

**SUBSTITUTION, RISK AVERSION AND ASSET PRICES:  
AN EXPECTED UTILITY APPROACH**

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## SUBSTITUTION, RISK AVERSION AND ASSET PRICES: AN EXPECTED UTILITY APPROACH

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The standard power utility function is widely used to explain asset prices. It assumes that the coefficient of relative risk aversion is the inverse of the elasticity of substitution. Here I use the Kihlstrom and Mirman (1974) expected utility approach to relax this assumption. I use time consistent preferences that lead to time consistent plans. In our examples, the past does not matter much for current portfolio decisions. The risk aversion parameter can be inferred from experiments and introspections about bets in terms of permanent consumption (wealth). Evidence about the change in the attitude towards bets over the life cycle may also restrict the value of the risk aversion parameter. Monotonic transformations of the standard power utility function do not change the predictions about asset prices by much. Both the elasticity of substitution and risk aversion play a role in determining the equity premium.

JEL codes: D11, D81, D91, G12

KEY WORDS: CONSUMPTION SMOOTHING, INTERTEMPORAL ELASTICITY OF  
SUBSTITUTION, RISK AVERSION, ASSET PRICES, EQUITY PREMIUM.

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## 1. INTRODUCTION

Since the discovery of the risk premium puzzle by Mehra and Prescott (1985) there has been a debate about the choice of the representative agent's utility function. In his presidential address Lucas (2003) followed Mehra and Prescott in using the standard power utility function:  $\sum_t \beta^t u(C_t)$  where  $u(C) = (\frac{1}{\rho})C^\rho$  for  $\rho \neq 0$  and  $u(C) = \ln(C)$  for  $\rho = 0$ . On the basis of evidence from interest rates across countries, he argues for a coefficient of  $\rho = 0$ : The logarithmic function.

The inter-temporal log utility function implies a relative risk aversion of 1 and an elasticity of substitution of 1. In general, the power utility function imposes a relationship between the inter-temporal elasticity of substitution and relative risk aversion: The coefficient of relative risk aversion is  $1 - \rho$  and the inter-temporal elasticity of substitution is:  $IES = \frac{1}{1-\rho}$ .

To appreciate the difficulty we may consider the following two thought experiments.

Thought experiment a: We consider the following two alternatives.

(1 a) Consuming in the next year 99,500 dollar worth with probability 1; (2 a) An equal chance of consuming in the next year 110,000 dollars worth or 90,000 dollars worth.

Thought experiment b: There is no uncertainty. Consumption in the current year is 100,000 dollars and consumption in the next year is expected to be 110,000 dollars (with probability 1). The consumer can lend and borrow at the real interest rate of 14.6%. His choices are: (1 b) be a borrower; (2 b) be a lender and (3 b) neither a borrower nor a lender be.

Under the log utility function with  $\beta = 0.96$ , the agent is indifferent between (1a) and (2a) and between (1b) and (2b). Under the more general class of the power utility function, a strict preference for the risk-free alternative (1a) implies a strict preference

for the borrowing alternative (*1b*). There are many people who will choose (*1a*) in the first thought experiments and choose to lend (at 14.6%) in the second.

This problem led to the generalization of the expected utility function by Selden (1978), Epstein and Zin (1989, 1991) and Weil (1989, 1990). I will refer to the approach taken by this literature as the "certainty equivalent approach".

Here I follow the expected utility approach used by Kihlstrom and Mirman (1974). By applying monotonic transformations to the utility function, this approach allows for changes in risk aversion that do not affect the ordinal properties of the utility function.<sup>2</sup> I use simple examples to understand the role of risk aversion and the Intertemporal Elasticity of Substitution (*IES*) in determining asset prices. Another objective is to understand the difference between the "certainty equivalence approach" and the expected utility approach. I do not attempt to solve the equity premium puzzle.

It is shown that under the expected utility approach there is still a connection between risk aversion and the Intertemporal Elasticity of Substitution (*IES*) but this connection is much less restrictive than under the standard power utility function. I thus relax the assumption that risk aversion is the reciprocal of *IES* rather than achieve a complete separation between the two as in the certainty equivalent approach.

A related difference is in the role of the *IES* in determining the equity premium. Under the expected utility approach taken here both risk aversion and *IES* play a role in determining the equity premium: A higher risk aversion and a lower *IES* lead to a higher equity premium. Roughly speaking if the representative agent has a lower *IES* he is willing to pay more for consumption smoothing and therefore will have a stronger demand for the risk-free asset that is a better smoothing tool. I shall elaborate on this point shortly.

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<sup>2</sup> It can be shown that the expected utility function considered here and the Epstein-Zin function, are both special cases of Kreps and Porteus (1978).

This is different from the implications of the certainty equivalent approach. In the two-periods numerical examples that I have worked out there is almost no effect of *IES* on the equity premium when using that approach.

Another and not less important difference is in the ease in which we can use experimental evidence and introspection to make direct "observations" about the risk aversion parameter. In their original paper Mehra and Prescott limit the risk aversion parameter to magnitudes between 0 and 10. They used an additive expected utility and in this case there is no difference between the attitude towards bets in terms of permanent consumption (wealth) and the attitude towards bets in terms of dated consumption. However a difference between the two emerges once we allow for non-additive expected utility functions.

Under the expected utility approach, the risk aversion parameter requires "observations" about the attitude towards bets in terms of permanent consumption or wealth. This is relatively easy and therefore most experiments ask questions about bets in terms of money (wealth) and not in terms of dated consumption. See Kahneman and Tversky (1979) for example.

The certainty equivalent approach requires introspection about bets in terms of dated consumption. This is much more difficult. To appreciate the difficulty, consider for example, a choice problem that has three possible outcomes: (a) dinner at McDonalds, (b) dinner at the best restaurant in town and (c) no dinner at all. When one faces the choice between (a) with probability 1 or an actuarially fair lottery with possible outcomes (b) and (c), he may choose to go to McDonalds if he did not have lunch and he is hungry and he may choose the bet if he is not so hungry. Similarly, we may consider a choice problem with the following possible outcomes: (a) vacation at home; (b) vacation in Hawaii and (c) no vacation at all. I may choose (a) with probability 1 if I did not have a vacation for a long time and I may choose a bet between (b) and (c) if I just came back from a vacation in Atlanta.

This difficulty emerges because dated consumption may not be well defined. Friedman (1969, in the Appendix) has argued for the role of memories. We go on vacation and then enjoy pictures and memories about it. He suggests to model consumption as the services provided by stocks, including the stocks of memories. (Once the stock of memories depreciates you rebuild it by taking another vacation). Indeed question about bets in terms of consumption become easier as we increase the length of the "period". It is much easier to answer a question about consumption in the next year than about consumption in the next month. And introspections about bets in terms of permanent consumption (wealth) are the easiest because they can be stated in money or wealth terms.

As was said before, unlike the certainty equivalent approach, the expected utility approach does not achieve a complete separation between the cardinal and the ordinal properties of the utility function. The separation is complete only at the beginning of the planning horizon. After that the ordinal properties start to play an increasing role in determining risk aversion. I show that if risk aversion to bets in terms of permanent consumption increases with age then risk aversion at age zero must be less than  $\frac{1}{IES}$ . This suggests to me fairly low values of the risk aversion parameter.

The Epstein-Zin approach is by now the standard approach to the problem of disentangling risk aversion from the elasticity of substitution. Comments that I got on earlier drafts suggest to me that many specialists in the field feel that "bad" things will happen if we use the expected utility approach: Plans may be time inconsistent or the past may play an important role in current choices. I try to address these issues in the simplest possible environment.

A key distinction is between time consistent preferences and time consistent plans. Recent work by Kihlstrom (2007) and Van den Heuvel (2007) illustrate the difference between the two. Both papers use the Kihlstrom-Mirman approach. Kihlstrom assumes that the consumer ignores past consumption in his current decisions while Van

den Heuvel assumes time consistent preferences that allow for the effect of past choices on current choices.

Time consistent preferences assume that the agent uses the same function each period and as he gets older choices that were made in the past become irreversible but are not forgotten and are still present in the objective function. Under the expected utility hypothesis time consistent preferences leads to plans that will be followed in the future. Thus time consistent preferences lead to time consistent plans. But time-consistent preferences may imply that past consumption matters for current decisions. If one tries to eliminate past consumption from the current utility, as Kihlstrom does, he may get plans that are not time consistent. Kihlstrom follows the "consistent planning" approach of Strotz (1956) and assumes that when making the current choice the consumer takes into account future "disobedience".

Here I consider an overlapping generations economy with consistent preferences finitely lived agents. The overlapping generation structure is used to deliver stationary asset prices when decision makers have finite lives. The finite life assumption matters and some of the results here are therefore different from the results obtained by Van den Heuven who considered the infinite horizon case.

As was said before, in the finite horizon case the attitude towards bets in terms of permanent consumption (wealth) changes with age. Over the life cycle the weight of the ordinal properties in determining the attitude towards such bets increases so that in the limit when age goes to infinity the attitude towards such bets depends on the ordinal properties only. The finite horizon case gives us therefore a richer theory in which both the cardinal and the ordinal properties of the utility function play a role in determining the attitude towards bets in terms of permanent consumption.

Another advantage of the overlapping generations economy is that past choices play a smaller role relative to the role they play in an infinite horizon model: A new generations does not take the consumption of older generations into account when

making current choices. This argument sounds like the most compelling reason for studying the overlapping generations alternative because the effect of past consumption on current choices has played a major part in discarding the expected utility approach. But in the body of the paper I argue that in the Intertemporal utility functions we use the past is not important for current portfolio choices. A more compelling reason for studying the finite horizon alternative is that it is different from the infinite horizon case and actual choices are made by finitely lived individuals.

I start with a monotonic transformation of the inter-temporal log (*IL*) utility function: The inter-temporal Cobb-Douglas (*ICD*). The *ICD* function allows for changes in risk aversion while holding constant the elasticity of substitution at the level of unity. It is shown that changes in risk aversion do not affect the expected rate of return on the market portfolio and have only a small effect on the risk free return and equity premium.

I then consider monotonic transformations of the standard power utility function: The constant elasticity (*ICE*) function. It is shown that also in this case asset prices do not change much under monotonic transformations but changes in the elasticity of substitution have a large effect on asset prices. In all the examples I worked out the equity premium is small and, as was said before, depends on both the elasticity of substitution and risk aversion.

The *ICE* utility function is time-non-separable. The habit persistent function used by Constantinides (1990) is another example of an expected utility time-non-separable function. Here I use the *ICE* utility function because it allows for a clean separation between the elasticity of substitution and risk aversion.

At the core of the Kihlstrom-Mirman approach is the distinction between the ordinal and the cardinal properties of the utility function. To appreciate the difference between the two, I now turn to distinguish between aversion to fluctuations and aversion to risk.



## 2. FLUCTUATIONS AVERSION, RISK AVERSION AND THE EQUITY PREMIUM

To understand the role of *IES* in determining the equity premium when using the expected utility approach I consider a consumer who lives for  $T + 1$  periods and faces a choice problem with the following possible outcomes:  $\{C_t = 3\}_{t=0}^T$  corresponding to the smooth consumption path *a* in Figure 1;  $\{C_t = 2\}_{t=0}^T$  corresponding to path *e* in Figure 1;  $\{C_t = 1\}_{t=0}^T$  (path *b*);  $C_t = 3.5$  in even periods and  $C_t = 2.5$  in odd periods (path *d*).

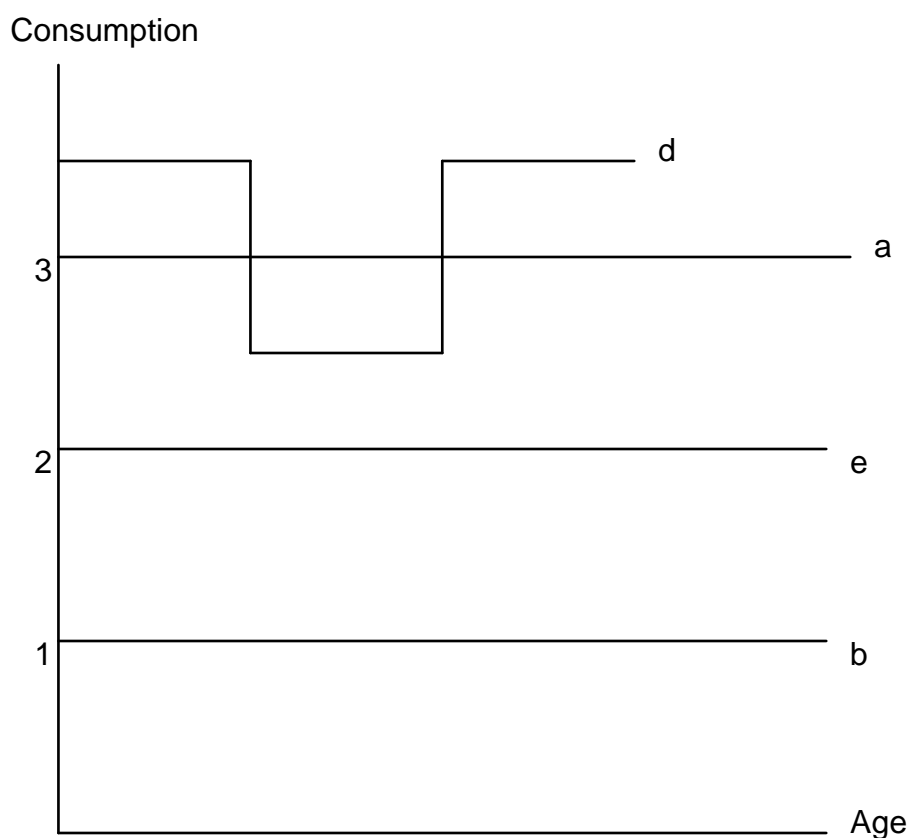


Figure 1

A consumer that prefers path *a* with probability 1 to path *d* with probability 1 has aversion to fluctuations or preference for smoothing. A consumer who prefers path *e* with probability 1 to an actuarially fair bet with paths *a* and *b* as possible outcomes, exhibits risk aversion to bets in terms of permanent consumption.

I consider now the case in which the representative agent is risk neutral to bets in terms of permanent consumption but show aversion to fluctuations (preference for smoothing). The representative agent gets an endowment of two trees. Both trees promise on average the same amount of fruits each period but one tree promises a random amount and the other is risk-free. The price of the risk-free tree must be lower because otherwise the representative agent will choose to hold only risk-free trees that allow for better smoothing. This is why we get an equity premium even in an economy that is populated by risk neutral agents.<sup>3</sup>

An agent with low *IES* has a greater demand for smoothing consumption and exhibits high aversion to fluctuations. Since the risk-free asset has an advantage in smoothing consumption, low *IES* contributes to high equity premium.

This is different from the certainty equivalent approach. To illustrate, consider an agent who lives for two periods and his utility function is:  $U(C_0, E(C_1))$ , where  $E(C_1)$  is the expected second period consumption. This risk-neutral consumer is willing to hold both trees at the same price regardless of the ordinal properties of the "aggregator function"  $U$ . Thus we will not get an equity premium in an economy that is populated by consumers who maximize  $U(C_0, E(C_1))$ .

Note that aversion to fluctuations need not imply aversion to bets in terms of permanent consumption. It is possible that a consumer does not like fluctuations because they require changes in durables. To implement the path  $d$  one needs to change his house every period or to suffer from a mismatch between his house size and other components

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<sup>3</sup> A similar argument is used in Eden (1977) to show that insurance does not require risk aversion.

of consumption. On the other hand if after a lottery between  $a$  and  $b$  he gets to know his permanent consumption early on he will make the optimal housing choice.<sup>4</sup>

I now turn to a related distinction.

### 3. BETS IN TERMS OF WEALTH (PERMANENT CONSUMPTION) AND BETS IN TERMS OF DATED CONSUMPTION

Bets in terms of wealth allow the agent to spread the gain or losses over time by lending and borrowing. In what follows I assume that perfect smoothing is desirable and therefore bets in terms of wealth are bets on permanent consumption. In terms of Figure 1, a bet with the outcomes the smooth path  $a$  or the smooth path  $b$  is a bet in terms of wealth. I use the terms wealth, permanent consumption and money synonymously and will often refer to bets in terms of wealth as money bets.

Bets in terms of dated consumption require a different thought experiment. We start from a non-random consumption path and then consider a bet that makes date  $t$  consumption a random variable holding consumption at all dates other than  $t$  constant.

The distinction between the two types of bets can be illustrated with the help of Figure 2 that assumes a two-period horizon ( $t = 0,1$ ) and a zero interest rate. The maximum utility that the consumer can get when having the wealth 3, 4 or 5 is  $a$ ,  $e$  and  $b$ , respectively. From observing the indifference map we know that:  $a < e < b$ . But we do not know by how much. The consumer will prefer a wealth of 4 with certainty to a random wealth {3 or 5 with equal probabilities} if  $e > (a + b)/2$ . This will occur for

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<sup>4</sup> A related argument is in Postlewaite, Samuelson and Silverman (2004). They show that consumption commitments can cause risk neutral agents to care about risk, creating incentives to both insure risks and bunch uninsured risks together.

example, if  $a = 2$ ,  $e = 9$  and  $b = 10$ . Otherwise, he will prefer the bet (if for example,  $a = 8.5$ ,  $e = 9$  and  $b = 10$ ).

This is different from a bet in terms of second period consumption that holds first period consumption constant. For example, a bet in terms of future consumption that holds current consumption at the level  $C_0 = 2$  and is of the same relative size as the money bet just described has the outcomes:  $C_1 = \{1.5 \text{ or } 2.5\}$ . In terms of Figure 2, the money bet has the possible outcomes point  $A$  or point  $B$ . The consumption bet has the possible outcomes: point  $G$  or point  $F$ .

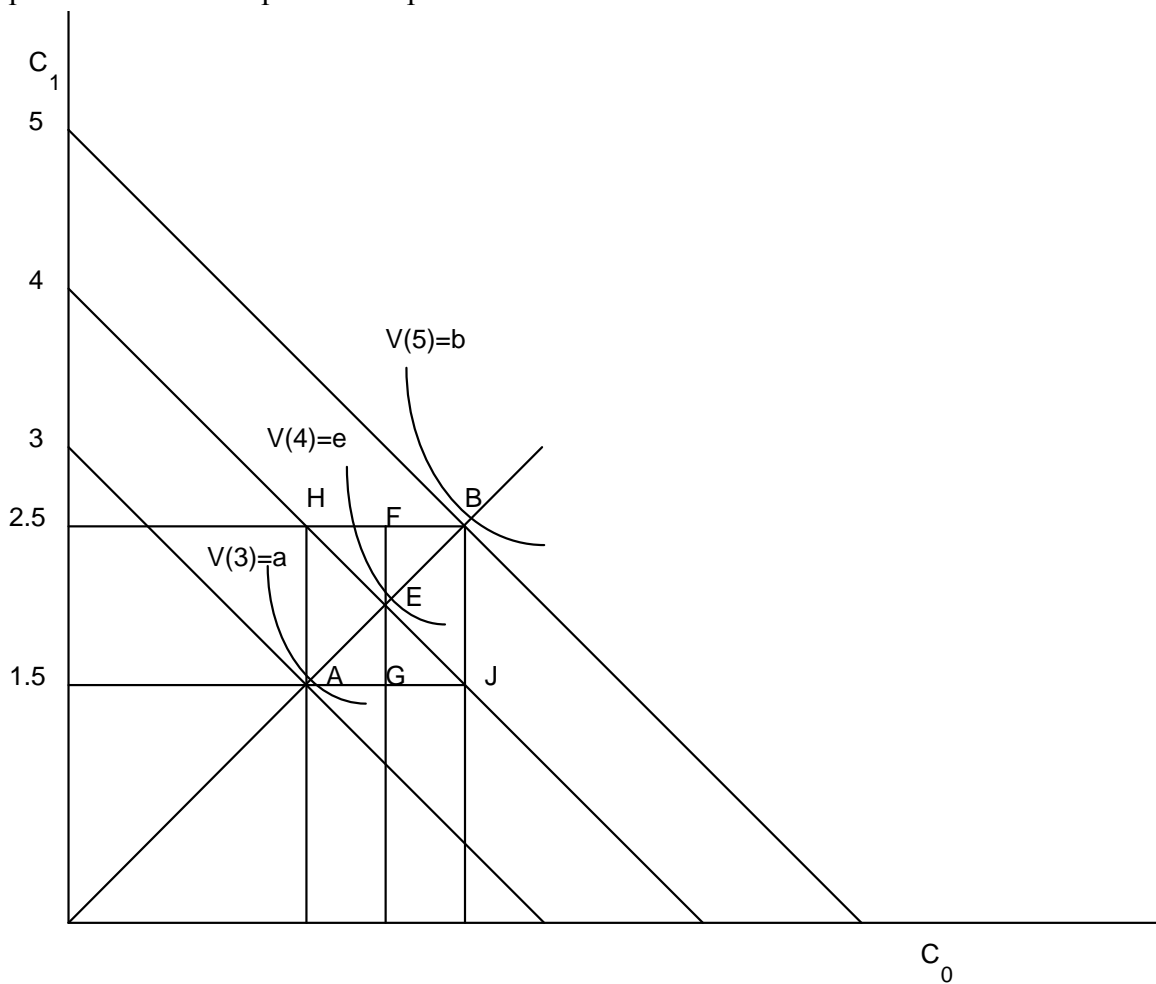


Figure 2

There is a connection between fluctuation aversion and risk aversion in terms of dated consumption. Someone who exhibits strong preference for smoothing should also exhibit risk aversion to bets in terms of dated consumption. For example, assume that the consumer has rectangular indifference curves that can be described by  $\min(C_0, C_1)$  or any monotonic function of it. In this case the information about the shape of the indifference curves is enough to tell us that the consumer will be averse to bets in terms of second period consumption. As we shall see the ordinal properties of the utility function play a role also in the aversion to money bets but this connection is not immediate.

#### 4. MEASURES OF RISK AVERSION AS A FUNCTION OF AGE

I now compare the relative risk aversion measures to money bets and consumption bets under the assumption that the market interest rate is equal to the subjective interest rate and therefore under certainty, the consumer wants to smooth consumption. To define the attitude towards money bet I consider the problem of a consumer who plans his consumption for  $T + 1$  periods under conditions of certainty.

$$(1) \quad V(w) = \max_{C_t} U(C_0, \dots, C_T) \text{ s.t. } \sum_{t=0}^T R^{-t} C_t = w.$$

Here  $U(C_0, \dots, C_T)$  is the utility function,  $R$  is the gross real interest rate and  $w$  is wealth. The attitude towards bets in terms of money is determined by the property of the value function  $V(w)$ .

I start from the time separable case:  $U(C_0, \dots, C_T) = \sum_t \beta^t u(C_t)$ , where  $u(C)$  is strictly concave and  $0 < \beta < 1$  is the discount factor. I assume  $R = 1/\beta$ . Under this assumption, the solution to (1) is the smooth path:  $C_t = kw$ , for all  $t$  where  $k = \left( \sum_{t=0}^T R^{-t} \right)^{-1}$ . Therefore:

$$(2) \quad V(w) = \sum_{t=0}^T \beta^t u(kw) = u(kw) \sum_{t=0}^T \beta^t = \frac{u(kw)}{k}$$

This leads to:

$$(3) \quad \frac{V''(w)w}{V'(w)} = \frac{u''(kw)kw}{u'(kw)} = \frac{u''(C)C}{u'(C)}$$

Thus under the time separable utility function, the relative risk aversion for bets in terms of money is the same as the relative risk aversion to bets in terms of consumption (at any date). An immediate implication is that relative risk aversion to money bets does not depend on age: When the individual advances with age, the horizon,  $T+1$ , gets shorter but consumption per period does not change and therefore relative risk aversion does not change.

I now turn to show that (3) is special to the time-separable case.

#### 4.1 THE COBB-DOUGLAS CASE

I consider the following utility function:

$$(4) \quad U(C_0, \dots, C_T; \alpha) = \frac{1}{\alpha} \prod_{t=0}^T (C_t)^{\alpha \beta^t}, \quad \alpha \neq 0, \quad \alpha < 1 \quad (ICD)$$

$$U(C_0, \dots, C_T; \alpha) = \sum_{t=0}^T \beta^t \ln(C_t), \quad \alpha = 0 \quad (IL)$$

where  $\alpha$  is a parameter. As we shall see, the assumption  $\alpha < 1$  implies risk aversion to consumption bets. As before I assume  $R = 1/\beta$  and therefore the solution to the consumer's maximization problem (1) is  $C_t = kw$  and the values functions are:

$$(5) \quad V(w) = (1/\alpha)(kw)^{\alpha \sum_{t=0}^T \beta^t} \text{ for } ICD; \quad V(w) = \ln(kw) \sum_{t=0}^T \beta^t \text{ for } IL$$

The coefficient of relative risk aversion to bets in terms of money ( $RAM$ ) is:

$$(6) \quad RAM = -\frac{V''(w)w}{V'(w)} = 1 - \alpha \sum_{t=0}^T \beta^t$$

The coefficient of relative risk aversion to bets in terms of consumption ( $RAC$ ) is:

$$(7) \quad RAC = -\frac{U_{tt}C_t}{U_t} = 1 - \alpha\beta^t$$

Note that  $RAM$  is different from  $RAC$  and the assumption  $\alpha < 1$  insures  $RAC > 0$ .

The difference between  $RAM$  and  $RAC$  can be illustrated in the two periods case with  $\beta = 1$ . In this case,  $RAC = 1 - \alpha$  and  $RAM = 1 - 2\alpha$ . When  $\alpha = 2/3$ ,  $RAM < 0$  and the consumer is willing to accept any actuarially fair money bet. But since  $RAC > 0$  he will buy any actuarially fair insurance to eliminate risk about future consumption. This is the argument used in Eden (1979) to account for the behavior of the insurance-buying gambler.

Note also that  $RAM$  changes with age. At age  $t$ ,  $RAM_t = 1 - \alpha \sum_{j=t}^T \beta^j$ . When  $\alpha > 0$ ,  $RAM$  increases with age reaching a maximum of  $1 - \alpha\beta^T$  in the last period of one's life.

When  $\alpha < 0$ ,  $RAM$  decreases with age reaching a minimum of  $1 - \alpha\beta^T$  in the last period of one's life. When  $\alpha$  approaches zero  $RAM$  approaches 1 (the log utility case).

Figure 3 illustrates the evolution of  $RAM$  and  $RAC$  as a function of age for two cases:  $\alpha = 0.1$  and  $\alpha = -0.1$ . In both cases the consumer is averse to bets in terms of consumption and  $RAC$  is close to unity. It increases with age when  $\alpha = 0.1$  and decreases with age when  $\alpha = -0.1$ . The  $RAM$  coefficient is more sensitive to age. When  $\alpha = 0.1$  the consumer starts with preference to bets in terms of money ( $RAM = -1$ ) and become risk averse at age 15. When  $\alpha = -0.1$  he starts his life with relatively strong risk aversion to money bets  $RAM = 3$ . Note that at the end of his life  $RAM = RAC \approx 1$  regardless of the choice of  $\alpha$ . We will see the same thing happening in the constant elasticity function.

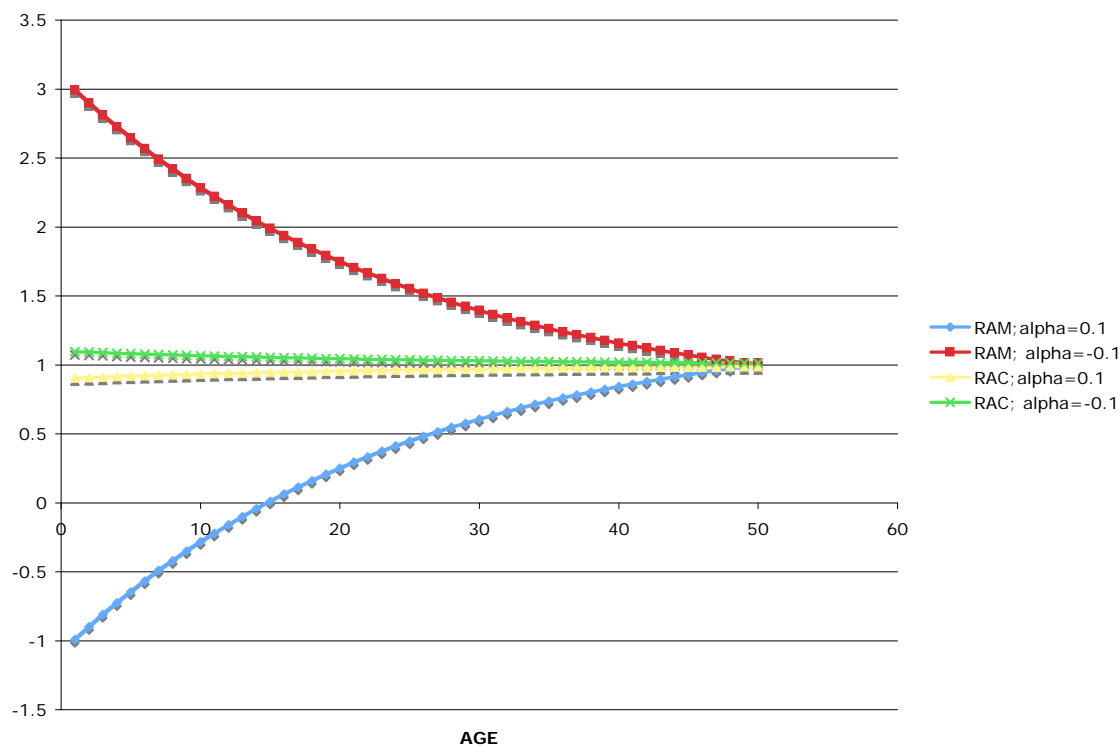


Figure 3: The evolution of  $RAM$  and  $RAC$  over the life cycle;  $ICD$  utility function;

$$\beta = 0.96; \alpha = 0.1 \text{ or } \alpha = -0.1$$



## 4.2 THE INTERTEMPORAL CONSTANT ELASTICITY (*ICE*) FUNCTION

I now consider the following function:

$$(8) \quad U(C_0, \dots, C_T) = \frac{1}{\psi} \left( \sum_{t=0}^T \beta^t (C_t)^\rho \right)^{\psi/\rho}; \quad \rho \neq 0 \text{ and } \psi \neq 0 \text{ (ICE)}$$

where  $IES = \frac{1}{1-\rho}$  is the intertemporal elasticity of substitution and  $\psi$  is a risk aversion parameter. As before I assume that  $R = 1/\beta$  and the optimal consumption under certainty is:  $C_t = kw$ . The value function is:

$$(9) \quad V(w) = (1/\psi)(kw)^\psi \left( \sum_{t=1}^T \beta^t \right)^{\psi/\rho}$$

The coefficient of relative risk aversion to bets in terms of money (*RAM*) is:

$$(10) \quad RAM = -\frac{V''(w)w}{V'(w)} = 1 - \psi$$

Thus at age 0, *RAM* does not depend on the ordinal properties of the utility function which are captured by the parameter  $\rho$ . This is not the case when the consumer advances in age. At age  $t$  the consumer has already chosen  $(\bar{C}_0, \dots, \bar{C}_{t-1})$  and his utility function is therefore:

$$(11) \quad U(\bar{C}_0, \dots, \bar{C}_{t-1}, C_t, \dots, C_T) = (1/\psi) \left( z_t + \sum_{j=t}^T \beta^j (C_j)^\rho \right)^{\psi/\rho},$$

where  $z_t = \sum_{j=0}^{t-1} \beta^j (\bar{C}_j)^\rho$ . Assuming  $C_j = kw$  the value function at age  $t$  is:

$$(12) \quad V(w, t) = (1/\psi) \left( z_t + y_t(w)^\rho \right)^{\psi/\rho}$$

where  $y_t = k^\rho \sum_{j=t}^T \beta^j$ . The coefficient of relative risk aversion at age  $t$  is therefore:

$$(13) \quad RAM_t = -\frac{V''(w, t)w}{V'(w, t)} = 1 - ((1 - \omega_t)\rho + \omega_t\psi),$$

where  $\omega_t = \frac{y_t(w)^\rho}{z_t + y_t(w)^\rho}$ . Along a smooth path when  $\bar{C}_j = kw$ ,

$$(14) \quad \omega_t = \frac{\beta^t - \beta^{T+1}}{1 - \beta^{T+1}} \text{ and } \lim_{T \rightarrow \infty} \omega_t = \beta^t.$$

Note that at age  $t=0$ ,  $\omega_t = 1$  and  $RAM_0 = 1 - \psi$  as in (10). Note also that  $\omega_t$  is decreasing in  $t$ . As a result the importance of the parameter  $\rho$  increases with age and the importance of the parameter  $\psi$  declines with age. In an infinite horizon model  $\omega_t$  is close to zero when  $t$  is large and the parameter  $\psi$  is no longer relevant. This suggests that in an infinite horizon model we can no longer separate between the elasticity of substitution and risk aversion and we are back to a single parameter model. Here I focus on the finite horizon case that allows for both parameters to play a role.

To separate the elasticity of substitution from risk aversion we write (13) as:

$$(15) \quad RAM_t = \omega_t RAM_0 + (1 - \omega_t)(\psi/\rho)$$

Since  $RAM_0 = 1 - \psi$  we can increase  $RAM_t$  for all  $t$  by reducing  $\psi$  without changing  $IES$ . The relationship (15) is also useful for determining changes in risk aversion over the life cycle. It implies:

$$(16) \quad RAM_t > RAM_{t-1} \text{ if } RAM_0 \leq \frac{1}{IES} \text{ and } RAM_t \leq RAM_{t-1} \text{ otherwise.}$$

Thus we need to assume that risk aversion is less than the inverse of the elasticity of substitution to get risk aversion that increases with age.

Figure 4 computes  $RAM_t$  under the assumption that  $IES = 0.5$  using (15). This is done for two cases  $RAM_0 = 0.5$  and  $RAM_0 = 3$ . In the first case,  $RAM$  increases with age. In the second it decreases with age. In both cases, it reaches approximately the same level of  $RAM = 2$  at the end of life because at this point only the elasticity of substitution matters. Figure 4 suggests that the agent converges to a behavior that can be described by a standard power utility function that imposes the restriction:  $RAM = \frac{1}{IES}$ .

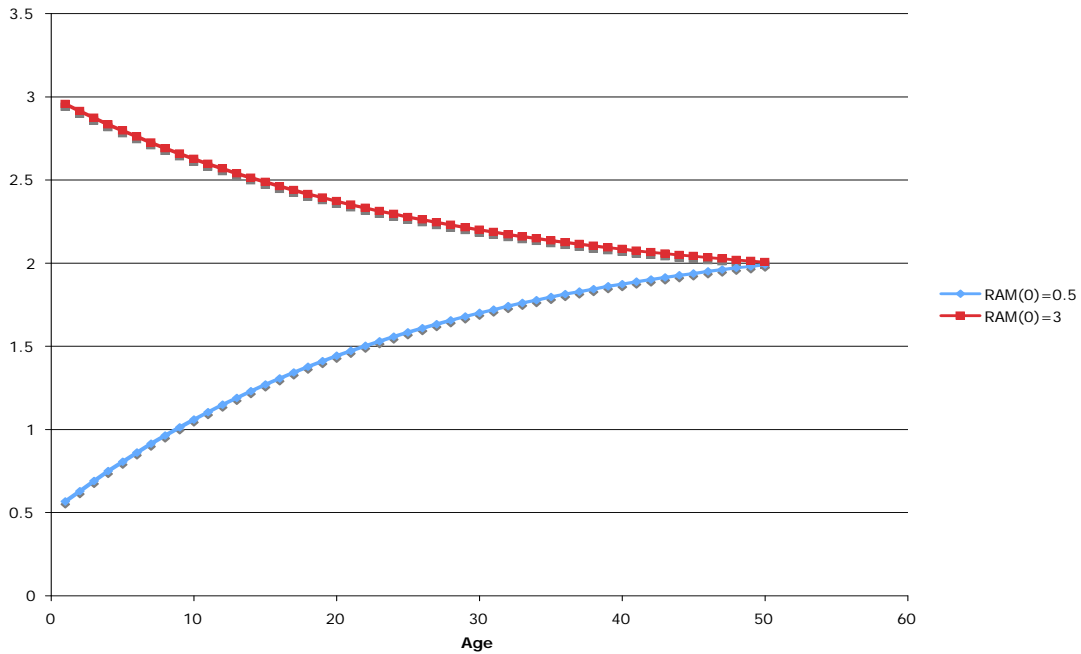


Figure 4:  $IES = 0.5, \beta = 0.96$

The relationship (15) may be useful for restricting the parameter  $RAM_0$ . For this purpose we must form an opinion, by introspection or by empirical evidence, about  $IES$  and the direction in which  $RAM_t$  evolves with age. Introspection about the evolution of risk aversion can be done as follows. We imagine that we consume along a smooth consumption path  $e$  in Figure 5. At age  $t$  we are offered a bet in terms of permanent consumption with possible outcomes  $a$  or  $b$ . How will our attitudes towards this bet change with  $t$ ?

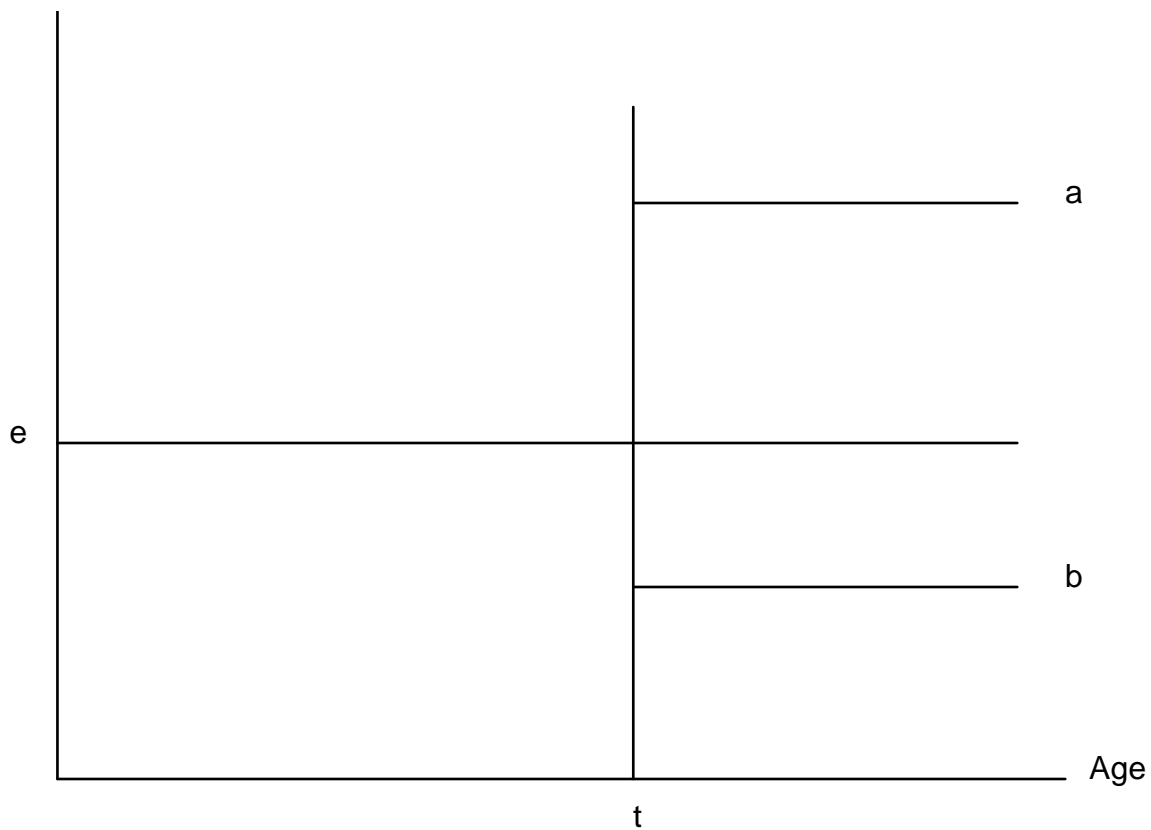


Figure 5: Introspection exercise about the relationship between  $RAM$  and age.

It seems that young people are more willing to take bets in terms of permanent consumption. For example, a war may be viewed as a bet on permanent consumption: There may be a draft but still some people volunteer and many who are drafted volunteer

to serve in special units. If you participate in a war that you believe in its cause you feel good about yourself for a long time (and you may tell stories about it). Of course the downward risk is that you get killed or wounded. If you get a desk job during the war there is no risk but you may feel bad for a long time. It seems that 18 years old are more willing to volunteer than 30 years old and the propensity to volunteer diminishes drastically after having kids. (Here we may view kids as irreversible consumption choices). In any event it seems that older people are more set in their ways and will therefore exhibit more aversion to bets in terms of permanent consumption. This suggests that  $RAM$  is increasing with age and therefore  $RAM_0 \leq 1/IES$ .

The literature provides various estimates of the  $IES$ . For example, Hansen and Singleton (1982) and Vissing-Jorgensen and Attanasio (2003) estimate  $IES > 1$ . Hall (1988), Campbell and Mankiw (1989) and Beaudry and Wincoop (1996) estimate  $IES < 1$ . Introspection of the type described by thought experiment  $b$  in the introduction, suggests to me  $IES \geq 1$ . Under the assumption that  $IES \geq 1$  and  $RAM$  increases with age, (15) implies  $RAM_0 \leq 1$ , but I do not impose this prior.

The importance of the past for current portfolio choices: In the  $ICD$  case  $RAM$  and the marginal rates of substitutions do not depend on past consumption. In the  $ICE$  case they do.

Epstein and Zin (1989) criticized the  $ICE$  function on the grounds that when  $\beta < 1$ , changes in  $C_0$  have a larger effect on risk aversion at age  $t > 1$  than changes in  $C_1$ . They consider the measure of  $RAC$ . But this is also true for our measure of  $RAM$  as is implied by (15) and the definitions:  $z_t = \sum_{j=0}^{t-1} \beta^j (\bar{C}_j)^\rho$  and  $\omega_t = \frac{y_t(w)^\rho}{z_t + y_t(w)^\rho}$ .

A different way of looking at the importance of the past is to ask what happens to the effect of a given shock over time. It is possible that the effect of a shock to  $C_0$  is larger than the effect of a shock to  $C_1$  but both becomes insignificant over time. Indeed this is

the case in the *ICE* utility function: Equation (15) implies that the effect of a given shock to  $C_0$  on  $RAM_t$  declines with  $t$ .

A third way is to look at the effect of a shock to permanent income. The effect of a permanent shock to consumption is different from the effect of a transitory shock. To see this point, we assume that the consumer learns that his permanent income has changed from  $k\omega$  to  $\lambda k\omega$ , where  $\lambda > 0$ . We consider two alternatives. In the first the new information arrived at  $t = 0$  and as a result  $C_0$  was changed as well as consumption in all other dates. In the second case the information arrived at  $t = 1$  and  $C_0$  was not changed. In the first case,  $\omega_t$  will not change and as a result  $RAM_t$  will not change. In the second case both will change. Thus information that arrives later has a larger effect on risk aversion.

We may therefore summarize the relative importance of shocks to past consumption on  $RAM_t$  in the following way.

1. At a given age  $t > 1$  a transitory shock to  $C_0$  has a larger effect on  $RAM_t$  than a transitory shock to  $C_1$ .
2. The effect of a transitory shock declines with age.
3. A shock to permanent income at age 0 has no effect on  $RAM_t$  but a shock to permanent income at age  $0 < j < t$  does affect  $RAM_t$ .

More important for our purpose is the effect of past consumption on the marginal rate of substitution (or the pricing kernel) that determines asset prices. This is discussed in detail in Appendix B where it is shown that (a) on average past consumption does not matter for current portfolio choices; (b) permanent shocks at  $t = 0$  do not affect current portfolio choices. Our examples suggest that transitory shocks to past consumption have very little (negligible) effect on asset prices while shocks to current consumption have large effect on asset prices.

Temporal risk aversion: Richard (1975) considers a bet between point  $J=(C_0 = 2.5, C_1 = 1.5)$  and point  $H=(C_0 = 1.5, C_1 = 2.5)$  in Figure 2 and defines temporal risk aversion as follows. A consumer who prefers the bet  $b_1=\{J \text{ or } H \text{ with equal probabilities}\}$  to the money bet  $b_2=\{A \text{ or } B \text{ with equal probabilities}\}$  has temporal risk aversion. Temporal risk neutrality and temporal risk preference are defined by indifference to the two bets and by strict preference of  $b_1$  over  $b_2$ .

Roughly speaking the preference between  $b_1$  and  $b_2$  depends on the relative importance of the ordinal properties of preferences (the desire to smooth consumption). I now show the following Claim: (a) A consumer with an additive utility function has temporal risk neutrality; (b) A consumer who is risk neutral to money bets and has strictly convex indifference curves, has temporal risk preference and (c) Under the *ICE* function risk aversion to money bets increases with age if the consumer has temporal risk preference and decreases with age otherwise.

To show part (a) note that:

$$(\frac{1}{2})[u(x) + \beta u(y)] + (\frac{1}{2})[u(y) + \beta u(x)] = (\frac{1}{2})[u(x) + \beta u(x)] + (\frac{1}{2})[u(y) + \beta u(y)] \text{ for all } x, y \geq 0.$$

I now use Figure 2 to show part (b). Point  $E$  with certainty is better than each of the two alternatives  $J$  and  $H$ . Therefore the consumer prefers the certainty point  $E$  to the bet  $b_1$ . Since a risk neutral to money bet is indifferent between  $b_2$  and the point  $E$  with certainty, it follows that the consumer prefers  $b_2$  to  $b_1$ . Thus, risk neutrality to money bets implies temporal risk preference.

Van den Heuven (2007) shows that in the *ICE* function the choice between  $b_2$  and  $b_1$  depends on the parameters  $\psi$  and  $\rho$ . When  $\rho > \psi$  the consumer exhibits temporal risk aversion. Using (13),  $\rho > \psi$  implies that *RAM* decreases with age. Since I do not

want to exclude the case in which *RAM* increases with age, I allow temporal risk preference.<sup>5</sup>

I now turn to the asset pricing implications of the above expected utility functions.

## 5. A TWO PERIODS SINGLE TREE ECONOMY

I now turn to assess the importance of the *RAM* and the *IES* for understanding asset prices. I start with a single-asset version of Lucas (1978) tree economy.

It is assumed that each period a new generation is born. Agents are identical and live for two periods. The representative agent gets an endowment of a tree in the first period of his life. The tree lives for two periods. It yields  $y$  units of consumption in the first period and  $d_s$  units in the second period, state  $s$ . There is a market for trees after the distribution of dividends. Note that only the young agents participate in the market for trees. The old agents have no trees after the distribution of dividends and have no reason to buy trees. Therefore, the overlapping generations structure is not important here. Its only role is to yield stationary asset pricing implications in a world with finitely lived agents.

The price of a tree is  $p$  and the representative consumer chooses (in the first period of his life) present consumption ( $C_0$ ) and the amount of trees ( $A$ ) subject to the budget constraint:

$$(17) \quad C_0 + pA = y + p$$

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<sup>5</sup> Introspection about temporal risk aversion seems to depend on the length of the period. I may not care about a series of uncorrelated small consumption bets but I may care about a mismatch between consumption in the first half and the second half of my life.



Consumption in the second period in state  $s$  is given by:

$$(18) \quad C_{1s} = Ad_s$$

The consumer chooses  $A$  and  $C_0$  to solve:

$$(19) \quad \max_{A, C_0} \sum_{s=1}^S \Pi_s U(C_0, C_{1s}) \text{ s.t. (17) and (18),}$$

where  $\Pi_s$  is the probability of state  $s$ . The first order conditions require:

$$(20) \quad \sum_{s=1}^S \Pi_s \left( U_{0s} - \frac{U_{1s} d_s}{p} \right) = 0$$

where  $U_{0s} = \partial U(C_0, C_{1s}) / \partial C_0$  and  $U_{1s} = \partial U(C_0, C_{1s}) / \partial C_{1s}$ .

The ICD-IL case: I now assume the Cobb-Douglas case:

$U(C_0, C_1) = (1/\alpha)(C_0)^\alpha (C_1)^\delta$ , where  $\delta = \alpha\beta$ . In this case:

$$(21) \quad U_{0s} - \frac{U_{1s} d_s}{p} = \frac{1}{\alpha} \left( \frac{\alpha}{C_0} - \frac{\delta}{y + p - C_0} \right) (C_0)^\alpha \left( \frac{(y + p - C_0) d_s}{p} \right)^\delta$$

Therefore the first order condition (20) requires

$$\left( \frac{\alpha}{C_0} - \frac{\delta}{y + p - C_0} \right) = 0 \text{ and } C_0 = \frac{\alpha(y + p)}{\alpha + \delta}.$$

To solve for  $p$  we substitute the market clearing condition  $C_0 = y$  in  $C_0 = \frac{\alpha(y + p)}{\alpha + \delta}$ . This

leads to:

$$(22) \quad p = (\delta/\alpha)y = \beta y.$$

The asset pricing formula (22) can also be obtained for the *IL* case. The rate of return on the asset is:

$$(23) \quad \frac{D}{p} = \frac{D}{\beta y} = \frac{G}{\beta},$$

where  $D = \sum_{s=1}^S \Pi_s d_s$  is expected dividends and  $G = 1 + g = D/y$  is the expected rate of consumption growth. Given the overlapping generations interpretation of the model, the aggregate consumption per period is constant and  $G$  is the rate of consumption growth over the lifecycle of the representative agent.

Since (6) implies  $RAM = 1 - \alpha(1 + \beta)$ , varying  $\alpha$  will change it without affecting the expected returns on the asset. We have thus shown,

Claim 1: When the representative agent's utility function is *ICD-IL*, the expected rate of return on the asset does not depend on the *RAM* measure of relative risk aversion and does not depend on the variance of the return. It depends only on the expected rate of consumption growth ( $G$ ) and the time preference parameter  $\beta$ .

Claim 1 is generalized in the Appendix to the more general finite horizon case. It follows directly from Kihlstrom and Mirman (1974) who show that in the *ICD* case uncertainty does not affect savings.

## 6. A TWO PERIODS MANY ASSETS ECONOMY

I now turn to the many assets case. I endow the representative agent with  $n$  trees that yield a total of  $y$  units of consumption (fruits) in the first period. Tree  $i$  yields  $d_{is}$

units in the second period in state  $s$ . The budget constraint of the representative agent is now:

$$(24) \quad C_0 + \sum_{i=1}^n p_i A_i = y + \sum_{i=1}^n p_i$$

$$(25) \quad C_{1s} = \sum_{i=1}^n d_{is} A_i$$

The agent problem is:

$$(26) \quad \max_{A_i} \sum_{s=1}^S \Pi_s U(C_0, C_{1s}) \text{ s.t. (24) and (25).}$$

The first order conditions for this problem are:

$$(27) \quad \sum_{s=1}^S \Pi_s (-U_{0s} p_i + U_{1s} d_{is}) = 0$$

I use  $D_s = \sum_{i=1}^n d_{is}$  for aggregate dividends in state  $s$ . I also assume that we can write the dividends of asset  $i$  in state  $s$  as a linear function of  $D_s$ :

$$(28) \quad d_{is} = a_i + b_i D_s + e_{is},$$

where  $\sum_{i=1}^n e_{is} = 0$  for all  $s$ ;  $\sum_{i=1}^n b_i = 1$  and  $\sum_{i=1}^n a_i = 0$ . The error terms  $e_{is}$  are determined by a zero sum purely distributive lottery, have zero mean and are independent of  $D_s$ .

Thus,  $\sum_{s=1}^S \Pi_s e_{is} = 0$  for all  $i$ . A risk free asset is an asset with non-random dividends  $d_{is} = 1$  (and  $a_i = 1, b_i = 0, e_{is} = 0$  for all  $s$ ). The market portfolio is an asset for which  $d_{is} = D_s$  (and  $a_i = 0, b_i = 1, e_{is} = 0$  for all  $s$ ).

The ICD case: Using the first order conditions (27), the market clearing conditions  $C_0 = y$ ,  $C_{1s} = D_s$  and (28) we arrive at the equilibrium condition:

$$(29) \quad p_i = \beta y \frac{\sum_{s=1}^S \Pi_s d_{is} (D_s)^{\alpha\beta-1}}{\sum_{s=1}^S \Pi_s (D_s)^{\alpha\beta}} = \beta y \frac{\sum_{s=1}^S \Pi_s (a_i + b_i D_s) (D_s)^{\alpha\beta-1}}{\sum_{s=1}^S \Pi_s (D_s)^{\alpha\beta}}$$

When  $a_i = 0$ ,  $p_i = \beta b_i y$ , and  $\frac{d_{is}}{p_i} = \frac{b_i D_s + e_{is}}{\beta b_i y}$ . Taking expectations leads to the following

Claim.

Claim 2: The expected gross rate of return on an asset with  $a_i = 0$  and  $b_i > 0$  is  $\frac{r}{\beta}$ .

Thus the expected rate of return on all assets with  $a_i = 0$  and  $b_i > 0$  is equal to the expected rate of return on the market portfolio.

I now turn to show that risk premium does not require risk aversion.

Claim 3: When  $\delta < 1$ , the rate of return on the risk free asset is less than  $\frac{r}{\beta}$ .

The proof of Claim 3 is in Appendix C. The intuition is as follows. When  $\delta < 1$ ,  $RAC = 1 - \delta > 0$  and the representative consumer is averse to uncertainty about future consumption. He will therefore hold the market portfolio rather than the risk free asset only if there is a risk premium. Note that when  $\delta = \alpha\beta < 1$  the coefficient of risk aversion  $RAM = 1 - \alpha(1 + \beta)$  may be positive or negative. For example,  $RAM = 0$  when  $\beta = 1$  and  $\alpha = 0.5$ . Therefore, risk premium does not require risk aversion to money bets.

I now turn to a numerical example that uses the following notation:

$R^b$  = the return on the risk-free asset (with  $a_i = 1$  and  $b_i = 0$ );

$R^1$  = the return on the market portfolio (with  $a_i = 0$  and  $b_i = 1$ ).

It is assumed that the rate of growth in aggregate dividends (consumption) is 1 or 1.04 with equal probabilities and  $\beta = 1$ . As we can see from Table 1 the rate of return on the market portfolio  $R^1$  does not depend on the *RAM* coefficient and is equal to  $\frac{g}{\beta} = 1.02$  in our example. The rate of return on the risk free asset is lower and decreases with our measure of risk aversion. The net rate of return on the risk free asset is 1.98% when  $RAM = 0$  and 1.92% when  $RAM = 3$ . The risk premium is accordingly, 0.02% when  $RAM = 0$  and 0.08% when  $RAM = 3$ .

Table 1: Expected gross rates of Returns under the *ICD-IL* utility function ( $\beta = 1$ )

$RAM = 1 - 2\alpha$	$R^1; d_i = \{1, 1.04\}$	$R^b; d_i = \{1, 1\}$	$100(R^1 - R^b)$
0	1.02	1.0198	0.02%
1	1.02	1.0196	0.04%
3	1.02	1.0192	0.08%
10	1.02	1.0179	0.21%

We may conclude that the *RAM* coefficient has very little effect on asset prices but has a considerable effect on risk premium: a change in *RAM* from 0 to 3 increases risk premium by 300%.

I now turn to the *ICE* utility function case.

## 7. THE CONSTANT ELASTICITY FUNCTION

I now consider the inter-temporal constant elasticity (*ICE*) utility function:

$$(30) \quad U(C_0, C_1) = \left(\frac{1}{\psi}\right) \left( (C_0)^\rho + \beta (C_1)^\rho \right)^{\frac{\psi}{\rho}} ; \rho \neq 0, \psi \neq 0$$

Note that the *ICE* function is a monotonic transformation of the standard power utility function that assumes:  $\psi = \rho$ . Recall that under the *ICE* function  $RAM = 1 - \psi$  and  $IES = \frac{1}{1-\rho}$ .

The asset pricing formula (26) is now:

$$(31) \quad p_i = \beta y^{1-\rho} \frac{\sum_{s=1}^S \Pi_s [(y)^\rho + \beta (D_s)^\rho]^{\frac{\psi}{\rho}-1} (D_s)^{\rho-1} d_{is}}{\sum_{s=1}^S \Pi_s [(y)^\rho + \beta (D_s)^\rho]^{\frac{\psi}{\rho}-1}}$$

I now turn to apply this formula for our example. As before, I assume that  $\beta = 1$  and consumption growth is: 1 and 1.04 with equal probabilities. Table 2 calculates the gross expected rate of return on the market portfolio for alternative values of  $RAM = 1 - \psi$  and  $IES = \frac{1}{1-\rho}$ . In this example, changes in the elasticity of substitution have a large effect on the gross expected interest rate while changes in risk aversion have a relatively small effect.

Table 2: Expected gross rates of returns on the market portfolio ( $\frac{D}{p}$ ) under the *ICE* utility function ( $\beta = 1$ )

	$RAM = 0$	$RAM = 1$	$RAM = 3$	$RAM = 10$
$IES=1.5$	1.0133	1.0134	1.0135	1.0139
$IES=0.99$	1.0202	1.0202	1.0202	1.0202
$IES=0.5$	1.0404	1.0402	1.0398	1.0384
$IES=0.333$	1.0612	1.0608	1.0600	1.0573

Table 3 computes the price of the risk-free asset (by substituting  $d_{is} = 1$  in [31]). The results support the claim that changes in *IES* are relatively more important also for the risk free return.

Table 3: The risk free return ( $R^b$ ) under the *ICE* utility function $(\beta = 1)$ 

	$RAM = 0$	$RAM = 1$	$RAM = 3$	$RAM = 10$
$IES=1.5$	1.0132	1.0130	1.0128	1.0119
$IES=0.99$	1.0200	1.0198	1.0194	1.0180
$IES=0.5$	1.0400	1.0396	1.0388	1.0361
$IES=0.333$	1.0606	1.0600	1.0588	1.0547

Table 4 subtracts Table 3 from Table 2 to get the risk premium. The risk premiums are small and depend on both the *IES* and the *RAM* coefficients. As suggested by the discussion in section 2, risk premium is positively related to *RAM* and negatively related to *IES*. Risk premium is positively related to *RAM* because an increase in *RAM* implies more demand for the elimination of risk. Risk premium is negatively related to *IES* because an increase in the elasticity implies less demand for smoothing.

Table 4: Risk premium in percentage terms ( $100[R^1 - R^b]$ ) under the *ICE* utility function $(\beta = 1)$ 

	$RAM = 0$	$RAM = 1$	$RAM = 3$	$RAM = 10$
$IES=1.5$	0.013	0.032	0.072	0.208
$IES=0.99$	0.020	0.039	0.079	0.215
$IES=0.5$	0.040	0.060	0.100	0.237
$IES=0.333$	0.062	0.082	0.122	0.260

Quantitatively the two effects are of equal importance in our example. A change in the  $RAM$  coefficient from 1 to 3 leads to a change in the risk premium from 0.04% to 0.08%. A "comparable" change in  $IES$  from 1 to 0.333 leads to roughly the same change in risk premium.

In previous drafts of this paper, I compared the implications of the  $ICD$  and  $ICE$  expected utility functions (Tables 1 - 4) with the implications of the generalized expected utility approach taken by Selden (1978) and Epstein and Zin (1991). The main difference is that in the Selden-Epstein-Zin approach, risk premium depends only on the risk aversion parameter.

## 9. CONCLUDING REMARKS

This paper studies the asset pricing implications of monotonic transformations of the standard power utility function. These monotonic transformations yield time-non-separable expected utility functions that allow for clear separation between  $IES$  and  $RAM$  at the beginning of the planning horizon. Then as the individual advances in age the  $IES$  plays an increasing role in determining  $RAM$ . We can still choose two parameters: One that determines  $IES$  and one that determines the path of  $RAM$  over the lifecycle. An assumption about the way  $RAM$  changes over the lifecycle imposes a restriction on  $RAM$  at age zero. If  $RAM$  increases with age then it must be the case that  $RAM_0 \leq 1/IES$ . This suggests low values of  $RAM_0$ .

In our examples, the expected rate of return on the market portfolio depends mostly on the  $IES$  and not on the  $RAM$  coefficient. Using the log utility as our benchmark (with  $IES=RAM=1$ ), we reduce  $IES$  from 1 to 0.333. This leads to an increase in the net rate of return on the market portfolio by about 200%. When we increase  $RAM$  from 1 to 3 the rate of return on the market portfolio does not change at all.



Similar results are obtained with respect to the risk free return. When *IES* changes from 1 to 0.333, the risk free rate goes up by about 200%. When *RAM* changes from 1 to 3 the net risk free rate goes down by about 2%. The risk free rate is close to the rate of return on the market portfolio and therefore the risk premium is small.

The equity premium depends on both the *IES* and the *RAM* coefficients in a more or less symmetric way. A reduction in the *IES* from 1 to 0.333 leads to the doubling of the equity premium from 0.04% to 0.08%. An increase in the *RAM* coefficient from 1 to 3 yields similar change in the equity premium.

The Epstein-Zin approach yields different results. In the two periods examples that I worked out there was almost no effect of *IES* on the equity premium when using the Epstein-Zin approach.

The standard power (SP) utility function yields the same predictions as the *ICE* utility function if we impose  $RAM = 1/IES$ . Since asset prices are not sensitive to changes in *RAM* this supports the interpretation that changes in the parameter of the SP utility function have large effect on asset prices because of the implied changes in *IES* and not because of the implied changes in *RAM*.

Our main results assume a *RAM* coefficient in the range 0 - 10. This is the range assumed originally by Mehara and Prescott (1985). It is possible that by allowing much higher *RAM* coefficients one can "solve" the original equity premium puzzle. Under the *ICD* function higher *RAM* coefficients will not change the rate of return on the market portfolio but will lower the risk-free rate. Therefore, very high *RAM* coefficients can lead to a large risk premium. This "solution" is possible because unlike the standard power utility function, the *ICD* function allows risk aversion and *IES* to be high simultaneously. See Kocherlakota (1996). However, there is a problem in assuming very high *RAM* coefficients because it implies that risk aversion may decrease sharply with age. There are other solutions to the puzzle that focus on taxes, liquidity and other frictions. See McGrattan and Prescott (2003).

Bansal and Yaron (2004) explain the equity premium puzzle by allowing for small persistent shocks to consumption growth and for changes in volatility. They use the Epstein-Zin approach and find a large effect of *IES* on the equity premium. But in their case a lower *IES* leads to a lower equity premium while under the expected utility approach used here it leads to a higher equity premium. It would be interesting to apply the expected utility approach to the consumption process assumed by Bansal and Yaron. I leave this to another paper.

## APPENDIX A: TIME CONSISTENT PREFERENCES AND TIME CONSISTENT PLANS

Some have argued that extending the analysis to many periods, must run into time inconsistency problems. Here I argue against this apparently widely held perception. I assume expected utility and time consistent preferences. An agent with time consistent preferences will treat past choices as irreversible and will continuously update the list of choice variables. He will also use Bayes' rule to update the probabilities of the states of nature. But the preference themselves do not change over time.

To illustrate, I assume a 3 periods horizon and a utility function:  $U(C_0, C_1, C_2)$ . Note that the discounting is implicit in this utility function and need not be exponential. It is assumed that only the list of choice variables changes over time. At  $t = 0$ , the consumer chooses  $(C_0, C_1, C_2)$  to maximize  $U(C_0, C_1, C_2)$ . At  $t = 1$  he chooses  $(C_1, C_2)$  to maximize  $U(\bar{C}_0, C_1, C_2)$ , treating his first period choice,  $\bar{C}_0$ , as given.

I follow the interpretation in Peleg and Yaari (1973) and argue that expected utility with time consistent preferences lead to plans that will be followed in future dates and are thus time consistent.

I start by visiting Strotz (1956) original article.

Age and distance from present discounting: Strotz (1956) assumes a time separable utility function. At time  $\tau$  the consumer discounts the instantaneous utility at time  $t$  by the discount function  $\lambda(t - \tau)$ . The discount function thus depends on the time-distance from the present. Strotz also allows for the effect of calendar time (or age) on the "instantaneous utility function".

In our three periods formulation Strotz's specification may be written as follows.

At  $\tau = 0$  the utility is given by:

$$U^0(C_0, C_1, C_2) = \lambda(0)u(C_0, 0) + \lambda(1)u(C_1, 1) + \lambda(2)u(C_2, 2)$$

At  $\tau = 1$  it is given by:

$$U^1(C_0, C_1, C_2) = \lambda(-1)u(C_0, 0) + \lambda(0)u(C_1, 1) + \lambda(1)u(C_2, 2)$$

Strotz argues that in general,  $U^0(C_0, C_1, C_2)$  is different from  $U^1(C_0, C_1, C_2)$  and therefore it will lead to time consistency problems unless  $\lambda(t - \tau) = \beta^{t-\tau}$ . In this exponential discounting case  $U^0(C_0, C_1, C_2) = \beta U^1(C_0, C_1, C_2)$  and therefore the change in the utility function does not change behavior. In our terminology, preferences are time-consistent only in the exponential discounting case.

Modern analysis typically uses  $u(C_t)$  instead of  $u(C_t, t)$ . In this case one must choose between age discounting and distant from present discounting. An age discounting specification is:

$$(A1) \quad U^0(C_0, C_1, C_2) = \lambda(0)u(C_0) + \lambda(1)u(C_1) + \lambda(2)u(C_2)$$

$$(A2) \quad U^1(\bar{C}_0, C_1, C_2) = \lambda(0)u(\bar{C}_0) + \lambda(1)u(C_1) + \lambda(2)u(C_2)$$

A distance from the present discounting is (A1) and

$$(A3) \quad U^1(\bar{C}_0, C_1, C_2) = \lambda(-1)u(\bar{C}_0) + \lambda(0)u(C_1) + \lambda(1)u(C_2).$$

Note that the preferences (A1) and (A2) are time-consistent. These preferences will lead to time-consistent plans because the marginal rate of substitution between  $C_1$  and  $C_2$  does not change when the consumer gets to  $t = 1$ . That is under (A2) we have:

$$(A4) \quad \frac{\frac{\partial U^0}{\partial C_1}}{\frac{\partial U^0}{\partial C_2}} = \frac{\frac{\partial U^1}{\partial C_1}}{\frac{\partial U^1}{\partial C_2}}$$

This does not hold under (A3) unless  $\lambda(t - \tau) = \beta^{t-\tau}$ . Thus the specification (A3) will lead to time consistent plans only in the special case of exponential discounting.

Uncertainty: In the case of uncertainty, the general formulation by Arrow (1964) may lead to a problem of time consistency because new information will typically lead to the updating of probabilities. But under the expected utility hypothesis the utility is linear in the probabilities and therefore updating the probabilities does not lead to time inconsistency problems.

To show this well-known claim, I assume a three periods horizon:  $t = 0, 1, 2$ . Events at each date may take  $S$  possible realizations. The probability that "state of nature"  $k$  will occur at  $t = 0$  is denoted by  $\pi_k$ . The probability that "state of nature"  $i$  will occur at date 1 given that "state of nature"  $k$  has occurred at  $t = 0$  is denoted by  $\pi_{ki}$  and the probability that "state of nature"  $j$  will occur at date 2 given that "state of nature"  $k$  has occurred at  $t = 0$  and "state of nature"  $i$  has occurred at  $t = 1$  is denoted by  $\pi_{kij}$ . Similarly,  $C_{0k}$  denotes consumption at  $t = 0$  state  $k$ ,  $C_{1ki}$  denotes consumption at  $t = 1$  state  $(k,i)$  and  $C_{2kij}$  denotes consumption at  $t = 2$  state  $(k,i,j)$ . The most general formulation used in Arrow (1964) assumes that the consumer evaluates consumption plans by the utility function:

$$(A4) \quad Z(C_{01}, \dots, C_{0S}; C_{111}, \dots, C_{1SS}; C_{2111}, \dots, C_{2SSS}).$$

At  $t = -1$  the consumer faces the budget constraint:

$$(A5) \quad \sum_{k=1}^S P_{0k} C_{0k} + \sum_{k=1}^S \sum_{i=1}^S P_{1ki} C_{1ki} + \sum_{k=1}^S \sum_{i=1}^S \sum_{j=1}^S P_{2kij} C_{2kij} = w,$$

where  $P$  are the prices of the contingent commodities. He maximizes (A4) subject to (A5). The first order conditions for this problem require:

$$(A6) \quad \frac{Z_{2msr}}{Z_{1ms}} = \frac{P_{2msr}}{P_{1ms}},$$

where  $Z_i = \partial Z / \partial \alpha_i$ . In this general formulation an agent that learns about the state at  $t = 0$  will update the probabilities and as a result the utility function  $Z$  will change. He will therefore want to change his plans.

I now turn to the expected utility case assuming that there exists a function  $U$  such that:

$$(A7) \quad Z = \sum_k \pi_k \sum_i \pi_{ki} \sum_j \pi_{kij} U(C_{0k}, C_{1ki}, C_{2kij})$$

In this case, the marginal utilities are:

$$(A8) \quad Z_{1ms} = \pi_m \pi_{ms} \sum_j \pi_{msj} U_{1ms}(C_{0m}, C_{1ms}, C_{2msj}), \quad Z_{2msr} = \pi_m \pi_{ms} \pi_{msr} U_{2msr}(C_{0m}, C_{1ms}, C_{2msr})$$

The marginal rate of substitution (MRS) is:

$$(A9) \quad \frac{Z_{2msr}}{Z_{1ms}} = \frac{\pi_{msr} U_{2msr}(C_{0m}, C_{1ms}, C_{2msr})}{\pi_{msr} U_{1ms}(C_{0m}, C_{1ms}, C_{2msr})}$$

Suppose now that at  $t = 0$  the consumer learns that  $k = m$ . Then his utility function will become:

$$(A10) \quad Z^1 = \frac{1}{\pi_m} \sum_i \pi_{mi} \sum_j \pi_{mij} U(\overline{C_{0m}}, C_{1mi}, C_{2mij})$$

Along the optimal plan when  $C_{0m} = \overline{C_{0m}}$ , the MRS,  $\frac{Z_{2msr}^1}{Z_{1ms}^1}$ , is the same as (A9). Thus the MRS does not change when at  $t = 0$  the consumer learns that state  $m$  has occurred. In this sense the expected utility assumption is sufficient for guaranteeing time consistency.

## APPENDIX B: THE IMPORTANCE OF PAST CONSUMPTION FOR CURRENT DECISIONS UNDER THE *ICE* UTILITY FUNCTION

I now assume the special case:

$$(B1) \quad \begin{aligned} Z &= \sum_k \pi_k \sum_i \pi_{ki} \sum_j \pi_{kij} U(C_{0k}, C_{1ki}, C_{2kij}) \\ &= (1/\psi) \sum_k \pi_k \sum_i \pi_{ki} \sum_j \pi_{kij} \{ (C_{0k})^\rho + \beta(C_{1ki})^\rho + \beta^2(C_{2kij})^\rho \}^{\psi/\rho} \end{aligned}$$

The marginal rates of substitution are now:

$$(B2) \quad MRS_{msr} = \frac{Z_{2msr}}{Z_{1ms}} = \frac{\pi_{msr} \beta (C_{2msr})^{\rho-1} \{ (C_{0m})^\rho + \beta(C_{1ms})^\rho + \beta^2(C_{2msr})^\rho \}^{\frac{\psi}{\rho}-1}}{(C_{1ms})^{\rho-1} \sum_j \pi_{msj} \{ (C_{0m})^\rho + \beta(C_{1ms})^\rho + \beta^2(C_{2msj})^\rho \}^{\frac{\psi}{\rho}-1}}$$

It is useful to express (B2) as a multiplication of two terms:

$$(B3) \quad A = \beta \pi_{msr} \left( \frac{C_{2msr}}{C_{1ms}} \right)^{\rho-1}; \quad B_{msr} = \frac{\{ (C_{0m})^\rho + \beta(C_{1ms})^\rho + \beta^2(C_{2msr})^\rho \}^{\frac{\psi}{\rho}-1}}{\sum_j \pi_{msj} \{ (C_{0m})^\rho + \beta(C_{1ms})^\rho + \beta^2(C_{2msj})^\rho \}^{\frac{\psi}{\rho}-1}}$$

The term  $A$  does not depend on  $C_{0m}$ . The term  $B$  is equal to unity on average (regardless of the magnitude of  $C_{0m}$ ):

$$(B4) \quad \sum_j \pi_{msj} B_{msj} = 1$$

To evaluate the relative importance of consumption in various dates I assume that consumption at date 1 and 2 are given by:

$$(B5) \quad C_1 = \{ C_0 \text{ or } (1+g)C_0 \text{ with equal probabilities} \},$$

$$C_2 = \{ (1+g)C_0 \text{ or } (1+g)^2 C_0 \text{ with equal probabilities} \},$$

where  $g$  is a parameter. I use  $R_{i,j} = \frac{1}{AB}$  to denote the price of current (date 1) consumption when the current state is  $i$ , in terms of next period consumption that will be delivered if the state in the next period is  $j$ . The variable  $R_{i,j}$  is thus the price divided by the probability of the date 2 state and is comparable to an interest rate. We may therefore think of  $R_{i,j}$  as an implicit gross interest rate.

Table *B1* provides the relevant calculations. The first column in the Table is the implicit interest rate when the current state is low and the contingent claim is on the low state in the next period. The second column is the implicit interest rate when the current state is high and the contingent claim is on the high state and so on. The first row is the choice of *IES*. The second is the base line case when  $C_0 = 1$ . The third row consider an increase in  $C_0$  to  $C_0 = 1.1$  and increasing all other consumption by the same percentage according to (B5). I refer to it as a permanent shock to  $C_0$ . The third row is a transitory shock to  $C_0$  that does not affect future consumption. (This is done by increasing  $C_0$  to 1.1 without implementing [B5]).

As we can see, the realization of  $C_1$  plays the major role in the determination of the implicit interest rate. When  $IES = 0.5$  (the first three rows), the implicit interest when buying a claim on the low state is 12% when the current state is low and 3.5% when the current state is high. But changes in  $C_0$  have almost no effect: A permanent change has no effect and a transitory change only a small effect.

Table B1: Prices (Implicit interest rates) of contingent claims at date 1

( $g = 0.04; \beta = 0.96$ )

	$R_{low,low}$	$R_{low,high}$	$R_{high,low}$	$R_{high,high}$
$IES = 0.5$				
$C_0 = 1$	1.119632035	1.226307539	1.035077176	1.133885187
Permanent shock: $C_0 = 1.1$	1.119632035	1.226307539	1.035077176	1.133885187
Transitory shock: $C_0 = 1.1$	1.119409257	1.226554727	1.034865662	1.134119958
$IES = 1.5$				
$C_0 = 1$	1.052992328	1.114814742	1.025884141	1.085969373
Permanent shock: $C_0 = 1.1$	1.052992328	1.114814742	1.025884141	1.085969373
Transitory shock: $C_0 = 1.1$	1.05316179	1.11462976	1.026047829	1.085790741

Table B1 suggests that the effect of the past on the implicit interest rate is negligible relative to the effect of current consumption. To examine whether the distant past plays a larger role than the more recent past I now consider the more general case in which the horizon is  $T + 1$  periods. The history from time 0 to time  $T - 2$  is denoted by



the index  $k$ . The history up to time  $T-1$  is denoted by the indices  $(k,i)$  and the history up to time  $T$  is denoted by the indices  $(k,i,j)$ . We can now replace (B1) by:

$$(B6) \quad Z = \sum_k \pi_k \sum_i \pi_{ki} \sum_j \pi_{kij} U(C_{0k}, \dots, C_{T-2k}, C_{T-1ki}, C_{Tkij}) \\ = (1/\psi) \sum_k \pi_k \sum_i \pi_{ki} \sum_j \pi_{kij} \left[ \sum_{t=0}^{T-2} \beta^t (C_{tk})^\rho + \beta^{T-1} (C_{T-1ki})^\rho + \beta^T (C_{Tkij})^\rho \right]^{\psi/\rho}$$

The marginal rate of substitution between consumption at time  $T$  and time  $T-1$  is:

$$(B7) \quad MRS_{msr} = \frac{Z_{Tmsr}}{Z_{T-1ms}} = \frac{\pi_{msr} \beta (C_{Tmsr})^{\rho-1} \left[ z + \beta^{T-1} (C_{T-1ms})^\rho + \beta^T (C_{Tmsr})^\rho \right]^{\frac{\psi}{\rho}-1}}{(C_{T-1ms})^{\rho-1} \sum_j \pi_{msj} \left[ z + \beta^{T-1} (C_{T-1ms})^\rho + \beta^T (C_{Tmsj})^\rho \right]^{\frac{\psi}{\rho}-1}},$$

where  $z = \sum_{t=0}^{T-2} \beta^t (C_{tm})^\rho$ . Also in this case  $MRS_{msr} = AB$ , where

$$(B8) \quad A = \pi_{msr} \beta \left( \frac{C_{Tmsr}}{C_{T-1ms}} \right)^{\rho-1}; \quad B_{msr} = \frac{\left[ z + \beta^{T-1} (C_{T-1ms})^\rho + \beta^T (C_{Tmsr})^\rho \right]^{\frac{\psi}{\rho}-1}}{\sum_j \pi_{msj} \left[ z + \beta^{T-1} (C_{T-1ms})^\rho + \beta^T (C_{Tmsj})^\rho \right]^{\frac{\psi}{\rho}-1}}$$

Note that also here:  $\sum_j \pi_{msj} B_{msr} = 1$ .

I now assume that consumption at time  $T$  and  $T-1$  depends on consumption at time  $T-2$  according to:

$$(B9) \quad C_t = C_{t-1} \text{ if } t \leq \tau \text{ and } C_t = 1.02C_{t-1} \text{ otherwise.}$$

The parameter  $\tau$  determines the date at which we start growing. Figure B1 assumes  $T = 52$  and  $\tau = 0, 10, 40$ . We see that changes in  $\tau$  produce large changes in the

path of past consumption relative to changes in  $C_{T-1}$  and  $C_T$ . (The Figure draws the two possible realizations of  $C_{T-1}$  and  $C_T$ ).

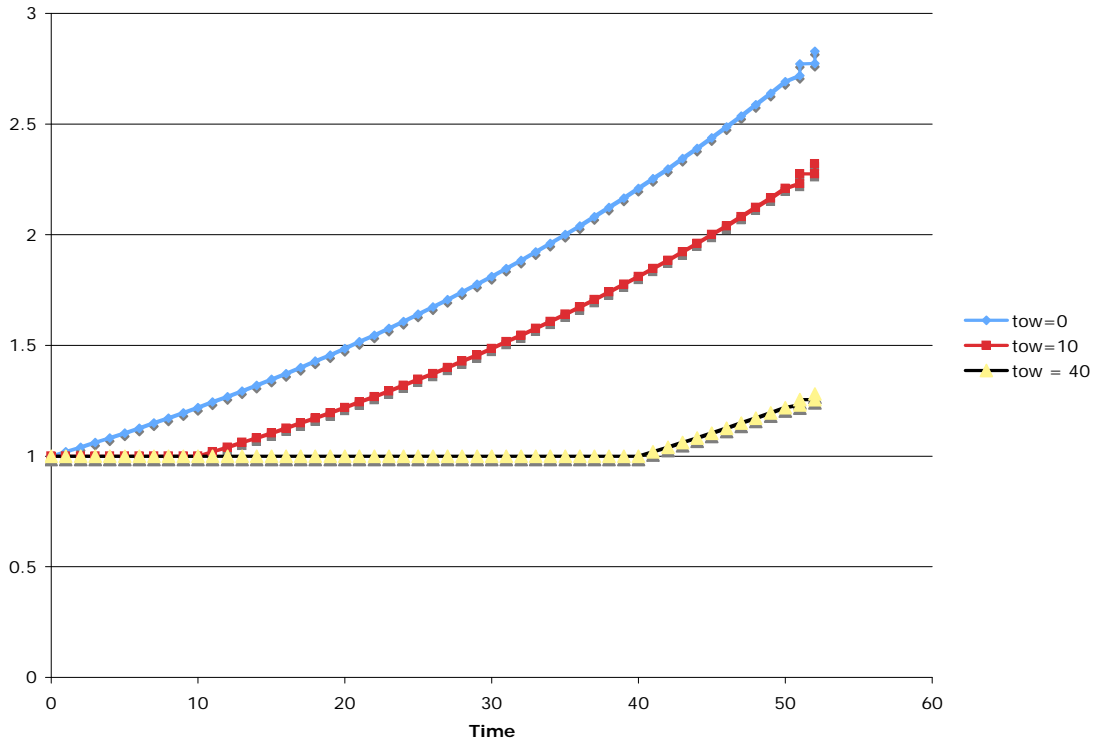


Figure B1: Growth starts at date  $\tau = 0, 10, 40$ . A shock to consumption occurs at  $t = 51$  and  $t = 52$

Figure B2 draws the implicit interest rates as a function of  $\tau$  assuming  $RAM = 3$  and  $IES = 1.5$ . The implicit interest rates are not sensitive to changes in  $\tau$  but are highly sensitive to changes in current consumption (the state at  $T-1$ ). When  $IES = 1.5$  and consumption at  $T-1$  is low the implicit gross interest rate for a claim on next period consumption in the low state is 1.05535. The implicit interest rate for the same contingent claim is 1.0416 when consumption at  $T-1$  is high. Thus the change in the time  $T-1$  state leads to a change in the net interest rate of about 33%. When we change  $\tau$  between 0 and 49 (which represents a huge change in past consumption) we get a change in the

implicit net interest rate of about 0.06%. This suggests that the state of past consumption is unimportant relative to the state of current consumption.

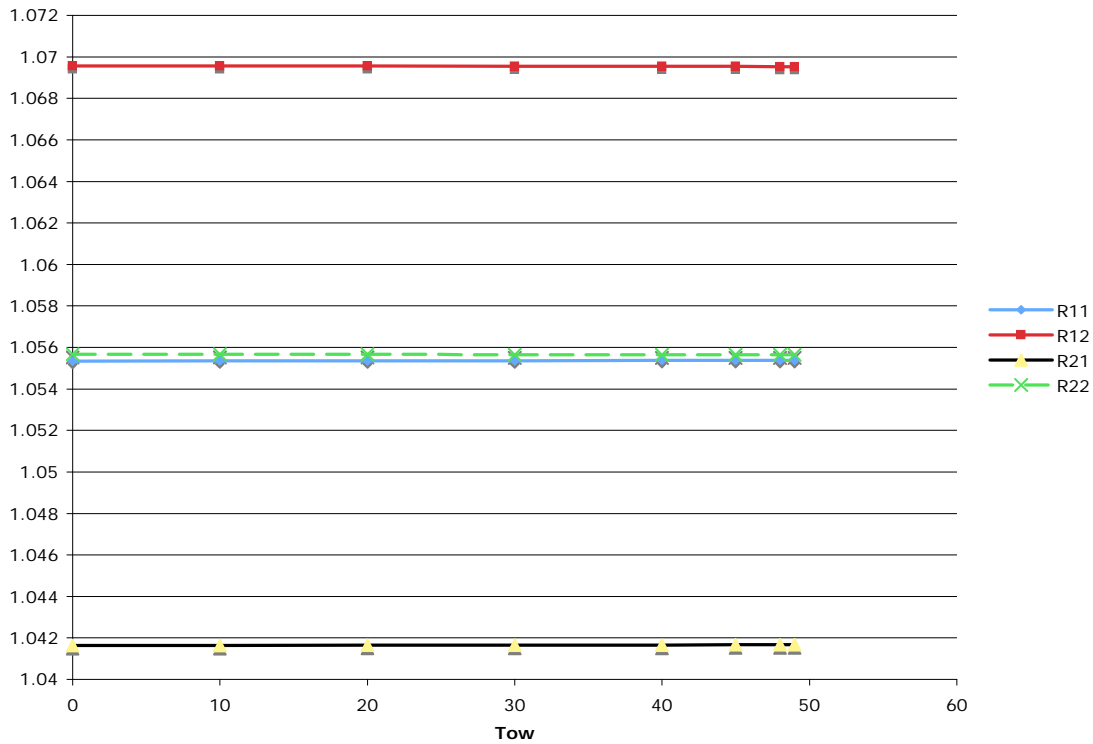


Figure B2: Implicit interest rates as a function of  $\tau$  ( $IES = 1.5, RAM = 3$ )

#### APPENDIX C: A FINITE HORIZON SINGLE ASSET *ICD* ECONOMY

I now consider an economy in which the representative agent lives for  $T$  periods. At  $t = 0$  he gets endowment of one tree that provides fruits for  $T$  periods and then dies (together with the agent).

I allow a general dividend (income) process. It is assumed that the representative agent at  $t = 0$  assigns positive probabilities,  $\pi_s$ , to all states  $s = 1, \dots, S$ . Over time he updates these probabilities when he learns that some states did not occur. The set of possible states at time  $t$  (the information available at time  $t$ ) is denoted by  $I_t$ . The

updated probability of state  $s$  is denoted by  $(\pi_s | I_t)$ . Note that  $(\pi_s | I_t) = 0$  if  $s \notin I_t$ . The agent also knows the information that he will have at time  $j > t$  if state  $s$  occurred. This information is denoted by  $I_{js}$ . At time  $t$  the choices of  $(A_0, \dots, A_{t-1})$  was already made. Since there is one tree per agent we assume  $A_j = 1$  for  $j < t$ . The agent chooses  $A_t$  and makes a contingent plan that specifies the amount of trees he will own at future dates:  $(A_{t+1s}, \dots, A_{T-1s})$ . The agent has to choose  $A_{js} = A_{j's}$  if at time  $j$  he cannot distinguish between the two states  $s$  and  $s'$ . Thus, he faces the informational constraint:  $A_{js} = A_{j's}$  if  $s, s' \in I_{js}$ . Assuming an *ICD* utility function we can state the time  $t$  problem as follows.

$$(C1) \quad V(k_{t-1}, I_t) = \max_{A_t, A_{t+1s}, \dots, A_{T-1s}}$$

$$k_{t-1} (d_t + p_t - A_t p_t)^{\alpha\beta^t} \sum_{s=1}^S (\pi_s | I_t) [A_t (d_{t+1s} + p_{t+1s}) - A_{t+1s} p_{t+1s}]^{\alpha\beta^t} k_{t+1s}$$

$$k_{t-1} = \prod_{j=0}^{t-1} (d_j)^{\alpha\beta^j} ; \quad k_{t+1s} = \prod_{j=t+2}^T [A_{j-1s} (d_{js} + p_{js}) - A_{js} p_{js}]^{\alpha\beta^j}$$

$$A_{js} = A_{j's} \text{ if } s, s' \in I_{js}$$

s.t.

I now define equilibrium as follows.

Equilibrium at time  $t$  is a vector

$(A_t, A_{t+1}, \dots, A_{T-1}, p_t, p_{t+1}, \dots, p_{T-1})$  such that

(a) given prices  $(p_t, p_{t+1}, \dots, p_{T-1})$ , the quantity vector

$(A_t, A_{t+1}, \dots, A_{T-1})$  solves (C1) and

(b) market clearing:  $A_t = 1$  and  $A_{js} = 1$  for all  $j > t$  and all  $s$ .

I now generalize the asset pricing formula (21) to the finite horizon case.

Claim C1: Equilibrium prices at time  $t$  are given by:

$$(C2) \quad p_t = (\beta + \beta^2 + \dots + \beta^{T-t})d_t \text{ and } p_{j_s} = (\beta + \beta^2 + \dots + \beta^{T-t})d_{j_s} \text{ for all } t < j < T$$

Note that when  $T = \infty$  (C2) implies  $p_t = \frac{d_t}{r^s}$ , where the subjective interest rate is defined by:  $1 + r^s = \frac{1}{\beta}$ . This formula is in the logarithmic preference example in Ljungqvist and Sargent (2000, page 239).

Proof: When  $T = 1$ , there is trade in the asset only in period

$t = T - 1 = 0$  and (C2) coincides with (21). We now proceed by induction. We assume that equilibrium prices when the horizon is  $T - t - 1$  (at time  $t + 1$ ) satisfy (C2) and show that equilibrium prices when the horizon is  $T - t$  (at time  $t$ ) satisfy (C2).

Given our induction hypothesis we can write the problem (C1) as:

$$(C3) \quad V(k_{t-1}; I_t) = \max_{A_t} k_{t-1} (d_t + p_t - A_t p_t)^{\alpha\beta^t} \sum_{s=1}^S (\pi_s | I_t) [A_t (d_{t+1s} + p_{t+1s}) - p_{t+1s}]^{\alpha\beta^{t+1}} k_{t+1s}$$

Now  $k_{t+1s} = \prod_{j=t+2}^T (d_{j_s})^{\alpha\beta^j}$  is a constant and  $p_{t+1s} = (\beta + \beta^2 + \dots + \beta^{T-t-1})d_{t+1s}$ . Note that the

assumption  $A_{t+1s} = 1$  follows from the induction hypothesis.

The first order conditions for the problem (C3) require:

$$(C4) \quad -\alpha\beta^t p_t (d_t + p_t - A_t p_t)^{\alpha\beta^t - 1} \sum_{s=1}^S (\pi_s | I_t) [A_t (d_{t+1s} + p_{t+1s}) - p_{t+1s}]^{\alpha\beta^{t+1}} k_{t+1s} \\ + (d_t + p_t - A_t p_t)^{\alpha\beta^t} \sum_{s=1}^S (\pi_s | I_t) \alpha\beta^{t+1} (d_{t+1s} + p_{t+1s}) [A_t (d_{t+1s} + p_{t+1s}) - p_{t+1s}]^{\alpha\beta^{t+1} - 1} k_{t+1s} = 0$$

Substituting  $A_t = 1$  and  $p_{t+1s} = (\beta + \beta^2 + \dots + \beta^{T-t-1})d_{t+1s}$  in (C4) leads to:

$$(C5) \quad p_t = (\beta + \beta^2 + \dots + \beta^{T-t})d_t$$

This completes the proof.  $\square$

We can now use Claim C1 to compute the rate of return on the asset as follows.

$$(C6) \quad \frac{d_{t+1s} + p_{t+1s}}{p_t} = \frac{(1 + \beta + \beta^2 + \dots + \beta^{T-t-1})d_{t+1s}}{(\beta + \beta^2 + \dots + \beta^{T-t})d_t} = \frac{d_{t+1s}}{\beta d_t}$$

Using  $G_t = \sum_{s=1}^S (\pi_s | I_t) \frac{d_{t+1s}}{d_t}$  to denote the expected consumption growth we can write the

expected rate of return at time  $t$  as:

$$(C7) \quad \frac{G_t}{\beta} = G_t(1 + r^s)$$

This is exactly the formula (22) that we got in the two periods horizon.

#### APPENDIX D: PROOF OF CLAIM 3

The rate of return on asset  $i$  is:

$$(D1) \quad (a_i + b_i D_s + e_{is}) / p_i = (1/\beta y)(a_i + b_i D_s + e_{is}) F(a_i, b_i),$$

where  $F(a_i, b_i) = \frac{1}{b_i + a_i \sum_{s=1}^S \Pi_s(D_s)^{\delta-1} / \sum_{s=1}^S \Pi_s(D_s)^\delta}$  is a non linear term. Since we

assume  $\delta < 1$ , the covariance between  $D$  and  $D^{\delta-1}$  is negative and

$$(D2) \quad F(1,0) = \frac{\sum_{s=1}^S \Pi_s(D_s)^\delta}{\sum_{s=1}^S \Pi_s(D_s)^{\delta-1}} = \frac{\sum_{s=1}^S \Pi_s(D_s)^{\delta-1} D_s}{\sum_{s=1}^S \Pi_s(D_s)^{\delta-1}} = \frac{Cov(D^{\delta-1}, D)}{\sum_{s=1}^S \Pi_s(D_s)^{\delta-1}} + \sum_{s=1}^S \Pi_s D_s$$

$$< \sum_{s=1}^S \Pi_s D_s.$$

Substituting this in (D1) and taking expectations leads to the conclusion that the expected rate of return on any asset with  $b_i = 0$  is less than  $G/\beta$ .  $\square$

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