and skilled labor in the production of human capital. Wen [6] looks at the contribution of using habit formation on leisure choice.

Here $k_t$ is the capital stock in period $t$, $\alpha$ is capital's share of aggregate income and $A_t$ is a random technology shock. Also, $q_t$ is the cost of producing a new unit of capital in period $t$, and $x_t$ represents the amount of spending in period $t$ on research.

Agents are able to lower the cost of producing capital in the future, by devoting real resources to research and development, which in turn lowers the value of the future cost of producing capital ($q_{t+1}$). This idea is captured by employing the following technology for research

$$q_{t+1} = B\left(q_t^\gamma\right)\left(x_t^\theta\right). \tag{3}$$

It is clear that the lower is this price of capital, the better it is for the agent because this implies a lower cost to producing future capital.[2] This specification is then consistent with secular fall in the real price of many types of capital goods, as documented by Greenwood, Hercowitz and Krusell [2]. As written, this equation implies that the price of capital is a function of the price in the previous period, and previous research spending. It is not necessary to have $\gamma = 1$ for there to be strong propagation of shocks, or even for there to be balanced growth.[3] The fact that $\gamma < 1$ seems then to imply that research spending has an effect that depreciates over time. This could be interpreted as meaning that research spending generates technological innovations which are productive in the immediate future, but the "long-run" impact can only be maintained with even more research spending. For this technology to make sense, it must be that $B > 0$, $\theta < 0$, so that $\frac{\partial q_{t+1}}{\partial x_t} < 0$.

There is no need to add labor to the model, since the main point can be captured without this feature. Keep in mind that $\alpha$ is capital's share, and we wish to keep this parameter fixed, say, at around $\alpha = .30$.

## 2.1 Solution of the Model

It will be assumed hereafter that the shocks $\log(A_t)$ are normally distributed. It is straightforward to lay out the dynamic programming problem for the model, and verify that the value function is written in the following manner:

$$V(k_t, q_t, A_t) = \pi_0 + \pi_1 \log(k_t) + \pi_2 \log(q_t) + \pi_3 \log(A_t),$$

where the $\pi_i$'s are constants. Furthermore it is easily seen that the decision rules for the problem can then be written as follows:

$$q_t k_{t+1} = (\beta\alpha) A_t k_t^\alpha \tag{4}$$

$$x_t = \left[\frac{\left(-\theta\alpha\beta^2\right)}{(1 - \gamma\beta)}\right] A_t k_t^\alpha \tag{5}$$

$$c_t = \left[1 - \alpha\beta + \frac{\left(\theta\alpha\beta^2\right)}{(1 - \gamma\beta)}\right] A_t k_t^\alpha \tag{6}$$

Of course, when $\theta = 0$, the model collapses to the usual one-sector growth model with logarithmic utility and Cobb-Douglas technology.

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[2] See Krusell [4] for another model in which the price of capital falls due to some endogenous R&D, but this model does not have uncertainty.

[3] In fact, equation (3) implies that the case in which $\gamma = 1$, and therefore $\theta = 0$, is really a case of exogenous growth.

## 2.2 The Dynamics of the Model

The substitution of equation (5) in to equation (3) yields the following equation describing the evolution of the price of capital

$$q_{t+1} = B \left[ \frac{(-\theta \alpha \beta^2)}{(1 - \gamma \beta)} \right]^{\theta} (q_t^{\gamma}) (A_t k_t^{\alpha})^{\theta}.$$

By substituting this expression into equation (4), it is possible to show that the evolution of the capital stock can be described by this second-order system:

$$(1 - \lambda_1 L)(1 - \lambda_2 L) \hat{k}_{t+1} = \left[ (1 - \gamma L) \hat{A}_t - \theta \hat{A}_{t-1} \right] = \left[ (1 - (\gamma + \theta) L) \hat{A}_t \right] \tag{7}$$

where '$L$' represents the Lag-operator, and where a '^' over a variable denotes the logarithm of that variable. Here the roots of this last equation are written as follows

$$\lambda_1, \lambda_2 = \frac{(\gamma + \alpha) \pm \sqrt{\gamma^2 + \alpha^2 - 2\alpha\gamma - 4\alpha\theta}}{2} \tag{8}$$

There are some special cases to note. First, if $\gamma = 1 - \left( \frac{\alpha\theta}{\alpha - 1} \right)$, then there is a unit root in equation (7). This is also the condition for balanced growth. If $(\gamma + \alpha) = 4 [\alpha (\gamma + \theta)] = 1$ then there are *two* unit roots in this process.[4] The necessary and sufficient condition for both roots to be positive is that $\gamma + \theta > 0$.

As is shown in equation (7), in general the capital stock will follow an ARMA(2,1) process, and there is certainly plenty of room for the propagation of exogenous shocks. It is also possible to see how this system responds to an exogenous 1% shock to the variable $A_t$, for t=1. This is show in Figure 1, which shows how aggregate output responds, for various parameter values. As is apparent, the greater is $\gamma$ or the smaller is $\theta$, the more prolonged is the response to this one-time shock to $A_t$.

For positive values of $\gamma$, output actually increases in the first, third and fifth periods, when compared with the periods immediately prior. The reason for this is as follows: A technology shock in period 1 directly generates an increase in output in that period. This produces an increase in research spending ($x_1$). This lowers the cost of producing capital in period 2, which generates a rise in output in period 3. Similarly, the increase in research spending ($x_3$) lowers the cost of producing capital in period 4, which generates a rise in output in period 5.

### Table 1

Serial Correlation of Output

| $\theta \backslash \gamma$ | 0 | .25 | .5 | .75 |
|---|---|---|---|---|
| 0 | .30 | .30 | .30 | .30 |
| -0.5 | .354 | .37 | .40 | .65 |
| -1.0 | .431 | .49 | .73 | - |
| -1.5 | .550 | .76 | - | - |
| $\left( \frac{\alpha-1}{\alpha} \right)$ | 1.00 | - | - | - |

[4] There can also be a *negative* unit root, which occurs if $\gamma = -1 - \left( \frac{\alpha\theta}{\alpha+1} \right)$. Since it necessary that $\gamma > 0$, this implies that $-\theta > \left( \frac{1+\alpha}{\alpha} \right) > 1$.

The serial correlation of aggregate output is a complicated function of the parameters $\alpha$, and $\theta$. Table 1 shows the serial correlation of output for various parameter values, while holding $\alpha = .30$. There is little serial correlation in the baseline model with $\theta = 0$, but lowering the value of $\theta$ certainly produces more correlation. Increasing the value of $\gamma$ also increases the serial correlation of output. However, one can only press this issue so far: In the table, the cells containing a '-' are ones for which there is a root in equation (7) that is greater than or equal to unity.

It is easy to see that as $\theta$ falls, the serial correlation in capital, and output gets higher, even when $\alpha$ is held fixed. Note the special case where $\theta \searrow \left(\frac{\alpha-1}{\alpha}\right)(1-\gamma)$, in which case

$$\gamma_1 = 1, \ \ and \ \ \gamma_2 = \alpha + \gamma - 1. \tag{9}$$

In other words, as the value of $\theta$ falls, the system approaches a state where there is a unit root in the process describing the evolution of capital, and therefore output as well. This is the case of balanced growth.

## 2.3   And the Solow Residuals?

Some business cycle models have been criticized because they appear to derive the serial correlation properties in output from the assumed serial correlation in the technology shocks, or Solow residuals. It is then of interest to calculate what the Solow residuals would look like in the present framework. First it is important to note that in the traditional approach, the capital stock is *not* re-normalized each period based on how the price $(q_t)$ has changed each period. The observer who did not taken into account that the relative price of capital were changing, would say that the amount of capital used in period t would be the amount of investment from the previous period: $(q_{t-1}k_t)$. Hence the Solow residual (apart from constants) would then be measured as follows:

$$SR_t \equiv \log(y_t) - \alpha \log\left(q_{t-1}k_t\right)$$

$$= \hat{A}_t + \alpha\theta\left[\frac{\hat{A}_{t-2}}{1-\gamma L}\right] + \alpha^2\theta\left[\frac{\left[(1-(\gamma+\theta)L)\hat{A}_{t-3}\right]}{(1-\lambda_1 L)(1-\lambda_2 L)(1-\gamma L)}\right] \tag{10}$$

Now remember that while $\theta < 0$, and $\lambda_1, \lambda_2, \gamma > 0$, this implies that these residuals are going to have some high serial correlation properties, even if the shocks themselves (i.e. $\log(A_t)$) are *iid.* It is easy to see that a sufficient condition for expression to be serially correlated is that $\gamma + \theta > 0$, since this implies that both $\lambda_1, \lambda_2 > 0$. After having viewed this equation, it would be surprising if there were not *considerable* persistence in these technology shocks.

A researcher who mistakenly assumes $\theta$ to be zero is going to attribute its influence to the exogenous technology shock $(A_t)$. But equation (5) shows that research spending is just proportional to output. Hence if one were to calculate the serial correlation of the Solow Residuals, one would find that they had exactly the same serial correlation properties as output itself. One might (wrongly!) conclude that output was deriving all its serial correlation properties from the assumed nature of the technology shocks. *However, in this model, the assumed technology shocks are iid and have no serial correlation at all.* There is no contradiction in having *iid* exogenous shocks, a modest value for $\alpha$, and highly correlated aggregate output.

Additionally, it might seem that there should be a tight relationship between the measured Solow residual in a given period $(SR_t)$, and the amount of research activity in the previous period $(x_{t-1})$. This makes intuitive sense

that resources devoted to generating new technologies, should result in technology in subsequent periods. The first row of Table 2 illustrates the correlation the measured Solow residuals, and non-federal research and development spending, measured on an annual basis, and it can be seen that there is little correlation between these measures. The next two rows also show the correlation between these two variables for the model, for various parameter values. It is noteworthy that there is also little relationship between these two variables in the model as well.

The second row of Table 2 also shows that in the model there is a moderately positive relationship between research activity in period $t$, and the subsequent period's output, measured on an annual level. However, as the table illustrates, there is a similar relationship in the data as well. These relatively weak correlations are all the more surprising when one remember that there is only one shock in the model.

<center><b>Table 2</b>[5]</center>

| | $corr(x_t, SR_{t+1})$ | $corr(x_t, y_{t+1})$ |
|---|---|---|
| *US data* | .195 | .398 |
| *Model ($\theta = -.5, \gamma = 0$)* | .230 | .353 |
| *Model ($\theta = -1.0, \gamma = 0$)* | .256 | .428 |
| *Model ($\theta = -.50, \gamma = 0.7$)* | .268 | .395 |

## 3   Final Remarks

The framework analyzed above is high stylized, and delivers simple decision rules that are easy to understand. As such, certain assumptions were made to bias the model *against* the possibility of overstating the amount of serial correlation in output generated from the exogenous shocks. First, capital is assumed to depreciate at 100% per period. Second, employment is absent from the model, and adding this feature to the model is also likely to add to the serial correlation of output since employment should also be procyclical.

Another interesting issue that has not been addressed in this model is the distinction between the solution to the planning problem studied here, and decentralized solution. It can be shown that if the research technology (equation 3) is specific (i.e. internal) to the firm, then the solution to the planning problem and the decentralized solution are identical. However, if there are spillovers so that research by one firm can affect others, then the resulting dynamics of the system are even more interesting, and this can further enhance the propagation of exogenous shocks.

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[5] The data used to calculate the Solow residuals is annual data from the Bureau of Labor Statistics from 1953-1998. The data on Non-Federal Research spending is from the National Science Foundation. Annual data was used because that is all that is available from the NSF. All data was detrended using the Hodrick-Prescott filter, using a smoothing parameter of 100.

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