

TAX COMPETITION RECONSIDERED

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Tax Competition Reconsidered¹

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ABSTRACT: In a classic model of tax competition, we show that the level of public good provision and taxation in a decentralized equilibrium can be efficient or inefficient with either too much, or too little public good provision. The key is whether there exists a unilateral incentive to deviate from the efficient state and, if so, whether this entails raising or lowering taxes. A priori, there is no reason to suppose the incentive is in either one direction or the other.

KEYWORDS: Efficiency, Nash equilibrium, over-provision, tax competition, under-provision.

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1 Introduction

According to received wisdom, competition between governments for mobile capital will result in a ‘race to the bottom.’ By taxing at a lower rate in order to attract capital from other jurisdictions, each government has an incentive to engage in wasteful competition, with the result that tax rates are set too low and public goods are underprovided.³ Zodrow and Mieszkowski (1986) and Wilson (1986) were the first to formalize the intuition of this argument, expounded by Oates (1972).

The purpose of this paper is to re-examine the standard conclusion of the tax competition literature, that when capital is mobile and a major source of taxation then a race to the bottom will be the result. Our analysis focuses on one of the models presented by Zodrow and Mieszkowski (1986) (henceforth Z-M) in which public goods are an input into the production process. Public good provision in a jurisdiction enhances the productivity of capital employed there. By varying one of Z-M’s key assumptions, we show that when public goods have a positive impact on productivity, then the presence of tax competition does not necessarily result in a race to the bottom.⁴ Instead, the outcome can be efficient, or there can be a ‘race to the top’ where there is over-provision of public goods and tax rates are too high. There is no reason, *a priori*, to suppose that one sort of outcome will prevail.

In the process of characterizing equilibrium and demonstrating this wider range of possible outcomes, we also prove existence of equilibrium. Our approach to the proof of equilibrium is novel in itself and offers a contribution to the literature on tax competition. This is noteworthy because the tax competition literature often leaves aside the question of existence, although important exceptions are provided by Rothstein (2003, 2004). Both of his papers extend and generalize models of tax competition, the question of existence being central to his analysis.

The intuition behind our analysis is as follows. Under the standard approach, the marginal product of capital schedule is declining in capital within a jurisdiction that faces a fixed

³The phrase ‘race to the bottom’ has more than one interpretation in the literature and in the policy debate more widely. In some places it is taken, literally, to imply an outcome where taxes and public good provision is non-existent. Here, as elsewhere, we give it the less literal interpretation of implying public good under-provision and taxes are set too low relative to the optimum. In parallel, we take a ‘race to the top’ to imply public good over-provision and taxes set too high.

⁴Our assumption (introduced as A5 in the analysis) changes the corresponding assumption in Z-M; it does not weaken it. However, our assumption is at least as plausible as Z-M’s and has its own pedigree from the literature on economic growth. Further explanation is given later in the introduction and in Section 4, where A5 is introduced.

world interest rate. An increase in the local tax raises the marginal cost of capital, causing some capital to flow out of the jurisdiction until marginal returns to capital are equated once again at the new higher marginal cost (holding the tax rate in the other jurisdiction(s) constant). But in our framework, if capital and the public good are complements then the tax facilitates provision of a local public good, which in turn shifts up the marginal product of capital schedule. If the degree of complementarity is sufficiently high, the marginal product of capital increases more than the marginal cost, causing capital to flow into the jurisdiction as a result. (This cannot happen when the tax finances a public consumption good, the case analyzed in most of the literature, because the marginal product of capital schedule is not affected by public good provision.)

Now it is possible that, starting at the first-best efficient level of public good provision, there is an incentive to raise the tax in order to increase the capital stock (holding the tax rate in the other jurisdiction(s) constant). But of course, if both jurisdictions act on the incentive to deviate from efficiency by raising taxes, neither necessarily ends up with any more capital, but both have over-provision in equilibrium. If the degree of complementarity is low, analogously, we have underprovision in equilibrium.

To help explain the results of our analysis further, we define the notion of *marginal public good valuation* ($mpgv$), which reflects the degree of complementarity between the public good and capital. For given capital, $mpgv$ measures the extent to which output is increased - through productivity enhancement - by the marginal unit of the public good. In the Z-M model, a race to the bottom depends on the assumption that (within a jurisdiction) the $mpgv$ is *everywhere* lower than the *marginal cost of the public good* (mcp_g). If $mpgv$ is below mcp_g at a given level of taxation, the government has an incentive to lower taxes; the tax revenue is worth more at the margin to citizens of that jurisdiction than the public good. If this holds everywhere then it must hold at the efficient point. Both jurisdictions have a unilateral incentive to deviate from efficiency by lowering taxes. Given that Z-M assume $mpgv$ is everywhere below mcp_g , a race to the bottom is not surprising.

We replace the assumption that the $mpgv$ is everywhere below mcp_g with an assumption that $mpgv$ is high at low levels of public good provision but falls monotonically as public good provision is increased. As it is increased, $mpgv$ eventually equals, then falls below, mcp_g . This is closely related to an assumption used in the growth literature; see Barro (1990) and Barro and Sala-i-Martin (1995 Chapter 4, Section 4).

Necessary and sufficient conditions on the model primitives for the efficient level of public good provision can be identified in the usual way. In addition, the level of public good provision at which $mpgv$ is equal to $mcpv$ can also be identified from model primitives. *A priori*, there is no reason to assume any relationship between these two levels. One could be greater than the other or they could coincide. If it so happens that the level of public good provision at which $mpgv$ is equal to $mcpv$ is lower than the efficient level then, at the efficient level, there will be a unilateral incentive to deviate by lowering taxes. A conventional race to the bottom results. But if on the other hand the level of public good provision at which $mpgv$ is equal to $mcpv$ is higher than the efficient level then there will be a unilateral incentive to deviate by raising taxes from the efficient level. Government is ‘too big’ in this case.⁵ And if $mpgv$ equals $mcpv$ at the efficient level of public good provision then there is no incentive to deviate and the equilibrium outcome is efficient.

1.1 Related Literature

Our results are in keeping with those of previous work in suggesting that there may be over-provision of public goods. For example, over-provision arises when there is tax exporting as in Gerking and Mutti (1981) or when policy-makers have Leviathan tendencies as in Mintz and Tulkens (1996). Our results are also in keeping with those of Noiset (1995) that over-provision may result in a Z-M model where public goods are an input in production.

We are not the first to show that the outcome of provision of local public goods can be efficient. It is worth emphasizing that in our framework the efficient outcome is a knife-edge case that lies between under-provision and over-provision. This contrasts with much of the literature on efficient public good provision, which studies mechanisms that guarantee efficiency. For example, Oates and Schwab (1988) present a model where the level of public service provision is chosen by the median voter. They demonstrate a Tiebout type mechanism in which voters vote with the vote rather than with their feet, and the same efficient outcome conjectured by Tiebout results. Black and Hoyt (1989) examine the process where jurisdictions bid for firms. They consider a situation where the marginal cost of providing a firm and its workers with public services is less than the tax revenue that the firm generates. In that case, a government may offer the firm subsidies that actually reduce the distortions

⁵In our model, government is too big when too many public goods are provided. A common illustration would be where there are too many roads. This is to be distinguished from the complaint that government is too big in the sense that public goods could be more efficiently provided by the private sector. Here in our model, the government is no less efficient a provider than the private sector would be.

that the average cost pricing of the public service creates, thus increasing efficiency. Wildasin (1989) shows that the inefficiency created by competition for mobile capital can be corrected by a subsidy.⁶ Wooders (1985) demonstrates that when local public goods are financed by lump sum taxation and consumers can ‘opt out’ to provide the public goods for themselves, then the outcome is near-optimal, where the closeness of the outcome to efficiency depends on the costs of opting out.⁷

Our modelling of government behavior is in the tradition of the so called ‘benevolent dictator,’ who maximizes the welfare of the representative citizen in his jurisdiction. This is in keeping with the approach adopted by Z-M and the papers that followed in the tax competition literature.⁸ The assumption that the government behaves as a benevolent dictator contrasts markedly with the approach taken in much of the more recent literature looking at taxation and public good provision, where the government is assumed to be employed as an agent by the electorate (the principal, in this setting). See Besley and Case (1995) and Rogoff (1990) for prominent examples demonstrating asymmetries of information that give rise to political agency problems. Since the novelty of our approach is to examine the incentive to deviate from an efficient state of the economy, and the conventional tax competition model is sufficient to motivate this, we leave aside the more complex ‘agency problem’ effects.

The overall point we make is that while other research has introduced a number of additional factors to the standard basic framework to derive a variety of outcomes, we show that in fact a number these outcomes are possible in the standard basic model.⁹

The results we present are of more than just theoretical interest. The empirical literature questions the extent to which a race to the bottom in tax rates actually occurs. The most comprehensive empirical investigation of this question at an international level has been undertaken by Devereux, Griffith and Klemm (2001). They bring together a number of different measures for ten or more OECD countries over the period 1970-1998. The universally quoted Statutory Tax Rate (STAT) is compared with others such as the Implicit Tax Rate on Corporate Profits (ITR-COR). The nature of their findings is summarized in the following

⁶In contrast, Wildasin (1988) shows that when governments compete using expenditures rather than taxes, the result is an even greater divergence from efficiency.

⁷See Wilson and Wildasin (2001) for a comprehensive review of the literature on ‘beneficial’ tax competition.

⁸The same approach has also been adopted in the closely related literature on commodity taxation in jurisdictions that are members of a federation; see for example Mintz and Tulkens (1986).

⁹Of course, this point does not extend to situations where the public good is provided for citizen-consumers, but funded by firms.

quote: “The differences in the development of STAT and ITR-COR over time is striking. The former clearly fell over time while the latter did not, and if anything rose.” Mintz and Smart (2001) present and examine evidence that corporate income tax rates have remained the same or increased slightly since 1986 across provinces in Canada. Baldwin and Krugman (2004) also present empirical evidence (as well as a theoretical model) which counters the idea that historically high taxation countries in the European Union have had to lower their capital tax rates across the board as capital markets have become more integrated. Higgott (1999) draws attention to a number of other papers which cast doubt over the pervasiveness of the ‘race to the bottom’ hypothesis.

1.2 Outline of the Paper

The paper proceeds as follows. In the next section the primitives of the model are set up and the conditions for efficiency, both necessary and sufficient, are stated. In Section 3 the strategic game played by jurisdictional governments is set up. Section 4 then restricts the general framework to consider the Z-M model. It is in this section that we demonstrate conditions under which the equilibrium state of the economy is either efficient, inefficient with too little public good provision, or inefficient with too much public good provision. As in Z-M, we focus on jurisdictions that are ex ante symmetric, yielding a symmetric interior equilibrium. Section 5 discusses three examples; Cobb-Douglas, Constant Elasticity of Substitution (C.E.S.) and an example that we develop specifically for this paper which, for brevity, we refer to as D.W.Z. It is shown that Cobb-Douglas production technology must exhibit strictly decreasing returns to scale in order to satisfy standard Z-M assumptions, and that the only possible equilibrium outcome under Cobb-Douglas is inefficient with too little public good provision; a race to the bottom. It is also shown that technology based on C.E.S. does not satisfy the standard Z-M assumptions. D.W.Z. technology can be used to demonstrate all three possible outcomes. This is done through a simple variation of a parameter that determines the change in the rate of return to capital from production (a primitive). Variation of the rate of return to capital from production affects, in turn, the degree of complementarity between capital and the public good and hence the incentive to deviate from efficiency. Thus the example provides insight that is not directly available from the general model. It gives a clear illustration of how variation in the underlying economic structure can change the nature of the equilibrium outcome. Finally, conclusions are drawn

in Section 6.

2 Primitives and Production Efficiency

The model is of just two jurisdictions, 1 and 2.¹⁰ Within each jurisdiction, the representative citizen acts as a consumer and a producer. The firms produce a homogeneous consumer good, the sole consumption good in the economy. In the present section, where we examine efficiency, we will assume that the level of public good provision is chosen centrally by a planner for both jurisdictions. In the next section public good provision is decentralized through taxation by a government in each jurisdiction.

Citizens of jurisdiction i own a quantity of capital \bar{k}_i , $i \in \{1, 2\}$. Total capital supply is denoted by $\bar{k} = \bar{k}_1 + \bar{k}_2 = 2\bar{k}$. Capital is perfectly mobile between jurisdictions. A *capital allocation* is a vector of capital demands across the two jurisdictions, $\mathbf{k} = (k_i, k_j) \in \mathbb{R}_+^2$, where k_i is capital demand in jurisdiction i . Because capital is perfectly mobile across jurisdictions, k_i may comprise domestically or foreign owned capital. The same is true of k_j .

2.1 Public Goods and the Means of Production

Production results in output of the consumer good, which will also serve as the numeraire. The production function is denoted by

$$f(k_i, y_i) \tag{1}$$

where $i \in \{1, 2\}$. The overall functional form represents a production technology that depends on the level of public good provision y_i as well as capital k_i . We make relatively mild assumptions about the functional form of (1):

A1. Let the function $f : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ be quasi-concave on the domain $k_i \in [0, \bar{k}]$. Moreover, for given $y_i > 0$ assume that f has a convex segment on the domain $k_i \in [0, \tilde{k})$, and a strictly concave, twice continuously differentiable (C^2) segment on the domain $k_i \in [\tilde{k}, \infty)$ where \tilde{k} is a unique point in the interval $\tilde{k} \in [0, \bar{k}]$; $\partial f(k_i, y_i) / \partial k_i \rightarrow \infty$ as $k_i \rightarrow \tilde{k}$, and $\partial^2 f(k_i, y_i) / \partial k_i^2 < 0$ for $k_i \in [\tilde{k}, \infty)$. Let $f(0, y_i) = 0$.

A2. Let $f : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ be C^2 with respect to y_i for given $k_i > 0$. Assume that $\partial f(k_i, y_i) / \partial y_i \rightarrow \infty$ as $y_i \rightarrow 0$, and $\partial^2 f(k_i, y_i) / \partial y_i^2 < 0$ for $k_i \in (0, \bar{k})$ and $y_i > 0$.

¹⁰The model and results could be generalized to n jurisdictions, but without adding insight (except perhaps for convergence results if n is taken as a variable).

Figure 1a illustrates technology that conforms to our assumptions for the case of strict concavity everywhere ($\tilde{k} = 0$). An example of this production technology is Cobb-Douglas:¹¹

$$f(k_i, y_i) = k_i^\alpha y_i^\beta, \quad \alpha, \beta > 0. \quad (2)$$

Figure 1b illustrates a more general “S-shaped” production technology ($\tilde{k} > 0$) pictured in most intermediate level textbooks. An example of a function for the case of first increasing then decreasing returns to capital is:

$$f(k_i, y_i) = \max \left\{ 0, \left(\frac{\alpha}{2} k_i^2 - \frac{\delta}{3} k_i^3 \right) \sqrt{y_i} \right\}$$

where $\alpha, \delta > 0$.¹²

Figure 1c shows an alternative functional form, where the segment $[0, \tilde{k}]$ is linear. This form has been used in the growth literature; see for examples Romer (1990a, b) and Askenazy and Le Van (1999). An example of the production function in Figure 1c is:

$$f_i(k_i, y_i) = \begin{cases} \beta k_i & \text{if } k_i \leq \tilde{k} \\ \beta \tilde{k} + (k_i - \tilde{k})^\alpha y_i^{1-\alpha} & \text{if } k_i > \tilde{k} \end{cases} \quad (3)$$

where $\tilde{k} \in (0, \bar{k}]$. This functional form proves to be very versatile and can be used to construct an example, under appropriate parameter restrictions, of all the cases that we discuss at a general level in the present paper.

After setting out our general analysis based on the assumptions A1 and A2 above (together with others that will be introduced in due course) we show in Section 5 that a race to the top cannot occur under Cobb-Douglas technology. We then show that under the technology given by (3), there may be a race to the top, or to the bottom, or equilibrium may be efficient. The key property that is present in (3) but that is absent from Cobb-Douglas technology appears to be that returns to capital are at first (weakly) increasing then eventually decreasing. In (3), the parameter \tilde{k} turns out to be important in that it (indirectly)

¹¹In Section 5 we show that if the exponents sum to 1 or more (i.e. constant or increasing returns) a key stability assumption due to Z-M (introduced below as A4) is violated. In other words we show that, in order to conform to the Z-M framework, Cobb-Douglas must have decreasing returns to scale. We also show that C.E.S. violates A4, being equivalent to constant returns Cobb-Douglas.

¹²Note that our assumption $\partial f(k_i, y_i) / \partial k_i \rightarrow \infty$ as $k_i \rightarrow \tilde{k}$ is not satisfied for this example. The assumption allows us to use standard technical apparatus to prove existence of equilibrium. But the example indicates that this assumption is not strictly necessary. Weakening this assumption would make the proof of existence much more involved without actually adding any insight. More broadly, our production technology conforms to the general class of production technology with non-convexities discussed by Mas-Colell, Whinston and Green (1995 Chapter 5).

parameterizes *mpgv*.¹³ When production technology is given by (3) it is sufficient to vary only \tilde{k} in order to obtain a race to the top, or to the bottom, or efficiency.¹⁴

Why should we accept (3) over more ‘standard’ examples such as Cobb-Douglas or Constant Elasticity of Substitution (C.E.S.)? Recognition that returns to a factor have the ‘at first increasing but eventually decreasing returns’ property captured by (3) goes all the way back to Adam Smith’s *Wealth of Nations*. Moreover, this property has been verified empirically in a wide variety of settings, as acknowledged by Frisch (1965, Section 6b, page 88);

“Empirical investigations into a number of different production processes show that production as a rule passes through a stage of rising and then a stage of diminishing returns, if we vary a single factor while allowing the others to remain constant. ... And it is possible to recognize this law in practically every sphere of agriculture as well as of industry.”

Thus, (3) is more empirically relevant than Cobb-Douglas or C.E.S. because these latter functions exhibit everywhere decreasing returns to a factor.¹⁵

Each of the illustrations in Figure 1 is drawn under the assumption that y_i is constant. The impact on the production function (drawn in k_i space) of a change in y_i is illustrated in Figure 2 (drawn for the production function pictured in Figure 1b). Here we see that for each given level of k_i the level of output is increasing in y_i . Because A2 implies that $f(k_i, y_i)$ is strictly concave in y_i , the amount by which output increases is decreasing in y_i .

We now consider the ‘production technology’ of the public good. From the point of view of the planner, the cost of y_i units of the public good is y_i units of the numeraire. For the decentralized problem, this technology is represented below in the more familiar form of the government revenue function; $y_i = t_i k_i$ (see A3).

¹³The concept of *mpgv* was discussed informally in the introduction. It will be made precise in Section 4.

¹⁴General analytical results linking \tilde{k} to *mpgv* are outside the scope of this paper. We believe that our set-up is reasonable since A1 and the assumption that determines the behavior of *mpgv* (A5) taken together, are mild assumptions for this literature, and facilitate a framework in which we are able to examine other important issues.

¹⁵In fact, functional forms represented by Figure 1b capture even more closely what Frisch describes. However, such functions turn out to be analytically intractable. Functional forms such as (3) are generally accepted as a tractable approximation.

2.2 The Efficiency of Production

The definition of production efficiency adapts a standard definition to the context of the present model.

Definition 1. A *plan*, consisting of a capital allocation $\mathbf{k}^E = (k_i^E, k_j^E) \in \mathbb{R}_+^2$ and vector of public goods $\mathbf{y}^E = (y_i^E, y_j^E) \in \mathbb{R}_+^2$, is *efficient* if:

- 1.1. $k_i^E + k_j^E = \bar{k}$;

- 1.2. for all other capital allocations $\mathbf{k} = (k_i, k_j) \in \mathbb{R}_+^2$ satisfying $k_i + k_j = \bar{k}$ and public goods $\mathbf{y} = (y_i, y_j) \in \mathbb{R}_+^2$, it holds that

$$\begin{aligned} & f(k_i^E, y_i^E) + f(k_j^E, y_j^E) - y_i^E - y_j^E \\ & \geq f(k_i, y_i) + f(k_j, y_j) - y_i - y_j. \end{aligned}$$

Under Definition 1, a plan is efficient if it entails the largest possible surplus for division between citizens in the two jurisdictions. It will be convenient to represent efficient capital allocations and public goods in terms of the allocations of capital and public goods to the respective jurisdictions; (k_i^E, y_i^E) and (k_j^E, y_j^E) . Thus, if $\mathbf{k}^E = (k_i^E, k_j^E)$ and $\mathbf{y}^E = (y_i^E, y_j^E)$ constitute an efficient plan, we also call the induced outcome $\mathcal{E} = \{(k_1^E, y_1^E), (k_2^E, y_2^E)\}$ efficient.

In general terms the planner's problem can be expressed as the maximization of the following objective function:

$$\begin{aligned} \max_{k_i, y_i, k_j, y_j} \Omega(k_i, y_i, k_j, y_j) &= f(k_i, y_i) + f(k_j, y_j) - y_i - y_j \\ &\text{subject to} \\ \bar{k} &= k_i + k_j \text{ where } (k_i, k_j) \in \mathbb{R}_+^2. \end{aligned}$$

Using the feasibility of total capital usage in the maximand the problem simplifies to

$$\max_{k_i, y_i, y_j} \Omega(k_i, y_i, y_j) = f(k_i, y_i) + f(\bar{k} - k_i, y_j) - y_i - y_j.$$

The following set of first order conditions are necessary when efficiency implies that output

is positive in both jurisdictions:

$$\frac{\partial f(k_i^E, y_i^E)}{\partial k_i} = \frac{\partial f(\bar{k} - k_i^E, y_j^E)}{\partial (\bar{k} - k_i)}; \quad (4)$$

$$\frac{\partial f(k_i^E, y_i^E)}{\partial y_i} = 1; \quad (5)$$

$$\frac{\partial f(\bar{k} - k_i^E, y_j^E)}{\partial y_j} = 1. \quad (6)$$

Because the capital feasibility condition $k_i^E + k_j^E = \bar{k}$ is used to substitute for k_j^E , solving the planner's problem for an efficient plan involves solving three first order conditions for the three unknowns, k_i , y_i and y_j .

The first condition, (4), states the familiar requirement that at an efficient plan the marginal unit of capital in each jurisdiction is equally productive. Conditions (5)-(6) state, for jurisdictions i and j respectively, that the marginal cost of foregoing a unit of the consumption good to produce the marginal unit of the public good must be equal to the marginal product of the public good in production. Conditions (4)-(6) are the standard first order conditions for \mathcal{E} to be an interior maximum.

The following lemma characterizes necessary and sufficient conditions under which efficiency implies that production takes place in *both* jurisdictions, i.e. the efficient plan is interior. To ensure that the point is indeed a global maximum, besides (4)-(6), we need the additional assumption that $\Omega(k_i, y_i, y_j)$ is strictly quasi-concave.¹⁶:

Lemma 1. *(Sufficiency) Assume that A1-A2 hold and that there exists a plan $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$ satisfying (4), (5) and (6). If the function $\Omega(k_i, y_i, y_j)$ is strictly quasi-concave then the plan \mathcal{E} is a unique symmetric interior efficient point.*

(Necessity) Assume that $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$ is an efficient plan and is interior. Then (4), (5) and (6) are satisfied.¹⁷

If such an interior maximum does not exist we may have a corner solution instead, where efficiency implies that all production occurs in just one jurisdiction. In this present paper we will maintain the focus of the previous literature on the situation where the efficient point is interior, that is, where production occurs in both jurisdictions. Efficiency at a corner solution is considered by Wooders et al (2001).

¹⁶The constraint set is convex.

¹⁷See Wooders et al (2001) for proof of Lemma 1.

3 Tax Competition in the Z-M Framework

In this section we determine the nature of tax competition between jurisdictions in terms of a decentralized equilibrium. The government in each jurisdiction sets the tax on capital expenditure to maximize the welfare of its representative citizen. Firms behave competitively, taking policies as given and choosing capital to maximize profits. Since citizens play the role of consumers and producers, they simply consume the resulting surplus. All of these aspects of the model will now be formalized for use in characterizing the decentralized equilibrium.

The literature has adopted two alternative assumptions with regard to the modelling of a government's attitude towards the price of capital. Z-M, on the one hand, treat each jurisdiction as a price taker. Jurisdictions cannot see through the effects of their taxes on the world net-of-tax price of capital. On the other hand, Wildasin (1988, 1989) and Bayindir-Upmann (1998) assume that each government is a strategic player rather than a price taker. In keeping with our focus on the Z-M model, we assume that each government is a price taker. We conjecture that the incentive to deviate from efficiency that we identify would also arise if governments took account of their impact on the world net-of-tax price of capital. To confirm this analytically may be significantly more complicated.

3.1 The Firm's View of Production

We begin by characterizing firm behavior. As already mentioned, the representative firm in jurisdiction i is assumed to behave competitively in capital and goods markets and to have no influence over policy making. It takes as given the level of public good provision, y_i , and the gross price of capital, p_i . Its objective is simply to maximize profits by choosing the appropriate level k_i^* of capital;

$$\max_{k_i \geq 0} \pi_i = f(k_i, y_i) - p_i k_i, \quad (7)$$

where p_i is the per unit gross price of capital. The first order condition of the firm in jurisdiction i is

$$\frac{\partial f(k_i, y_i)}{\partial k_i} - p_i = \begin{cases} 0 & \text{for } k_i^* \in [0, \bar{k}] \\ \geq 0 & \text{for } k_i^* = \bar{k}. \end{cases} \quad (8)$$

The first order condition holds with equality at an interior solution. But at a corner solution $k_i^* = \bar{k}$, it may be the case that $\partial f(k_i, y_i) / \partial k_i > p_i$ and the firm could increase profits were it able to increase its capital use. Define $\underline{p}_i = \partial f(\bar{k}_i, y_i) / \partial \bar{k}_i$. At any price $p_i \leq \underline{p}_i$, we have

$k_i^* = \bar{k}_i$. The gross price of capital is given by the identity

$$p_i = p_i(r, t_i) \equiv r + t_i.$$

Because capital is perfectly mobile between jurisdictions there is a single world price of capital r . Tax revenue is deducted on the destination basis, in the sense that the local tax rate must be paid on all the capital used in production within the jurisdiction, whether of local or foreign origin.¹⁸

By standard arguments, because the production function is continuous and strictly concave for $k_i \geq \tilde{k}$, a profit maximizing solution k_i^* does indeed exist. Moreover, providing that the average revenue from production is greater than the average cost, the solution k_i^* is unique and greater than zero.

Given y_i , let \hat{k}_i denote the maximum feasible average product of capital, defined by $\hat{k}_i(y_i) = \arg \max_{k_i} (f(k_i, y_i)/k_i)$.¹⁹ Under our assumptions (A1-A2) and with the firm taking r , t_i and y_i as fixed and positive, if $f(\hat{k}_i, y_i)/\hat{k}_i > p_i$ then $k_i^* > \hat{k}_i > 0$ and $\pi_i > 0$. Define $\bar{p}_i = f(\hat{k}_i, y_i)/\hat{k}_i$. Then, from (A.1), k_i^* converges continuously to \hat{k}_i as p_i is increased. This is illustrated in Figure 3. Clearly, if p_i is increased to the point where $f(\hat{k}_i, y_i)/\hat{k}_i = p_i$ then $k_i^* = \hat{k}_i$ and $\pi_i = 0$. At the point where $f(\hat{k}_i, y_i)/\hat{k}_i = p_i$ and $\pi_i = 0$ there obviously also exists the solution $k_i^* = 0$ to the firm's problem. However, we will assume that in this situation the firm chooses $k_i^* = \hat{k}_i$. Consequently, we have a well defined function that maps p_i into k_i^* : if $p_i \leq \underline{p}_i$, then $k_i^* = \bar{k}_i$; if $\underline{p}_i < p_i < \bar{p}_i$ then k_i^* is the unique k_i that solves $\partial f(k_i, y_i)/\partial k_i = p_i$; if $p_i > \bar{p}_i$ then $k_i^* = 0$. This function between p_i and k_i^* will be important when considering a firm's response to tax setting.

We now have a complete characterization of the solution to the firm's problem for a fixed level of the public good. Define the feasible set for r and t_i , given y_i , as

$$F_{y_i} = \{r \geq 0, t_i \geq 0 \mid \frac{f(\hat{k}_i, y_i)}{\hat{k}_i} \geq r + t_i\}.$$

3.2 Capital Market Clearing for Given Public Good Levels

Capital market clearing is the result of competitive behavior by the firms in both jurisdictions. Equilibrium in the capital market is defined as follows:

¹⁸Here the tax is modelled as specific. From the work of Lockwood (2001) and Lockwood and Wong (2000), we anticipate that our results would change quantitatively but not qualitatively if an ad valorem and/or origin based regime were modelled instead.

¹⁹That is, for ease of notation, when the given value of y_i is clear we will abbreviate $\hat{k}_i(y_i)$ to \hat{k}_i .

Definition 2. A *capital market equilibrium* for a given level of public good y_i is a pair (r^*, \mathbf{k}) , where r^* is the equilibrium rate of interest and \mathbf{k} is a capital allocation $\mathbf{k} = (k_i^*, k_j^*) \in \mathbb{R}_+^2$, such that:

- (i) for each $i = 1, 2$, $k_i^* \in \arg \max_{k_i} \pi_i(r^*, t_i, y_i)$;
- (ii) $\sum_{i=1,2} k_i^* \leq \bar{k}$ with strict equality if $r^* > 0$.

Section 3.1 stated the firm's problem for a single jurisdiction. The first condition says that one such problem must be solved by each firm in each jurisdiction. The second is a feasibility condition, saying that (for a positive price of capital) total demand for capital must be equal to supply.

3.3 Welfare and the Feasibility of Consumption

The output available for consumption by citizens of jurisdiction i is

$$c_i \leq \pi_i + r\bar{k}_i. \quad (9)$$

That is, citizens of jurisdiction i receive the profits from the firm in that jurisdiction and also the revenue from rental of their endowment of capital. We assume that consumers have monotonically strictly increasing preferences for output.

3.4 The Government's View of Production

Government revenue is determined by taxes on capital.

A3. The availability of the public good within a jurisdiction is given by the government budget condition:

$$y_i = t_i k_i. \quad (10)$$

The feasible level of public good provision is given by the same function as for the planner (see Section 2.2).²⁰

The decentralized problem for the governments, described in this section, differs from that of the planner in that the opportunity cost of the public good is expressed in terms of the capital tax base and the tax rate, as in a standard tax competition model. This reflects the fact that, while a planner can simply pick the optimal level of public good provision y_i

²⁰More generally, the government budget condition need not hold with equality. But in the present context the government would always want to use all of the revenue it collects to provide the public good.

(subject to feasibility conditions) the government has to raise the revenue through taxation in order to produce the public good.

The government sets taxes in order to maximize consumption of the representative citizen, determined by equation (9), taking as given the actions of the government in the other jurisdiction:

$$\begin{aligned} & \max_{t_i} \pi_i(t_i, y_i, r) + r\bar{k}_i \\ & = \max_{t_i} f(k_i, y_i) - (r + t_i)k_i + r\bar{k}_i \end{aligned}$$

subject to the constraints that follow.

1. The firm in jurisdiction i maximizes profit at k_i^* given y_i . Thus,

$$\partial f(k_i^*, y_i) / \partial k_i \geq r + t_i.$$

2. The production of the public good is feasible;

$$y_i = t_i k_i^*.$$

3. $r + t_i \in F_{y_i}(r, t_i)$.

When we substitute the budget constraint and the firms' demand for capital k_i^* into the production function the above maximization problem is equivalent to maximization with respect to t_i only. Although we showed that a solution to the firm's problem k_i^* can always be found, it is not obvious that we can always find a C^1 solution k_i^* that is consistent with conditions 1 and 2 of the government's problem. So we introduce below assumptions A4 and A5 in order to guarantee a unique solution k_i^* for given r and t_i . The following assumption is taken directly from Z-M.

A4. (Z-M assumption)²¹

$$t_i(\partial^2 f / \partial k_i \partial y_i) - \partial^2 f(k_i, y_i) / \partial k_i^2 < 0, \text{ where } t_i = y_i / k_i. \quad (11)$$

Given the importance of the term to this assumption, we define the *overall marginal productivity of capital in i* ($ompk_i$) as

$$ompk_i: t_i(\partial^2 f(k_i, y_i) / \partial k_i \partial y_i) - \partial^2 f(k_i, y_i) / \partial k_i^2.$$

²¹The following condition appears in Zodrow and Mieszkowski (1986) as equation (17).

Equation (11) reflects the responsiveness of the (diminishing) $ompk_i$ to a change in taxes with an accompanying change in public good provision. Note that (11) is the second derivative of the production function with respect to capital. Thus, the negativity of $ompk_i$ implies that the derivative of the production function with respect to capital, which (from A1) is a continuous function, is downward sloping. To put this intuitively, even though capital is used for production of output and output can be used to produce public goods, which further enhance marginal productivity of capital, under A4 the production function exhibits *diminishing ompk*. This is a natural assumption.²² It rules out a destabilizing ‘virtuous circle’ in which more capital facilitates more public good provision which enhances productivity to the extent that the demand for capital increases, and so on.

We are now able to establish that, under A4, a unique solution exists to the firm’s problem and this solution can be expressed as a function of r and t_i . Once we take the budget constraint (holding with equality) into account, $\bar{p}_i(k_i) = f(\hat{k}(t_i k_i), t_i k_i) / (\hat{k}(t_i k_i))$. This can be interpreted as the overall average product of capital for a given tax rate. Note that, by A4, $\bar{p}_i(k_i)$ is always higher than the overall marginal product and always decreasing as k_i increases.

Proposition 1. *Assume A1-A4. For given $r > 0$, $t_i > 0$ in the feasible set F_{y_i} , there always exists at least one solution k_i^* that satisfies both conditions 1 and 2 of the government’s problem. Moreover if A4 is satisfied globally, then this solution k_i^* is uniquely defined for every r, t_i in F_{y_i} and the function $k_i(t_i, r)$ is C^1 .*

Proof. Note that $\partial f(k_i, y_i) / \partial k_i$ is C^1 (by A1), and hence $df(k_i, t_i k_i) / dk_i$ is C^1 (by A1 and A2). And by A4, for any $t_i > 0$ it holds that $df(k_i, t_i k_i) / dk_i$ is monotonically decreasing in k_i . Since A4 is satisfied globally, and with the firm taking r and t_i as fixed and positive, $\bar{p}_i(k_i) = f(\hat{k}_i, k_i t_i) / \hat{k}_i \geq p_i = r + t_i > 0$. It follows that there exists a solution k_i^* with $k_i^* \geq \hat{k}_i > 0$ for which $\pi_i(t_i, r) \geq 0$. \square

The proof of Proposition 1 is illustrated in Figure 4. Along the line $\bar{p}_i(k_i)$, the representative profit-maximizing firm makes zero profits; if the cost of capital were equal to $\bar{p}_i(k_i)$ then average cost equals average product and profits are zero. Above the line $\bar{p}_i(k_i)$, profits are negative, which we have ruled out by assuming that $r > 0$, $t_i > 0$ are in the feasible set F_{y_i} . Below the line $\bar{p}_i(k_i)$, profits are positive. Provided the line $df(k_i, k_i t_i) / dk_i$ intersects

²²This is not the same as the standard assumption of diminishing marginal productivity of capital, that $\frac{\partial f_i(k_i, y_i)}{\partial k_i}$ is negatively sloped.

the line p_i below the line $\bar{p}_i(k_i)$, the firm makes positive profits. By A4, the average product $\bar{p}_i(k_i)$ is always higher than the marginal product, hence the intersection is always below $\bar{p}_i(k_i)$.

The fact that the government views production in a different way than firms should be highlighted. While firms take the level of the public good as given, governments account fully for the impact of providing the public good in making decisions on the level of provision, and the requisite level of taxation.

Under conditions of Proposition 1, the function $\pi_i(t_i, y_i, r)$ can be written as a function of r and t_i only; $\pi_i(t_i, r)$. Holding r constant, the problem of the government in jurisdiction i is then solved using the following first order condition;

$$\begin{aligned} \frac{dc_i}{dt_i} &= \left(\frac{\partial f}{\partial k_i} - (t_i + r) \right) \frac{\partial k_i}{\partial t_i} \\ &+ \frac{\partial f}{\partial y_i} \left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} \right) - k_i = 0. \end{aligned} \quad (12)$$

Assumption A5, introduced below, ensures that this problem has a unique maximum.²³

Implicit in the formulation of the government's problem is the assumption that lump-sum transfers are not available as a policy instrument. This is a standard assumption in the tax competition literature. It is well recognized that if lump-sum transfers are possible then efficiency can be achieved directly, and the whole efficiency question of tax competition vanishes.

3.5 Definition of Equilibrium

We now introduce a formal definition of equilibrium:

An *equilibrium* is defined by taxes t_i^* , a rate of interest r^* and capital allocations k_i^* , $i = 1, 2$, such that:

- (i) $t_i^* = \arg \max_{t_i} (c_i(t_i, r))$, $i = 1, 2$;
- (ii) there exists a rate of interest r^* and a capital allocation $\mathbf{k} = (k_1^*, k_2^*)$ such that the capital market is in equilibrium;
- (iii) budgets balance - $y_i^* = t_i^* k_i^*$, $i = 1, 2$.

²³The proof of Theorem 1 shows that the solution to the government's problem is unique for the case where equilibrium is efficient. Uniqueness of the solution to the government's problem is shown for the cases of under-provision and over-provision in the proof of Theorem 2.

We now have a complete statement of equilibrium: each government sets its tax to maximize the welfare of its representative citizen (taking r as given); firms behave competitively in their production decisions, taking interest rates, taxes and public good provision as given and choosing capital to maximize profits; capital markets clear.

4 Our Alternative Assumption and Equilibrium

It is generally understood from Z-M that when capital is mobile the incentive to attract capital through a reduction of taxes will bring about an under-provision of the public good, even when its role is to enhance the marginal productivity of capital. It is in this section that we show how a wider range of (mutually exclusive) alternatives may arise if we use an alternative assumption to describe the impact of public good provision on the marginal productivity of capital.

One of the properties that carries over to our model from the Z-M model, where the primitives are ex ante symmetric across jurisdictions and production occurs in both jurisdictions, is that the equilibrium is symmetric. Taxes and public good provision are the same across jurisdictions, whether efficient or inefficient in equilibrium. As mentioned above, we will focus on this type of symmetric equilibrium in our analysis as well.

We have already invoked Assumption A4 which was taken from the Z-M model. Now we introduce a variation of the other key Z-M assumption.

A5. Fix the capital in each jurisdiction at $\bar{k}/2$; set $k_i = k_j = \bar{k}/2$. Assume that the function

$$1 - k_i \frac{\partial^2 f(k_i, y_i)}{\partial k_i \partial y_i}$$

is monotonically increasing in y_i and that the following two conditions hold:

$$\begin{aligned} \lim_{y_i \rightarrow 0} \left(1 - k_i \frac{\partial^2 f(k_i, y_i)}{\partial k_i \partial y_i}\right) &< 0; \\ \lim_{y_i \rightarrow \infty} \left(1 - k_i \frac{\partial^2 f(k_i, y_i)}{\partial k_i \partial y_i}\right) &> 0. \end{aligned}$$

An implication of A5 is that $k_i \partial^2 f(k_i, y_i) / \partial k_i \partial y_i$ declines monotonically with y_i . A5 replaces the assumption made by Z-M that²⁴

$$1 - k_i \frac{\partial^2 f(k_i, y_i)}{\partial k_i \partial y_i} > 0 \text{ for all } y_i.$$

²⁴This assumption appears as equation (16) in Zodrow and Mieszkowski (1986).

We now explain assumption A5. in more detail, and compare it to the Z-M assumption that it replaces. Given the importance of the term to both assumptions, we define the *marginal public good valuation in i* ($mpgv_i$) as

$$mpgv_i: k_i \partial^2 f(k_i, y_i) / \partial k_i \partial y_i.$$

This valuation is made from the viewpoint of the jurisdiction by its representative citizen or government, and not for the economy as a whole (that is, from the viewpoint of a planner). The $mpgv$ measures the extent to which output is increased - through productivity enhancement - by the marginal unit of the public good. Our assumption says that the marginal value of the public good is higher than the marginal cost when public good provision is relatively low, but then declines as public good provision is increased, eventually falling below marginal cost when public good provision is relatively high. This contrasts with the assumption originally made by Z-M which stipulates that the marginal value of the public good is *never* as high as the marginal cost. By assumption, the marginal cost of the public good is equal to 1.

Assumption A5 essentially says that the public good becomes less effective at enhancing the marginal productivity of capital as public good provision increases. This is similar to the assumption made by Barro (1990) and Barro and Sala-i-Martin (1995 Chapter 4, Section 4). They too focus on a change in the marginal impact of the public good on the marginal productivity of capital but they look at its variation with respect to the tax rate, holding output constant.

In effect, Z-M introduce the assumption that, starting from any efficient level of public good provision, a jurisdiction can make itself unilaterally better off by lowering taxes. Consequently, the state of the economy *must* be inefficient in equilibrium with underprovision of the public good, the familiar ‘race to the bottom’. It is our alternative assumption, embodied in A5, that underpins a wider range of possibilities. As explained above, three situations can arise. One is the same as Z-M’s; there is a unilateral incentive to deviate from the efficient plan by reducing taxes. Another equally valid situation is that there is a unilateral incentive to deviate from the efficient plan by *raising* taxes. This introduces the possibility that equilibrium supports inefficiently high levels of taxation and public good provision - ‘government is too big’. Finally, it is possible that the state of the economy is efficient in equilibrium. Recall that this is a knife-edge case that lies between the other two possibilities.

Having introduced A5, the following lemma establishes that there must exist a level of capital usage at which the associated increase in output due to the increased marginal productivity of capital is exactly equal to the marginal cost:

Lemma 2. *Assume A1, A2, and A5 and that $f(k_i, y_i)$ is C^2 on the compact set $k_i \in [\tilde{k}, \bar{k}]$ and C^2 on the set $y_i \in \mathbb{R}_+$. Then by A5, there exists, for $k_i = \bar{k}/2$, a value $y_i^I \in \mathbb{R}_+$ such that*

$$mpgv_i = k_i \frac{\partial^2 f(k_i, y_i^I)}{\partial k_i \partial y_i} = 1.$$

Proof: Follows from a straightforward application of the intermediate value theorem. \square

By following exactly the same steps as in Z-M, we can derive the change in capital demand within a jurisdiction due to a change in t_i . Differentiate (8) and (10) and combine the results to yield

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - mpgv_i}{ompk_i}. \quad (13)$$

The sign of this expression depends on the level of capital usage and public good provision. The denominator is negative by A4. If the public good had no impact on the marginal productivity of capital (as is usually assumed) then we would have $mpgv_i = 0$. The numerator compares the marginal value of the public good to the marginal cost. By Lemma 2 we know that there exists a value $y_i^I \in \mathbb{R}_+$ for which $\partial k_i / \partial t_i = 0$. This corresponds to a unique value of t_i denoted $t_i^I = \frac{y_i^I}{k_i}$ where $k_i = \bar{k}/2$.

4.1 Main Results and Intuition

We first present our two main results, and then we present a simple framework for understanding how the results work, which will also illuminate how the results relate to each other.

First, we introduce some helpful terminology.

Let (k_i^E, y_i^E) and (k_j^E, y_j^E) describe an efficient plan. Let the *policies induced by an efficient plan* (t_i^E, y_i^E) and (t_j^E, y_j^E) be given by $t_i^E = y_i^E / k_i^E$ and $t_j^E = y_j^E / k_j^E$.

Let (t_i^*, y_i^*) and (t_j^*, y_j^*) denote taxes and the induced public goods provision, given that $k_i^* = k_j^* = \frac{k}{2}$ in the decentralized equilibrium. The following Theorem demonstrates conditions under which the equilibrium is efficient. Symmetry in the following result means a solution in which $k_i = k_j$, $y_i = y_j$, $t_i = t_j$ etc.

Theorem 1. Assume A1-A5 and that there exists a symmetric interior efficient plan $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$.²⁵ Assume that $\tilde{k} < \bar{k}/2$. Assume $y_i^I = y_i^E$. Then there exists a decentralized equilibrium with taxes (t_i^E, t_j^E) , $r^* = f'(k_i^E, y_i^E) - t_i^E$, and capital allocation $k_i^* = k_j^* = \bar{k}/2$. The induced public good supply is y_i^E, y_j^E .

The assumption that $\tilde{k}_i < \bar{k}/2$ implies that diminishing returns to capital sets in for relatively low levels of capital use. Theorem 1 relates to a special ‘knife edge’ case in which $y_i^I = y_i^E$. There is no reason to expect, a priori, that y_i^I and y_i^E will coincide. The remaining possibilities are now considered. To exhaust the full range of possibilities, we need to consider the situation where $y_i^I < y_i^E$ and where $y_i^E < y_i^I$ as well. These two possibilities are dealt with simultaneously in the following theorem. We continue to assume that A1-A5 hold.

Theorem 2. Assume A1-A5 and that there exists an interior symmetric efficient plan $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$. Assume that $\tilde{k} < \bar{k}/2$. Then there exists a symmetric interior decentralized equilibrium with $t_i^* = t_j^* = t^*$, $k_i^* = k_j^* = \bar{k}/2$ and r^* , and induced level of public good provision y_i^* . The equilibrium may be characterized as follows:

(i) If $y_i^I < y_i^E$ then $y_i^I < y_i^* < y_i^E$ so the equilibrium outcome is inefficient, with under-provision of the public good.

(ii) If $y_i^E < y_i^I$ then $y_i^E < y_i^* < y_i^I$ so the equilibrium outcome is inefficient, with over-provision of the public good.

The intuition behind both of these results can be seen easily by using (13) to determine *the incentive to deviate from the efficient solution*.²⁶ By assumption, the government anticipates that in equilibrium $\partial f(k_i^*, y_i)/\partial k_i = r + t_i$. Also, by the government budget constraint (10), $\partial y_i/\partial t_i = k_i$ and $\partial y_i/\partial k_i = t_i$. Using these facts in (12), we obtain the following simplified form:

$$\frac{dc_i}{dt_i} = \frac{\partial k_i}{\partial t_i} \frac{\partial f(k_i, y_i)}{\partial y_i} t_i + k_i \left(\frac{\partial f(k_i, y_i)}{\partial y_i} - 1 \right).$$

Now, by (5) at an efficient allocation $\partial f(k_i, y_i)/\partial y_i = 1$. So, using this and (13), the incentive to deviate from an efficient solution is given by:

$$\frac{dc_i}{dt_i} = \left(\frac{1 - mpgv_i}{ompk_i} \right) t_i. \quad (14)$$

The incentive to deviate can then be determined directly from the relationship between y_i^E and y_i^I . If $y_i^E < y_i^I$ then because $mpgv_i$ is declining in y_i this implies that $mpgv_i > 1$ at y_i^E .

²⁵Symmetry here means a solution in which $k_i = k_j = \bar{k}/2$, $y_i = y_j$, $t_i = t_j$ etc.

²⁶We are grateful to an anonymous referee for suggesting the following exposition of our results.

Then from (13) we have $\partial k_i / \partial t_i > 0$ and from (14) we have $dc_i / dt_i > 0$. This would generate a race to the top.²⁷ In summary, at y_i^E we have:

$$y_i^E < y_i^I \Leftrightarrow mpgv_i > 1 \Leftrightarrow \frac{\partial k_i}{\partial t_i} > 0 \Leftrightarrow \frac{dc_i}{dt_i} > 0.$$

Obviously, by the same logic, if $mpgv_i < 1$ at the efficient solution then there is an incentive to deviate by lowering taxes, which would generate a race to the bottom. And if $mpgv_i = 1$ at the efficient solution then there is no incentive to deviate from the efficient solution, so the equilibrium is efficient.

5 Examples

First we look at a Cobb-Douglas example. We show that in order to satisfy A4 the Cobb-Douglas production technology must exhibit decreasing returns to scale. Then it is easy to show that $y_i^E > y_i^I$; by Theorem 2 there must be a race to the bottom under Cobb-Douglas. We also briefly consider C.E.S., and show why it cannot be used as an example of production technology within the Z-M modelling framework because Assumption A4 fails. Finally, an example of (3) is used to demonstrate and compare (mutually exclusive) equilibria with all possible outcomes; a race to the top, a race to the bottom and efficiency.

Cobb-Douglas. Consider Cobb-Douglas production technology, as specified in equation (1). Following the derivation set out in Section 4, we have that

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - mpgv_i^{CD}}{ompk_i^{CD}}$$

where

$$\begin{aligned} mpgv_i^{CD} &= \alpha \beta k_i^\alpha y_i^{\beta-1}; \\ ompk_i^{CD} &= \alpha k_i^{\alpha-2} y_i^{\beta-1} (\beta t_i k_i - (1 - \alpha) y_i) \end{aligned}$$

²⁷An alternative but equivalent approach would be to focus on how the demand for capital responds to a (balanced budget) change in public good provision. The first order condition for profit maximization, $\partial f(k_i, y_i) / \partial k_i - r + t_i = 0$ together with the government budget constraint $y_i = t_i k_i$ implicitly defines the demand for capital as a function of y_i ; $k(y_i, t_i(k_i))$. Then, differentiating both equations with respect to y_i yields

$$\frac{dk_i}{dy_i} = \frac{(1 - mpgv_i)}{k_i (\partial^2 f / \partial k_i^2 + t_i)}.$$

The basic analysis would proceed in the same way, but could be explained in terms of how public good provision itself affects the demand for capital. A race to the top can then be understood in the following terms: The government will increase the level of public good provision beyond the efficient level if it actually increases capital within the jurisdiction, thereby increasing its tax base.

The first thing to check is the conditions under which A4 is satisfied. Notice that for $t_i k_i = y_i$, which is a requirement of A4, then $ompk_i^{CD} < 0$ if and only if $\alpha + \beta < 1$; A4 is satisfied if and only if $\alpha + \beta < 1$. Hence, the Cobb-Douglas production function must satisfy decreasing returns for the stability condition A4 to be satisfied. Notice that $\partial k_i / \partial t_i$ is undefined for $\alpha + \beta = 1$ since $ompk_i^{CD} = 0$. A destabilizing cycle results in which an increase in taxation facilitates more public good provision which enhances productivity to the extent that the demand for capital increases, and so on.

We solve for a symmetric equilibrium, normalized so that $\bar{k} = 2$. It is straightforward to solve for y_i^E and y_i^I .²⁸ We have²⁹

$$\begin{aligned} y_i^E &= \left(\frac{k_i^{-\alpha}}{\beta} \right)^{\frac{1}{\beta-1}}; \\ y_i^I &= \left(\frac{k_i^{-\alpha}}{\alpha\beta} \right)^{\frac{1}{\beta-1}}. \end{aligned}$$

By inspection, $y_i^I < y_i^E$ for $\alpha + \beta < 1$. Then by Theorem 2, a race to the bottom occurs under Cobb-Douglas with decreasing returns to scale.³⁰

C.E.S. We now show why C.E.S. production technology cannot be used as an example. Let the production technology be given by

$$f(k_i, y_i) = (\delta_k k_i^\rho + \delta_y y_i^\rho)^{\frac{1}{\rho}}.$$

Then, again, following the derivation set out in Section 4, we have that

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - mpgv_i^{CES}}{ompk_i^{CES}}$$

where

$$\begin{aligned} mpgv_i^{CES} &= (1 - \rho) \delta_k \delta_y y_i^{\rho-1} (\delta_k k_i^\rho + \delta_y y_i^\rho)^{\frac{1}{\rho}-2}; \\ ompk_i^{CES} &= (1 - \rho) \delta_k \delta_y k_i^{\rho-2} y_i^{\rho-1} (\delta_k k_i^\rho + \delta_y y_i^\rho)^{\frac{1}{\rho}-2} (y_i - t_i k_i). \end{aligned}$$

Notice that under the balanced budget restriction, which is applied under A4, $y_i = t_i k_i$ and so $ompk_i^{CES} = 0$. As a consequence, $\partial k_i / \partial t_i$ is undefined under C.E.S.. This follows from

²⁸A full derivation of equilibrium is carried out for D.W.Z. technology below. The derivation, for Cobb-Douglas, of y_i^E and y_i^I is carried out in exactly the same way but is omitted since it is more straight-forward.

²⁹It is noteworthy that y_i^I exists for Cobb-Douglas technology. Therefore, Cobb-Douglas does not conform to the *original* Z-M assumption (*not* our replacement A5) that $mpgv_i < 1$ for all y_i .

³⁰It is not possible to obtain a general closed form solution for y_i^* . Simulation results confirm that $y_i^I < y_i^* < y_i^E$ for $\alpha + \beta < 1$, that $r^* > 0$, and that firms make positive profits in equilibrium.

the fact that, like for Cobb-Douglas where $\alpha + \beta = 1$, C.E.S. exhibits constant returns to scale.

D.W.Z. Now we have our example which illustrates all of the situations characterized in Theorems 1 and 2. The example is based on the production function (3). We will work with a particular form of this function for which we set parameter values at $\alpha = \beta = 1/2$. In that case

$$f(k_i, y_i) = \begin{cases} k_i/2 & \text{if } k_i \leq \tilde{k} \\ \tilde{k}/2 + (k_i - \tilde{k})^{\frac{1}{2}} y_i^{\frac{1}{2}} & \text{if } k_i > \tilde{k} \end{cases}$$

with $\tilde{k} \in [0, \bar{k}/2)$. The restriction that $\tilde{k} < \bar{k}/2$ is imposed to allow a symmetrical equilibrium in which each jurisdiction demands $\bar{k}/2$ and firms make non-negative profits.³¹ This will enable us to compare an efficient solution to a symmetrical decentralized equilibrium solution, for which the representative firm in each jurisdiction has an interior solution to its problem. Again, to keep presentation tidier we will impose the additional parameter restriction $\bar{k} = 2$.

It is first worth noting that the public good has no impact on the level of output for $k_i \leq \tilde{k}$. Thus, strictly speaking assumption A2 holds only for $k_i > \tilde{k}$. This is in fact the only range that matters, though, as the firm will only operate over the range where returns are decreasing, i.e. where $k_i > \tilde{k}$. The advantage with this functional form is that it yields highly tractable solutions as we shall see.

We will first characterize efficiency. For the present example it turns out that there is a locus of efficient points. Note that although Lemma 1 identifies sufficient (as well as necessary) conditions for a unique symmetrical efficient point, Theorems 1 and 2 require only that a symmetrical efficient point exists; it does not have to be unique.

The first step in characterizing efficiency is to solve for y_i^E which can be done using (5). Differentiating $f(k_i, y_i)$ with respect to y_i we obtain

$$\frac{\partial f(k_i, y_i^E)}{\partial y_i} = \frac{(k_i - \tilde{k})^{\frac{1}{2}}}{2(y_i^E)^{\frac{1}{2}}}.$$

Setting $(k_i - \tilde{k})^{\frac{1}{2}} / (2(y_i^E)^{\frac{1}{2}}) = 1$ as required by (5), and solving for y_i^E , we obtain

$$y_i^E = (k_i - \tilde{k})/4. \tag{15}$$

³¹If a firm is on the concave segment of its production function at $\bar{k}/2$ then this is sufficient for a profit maximizing solution.

Using (15), the constraint $k_j = 2 - k_i$ and $f(k_i, y_i) = \tilde{k}/2 + (k_i - \tilde{k})^{\frac{1}{2}} (y_i)^{\frac{1}{2}}$ in the planner's problem we obtain $\Omega = (1 + \tilde{k})/2$. This solution holds for any $k_i \in (\tilde{k}, 2 - \tilde{k})$.³² The point $\bar{k}/2 = 1$ is included in the interval $(\tilde{k}, 2 - \tilde{k})$, and so a symmetric efficient solution exists as required.

We now characterize the decentralized equilibrium. The solution to the firm's problem can be found in the usual way and can be expressed as follows:

$$k_i^* = \tilde{k} + \frac{y_i}{4p_i^2}.$$

Notice from this solution that $k_i^* > \tilde{k}$ as we should expect, to a degree dependent upon p_i and y_i .

Since the analysis of Theorems 1 and 2 essentially requires a comparison of symmetrical solutions at which $k_i = k_j = \bar{k}/2 = 1$, it will be helpful to solve for the value of p_i at which $k_i^* = 1$. Setting $\tilde{k} + y_i/(4p_i^2) = 1$ and rearranging, we obtain

$$p_i = \pm \frac{(y_i)^{\frac{1}{2}}}{2(1 - \tilde{k})^{\frac{1}{2}}}.$$

Clearly, it is only the positive root that is of interest to us. We can now solve for a symmetric decentralized capital market equilibrium in which $k_i^* = k_j^* = 1$ providing $p_i = (y_i)^{\frac{1}{2}} / \left(2(1 - \tilde{k})^{\frac{1}{2}}\right)$ and $p_j = (y_j)^{\frac{1}{2}} / \left(2(1 - \tilde{k})^{\frac{1}{2}}\right)$. As r is common to both jurisdictions, a symmetrical decentralized equilibrium requires that both jurisdictions have the same level of taxation.

It will turn out to be most convenient to solve for the level of taxation using the government's budget constraint; $t_i = y_i/k_i$. So we next need to solve for the equilibrium value of public good provision y_i^* . By the proof of Theorem 2, we have that in the decentralized equilibrium,

$$\frac{\partial f(k_i^*, y_i^*)}{\partial y_i} = \frac{1}{1 + (t_i^*/k_i^*) (\partial k_i/\partial t_i)}.$$

We can solve for $\partial k_i/\partial t_i$ using (13). Doing so, we obtain

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - mpgv_i^{DWZ}}{ompk_i^{DWZ}}.$$

³²It is straight forward to verify that this solution is indeed efficient, and that a higher level of output cannot be obtained for $k_i \notin (\tilde{k}, \bar{k} - \tilde{k})$.

where

$$\begin{aligned} mpgv_i^{DWZ} &= \frac{k_i}{4 \left(k_i - \tilde{k} \right)^{\frac{1}{2}} (y_i)^{\frac{1}{2}}} \\ ompk_i^{DWZ} &= \frac{t_i k_i - y_i - t_i \tilde{k}}{4 \left(k_i - \tilde{k} \right)^{\frac{3}{2}} (y_i)^{\frac{1}{2}}} \end{aligned}$$

There are two things worth noting here. First, we are able to gain useful insight into how \tilde{k} affects $mpgv_i^{DWZ}$. Recall that $k_i^* = 1$ in equilibrium. So the only part of $mpgv_i^{DWZ}$ that varies with \tilde{k} is $\partial^2 f / \partial k_i \partial y_i = 1 / \left(4 \left(k_i - \tilde{k} \right)^{\frac{1}{2}} (y_i)^{\frac{1}{2}} \right)$. From this we see that $mpgv_i^{DWZ}$ is increasing in \tilde{k} because, as \tilde{k} increases, k_i and y_i become stronger complements. It is in this sense that we can say \tilde{k} (indirectly) parameterizes $mpgv_i^{DWZ}$.

Second, note that (given $k_i^* = 1$) if $\tilde{k} > 0$ and $t_i > 0$ then $ompk_i^{DWZ} < 0$ and so A4 is satisfied.

Substituting the expression for $\partial k_i / \partial t_i$ into the expression for $\partial f(k_i^*, y_i^*) / \partial y_i$ we obtain

$$\frac{\partial f(k_i^*, y_i^*)}{\partial y_i} = \frac{1}{1 - \frac{t_i^* \left(k_i^* - 4(k_i^* - \tilde{k})^{\frac{1}{2}} (y_i^*)^{\frac{1}{2}} \right) (k_i^* - \tilde{k})}{k_i^* (k_i^* t_i^* - y_i^* - t_i^* \tilde{k})}}$$

From the production function,

$$\frac{\partial f(k_i, y_i)}{\partial y_i} = \frac{\left(k_i - \tilde{k} \right)^{\frac{1}{2}}}{2y_i^{\frac{1}{2}}}.$$

Setting these two functions equal to each other and solving simultaneously for y_i^* and $t_i^* = y_i^* / k_i$ we obtain the following:

$$\begin{aligned} t_i^* &= \frac{k_i^3 \left(k_i - \tilde{k} \right)}{4 \left(2k_i^2 - 3\tilde{k}k_i + 2\tilde{k}^2 \right)^2}; \\ y_i^* &= \frac{k_i^4 \left(k_i - \tilde{k} \right)}{4 \left(2k_i^2 - 3\tilde{k}k_i + 2\tilde{k}^2 \right)^2}. \end{aligned} \tag{16}$$

We can now obtain a complete characterization of a decentralized symmetrical equilibrium. Setting $k_i^* = 1$, we can use the above solutions to work out values of r , t_i^* and y_i^* consistent

with a symmetric decentralized equilibrium in the capital market. We have:

$$y_i^* = t_i^* = \frac{1 - \tilde{k}}{4 \left(2 - 3\tilde{k} + 2\tilde{k}^2 \right)^2};$$

$$r = \frac{1}{4} \left(\frac{\sqrt{\frac{1 - \tilde{k}}{(2 + \tilde{k}(2\tilde{k} - 3))^2}}}{\sqrt{1 - \tilde{k}}} - \frac{1 - \tilde{k}}{\left(2 + \tilde{k} (2\tilde{k} - 3) \right)^2} \right)$$

It is possible to verify that y_i^*, t_i^* and r are positive for all $\tilde{k} \in (0, 1)$. This is convenient because it will enable us to compare equilibrium values of y_i^* to the efficient level of provision y_i^E across a range of values for \tilde{k} .

In order to fully characterize Theorems 1 and 2 for this example, we also need to solve for y_i^I . An expression for $mpgv$ is obtained from the cross partial derivative of the production function multiplied by k_i :

$$k_i \frac{\partial^2 f(k_i, y_i)}{\partial k_i \partial y_i} = \frac{k_i}{4 \left(k_i - \tilde{k} \right)^{\frac{1}{2}} \left(y_i \right)^{\frac{1}{2}}}.$$

To find y_i^I , we solve for the value of y_i that sets $k_i \partial^2 f(k_i, y_i) / \partial k_i \partial y_i = 1$. This yields the following expression:

$$y_i^I = \frac{k_i^2}{16 \left(k_i - \tilde{k} \right)}. \quad (17)$$

We now have expressions for y_i^E , y_i^* and y_i^I which we can compare across a range of values for \tilde{k} at symmetric equilibrium solutions. It is quite legitimate to compare outcomes as \tilde{k} is varied. It may be worth emphasizing that the parameter \tilde{k} is essentially a technological parameter that measures the level of capital use at which returns to capital cease to be increasing and, for further increases in capital, returns to capital diminish. It is easy to see from (15) that y_i^E is decreasing in \tilde{k} , and from (17) that y_i^I is increasing in \tilde{k} . The behavior of y_i^* is a little more difficult to see by inspection of (16). Therefore, a comparison of y_i^E , y_i^* and y_i^I is made graphically in Figure 5.

From Figure 5 we can see that Theorem 1 is characterized when $\tilde{k} = \frac{1}{2}$; the values for y_i^E , y_i^* and y_i^I coincide. For $\tilde{k} < \frac{1}{2}$ we see that $y_i^I < y_i^* < y_i^E$, which is the first case considered in Theorem 2; the case of underprovision of the public good in equilibrium. Note that $y_i^I < y_i^E$ for the standard example of Cobb-Douglas technology, which is given by setting $\tilde{k} = 0$.

For $\tilde{k} > \frac{1}{2}$ we see that $y_i^I > y_i^* > y_i^E$, the second case considered in Theorem 2; that of over-provision of the public good in equilibrium.

To better appreciate how this example operates, let us relate its specific workings to the general framework set out in previous sections. In particular, let us look in more detail at how changing \tilde{k} affects the incentive to deviate from efficiency, which depends on how \tilde{k} affects y_i^E and y_i^I .

First, it is easy to see why y_i^E is inversely related to \tilde{k} . As \tilde{k} is increased, this increases the domain over which the production function is (weakly) convex and, as a result, increases the marginal productivity of capital at any point on the concave segment. This in turn mandates a shift of resources away from public good production at all production levels, including efficiency.

On the other hand, y_i^I is increasing in \tilde{k} . Intuitively, an increase in \tilde{k} has the effect of increasing the complementarity between the public good and capital, and as a result the incentive to deviate from efficiency is increased. The value of k_i is fixed at $k_i = \bar{k}/2$. So the only term which can vary with \tilde{k} is $\partial^2 f(k_i, y_i) / \partial k_i \partial y_i$. As \tilde{k} is increased, this has the effect of increasing $\partial f(k_i, y_i) / \partial k_i$ at any given value of $k_i > \tilde{k}$, and hence increasing $\partial^2 f(k_i, y_i) / \partial k_i \partial y_i$ as well. This is illustrated in Figure 6, which shows that $1 - k_i \partial^2 f(k_i, y_i) / \partial k_i \partial y_i$ is increasing in y_i and decreasing in \tilde{k} . Recall that $y_i = y_i^I$ when $1 - k_i \partial^2 f(k_i, y_i) / \partial k_i \partial y_i = 0$. So the value of y_i^I must be decreasing in \tilde{k} .

It may appear at first sight that on one level we simply confirm the results of Z-M by showing that a standard Cobb-Douglas/decreasing returns production function gives rise to underprovision in equilibrium. However, Z-M do not discuss the relationship between the technology assumed and the equilibrium outcome as we do here. Thus, the example we present is useful in clarifying how this key set of interactions work.

6 Conclusions

The purpose of this paper has been to show that, within the context of a standard tax competition model, a wider set of outcomes than has previously been suggested is possible. The past literature tends to focus on a ‘race to the bottom’ of tax rates and public good provision. Where other outcomes such as efficient taxation or a ‘race to the top’ are shown to arise, this is due to the presence of other mechanisms, for example a type of Tiebout mechanism where the representative citizen is able to vote for their preferred policies. In

contrast, we show that all of these (mutually exclusive) outcomes are possible within the same standard tax competition framework.

We use the version of the standard model where the public good enters the production function of firms. This is distinguished from the more familiar approach of simply assuming that the good produced by the government enters the utility function. The way that we obtain our broader set of results is to vary a standard assumption. In the past literature it has been assumed that the additional output obtained from provision of the public good through taxation is never as great as the opportunity cost in terms of tax revenue. Therefore, in the conventional set-up there is always a unilateral incentive to deviate from the efficient level of public good provision by lowering taxes. Under our alternative assumption there may be a unilateral incentive to deviate upwards, downwards or not at all from the efficient level of public good provision. Thus, all three possibilities can arise in equilibrium.

The analysis of this paper focuses on a model where taxation provides firms with a good that they value, because it increases the productivity of their capital. The model could be cast in a consumer setting by looking at taxation associated with consumption. There is already a literature on this area, which looks at how the ‘earmarking’ of taxes for specific purposes valued by consumers can reduce the free rider problem. See, for example, Dhillon and Perroni (2001). The analysis of this present paper suggests that in a situation where consumers value the public good being provided along parallel lines to the valuation placed on public goods by firms, the conventional free rider problem may under certain circumstances disappear completely.

An original aspect of our results, which we present in the proofs contained in the Appendix, is that the existence of an efficient state implies the existence of a symmetric Nash equilibrium. At the efficient state, certain first order conditions are satisfied. At the state where the marginal cost to a jurisdiction of a unit of the public good equals its marginal value, another set of first order conditions is satisfied. A comparison of these two points (and the Intermediate Value Theorem) indicates whether the equilibrium outcome is efficient, or inefficient with over or under provision of the public good. In our model, the relationship between these two sets of first order conditions is determined by the degree of complementarity between capital and the public good, a feature that is illustrated most clearly in the example that we present. We believe that this mode of analysis, examining how variation in the underlying economic structure affects the incentive to deviate from efficiency, represents

a novel and useful way to understand a broader class of policy games.

A Appendix

Theorem 1. *Assume A1-A5 and that there exists a symmetric interior efficient plan $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$. Hence $k_i = k_j = \bar{k}/2$. Assume $y_i^I = y_i^E$. Assume that $\tilde{k} < \bar{k}/2$. Then there exists a decentralized equilibrium with taxes (t_i^E, t_j^E) , $r^* = f'(k_i^E, y_i^E) - t_i^E$, and capital allocation $k_i^* = k_j^* = \bar{k}/2$. The induced public good supply is y_i^E, y_j^E .*

Proof of Theorem 1: Let $t_i = t_j = t^E$ and $r = f(\bar{k}/2, y^E) - t^E$. Note that since the efficient point is symmetric, it is interior and hence $y_i^E = y_j^E > 0$. Then by construction the firm's demand for capital at $y_i = y_i^E$ is $\bar{k}/2$. Since we assume A.1 and $\bar{k}/2 > \tilde{k}_i$ for any $y_i > 0$, the production function is concave at the efficient point $\bar{k}/2, y_i^E$, and the second order conditions for a maximum are satisfied at the efficient point. Note that the firm's profits are non-negative at k_i^E since the production function is concave at the efficient point, so $r + t_i^E = f'(k_i^E, y_i^E) \leq \frac{f(k_i^E, y_i^E)}{k_i^E}$.

Budget Balance is clearly satisfied since we have chosen taxes such that $t_i^E = \frac{y_i^E}{k_i^E}$.

It remains to show the government's payoffs are maximized at taxes given by t^E . The first order condition for the government in jurisdiction i is (12), reproduced here:

$$\begin{aligned} \frac{dc_i}{dt_i} &= \left(\frac{\partial f}{\partial k_i} - (t_i + r) \right) \frac{\partial k_i}{\partial t_i} \\ &\quad + \frac{\partial f}{\partial y_i} \left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} \right) - k_i = 0. \end{aligned}$$

Note that $y_i^E = y_i^* = y^I$. At y^I , $k_i \partial^2 f(k_i^E, y_i^I) / \partial k_i \partial y_i = 1$; we know that such a point exists by Lemma 2. Thus, $\frac{\partial k_i}{\partial t_i} = 0$ and $\frac{\partial f(k_i^E, y_i^E)}{\partial y_i} = 1$. Moreover the first order conditions for the firm imply that $\frac{\partial f(k_i^E, y_i^E)}{\partial k_i} = (t_i^E + r)$. Finally $\frac{\partial y_i(k_i^E, t_i^E)}{\partial t_i} = k_i^E$ so the first order condition for the government is clearly satisfied. To confirm that this is indeed a maximum, first note that we can rewrite the government's objective function as $c_i = f(k_i, y_i) - t_i k_i + r(\bar{k}_i - 1) = f(k_i, y_i) - y_i + r(\bar{k}_i - 1)$ (by budget balance). Since r is taken as given by the government, this is equivalent to maximizing $c'_i = f(k_i, y_i) - y_i$. Next, notice that the efficient solution is a symmetric interior maximum. Thus if c_i is not maximized at t_i^E , this implies that c'_i must be either a minimum or a point of inflection (since we have shown that it is a critical point). But then by symmetry, c'_j is also a minimum or point of inflection, contradicting the fact that $c'_i + c'_j$ is maximized at $k_i^E, k_j^E, y_i^E, y_j^E$. Thus, t_i^E must maximize c'_i and hence c_i .

This proves that the efficient point is indeed a decentralized equilibrium \square

Theorem 2. Assume A1-A5 and that there exists an interior symmetric efficient plan $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$. Assume that $\tilde{k} < \bar{k}/2$. Then there exists a symmetric interior decentralized equilibrium $t_i^* = t_j^* = t^*$, $k_i^* = k_j^* = \bar{k}/2$ and r^* , and induced level of public good provision y_i^* . If $y_i^I < y_i^E$ then $y_i^I < y_i^* < y_i^E$ and if $y_i^E < y_i^I$ then $y_i^E < y_i^* < y_i^I$ so the equilibrium outcome is inefficient, with under- (over-) provision of the public good.

Proof of Theorem 2: The firm's profits are non-negative at $k_i = \bar{k}/2$ since the production function is concave at $k_i > \tilde{k}$ and, by assumption, $\bar{k}/2 > \tilde{k}$. So we can find a feasible r^* and t_i^* such that $r^* + t_i^* = f'(\bar{k}/2, y_i^*)$. The first order condition for the government's objective function is the following:

$$\frac{dc_i}{dt_i} = \left(\frac{\partial f(k_i^*, y_i^*)}{\partial k_i} - (t_i + r) \right) \frac{\partial k_i}{\partial t_i} + \frac{\partial f(k_i^*, y_i^*)}{\partial y_i} \left(\frac{dy_i}{dt_i} \right) - k_i^* = 0.$$

In equilibrium the first order conditions for the firm imply that $\frac{\partial f(k_i^*, y_i^*)}{\partial k_i} - (t_i + r) = 0$. Moreover $dy_i/dt_i = t_i \partial k_i / \partial t_i + k_i$. Using both of these facts, the above condition becomes

$$\frac{dc_i}{dt_i} = \frac{\partial f(k_i^*, y_i^*)}{\partial y_i} \left(t_i \frac{\partial k_i}{\partial t_i} + k_i^* \right) - k_i^*.$$

Consider the case where $y_i^I < y_i^E$. We will now show that while at y_i^E it is the case that $dc_i/dt_i < 0$, we have that at y_i^I it is the case that $dc_i/dt_i > 0$, so there must exist (by continuity) a point at which $dc_i/dt_i = 0$.

At y_i^E , note that $\partial f(k_i^*, y_i^*) / \partial y_i = 1$. Therefore, $dc_i/dt_i = t_i \partial k_i / \partial t_i$. We know that $\partial k_i / \partial t_i < 0$ at y_i^E : its numerator is equal to 0 at y_i^I and therefore positive at $y_i^E > y_i^I$ by A5; its denominator is negative by A4 (see 13). It follows immediately that $dc_i/dt_i < 0$ for $y_i = y_i^E$.

At y_i^I , note that $\partial f(k_i^*, y_i^*) / \partial y_i > 1$ by A2 (concavity of $f(k_i, y_i)$ in y_i). In addition, by Lemma 2, $\partial k_i / \partial t_i = 0$ at y_i^I . It follows immediately that $dc_i/dt_i > 0$ for $y_i = y_i^I$.

Moreover, dc_i/dt_i is continuous in y_i , following from A2 and A5. Therefore, there must exist at least one point at which $dc_i/dt_i = 0$ and if there is more than one such point, then the one for which c_i is largest is the global maximum.

The argument for the case where $y_i^I > y_i^E$ follows along the same lines. \square

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Figure 1a

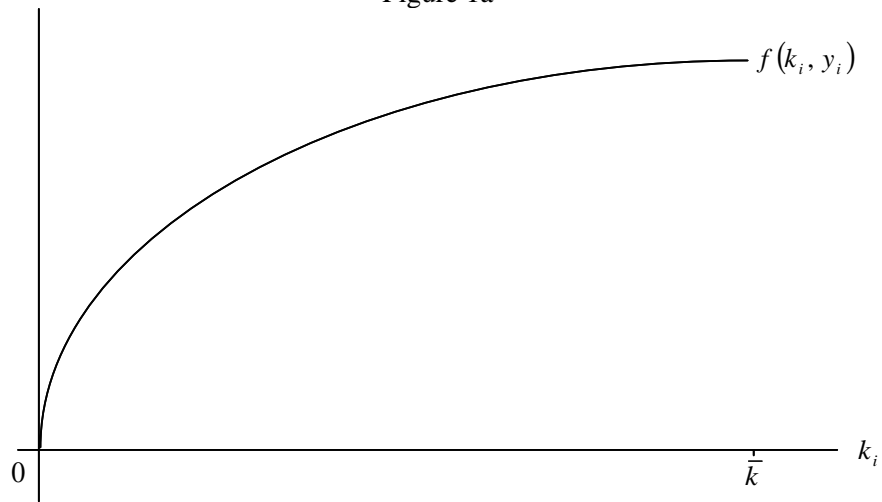


Figure 1b

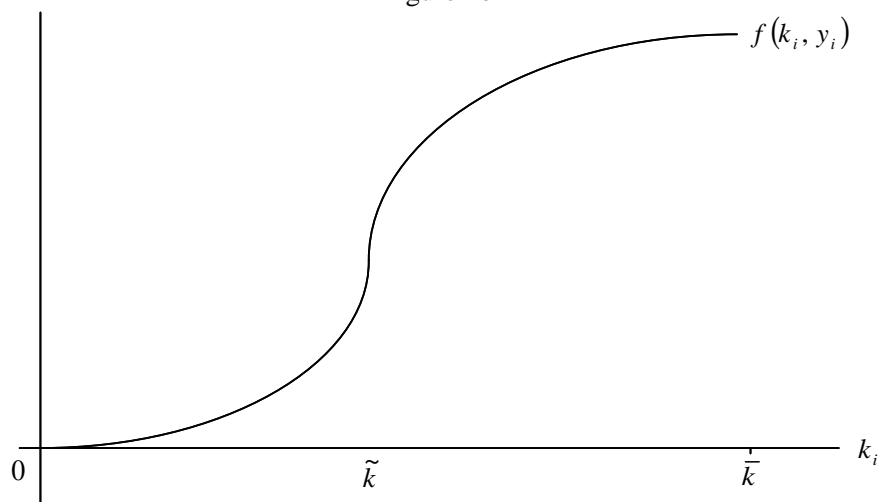


Figure 1c

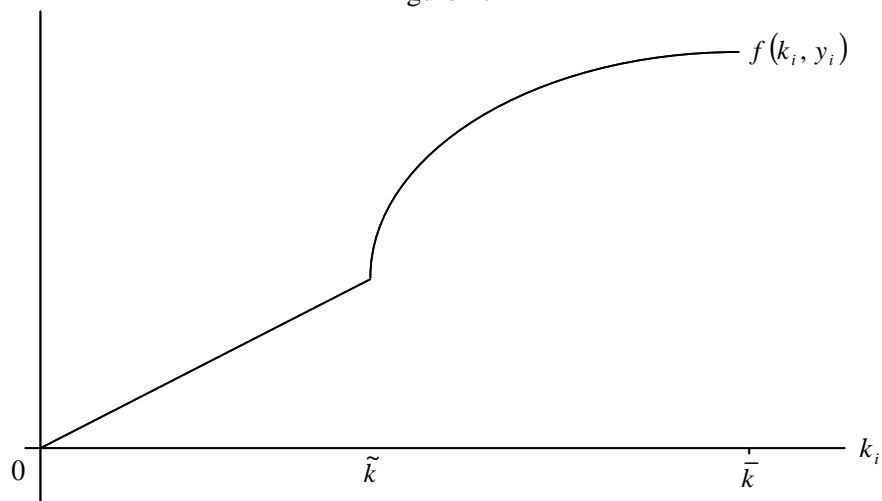


Figure 2

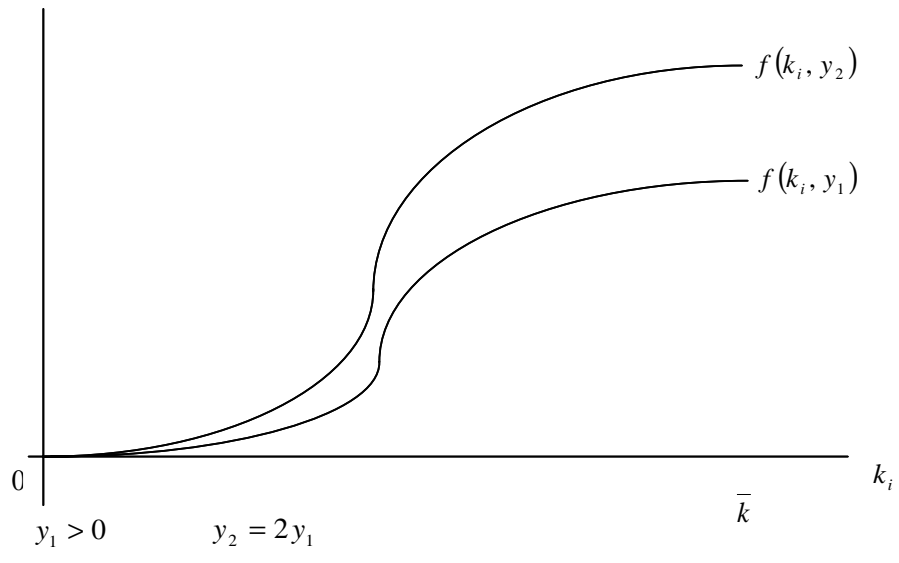


Figure 3

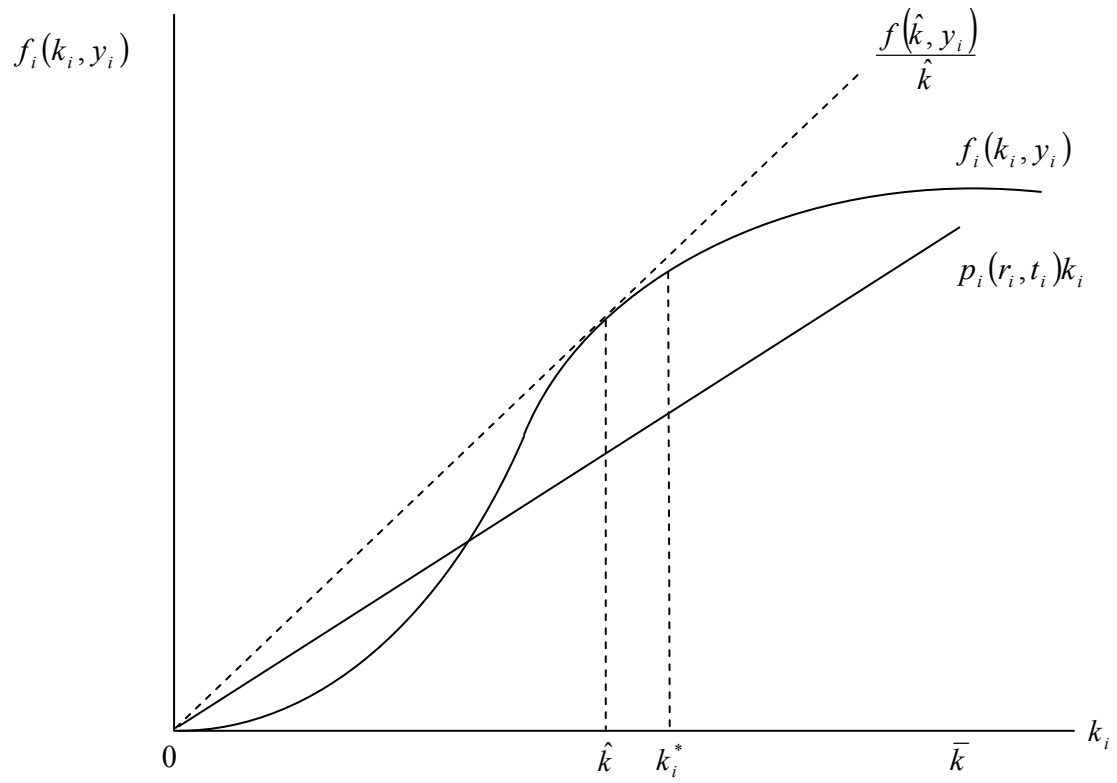


Figure 4

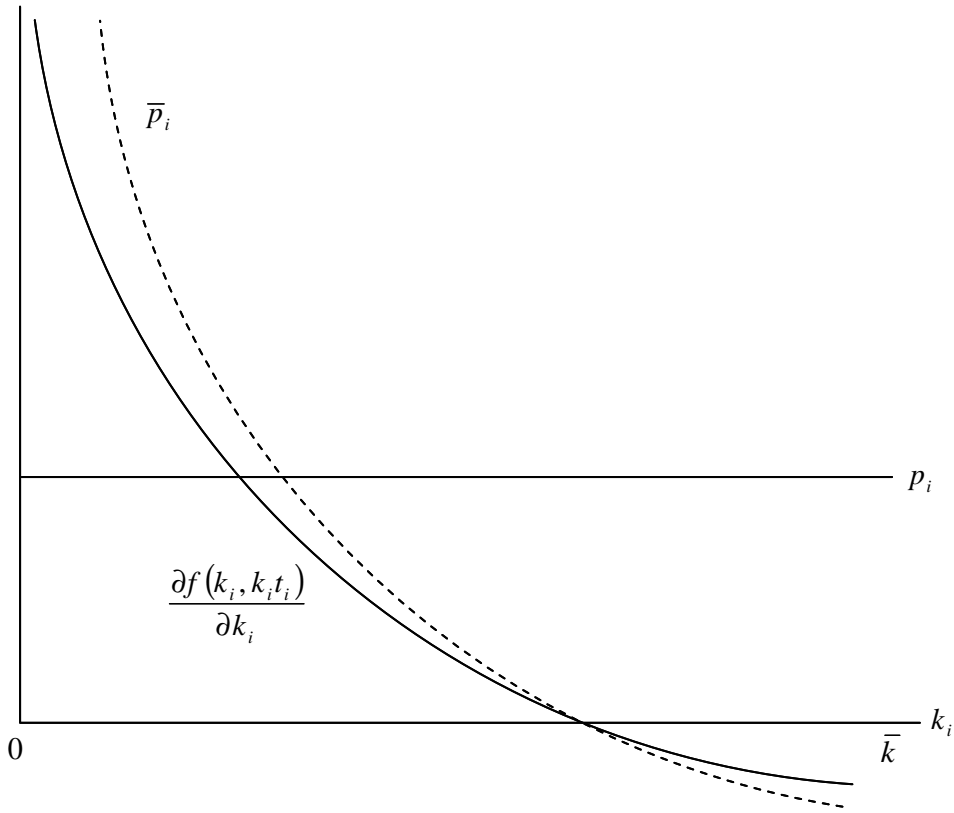


Figure 5

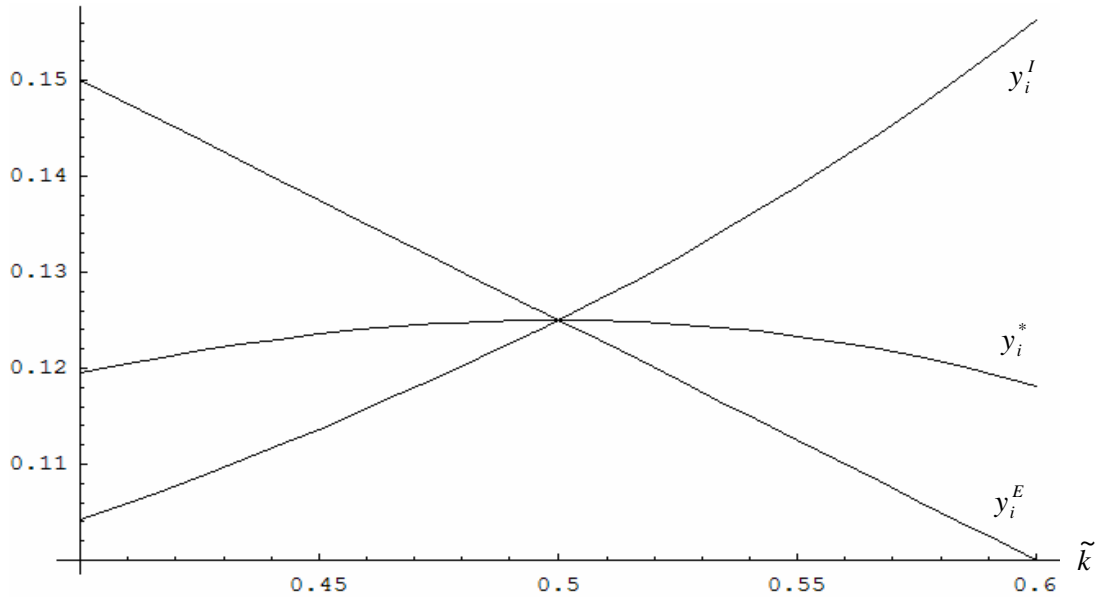
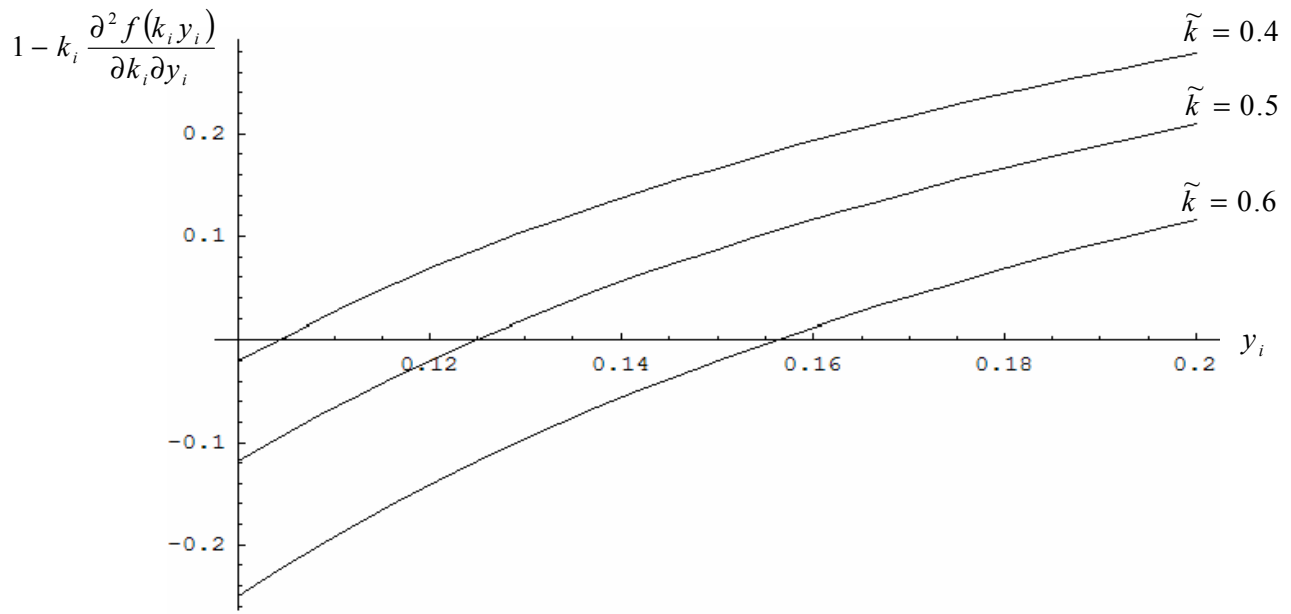


Figure 6



$$k_i = \bar{k} / 2 = 1$$