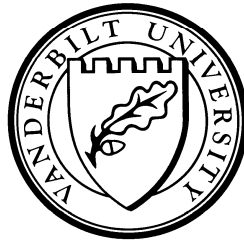


**INTERGENERATIONAL INTERMEDIATION AND ALTRUISTIC PREFERENCES**

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# INTERGENERATIONAL INTERMEDIATION AND ALTRUISTIC PREFERENCES

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The paper analyzes the intermediation role of government under the assumption that it has an advantage over the private sector in collecting uncollateralized loan payments. It is shown that a government loan program may improve the welfare of all generations (including the current old generation) if agents care about future generations in the time inconsistent manner originally proposed by Phelps and Pollak (1968). Numerical examples suggest that the welfare gains from intervention may be quite large and depends on the degree of altruism as defined by Phelps and Pollak. The welfare gains are large when agents are relatively “egoistic” because in this case the time inconsistency problem is more severe and there is more room for intervention.

Key words: Intermediation, Inconsistent Altruistic Behavior, Investment in Children, Government Loan Program.

JEL codes: E21, E42, E52

## 1. INTRODUCTION

The role of government in the capital market has been a subject of the debate that followed the recent financial crisis. Here I focus on the role of government in intermediating between generations.

Although the main model assumes altruistic behavior I start with the standard overlapping (OG) model that assumes egoistic preferences. One widely used version of the model assumes that agents live for two periods and get an endowment of a non-storable good only in the first. In this setting trade between generations is not possible in the absence of government intervention. Samuelson (1958) has shown that money (or more generally, a bubble asset) can solve the problem: When every young generation wants to save and is willing to accept money, the old may use the money they saved in the first period of their life to buy goods from the young.

The money solution will not work when agents get income only in the second period of their life, because the young have nothing to give to the old. More generally, it will not work in any (realistic) setting in which agents live for a finite number of periods and want to borrow at the beginning of the life cycle. What may work in these cases is a government loan program (or “negative money”). I find that a government loan program may improve welfare in the steady state but the current old generation must reduce consumption to start it.

The current old may support a government loan program if they care about the welfare of future generations in an inconsistent manner as in the classic paper of Phelps and Pollak (1968). Caring about future generations in a consistent manner is not enough because the parent does not need the government to give money to his child. For example, when altruism is modeled by introducing the child’s utility function as an argument in the utility function of the parent as in Barro (1974) and Aiyagai et.al. (2002),

there is no tension between the parent and the child: Whatever the child chooses to do is optimal from the parent's point of view. Under Barro's formulation the intergenerational consumption plan is analogous to a time consistent intertemporal plan that an infinitely lived agent with exponential discounting will choose. See, Strotz (1956) for the time consistency issues that arise when agents live forever and the discount is not exponential.

With inconsistent preferences, the father may not like the plan of his son. He may want his son to spend more on his grandson than he actually does. Therefore a small increase in my spending on my son will increase the welfare of both my son and my father (who cares about his grandchild). But I fail to take the effect on my father's welfare when making the spending choice. This external effect is absent in the Barro model because a small increase in the spending on my son does not affect my utility (because of the envelope theorem) and therefore does not affect the utility of my father.

This external effect is important if the representative agent is relatively egoistic in the sense that he cares about himself much more than about both his son and his grandson but at the same time he does not have strong preference for his son over his grandson. In this case a government loan program can increase welfare by a substantial amount.

Thus the current old generation will benefit from a government loan program only if it cares about the welfare of future generations but surprisingly, the benefits are small if agents care a lot about future generations and is large if they are relatively egoistic. To understand this paradox we may note that when agents are "perfectly altruistic" according to the definition of Phelps and Pollak, they have exponential discounting and the plan is time consistent. In this case there are no external effects and there are no benefits from a government loan program.

The plan of the paper is as follows. I start with a steady state analysis under the assumption of egoistic preferences. This section is used to make an analogy between a loan program and money: both are used to facilitate trade between generations and both may be viewed as a bubble asset. I then turn to the case of altruistic preferences. I

compare the first best plan from the point of view of the current old generation (the planner's solution) to autarky. I also ask under what conditions will the planner's program lead to a Pareto improvement. Numerical examples are used to judge the importance of the external effect problem.

## 2. INTERMEDIATION UNDER EGOISTIC PREFERENCES

In this section I focus on the intermediation role of government under the assumption of egoistic preferences. I show that a government loan program may increase welfare in the steady state but it will reduce the consumption of the current old who must provide the initial funds to start the program. The current old must therefore have some altruistic motive to support such a program. Although I believe that this section provides some important intuition about the working of a loan program, it is not necessary for the main model in section 3. The reader may therefore choose to go directly to the main model.

I assume a single non-storable good OG economy in which agents get an endowment of 1 unit in each of the two periods of their life. The utility function of the representative agent is:  $\omega U(y) + U(x)$ , where  $y$  is consumption when young,  $x$  is consumption when old and  $\omega \geq 0$  is a time preference parameter ( $\omega = 1/\beta$ ). The period utility  $U$  has the standard properties ( $U' > 0, U'' < 0$ ).

I start from the problem of a planner who wants to maximize welfare in the steady state. Since at each period there is 1 unit per agent and there are two representative agents (young and old) the resource constraint is:

$$(1) \quad x + y = 2$$

The planner's problem is:

$$(2) \quad \max_{x,y} \omega U(y) + U(x) \quad \text{s.t. (1)}.$$

The first order condition that an interior solution to this problem must satisfy is:

$$(3) \quad \omega U'(y) = U'(x)$$

Figure 1 illustrates the planner's solution. The endowment (autarky point) is at point  $E$  and consumption is at point  $B$ . The intuition is as follows. From the planner's point of view the shadow price of consumption to the young in terms of consumption to the old is 1: If he wants to give an additional unit to the young he must take it from the old. In the steady state the planner therefore equates the marginal utility over the lifetime.

Decentralization: The planner can implement the allocation by making an offer to lend and borrow at a zero interest rate. The debt of the old agents to the government is  $\tilde{y} - 1$ , where  $(\tilde{y}, \tilde{x})$  is the solution to the planner's problem. In the steady state the government collects this debt and transfer it to the young as a loan. The debt of the old generation is positive if  $\tilde{y} > 1$  as in Figure 1B and is negative otherwise, as in Figure 1A.

Using money: Money can play a role if the young want to save when the interest rate is zero. To see this point, assume that the government promises to buy and sell any amount of the good for 1 dollar per unit. In this case, a young agent may sell  $1 - y$  units for  $M = 1 - y \geq 0$  dollars. Consumption in the next period is equal to the amount of money accumulated in the first period plus the second period endowment:  $x = M + 1$ .

Substituting  $M = 1 - y$  in the last equation leads to:  $x + y = 2$ . The consumer can therefore choose any consumption pair  $(y, x)$  that satisfies:

$$(4) \quad x + y = 2 \text{ and } 1 - y \geq 0.$$

When the non-negativity constraint is not binding, as in Figure 1A, the consumer will choose the planner's solution  $(\tilde{y}, \tilde{x})$ . Money works in this case.<sup>2</sup>

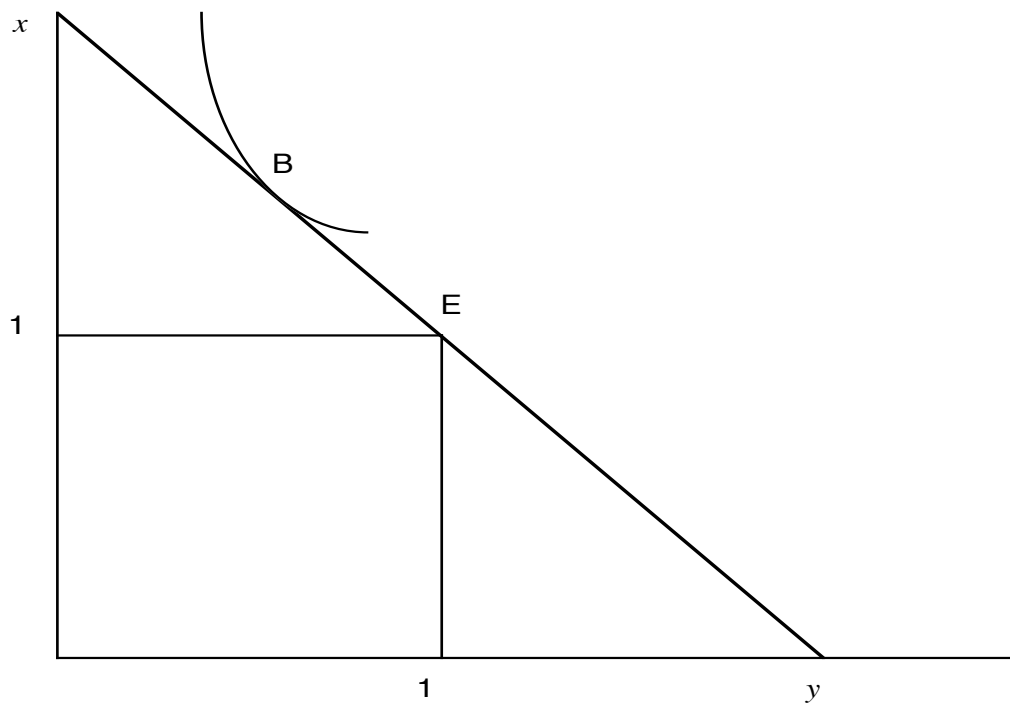
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<sup>2</sup> Some prefers to call the bubble asset in the OG model bond rather than money, because there is no transaction motive for holding it. Under the Friedman rule, there is no sharp distinction between money and bonds: the "last" dollar held as money does not provide liquidity services and is indistinguishable from bonds.

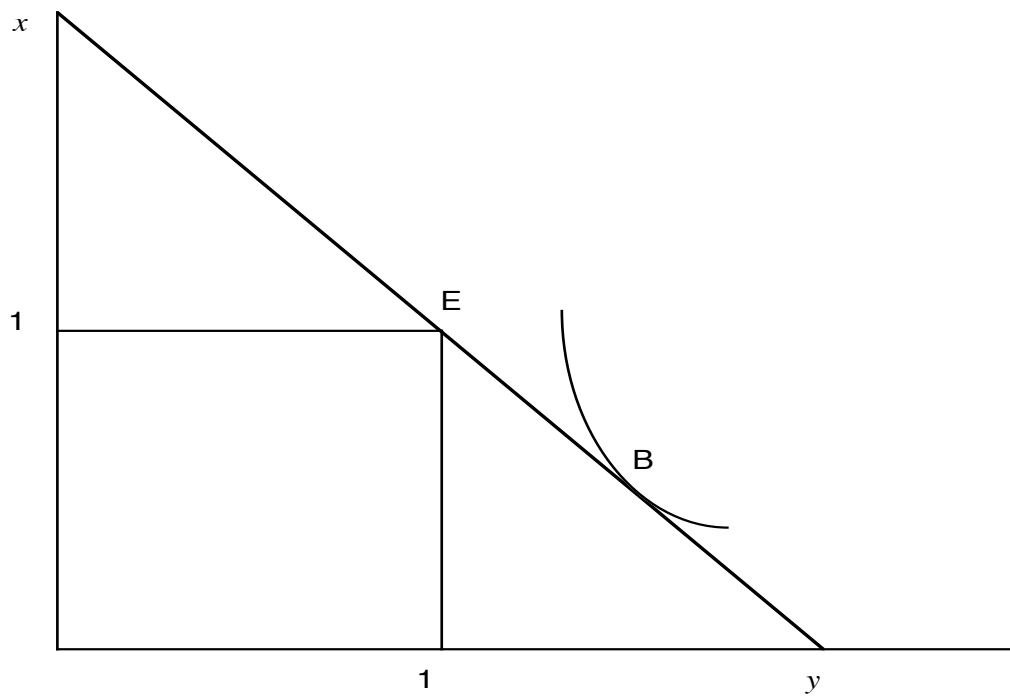
But when the consumer wants to choose a point to the right of the endowment point, as in Figure 1B, the non-negativity constraint is binding and introducing money will not help. In this case a Government loan program is required.

Negative money: Money in Samuelson's model is a bubble asset held by the public. A government loan is an asset held by the government but in the steady state the government does not use it to finance real spending. In this sense, a government loan is also a bubble asset.

Alternatively, we may think of government loans as negative money. From a mathematical point of view, there is little difference between the positive money holdings that occur when  $\tilde{y} \leq 1$  and the negative money holdings that occur when  $\tilde{y} > 1$ . But the involvement of the government is different. In the first case we need the government only for the initial step of introducing money: In the steady state, the old will simply give the dollar bills (which says that "the government owes the owner of this bill, 1 unit of consumption") directly to the young in exchange for goods. To make negative money work we need the government involvement in all periods: The government must collect the debt from the old and transfer it to the young as a loan (because the young will not directly accept pieces of paper that say "The owner of this paper owe the government 1 unit of consumption").



A. The Consumption Point  $B$  is to the Left of the Endowment Point



B. The Consumption Point  $B$  is to the Right of the Endowment Point

Figure 1: The First Best Steady State Outcome in an Endowment Economy



## 2.1 Productive Capital

I now introduce the possibility of productive investment and show that a government loan program may improve steady state welfare even when young agents want to save (consume less than their endowment).

I assume that the representative young agent gets an endowment of one unit of corn in the first period of his life and no endowment in the second period of his life. He can sow a fraction of his endowment and eat the rest. Let  $k$  denotes the amount he chooses to invest (sow). Then his first period consumption is:

$$(5) \quad y = 1 - k$$

The amount he harvest and consume in the second period is:

$$(6) \quad x = F(k)$$

where  $F$  has the standard properties of a production function ( $F' > 0, F'' < 0$ ). He solves:

$$(7) \quad \max_{x,y,k} \omega U(y) + U(x) \quad \text{s.t. (5) and (6)}.$$

The first order condition for this problem is:

$$(8) \quad \frac{\omega U'(y)}{U'(x)} = F'(1 - y)$$

It requires that the slope of the indifference curve is equal to the slope of the production possibility frontier as described by point  $A$  in Figure 2.

A planner in this economy who wants to maximize welfare in the steady state will face the resource constraint:

$$(9) \quad x + y = 1 - k + F(k)$$

Note that all the choices satisfying (5) and (6) also satisfy (9). But there are some choices that satisfy (9) but do not satisfy (5) and (6). Therefore the planner will in general be able to improve over the autarkic outcome.

The planner's problem is:

$$(10) \quad \max_{x,y,k} \omega U(y) + U(x) \quad \text{s.t. (9)}.$$

The first order conditions for this problem are:

$$(11) \quad F'(k) = 1 \text{ and } \omega U'(y) = U'(x)$$

Figure 2 illustrates the solution  $(\tilde{k}, \tilde{x}, \tilde{y})$  to (10), where  $P$  is the production point and  $B$  is the consumption point.

We may now distinguish between two cases. If point  $B$  is to the left of the production point  $P$  then conventional money can be used to implement the solution. But if it is to the right as in Figure 2B, conventional money will not work and a government loan program is required.

The planner can implement the solution by offering zero interest loans to the young. The young will take a loan equal to:

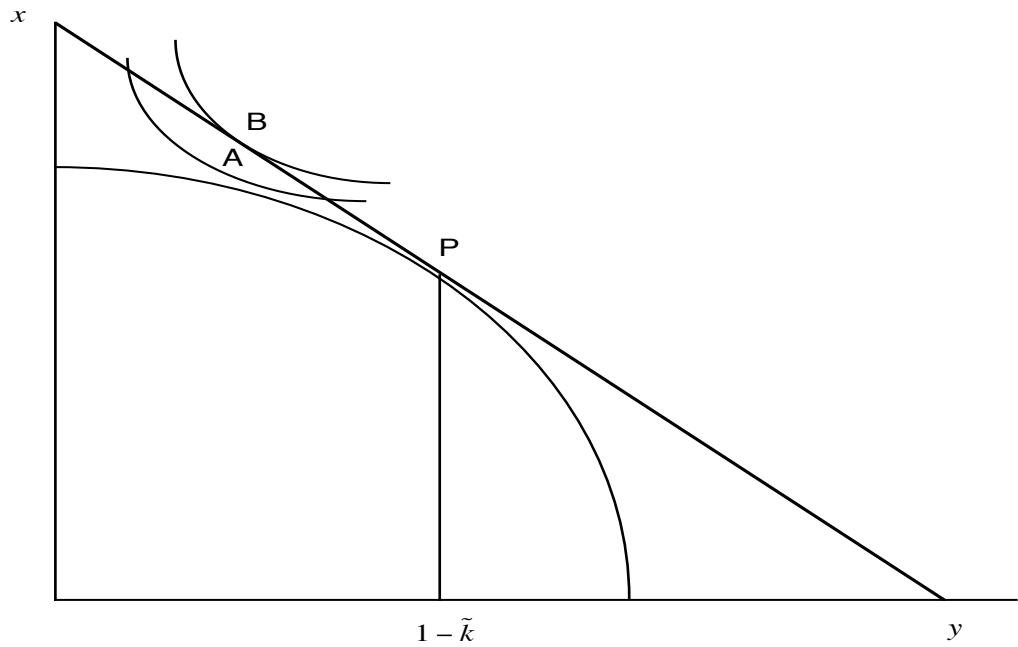
$$(12) \quad L = y + k - 1$$

And his second period consumption will be:

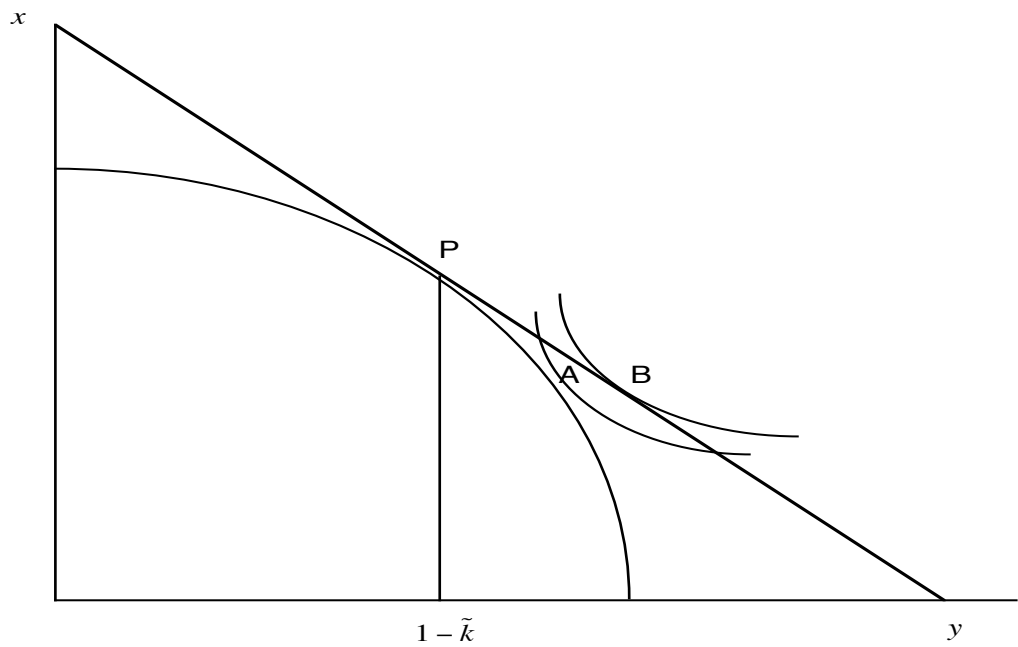
$$(13) \quad x = F(k) - L.$$

Substituting (12) in (13) leads to the resource constraint (9). Therefore when offered the opportunity to borrow at zero interest rate the consumer's budget constraint coincides with the planner's constraint and the consumer will therefore adopt the planner's solution.

Note the similarity between Figure 2 and the standard diagram that illustrates the benefits from international trade. Here the loan program opens the opportunity for trade between generations and therefore the production point  $P$  can be different from the consumption point.



A. The Optimal Consumption Point  $B$  is to the Left of the Optimal Production Point



B. The Optimal Consumption Point  $B$  is to the Right of the Optimal Production Point

Figure 2: Autarky and the First Best Steady State Outcome When Investment in Productive Capital is Possible

A government loan program may thus be necessary to maximize welfare in the steady state.<sup>3</sup> But getting to the steady state is a problem because the program reduces the consumption of the current old generation.

We may be tempted to say that when the current old generation cares enough about the welfare of future generations it will support a government loan program that maximizes steady state welfare. As was said in the introduction, caring about future generations does not imply a role for the government: The non-cooperative outcome is efficient when parents care about the welfare of their children in a consistent manner. A role for the government may arise when preferences are inconsistent. I now turn to analyze this case.

### 3. INCONSISTENT ALTRUISTIC BEHAVIOR

In the classic model of Phelps and Pollak (1968), parents care about their children but the children will not follow the parents' plan. As a result, the parents will want to affect the allocation of resources after their death and government intervention can help in achieving that. I now elaborate paying particular attention to the conditions under which the choice of the current old under full commitment will lead to a Pareto improvement.

Similar to Phelps and Pollak, I assume that the utility function of the old generation at time  $t$  (born at time  $t-1$ ) is:

$$(14) \quad U(x_{t-1}) + \lambda(\omega U(y_t) + U(x_t)) + \lambda\alpha(\omega U(y_{t+1}) + U(x_{t+1})) + \lambda\alpha^2(\omega U(y_{t+2}) + U(x_{t+2})) + \dots$$

$$0 < \lambda < 1 \text{ and } 0 < \alpha < 1$$

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<sup>3</sup> The old will not lend to the young because they will be dead by the time the loan is due. To examine another possibility, suppose that production is done by firms and the old own the firms. To support the first best steady state solution the price of the firm must be  $1 - \tilde{y}$  and the firm must pay  $\tilde{x}$  as dividend. This is not possible because  $\tilde{x} > F(1 - \tilde{y})$ .

Here  $(y_t, x_t)$  is the consumption of an individual born at  $t$  in the first and the second period of his life. The utility of the generation born at time  $t + \tau$  from his own consumption is:  $\omega U(y_{t+\tau}) + U(x_{t+\tau})$ , where  $\omega = \frac{1}{\beta}$  is the weight on first period consumption. The current old generation (born at  $t - 1$ ) discounts it by  $\lambda \alpha^\tau$ . The constant  $\lambda$  applies to all “other” generations and the constant  $\alpha$  is used to discount “remoteness”.

Phelps and Pollak assume that agents live for one period ( $\omega = 0$ ) and  $\lambda = \delta \alpha$ , where  $0 < \delta < 1$  measures “altruism”. They refer to the case  $\delta = 1$  ( $\lambda = \alpha$ ) as “perfect altruism”. Thus, “perfect altruism” occurs if “remoteness” is the only reason for discounting the child’s utility. The other extreme of egoistic behavior is:  $\delta = 0$  ( $\lambda = 0$ ). Note also that the standard exponential discounting occurs when  $\delta = 1$  ( $\lambda = \alpha$ ). In the calibration section I adopt the assumption  $\delta < 1$  ( $\lambda < \alpha$ ).

To simplify, I assume that only the old generation works and that they use their income to finance their own consumption, their children’s consumption and to invest in their children’s ability to produce when old. The investment in the children’s ability need not be in the form of human capital exclusively. It is possible that the parent will invest in a machine that he gives to his son when the son reaches adulthood. Thus, I do not distinguish here between human and physical capital. Formally, capital is used with a one period lag and fully depreciates after one period. The amount produced by the old generation at time  $t$  is  $F(k_{t-1})$ , where  $k_{t-1}$  is the amount of capital created in period  $t - 1$  when the current old were young. The resource constraint is:

$$(15) \quad x_{t-1} + y_t + k_t = F(k_{t-1})$$

The planner’s problem: I start from the problem of a social planner who chooses  $\{y_t, k_t\}_{t=0}^\infty$  to maximize the utility of the current old, born at  $t = -1$ , subject to (15). Substituting the constraint in the objective function (14) leads to the following maximization problem.

$$(16) \quad \max_{y_t \geq 0, k_t \geq 0} U(F(k_{-1}) - y_0 - k_0) + \lambda \sum_{t=0}^{\infty} \alpha^t [\omega U(y_t) + U(F(k_t) - y_{t+1} - k_{t+1})]$$

Using dynamic programming we can write the problem (16) as follows.

$$(17) \quad \max_{y_0, k_0} U(F(k_{-1}) - y_0 - k_0) + \lambda \omega U(y_0) + \lambda V(k_0)$$

where,

$$V(k) = \max_{y', k'} U(F(k) - y' - k') + \alpha \omega U(y') + \alpha V(k')$$

The function  $V(k)$  is the value of the investment to the current old, if all future generations behave as if their discount factor is  $\alpha$ . When  $\alpha > \lambda$ , parents want their children to behave as if they are more patient than they really are. The opposite is true when  $\alpha < \lambda$ .

I use  $\{\bar{y}_t, \bar{k}_t\}$  to denote the solution to (17) and for notational convenience I also use:  $\bar{k}_{-1} = k_{-1}$ . The solution to (17) must satisfy the first order conditions:

$$(18) \quad U'(F(\bar{k}_{t-1}) - \bar{y}_t - \bar{k}_t) = b \omega U'(\bar{y}_t) = b U'(F(\bar{k}_t) - \bar{y}_{t+1} - \bar{k}_{t+1}) F'(\bar{k}_t)$$

where  $b = \lambda$  for  $t = 0$  and  $b = \alpha$  for  $t > 0$ .

In Appendix A, I provide some comparative statics and show that under certain conditions, an increase in  $k_{t-1}$  will lead to an increase in all the three possible ways of spending income:  $x_{t-1}, y_t, k_t$ . Note that for large  $t$  we may achieve a steady state in which  $\bar{y}_t = \bar{y}_{t+1}$  and  $\bar{k}_t = \bar{k}_{t+1}$ . The marginal product of capital in this first best steady state is:

$$(19) \quad F'(\bar{k}) = \frac{1}{\alpha}$$

Autarky: As was said above, the current old generation wants future generations to behave as if their discount factor is  $\alpha$ . Since future generations will use their true discount factor to make choices, in the absence of intervention they will not follow the planner's solution. How will they behave?

Phelps and Pollak (1968) describe the behavior of the parents by the Nash equilibrium outcome of a "game" in which each generation chooses the ratio of savings to income. They assumed that each generation lives for one period only, has a power

utility function and a linear production function. Here I follow their approach using insights from Barro (1999) who analyzes the effects of a variable rate of time preference in a neoclassical growth model.

To simplify, I assume that  $\omega = 0$  and consumption occurs only in the second period of one's life. This assumption will be used whenever the first period consumption does not play an important role. Each generation chooses investment as a function of its own capital:  $k' = s(k)$  where the function  $s$  is weakly monotone and differentiable. Under this assumption, the old generation at time  $t$  can perfectly predict the investment of future generations as a function of his investment:  $k_{t+1} = s(k_t)$ ,  $k_{t+2} = s[s(k_t)] = s^2(k_t)$ ,  $k_{t+3} = s[s[s(k_t)]] = s^3(k_t)$  and so on. The problem of the old generation at time  $t$  is therefore:

$$(20) \quad s(k_{t-1}) = \arg \max_k U(F(k_{t-1}) - k_t) + \lambda U(F(k_t) - s(k_t)) + \lambda \alpha U(F[s(k_t)] - s^2(k_t)) \\ + \lambda \alpha^2 U(F[s^2(k_t)] - s^3(k_t)) + \dots + \lambda \alpha^n U(F[s^n(k_t)] - s^{n+1}(k_t)) + \dots$$

The function  $s$  defined recursively by (20) is a Symmetric Nash equilibrium in a game in which each generation chooses investment as a function of his capital. It can be shown that  $s'(k) < F'(k)$ . Thus an increase in the capital of the parent is used to increase both the parent's consumption and the parent's investment.

Using  $x_{t-1} = F(k_{t-1}) - k_t$  to denote the consumption of the generation born at  $t-1$ , the first order condition for the problem in (20) is:

$$(21) \quad U'(x_{t-1}) = \lambda U'(x_t)(F'(k_t) - s'(k_t)) + \lambda \alpha U'(x_{t+1})(F'[s(k_t)]s'(k_t) - s'[s(k_t)]s'(k_t)) + \dots$$

In the steady state when  $k = s(k)$ , we can write (21) as:

$$(22) \quad F'(k) = \frac{1 - (\alpha - \lambda)s'(k)}{\lambda}$$

We can also write  $s'(k) = \sigma(k)F'(k)$ , where  $0 < \sigma(k) < 1$  is the marginal propensity to save out of income. Substituting this in (22) leads to:

$$(23) \quad F'(k) = \frac{1}{\sigma(k)\alpha + (1 - \sigma(k))\lambda}$$

Thus, in the steady state, agents behave as if their discount rate is a weighted average of the discount of “others” and the discount of “remoteness”.<sup>4</sup> The weights in the steady state are determined entirely by the marginal propensity to invest  $\sigma$ . This does not coincide with the preference of the current old generation (the planner’s solution) who wants a steady state in which agents behave as if their discount factor is  $\alpha$ .

The possibility of a Pareto improvement: It is well known that in the Diamond (1965) model, we can get a Pareto improvement when there is over-accumulation of capital. See Benassy (2011, chapter 8). Here (23) and the assumption  $0 < \alpha, \lambda < 1$  imply that in the autarkic steady state,  $F'(k) < 1$  and there is no over-accumulation of capital.

Nevertheless, it may be possible to increase the welfare of all generations because of the assumed inconsistent preferences.<sup>5</sup>

Inconsistent preferences lead to an external effect because agents fail to take into account the effect of their decisions on previous generations. I focus on three generations: the grandfather (born at  $t - 1$ ), the father (born at  $t$ ) and the child (born at  $t + 1$ ). A Pareto improvement is in general possible because the father neglects the effect of his choice on the welfare of the grandfather.

To derive the conditions under which a Pareto improvement is possible, I start from autarky and assume the following deviation: The father changes his investment by  $dk_{t+1}$  units and the grandfather changes his investment by  $dk_t$  units so that the utility of

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<sup>4</sup> Barro (1999) allows for a more general variable discount factor and shows that under a logarithmic utility function agents behave as if they have a constant discount factor that is a weighted average of the variable discounts applied to different points in the future.

<sup>5</sup> Another possibility for Pareto improvement may arise if agents from the same generation are caught in a prisoners’ dilemma type situation. In this case each individual has a dominant strategy and the pursuit of this by each produces an overall result that is not efficient. See Sen (1967) and the literature cited by him for the conditions under which this type of inefficiency can arise. Here I assume that each generation is represented by a single individual so that a collusion among members of the same generation cannot lead to a Pareto improvement.



the father remains at the autarkic level. All other generations do not change their choices.

Thus,

$$(24) \quad (F'(k^{au})dk_t - dk_{t+1})U'(C^{au}) + \lambda F'(k^{au})dk_{t+1}U'(C^{au}) = 0$$

where  $k^{au}$  denotes the solution to (23). This and (23) leads to:

$$(25) \quad dk_t = \frac{(1 - \lambda F'(k^{au}))}{F'(k^{au})} dk_{t+1} = \sigma(\alpha - \lambda) dk_{t+1}$$

Note that in the exponential discounting case  $\alpha = \lambda$  the grandfather does not have to compensate the father for the change in his investment. The intuition is that in the consistent case, small changes do not change the objective functions of both the father and the grandfather.

The welfare of the grandfather will increase if:

$$(26) \quad -U'(C^{au})dk_t + \lambda U'(C^{au})(F'(k^{au})dk_t - dk_{t+1}) + \lambda \alpha U'(C^{au})F'(k^{au})dk_{t+1} > 0$$

Substituting (25) in (26) leads to:

$$(27) \quad A(dk_{t+1}) > 0,$$

where,  $A = (\alpha - \lambda) \left( \frac{1}{\sigma(\alpha - \lambda) + \lambda} - \frac{\sigma}{\lambda} \right)$ . A Pareto improvement is possible when  $A \neq 0$ . I

consider the case in which  $\lambda > \sigma^2(\alpha - \lambda) + \lambda\sigma$  and the expression in the second bracket is strictly positive. When  $\alpha < \lambda$ ,  $A < 0$  and a Pareto improvement can be achieved by choosing  $dk_{t+1} < 0$ . In this case (25) implies  $dk_t > 0$ , and the grandfather compensates the father for reducing his investment. When  $\alpha > \lambda$ ,  $A > 0$  and a Pareto improvement can be achieved by choosing  $dk_t > 0$  and  $dk_{t+1} > 0$ . In this case, the grandfather compensates the father for increasing his investment. I will focus on the case  $\alpha > \lambda$  because it seems that most grandparents would like their son to invest more in the grandson rather than less. As was said before this is also the assumption used by Phelps and Pollak.

Note that  $A = 0$  when  $\alpha = \lambda$  and therefore a Pareto improvement is not possible in this case of exponential discounting. See Strotz (1956).

What will happen to the welfare of all generations if we adopt the planner's solution? This is not the same question as the efficiency question discussed above because the proposed deviation from autarky did not coincide with the planner's solution. The question is of interest because the current old are the voters in our economy and they will want to implement their preferences. I now turn to this issue and to the comparison of the planner's and the autarky solutions.

Autarky and the planner's solution: Note that the first order condition for the planner's problem (18) implies:

$$(28) \quad U'(x_t) - U'(x_{t-1}) = U'(x_t)(1 - bF'(k_t)) \quad ; \quad b = \lambda \text{ for } t = 0 \text{ and } b = \alpha \text{ for } t > 0.$$

Since (23) and  $\alpha > \lambda$  imply that under autarky  $\frac{1}{\alpha} < F' < \frac{1}{\lambda}$ , it follows that the marginal utility first increases and then declines until we get to the planner's steady state (19). This leads to:  $x_0 < x_{-1}$  and  $x_t \geq x_{t-1}$  for all  $t > 0$ . Thus the consumption of the agents born at  $t = 0$  is less than the consumption of their parents. But all future generations enjoy consumption growth until we get to the planner's steady state (19). It follows that the welfare of the current young is less than the welfare of all future generations and this leads to the following Claim.

Claim 1: Implementing the planner's solution will lead to a Pareto improvement if it leads to an increase in the welfare of the current young.

I now turn to two special cases.

The linear utility case:  $U(x) = x$ . Under autarky the solution to (20) satisfies<sup>6</sup>:

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<sup>6</sup> To check that this is indeed a solution, note that under (29),  $s'(k) = 0$  and the investment in human capital does not depend on the human capital of the parent. Substituting  $s'(k) = 0$  in (22) leads to (29).

$$(29) \quad F'(k_t) = \gamma_\lambda$$

Note that this is a dominant strategy: The amount of investment does not depend on the choice of future generations.

The first order condition for the planner's problem (18) requires:

$$(30) \quad F'(\bar{k}_0) = \gamma_\lambda, \quad F'(\bar{k}_t) = \gamma_\alpha \text{ for } t > 0.$$

Thus, when  $\lambda < \alpha$ , each generation would like their children to invest more than they did. This of course will not happen under autarky.

In the linear utility case  $\sigma = 0$  and  $A = \frac{(\alpha-\lambda)\gamma_\lambda}{\lambda} > 0$ . Condition (27) therefore implies that a Pareto improvement over autarky is possible. But as was said before this does not imply that adopting the planner's solution will lead to a Pareto improvement.

I now turn to the conditions under which the planner's solution will lead to a Pareto improvement over autarky. I use  $k(R)$  to denote the solution to  $F'(k) = R$  and  $C(R) = F(k(R)) - k(R)$ . Note that  $C'(R) < 0$  for all  $R > 1$ .<sup>7</sup> Under autarky investment is  $k(\gamma_\lambda)$ , consumption is  $C(\gamma_\lambda) = F(k(\gamma_\lambda)) - k(\gamma_\lambda)$  and welfare is:

$$(31) \quad AU = C(\gamma_\lambda) + \frac{\lambda}{1-\alpha} C(\gamma_\lambda)$$

Under the planner's solution the investment of the current old is  $k(\gamma_\lambda)$  and the investment of the current young and all subsequent generations is  $k(\gamma_\alpha) > k(\gamma_\lambda)$ . The consumption of the current old is  $C(\gamma_\lambda)$ , the consumption of the current young is  $F(k(\gamma_\lambda)) - k(\gamma_\alpha) < C(\gamma_\lambda)$  and the consumption of subsequent generations is:  $C(\gamma_\alpha) > C(\gamma_\lambda)$ . The welfare of the current old generation is higher than under autarky because the planner maximizes their utility. The welfare of the current young generation is:

$$(32) \quad F(k(\gamma_\lambda)) - k(\gamma_\alpha) + \frac{\lambda}{1-\alpha} C(\gamma_\alpha)$$

The welfare of all future generations (born at  $t > 0$ ) is:

$$(33) \quad C(\gamma_\alpha) + \frac{\lambda}{1-\alpha} C(\gamma_\alpha) > AU$$

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<sup>7</sup> This follows from the first order condition to  $\max_k F(k) - k$  and the concavity of  $F$ .

Thus, the welfare of all generations other than the current young is above the autarkic level. The welfare of the current young may be above or below autarky: Their consumption is lower but their utility maybe higher because the consumption of future generations is higher. Figure 4 illustrates the differences between autarky and the planner's solution.

The welfare of the current young will improve under the planner's solution, if the benefit of having higher consumption for future generations is greater than the cost of increasing investment:

$$(34) \quad \frac{\lambda}{1-\alpha} \{C(\frac{1}{\alpha}) - C(\frac{1}{\lambda})\} > k(\frac{1}{\alpha}) - k(\frac{1}{\lambda})$$

Since  $\frac{C(\frac{1}{\alpha}) - C(\frac{1}{\lambda})}{k(\frac{1}{\alpha}) - k(\frac{1}{\lambda})} > F'(k(\frac{1}{\alpha})) = \frac{1}{\alpha}$ , the inequality (34) is satisfied if  $\frac{1}{\alpha} > \frac{1-\alpha}{\lambda}$  or

$$(35) \quad \lambda > \alpha(1-\alpha)$$

Since  $\alpha(1-\alpha) \leq 0.25$ , assuming  $\alpha > \lambda > 0.25$  is sufficient. Thus, following the planner's solution is likely to lead to a Pareto improvement.

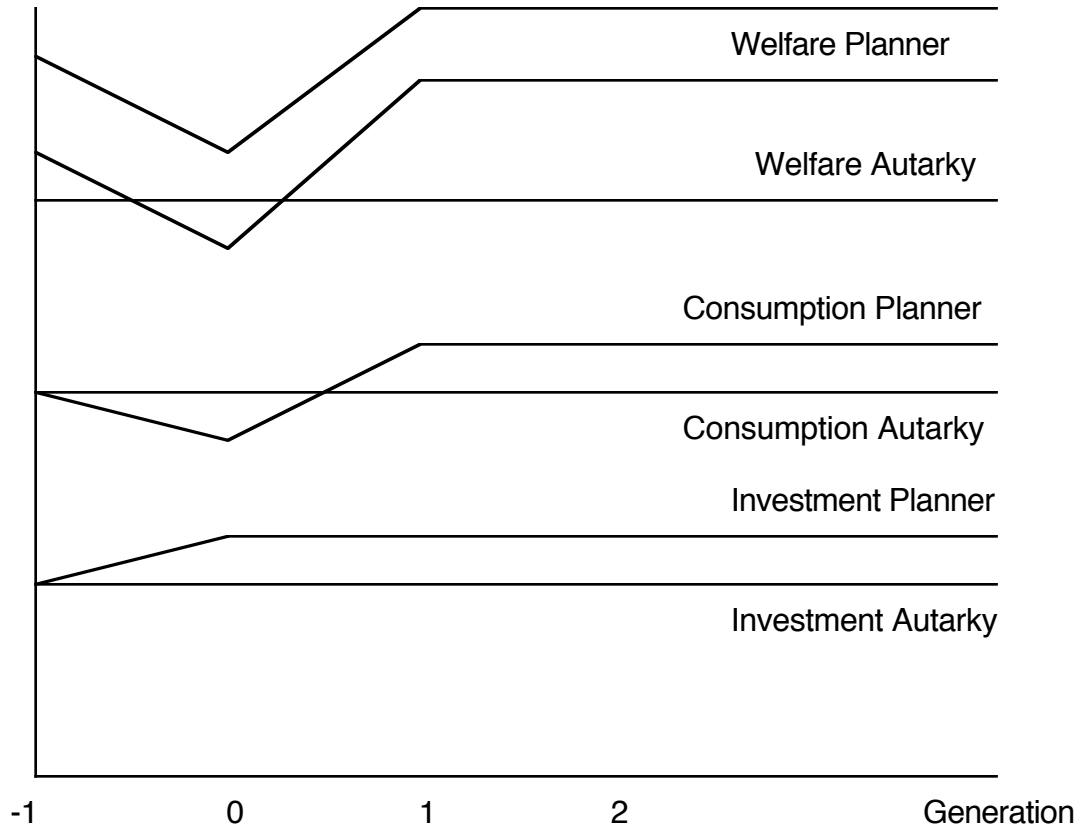


Figure 4: Autarky and the Planner's solution under linear utility function.

The log utility, Cobb-Douglas case:  $U(x) = \ln(x)$  and  $F(k) = k^\phi$ , where  $0 < \phi < 1$  is the elasticity of output with respect to the investment in children. I start with the autarkic case, and show that in this special case the marginal propensity to save out of income is a constant:  $\sigma(k) = \sigma$  for all  $k$ .

To show this claim, I write the problem (20) as:

$$(36) \quad \max_{k_t} U(F(k_{t-1}) - k_t) + \lambda U((1 - \sigma)F(k_t)) + \lambda \alpha U((1 - \sigma)F[\sigma F(k_t)]) \\ + \lambda \alpha^2 U((1 - \sigma)F\{\sigma F[\sigma F(k_t)]\}) + \dots$$

The first order condition for this problem is:

$$(37) \quad U'(F(k_{t-1}) - k_t) = \lambda U'((1 - \sigma)F(k_t))(1 - \sigma)F'(k_t) \\ + \lambda \alpha U'((1 - \sigma)F[\sigma F(k_t)])(1 - \sigma)F'[\sigma F(k_t)]\sigma F'(k_t) \\ + \lambda \alpha^2 U'((1 - \sigma)F\{\sigma F[\sigma F(k_t)]\})(1 - \sigma)F'\{\sigma F[\sigma F(k_t)]\}\sigma F'[\sigma F(k_t)]\sigma F'(k_t) + \dots$$

When  $U(x) = \ln(x)$  and  $F(k) = k^\phi$ , (37) can be written as:

$$(38) \quad \left( (k_{t-1})^\phi - k_t \right)^{-1} = \lambda \phi (k_t)^{-1} + \lambda \alpha \phi^2 (k_t)^{-1} + \lambda \alpha^2 \phi^3 (k_t)^{-1} + \dots = \frac{\lambda \phi}{k_t (1 - \alpha \phi)}$$

This leads to:

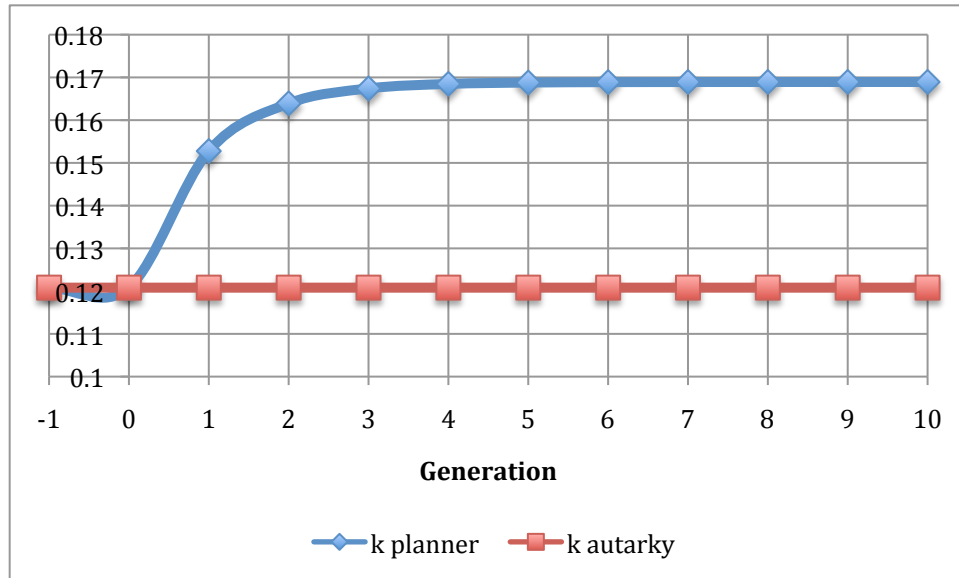
$$(39) \quad k_t = \frac{\lambda \phi}{1 - \phi(\alpha - \lambda)} (k_{t-1})^\phi$$

Thus, when the generation born at  $t - 1$  expects other generations to invest a fraction of their income, they will invest a fraction of their income that is equal to:

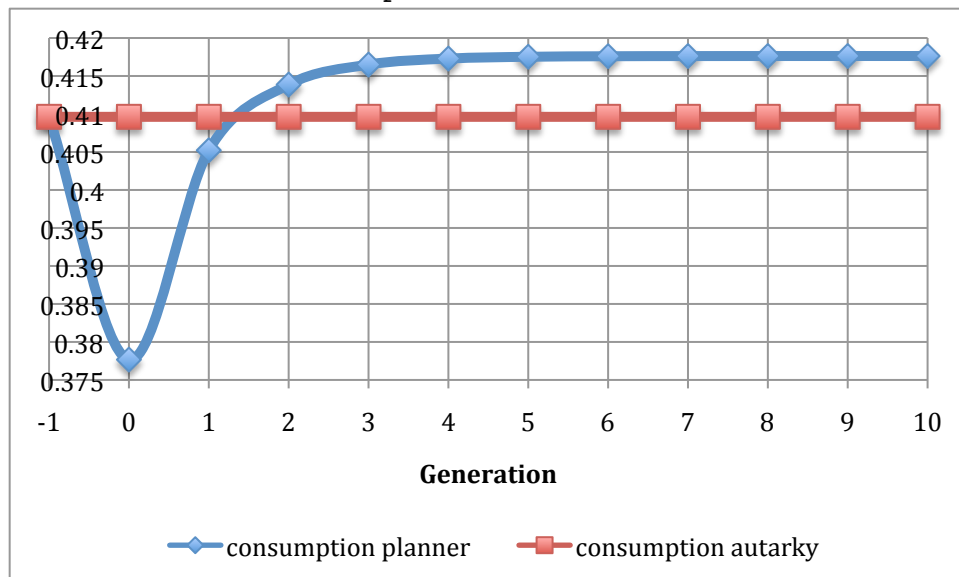
$$(40) \quad \sigma = \frac{\lambda \phi}{1 - \phi(\alpha - \lambda)}$$

It follows that the fraction (40) is a symmetric Nash equilibrium.

The planner's solution for this case is to have the current old invest the fraction (40) of their income and then to have the following generations invest a fraction  $\sigma^p = \phi \alpha$  of their income. Figure 5 compares the planner's solution to autarky.



A. Capital accumulation



B. Consumption

Figure 5: The log utility, Cobb-Douglas case: Planner and Autarky for the parameters:  $(\alpha = 0.96, \lambda = 0.7, \phi = 0.3)$

Changes in Technology: I assume now that output at time  $t$  is given by:

$$(41) \quad Y_t = \theta^t F(k_{t-1}) = \theta^t (k_{t-1})^\phi,$$

where  $\theta \geq 1$  is the gross rate of change in productivity. It is shown in Appendix B that under the log utility function the calculations of  $\sigma$  and  $\sigma^p$  do not depend on  $\theta$ . Under autarky, capital at time  $t$  is given by:  $k_t = \sigma \theta^t (k_{t-1})^\phi$ . Dividing by  $k_{t-1} = \sigma \theta^{t-1} (k_{t-2})^\phi$  we obtain the gross rate of change in capital:

$$(42) \quad \frac{k_t}{k_{t-1}} = \theta \left( \frac{k_{t-1}}{k_{t-2}} \right)^\phi$$

In the steady state:  $\frac{k_t}{k_{t-1}} = \frac{k_{t-1}}{k_{t-2}}$  and the gross rate of growth is given by:

$$(43) \quad \frac{k_t}{k_{t-1}} = \theta^{1/(1-\phi)}$$

This is also the rate of growth in output. Note that the steady state rate of growth depends only on  $\phi$  and  $\theta$ .

### 3.1 Time Preference and Generation Preference

The parameters  $(\lambda, \alpha)$  are critical for evaluating the welfare gain from adopting the planner's problem. But these parameters are not directly related to the discounting parameters in infinitely lived agents models.

To make this point, it is useful to compare the Phelps-Pollak (1968) utility function and the Laibson (1997) utility function. From a mathematical point of view they are the same but the interpretation is different. The Phelps-Pollak discounting parameters describe the value that individuals place on the utility of future generations, while the Laibson discounting parameters describe the value that they place on the utility from their own future consumption.

There need not be a connection between the two. We may have individuals that put a high value on their own future consumption but do not care much about their



children and may even choose not to have children. We may also have families that care about their children, but do not seem to care much about their own future consumption.

We may therefore want to distinguish between time (life cycle) preference and generation preference. The first refers to preference about dated consumption of the same individual. The second refers to preference about the consumption of different generations. To make this distinction, I assume now that each agent lives for 30 years and produce a heir upon his death. The utility of the generation labeled zero is:

$$(44) \quad U(c_0^0) + \gamma \sum_{t=1}^{29} \beta^t U(c_t^0) + \lambda \left( U(c_0^1) + \gamma \sum_{t=1}^{29} \beta^t U(c_t^1) \right) \\ + \lambda \alpha \left( U(c_0^2) + \gamma \sum_{t=1}^{29} \beta^t U(c_t^2) \right) + \lambda \alpha^2 \left( U(c_0^3) + \gamma \sum_{t=1}^{29} \beta^t U(c_t^3) \right) + \dots$$

The utility of generation  $n$  from his own consumption is:  $U(c_0^n) + \gamma \sum_{t=1}^{29} \beta^t U(c_t^n)$ . The time preference parameters are  $\gamma$  and  $\beta$ : At age zero, the individual discounts the utility derived from consumption at age  $t \geq 1$  by  $\gamma \beta^t$ . The generation preference parameters are  $\lambda$  and  $\alpha$ : At age  $t$ , generation zero discounts the utility from the consumption of generation  $n$  at age  $t$  by  $\lambda \alpha^{n-1}$ . The generation preference parameters are thus used to discount the consumption of future generations holding age constant.

The time between generation zero age zero to generation  $n$  age  $t$  is:  $\tau = 30n + t$ .

We can write the utility of generation zero at age zero as:

$$(45) \quad \sum_{\tau=0}^{\infty} d(\tau) U(c_\tau),$$

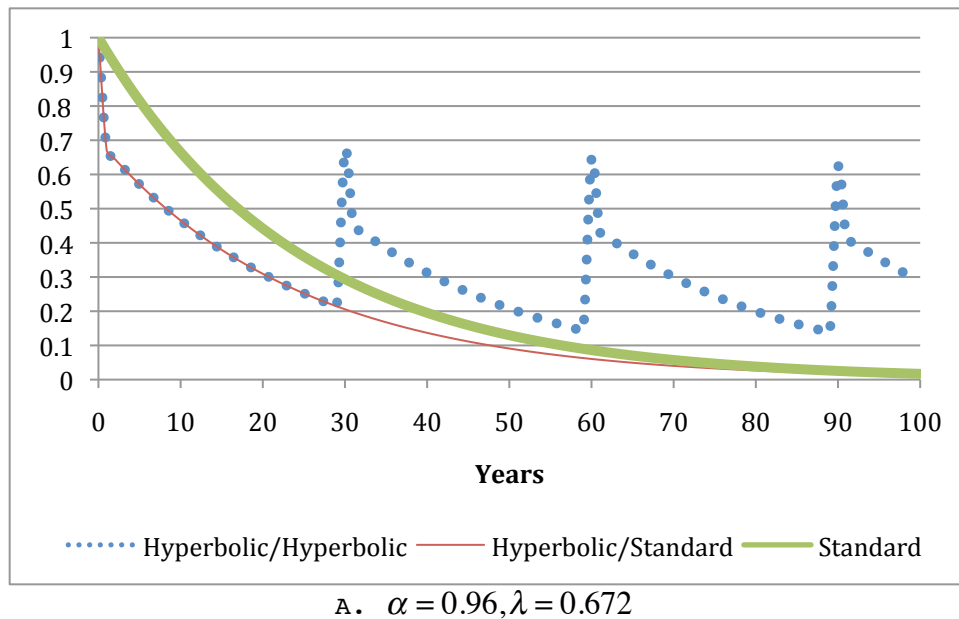
where here  $c_\tau$  is the consumption at time  $\tau$  with no reference to who is enjoying it. We

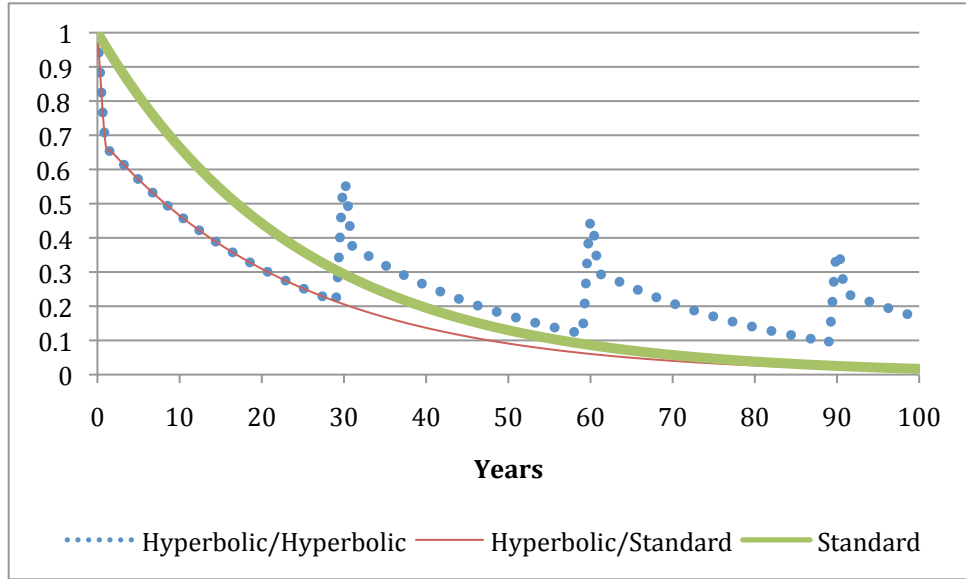
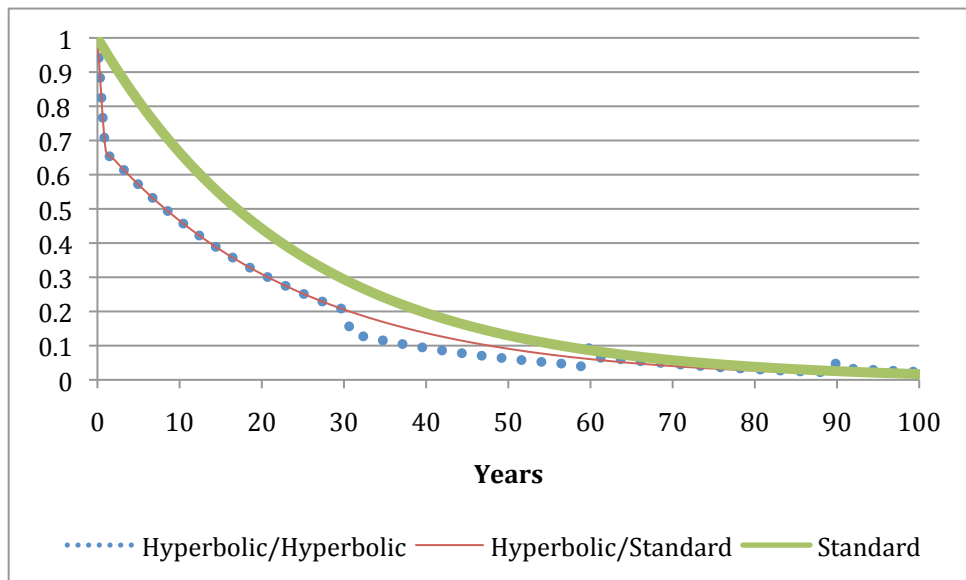
can now compute  $d(\tau)$  as a function of the time and generation preferences. We can get the standard exponential discounting,  $d(\tau) = \beta^\tau$ , by assuming:  $\gamma = 1$  and  $\lambda = \alpha = \beta^{30}$ .

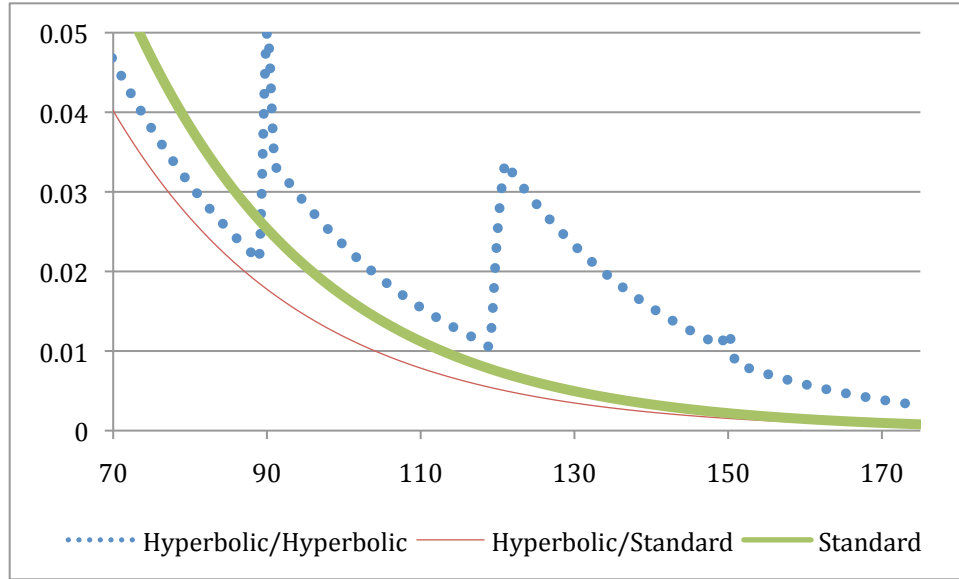
Laibson (1997) assumes:  $d(0) = 1$  and  $d(\tau) = \gamma \beta^\tau$  for  $\tau > 0$ . I refer to his formulation as hyperbolic time discounting and standard generation discounting. This can be obtained by

assuming:  $\gamma < 1$ ,  $\lambda = \alpha = \beta^{30}$ . Hyperbolic time discounting and hyperbolic generation discounting imposes no restrictions on the four parameters.

Figure 6 illustrates. For the standard discounting, I use  $\beta = 0.96$ . For the hyperbolic/standard discounting, I use  $(\beta = 0.96, \gamma = 0.7)$  which are the parameters used by Angeletos et al. (2001). The three cases in the Figure use different generation preference parameters. Figure 6A uses  $(\alpha = 0.96, \lambda = 0.672)$ , Figure 6B uses  $(\alpha = 0.8, \lambda = 0.56)$  and Figure 6C uses  $(\alpha = 0.5, \lambda = 0.25)$ . Figure 6D uses the same parameters as 6C but focus on the distant future. It shows that although the hyperbolic/hyperbolic case tracks the hyperbolic/standard case very well, there are non-trivial differences when looking at the relatively distant future.



B.  $\alpha = 0.8, \lambda = 0.56$ C.  $\alpha = 0.5, \lambda = 0.25$



D. The distant future when  $\alpha = 0.5, \lambda = 0.25$

Figure 6: The discount factor  $d(\tau)$ . Standard discounting uses:  $\gamma = 1$ ,  $\beta = 0.96$  and  $\alpha = \lambda = \beta^{30} = 0.29$ . Hyperbolic/Standard discounting uses:  $\gamma = 0.7$ ,  $\beta = 0.96$  and  $\alpha = \lambda = \beta^{30} = 0.29$ . Hyperbolic/Hyperbolic discounting uses:  $\gamma = 0.7$ ,  $\beta = 0.96$  and various generation preference parameters.

Note that when holding age constant, the parent values his utility by more than his son's utility. But this is not the case when the ages of the parent and the son are different. In cases A and B the parent value the utility of his son at age 1 more than his own utility at age 29. The opposite is true in case C.

### 3.2 Welfare Gain

How much can we gain by adopting the planner's solution? To answer this question I now turn to a calibration exercise.

I interpret the first period of life in the model as the time from birth until age 30. I choose  $\theta = 1.02^{30}$  and  $\phi = 0.33$ . This choice leads to a 3% annual steady state rate of growth in GDP.

In the model the parents spend money on the child's consumption, on his human capital and on physical capital that will be transferred as bequest. In practice, it is difficult to distinguish between expenditures on children's consumption and investment in children. Is buying a toy for a child, an investment or consumption? I proceed by assuming that all expenditures on children have both an investment and a consumption component. Under this assumption, the rate of return on the parents' expenditure on children should be relatively low because of the consumption component of expenditures.

Table 1 computes the implications of choosing  $\alpha = 0.9$  and various levels of  $\lambda$ . The first row is the annual rate of return under autarky:  $(F'(k^{au}))^{1/30} - 1$ . This is decreasing in  $\lambda$ . The second row is the annual steady state rate of return under the planner's solution. This does not depend on  $\lambda$  and is equal to 1.3% for this example. The third row is the steady state fraction of income spent on children under autarky. It increases with  $\lambda$ . The fourth row is the steady state fraction of income spent under the planner's solution: It does not depend on  $\lambda$  and is equal to 0.3 for this example. The last row is the welfare gain that occurs when starting from autarky and going to the planner's solution.

This is computed as the percentage increase in the autarkic steady state consumption that yields the same utility to the current old as the adoption of the planner's program. As can be seen the welfare gain is decreasing in  $\lambda < \alpha$ . The intuition is that holding  $\alpha$  constant, the external effect problem is more severe when  $\lambda$  is low.

Table 1: Implications when  $\alpha = 0.9$

$\lambda$	0.8	0.7	0.6	0.5	0.4	0.3
$r^{au}$	1.6%	2.0%	2.4%	2.9%	3.5%	4.3%
$r^p$	1.3%	1.3%	1.3%	1.3%	1.3%	1.3%
$\sigma^{au}$	0.27	0.25	0.22	0.19	0.16	0.12
$\sigma^p$	0.3	0.3	0.3	0.3	0.3	0.3
Gain	0.2%	0.8%	2.0%	4.0%	7.1%	12.0%

Notes: In this and in all other tables, I assume: ( $\phi = 0.33, \theta = 1.02^{30}$ ) and the log utility function.  $r^{au}$  is the (annual steady state) rate of return under autarky,  $r^p$  is the rate of return under the planner's solution,  $\sigma^{au}$  is the fraction of income spent on children under autarky,  $\sigma^p$  is the fraction of income spent under the planner's solution and "Gain" is the welfare gain from adopting the planner's solution computed as the equivalent percentage increase in the steady state autarkic consumption.

To select values of the generation preference parameters ( $\lambda, \alpha$ ) we may use estimates on the fraction of income spent on children. According to the estimate of the United States Department of Agriculture the cost of raising 2 children from the age of 1-17 is 42% of expenditures.<sup>8</sup> Under the assumption that agents have children at home half of their adult life (age 30+) and with no discounting, the expenditures on children are about 21% of lifetime expenditures. This is likely to be a lower bound for the following reasons. Expenditures on children occur at the early phase of adulthood (say 30 to 50) and total expenditures fall after the children leave home. In addition the Department of Agriculture estimate exclude costs related to childbirth and prenatal health care, the cost of college education, and time costs. It also excludes gifts made by the parents to their

<sup>8</sup> "Expenditures on Children by Families, 2006" United States Department of Agriculture, Center for Nutrition, Policy and Promotion Miscellaneous Publication Number 1528-2006 (April 2007).

adult children (either in their lifetime or as a bequest). I thus regard parameters that lead to  $\sigma^{aut} \geq 0.21$  as more “reasonable” than parameters that lead to  $\sigma^{aut} < 0.21$ .

Table 2 describes the fraction of lifetime income spent on children as a function of  $(\alpha, \lambda = \alpha - d)$  where the difference  $d$  takes the values:  $d = 0.3, 0.2, 0.1$ . Note that the planner wants to spend more than what is spent under autarky and the difference between the planner’s spending and the autarkic spending is increasing in the distance  $d = \alpha - \lambda$ . The fraction of income spent seems too low when  $\alpha \leq 0.7$ .

Table 2: The fraction of lifetime income spent on children under autarky ( $\sigma^{aut}$ ) and (in the last column) under the planner’s solution ( $\sigma^p$ )

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$	$\sigma^p = \alpha\phi$
0.9	22%	25%	27%	30%
0.8	18%	21%	24%	26%
0.7	15%	18%	20%	23%
0.6	11%	14%	17%	20%
0.5	7%	11%	14%	17%
0.4	4%	7%	10%	13%

Table 3 reports annual rates of return on the expenditures on children. Since there is a consumption element in raising children, it seems that high  $\alpha$  with lower rates of return are reasonable. Also here the planner wants a lower rate of return than under autarky and the difference between the autarkic rate and the planner’s rate is increasing in  $d = \alpha - \lambda$ .

Table 3: The annual steady state rate of return under autarky ( $r^{aut}$ ) and (in the last column) under the planner’s solution ( $r^p$ )

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$	$r^p$
0.9	2.4%	2.0%	1.6%	1.3%
0.8	3.0%	2.5%	2.1%	1.7%
0.7	3.8%	3.1%	2.6%	2.2%
0.6	4.8%	3.9%	3.2%	2.7%
0.5	6.2%	4.9%	4.0%	3.3%
0.4	8.7%	6.3%	5.0%	4.1%

Tables 4-7 calculate the welfare gains from the point of view of the generation born at  $t = -1, 0, 1$  and the generation born when the economy reaches a steady state ( $t \geq 10$ ). When  $\alpha = 0.9$  and  $\lambda = 0.6$ , the welfare gain is 2% from the point of view of the current old, 1.7% from the point of view of the current young, 3.6% from the point of view of the generation born at  $t = 1$  and 4.5% from the point of view of generations born at  $t \geq 10$ .

As was said before, Angeletos et.al. (2001), use the time preference parameters ( $\beta = 0.96, \gamma = 0.7$ ). The symmetric case in which the generation preference parameters are equal to the time preference parameters has some appeal. It says that people think about generations and years in a similar way. Assuming ( $\alpha = 0.96, \lambda = 0.7$ ) yields:  $\sigma^{au} = 0.25, \sigma^p = 0.31, r^{au} = 1.9\%, r^p = 1.1\%$ . The welfare gain is 1.4%, 1.4%, 1.9% and 2.2% from the point of view of the generation born at  $t = -1, t = 0, t = 1$  and  $t \geq 10$ .

I also computed the implication of increasing the elasticity  $\phi$  holding ( $\alpha, \lambda$ ) constant. The main effect is to increase the fraction of income spent on children and the welfare gains from adopting the planner's program. For example, when ( $\phi = 0.7, \alpha = 0.96, \lambda = 0.7$ ),  $\sigma^{au} = 0.6$  and the welfare gain from the point of view of the current old generation is 3.3%. These magnitudes are more much larger than the 0.25 and the 1.4% that we get when  $\phi = 0.33$ .



Table 4: Welfare gains from the point of view of the generation born at  $t = -1$ 

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$
0.9	2.00%	0.80%	0.20%
0.8	1.94%	0.78%	0.18%
0.7	1.90%	0.77%	0.18%
0.6	1.78%	0.75%	0.18%
0.5	1.62%	0.73%	0.17%
0.4	1.31%	0.68%	0.17%

Note: The gain here and in the following tables is from adopting the planner's solution (that represents the point of view of the generation born at  $t = -1$ ).

Table 5: Welfare gains from the point of view of the generation born at  $t = 0$ 

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$
0.9	1.7%	0.7%	0.2%
0.8	1.3%	0.5%	0.1%
0.7	0.7%	0.4%	0.1%
0.6	0%	0.1%	0%
0.5	-1.3%	0%	0%
0.4	-3.4%	-0.8%	0%

Table 6: Welfare gains from the point of view of the generation born at  $t = 1$ 

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$
0.9	3.6%	1.7%	0.6%
0.8	5.7%	2.9%	1.1%
0.7	8.6%	4.5%	1.8%
0.6	13.2%	6.8%	2.7%
0.5	21.4%	10.5%	4.1%
0.4	40.7%	17.1%	6.3%

Table 7: Welfare gains from the point of view of generations born at  $t \geq 10$ 

$\alpha$	$\lambda = \alpha - 0.3$	$\lambda = \alpha - 0.2$	$\lambda = \alpha - 0.1$
0.9	4.5%	2.2%	0.8%
0.8	7.9%	4.0%	1.6%
0.7	12.7%	6.6%	2.6%
0.6	20.4%	10.2%	4.1%
0.5	34.4%	16.1%	6.2%
0.4	69.4%	27.1%	9.6%

### 3.3 The Role of Government

In principle the parents can implement the planner's solution  $(\bar{y}_t, \bar{k}_t)$  by offering their son the following contract: "I will give you  $(\bar{y}_t, \bar{k}_t)$  in exchange for a promise to give your son (my grandson) the amounts  $(\bar{y}_{t+1}, \bar{k}_{t+1})$ ". But there is a problem of enforcement of uncollateralized contracts, especially after the death of the lender.<sup>9</sup>

The government can play a role, if it has a better commitment technology. In general, some governments are more credible than others. Norway may serve as an example of a relatively credible government that attempts to affect the intergenerational allocation of resources. It invests the very large profits from its oil exploration in a "petroleum fund" (later changed its name to "pension fund") that is run by the central bank. It could have used the profits to reduce taxes and let its citizen choose the amount of bequest but it chose the more paternalistic approach, maybe because it has a better commitment technology. I assume here a relatively credible government.

Government programs that lead to an increase in the total spending on children, like public schools, may be used as a commitment device because, once the program is in place, it is difficult to eliminate it. Such a program is analogous to a loan: The children who get the benefits of the program are committed to give similar benefits to their children.

Here I focus on an explicit loan program as a device that can be used by the parents to affect the allocation of resources after their death. I follow Friedman (1960) in assuming that the government has an advantage over the private sector in enforcing

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<sup>9</sup> In principle, parents can affect the allocation of resources after their death by leaving a bequest to their grandchildren. But to affect the allocation the bequest must be sufficiently large. For example, suppose that the parents (grandparents) want to allocate 25% of their kids' wealth to the consumption and education of their grandchildren while the kids want to spend only 20%. To affect the allocation the parents (grandparents) must leave a bequest that is greater than 20% of their kids' wealth. It seems that only the super rich can leave an intentional bequest that is larger than what their kids want to spend on their grandchildren.

uncollateralized loan contracts and consider an extension of his proposal for educational loans to general-purpose loans aimed at facilitating intergenerational trade.

Formally, the implementation of the planner's solution  $(\bar{y}_t, \bar{k}_t)$  works as follows. The current old generation pays a lump sum tax equal to  $T = \bar{y}_0 + \bar{k}_0$ . The tax revenues finance a loan to the current young (born at  $t = 0$ ) at the interest rate  $R_0 = F'(\bar{k}_0)$ . The current young make a choice between consumption and investment. At  $t = 1$  they pay the loan and this is used to finance the loan for the newly born.

I now turn to describe the implementation in detail. I assume that under the proposed loan program the government crowds out altruistic behavior and agents behave as if they care only about the utility derived from their own consumption. This will be shown later as a result. The young born at  $t$ , chooses the size of the loan  $L_t$  and the amount of investment,  $k_t$ , by solving:

$$(46) \quad \max_{k,L} \omega U(L_t - k_t) + U(F(k_t) - R_t L_t + S_t),$$

where  $S_t$  is a lump sum subsidy (tax) paid when old and  $R_t$  is the gross interest rate on the loan.

The first order conditions for this problem are:

$$(47) \quad \omega U'(L_t - k_t) = U'(F(k_t) - R_t L_t + S_t) R_t = U'(F(k_t) - R_t L_t + S_t) F'(k_t)$$

Claim 2: The government can implement the first best if it charges the interest rate  $R_t = F'(\bar{k}_t)$  on the loan to the generation born at  $t$  and give them a lump sum transfer (when they reach old age) equal to:

$$(48) \quad S_t = (R_t - 1)(\bar{y}_t + \bar{k}_t) - (\bar{y}_{t+1} + \bar{k}_{t+1} - \bar{y}_t - \bar{k}_t)$$

Proof: Substituting  $R_t = F'(\bar{k}_t)$  and (48) in (47) leads to (18).  $\square$

Note also that the transfer to the old is equal to the interest payment on the loan,  $(R_t - 1)(y_t + k_t)$ , minus the change in the loan,  $L_{t+1} - L_t = k_{t+1} + y_{t+1} - k_t - y_t$ . In the steady

state, when the size of the loan does not change, the transfer is equal to the interest payment and the optimal gross interest rate is  $\frac{1}{\alpha}$ .

The crowding out assumption: In formulating the problem (46) I assumed that after paying the loan parents do not want to give additional funds to their children. To see that this is indeed a Nash equilibrium behavior, note that under the loan program investment is determined by (18) and  $U'(\bar{x}_{t-1}) = \alpha\omega U'(\bar{y}_t)$ . Since  $\alpha > \lambda$ , it follows that  $U'(\bar{x}_{t-1}) > \lambda\omega U'(\bar{y}_t)$  and the parent has no incentive to deviate from the Nash strategy of no additional funds.

Voting for a “social contract”: As can be seen from Table 5, adopting the planner’s program will lead to a Pareto improvement in most cases. This means that future generations will not want to repeal the loan program if the alternative is autarky. But since  $U'(\bar{x}_{t-1}) > \lambda\omega U'(\bar{y}_t)$  each generation will want to modify the law and reduce the transfer to the current young for one period only (say by increasing the interest rate on the loan for one period).

This will not occur if any deviation is followed by a sufficiently long punishment period in which agents revert to non-cooperative behavior as in Friedman (1971).

#### 4. CONCLUDING REMARKS

In the steady state, intergenerational intermediation may be viewed as a social contract of the following type: The old give something to the young who in exchange agree to give something to the next generation. When agents use exponential discounts ( $\lambda = \alpha$ ) the optimal social contract is enforced by “perfect” altruism. But when altruism is less than “perfect” and  $\lambda < \alpha$ , it is in general possible to get a Pareto improvement if all generations agree to increase investment.

I focus on the program that maximizes the utility of the current old. Under mild assumptions this program will lead to a Pareto improvement. And the benefits from adopting it may be quite large if agents are relatively “egoistic” and the distance  $\alpha - \lambda$  is large.

To realize these benefits we need a commitment device. In general a commitment device can be described as a loan contract: The expenditures on the son are given as a loan and the loan payments are used to finance the loan to the grandson. It seems that the government has an advantage in enforcing this type of uncollateralized loan contracts.

Will our conclusion change if agents live for more than two periods and suffer from Laibson’s time inconsistency problem (having the utility function [44] with  $\gamma < 1$ )? I think that in this case social security and a government loan program can both be used to address inconsistencies. Social security will address life cycle or time inconsistency while a government loan program will address the inconsistency when planning consumption over generations. To see how this will work, note that the government loan program crowds out altruistic behavior. The individual who gets the loan in the first period of his life will thus worry only about ways to implement his choice of consumption over the lifecycle. He may use social security and other illiquid assets as commitment devices to address the time inconsistency issue.

## APPENDIX A: CHARACTERIZING THE SOLUTION TO (16)

As was said in the text, we can write the problem (16) as:

$$(A1) \quad \max_{y_0, k_0} U(F(k_{-1}) - y_0 - k_0) + \lambda \omega U(y_0) + \lambda V(k_0)$$

where

$$(A2) \quad V(k) = \max_{y', k'} U(F(k) - y' - k') + \alpha \omega U(y') + \alpha V(k')$$

The first order conditions for the problem (A2) are:

$$(A3) \quad U'(F(k) - y' - k') = \alpha \omega U'(y')$$

$$(A4) \quad U'(F(k) - y' - k') = \alpha V'(k')$$

$$(A5) \quad V'(k) = U'(F(k) - y' - k') F'(k)$$

Note that

$$(A6) \quad V''(k) = U''(F(k) - y' - k') (F'(k))^2 + U'(F(k) - y' - k') F''(k) < 0$$

We can write the solution to (A2) as  $y' = g(k)$  and  $k' = h(k)$ .

Claim A1: When  $F'(k) > 1$  the functions  $g(k)$ ,  $h(k)$  and  $x(k) = F(k) - g(k) - h(k)$  are increasing.

Proof: Taking full differential of (A3) leads to:

$$(A7) \quad U''(F(k) - y' - k') F'(k) - U''(F(k) - y' - k') \frac{dk'}{dk} = [\alpha \omega U''(y') + U''(F(k) - y' - k')] \frac{dy'}{dk}$$

Taking full differential of (A4) leads to:

$$(A8) \quad U''(F(k) - y' - k') F'(k) - U''(F(k) - y' - k') \frac{dy'}{dk} = [\alpha V''(k') + U''(F(k) - y' - k')] \frac{dk'}{dk}$$

Using (A7) we get:

$$(A9) \quad \frac{dy'}{dk} = \frac{U''(F(k) - y' - k') F'(k) - U''(F(k) - y' - k') \frac{dk'}{dk}}{\alpha \omega U''(y') + U''(F(k) - y' - k')}$$

Substituting (A9) in (A8) leads to:

$$(A10) \quad \frac{dk'}{dk} = ZF'(k),$$

where

$$Z = \frac{U''(F(k) - y' - k')}{[\alpha V''(k') + U''(F(k) - y' - k')] + U''(F(k) - y' - k') \frac{U''(F(k) - y' - k')(F'(k) - 1)}{\alpha \omega U''(y') + U''(F(k) - y' - k')}}}$$

Assuming that  $F'(k) > 1$  we get:

$$(A11) \quad 0 < Z < 1$$

Thus, an increase in  $k$  leads to an increase in the investment in human capital in an amount that is less than the increase in the income of the old.

We can now write (A9) as:

$$(A12) \quad \frac{dy'}{dk} = \frac{U''(F(k) - y' - k')F'(k)(1 - Z)}{\alpha \omega U''(y') + U''(F(k) - y' - k')} > 0.$$

Since an increase in  $k$  leads to an increase in  $y'$ , (A3) implies that it must also lead to a decrease in  $U'(x)$  and therefore to an increase in  $x$ .  $\square$

## APPENDIX B: TECHNOLOGICAL CHANGES

In this Appendix I assume  $Y_t = \theta' F(k_{t-1}) = \theta' (k_{t-1})^\phi$  and a log utility function. I start from showing the following Claim.

Claim B1: Under autarky, the agent born at  $t - 1$  will invest the fraction (40) of his income if he expects that all future generations will also follow this rule.

The proof uses the following steps.

$$(B1) \quad \max_{k_t} U(\theta' F(k_{t-1}) - k_t) + \lambda U((1 - \sigma)\theta^{t+1}F(k_t)) + \lambda \alpha U((1 - \sigma)\theta^{t+2}F[\sigma\theta^{t+1}F(k_t)]) \\ + \lambda \alpha^2 U((1 - \sigma)\theta^{t+3}F\{\sigma\theta^{t+2}F[\sigma\theta^{t+1}F(k_t)]\}) + \dots$$

$$(B2) \quad U(\theta' F(k_{t-1}) - k_t) = \lambda U((1 - \sigma)\theta^{t+1}F(k_t))(1 - \sigma)\theta^{t+1}F'(k_t) \\ + \lambda \alpha U((1 - \sigma)\theta^{t+2}F[\sigma\theta^{t+1}F(k_t)])(1 - \sigma)\theta^{t+2}F'[\sigma\theta^{t+1}F(k_t)]\sigma\theta^{t+1}F'(k_t) + \dots$$

$$(B3) \quad (\theta' (k_{t-1})^\phi - k_t)^{-1} = \lambda ((1 - \sigma)\theta^{t+1}(k_t)^\phi)^{-1} (1 - \sigma)\theta^{t+1}\phi(k_t)^{\phi-1} \\ + \lambda \alpha ((1 - \sigma)\theta^{t+2}[\sigma\theta^{t+1}(k_t)^\phi]^\phi)^{-1} (1 - \sigma)\theta^{t+2}\phi[\sigma\theta^{t+1}(k_t)^\phi]^{\phi-1} \sigma\theta^{t+1}\phi(k_t)^{\phi-1} + \dots$$

$$(B4) \quad \left(\theta^t(k_{t-1})^\phi - k_t\right)^{-1} = \lambda\phi(k_t)^{-1} + \lambda\alpha\phi^2(k_t)^{-1} + \dots = \frac{\lambda\phi}{k_t(1-\alpha\phi)}$$

$$(B5) \quad k_t = \frac{\lambda\phi}{1-\phi(\alpha-\lambda)}\theta^t(k_{t-1})^\phi$$

So the fraction (40) is a symmetric Nash equilibrium also in this more general case.

Claim B2: The fraction of income invested according to planner's solution does not depend on  $\theta$  and is given by (40) for the generation born at  $t = -1$  and by  $\sigma^p = \phi\alpha$  for the generations born at  $t \geq 0$ .

The proof uses the following steps. I start by checking the first order condition (18) for

$t = 0$ :

$$\frac{1}{(k_{-1})^\phi - k_0} = \lambda\left(\theta(k_0)^\phi - k_1\right)^{-1}\theta\phi(k_0)^{\phi-1}$$

Substituting  $k_1 = \alpha\phi\theta(k_0)^\phi$

$$k_0 = \frac{\lambda\theta\phi}{\theta - \alpha\phi\theta + \lambda\theta\phi}(k_{-1})^\phi = \sigma(k_{-1})^\phi$$

Checking the first order conditions for  $t > 0$ :

$$\frac{1}{\theta^t(k_{t-1})^\phi - k_t} = \alpha\left(\theta^{t+1}(k_t)^\phi - k_{t+1}\right)^{-1}\theta^{t+1}\phi(k_t)^{\phi-1}$$

Substituting:  $k_{t+1} = \alpha\phi\theta^{t+1}(k_t)^\phi$  leads to:

$$k_t = \alpha\phi\theta^t(k_{t-1})^\phi$$

Thus the fraction of income invested under the planner's program does not depend on  $\theta$ .



## REFERENCES

- Aiyagari Rao S., Jeremy Greenwood and Ananth Seshadri., “Efficient Investment in Children” Journal of Economic Theory, February 2002, 102(2), pp. 290-321.
- Angeletos, George-Marios, David Laibson, Andrea Repetto and Jeremy Tobacman “The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation” The Journal of Economic Perspectives, Vol. 15, No. 3 (Summer, 2001), pp. 47-68.
- Barro, R.J., 1974: “Are Government Bonds Net Wealth?,” Journal of Political Economy, 82 (6), 1095–1117.
- \_\_\_\_\_ 1999: "Laibson Meets Ramsey in the Neoclassical Growth Model." Quarterly Journal of Economics. 114:4, pp. 1125-52.
- Benassy Jean-Pascal., Macroeconomic Theory, Oxford University Press, 2011.
- Diamond, Peter, A., 1965: “National Debt in a Neoclassical Growth Model” The American Economic Review, Vol. 55, No. 5, Part 1, 1126-1150.
- Friedman, J. W.: "A Non-cooperative Equilibrium for Supergames," Review of Economic Studies, 28 (1971), 1-12.
- Friedman Milton, Capitalism and Freedom, Fortieth Anniversary Edition, The University of Chicago Press.
- Laibson, David I. 1997. "Golden Eggs and Hyperbolic Discounting." Quarterly Journal of Economics. 62:2, pp. 443-78.
- Phelps E. S. and R. A. Pollak., “On Second-Best National Saving and Game-Equilibrium Growth” The Review of Economic Studies, Vol. 35, No. 2 (Apr., 1968), pp. 185-199.
- Samuelson, Paul A., 1958: “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money,” Journal of Political Economy, 66 (6), 467–82.
- Sen, Amartya K., 1967: “Isolation, Assurance and the Social Rate of Discount” The Quarterly Journal of Economics, Vol. 81, No. 1, Feb. pp. 112-124.
- Strotz, Robert H., 1956: "Myopia and Inconsistency in Dynamic Utility Maximization," Review of Economic Studies, XXIII, 165-180.