Essays in Macroeconomic Models with Financial Frictions

By

Frank A. Ciarliero

Dissertation

Submitted to the Faculty of the

Graduate School of Vanderbilt University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

August 7th, 2020

Nashville, TN

Approved:

Peter L. Rousseau, Ph.D.

Mario J. Crucini, Ph.D.

Robert A. Driskill, Ph.D.

Kevin X. D. Huang, Ph.D.

David C. Parsley, Ph.D.

for the First Peoples of the continent of Australia...

ACKNOWLEDGMENTS

The completion of this dissertation is the culmination of many years of education at all the possible levels. I extend my gratitude to all the educators who have taught and nurtured me throughout.

I thank my advisor Peter Rousseau for comments and helpful guidance throughout my years in graduate school. Above all, his belief in my abilities and talents has inspired me to reach the finish line.

I recognize the assistance of Vanderbilt University in providing supporting funds throughout my tenure here. In particular, enrolling in this program has allowed me to learn from my committee members Mario Crucini, Robert Driskill (who I am implored to call Bob), Kevin Huang, and David Parsley, at various stages, both in research and in the classroom. I profoundly thank them.

Beyond my committee, I thank the other faculty members who guided me in various ways throughout and presented me with opportunities, particularly Eric Bond, Greg Huffman, Greg Leo, Kathleen McKiernan, Hyunseung Oh, Joel Rodrigue, and John Weymark. Special thanks to Kathleen Finn, whose kindness personifies the pleasant time I have had in Nashville.

I have met many wonderful and diverse classmates in the Vandy economics graduate cohort. To my perennial office mates Nicolas Mäder and Zeeshan Samad, along with Caroline Abraham, Daniel Mangrum, Paul Niekamp, Oscar O'Flaherty, and Jonah Yuen, I say thanks for being great friends.

Above all, I thank my family and wonderful partner Rebekah Austin. The greatest contribution they made among many, was providing the implicit knowledge that irrespective of the outcome of my brash choice to move to the United States or the outcome of this research, that I would be loved unconditionally.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS AND NOMENCLATURE

Aaa Bond	Moody's investment grade rating for corporate and sovereign debt.
Baa Bond	Moody's junk grade rating for corporate and sovereign debt.
Capital Structure	The mix of debt and equity a firm employs to finance its operations.
DSGE	Dynamic Stochastic General Equilibrium Model
Extensive Margin	A decision taken whereby the choice is between undertaking an activity or not.
GDP	Gross Domestic Product
GMM	Generalized Method of Moments
Inflation	The rise in prices from period to period.
Intensive Margin	A decision taken whereby the intensity of an activity is chosen.
Quantitative Easing	Monetary Policy conducted to target long-term interest rates and implemented via bond purchases.

INTRODUCTION

This dissertation explores the consequences, or lack thereof, of the presence of financial frictions in micro-founded models of the macroeconomy of the United States (US) and in the practice of monetary policy by the Federal Reserve. I primarily explore the effects of different types of firm borrowing, focusing on long-term collateralized debt in Chapter I and its general equilibrium effects where household, firm, and banking decisions are made on the intensive margin. In Chapter II, I specifically consider extensive margin decisions and how they are affected by a debt structure that is unsecured. Chapter III offers an exploration of US Monetary Policy following the Great Recession. In this time period, overnight rates were held for many years at the zero lower bound and questions arose regarding central bankers' ability to pursue their policy objectives.

The research illuminates the specific transmission mechanisms in the economy of policy changes and aggregate shocks when financial frictions are present and modeled in a structural manner. Moreover, I shed light on which time series in the data carry identification power for these underlying shocks.

In Chapter I, firms and banks search for counterparts to execute long-term loan contracts with. I find that financial shocks to the collateral quality of existing capital create incentives in the firm to increase investment so that they may maintain their desired capital structure. The general equilibrium effects into household balance sheets from this behavior occur via wealth effects, substantially depressing consumption in financial recessions, boosting work incentives, and depressing real wages. The series of bank marketable securities provides the identification power that allows us to distinguish a recession that originates in the financial as opposed to the productive sector of the economy. I model marketable securities as an outlet for banks to invest funds in that could otherwise be directed to lending. The timing of increases in this series sheds light on the underlying state of the economy. An increase in marketable securities absent a wider economic slowdown suggests a relative withdrawal of lending liquidity by banks; a financial recession. In contrast, when marketable securities rise following a wider productive downturn, we recognize a lack of demand for funds from the private sector. This suggests a recession with its genesis in the real sector.

While Chapter I features intensive margin decisions being made by the economic agents, a substantial body of work demonstrates that many, and possibly all, crucial firm decisions are made on the extensive margin. Chapter II delves into models of lumpy investment. Models which to date have captured the firm-level dynamics quite well, but not necessarily aggregate dynamics. By adding an element of financial frictions via revolving debt, I am able to induce investment substitution along the extensive margin. Firms elect to replace capital, a large investment outlay, in a strong aggregate state of the economy, and when financial conditions are conducive. When debt funding capacity is lacking, firms substitute to relatively more modest maintenance investment. The operation of this maintenance channel allows the enhanced lumpy investment model to exhibit more accurate aggregate properties and also improves the model's fit at the firm-level.

In terms of policy implications, the results in Chapter I allow policymakers to better target their responses as a result of being able to discern the underlying cause of any downturn in real time. Arising in Chapter II are potentially new implications for stimulating investment. Aggregate investment is highly path dependent in the model, with replacement investment responding strongly in volatile periods. Long periods of aggregate stability can lead firms to enter an apparent stasis where most investment outlays are in the form of maintenance. This is an insight that should be taken into consideration.

Chapter III considers an institutionally induced financial friction, the zero lower bound on nominal interest rates. A debate has emerged following the Great Recession with one side viewing monetary policy as constrained by the zero lower bound in the time period ensuing. The competing view is that the Federal Reserve has been able to implement its goals via alternate means. The results in this chapter support the latter viewpoint. I specifically find that the zero lower bound was binding as a constraint in the post Great Recession period. However, this did not limit the ability of central bankers to achieve their price stability and full employment mandate. Rather, the Federal Reserve was able to pivot towards a practice that effectively targeted money velocity, implementing its implied policy stance therein.

The following dissertation elucidates the above in greater detail, describing the modeling approaches and innovations, and detailing the empirical methodologies employed.

CHAPTER I

Money on the Sidelines? The Role of Costly Search and Weak Loan Demand in the Slow Recovery

1. Introduction

The financial crisis that triggered the Great Recession saw sizable reductions in the depth of credit markets. The provision of loans fell sharply while credit spreads widened, making the experience seem like a financial supply shock. The policy response was unconventional, with the Federal Reserve directing generous sums of liquid funds to banks. Banks were well capitalized following successful rounds of capital raising and laden with funds, yet lending was subdued. This is known colloquially as 'money on the sidelines', a phrasing which suggests a lack of willingness to lend by banks. Rather than accepting that interpretation, this paper develops an endogenous channel where new frictions in loan markets allow for the identification of periods of weak loan demand as distinct from a financial crisis where loan supply is reduced.

The Great Recession was sizable with a longer duration than other downturns in the post World War II experience, coupled with a particularly slow recovery. This is a peculiar happenstance in the context of existing models and the practice of unconventional monetary policy during and following the crisis. The expectation would be that a large injection of funds to banks would flow quickly as loans to firms and stimulate economic activity. However, the lived experience was that bank credit to the private sector fell dramatically just as liquid bank assets were growing strongly. In particular, this growth was seen in excess reserves, suggesting a gap in existing models that neglect to define channels whereby banks can invest available funds rather than loaning them. The policy implications are stark. A government authority seeking to alleviate the recession and quicken the pace of recovery should not only concern itself with the provision of funds to banks, but also their dissemination as loans. The evidence in this paper suggests that policy during the recession successfully restored financial markets quickly but neglected the private sector, which by the latter part of the crisis and into the recovery phase saw firms reducing their loan demand due to a lack of available investment avenues.

As conjectured in Kashyap and Zingales (2010), meaningful financial spillovers of the type motivated by the experience of the Great Recession are not possible when debt adjustment is costless.[1] In this paper, capital structure decisions are non-neutral and result in a productive cost to adjusting the debt to equity mix for firms. The first source of non-neutrality is an intermediation friction in loan markets that I model with a search and matching approach. Secondly, loans are repaid over multiple periods according to an annuity formulation.[2] This has the effect of front loading the weight of the interest component of any repayment, making the firm more susceptible to adverse changes in interest rates following large loan acquisitions.

This loan market is embedded in the workhorse New Keynesian model with loans being supplied from leverage ratio optimizing banks.[3] Costly search for long-term loans leads to an endogenous term structure arising from the model. Such a channel expands the empirical reach of DSGE models, allowing us to account for the counter-cyclically widening credit spread observed in the data, which was a prescient feature of the Great Recession.

Additionally, imperfect matching in the loan market endogenously generates distinct categories of bank assets. Successful matching results in the creation of a loan, while a failure to match forces banks to divert available funds towards marketable securities. This feature allows the model to account for the build-up of marketable securities seen in the Great Recession and to a lesser extent in recessions prior. Two competing forces can account for increasing marketable securities in this setup. Either firms reduce loan search or banks withdraw some leverage. Both have the effect of inducing less loan matching, however, whether this is a demand or supply driven phenomenon is critical in deciding the appropriate policy response.

The key identifying relationship in the model with the addition of the long-term loan market is the cyclical behavior of investment. As opposed to standard treatments where investment is strongly pro-cyclical in response to all shocks, firm behavior in this model sees investment responding most when the investment-specific environment is strong. This is the case of an investment efficiency

[1] New Keynesian models such as Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), and Justiniano, Primiceri and Tambalotti (2011), implicitly assume that real aggregates are invariant to capital structure decisions.

[2] See Alpanda and Zubairy (2017) and Quint and Rabanal (2017) for works that feature long-term loans in other macroeconomic contexts.

[3] I use the structure of Gertler and Karadi (2011) to model bank behavior.

shock. Here, the firm requires financing for its investment, building up loans slowly and meeting any shortfall with equity. In contrast, in response to a positive financial shock, the firm will bias its behavior toward loan search, taking advantage of looser lending standards to finance itself with relatively more debt than equity. Here, investment is allowed to drift, exhibiting short-term counter-cyclical behavior.

That the model produces these distinct investment paths is crucial to its micro-foundational validity. In times when the investment response is subdued, so too is the response of its price, the risk-free real interest rate. As a result, positive shocks do not necessarily induce household saving behavior, which normally require a strong labor hours response to wage changes to produce the coincidentally pro-cyclical pattern of output, consumption, investment, and labor hours movement seen in the data. In contrast, the channel in this paper is a strong household wealth effect stemming from household ownership of firms that substitute to equity financing when debt demand is unsatisfied in the short-term.

The spillover of loan decisions into productive outcomes places this paper in the class of models that contain financial frictions.[4] The theoretical contribution of this paper in this context is to explore the interplay between banks and firms in general equilibrium, capturing the disaggregated bank asset reallocations between loans and marketable securities. These additions are embedded in the existing framework, allowing the model to exhibit the empirically observed co-movements concurrently across real and financial variables.

The augmented model is estimated via Bayesian methods, utilizing an expanded set of observables that additionally include a measure of aggregate marketable securities held by banks and a measure of the credit spread. This approach facilitates an empirical evaluation of the model on the supply and demand sides of the real economy, labor market, and newly in this chapter, the loan functions of banks in the US economy. The simulation at the posterior mean provides a rich business cycle narrative of the post-Volcker period of the US economy. The 1991 and 2001 recessions were both caused by a combination of monetary policy, technology, and investment efficiency shocks. Quite distinctly, the Great Recession is financial in nature at the onset, becoming a real

[4]See Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997), for early incarnations of such models, with Ajello (2016), Akinci and Queralto (2016), Bigio (2015), Christiano, Motto and Rostagno (2014), Gertler, Kiyotaki and Prestipino (2018), Gomes, Jermann and Schmid (2016), Jermann and Quadrini (2012), Nuño and Thomas (2017) being recent examples of general equilibrium developments in financial frictions macroeconomic models.

recession caused by a weak investment environment during its second phase. Moreover, the simulated shock series exhibits rising aggregate productivity during the Great Recession, consistent with Anzoategui et al. (2019), who show such technology movement.[5]

Under the propagation mechanism of the model, a period of weak investment-specific productivity would produce subdued loan demand from firms. This is consistent with micro-level surveys of senior loan officers in US banks, who reported decreased demand for loans in the latter part of the recession. The other main finding in that survey is the reporting of tightening lending standards in the first half of the crisis, which the model captures via a negative financial shock.

In addition to examining the model for its adherence with micro-evidence, I also assess the parameter fit and likelihood. Can the model implied simulation of the observed time series match the data for plausible parameter values? Does the model fit the data at the posterior mean with a high likelihood? Unfortunately, employing an expanded set of observables that extend beyond the scope of the basic New Keynesian model renders it impossible to perform a comparison of model likelihoods. On the other hand, the question of reasonable parameter estimates can be addressed.

Following Bayesian estimation, the parameter estimates broadly conform with existing studies in investment and capital utilization adjustment costs, however, the model requires substantially lower levels of nominal frictions and habit formation to generate the observed data.[6] This is a desirable outcome because many of those frictions are considered to be of dubious grounding at the microeconomic level. Additionally, the Frisch elasticity of labor supply is estimated to be higher than seen in typical studies, which reflects recent contributions concerning the interpretation of labor supply made in Christiano, Trabandt and Walentin (2011).

Having estimated the model, I then turn to the question of addressing the extent to which each shock explains the US business cycle. By way of variance decomposition at different horizons, I find that investment efficiency and technology shocks account for the majority of the US business cycle at long-term horizons. However, financial shocks captured by a fall in the value of existing capital as collateral, account for the majority of short-term business cycle fluctuations. That

[5]The results hold whether technology is measured by labor productivity or capacity-adjusted total factor productivity.

[6]New Keynesian DSGE models feature habit formation in consumption, sticky prices and wages, wage and price indexation, costly investment adjustment, and costly capital utilization adjustment to fit the data. See Christiano, Eichenbaum and Trabandt (2016) for a survey of the micro-level evidence of the plausibility of some of these frictions. More recent work by Keen and Koenig (2018) explores the plausibility of wage and price indexation, while Chetty and Szeidl (2016) explores habit formation.

result is indicative of the role of financial frictions in firm-level behavior. Short-term denials of financing severely dampen a firm's productive capabilities. In the long-run, the ideal debt position becomes attainable, and business cycle fluctuations are the result of persistent shifts in productivity, particularly of new investment.

Beyond the contribution to the US business cycle literature, I address criticisms in Chari, Kehoe and McGrattan (2008) and Canova and Sala (2009) that question the exogeneity of shocks to the price markup and risk premium. These shocks potentially capture endogenous shifts in competition and also 'flight-to-quality' responses, respectively. This critique is not trivial, with their inclusion potentially biasing parameter estimates and limiting identification. To overcome these issues, I utilize alternative shocks in the model that allow me to continue to include the full set of observables used in prior studies, while continuing to identify all parameters and shock series.[7]

Specifically, I assume exogenous variation to the parameterized wealth effect of real wage shifts, and a Nash bargaining weight that captures the relative importance of banking competition or intermediation costs in determining the credit spread. The former of these is a deep preference parameter for which to my knowledge, there is no evidence of endogenous variation. The second shock is potentially deficient in the same manner as the excluded types. For this to be the case, the business cycle would need to endogenously affect the cost of bank intermediation or the level of bank competition. The latter is unlikely as banking merger and acquisition activity is highly regulated, and the implicit existence of 'too-big-to-fail' type policies render network wide banking failures unlikely. However, intermediation costs do plausibly suffer from endogeneity issues. The view taken in this paper then is that the inclusion of these two new shocks leads to no worse an outcome than the standard estimation approach, and perhaps an improvement. The paper serves to illuminate the possibilities of utilizing alternate shocks.

The endogenous generation of a credit spread in the model allows the paper to make a policy contribution. Specifically, credit spreads can be added to the standard Taylor Rule as suggested by Cúrdia and Woodford (2016), and as empirically supported in the post recession period by

[7] The other alternatives to including potentially endogenous shocks is to estimate the model with fewer observed time series, limiting and sometimes eliminating identification of some parameters. Alternatively, measurement errors can be included in the measurement equations that map the model variables to their data counterparts. This approach is problematic where such errors are not warranted by the data. See Canova, Ferroni and Matthes (2014) for a full analysis of these issues. Cuba-Borda et al. (Forthcoming) expand upon other issues with measurement error use.

Caldara and Herbst (2019).[8] I include such an augmented rule in the model and assign a partially informative prior to the Taylor Rule weight on the credit spread. In particular, the prior is slightly biased towards the range of values that suggest little or no monetary policy response to credit spreads. Following estimation, the posterior distribution drifts away from the prior mean and towards the range of values implying monetary intervention following credit spread shifts.

The remainder of this paper is organized as follows, Section 2 provides greater detail about the aforementioned stylized facts that are utilized in estimation. Section 3 describes the structure of the loan market. Section 4 describes the model and optimization problems for each of the agents, as well as the specification of monetary and fiscal policy. Section 5 describes the data and variable construction used in the Bayesian estimation. Section 6 presents the results, and Section 7 concludes.

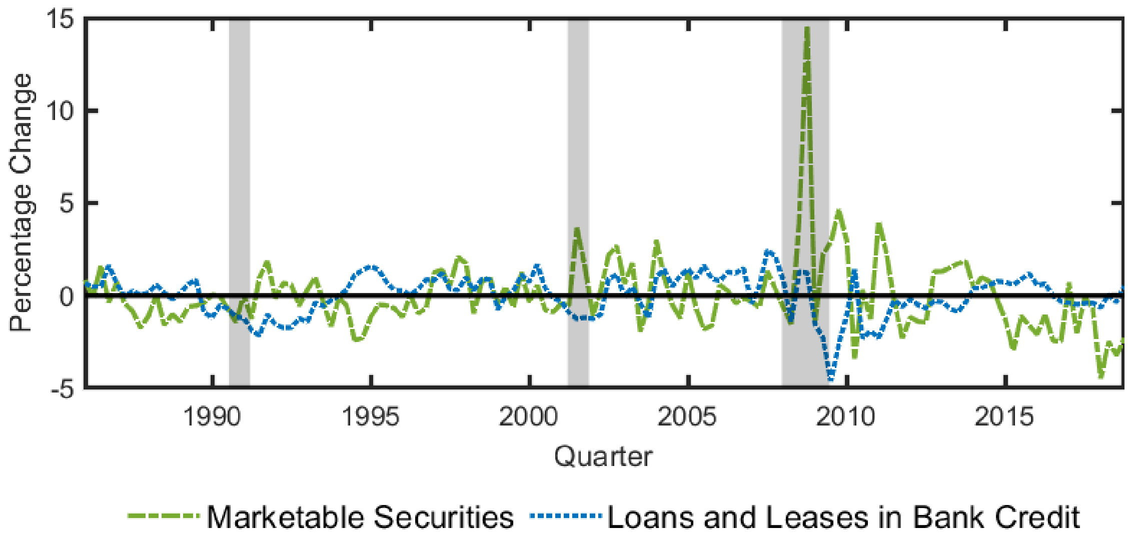
2. Stylized Facts from Key Financial Aggregates

The key empirical regularity that is exploited in this paper is bank substitution between loans to the private sector and marketable securities. Loans to the private sector (Loans) is defined herein as the sum of the value owed to banks from lending contracts with a borrowing counter-party. In contrast, marketable securities are the sum of the value of all assets held that are tradeable, making these liquid assets. They can be excess cash that are not a part of required reserves, loans to other banks, and holdings of treasury and mortgage backed (MBS) securities. The balance sheet identity for banks on the assets side is,

$$Assets = Loans + Marketable\ Securities + Required\ Reserves$$

All bank assets are loans or required reserves at the aggregate level, however, Figure 1 demonstrates a clear distinction in the cyclical behavior of the disaggregated component parts. Marketable securities are starkly counter-cyclical, particularly during the Great Recession, and to a lesser extent in the other two recessionary episodes. Loans experience periods of growth during non-recessionary periods. Moreover, this growth occurs with a substantial lag following recessions.

[8] The standard Taylor Rule is assumed to follow the specification empirically observed in Clarida, Galí and Gertler (2000).



* Source: Federal Reserve Board of Governors Release H.8 Assets and Liabilities of Commercial Banks in the United States. Quarterly data from 1986 Q1 to 2019 Q1. ΔMS is the growth rate of log per-capita real marketable securities, where marketable securities is the sum of bank holdings of treasury and mortgage-backed securities, cash, loans to other banks, and other assets, less required reserves. ΔL is the quarterly growth rate of log per-capita real loans and leases in bank credit. Both series are de-meaned.

Figure 1. Assets in Aggregate US Banks' Balance Sheet

Table 1 shows that the growth rates of marketable securities and loans are more volatile than real GDP, with marketable securities representing the most volatile series. Also, while marketable securities growth is most negatively correlated with GDP growth contemporaneously, loans are most positively correlated with GDP at a lag of eight quarters. Additionally, marketable securities and loans are negatively correlated, and this is strongest for eight quarter lags of marketable securities.

Figure 2 gives us an indication as to the micro-level behavior underlying the cyclical movement of loans and marketable securities. Senior loan officers are asked to report whether their bank (i) raised lending standards and (ii) faced stronger loan demand. The movement of these series displays a striking resemblance to the aggregate movements of the two categories of bank assets, suggesting that a model that features a channel for firms to vary their loan demand and for banks to change lending standards could reconcile the co-movement of real and financial aggregates. Moreover, the differences in the senior loan officer responses across the two recessionary periods sheds light on the financial conditions of the economy during both periods. In 2001, standards and demand exhibit near perfect negative correlation. In contrast, in the Great Recession, standards are first rising

Table 1. Cyclical Relationships of Bank Balance Sheet Aggregates

x	St. Dev.	Corr$(x, \Delta GDP)$	Corr$(x, \Delta GDP_{-8})$	Corr$(x, \Delta MS)$	Corr$(x, \Delta MS_{-8})$
ΔGDP	0.0057	1	-0.0583	-0.2155	-0.0549
ΔMS	0.0203	**-0.2155**	-0.1051	1	-0.0682
ΔL	0.0111	0.0112	**0.2815**	-0.073	-0.1555

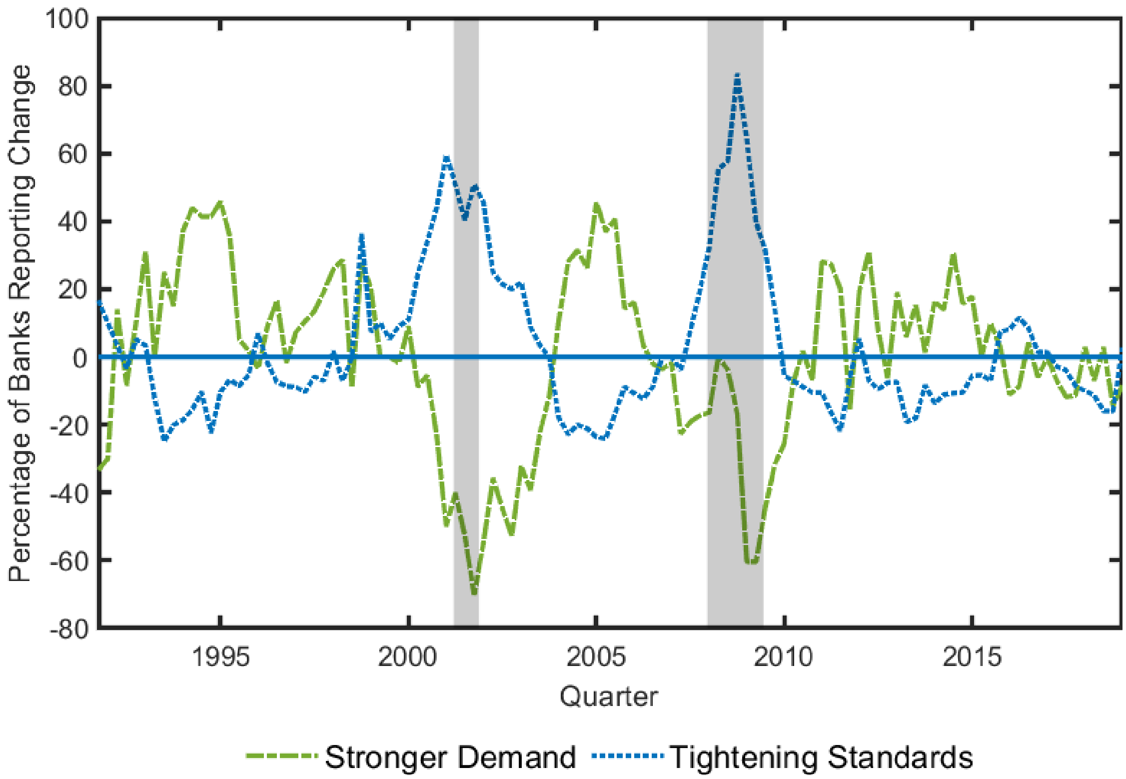
* Source: Federal Reserve Board of Governors Release H.8 Assets and Liabilities of Commercial Banks in the United States. Quarterly data from 1986 Q1 to 2019 Q1. ΔMS is the quarterly growth rate of de-meaned log per-capita real marketable securities, where marketable securities is the sum of bank holdings of treasury and mortgage-backed securities, cash, loans to other banks, and other assets, less required reserves. ΔL is the quarterly growth rate of de-meaned log per-capita real loans and leases in bank credit.

** Source: US Bureau of Economic Analysis National Income and Product Accounts. Quarterly data from 1986 Q1 to 2019 Q1. ΔGDP is the quarterly growth rate of de-meaned log per-capita real GDP.

in the middle part of the recession while demand is not changing, suggesting a specific financial shock. Demand is then starkly weaker for two consecutive quarters while standards were beginning to moderate. The results in this paper suggest that the late recession fall in loan demand is the result of an investment-specific shock that follows a financial shock, rather than a response to banks withholding funds due to tighter standards alone.

We seek a modeling structure that is able to incorporate these stylized facts in the financial sector of the economy, while continuing to capture the extensively documented standard business cycle stylized facts on the real side of the economy. Search and matching frictions that incorporate costly loan search are appealing. Such a structure endogenously prevents banks from loaning all available funds, reflects information asymmetries inherent in the loan market by way of a reduced form matching function, and allows for the slow accumulation of loans, as not all requests for financing are immediately satisfied.

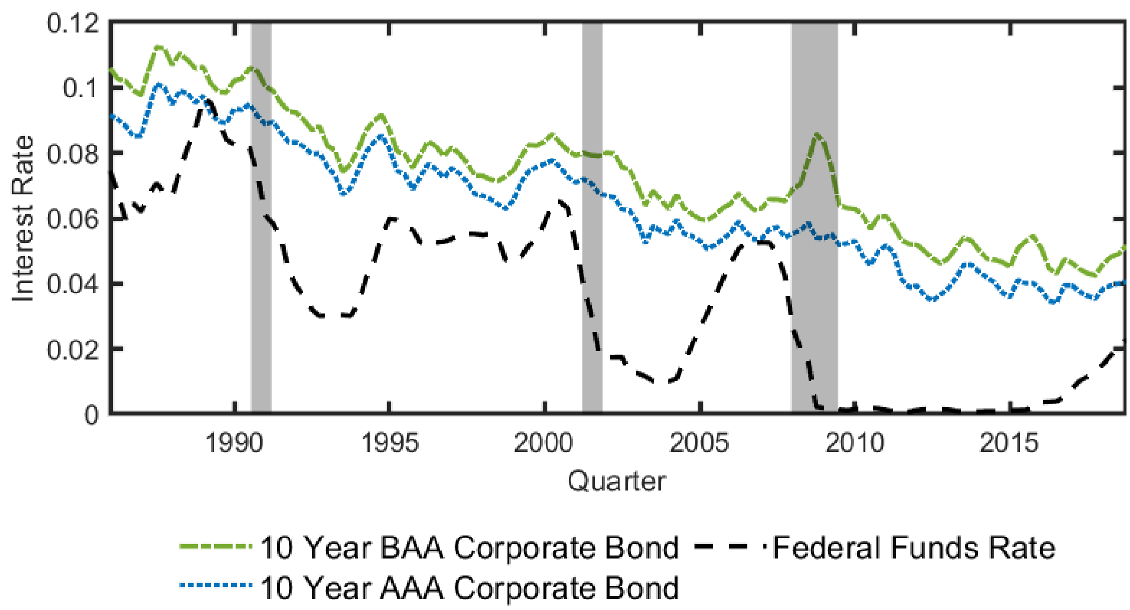
The second prescient stylized fact motivating this paper is the evolution of interest rates and credit spreads over the cycle. The Great Recession is distinct to the others in that the Baa corporate bond rate moves counter-cyclically while the federal funds rate is adjusting in a highly cyclical manner. This effect appears to be related to the underlying riskiness of such loans, with the Aaa rate remaining relatively flat in contrast.



* Source: Federal Reserve Board of Governors Senior Loan Officer Opinion Survey on Bank Lending Practices. Quarterly data from 1991 Q4 to 2019 Q1. Series are (i) Net Percentage of Banks Reporting Stronger Demand for Commercial and Industrial Loans for Large and Middle-Market Firms, (ii) Net Percentage of Banks Reporting Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms.

Figure 2. Senior Loan Officer Opinion Survey on Bank Lending Practices

Under the search and matching structure, interest rate and quantity determination are decoupled. A Nash bargaining surplus sharing rule will instead determine the interest rate. This modeling choice allows for distinct channels that can capture the interest rate dynamics and movements in the quantity aggregates. That interest rates do not clear the loan market is not a novel concept, beginning with Stiglitz and Weiss (1981) where credit rationing is the channel through which the loan market clears.



* Source: Board of Governors of the Federal Reserve System. Quarterly average of monthly rates. Units are annualized.

Figure 3. Interest Rates by Risk Level

3. The Market for Loans to Non-Financial Firms

Before commencing with a description of the loan market, it is useful to first consider why firms would hold an outstanding amount of debt at all. Most closed economy representative agent models have the feature that there is no net aggregate debt holding in equilibrium.[9] Alternatively, non-zero levels of bond holdings in equilibrium can be induced by assuming some heterogeneity in levels of patience among different agents. In such models, market forces endogenously sort the agents such that the relatively patient loans funds in equilibrium to the relatively impatient. This is the case among different households in Kiyotaki and Moore (1997), among entrepreneurs and productive firms in Bernanke, Gertler and Gilchrist (1999), and among consuming households and bankers in Gertler and Karadi (2011). While these approaches are technically convenient, they shed little light on the micro-foundations of lending and borrowing behavior in the economy. Jermann and Quadrini (2012), model an explicit micro-foundation whereby productive firms borrow short-term

[9]A canonical example would be Carlstrom and Fuerst (2003). The open economy literature does not have this feature and funds flow across countries as in Baxter and Crucini (1993).

funds from households. It is assumed that firms earn a tax shield derived from interest payments and so after tax is considered, firms face an effective interest rate that is lower than their willingness to pay.

While the mechanism just described is successful at inducing equilibrium lending and borrowing behavior, it suffers from implying that the spread on savings and borrowing rates is positive, which is not what is observed empirically and that firms can borrow effectively at below the risk-free interest rate. To avoid these deficiencies, I use the search and matching structure such that structural inefficiencies induce a surplus in the loan market for both the firms that demand funds and the banks that supply them. Consequently, banks are able to be profitable in equilibrium and firms are able to borrow at a rate that is less costly than the effective cost of their alternative funding strategy, which is equity.

The search mechanism used in this chapter alleviates two of the concerns raised in these models' primary use describing labor markets. An annuity structure for loans naturally dictates the amount of repayment per period and ensures that a loan will be repaid over time. As a result, repayment evolves endogenously rather than at a constant rate as job separations do in labor search models. The matching function employed to clear the market can be considered a reduced form mechanism that describes frictions associated with financial intermediation. Specifically, those frictions should be considered of the type in Greenwood and Jovanovic (1990), where banks pool risks eliminating idiosyncratic risk, but in a costly way. The other typical approach seen in Holmstrom and Tirole (1997) or Bigio (2015), postulates the role of intermediation as placing limits on moral hazard type behavior. Such concerns are not modeled in this paper.

3.1 Search and Matching

The search and matching structure is derived from the analogous structure used in the labor market by Gertler, Sala and Trigari (2008), but adapted to reflect the specific features of long-term loans. Specifically, a loan contract is a triplet $\langle L_t, Rep_t, r_t^f \rangle$. Denote L_t as the amount of loans that are outstanding at time t. Firms are required to make repayments on loans that follow an annuity structure over $T > 1$ periods. The repayment, Rep_t, that prevails when the loan interest rate is given by r_t^f is given by the standard annuity formula,

$$Rep_t = r_t^f L_t \left(1 - \frac{1}{(1+r_t^f)^T}\right)^{-1} \tag{I.1}$$

Any repayment is then made up of a component, $\mathfrak{P}_t$ that repays the principal outstanding on the loan and an interest component. The repayment is therefore decomposed as $Rep_t = \mathfrak{P}_t + r_t^f L_t$. Herein, it is convenient to define the proportion of existing loans that are still outstanding as, $z_t \in (0,1)$, and express the repayment proportion as,

$$\frac{Rep_t}{L_t} = 1 - z_t + r_t^f \tag{I.2}$$

At any time then, the amount of existing loans that are outstanding is given by $z_t L_t$.

In order to acquire new loans, firms must engage in costly search. This reflects that lending is an activity that requires intermediation services to be provided to the lender and in order to reflect this, the origination of a loan is modeled as being a costly process which falls on the firm to pay.[10] Specifically, the firm must make an application for the loan amount, S_t. Given that application, the probability that the amount requested is originated by the banking sector is given by Q_t. The expected amount of new loans then is given by $Q_t S_t$. Again, it is convenient to denote the proportion of loans that are newly issued as $x_t \in (0,1)$. By the law of large numbers, it must be that,

$$x_t = \frac{Q_t S_t}{L_t} \tag{I.3}$$

The law of motion for loans can then be expressed as next period's loans being the sum outstanding loans and newly originated loans, and in our notation that is,

$$(x_t + z_t)L_t = L_{t+1} \tag{I.4}$$

At any time, banks have G_t funds available to be loaned to firms.[11] Available funds can be

[10]The payment of an adjustment cost by firms can be thought of as purchases of banking services. Their purchase facilitates banking activities such as vetting, management of accounts, and payments to bankers to form client relationships.

[11]The endogenous derivation of G_t is left to Section 4.3, where the bank's optimization problem is formulated.

directed to either the loan market or invested as marketable securities, which in the model is equivalent to holdings of one period government bonds, denoted as MS_t.[12] At any time, funds held as marketable securities can be considered to be funds that are available to be loaned, the analog of unemployed workers in the standard search and matching model. It follows then that,

$$G_t = L_t + MS_t \tag{I.5}$$

Marketable securities then have probability F_t of being converted into loans. Both the probabilities of loan finding, Q_t, and of loan filling, F_t, are taken as given by the firm and bank respectively, and are determined in equilibrium by a reduced form matching function given by,

$$m_t = \Xi(MS_t)^{\varpi}(S_t)^{1-\varpi} \tag{I.6}$$

where $\varpi \in (0, 1)$ captures the relative level of congestion in the loan market, and $\Xi > 0$ is a scale parameter that is targeted in equilibrium to set a desired steady state ratio of other variables. It structurally captures the degree of intermediation efficiency. The probabilities of loan finding and loan filling are then given in equilibrium by,

$$F_t = \frac{m_t}{MS_t} \tag{I.7}$$

$$Q_t = \frac{m_t}{S_t} \tag{I.8}$$

In order to submit an application for a loan to the bank, the firm must pay an adjustment cost that has the effect of neutralizing the capital structure irrelevancy implication of the standard New Keynesian model. The cost of acquiring loans follows the quadratic cost structure in Gertler, Sala and Trigari (2008) and is given as $\frac{\kappa_L}{2}x_t^2 L_t$. In this specification, the parameter $\kappa_L > 0$ magnifies the level of the adjustment cost faced by the firm, and can be interpreted as the price of financial services.

The aforementioned structure serves as a sub-model of the full DSGE model. The supply of funds from banks and the loan interest rate are taken as given by the participants in the loan

[12] Empirically the distinction is captured in a bank balance sheet, with assets being either loans and leases to the private sector, or holdings of treasury securities, mortgage backed securities, cash, excess reserves, other assets, and loans to other banks.

market and then the level of newly originated loans are determined, along with the loan finding and loan filling probabilities. Given those, the level of marketable securities in the bank's loan portfolio is also determined at this stage.

3.2 Interest Rate Bargaining

The nominal loan interest rate, i_t^f is determined via a Nash Bargaining game between a representative bank and a representative firm. The model is structured such that when a firm successfully applies for new loans, the entire loan portfolio is refinanced and the interest rate that prevails on the day of refinancing is applied to the entire loan portfolio, both existing and new loans.

Nash bargaining functions by maximizing the size of the total surplus, where that is a Cobb-Douglas aggregate of the surpluses of the two participants in the game. In this case, those are the bank and firm, and the Nash Bargaining weight is given by $\vartheta \in (0,1)$, where this is the elasticity on the bank's surplus.

Subsequent to successfully matching, the bank and firm meet to bargain over an interest rate that will prevail over the loan. Normalizing the nominal size of the loan to 1, if the bank agrees to the proposed interest rate, it is implemented and the loan proceeds. If not, the bank withdraws from the match and funds are diverted to marketable securities, which earn the risk free interest rate i_t, which is taken as given, and determined by the policy stance of the monetary authority. Subsequently, the bank can return to the search market in the following period and from their perspective in the current bargaining period, will successfully agree on an interest rate if a match occurs. This motivates the value of the outside option to the bank, V_t^u being given by,

$$V_t^u = i_t + \beta\Lambda_{t,t+1}^n \left(F_{t+1}V_{t+1}^L + (1 - F_{t+1})V_{t+1}^u\right) \tag{I.9}$$

where $\beta\Lambda_{t,t+1}^n$ is a nominal stochastic discount factor for the bank that is determined from the Euler Equation in the forthcoming household problem. We now need to define the value of executing the loan to the bank, V_t^L. When a loan is executed, the bank will receive nominal interest on the full amount in the current period, and then on the outstanding proportion in each subsequent period until the loan is repaid. I assume that the principal component of a loan repayment is placed into marketable securities in the period in which it is repaid, subsequently being made available to be loaned again. Therefore, the value of a loan to the bank is given by,

$$V_t^L = i_t^f + \Lambda_{t,t+1}^n \left(z_t V_{t+1}^L + (1 - z_t) V_{t+1}^u \right) \tag{I.10}$$

The value to the firm of executing a loan is given by the Lagrange multiplier on the law of motion for loans, (I.4). That is, the shadow value of loosening the law of motion constraint. Given that firms cannot affect the repayment amount, as the repayment amount is a feature of the loan contract and assumed to not be able to be varied period to period, the only remaining method available to loosen that constraint is to add loans. Denote the nominal Lagrange multiplier on the law of motion constraint as λ_t^n. The specification of which will be determined in the firm optimization problem in Section 4.2.

As both the net surplus of the bank and the surplus of the firm are now specified, we can proceed to determining the surplus sharing rule arising from Nash Bargaining. This rule implicitly determines the nominal loan interest rate. To do so, we maximize the Cobb-Douglas aggregate of the total surplus, given by $(V_t^L - V_t^u)^{\vartheta_t} (\lambda_t^n)^{1-\vartheta_t}$. Optimization then proceeds to yield the surplus sharing rule as,

$$\frac{\vartheta_t}{1 - \vartheta_t} = \frac{V_t^L - V_t^u}{\lambda_t^n} \tag{I.11}$$

The elasticity parameter can be interpreted as capturing the degree of market power of banks in the loan market. As this parameter approaches its lower bound of zero, the implication is that firms enjoy relatively more market power, and the converse is true as the upper bound of one is approached. Structurally, the parameter captures the degree to which these competing forces explain the credit spread. Where firms enjoy more market power, the spread is the result of intermediation costs, while less bank competition explains the spread in the other polar case.

In addition, the bargaining elasticity parameter is subject to exogenous shocks and evolves according to the stationary AR(1) process,

$$\log \vartheta_t = (1 - \rho_\vartheta) \log \vartheta + \rho_\vartheta \log \vartheta_{t-1} + \varepsilon_t^\vartheta \tag{I.12}$$

Shocks to this elasticity therefore capture exogenous shifts in the relative balance between bank competition and intermediation costs in determining the credit spread. The former at an aggregate level is likely the result of government regulations in this highly regulated sector. Bank entry and exit is subject to comprehensive regulations in the United States that encompass macroprudential standards and branch network requirements. The exit process is acutely affected by 'too-big-to-fail' type policies. I argue this is more so a function of the prevailing political atmosphere at any particular time rather than being affected by the business cycle. That is, in some recessions banks will be allowed to fail and in others that will not be the case.

This shock is included in the model at the expense of price markup shocks. Those capture shifts in the entry and exit process in the goods market. Given the lack of regulation in goods markets relative to banking, we would expect the bank bargaining elasticity shock to at worst be less affected by endogeneity than price markup shocks and at best, to be completely exogenous.

4. The Model

The economy is populated by households, firms, banks, and a government which is split into a fiscal and a monetary authority. Households are standard as in the DSGE literature apart from utilizing the Schmitt-Grohé and Uribe (2012) preference structure that parameterizes the strength of the income effect of wage changes on labor supply. Firms also operate as has become standard in New Keynesian models, however, I will embed the aforementioned loan market constraints into the firm problem so that firms must optimize their capital structure as well as making productive input and investment decisions, trading off higher output for an ideal debt to equity mix, and vice versa. Banks are as in Gertler and Karadi (2011) with adjustments to reflect the long-term nature of loans, similar to how Quint and Rabanal (2017) approach the problem. Lastly, fiscal policy is specified as a spending rule as a percentage of GDP, while monetary policy follows a modified Clarida, Galí and Gertler (2000) Taylor Rule, where the monetary authority is empowered to adjust monetary policy in response to changes in credit spreads, as suggested by Cúrdia and Woodford (2016).

4.1 Households

Households are structured as a family that pools incomes and in so doing, provide insurance to each of the members, who either supply differentiated labor to firms, or act as bankers. The objective of the household is to maximize discounted lifetime utility,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{V_t^{1-\sigma} - 1}{1-\sigma} \tag{I.13}$$

where utility is derived positively from consumption of a bundle of non-durable and non-storable goods, c_t, and negatively from the supply of labor hours, h_t. Each period's utility is discounted geometrically by the discount factor $\beta \in (0,1)$ and the period utility is given by,

$$V_t = c_t - Hc_{t-1} - \Psi_t h_t^{1+\frac{1}{\varepsilon}} S_t \tag{I.14}$$

with,

$$S_t = (c_t - Hc_{t-1})^{\gamma_t} S_{t-1}^{1-\gamma_t} \tag{I.15}$$

where $H \in [0,1)$ governs the degree of internal habit formation in consumption, $\varepsilon > 0$ is the Frisch elasticity of labor in the special case where $H = \gamma_t = 0$. Also, in the case where $H = 0$ and $\gamma_t = \gamma$, we have the polar cases of Greenwood, Hercowitz and Huffman (1988)(GHH) preferences when $\gamma = 0$ and King, Plosser and Rebelo (1988)(KPR) preferences when $\gamma = 1$. Therefore, γ captures the strength of the income effect on labor supply in response to real wage shifts. The respective exogenous processes γ_t and Ψ_t evolve following stationary AR(1) processes given by,

$$\log \gamma_t = (1-\rho_\gamma)\log \gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma \tag{I.16}$$

$$\log \Psi_t = (1-\rho_\Psi)\log \Psi + \rho_\Psi \log \Psi_{t-1} + \varepsilon_t^\Psi \tag{I.17}$$

The typical preference shock structure in the literature features a multiplicative shock on the discount factor that has the effect of making consumer rates of time preference time varying. It is argued in Chari, Kehoe and McGrattan (2008) that this shock actually captures 'flight-to-quality' behavior in recessions rather than any exogenous shifts in consumers' patience levels. Additionally, Canova and Sala (2009) argue that this shock is poorly identified, and Canova, Ferroni and Matthes (2014) postulate that such a shock captures model mis-specification. To avoid these issues, I instead embed preference shocks in the parameter that captures the strength of the income effect.

Given that exogenous disturbances need to occur in a completely random manner, unrelated to the underlying state of the economy, I argue that it is more plausible that such a condition would be satisfied by shifts in the strength of the income effect rather than rates of time preference. This is because rates of time preference are typically approximated by the long-term risk free real interest rate, a relationship which derives from the Euler Equation. A specification where rates of time preference are exogenously time varying in the short-run implies some empirical linkage between asset prices and consumption-saving behavior is unexplained by the model (mis-specification) and/or time varying in a manner unrelated to the business cycle, which seems unlikely.

The households are constrained in their efforts to maximize utility by a budget constraint. While consumption goods are non-durable and non-storable, default-risk-free deposits, b_t can be made with the bank which earn real interest r_t with certainty. Households are also the owners of firms and are paid a dividend, d_t from those entities. Banks, as will be seen later, reinvest all profits and are taxed by the government to constrain their growth, and so dividends from banks are not paid to households. Lastly, the government is partially funded by lump sum taxes, tax_t. The budget constraint is then given by,

$$c_t + tax_t + b_t = w_t h_t + b_{t-1}(1 + r_{t-1}) + d_t \tag{I.18}$$

Individual household members are uniformly distributed along the interval $[0, 1]$ and heterogenous in their labor productivity. I assume that firms cannot distinguish the types and pay a common wage that is distributed to the workers by a union. The union acts to aggregate the various types into a single representative worker. It is only able to change the wage it demands from the firm with probability $1 - \phi_w$. If it is able to re-optimize the wage, it is set to $\widetilde{w}_t$. In periods where re-optimization is not possible, the prior period's wage is indexed to inflation so that the prevailing wage is given by $w_{t-1}(1 + \pi_{t-1}^{\iota_w})/(1 + \pi_t)$, where $\iota_w \in (0, 1)$ captures the degree of wage indexation. Denote $\eta_w > 1$ as the elasticity of substitution. Then solving the union's implicit household utility maximization problem yields the law of motion for the real wage,[13]

$$w_t^{1-\eta_w} = (1 - \phi_w)\widetilde{w}_t^{1-\eta_w} + \phi_w w_{t-1}^{1-\eta_w} \left(\frac{1 + \pi_{t-1}^{\iota_w}}{1 + \pi_t} \right)^{1-\eta_w} \tag{I.19}$$

[13]See the appendix for the full derivation.

Any and all union profits are distributed to the household as a lump sum. That union profits exist implies that the sticky wage specification has created a wedge in the labor market. Exogenous fluctuations in the wedge are captured by the innovations to Ψ_t, which is therefore a labor dis-utility or wage markup shock. The innovations here can be thought of as capturing micro-level structural changes in the labor market, perhaps caused by policy switches, that induce non-market driven shifts in the labor market wedge. The level of labor dis-utility evolves according to the stationary AR(1) process,

$$\Psi_t = (1 - \rho_\Psi)\Psi + \rho_\Psi \Psi_{t-1} + \varepsilon_t^\Psi \tag{I.20}$$

4.2 Firms

The model features both wholesale and retail firms. Retail firms are merely a unit that determines the price subject to Rotemberg adjustment costs, while the key input and capital structure decisions occur at the wholesale level. Wholesale firms are assumed to own the retail firms, so that all retail profits are returned lump sum to the wholesale firms.

Wholesale firms have access to a production technology that produces output y_t, given by,

$$y_t = A_t(u_t k_t)^\alpha h_t^{1-\alpha} \tag{I.21}$$

where A_t is the technology level, u_t is the utilization level of capital k_t, and h_t represents labor hours. The parameter $\alpha \in (0,1)$ is an elasticity that weighs the relative importance of each input in the production process. The technology level is subject to stationary shocks following,

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A \tag{I.22}$$

Changing the level of capital utilization is costly, with adjustment costs given by $\Psi(u_t) = \vartheta_u(u_t - 1)^{1+\chi}/(1+\chi)$, with ϑ_u chosen such that the steady state level of utilization is unity. The parameter χ has the property that $\Psi''(1)/\Psi'(1) = \chi$.[14]

[14] In order to estimate the utilization cost elasticity such that the posterior distribution is bounded above, I follow the established method in the literature and estimate the parameter $\psi \in (0,1)$ where $\chi = \frac{\psi}{1-\psi}$. Utilization changes become more costly as $\psi \to 1$.

Capital is a state variable which depreciates at the rate $\delta \in (0,1)$ and requires investment, inv_t in order to grow. However, investment is subject to adjustment costs such that the effective amount of investment is given by $\Upsilon(\frac{inv_t}{inv_{t-1}}) = (1 - \frac{\varrho}{2}(\frac{inv_t}{inv_{t-1}} - 1)^2)inv_t$. Effective investment is thus investment less the cost of investment adjustment, with $\varrho > 0$ being the elasticity of the investment adjustment cost function.[15] The law of motion for capital is thus given by,

$$k_{t+1} = (1-\delta)k_t + \zeta_t^I \Upsilon\left(\frac{inv_t}{inv_{t-1}}\right) \tag{I.23}$$

The capital law of motion is subject to shocks that affect the process of effective investment being transformed into capital. The marginal efficiency of investment evolves according to the stationary AR(1) process,

$$\log \zeta_t^I = \rho_I \log \zeta_{t-1}^I + \varepsilon_t^I \tag{I.24}$$

The above describes the productive side of the firm, which involve investment, labor demand and capital utilization decisions. In addition, firms must decide how they are to fund their operation by selecting a debt and equity mix. In order to prevent an equilibrium emerging where firms are incentivized to exit the market rather than take on debt, I assume that the firm's objective is to maximize the firm value, the sum of the equity and debt values. That behavior in this model absent this assumption leads to firms paying excessive dividends suggests an agency problem where debtholders' interests are not taken into account by management, who would otherwise be acting solely to maximize the return to equityholders. The assumption that managers maximize the firm value can therefore be viewed as a mechanism to lessen the negative effects on banks, acting as principals, in loaning funds to an entity whose interests may not be aligned with theirs.

The value of equity is defined in the standard way, as a discounted stream of dividends, d_t. I define the value of debt as a discounted stream of debt coupons, which were earlier denoted as

[15] Specifying the investment adjustment cost function in this way ensures that Tobin's q is equal to unity in steady state.

Rep_t. The firm value, FV, is defined over an expected T period life of the firm as,

$$FV(inv_{t-1}, k_t, L_t) = \max \mathbb{E}_t \sum_{j=0}^{T} \beta^j \Theta_{t,t+j}(d_{t+j} + Rep_{t+j}) \tag{I.25}$$

In this formulation, the augmented firm discount factor, $\beta^j \Theta_{t,t+j}$ is the product of the price of a consumption based risk-free security and the proportion of unpaid loans. Given by, $\Theta_{t,t+j} = z_{t-1+j}\frac{\Lambda_{t+j}}{\Lambda_t}$. The firm therefore operates under the assumption that it will maintain its chosen debt to equity ratio over its expected life. Under the assumption that the firm can refinance its loan position such that loans have a time to maturity of T when they are refinanced, we can let $T \to \infty$ and denote the firm value in recursive form as,

$$FV(inv_{t-1}, k_t, L_t) = \max \ d_t + Rep_t + \beta \mathbb{E}_t \Theta_{t,t+1} FV(inv_t, k_{t+1}, L_{t+1}) \tag{I.26}$$

Such an assumption amounts to managers holding beliefs at all time periods that the firm will be a going concern. Given the standard assumptions on the household discount factor β and that loans must be repaid, so that $z_t \in (0, 1)$, it follows that the above is a contraction mapping.

In order to constrain firm borrowing, I specify a firm borrowing limit where some proportion $\zeta_t \in (0, 1)$ of the firm's future capital stock can be used as collateral against loans.[16] Loans are either the long-term loans acquired through the long-term loan market or, a zero interest intra-period working capital loan, secured against the firm's production, as in Jermann and Quadrini (2012). The borrowing constraint is then,

$$\zeta_t k_{t+1} - L_{t+1} = y_t \tag{I.27}$$

The inclusion of this working capital loan has the effect of distorting the firm's labor hiring choice. The optimal firm labor demand has the firm setting the wage such that it is equal to the marginal product of labor net of the marginal tightening of the borrowing constraint induced by requiring an additional working capital loan.[17]

The proportion of capital that can be used as collateral is subject to shocks, deemed to be

[16] Similar assumptions are made in Kiyotaki and Moore (1997), and variants thereof. As in such papers, I assume that the borrowing constraint is always binding.

[17] See the appendix for the full derivation.

collateral quality shocks.[18] The evolution of the level of collateral quality follows the stationary AR(1) process,

$$\log \zeta_t = (1 - \rho_\zeta) \log \zeta + \rho_\zeta \log \zeta_{t-1} + \varepsilon_t^\zeta \tag{I.28}$$

Lastly, the firm is required to generate sufficient after-tax operating profit to cover its financing expenses. In particular, debt adjustment is subject to adjustment costs as already discussed, as is dividend adjustment, in a manner similar to Jermann and Quadrini (2012). Specifically, the effective dividend outlay is given by $\varphi(d_t) = (1+\tau)d_t + \frac{\kappa_d}{2}(d_t - d)^2$. This function reflects that in addition to outlaying a dividend, the firm must pay tax in the amount τd_t, with $\tau \in (0,1)$, and an adjustment cost where the dividend deviates from its steady state level.[19] In addition, taxes are levied on investment, at the same rate as those on dividends, reflecting that capital expenditure must be made from after-tax profits under the US tax code. However, I allow for depreciation expenses to be tax deductible. The firm's profit constraint is thus given by,

$$y_t = w_t h_t + inv_t(1+\tau) - \tau\delta k_t + \Psi(u_t)k_t + \varphi(d_t) + r_t^f L_t + \frac{\kappa_L}{2} x_t^2 L_t \tag{I.29}$$

Definition. The problem of the firm is to choose $(y_t, h_t, k_{t+1}, inv_t, u_t, d_t, Rep_t, L_{t+1}, S_t)$ to maximize (I.26) subject to (I.2), (I.3), (I.4), (I.21), (I.23), (I.27), and (I.29), while taking all prices, interest rates and the probability of loan finding as given, and also taking the proportion of unpaid loans as given.

Optimization then yields the first order conditions that determine the firm's demand for labor,

[18] These shocks have been shown by Jermann and Quadrini (2012) to be a significant driver of US business cycles.

[19] As developed by Jermann and Quadrini (2012), this formulation reflects the costs of equity issuance, or manager preferences for stable dividends for signaling reasons.

capital, investment, utilization, and loans respectively as,[20]

$$(1-\alpha)\frac{y_t}{h_t}\left[1-\mu_t\varphi'(d_t)\right] = w_t \tag{I.30}$$

$$q_t - \zeta_t\mu_t = \beta\mathbb{E}_t\Theta_{t,t+1}\left[\left(\alpha\frac{y_{t_1}}{k_{t+1}} - \Psi'(u_{t+1}) + \tau\delta\right)\left(\varphi'(d_{t+1})\right)^{-1} - \alpha\mu_{t+1}\frac{y_{t+1}}{k_{t+1}} + q_{t+1}(1-\delta)\right] \tag{I.31}$$

$$\frac{1+\tau}{\varphi'(d_t)} = \zeta_t^I q_t \Upsilon_{\uparrow,t} + \beta\mathbb{E}_t\Theta_{t,t+1}\zeta_{t+1}^I q_{t+1}\Upsilon_{\downarrow,t+1} \tag{I.32}$$

$$\alpha\frac{y_t}{k_t}\left[1-\mu_t\varphi'(d_t)\right] = \Psi'(u_t)u_t \tag{I.33}$$

$$\kappa_L x_t + \mu_t\varphi'(d_t) = \beta\mathbb{E}_t\Theta_{t,t+1}\frac{\varphi'(d_t)}{\varphi'(d_{t+1})}\left[\frac{Rep_{t+1}}{L_{t+1}}\varphi'(d_{t+1}) - r_{t+1}^f + \frac{\kappa_L}{2}x_{t+1}^2 + \kappa_L x_{t+1}z_{t+1}\right] \tag{I.34}$$

In the above, denote q_t as the Lagrange multiplier on the firm's capital law of motion, which in the literature is Tobin's q, the ratio of a firm's market price of capital to its book value. Also, the term μ_t is the Lagrange multiplier on the borrowing constraint and is a measure of how 'tightly' the constraint binds.

In particular, the use of equity adjustment frictions introduces distortions into the labor demand, utilization, and capital demand dimensions of the firm's optimization problem. Rather than equating the marginal cost of an additional input to its marginal product, the firm now equates marginal cost to marginal product net of the marginal tightening of the borrowing constraint induced by adding a unit of input.

In turn, the last condition, which is the loan demand equation, determines the tightness of the borrowing constraint as a function of the cost of equity adjustment, loan search costs, rate of repayment of a loan, the rate of loan acquisition, and the loan interest rate. As a result, for any input decision, the firm is considering the effect such a change would have on its capital structure, and is attempting to smooth loan acquisition costs over time. If not, sudden jumps in x_t, the rate of loan acquisition, result in a tightening of the borrowing constraint μ_t, and thus reduced marginal product of capital and labor.

Implicit in the above derivation is that the marginal value of loosening the loan law of motion constraint, λ_t, is given by $\lambda_t = \kappa_L x_t/\varphi'(d_t)$, or that the marginal willingness to pay to acquire additional loans for firms is equal to the ratio of marginal costs of loan acquisition to the marginal

[20] I denote $\Upsilon_{\uparrow,t}$ as the partial derivative with respect to the numerator and $\Upsilon_{\downarrow,t}$ as the partial derivative with respect to the denominator. The time subscript reflects the time period of investment in the numerator.

cost of equity adjustment. This relationship then specifies the firm's surplus in the interest rate bargaining process described earlier.

After the firm chooses output, it is sold to the retail sector, who perform costless packaging and resell the product to households subject to Rotemberg adjustment costs.[21] Denote $\eta_p > 1$ as the elasticity of substitution among varieties of goods, and $\iota_p \in (0, 1)$ as the degree of price indexation allowed for retail firms before price adjustment costs are incurred. Then I present the pricing first order condition as,

$$\begin{aligned} 1 - \eta_p + \eta_p mc_t - \phi_p \frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} \left(\frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} - 1 \right) \\ + \beta \mathbb{E}_t \Theta_{t,t+1} \phi_p \frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_p}} \left(\frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_p}} - 1 \right) \frac{y_{t+1}}{y_t} = 0 \end{aligned} \tag{I.35}$$

The parameter $\phi_p > 0$ captures the level of cost associated with price adjustment, while mc_t is a variable capturing marginal cost, the real price that the wholesale firm sells its output to the retail firm. I include the above pricing frictions so that monetary policy shocks are able to have real effects on the economy.[22]

4.3 Banks

Banks take the results of the search and matching market as given, and also the result of the bargaining game over the long-term interest rate. The function of banks in the model is to endogenously determine the leverage ratio, lev_t, and in so doing, the total amount of bank assets G_t. The search and matching market then, in determining the amount of loans, also determines the amount of marketable securities held by banks.

To achieve that end, I modify the modeling structure for banks developed in Gertler and Karadi (2011), incorporating some of the modifications in Quint and Rabanal (2017), with all other modifications made to reflect the specific features of this model. Bank assets are financed by deposits from households b_t and bankers' own net worth, nw_t. In addition, a proportion $req \in (0, 1)$ of deposits must be held as reserves, and cannot be made available to the search and matching market.[23] The

[21]The derivation follows the standard method and is left to the appendix.

[22]Under pricing frictions, when the nominal short term interest rate is varied by the monetary authority, inflation does not instantly respond, leading to changes in the real interest rate and thus the implicit price of investment and saving in the economy.

[23]The reserves channel is included to reflect that some proportion of cash assets held by US banks is required

bank balance sheet is thus given by,

$$G_t = nw_t + (1 - req)b_t \tag{I.36}$$

where bank assets are the sum of loans and marketable securities as given in (I.5). As banks are able to earn an economic profit because of the frictions present in the search mechanism, we have to in some way limit the growth of banks to keep the model stationary. I achieve this by specifying a tax on bankers' net worth, levied at the constant rate $\tau^b \in (0, 1)$.[24] Given this, bankers' net worth evolves according to,

$$nw_t = (1 - \tau^b)\left(R^G_{t-1}G_{t-1} - R_{t-1}(1 - req)b_{t-1}\right) \tag{I.37}$$

The gross interest rate R^G_t is taken as given by the bank, and is a weighted average return to lending given by,

$$R^G_t = \frac{(x_t + z_t + r^f_t)L_t + (1 + r_t)MS_t}{G_t} \tag{I.38}$$

Reserves and marketable securities are paid interest at the risk-free rate, and the underlying funds are loaned to and held by the government. Additionally, deposits are paid interest at the risk-free rate, and such interest payments are an expense to banks.

Defining the leverage ratio $lev_t = \frac{G_t}{nw_t}$, and using the bank balance sheet, (I.36), we can express the bankers' net worth law of motion as,

$$nw_t = (1 - \tau^b)\left[lev_{t-1}(R^G_{t-1} - R_{t-1}) + R_{t-1}\right] nw_{t-1} \tag{I.39}$$

Owing to search frictions, it must be that the gross credit spread earned by banks $R^G_{t-1} - R_{t-1}$ is positive, and so absent the tax on net worth, bankers would be able to increase their net worth in an unbounded way by increasing the leverage ratio.

Banks have a survival probability of $\theta^B \in (0, 1)$, and in the event of exit, bankers transfer net worth back to the household. At the same time, new bankers enter the market, endowed with

reserves, and not available for loan transactions.

[24] I target the steady state leverage ratio and set the tax rate such that the desired leverage ratio is achieved.

a transfer from the household as start-up funds, and the existing loan portfolio of the exiting bankers. In this way, the flow of bank funds is not disrupted by the entry/exit process, while bankers' incentives can be expressed in a manner where the resulting value function is a contraction mapping.

Specifically, the objective of bankers is to maximize their expected terminal net worth. Conditional on survival to the next period, the value of banking operations, $\mathcal{V}_t$, is given by,

$$\mathcal{V}_t = (1-\tau^b)(1-\theta^B)\mathbb{E}_t \sum_{j=0}^{\infty} (\theta^B)^j \beta^{j+1} \Theta^B_{t,t+j} nw_{t+1+j} \tag{I.40}$$

where $\Theta^B_{t,t+1+j} = \frac{\Lambda_{t+1+j}}{\Lambda_t}$, is the standard stochastic discount factor derived from the household Euler Equation, and is the price of a risk-free security. We can then express the above recursively, noting that the state variables are deposits and assets, as,[25]

$$\mathcal{V}_{t-1}(G_{t-1}, b_{t-1}) = \beta \mathbb{E}_{t-1} \Theta_{t-1,t} \left[(1-\tau^b)(1-\theta^B) nw_t + \theta^B \max \mathcal{V}_t(G_t, b_t) \right] \tag{I.41}$$

Bankers seek to maximize (I.41) but face a household participation constraint that must be satisfied that is derived from a Diamond and Dybvig (1983) type agency friction in the banker/depositor relationship. Specifically, bankers can abscond with a proportion of bank assets, $\omega \in (0, 1)$.[26] In order to prevent such banker behavior, the participation constraint is set such that bankers' incentives are to optimally not divert funds. The participation constraint is thus,

$$\mathcal{V}_t \geq \omega G_t \tag{I.42}$$

where the value of bank operations is at least as high as the value of absconding.[27] Having specified the banking structure, we can define the bankers' optimization problem below.

Definition. The problem of the bankers is to choose (G_t, nw_t, b_t) to maximize (I.41) subject to (I.42), while taking all interest rates as given, and taking the allocation of loans and marketable securities arising from the search market as given.

[25] See the appendix for details.

[26] The value of ω is chosen in order to target the steady state bank assets to GDP ratio.

[27] I select the parameters such that the participation constraint always binds, however, Gertler, Kiyotaki and Prestipino (2018) explore the dynamics of the economy when this constraint is allowed to occasionally bind.

Arising out of bankers' optimization is a condition for the optimal leverage ratio, given by,[28]

$$lev_t = \frac{\mathbb{E}_t \beta \Theta_{t,t+1} \Omega_{t+1} (1-\tau^b) R_t}{\omega - \mathbb{E}_t \beta \Theta_{t,t+1} \Omega_{t+1} (1-\tau^b)(R^G_t - R_t)} \tag{I.43}$$

Bank leverage is decreasing in the proportion of funds that can be diverted, decreasing in banking taxes, and increasing in the gross credit spread, $R^G_t - R_t$. In a standard Gertler and Karadi (2011) model, an increase in the credit spread would lead to higher leverage and an increase in credit flow to the private sector. This is because all bank assets in standard treatments are loaned. The difference induced under the extensions in this paper do not result in such propagation. Bank leverage increases under a widening credit spread, however, that increase *ceteris paribus* reduces the probability of loan filling. Moreover, the widening credit spread reduces the appeal of loan financing for firms relative to equity, and increases repayments on existing debt. Combined, these movements serve to limit firm appetite for finding new financing, *ceteris paribus* reducing the probability of loan finding. Combining both effects, the increase in bank leverage following a credit spread widening will disproportionately flow into marketable securities rather than loans.

4.4 Government

The government is comprised of a fiscal and a monetary authority. Fiscal policy is determined exogenously with government spending g_t being tailored to follow a rule based on its proportion in output according to the stationary AR(1) process,

$$g_t = \left(1 - \frac{1}{G^y_t}\right) y_t \tag{I.44}$$

where the government spending to output ratio, $G^y_t > 1$, evolves exogenously and subject to shocks according to,

$$\log G^y_t = (1-\rho_G) \log G^y + \rho_G \log G^y_{t-1} + \varepsilon^G_t \tag{I.45}$$

[28]The derivation can be found in the appendix. The term $\Omega > 0$ is a function of parameters derived in the appendix.

The government budget constraint then aggregates the three tax sources, which are lump sum taxes levied on households, firm distribution and investment taxes, and taxes on bankers' net worth, in order to fund government spending and interest payments to banks on reserves and marketable securities. The bank tax is given by $bt_t = \tau^b(lev_{t-1}(R^G_{t-1} - R_{t-1}) + R_{t-1})nw_{t-1}$. I thus specify the government budget constraint as,

$$tax_t + \tau d_t + \tau inv_t - \tau\delta k_t + bt_t - (MS_t + req \cdot b_t)(1 + r_t) = g_t - MS_{t+1} - req \cdot b_{t+1} \tag{I.46}$$

Lastly, to close the model, I specify a modified Taylor rule for the conduct of monetary policy. The rule is of the form in Clarida, Galí and Gertler (2000), with additions as proposed in Cúrdia and Woodford (2016), such that the monetary policy instrument, the nominal short term interest rate, responds to inflation, the output gap, and now, credit spread deviations from the steady state. The rule is then,

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{r_i} \left[\left(\frac{1+\pi_t}{1+\pi^*}\right)^{r_\pi} \left(\frac{mc_t}{mc}\right)^{r_y} \left(\frac{Sp_t}{Sp}\right)^{r_s}\right]^{1-r_i} \exp(fed_t) \tag{I.47}$$

In particular, the parameter $r_i \in (0, 1)$ captures the degree of interest rate smoothing, while the parameters r_π, r_y and r_s capture the degree of response to inflation, the output gap, and the credit spread respectively. $\pi^* = 0$ is the inflation target, and is set as such as the model is stationary. Due to the complexity of the model, it is unnecessarily computationally burdensome to specify the output gap in the usual way, by tracking a parallel economy that evolves contemporaneously but absent nominal frictions, instead, I approximate the output gap as the deviations in marginal cost from steady state.

I define the credit spread Sp_t, implicitly as,

$$1 + i^f_t = (1 + i_t)Sp_t \tag{I.48}$$

where the rule used nests the standard Clarida, Galí and Gertler (2000) rule when $r_s = 0$, and also nests the case where the monetary authority is fully responsive to credit spread changes, when $r_s = 1$. Both occur as polar cases.

Deviations from the above rule occur in a stationary way, are captured by the variable fed_t, and evolve exogenously following the stationary AR(1) process,

$$fed_t = \rho_i fed_{t-1} + \varepsilon_t^i \tag{I.49}$$

5. Data and Variable Construction

I estimate the model using Bayesian methods as in An and Schorfheide (2007) utilizing a subset of the US aggregate data which has a direct concordance to the model's variables.[29] Eight time series are employed to estimate the parameters that are in common with this model and Smets and Wouters (2007), as well as the adjustment cost parameters for equity and debt issuance, the Nash bargaining weight in the nominal long-term interest rate surplus rule, the Taylor Rule weight placed on the credit spread, and the auto-regressive and shock standard deviations for each of the 8 exogenous processes present in the model.

The estimation procedure utilizes Bayes' rule to form a posterior distribution on the model's parameters conditional on the data. Denote X as the complete set of model parameters and Y as the complete set of model variables. We update a prior distribution on the subset $x \subset X$ of the model's parameters using a subset of observed model variables $y \subset Y$. Denote $p(\theta)$ as the prior distribution, a function of a draw on the values of the parameters x, denoted by θ. Also denote $p(y|\theta)$ as the likelihood function of the data given a draw. The posterior distribution $p(\theta|y)$ over the values of the parameters given the data is then calculated using Bayes rule as,

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}$$

where $p(y)$ is the marginal likelihood of the model.

Given the scale of the model, an analytic characterization of the posterior distribution is not feasible and so numerical methods are utilized. Specifically, we seek to draw proposal parameter values from a proposal distribution and evaluate the likelihood function. The posterior distribution is then approximated as the product of the prior and likelihood evaluated at the proposal.

The sampling algorithm chosen to accomplish this is the Random Walk Metropolis Hastings (RWMH) algorithm with the proposal distribution being a multivariate normal. The algorithm

[29] A textbook treatment is available in Herbst and Schorfheide (2015).

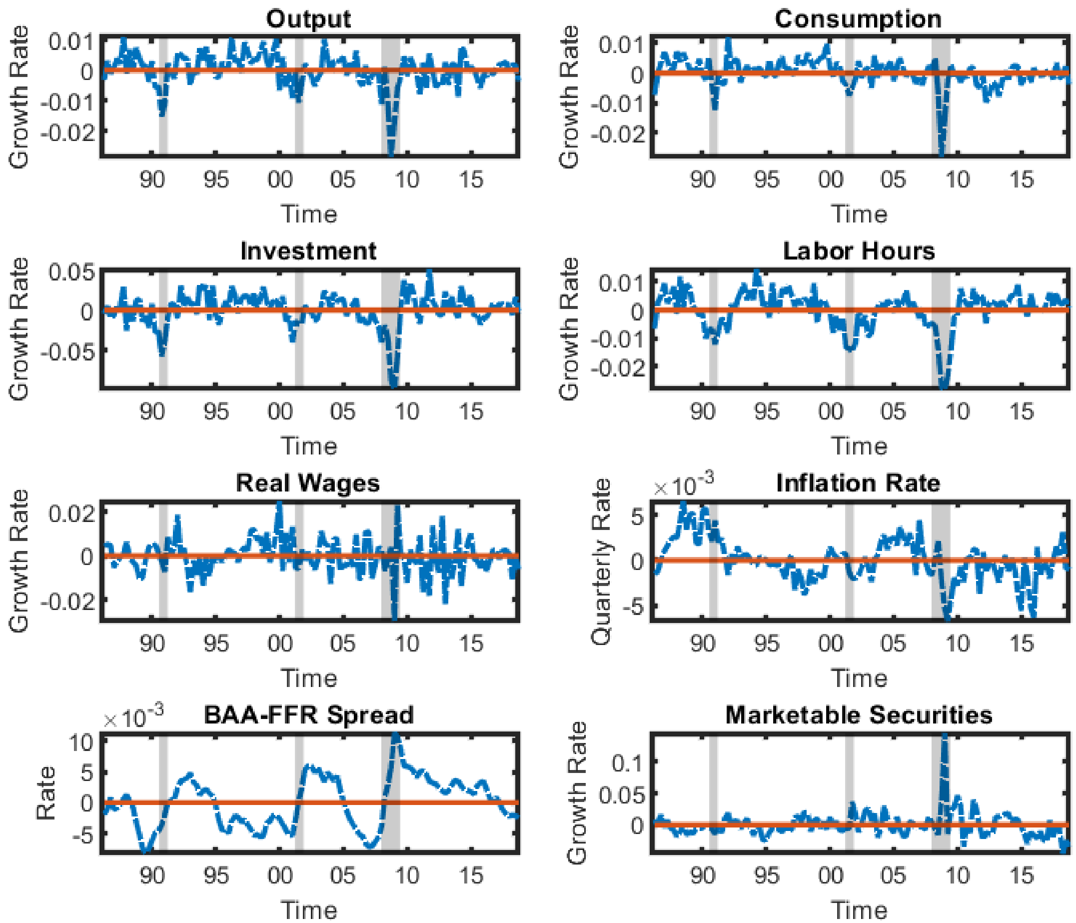
works by amassing serially correlated draws from the proposal distribution and accepting a draw with certainty if the density of the posterior evaluated at the candidate draw is higher than the density at the previously accepted draw. In the event that this is not the case, the acceptance probability is given by the odds ratio between those two objects.

To initialize the algorithm, a variance-covariance matrix for the proposal distribution is required. I choose this as the inverse Hessian evaluated at the posterior mode, which is approximated by a numerical procedure. This is scaled such that the ergodic acceptance rate is in the optimal range suggested in the literature as between 0.25 and 0.33. Running the algorithm over a sufficiently large number of draws leads to convergence to the true posterior distribution, assuming the posterior has a unique mode.[30]

The raw data used in the estimation is at quarterly frequency for the US economy over the period 1986Q1 to 2019Q1. Evidence in Clarida, Galí and Gertler (2000) suggests that a structural break occurred during the Volcker chairmanship of the Federal Reserve, as the central bank shifted monetary policy from being accomodative of inflation, to a hawkish stance. The sample period is chosen accordingly in order to avoid the structural break. Data for seasonally adjusted nominal gross domestic product, personal consumption, and gross private domestic investment are taken from the US Bureau of Economic Analysis (BEA) National Income and Product Account (NIPA) tables. I define consumption as the sum of Personal Consumption Expenditures on Nondurable Goods and Services, while the investment variable is the sum of Gross Private Domestic Investment and Personal Consumption Expenditures on Durable Goods, following Justiniano, Primiceri and Tambalotti (2011). I then divide these by the US Bureau of Labor Statistics (BLS) measure of the Civilian Non-Institutional Population Level of Persons 16 Years and Over. Additionally, I deflate these variables using the GDP Implicit Price Deflator taken from NIPA. The observed variables are assigned the names, $gdp_t, cons_t, invest_t$ respectively.

I also extract seasonally adjusted data on labor hours as the BLS Index of Nonfarm Business Sector: Hours of All Persons. This is then divided by population to extract a measure of hours per worker. The real wage series is the BLS index of Nonfarm Business Sector: Compensation per

[30] I check for a bimodal posterior by initializing the posterior mode approximating algorithm at different starting values and checking for convergence. A second check involves comparing the posterior mean and ensuring that it is close to the approximated mode across the parameters. Both of these checks suggest that the posterior is evaluated at the unique mode.



* De-meaned time series used in estimation from 1986-Q1 to 2019-Q1. Each quantity series is in real per capita units. Wages are in real units. A full description of the data construction is available in the appendix.

Figure 4. De-meaned Time Series Used in Estimation

Hour, which I deflate. The inflation rate is set to be the growth rate of the GDP Deflator. The respective names of the observables are assigned as $hours_t, wage_t, infl_t$.

The Federal Funds Rate is employed as the proxy for the short-term rate i_t, and the Baa 10 Year Corporate Bond rate as the proxy for i_t^f. The data analog of the credit spread is then defined according to (I.48). Lastly, seasonally adjusted aggregate bank balance sheet data for all US commercial banks is extracted from the Board of Governors of the Federal Reserve System (BOARD) H.8 Release. Specifically, the variables Securities in Bank Credit, Cash Assets, Loans to Commercial Banks and Other Assets Including Trading Assets are utilized from that release.

In addition, I extract the Required Reserves of Depository Institutions from the BOARD's H.3 release. My proxy for the variable MS_t in the model is then the sum of Securities, Cash, Loans to Banks, and Other Assets, less Required Reserves. The respective names are $spread_t, sec_t$. All of above variables are de-meaned to complete their construction.

Figure 4 displays the observed path of each de-meaned variable chosen for estimation. There are three recessionary episodes in the time period and a strongly pro-cyclical relationship can be observed between output, consumption, investment and labor hours in that time. The slow recovery following the Great Recession can be seen in the US GDP growth being rate being below its sample average for the majority of the period following the recession's end.

A number of other features are worthy of emphasis at this stage. Investment growth is strong in all non-recessionary periods. This is the case for consumption also in the boom periods prior to the Great Recession, though consumption growth is markedly weak following that recession. This is despite labor hours growing strongly following 2009, which is not the case in the other two post-recessionary phases. That consumption growth is weak in the wake of a strong growth in work effort through labor hours is indicative of substantial wealth effects dictating consumer behavior. In the model, there are three potential sources of wealth effects in household budgets. These are through wages, dividends, and lump sum taxes. In the case where the wealth effect parameter γ_t is close to zero, the wealth effect of wage changes on labor supply is absent, leaving only the remaining two channels as possible sources.

The two observables added in this study are the credit spread and marketable securities measures. Credit spreads widen in both the 2001 and Great Recessions and marketable securities see an acceleration of growth during and following these recessions. Those relationships are less pronounced in the 1991 recession. These added series allow for the identification of the two financial shocks in the model, collateral quality, where a negative disturbance tightens the credit spread, and the bank bargaining weight shock where an increase in bank bargaining power widens the spread.

The model contains equal numbers of shocks and observed time series and no equations that imply perfect colinear relationships between the observed time series. Therefore, there are no stochastic singularity problems. The observed time series and model equivalent variables are related by eight measurement equations. These are presented below. As we do not require measurement errors to overcome stochastic singularity, measurement errors are given by the 8×1 vector $M =$

$[0]_{8\times 1}$, in the measurement equations. These are,

$$\begin{bmatrix} \log \frac{gdp_t}{gdp_{t-1}} \\ \log \frac{cons_t}{cons_{t-1}} \\ \log \frac{invest_t}{invest_{t-1}} \\ \log \frac{hours_t}{hours_{t-1}} \\ \log \frac{wage_t}{wage_{t-1}} \\ \log infl_t \\ \log spread_t \\ \log \frac{sec_t}{sec_{t-1}} \end{bmatrix} = \begin{bmatrix} \log \frac{y_t}{y_{t-1}} \\ \log \frac{c_t}{c_{t-1}} \\ \log \frac{inv_t}{inv_{t-1}} \\ \log \frac{h_t}{h_{t-1}} \\ \log \frac{w_t}{w_{t-1}} \\ \pi_t \\ Sp_t - 1 \\ \log \frac{MS_t}{MS_{t-1}} \end{bmatrix} + M$$

I calibrate a subset of the parameters as in Table 7.[31] The loan length of T is set to 40 quarters to match the Baa Corporate Bond length of 10 years. All steady state ratios are set according to the observed sample average.[32] I set the household discount factor of β taking advantage of the steady state Euler Equation's implication that the rate of time preference is equal to the steady state real short-term real interest rate. The empirical analog of which is the sample average for the real Federal Funds Rate. This implies an annualized real risk-free rate of approximately 1.45%. The corporate tax rate is set to the prevailing rate over the majority of the sample. I set the bank exit rate to a number close to 1, of 0.995.[33] Likewise, the reserve requirement is set to the statutory limit. All other parameters are calibrated in the typical range seen in the literature.

In order to initialize the RWMH algorithm such that it begins both to sample from a reasonable area of the parameter space, and has some initial curvature from which to accept or reject the early draws, I mostly specify non-uniform prior distributions on each parameter being estimated. In this context, a reasonable area of the parameter space is both one which satisfies the Blanchard-Kahn conditions for dynamic model stability and where the parameters have plausible structural interpretations. Broadly, I specify a beta distribution for parameters that are naturally bounded

[31]Note that I have already implicitly calibrated the coefficient of relative risk aversion to unity, as is standard in the literature.

[32]I target the steady state labor dis-utility term, the steady state proportion of capital that can be held as collateral, the share of bank assets that can be diverted, and the tax on bankers' net worth to achieve the steady state targets for labor hours, debt to GDP, bank assets to GDP, and the leverage ratio respectively.

[33]The assumption of some positive bank exit is made for technical reasons to ensure stationarity rather than from a microfoundation. Estimation of that parameter suggests its value is close to 1, however, including such a parameter in estimation leads to other technical issues, so that approach is avoided.

Table 2. Calibrated Parameters and Steady State Targets

Parameter	Description	Target
α	Capital Share of Income	0.38
β	Discount Factor	0.9965
σ	Relative Risk Aversion Coefficient	1
δ	Depreciation Rate	0.025
h	Steady State Hours	0.3
g/y	Government Spending to GDP Ratio	0.1928
S/y	Financial Services Consumption to GDP Ratio	0.0479
L/y	Debt to GDP Ratio	0.3716
G^y/y	Bank Assets to GDP Ratio	$1.3\frac{L}{y}$
T	Loan Length	40
lev	Leverage Ratio	13.75
req	Reserve Requirement	0.1
θ_b	Bank Exit Rate	0.995
τ_d	Corporate Tax Rate	0.35

in the $(0, 1)$ interval. All shock standard deviations are given inverse gamma priors, following the standard in the literature. Otherwise, I implement any of gamma, inverse gamma, normal, or uniform priors as appropriate to the particular parameter.

6. Results

The model is solved via first order approximation methods after linearizing each model equation around the steady state. This section presents the results of Bayesian estimation. I first present the posterior means and 95% Highest Posterior Density (HPD) confidence intervals for each parameter. We then analyze the impulse response functions for each of the economically significant shocks.[34] Having established the dynamic functioning of the model at the posterior mean, we direct attention to the implications for our understanding of US business cycles. A simulation of the eight shocks is presented, as well as an analysis of the variance decomposition of those shocks for the estimated

[34] The technology, investment efficiency, monetary policy, and collateral quality responses are shown in Section 6.2. The remaining four impulse responses are included for reference in the appendix.

variables. Lastly, we assess the model's fit against a subset of empirical and model moments.

6.1 Parameter Estimates

The parameter estimates broadly reflect that the model relies less on the nominal and real frictions introduced in the canonical works, suggesting an important role for the enhanced propagation mechanisms introduced in this paper in explaining the dynamics of the US economy in response to exogenous shocks.

The distinguishing parameter estimates are the inverse Frisch elasticity, income effect preference parameter, and the habit formation term. The inverse Frisch elasticity is estimated at close to 0.1. This is well away from the prior mean motivated by past macro studies of two, and considerably less than the higher range estimate of five seen in Schmitt-Grohé and Uribe (2012). The consequence is a high elasticity of labor hours to wage changes which accords well with recent new interpretations of labor supply made in Christiano, Trabandt and Walentin (2011). They argue that the majority of fluctuations in total labor hours are the result of decisions from workers at the margin, who have very high elasticity to wage changes. We will see that despite this strong response to wage changes in labor hours, that actually hours are driven more so by large wealth effects of firm dividend policy.

We can see from the estimate of the steady state income effect γ, that such wealth effects are not arising from income effects in the labor supply decision. This parameter is estimated to be very close to zero, the polar case of GHH preferences, where there is no income effect at all absent habit formation. Prior evidence on this parameter is obtained in Schmitt-Grohé and Uribe (2012) and Dey (2014). The former, in a context with news shocks finds this parameter to also be close to zero, while the latter, in a New Keynesian context estimates it at closer to one.

That wealth effects barely arise from labor supply decisions leaves only lump sum tax changes and shifts in dividend payouts by firms as possible sources of such effects. Foreshadowing the results in Section 6.2, we will see that as firms shift to a more debt-laden capital structure, they will pay large special dividends. This occurs in response to positive technology and financial shocks. Moreover, longer term disturbances to dividend payouts will be experienced when the investment environment changes, with a negative shift in policy occurring in response to negative investment efficiency shocks.

Table 3. Bayesian Parameter Estimates

		Prior	Post. Mean	Post. SD	95% HPD Int.	
Deep Structural Parameters:						
ε^{-1}	Inverse Frisch Elasticity	$\mathcal{G}(2, 0.75)$	0.118	0.044	0.04	0.201
γ	Income Effect in Steady State	$\mathcal{B}(0.5, 0.15)$	0.028	0.007	0.015	0.041
H	Habit Formation	$\mathcal{B}(0.5, 0.15)$	0.313	0.04	0.235	0.391
ψ	Utilization Cost Elasticity	$\mathcal{B}(0.5, 0.15)$	0.923	0.03	0.863	0.976
ϱ	Investment Adjustment Elasticity	$\mathcal{N}(4, 1.5)$	5.402	0.722	3.987	6.797
ϕ_p^{-1}	Inv. Rotemberg Price Adjustment	$\mathcal{IG}(0.1, 0.3)$	0.02	0.003	0.015	0.025
ϕ_w	Calvo Wage	$\mathcal{B}(0.66, 0.15)$	0.594	0.03	0.534	0.652
λ_p	SS Price Markup	$\mathcal{G}(0.2, 0.05)$	0.294	0.041	0.216	0.375
λ_w	SS Wage Markup	$\mathcal{G}(0.2, 0.05)$	0.137	0.036	0.07	0.208
ι_p	Price Indexation	$\mathcal{B}(0.5, 0.15)$	0.172	0.067	0.053	0.304
ι_w	Wage Indexation	$\mathcal{B}(0.5, 0.15)$	0.417	0.142	0.149	0.69
$\nu_d{}^a$	Equity Adjustment Elasticity	$\mathcal{B}(0.15, 0.1)$	0.1	0.067	0.019	0.23
κ_L	Loan Search Cost	$\mathcal{N}(5, 0.5)$	6.99	0.392	6.233	7.758
ϑ	Nash Bargaining Weight	$\mathcal{B}(0.5, 0.15)$	0.027	0.004	0.019	0.035
r_i	Taylor Rule Smoothing	$\mathcal{B}(0.78, 0.1)$	0.541	0.049	0.442	0.631
r_π	Taylor Rule Inflation Weight	$\mathcal{G}(1.7, 0.5)$	1.93	0.543	1.059	2.99
r_y	Taylor Rule Output Gap Weight	$\mathcal{G}(0.125, 0.1)$	1.68	0.188	1.339	1.998
r_s	Taylor Rule Credit Spread Weight	$\mathcal{B}(0.5, 0.15)$	0.532	0.147	0.251	0.814

[*] Distribution Key: $\mathcal{B}$—Beta, $\mathcal{G}$—Gamma, $\mathcal{IG}$—Inverse Gamma, $\mathcal{N}$—Normal, $\mathcal{U}$—Uniform. 3 million draws from the posterior distribution per Metropolis-Hastings chain. I use 4 chains in the estimation with an average acceptance rate of approximately 0.275. I estimate a transformation of κ_d such that $\kappa_d = \frac{\nu_d}{1-\nu_d}$. As $\nu_d \to 1 \Rightarrow \kappa_d \to \infty$, and as $\nu_d \to 0 \Rightarrow \kappa_d \to 0$.

Table 3. Bayesian Parameter Estimates (continued)

		Prior	Post. Mean	Post. SD	95% HPD Int.	
Autoregressive Diagonal Parameters:						
ρ_A	Technology	$\mathcal{B}(0.6, 0.2)$	0.943	0.016	0.911	0.974
ρ_z	Collateral Quality	$\mathcal{B}(0.6, 0.2)$	0.983	0.005	0.973	0.993
ρ_i	Monetary Policy	$\mathcal{B}(0.5, 0.2)$	0.731	0.061	0.61	0.848
ρ_I	Investment Efficiency	$\mathcal{B}(0.6, 0.2)$	0.639	0.044	0.554	0.725
ρ_g	Government Spending	$\mathcal{B}(0.6, 0.2)$	0.855	0.025	0.806	0.904
ρ_Ψ	Wage Markup	$\mathcal{B}(0.5, 0.2)$	0.639	0.041	0.558	0.72
ρ_γ	Income Effect	$\mathcal{B}(0.5, 0.2)$	0.375	0.032	0.312	0.437
ρ_ϑ	Bank Bargaining	$\mathcal{B}(0.5, 0.2)$	0.787	0.02	0.747	0.826
Autoregressive Off-Diagonal Parameters:						
ρ_{gA}	Gov/Tech Spillover	$\mathcal{U}(-1, 1)$	-0.061	0.018	-0.097	-0.027
ρ_{zA}	CQ/Tech Spillover	$\mathcal{U}(-1, 1)$	-0.002	0.007	-0.016	0.012
Shock Standard Deviations:						
σ_A	Technology	$\mathcal{IG}(0.005, 0.15)$	0.0047	0.0003	0.0041	0.0053
σ_z	Collateral Quality	$\mathcal{IG}(0.005, 0.15)$	0.0027	0.0002	0.0023	0.0031
σ_i	St. Dev. Monetary Policy	$\mathcal{IG}(0.005, 0.15)$	0.0056	0.0011	0.0036	0.0078
σ_I	St. Dev. Inv. Efficiency	$\mathcal{IG}(0.005, 0.15)$	0.0486	0.0088	0.0319	0.066
σ_g	St. Dev. Gov. Spending	$\mathcal{IG}(0.005, 0.15)$	0.0032	0.0002	0.0028	0.0036
σ_Ψ	St. Dev. Wage Markup	$\mathcal{IG}(0.005, 0.15)$	0.0378	0.0044	0.0292	0.0465
σ_γ	St. Dev. Wealth Effect	$\mathcal{IG}(0.005, 0.15)$	0.0521	0.0071	0.0389	0.0665
σ_ϑ	St. Dev. Bank Bargaining	$\mathcal{IG}(0.005, 0.15)$	1.6611	0.2537	1.1906	2.1719

* Distribution Key: $\mathcal{B}$—Beta, $\mathcal{G}$—Gamma, $\mathcal{IG}$—Inverse Gamma, $\mathcal{N}$—Normal, $\mathcal{U}$—Uniform. 3 million draws from the posterior distribution per Metropolis-Hastings chain. I use 4 chains in the estimation with an average acceptance rate of approximately 0.275. I estimate a transformation of κ_d such that $\kappa_d = \frac{\nu_d}{1-\nu_d}$. As $\nu_d \to 1 \Rightarrow \kappa_d \to \infty$, and as $\nu_d \to 0 \Rightarrow \kappa_d \to 0$.

In particular, that progressive shift in dividend policy in response to investment efficiency shocks supports staggered consumption growth over time. This version of 'hump-shaped' consumption changes is normally induced in the literature by high levels of habit formation in consumption preferences. Again we see that the estimate of the habit formation parameter is lower than the prior mean, a prior that is motivated by the high habit formation levels normally chosen or estimated by existing studies.

The adjustment of consumption on the wealth dimension rather via habit formation is supported by the empirical findings in Chetty and Szeidl (2016). They suggest that 'consumption commitments' induced by fixed costs of housing and similar assets that are difficult to adjust are the mechanism that cause the typically observed consumption adjustment pattern. The mechanism here is similar, but rather than fixed costs of assets affecting consumption, it is the household's inability to impose a fixed dividend payout or capital structure on the firms they own that makes their consumption susceptible to wealth changes.

Considering the remaining parameters. We see that the degree of price and wage indexation is lower than typically found in the literature, which is a strength of the paper given that such nominal frictions have been viewed to be of dubious micro-based merit. Evidence that indexation is not a feature of the US economy in the post-Volcker period can be found in Keen and Koenig (2018).

The price and wage markups are in the typical range seen in the literature. Below the higher end of the range seen in Dey (2014) and above the lower end seen in Jermann and Quadrini (2012). Additionally, the Rotemberg cost of price adjustment is substantially lower than usually seen, implying a posterior mean of $\phi_p \approx 50$, where typically results greater than 100 are needed to match empirical studies on the frequency of price adjustment reflected in the Calvo equivalent parameter. The Calvo wage adjustment parameter implies an expected duration of a wage bargain of between 2 to 3 quarters, less than the greater than a year period usually estimated. Combined, these two estimates suggest that nominal pricing frictions have reduced in the US economy, and also that the model is less reliant on pricing frictions to generate responses to monetary policy shocks that accord with the VAR evidence.

The utilization adjustment elasticity is in line with estimates of it seen in Gertler, Sala and Trigari (2008) and Jermann and Quadrini (2012), suggesting that utilization adjustment continues to be a substantial feature of firm input decisions despite the extension of the sample and change in propagation mechanism. The investment adjustment elasticity is in the midpoint of the typically estimated range.

Equity adjustment costs are estimated to be smaller than is the case in Jermann and Quadrini (2012), however, these are still found to be positive. This lower estimate coupled with a role for costly debt adjustment suggests that the debt adjustment channel is relatively more so the cause of capital structure non-neutrality than costs to equity adjustment.

In addition, the estimates suggest that the credit spread observed in the US economy is the result of intermediation costs in the lender/borrower relationship as opposed to the alternate explanation in the model, which would be a lack of bank competition. We can see this in the bargaining weight parameter estimated being close to zero, thus placing little weight on the bank's surplus in setting the interest rate on longer term debt.

Lastly, the weights in the Taylor Rule for interest rate smoothing, inflation, the output gap, and the credit spread are estimated. As noted in Canova and Sala (2009), these parameters suffer from weak identification in New Keynesian estimations, and that is similarly a weakness in this study. Additionally, the zero lower bound period which occurred for a subset of the sample period potentially biases parameter estimates in the Taylor Rule. In a model such as Gust et al. (2017), if the assumption that the monetary authority is constrained in its ability to implement monetary policy is true, then failing to account for it introduces substantial bias into the parameter estimates. Alternatively, if it is the case that the monetary authority is able to continue to implement its objectives via unconventional type monetary policies, then the zero lower bound period is of less consequence in terms of biasing the parameter estimates.

The estimates in my model suggest a level of inflation hawkishness comparable to that found in prior studies, originating with Clarida, Galí and Gertler (2000). However, I find substantially less smoothing of interest rates by the Federal Reserve, a higher weight on the output gap, and evidence that the Federal Reserve loosens monetary policy in response to widening credit spreads. This latter result supports recent VAR evidence from Caldara and Herbst (2019), and suggests that a more empirically truthful representation of the Taylor Rule would follow the Cúrdia and Woodford (2016) specification, as in this paper.

For the exogenous shock processes, the parameters broadly accord with prior studies. Differences are observed in the persistence of monetary policy shocks, with this being estimated much higher than the usual level. Alternatively, the newly inserted shocks to the income effect parameter and the Nash bargaining weight are estimated with little and mid-range persistence respectively. No persistence parameters are estimated near the prior bounds, which is a positive sign that the model does not exhibit any unit root type behavior.

The off-diagonal autoregressive parameters are included because both have been estimated in the past as being non-zero. The government/technology spillover is quite small and negative, which is more in line with the result in Herbst and Schorfheide (2015), rather than the large values seen in Smets and Wouters (2007). This result suggests a small role for automatic stabilizers in fiscal policy. The off-diagonal term between collateral quality shocks and technology is estimated at zero. The mild negative spillover found in Jermann and Quadrini (2012) does not seem to be present under this specification.

6.2 Impulse Response Functions

Bayesian impulse response functions evaluated at the posterior mean and the 95% Highest Posterior Density (HPD) intervals are presented in Figures 5 to 8 and isolate how each aggregate of interest in the model responds to a shock.[35] The impulse response graphs capture the percentage deviation in variable x from its steady state in response to a one standard deviation shock to variable y.[36] The assumption is that the shock occurs at time 0, with no further shocks at future periods and no other shocks at any other time. We therefore can trace the response of each presented variable to each isolated shock over time.

The analysis that follows is used to explain the theoretical effects of each individual shock on the variables of interest in the model and to compare and contrast which shocks cause the largest disturbances to the economic aggregates. We first consider the responses to a technology shock. This is the case of a positive innovation that improves the productivity of the existing capital stock and labor inputs. For the same amount of input, firms can now produce a greater amount of output. This shock represents an intermediate case between financial and investment efficiency

[35] I do not present the impulse responses for government spending, income effect, wage markup, and interest rate bargaining shocks as these are less economically interesting in that they either have small effects or only have significant effects when they are very large. They are available in the appendix.

[36] See Section 6.1 for the estimated standard deviations of each shock process.

shocks, where the responses strongly bias towards an increase in the variable directly affected by the shock.

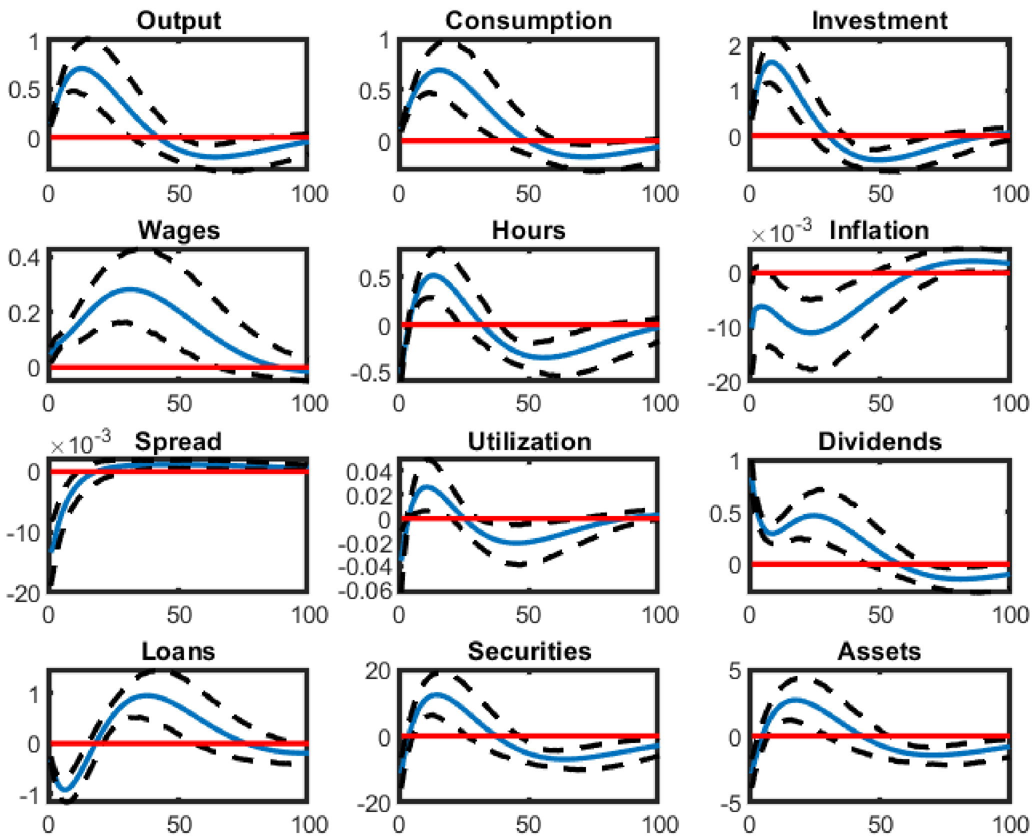


Figure 5. Impulse Responses—Technology Shocks

In response to a technology shock, firms lower their input usage by reducing capital utilization and labor demand. Output is still able to mildly rise due to increased productivity of the inputs. Instead, the firm moves to increase the debt proportion of its capital structure by adding loans. In the short-run, loan matching is weak due to the rise in consumption that reduces bank assets. That consumption rise is caused by two competing forces. Firstly, investment increases which puts upward pressure on the real interest rate, inducing less consumption. However, to support their desire for more debt, firms reduce equity by paying a special dividend. This increases household wealth and raises consumption. The wealth effect of the dividend is larger than the savings biasing effect of the real interest rate, and so consumption rises on impact and bank assets from deposits are reduced.

Over time, the firm tapers these special dividends and and households begin to work more to replace that lost income and support their habit induced higher consumption levels. More of this income begins to be saved, which flows into increased loans to firms. As the new loan position is reached, the firm begins to taper investment rapidly and again starts to reduce equity by issuing a second wave of special dividends.

Noticeably, households' labor supply is quite unresponsive to wage changes in the short run. Wages experience a slight rise on impact but labor hours are reduced, and hours grow most rapidly during the fastest dividend tapering period seen early in the horizon. The other main feature we observe is the operation of the loan market. Rather than adjust to the optimal higher debt level immediately, the process is slow, and we see increased marketable securities flowing into increased loans over time.

Investment efficiency shocks are innovations to the productivity of new capital in the form of investment, as opposed to the entire capital stock in the case of technology shocks. Firms strongly boost investment here, and increase utilization. This causes a rise in the price of investment, the real risk free rate, which all else equal induces savings behavior from households. However, we again see the wealth effect of firm dividend policy at play, as reductions in dividends to finance increased investment reduce the wealth of households. Again, that wealth effect of dividend changes is strong, inducing an increase in labor hours from households. Over time, the firm is able to reduce its reliance on equity to fund the investment projects, and instead starts to increase debt as higher deposits from household savings flow into loans to firms.

Investment is observed to be more volatile than output in the data. We see that such volatility is much more so a feature of the investment response relative to output for the investment efficiency shock compared to technology. This particular feature is the main driving force that explains the prominence of investment efficiency shocks in explaining observed investment growth.

The key novel feature present in the impulse responses to monetary policy shocks is the presence of the Bernanke (1993) credit channel. A monetary policy shock is a tightening in monetary policy that occurs unilaterally, and in violation of the monetary policy rule. Such an event is contractionary, but more so than in standard models due to the credit channel, which we can see in operation in the impulse response for firm loans. The unjustified monetary tightening widens

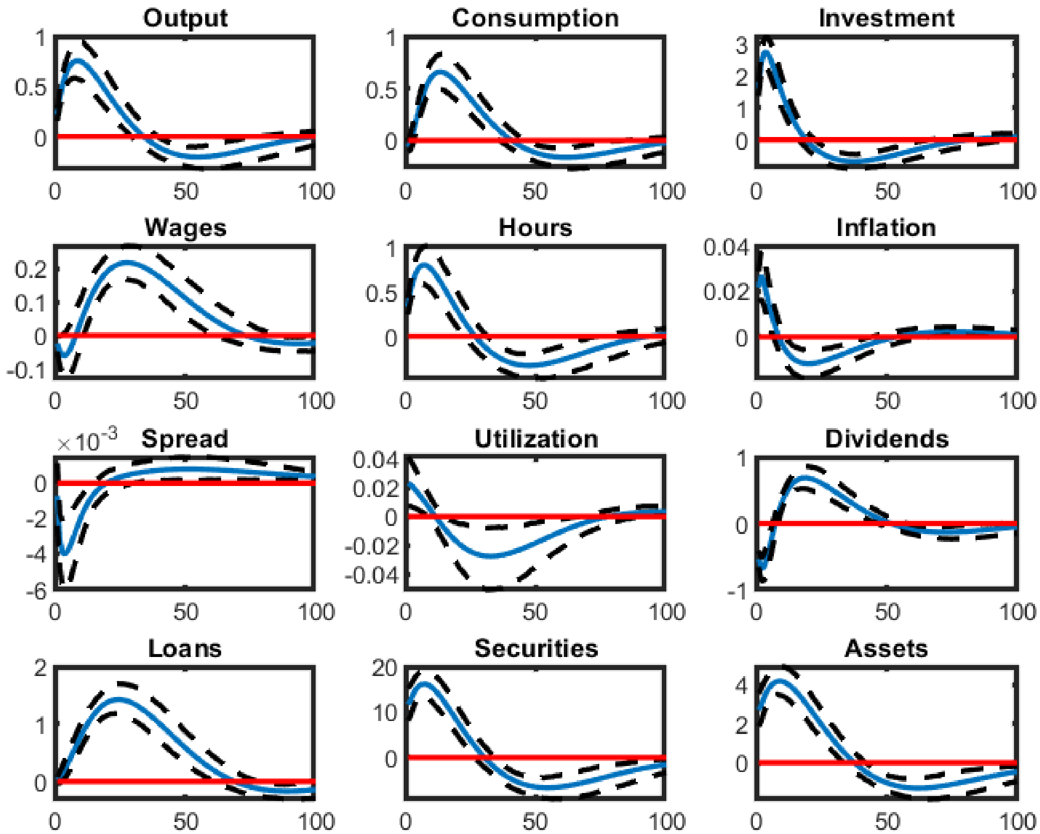


Figure 6. Impulse Responses—Investment Efficiency Shocks

the credit spread, and thus increases the burden of existing debt. This is because the interest component of repayments is now relatively larger, and so a larger repayment is needed to maintain the pace of loan repayments. This ensuing tightening of the borrowing constraint thus amplifies the drop in output.

Also, we can see that increased rates result in more deposits into banks, as households bias their behavior more towards saving. This supports loan matching in the short run, further depressing labor demand, utilization and investment in the short run. Taken together, these amplified monetary policy effects explain the reduced reliance of the model on nominal frictions. These were seen in the prior section, with lower than normally estimated levels of indexation and nominal stickiness.

Lastly, I present the impulse response for collateral quality shocks, which were shown by Jermann and Quadrini (2012) to be an important driver of US business cycles. In their model, firms

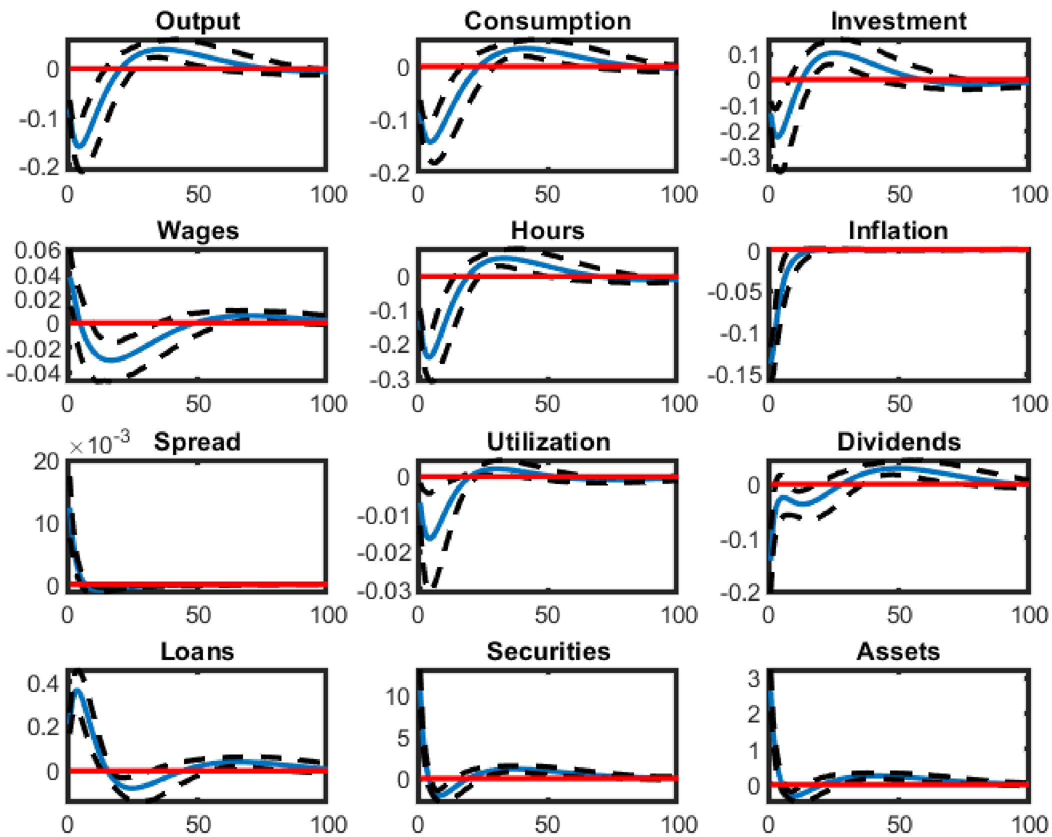


Figure 7. Impulse Responses—Monetary Policy Shocks

can freely adjust one period debt. In response to a shock of this type, firms are able to increase their working capital loans and boost output. In so doing, both investment and hours are strongly procyclical. In this model, the response differs in economically significant ways.

The clearest distinction is in the response of investment. Rather than being pro-cyclical, firms firstly maintain and then begin to reduce investment in response to a collateral quality shock. Instead, firms strongly pursue a more debt-laden capital structure. Investment begins to increase in the medium term as the shock dissipates and the firm needs to repair its collateral position as depreciation reduces the capital stock.

Collateral quality shocks strongly improve the wealth of households in the short run. Firms pay a large special dividend as they reduce equity financing. The strength of the ensuing wealth effect is quite pronounced and noticeable in the consumption response. Despite the presence of a small

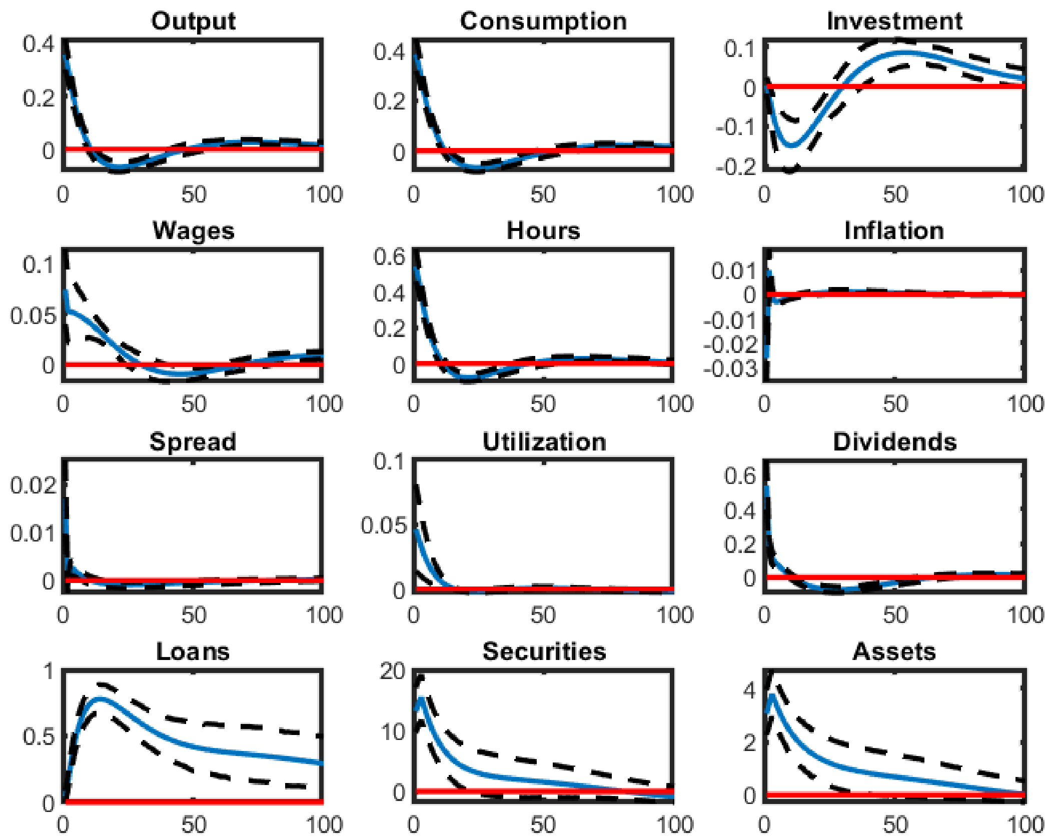


Figure 8. Impulse Responses—Collateral Quality Shocks

level of habit formation in preferences, households do not grow consumption over time and instead a large increase on impact is seen.

6.3 Estimated Shocks

Figure 9 presents the simulated shock series implied by the model at the posterior mean. These can be exploited to understand the causes of each recession in the sample. The 1991 recession is the result of falls in productivity with the recovery being aided by positive government spending increases and looser monetary policy. Financial shocks are not a feature here, with collateral quality remaining high, and bank bargaining changes being minimal. The slow recovery in the labor market is explained by a series of negative shifts in investment efficiency in the recovery phase.

The 2001 recession is similar, with sustained falls in productivity in the lead up to the recession.

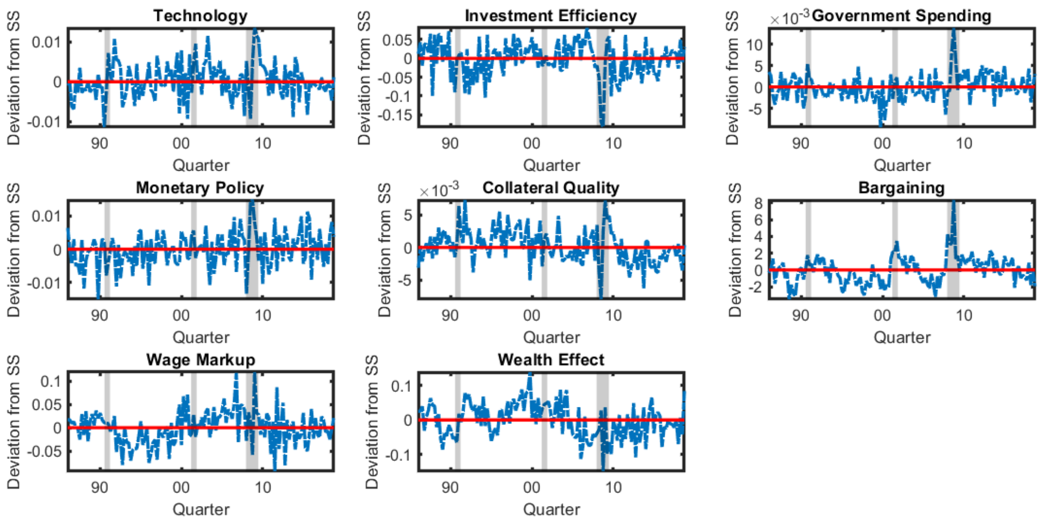


Figure 9. Estimated Shocks

In contrast to 1991, monetary policy was overly tight in this period and government spending weak. Again, financial factors appear to be limited, with little negative movement in collateral quality. Although we do see a slight increase in bank bargaining power, accounting for the widening credit spread.

Where financial factors do induce a recession is in the Great Recession. The time period preceding it is similar to the experiences in 91 and 01. Similar sized productivity contractions occur, however, a drop in collateral quality ensues. This is identified around the time of the Bear Stearns collapse. Notably, the fall in collateral quality is not persistent as shown in the sharp rise from the trough in early 2008. Productivity also quickly recovers. Instead, a large fall in investment efficiency occurs in late 2008. This corresponds to the time period where Lehmann Brothers collapses and where the Troubled Asset Relief Program (TARP) is instituted. The latter captured by the large positive government spending shock around that time. Further positive government spending shifts late in the recession are also captured in that series. These correspond to the early Obama-era stimulus package that included tax cuts and is popularly identified by the 'Cash for Clunkers' program.

The bank bargaining series captures the two main banking collapses. The first being Bear Stearns and the second, larger shift being Lehmann Brothers. That the second upward shift in bank bargaining power is also accompanied by an upward shift in collateral quality is strongly indicative of banks having high lending capacity at that time. Coupled with the sharp declines in investment efficiency, the simulations suggest that weak lending activity at the time was a result of subdued demand for loans from the private sector. The transformation of investment to capital was depressed, and credit spreads were high, suggesting a low payoff to firms from taking on more debt, and an increased burden from debt already in place.

6.4 Variance Decompositions

There is considerable debate in the literature as to which shocks are most important in explaining the US business cycle. The original Real Business Cycle literature emphasizes the importance of technology shocks. Papers derived from GHH suggest that investment efficiency shocks play the most important role. Such papers include Gertler, Sala and Trigari (2008) and Justiniano, Primiceri and Tambalotti (2011). In contrast, more recent work in Jermann and Quadrini (2012) and Christiano, Motto and Rostagno (2014), emphasize the role of various types of financial shocks. In this paper, those are captured by collateral quality shocks. I analyze the relative contribution of shocks with the use of a variance decomposition, with the results presented in Table 4.

The results of that variance decomposition indicate that the additional long-term loan channels introduced into the New Keynesian model have resulted in collateral quality shocks explaining less of the aggregate data in the long-run than was the case in prior work. Their role remains quite large in explaining short-run output, consumption and hours movements. Instead, traditional productivity and investment efficiency shocks are the main longer term drivers of US business cycles.

The traditional shocks struggle to account for the movements in the credit spread, where the bargaining weight shock rather than collateral quality shocks accounts for the majority of its variation. This suggests an avenue for further research that expands upon the causes of loan origination costs and further develops that propagation channel.

However, the traditional shocks account for the majority of the movement in marketable securities, suggesting that the internal asset re-allocations taking place in banks are an important channel through which business cycle variations are transmitted through the economy. This is the case both at short and long-term frequencies.

Table 4. Variance Decomposition (On Impact, Long Run)[a]

	Technology	Investment Efficiency	Monetary Policy	Collateral Quality	Others[b]
Impact Horizon:					
Output	8.1	20.8	3.2	57.9	9.4
Consumption	4	1.5	2.3	45.2	47[c]
Investment	6.3	76.4	0.6	0	16.1
Wages	0.2	0.1	0.2	0.8	98.8[d]
Hours	35.9	14.4	2.3	39.4	8
Inflation	0.3	1	40	1.6	57.2
Marketable Securities	11.5	12.2	10.3	15.3	50.8
Credit Spread	0.6	0	0.5	1.1	97.8[e]
Long-Run Horizon:					
Output	44	39.6	1.2	2.1	13
Consumption	49.6	34.4	1.1	2.8	12.1
Investment	34.8	52.5	0.5	0.5	11.8
Wages	36	17	0.3	0.7	46
Hours	30.6	41.1	2.2	4.1	46
Inflation	4.6	5.8	54.8	1	33.8
Marketable Securities	16.3	17.7	0.7	9	56.3
Credit Spread	1.2	0.2	0.4	0.6	98.9[e]

[a] Variance decomposition is calculated as the average of 10 000 draws from the posterior distribution. The units are percentages. Columns on the left are calculated with an impact horizon. Columns on the right are calculated over the entirety of the sample and thus capture the long run horizon.

[b] The 'Others' column is an aggregate of the remaining shocks in the model.

[c] The majority of impact consumption variance is explained by the government spending shock.

[d] The majority of the impact wages variance is explained by the wage markup shock.

[e] The majority of the variance at both horizons for the credit spread is explained by the shock to the Nash bargaining weight parameter.

Unsurprisingly, inflation movements are largely captured by deviations from the Taylor Rule, captured by monetary policy shocks. Long-run wage shifts are reflected in productivity changes brought on by both aggregate productivity shifts and from newly acquired investment. However, in the short-run, wage changes are almost entirely explained by wage markup and wealth effect shocks.

The decomposition results for short-run consumption changes are indicative of the modeling consequences of removing risk premium shocks. Instead in this model, government spending shocks account for the majority of short run shifts in consumption. The implication is thus that government spending shifts, at least of the nature experienced in the sample, have a strong crowding out effect on consumption.

6.5 Model Fit

Table 5. Model Fit Measures

	St. Deviation		First Order		Second Order	
Growth Variable	Model	Data	Model	Data	Model	Data
In-Sample:						
Output	0.67	0.57	0.55	0.35	0.41	0.31
Consumption	0.73	0.47	0.29	0.42	0.21	0.13
Investment	2.33	2.15	0.58	0.51	0.32	0.38
Labor Hours	1.11	0.69	0.28	0.68	0.18	0.61
Wages	1.05	0.84	0.18	-0.15	-0.06	0.1
Inflation	0.3	0.24	0.60	0.63	0.43	0.5
Credit Spread	0.26	0.4	0.73	0.94	0.57	0.84
Marketable Securities	38.83	2.03	0.03	0.29	-0.01	0.1
Out-of-Sample:						
Loans	0.94	1.11	0.81	0.67	0.65	0.43

* Model moments calculated at the posterior mean parameter set.

Table 5 presents the in-sample standard deviation and auto-correlation comparisons in the data and the model for each variable used in estimation. Additionally, I present the same analysis for the loans variable, which is an out-of-sample comparison. The model's fit is strongest in investment and the out-of-sample loans variable, while being reasonably poor for marketable securities. Such an outcome is suggestive that further modeling work is needed or that an additional shock in the banking or household savings channels are needed to facilitate matching its empirical movement. That such a shock is not present in the model suggests that the inclusion of marketable securities in estimation assists in estimating collateral quality shocks. This facilitates the strong out-of-sample fit for the loans variable.

The consumption standard deviation is over predicted by the model. This is likely a consequence of removing risk premium shocks, which typically capture a substantial part of the consumption movement. The other variable that is matched relatively poorly is labor hours, although this is usually addressed by including search and matching type frictions in the labor market. The model continues to match the output variations well, as established in prior literature.

The strong match in investment is noteworthy given the substantial deviations in shock propagation on the investment dimension that was introduced by the model's extensions. That these extensions do not tie labor hours and wages together closely causes deviations in the model predicted path of wages and hours. Further labor market work is therefore required.

7. Conclusion

The New Keynesian model was extended to facilitate long-term borrowing by firms, where the origination of loans is costly and involves a search process that slows the pace of origination. A failure to match from that search process endogenously creates excess bank funds which we termed marketable securities. Exploiting empirical variation in that series and the BAA ten year corporate bond/federal funds rate spread allowed us to distinguish between tight financial conditions and periods where firms face high costs to investment and loan acquisition with concurrently low rewards to investment.

The estimation performed revealed that both conditions were present in the Great Recession. The recession was induced by productivity contractions in much the same way as the two prior US recessionary episodes. However, the Great Recession then featured a financial tightening at first, followed by a fast upswing in financial conditions. That lending was weak in the latter part of the recession and recovery period was not caused by a lack of bank willingness to lend. Instead, weak loan demand conditions were created by high borrowing costs and lower returns to investment.

The model is able to generate the key co-movements in the aggregate variables of interest with a high inverse Frisch elasticity, with reduced reliance on habit formation, and with a barely present income effect on labor hours of wage changes. This results in wealth effects arising from firm capital structure decisions strongly influencing the consumption and labor responses of households. Firms favor investment increases when the return to investment is high, and debt increases in other periods of vibrant economic activity, particularly in strong financial times. This is the key departure from the usual propagation seen in existing models.

The results serve to guide future policy responses to recessions that feature a financial component. Central banks must concern themselves with not only providing liquid funds to banks, but also promoting demand side confidence to borrow those funds. A path to accomplishing this is in keeping credit spreads from widening while the fiscal authority focuses on creating investment opportunities. The model in this paper serves to assist policymakers in better distinguishing financial and real shocks and responding accordingly.

CHAPTER II

Investment on the Extensive Margin. Replacement, Maintenance, and the Role of Credit.

1. Introduction

A consensus has emerged that modern macroeconomic models need to be micro-founded in order to impart relevant policy advice with predictable effects. It is striking then that the leading DSGE models almost exclusively employ mechanisms whereby most meaningful economic adjustments at the micro-level occur on the intensive margin. It is largely accepted though that the majority of the shifts in many aggregates can be accounted for mainly from agents' decisions taken on the extensive margin. This is the case in investment, hours worked, and financial aggregates to name but a few. The debate now largely centers on the question of whether firm-level 'lumpiness' in decisions manifest significantly at the aggregate level or if general equilibrium effects act to collapse the aggregate outcomes to those predicted by models that solely feature intensive margin adjustment.

In this chapter, heterogenous firms simultaneously choose investment and financing policies along the extensive margin. Investment can take the form of replacement of existing capital as in Cooper, Haltiwanger and Power (1999)(henceforth, CHP), or maintenance, a less costly alternative to replacement which produces a lower productivity boost and preserves capital at the present vintage. The financing mechanism takes the form of revolving debt. The most commonly available form of which is credit card debt. Under this financing structure, firms can choose to carry a positive debt balance to the next period and incur relatively high interest payments.[1] Alternatively, they may elect to eliminate their outstanding balance and have the payment of in arrears interest waived.

Including these additional discrete choices enhances existing models of the firm on a number of dimensions and in a manner consistent with contributing to the wider debate named above. The question of whether firm level lumpiness of investment decisions affects investment aggregates

[1] Credit card interest rates often feature a spread over prime rates for short-term borrowing.

originates in the finding of spikes and periods of inaction along the investment dimension at the firm-level by Doms and Dunne (1998). With models of lumpy investment emerging in Caballero and Engel (1999) and CHP, and empirical validation of these in Cooper and Haltiwanger (2006). However, these papers suffer from aggregation issues largely arising from excessive clustering of replacement decisions by firms at the time of an aggregate state switch. The model in this chapter corrects two of the resulting micro-level issues arising from excessive clustering. The firms in this paper do not experience long periods of investment inactivity consistent with the Dutch firms analyzed in Letterie and Pfann (2007). Secondly, the age distribution of capital in this chapter is consistent with the plant-level evidence from US firms in CHP, featuring fat tails at the new and older ends of the capital age spectrum.

The addition of a maintenance decision in this paper in part assists in producing these desirable micro features and is justified by noting that maintenance is the most common investment expenditure carried out by plants in CHP. McGrattan and Schmitz (1999) also document the extensive amount of maintenance and repair that occurs in the Canadian economy.[2] Maintenance is performed in this paper disproportionately by credit constrained firms, is pro-cyclical, and occurs at the aggregate level in all states of the world. This acts to provide a baseline level of investment in the economy and acts to keep older capital stocks productive such that older vintages have an effective younger and more productive age.

While maintenance induced co-movements assist in improving the aggregate and micro-level implications of lumpy investment models, the problem of excessive clustering of investment decisions persists absent further structure. To motivate the financial frictions in this paper, it is firstly instructive to consider the arguments made to date in the broader debate on whether general equilibrium effects wash away lumpy micro decisions at the aggregate level. That case is made in Thomas (2002) and Khan and Thomas (2008).[3] Relevant for the modeling choices made in this paper is the retort in Bachmann and Ma (2016), where it is shown that the market clearing

[2] Albonico, Kalyvitis and Pappa (2014) extend a DSGE model to feature a maintenance dimension on the intensive margin and analyze the aggregate implications. Maintenance is pro-cyclical in their results. Saglam and Veliov (2008) also suggest that maintenance is pro-cyclical.

[3] An authoritative retort to this position is made in Bachmann, Caballero and Engel (2013), noting that lumpy investment models feature pro-cyclical aggregate investment volatility. Another counter-point is found in Winberry (2020) where observed real interest rate dynamics do not match what would be needed to counter the effects of micro-level lumpy investment. Fiori (2012), Gourio and Kashyap (2007), Jovanovic and Tse (2010), Reiter, Sveen and Weinke (2013), and Šustek (2011) all document aggregate features of the business cycle that can be explained by lumpy investment at the firm-level.

mechanism is relevant for the aggregate effects of lumpy micro decisions. This insight motivates the modeling of revolving debt in this chapter, where substantial evidence exists that prices (the interest rate on credit card debt) do not clear the market. For example, Ausubel (1991) documents the near invariance of credit card interest rates to the underlying cost of funds while Gathergood et al. (2019) reveal that credit cards are repaid in a manner consistent with equalizing the balance outstanding across open card accounts. Both results suggest an alternate market clearing mechanism other than prices. Furthermore, Herbst-Murphy (2012) reveals the predominance of credit cards as both payment and borrowing methods in the small business sector of the US economy.

The credit card friction in this model creates a disconnect between the time when expenditure is incurred that leads to an investment activity and the time when payment for that expenditure occurs. Firms can elect to pay their debt in full or in part, incurring interest at a higher than risk-free rate if the latter is chosen. The mechanism creates lumpiness not only in the investment decision but also in financing.[4] In the model, firms make replacement decisions when debt is low and perform maintenance when they are debt constrained.

That the model features both investment and financing dimensions places this paper in the literature that finds that financial factors affect investment decisions originating in Fazzari, Hubbard and Petersen (1988).[5] Moreover, the dynamics of debt and investment in this chapter capture the stylized features of these quantities at the micro-level outlined in Hennessy and Whited (2005). Specifically, firms experience periods over their life-cycle of being heavily leveraged and other times of having very low debt, with no particular target leverage ratio. Newly in this chapter, firms are more likely than not to be in a low debt state over their life-cycle. That is, fully repaying credit card debt does not lead immediately to replacement activity. Rather, any replacement following full repayment occurs with often substantial lag.

In light of recent policy developments in the US economy owing to the COVID-19 pandemic, I embed in the model an option to request loan forbearance on the part of firms. This manifests as the option to skip a required minimum payment for one period conditional on forbearance being granted. Under standard calibrations of aggregate productivity, the region where forbearance

[4]Bazdresch (2013) achieves concurrent lumpy investment and financing decisions by introducing non-convex adjustment costs on the financing margin.

[5]A contrary viewpoint is offered in Gomes (2001), supporting the canonical Hayashi (1982) model where Tobin's q is all that affects investment.

would be chosen by firms is never reached in simulations. Specifically, the model predicts that firms with relatively high debt and an older capital stock would request forbearance and that this option displaces the choice of bankruptcy rather than productive investment activity. Having this structure in place will allow this model to capture the most recent dynamics when data is released and when a distribution of idiosyncratic productive halts can be determined.

The remainder of this chapter is organized as follows. Section 2 depicts the modeling environment. Section 3 provides a discussion of the results, covering the solution method and resulting policy functions, the testable predictions arising from the model, and a discussion of the simulations with implications at the aggregate and firm-level. Section 4 concludes.

2. The Model

There are $N > 1$ firms operating in a competitive market who produce a non-durable good that is sold into the marketplace. At the beginning of each period indexed by t, the jth firm idiosyncratically draws a productivity realization $\varepsilon_{j,t} \in \mathcal{E}$ and simultaneously an upper limit for their credit card balances $\overline{cc}_{j,t} \in \overline{CC}$. The idiosyncratic exogenous state is thus an ordered pair $(\varepsilon, \overline{cc}) \in \mathcal{E} \times \overline{CC}$, and we shall apply the short-hand $s_{j,t}$ to this ordered pair. The distributions of the idiosyncratic states are independent and do not exhibit any persistence. The aggregate productivity state is given by A_t which follows a stationary AR(1) process.

2.1 Capital

The jth firm enters any time period with a capital stock $k_{j,t}$ and an amount outstanding of credit card debt $cc_{j,t}$, representing the endogenous state variables in the system. Firms must make two discrete choices in each period that independently dictate the evolution of each endogenous state to the next period. The capital choice allows for replacement $R_{j,t} \in \{0, 1\}$ or alternatively, maintenance $M_{j,t} \in \{0, 1\}$. I normalize the capital stock such that new capital is given by $k_{j,t} = 1$. Where neither maintenance nor replacement are chosen, capital depreciates at the rate $\delta \in (0, 1)$. Capital evolves according to,

$$k_{j,t+1} = \mathbb{1}(R_{j,t} = 1) + [1 - \mathbb{1}(R_{j,t} = 1)]\,[(1 - \delta)k_{j,t} + \mathbb{1}(M_{j,t} = 1)\delta k_{j,t}] \qquad \text{(II.1)}$$

All adjustment to capital occurs on the extensive margin with capital being restored to its as new state when replacement occurs, being preserved in its current condition when maintenance occurs, and depreciating when neither of the investment alternatives occur.

2.2 Credit Cards

A credit card is a contract comprised as a triplet $\langle r^c, \underline{z}, \overline{cc}_{j,t} \rangle$. Respectively an interest rate, a minimum payment expressed as a percentage of the outstanding balance with the condition $\bar{z} > r^c$, and a soft credit limit $\overline{cc}_{j,t}$ which is idiosyncratically changing over time. Firms take interest rates as given and for the purposes of this model, interest rates will remain fixed over time. The minimum payment percentage is also taken as given and fixed over time. The credit limit operates as a soft limit in the sense that a firm whose balance outstanding exceeds the limit is not required to de-leverage down to the limit. Instead they are simply not able to add any new spending to the existing balance should they be over the limit or are unable to incur spending that would lead to the limit being exceeded.

The discrete choice variable $B_{j,t} \in \{0, 1\}$ reflects the decision to fully repay the outstanding balance. Where this does not occur, the firm must either transmit the minimum repayment given by $\underline{z}cc_{j,t}$ or must apply for forbearance, where the firm is allowed to skip the current periodic minimum payment. Forbearance involves a search for a sympathetic banker with the probability of it being granted given by a well defined matching function in the probabilistic sense, that is an increasing function of both the aggregate and idiosyncratic productivity states. The probability of being granted forbearance is denoted $p^f_{j,t}$ and is given by,

$$p^f_{j,t} = \frac{\varepsilon_{j,t} A_t}{(\varepsilon^{\zeta}_{j,t} + A^{\zeta}_t)^{\frac{1}{\zeta}}}, \qquad 0 < \zeta < \bar{\zeta} \tag{II.2}$$

Where forbearance is not granted, the firm must satisfy the minimum payment requirement.[6] The decision to request forbearance is reflected in the discrete choice variable $F_{j,t} \in \{0, 1\}$.

Conditional on forbearance not being requested and no new spending occurring, the evolution

[6]I set ζ in a range such that for each pair of idiosyncratic and aggregate states, the probability of forbearance being granted is well defined.

of credit card balances in then given by,

$$cc_{j,t+1} = [1 - \mathbb{1}(B_{j,t} = 1)](1 + r^c - \underline{z})cc_{j,t} \tag{II.3}$$

where fully repaying the existing balance incurs no further interest charges and the balance next period is zero. In contrast, choosing the minimum payment route reduces the balance owing due to the minimum payment fully covering the interest component by assumption, however, further interest is charged that is to be paid the following period.[7]

2.3 Bankruptcy

In addition to the capital and credit card binary choices facing firms, we also allow for bankruptcy so that a firm that has faced a series of negative shocks preventing them from ever again attaining a new capital stock or repaying their debts in full, can instead choose a lottery which if successful, allows them to begin anew. Let the bankruptcy binary choice be denoted by the discrete choice variable $X_{j,t} \in \{0, 1\}$.

2.4 Value Functions

Table 6 summarizes each binary choice and its impact on the relevant endogenous state variable.

The objective of firms is to maximize their discounted stream of net income subject to the institutional features described above that govern the evolution of capital and debt balances. In addition, the ability to tap equity markets in the short-run is restricted, so that net income must be non-negative in each period. Income is generated via the sale of goods, $y_{j,t}$, which are produced with capital and labor $h_{j,t}$ inputs. Firm output is given by,

$$y_{j,t} = \varepsilon_{j,t} A_t \lambda^{\mathbb{1}(R_{j,t}=1)} f(k_{j,t}, h_{j,t}) \tag{II.4}$$

The production function f is homogenous of degree 1, is increasing in each input, and is concave in each input.[8] The parameter $\lambda \in (0, 1]$ is a productivity cost incurred when replacement is chosen

[7] An alternate scheme for minimum payment determination practiced in the marketplace is to pay a fixed percentage plus any finance charges. I avoid modeling this scheme to economize on computation.

[8] I use a Cobb-Douglas functional form such that $f(k, h) = k^\alpha h^{1-\alpha}$ with $0 < \alpha < 1$.

Table 6. Summary of Discrete Choices Available to Firms

Choice	Description	Affected State Variable	Next Period State
R	Replacement	k	1
M	Maintenance	k	k
$\neg R \cap \neg M$	Depreciation	k	$(1-\delta)k$
B	Full Repayment	cc	0
F	Forbearance Accepted	cc	$cc(1+r^c)$
	Forbearance Rejected	cc	$cc(1+r^c-\underline{z})$
$\neg B \cap \neg F$	Minimum Repayment	cc	$cc(1+r^c-\underline{z})$
X	Bankruptcy	k	k^{s} [a]
		cc	0

[a] k^s is a parameter that sets the age of capital for new entrants.

as an investment strategy. This reflects the contemporaneous slow-down in production observed by CHP when firms conduct replacement activities. The parameter can be interpreted as an integration cost of new investment.

Firms incur costs that can be assigned to a credit card and others that cannot be and must be paid with cash in the period in which they are incurred. We will restrict wage expenses $wh_{j,t}$ to be cash-only costs while expenses related to investment, replacement and maintenance costs, are eligible to be paid for with credit. The wage $w > 0$ is taken as given by the firm and for the purposes of this partial equilibrium analysis, shall remain fixed over time. The replacement cost $\phi > 0$ is fixed and is eligible to be added to the credit card in full or in part. Define the function v as an indexing function that returns the age of capital $k_{j,t}$. Then the maintenance cost ψ is a composition of functions over the age of the firm's capital. We add the regularity conditions that $\psi(v(1)) = \phi$, $\psi(.) > 0\ \forall k$, $\psi(v(k)) = \underline{\psi}$ when $v(k) > v(\overline{k})$, and $\psi'(v(k)) < 0$ when $v(k) \leq v(\overline{k})$. Respectively, maintenance cost is equal to replacement cost for new capital, the maintenance cost is positive for all capital levels, the maintenance cost is fixed whence capital exceeds the cut-off age $\overline{k}$, and the maintenance cost is decreasing in capital whence capital is not above the cut-off level.[9]

I assume that when the firm chooses replacement or maintenance and has debt capacity available, that the cost of that decision will be paid via an addition to credit card debt. Where credit

[9] I use the functional form $\psi(v(k)) = \max\{\nu, \phi - \eta v(k)\},\ \nu, \eta > 0$ so that the maintenance cost is linearly decreasing in the age of capital until the floor on the maintenance cost is reached.

card financing is not available (credit limit is already exceeded) or where paying the full cost on a credit card would lead to the limit being exceeded, I allow for the portion of the cost that would violate the credit limit to be paid in cash. Denote $Cash_{j,t} \geq 0$ as the cash component of any payment of costs towards replacement or maintenance.

I use $a = R, M, D$ to index the investment decision, respectively replacement, maintenance, or depreciation. Also, $b = B, P, F$ to index the debt decision, respectively full repayment, minimum repayment, or forbearance. The variable X continues to indicate bankruptcy. The value function V is defined over the endogenous and exogenous states, and is given by,

$$V(cc, k; s, A) = \max \left\{ \left[V^{a,b} \right], V^X \right\}, \qquad \forall\ (a, b) \tag{II.5}$$

The individual value functions reflecting each decision can then be defined in turn. We denote the discount factor $\beta \in (0, 1)$, which is taken as given by firms. Firstly, given any debt decision incurring repayment expenditures q_t^b, the individual value functions reflecting each investment decision are given by,

$$V^{R,b}(cc, k; s, A) = \max_{h>0, b\in\{B,P,F\}} \varepsilon A \lambda f(k, h) - wh - Cash^R - q^b \tag{II.6}$$

$$+ \beta \mathbb{E} V(cc' + \phi - Cash^R, 1; s', A')$$

$$V^{M,b}(cc, k; s, A) = \max_{h>0, b\in\{B,P,F\}} \varepsilon A f(k, h) - wh - Cash^M - q^b \tag{II.7}$$

$$+ \beta \mathbb{E} V(cc' + \psi - Cash^M, k; s', A')$$

$$V^{D,b}(cc, k; s, A) = \max_{h>0, b\in\{B,P,F\}} \varepsilon A f(k, h) - wh - q^b + \beta \mathbb{E} V(cc', (1 - \delta)k; s', A') \tag{II.8}$$

We can now take the investment decision as given and define the value functions for each debt repayment decision. Let q^c denote cash expenditures from the given investment decision and q^{cc} as credit card expenditures from the given investment decision. The value functions are then,

$$V^{a,B}(cc, k; s, A) = \max_{h>0, a\in\{R,M,D\}} \varepsilon A\lambda^{\mathbb{1}(a=R)}f(k,h) - wh - cc - q^c + \beta\mathbb{E}(q^{cc}, k'; s', A') \tag{II.9}$$

$$V^{a,P}(cc, k; s, A) = \max_{h>0, a\in\{R,M,D\}} \varepsilon A\lambda^{\mathbb{1}(a=R)}f(k,h) - wh - \underline{z}cc - q^c \tag{II.10}$$

$$+ \beta\mathbb{E}V(cc(1 + r^c - \underline{z}) + q^{cc}, k'; s', A')$$

$$V^{a,F}(cc, k; s, A) = p^f\Big[\max_{h>0, a\in\{R,M,D\}} \varepsilon A\lambda^{\mathbb{1}(a=R)}f(k,h) - wh - q^c \tag{II.11}$$

$$+ \beta\mathbb{E}V(cc(1 + r^c) + q^{cc}, k'; s', A')\Big] + (1 - p^f)V^{a,P}$$

The characterization of value functions is complete with a definition of the value of bankruptcy. When a firm chooses bankruptcy they will not produce, forego any existing debt obligations, and have their capital stock seized. There exists a non-trivial probability p^s of re-opening in the next period with the capital stock k^s and no debt. The value of bankruptcy is then given by,

$$V^X(cc, k; s, A) = p^s\beta\mathbb{E}V(0, k^s; s, A) \tag{II.12}$$

Firms observe the aggregate state and their own idiosyncratic state at the beginning of the period, noting that the idiosyncratic state is two-dimensional and comprised of a credit limit and productivity draw. Subsequently, each firm simultaneously chooses whether to enter bankruptcy and conditional on continuing as a going concern, will simultaneously choose its labor demand on the intensive margin, along with credit repayment and investment decisions on the extensive margin.

2.5 Aggregation

Investment aggregates are derived from changes in the capital law of motion summed over each firm. We define three relevant investment aggregates being capital consumption (Kc), maintenance (Km), and replacements (Kr). The three are aggregations of the value changes of capital resulting from the choices of firms in each period. Given these definitions, the expressions of these aggregates are,

$$Kc_t = \sum_{j=1}^{N_t} \mathbb{1}(R_{j,t} = 0 \cap M_{j,t} = 0)\delta k_{j,t} \tag{II.13}$$

$$Km_t = \sum_{j=1}^{N_t} \mathbb{1}(M_{j,t} = 1)\delta k_{j,t} \tag{II.14}$$

$$Kr_t = \sum_{j=1}^{N_t} \mathbb{1}(R_{j,t} = 1)(1 - k_{j,t}) \tag{II.15}$$

We can then define aggregate gross investment I_t as being the sum of maintenance and replacement expenditures $I_t = Km_t + Kr_t$ while a net investment concept would subtract depreciated capital.

The remaining relevant aggregated measures are output (Y), the aggregate credit outstanding (CC), and the aggregate repayments on credit cards (Rep). These are calculated by summing over the individual firms and given by,

$$Y_t = \sum_{j=1}^{N_t} y_{j,t} \tag{II.16}$$

$$CC_t = \sum_{j=1}^{N_t} cc_{j,t} \tag{II.17}$$

$$Rep_t = \sum_{j=1}^{N_t} \mathbb{1}(B_{j,t} = 1)cc_{j,t} \tag{II.18}$$

2.6 Calibration

The model is solved by way of Value Function Iteration. The state space is discretized into a four-dimensional grid.[10] Both of the idiosyncratic states follow i.i.d. uniform distributions and the aggregate state follows an AR(1) process. I use the method of Rouwenhorst (1995) to discretize the aggregate productivity processes. Parameters for the aggregate process are as in Chapter I.

The capital grid is discretized such that the ith entry is given by $k^i = (1-\delta)k^{i-1}$, with $k^1 = 1$. I assign two grid points to the credit limit variable, allowing one limit $\overline{cc}^1$ to exceed the fixed cost of investment while the other, $\overline{cc}^2$, is set to be less than the fixed cost of investment. In this way

[10]There are 250 grid points for credit card debt, 9 for idiosyncratic productivity, 2 for the credit limits, 5 for aggregate productivity, and 72 for capital. The capital grid allows for capital to depreciate to be 18 years old. This is in line with typical cut-offs used in the perpetual inventory method.

firms facing the lower limit are credit constrained. The credit card balance grid is evenly spaced with a lower bound of zero and an upper bound exceeding the highest possible credit limit, so as to give some mass in the algorithm to forbearance requests when holding high amounts of debt. Recall that that the credit limit is a soft limit that can be exceeded, with restrictions applying to new purchases being placed on credit cards if the limit is exceeded.

Table 7. Calibrated Parameters

Parameter	Description	Target	Source
α	Capital Share	0.352	Chapter I
β	Discount Factor	0.9864	SS Target
δ	Depreciation Rate	0.025	Chapter I
r^c	Real Credit Card Interest Rate	0.0303	SS Target
λ	Productivity loss with Replacement	0.75	CHP
ϕ	Fixed Cost of Replacement	0.2	CHP
η	Coefficient on Age in Maintenance Cost	0.02	See Description
ν	Minimum Maintenance Cost	0.01	See Description
$\underline{z}$	Minimum Repayment Percentage	0.09	Institutional
ζ	Forbearance Matching Function	1.9775	See Description
p^s	Probability of New Entry	0.125	See Description
ρ^A	Autoregressive Parameter for Agg. State	0.943	Chapter I
σ^A	Standard Deviation for Agg. State	0.016	Chapter I
$\overline{cc}^1$	Upper Credit Limit	1.25ϕ	See Description
$\overline{cc}^2$	Lower Credit Limit	0.5ϕ	See Description

The remaining parameters are set in Table 7. Where a parameter is estimated or calibrated in Chapter I, I use that value except in the case of the discount factor. As we are targeting a small business calibration here, we seek a target rate series that better captures small firms' internal rate of return requirements. I choose the Baa corporate bond series as such a rate. Other parameters are taken directly from CHP, where they are calibrated according to their plant-level data for US firms. I calculate the real credit card interest rate as a long-term average from the series of credit card interest rates across commercial banks from the Federal Reserve Board of Governors, deflated by the price index for Gross Value Added in GDP by Nonfarm Business.[11] I similarly deflate the series of Baa Corporate Bond rates. Taking the long-run average over the same sample period as the credit card interest rate series yields the long-run estimate of the internal rate of return. This implicitly sets the discount factor, β.

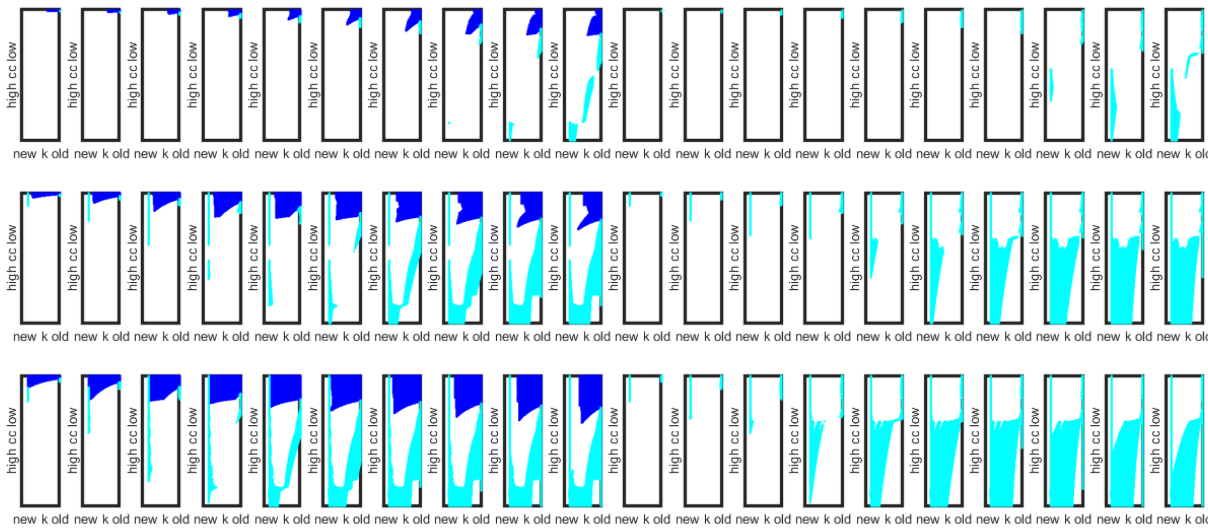
Maintenance costs are calibrated such that they decline linearly and a constant maintenance cost in age is reached when capital has an effective age of 2.5 years. The minimum repayment percentage on credit card debt is based on institutional features of credit cards. Minimum payments typically range from 2-4% of the outstanding balance per month. I choose the midpoint in this range and convert that monthly percentage to a quarterly one. The forbearance matching function parameter is set such that a firm operating with the highest idiosyncratic and aggregate productivity has a probability of being granted forbearance of 1. Lastly, I set the upper credit limit such that the cost of replacement would represent 80% of the credit limit, while the lower bound is set such that incurring the maintenance expense when capital is 2 years old would represent 40% of the credit limit.

3. Results

3.1 Policy Functions

The policy functions take the form of binary choices for each option available along the investment and credit card repayment dimensions. We denote a policy function over the decision Z as $Z(cc, k; s, A)$. Figure 10 shows the results for investment decisions. Decisions are made in non-monotonic ways and the credit constraint causes substantive switches in the magnitude of investment. Figure 11 show the results for the repayment decision. A series of remarks are made to describe the results below.

[11] The source is the G.19 Consumer Credit release from the Federal Reserve Board of Governors—Commercial Bank Interest Rate on Credit Card Plans, All Accounts. The sample starts in November 1995.



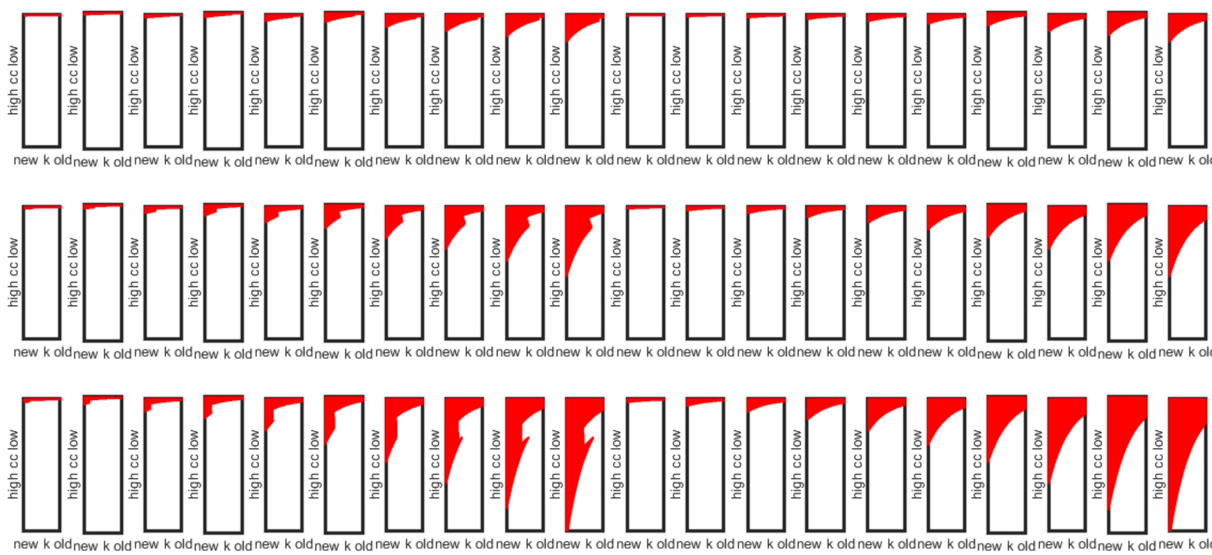
* Blue—Replacement, Cyan—Maintenance. Row 1—Lowest Aggregate Productivity, Row 2—Median Aggregate Productivity, Row 3—Highest Aggregate Productivity. Idiosyncratic Productivity is increasing in the columns in blocks of 10. The first 10 columns reflect high credit limits. The second 10 reflect low credit limits. The x-axis displays the capital grid from newest to oldest. The y-axis displays the credit card grid from highest debt to lowest.

Figure 10. Investment Policy Function

Remark. Replacement decisions skew towards being more likely as capital is older, credit limits are high, the existing credit outstanding is low, and the aggregate productivity state is higher.

The answer to the question of when firms replace capital is a complex one in this model. It is quite clear that this type of investment is more likely as aggregate productivity increases and as capital ages, however, new and disparate dynamics ensue at the idiosyncratic level. Firms do not engage in replacement behavior when they are credit constrained. Be that from low credit limits or high debt outstanding. In so doing, they display a clear preference for funding with the debt instrument available to them in this model. This is due to firms being risk-neutral and impatient. The result is that firms would rather take on the risk that a negative future shock prevents their full credit card repayment in exchange for the decreased expected cost in present value terms of investment associated with debt rather than cash payment. This outcome is preferred to the cost certainty associated with immediate cash payment.

It is also worth noting that debt in this model is unsecured and so additional investment in capital does not exhibit the dual benefits of additional collateral and greater productive capacity. Consequently, a negative credit shock in this model acts to suppress investment behavior on the replacement margin. Firms simply wait for better credit conditions to invest. This is in contrast to the results in Chapter I where a negative credit shock induces greater investment. There capital acts as collateral and so when borrowing capacity is exogenously reduced, firms reduce the impact of that shock by endogenously boosting their borrowing capacity by increasing investment. That motivation is absent in an unsecured credit setting as is the case here.



* Red—Full Repayment. Row 1—Lowest Aggregate Productivity, Row 2—Median Aggregate Productivity, Row 3—Highest Aggregate Productivity. Idiosyncratic Productivity is increasing in the columns in blocks of 10. The first 10 columns reflect high credit limits. The second 10 reflect low credit limits. The x-axis displays the capital grid from newest to oldest. The y-axis displays the credit card grid from highest debt to lowest.

Figure 11. Repayment Policy Function

Remark. Full repayment of credit card debt is less likely as debt increases, and more likely as idiosyncratic and aggregate productivity rises. The effects of capital age and credit limits are non-monotonic.

We see that as aggregate productivity and idiosyncratic productivity rise, an increasing number of grid points on the debt dimension are able to be fully repaid. Full repayment becomes less likely as capital ages. This is particularly so for lower levels of debt where the repayment decision comes into tension with the replacement decision. All else equal, debts are more likely to be repaid with older capital when credit limits are low. Firms have less of an urge to replace capital with low credit limits and so the immediate burden of investment is greater, tilting the decision towards credit repayment. However, when credit limits are high, firms strongly desire replacement of existing older capital and so repayment is postponed at lower debt levels.

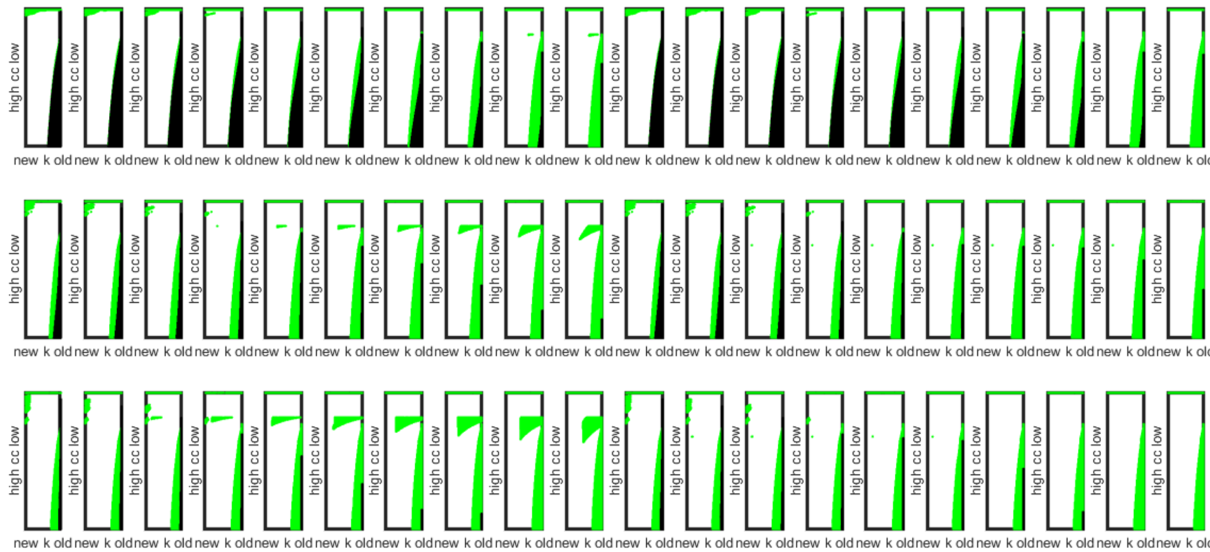
Remark. Maintenance is more likely when credit limits are low, debt is high and capital is newer. As productivity increases, both idiosyncratic and aggregate, maintenance encompasses more older capital but when credit limits are high, maintenance encompasses more newer capital.

Maintenance expenditures are prominent in studies that use firm-level data but not explicitly collected for the national accounts. The results in this model suggest it is disproportionately performed by productive firms with newer capital when credit limits are low and debt outstanding is high. This is the case in the time periods after replacement has occurred where firms will have high credit balances. The financing dimension is therefore critical in inducing firms to maintain their capital stocks. The pressure to replace capital grows as capital ages and firms need to be in a position to take on debt before undertaking any replacement investment, as outlined in the prior remarks. They respond by maintaining capital while it is relatively new. This keeps the effective age of capital down and allows firms time to wait for productive times where they can lower their debt and prepare for replacement.

Remark. Holding aggregate productivity constant and assuming credit limits are high, the likelihood of replacement is hump-shaped in idiosyncratic productivity.

When replacement occurs, firms not only incur the fixed cost of investment but also a concurrent productivity reduction. This can be quite burdensome when idiosyncratic productivity is high. As a result, the replacement response to idiosyncratic productivity exhibits a hump-shape where the greatest proportion of grid points where replacement is chosen occur when productivity is near average. Firms have limited financial capacity to replace capital when idiosyncratic productivity is low and experience the concurrent productivity slow-down more so when idiosyncratic productivity is high.

These dynamics are distinct to previous studies like CHP, where replacement is decreasing in idiosyncratic productivity. There, the effect of concurrent slow-downs is the dominating factor and firms simply draw on equity to fund investment in those bad times that are specific to them. The equity funding channel is disallowed in this model and so low productivity firms perform less replacement, preferring to wait for reversion to the mean.



* Green—Request Forbearance, Black—Bankruptcy. Row 1—Lowest Aggregate Productivity, Row 2—Median Aggregate Productivity, Row 3—Highest Aggregate Productivity. Idiosyncratic Productivity is increasing in the columns in blocks of 10. The first 10 columns reflect high credit limits. The second 10 reflect low credit limits. The x-axis displays the capital grid from newest to oldest. The y-axis displays the credit card grid from highest debt to lowest.

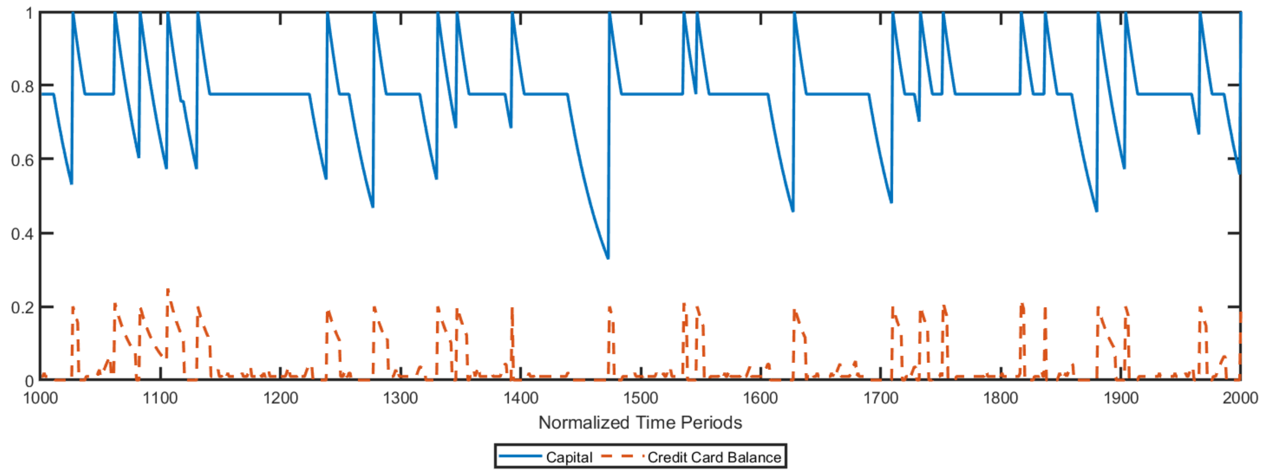
Figure 12. Forbearance and Bankruptcy Policy Functions

Remark. Forbearance is requested in two economically significant zones. (i) As a substitute to bankruptcy when idiosyncratic or aggregate productivity is relatively high. (ii) As a substitute to making a minimum payment so as to allow replacement to occur at relatively low debt levels and when capital is of an age where maintenance is not viable given the low debt level.

3.2 Simulations

Overview

I simulate the model over 2000 periods with 200 firms initially operating and discard the first 1000 time periods. Figure 13 depicts this simulation for a single firm. We see that capital exhibits a lumpy evolution with long or short periods where the effective age is held steady. This is where maintenance is optimally performed. The outstanding credit card balance also exhibits lumpiness. Compared with a model where the only dimension for investment is the extensive margin, replacement occurs much less frequently over the life-cycle. Two factors contribute to this with maintenance allowing the firm to keep capital productive for long periods, allowing the actual age



* Model is simulated over 2000 quarters with the first 50% of observations discarded.

Figure 13. Simulation of a Single Firm

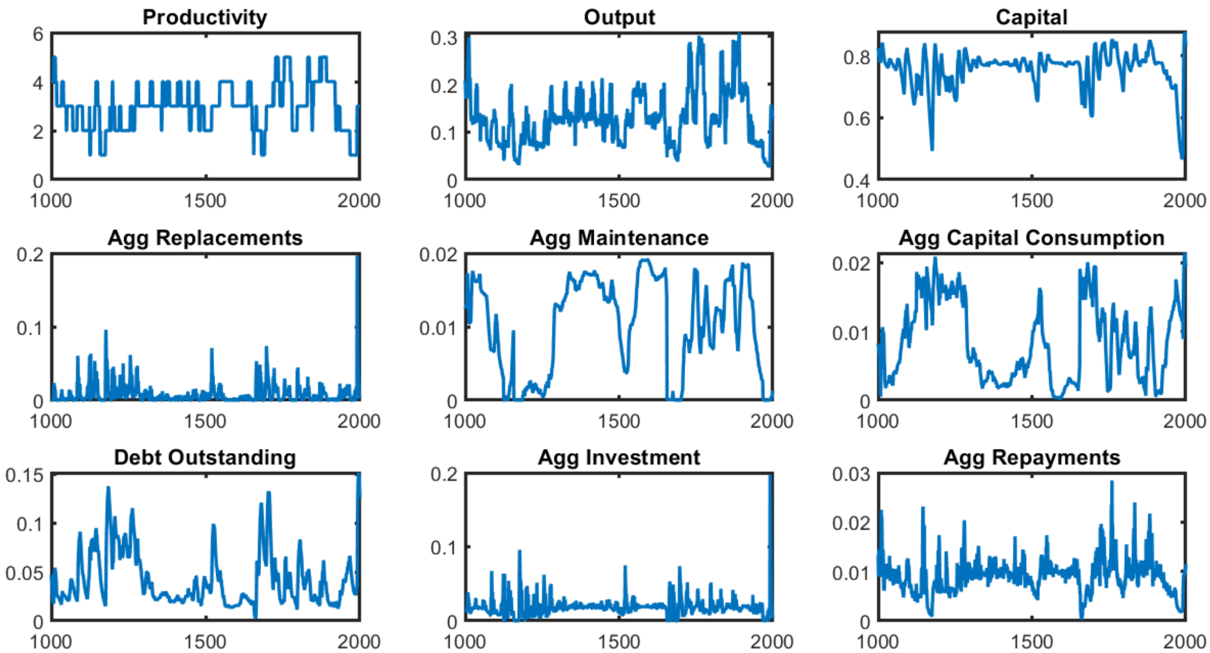
of capital to rise. The second factor is variable credit limits, where the combination of a move to a high aggregate state when credit limits are low suppresses replacement investment that would otherwise have occurred.

Figure 14 presents the simulation of the whole economy with each variable being a sample average of the aggregates defined in Section 2.5.[12] It is noteworthy that investment, the sum of replacements and maintenance, is positive in all periods. This is produced because maintenance expenditures are strongly pro-cyclical and positive in each period of the simulation. Outstanding debt is pro-cyclical in periods where replacement investment is responding strongly to upticks in productivity. Firms fund this expenditure through credit card debt and this causes the pro-cycical pattern of debt outstanding in those periods.

In contrast, periods of relative stability, such as the second quartile, see low periods of debt. As replacements fall and firms shift their investment towards maintenance, so too does outstanding debt fall. Repayments though remain relatively high in such periods, indicating most firms enter a stable cycle of fully repaying credit card debt and maintaining capital during longer stable economic periods.

Further adding to the richness of the debt dynamics is the experience in other quartiles. The

[12] Presenting the raw aggregates would necessitate additional structure that determines the equilibrium number of firms operating. This is not done in order to economize on costly computation.



[*] Model is simulated over 2000 quarters with the first 50% of observations discarded. Each series represents the average across active firms.

Figure 14. Simulation of at the Industry-Level

first quartile sees short bursts into higher productivity zones which are accompanied by surges in replacements, leading to higher debt. We can see that the debt outstanding in this period remains high as productivity declines. Most of that debt is eventually repaid once the second quartile is reached, which is relatively more stable at higher productivity levels.

The experience in the fourth quartile is quite the opposite. Here, the productivity series is quite volatile but skewed to the higher end. We see that replacement activity peaks early in that time period, with higher levels of productivity not being accompanied by high levels of replacement as the quartile progresses. Instead maintenance expenditures rise quickly and persist at high levels, while the large debt build-up in the early part of the quartile is quickly repaid and remains low thereafter.

Capital consumption exhibits a mildly counter-cyclical bent during the sample whereas the equivalent measure in the US economy is mildly pro-cyclical. Adding a capital utilization dimension could improve the performance of the model on this dimension.

Testable Predictions

A number of testable predictions arise from these simulations which I will outline in a series of remarks.

Remark. Firm level debt should rise sharply in periods of fast aggregate growth, be low in longer stable periods and high in volatile periods.

This follows from the experiences in the first, second, and fourth quartiles with debt shifting between being pro and counter-cyclical and displaying significant signs of path dependence.

Remark. Maintenance expenditures should be highest in periods of low debt and aggregate stability. Maintenance should be lowest in recessions

This follows from the contrasting experiences of the second and fourth quartiles. In the second, maintenance is persistently high and accompanies a relatively stable period in the productivity series. Instead in the fourth, maintenance reaches the same high levels but fluctuates much more so despite the average productivity level over the fourth quartile being very similar to that over the second.

The third quartile sees the longest recessionary period, which is also accompanied by near zero levels of maintenance. This leads the to final testable prediction ahead.

Remark. Maintenance is highly volatile over medium term business cycles while replacement is most volatile in the short-term.

We again contrast quartiles to see this. Replacement volatility is quite low over the stable period in the second quartile, while being highly volatile in the short cycles of the first and fourth quartile. In contrast, maintenance volatility is observed over longer horizons. In the language of medium-term business cycles as in Comin and Gertler (2006), the first quartile is skewed towards lower productivity while the second is skewed towards higher productivity. We see that maintenance experiences low troughs and high peaks that are sustained over these medium-term phases.

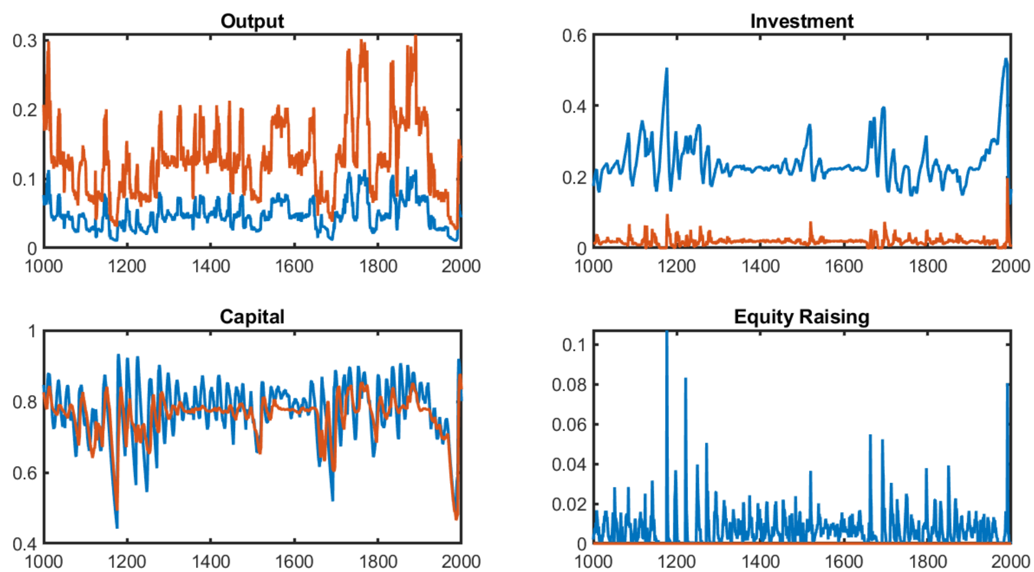
Long-Term Aggregate Ratios

Table 8 displays sample averages of long-term ratios of interest as a method of analyzing the aggregate fit of the model. The productivity series is mean-reverting as expected, a biased sample

Table 8. Long-Term Aggregate Ratios in Simulations

Ratio	Description	Average	Target
A	Productivity	1.0056	1
I/Y	Investment/Output	0.164	0.2
CC/Y	Debt Outstanding/Output	0.416	N/A
K/Y	Capital/Output	6.94	9

can be ruled out. The investment to output variable is quite close to the typically calibrated value. The amount of debt outstanding as a percentage of output is highly data dependent and so I do not offer a target. It is the case however that the ratio being close to 40% appears reasonable for the US economy. The capital to output ratio is relatively small, but an improvement over those implied in past studies.



* Red—Model with Maintenance and Credit Card Frictions, Blue—CHP Model. Model is simulated over 2000 quarters with the first 50% of observations discarded.

Figure 15. Model Comparison

As a point of comparison to CHP, Figure 15 shows the simulated series that overlap this model with that of CHP that are of interest.[13] We see that the CHP simulation implies an investment to output ratio that is greater 1, consistent equity raisings to fund investment, and a highly volatile

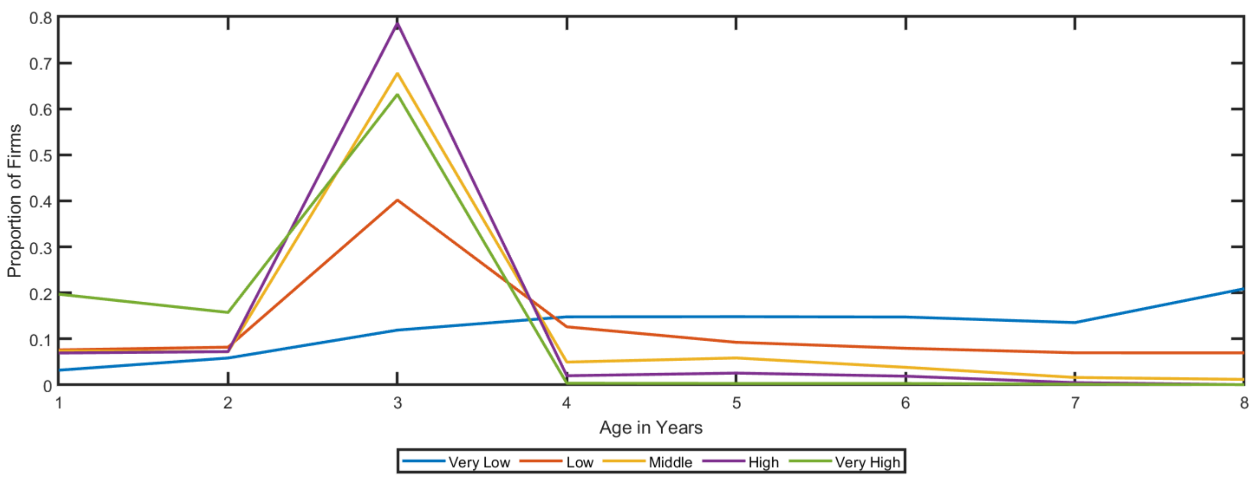
[13]See the appendix for a description of the model in CHP.

aggregate capital stock.

The equity raising channel is completely shut-down in this paper but is over-active in the CHP model. While both are extremes, it should be noted that a preference for stable dividends among managers and the negative market signals associated with equity raisings typically limit their use relative to debt. For that reason,adjustment on the debt rather than equity margin featured in this paper is likely preferable.

Vintage Distribution

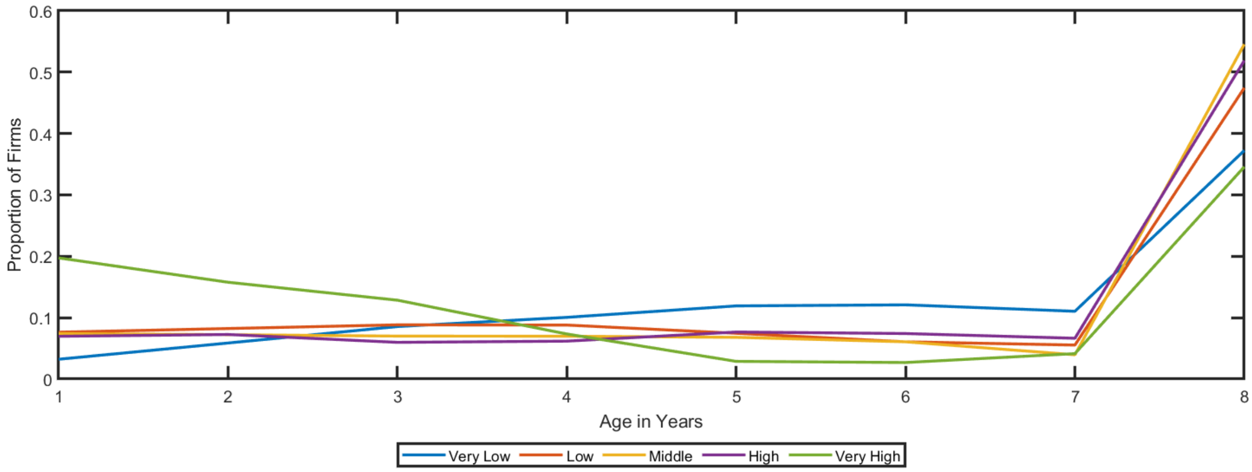
Heretofore we have analyzed the fit of the model against aggregate measures. On the firm-level, we are able to speak to the age/vintage distribution of capital. CHP in their Figure 9, present an age distribution of capital in their plant-level data for each of the years 1983 through 1987. The distribution is fat-tailed with most mass at the new and older end of the spectrum. Their model however, is unable to replicate the high proportion of firms with older capital, 8 years or older.



* Model is simulated over 2000 quarters with the first 50% of observations discarded. Legend refers to aggregate productivity state.

Figure 16. Effective Vintage Distribution

For the purposes of this section, we distinguish between the effective working age of capital and its actual age. Figure 16 shows the distribution for each aggregate state simulated in this paper. Due to the presence of maintenance, the effective age is heavily clustered at an effective age of 3 years for most productivity levels. The exception to this is the lowest productivity level, where maintenance is rarely performed. Combined with the fact that very low amounts of replacement occur in that state, the effective age skews towards the older end of the spectrum when the economy is performing poorly.



* Model is simulated over 2000 quarters with the first 50% of observations discarded. Legend refers to aggregate productivity state.

Figure 17. Actual Vintage Distribution

When we consider the actual age distribution in Figure 17, we see that the majority of capital at all ages is older capital.[14] When compared to the analogous figure in CHP, the distribution at the highest productivity is the closest to matching their distribution. The 1983-87 sample period in CHP was a high growth period in the US economy, following the Volcker-Disinflation recessions. It is therefore fair to compare this high productivity state in this paper to that period of history in the United States.

4. Conclusion

I have augmented a model of lumpy investment such that investment on the extensive margin can occur as replacement or maintenance. Furthermore, a second dimension is added whereby firms can adjust their debt on the extensive margin also, introducing a debt repayment and investment trade-off. Distinctly to Chapter I, a negative credit shock here lowers investment, inducing substitution away from replacement and towards maintenance. The reason for this difference is the unsecured

[14] Each firm's capital is tracked by age to recover the actual age distribution. Capital is taken to age by one period regardless of any maintenance activity.

nature of credit here, being revolving debt, as opposed to the secured nature of credit in Chapter I.

The model has a number of features that improve and expand the ability of lumpy investment models to capture both the aggregate and firm level data. At the firm-level, we generate lumpy debt repayment, persistent low debt and high debt periods, and an age distribution of capital that is comparable to that seen in prior studies. A distribution with mass clustered at the newer and older ends of the age spectrum.

At the aggregate level, the long-run ratios of aggregates better reflect what is typically the case in the data. We expand the coverage of the model to reflect pro-cyclical maintenance expenditures and low levels of equity raising activity across the business cycle for the purposes of replacement investment. The aggregates evolve in path dependent ways, with volatility being dependent on the length of business cycles. Long periods of steady growth like the Great Moderation and post Great Recession period should exhibit low investment volatility and high maintenance investment. More volatile historical periods like the post WWII period should see replacement investment being highly volatile.

Further work can continue to refine the aggregation of lumpy investment models. Extensions to include variable capacity utilization, secured and unsecured forms of debt, and retained earnings funding are all viable and would be likely to add value to this literature.

CHAPTER III

Velocity Targeting at the Zero Lower Bound

1. Introduction

The zero lower bound appears to constrain the ability of central banks to achieve monetary objectives. A further loosening of monetary policy at the zero lower bound would necessitate negative short-term rates. While this has been the experience recently in some countries, the US experience has seen the Federal Funds Rate (FFR) operate extremely close to zero for the entire period from 2008 until the first quarter of 2016. Logically, this suggests that either the Federal Reserve's (Fed) monetary policy rule has dictated that it should hold overnight rates at essentially zero for the period described or that the Fed's rule would dictate negative rates, but that the Fed is constrained in implementing this objective.

The first part of this chapter thus seeks to provide an answer to this question; should the Fed be holding rates at zero under its established rule or is it constrained by the zero lower bound. Conditional on the latter, the second question is to identify the monetary instrument of the Fed beyond 2007 and if their post 2007 objectives are consistent with what they were before 2007.

The Fed's goals mandated by legislation are to target the output gap and inflation. These goals are translated into a rule, being the Taylor Rule. It dictates that the Fed sets short term interest rates to react in greater magnitude than changes in inflation while also stabilizing output gap. A relatively more empirically sound version of the Taylor Rule is the variant introduced and tested by Clarida, Galí and Gertler (2000) (CGG). This is a forward looking version of the Taylor Rule with a role for interest rate inertia. So that the Fed targets rates according to expectations of future inflation and output gap but only smooths rates over time to reach the target rate.[1]

In order to answer the question, I firstly estimate the CGG specification for the period 1979 to 2007, which corresponds to the beginning of the Volcker Fed chairmanship to the end of the Great Moderation. I find that the structural parameters have not changed in an economically significant

[1] Consolo and Favero (2009) correct for weak instrumentation in the CGG analysis and find that the estimated degree of smoothing is lower while the other parameters are quantitatively as originally estimated.

way from that which was estimated by CGG for the period to 1996. The inflation target is slightly lower, the Fed still acts to tighten in response to increased expected inflation and the output gap and the smoothing parameters are essentially unchanged.

I take this result as evidence that the Fed has been operating with essentially the same policy objectives since the chairmanship of Volcker until 2007. The inflation target is around 2.5% and the Fed is quite hawkish on inflation. Given this, the next part of the exercise is to extrapolate the interest rate target for the post 2007 period. I do this by applying the parameter estimates from the 79-07 period to the post 2007 data. The results indicate that the Fed would be operating with negative interest rates post 2007. The fact that it has not, I take as evidence of the zero lower bound binding.

Armed with this knowledge, I then seek to resolve the issue of whether the Fed has changed its objective or rather its instrument in response to the binding zero lower bound. This question, in particular, has implications for future DSGE modelling which seeks to capture the comovements of aggregates in the post 2007 US economy.[2] I provide evidence that the instrument of the Fed has changed but the underlying objectives are the same. Thus, unlike the CGG research which shows a structural break in monetary policy in 1979, my research does not indicate such a break in 2008.

I focus on money velocity as a potential instrument of monetary policy after 2007. Regressions of money velocity measures indicate that it is non-stationary over the period concluding in 2007. This ceases to be the case post 2007, as the interest rate target of the Fed has a positive and statistically significant small effect on money velocity contemporaneously. This effect is non-trivial, accounting for a substantial part of the cyclical component of money velocity.[3]

The notion that the Fed may optimally switch to targeting a money aggregate off equilibrium is explored theoretically in Atkeson, Chari and Kehoe (2010). They find that one-period switches to money targeting constitute an optimal response whenever interest rate targeting is not possible.[4] The results in this paper provide empirical support that such a regime is implementable. In contrast, a model like Adam and Billi (2006) is unsupported by these results. They optimally obtain that

[2] Gust et al. (2017) suggest that estimating DSGE models during ZLB periods without considering the nonlinear implications introduces substantial error and bias into any results. Wu and Xia (2016) and Wu and Zhang (2019) provide methods to correct for any issues arising from the ZLB.

[3] The trend and cyclical decomposition is obtained via a Hodrick-Prescott (HP) filter.

[4] The model abstracts from an endogenous propagation mechanism that would generate aggregate persistence in a stationary setting such as capital.

the central bank should over-react to inflation at the zero lower bound when overnight rates are the sole monetary instrument. Were this to be the case, we would expect to observe parameter drift in the post 2007 estimation and this result is not forthcoming.

The findings in this paper are consistent with the view that 'quantitative easing' has been pursued by the Fed post 2007 as described in Bernanke (2020). Moreover, the results support a growing view in the literature that the zero lower bound is not a constraint on monetary policy. A survey can be obtained in Debortoli, Galí and Gambetti (2019).

The remainder of the chapter is arranged as follows. Section 2 introduces the structural model. Section 3 discusses the data and its application of the structural model as well as the estimation strategy. Section 4 presents the results and Section 5 concludes.

2. The Model

2.1 Derivation of the Clarida, Galí and Gertler (2000) Rule

I begin this section by first considering the standard Taylor Rule with interest rate targeting dictated by inflation and the output gap,[5]

$$r_t^* - r^* = \beta(\pi_t - \pi^*) + \gamma x_t \tag{III.1}$$

Under this specification. the current interest rate deviation from the long term target rate $r_t^* - r^*$ is a function of the current inflation deviation from target $\pi_t - \pi^*$ and the output gap x_t.[6] The CGG rule follows in this spirit but makes the assumption that the central bank bases its interest rate decisions on forecasts of inflation and the output gap so that the rule for the target rate becomes,

$$r_t^* = r^* + \beta\left(\mathbb{E}_t[\pi_{t,k}|\Omega_t] - \pi^*\right) + \gamma \mathbb{E}_t[x_{t,q}|\Omega_t] \tag{III.2}$$

Specifically, the central bank makes a forecast k periods ahead of t of inflation and q periods ahead of t of the output gap, based on information which is available and contained in the information set Ω_t.

[5] See Koenig, Leeson and Khan (2012) and Woodford (2003) for a survey of the Taylor Rule.

[6] The assumption is that the long-term output gap target is to eliminate the output gap so that acutal GDP is equal to potential GDP. Thus the long run target is zero.

Define the forecast error of y at time t as $\eta_t^y := y_t - \mathbb{E}_{t-1}[y_t]$. Additionally, the Fischer relationship gives an approximate relationship between real and nominal rates as, $rr_t^* = r_t^* - \mathbb{E}_t[\pi_{t,k}|\Omega_t]$, which in steady state is $rr^* = r^* - \pi^*$. We can now substitute for r^* and the expectation terms using the steady state Fischer equation and the respective forecast errors. Simple algebra then yields,

$$r_t^* = rr^* - (\beta - 1)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q} - \beta\eta_{t,k}^{\pi} - \gamma\eta_{t,q}^{x} \tag{III.3}$$

We now have an expression for the time t nominal rate target in terms of long-run targets and observed inflation and the output gap.

Rather than assume that the central bank sets rates to this target at all times, some level of inertia in the process is assumed so that the observed rate is a linear combination of past rates and the target of the form,

$$r_t = \rho(L)r_{t-1} + (1-\rho)r_t^* \tag{III.4}$$

Where $\rho(L) = \rho_1 + \rho_1 L + \cdots + \rho_p L^{p-1}$ with $\rho = \rho(1)$.

Finally, substituting the interest rate target into the inertia equation and defining a composite error term as $\epsilon_t := -(1-\rho)(\beta\eta_{t,k}^{\pi} + \gamma\eta_{t,q}^{x})$, we have the equation which yields the nominal rate at time t as,

$$r_t = \rho(L)r_{t-1} + (1-\rho)\left[rr^* - (\beta - 1)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q}\right] + \epsilon_t \tag{III.5}$$

We now have a structural model which is presented in terms of observed data and an error term which is a linear combination of forecast errors. It is this equation which will be estimated in Section 3.

2.2 Derivation of Target Money Velocity

My hypothesis is that the central bank is constrained by the zero lower bound at times in setting interest rates. That is that $r_t' = \max\{0, r_t\}$, so that the observed interest rate r_t' cannot become negative.

In such a time where the zero lower bound binds, the central bank may wish to continue to express its monetary goals, in this case r_t^* by manipulation of another variable that it can influence which is not constrained. Such a variable is money velocity, so we now need to obtain a structural relationship between money velocity and interest rates. To accomplish this, I consider a DSGE model as seen in Carlstrom and Fuerst (2003) with a cash-in-advance constraint.

In the model, households maximize discounted lifetime utility, a function of a non-durable consumption good C_t and real money balances A_t/P_t. Income Y_t is exogenously given by a non-stationary $AR(1)$ process. Additionally, households have a stock of money M_t, which earns no nominal interest, and access to government bonds B_{t-1}, which are traded in a complete and frictionless market earning gross interest R_{t-1}. Finally, households are subject to a cash-in-advance constraint of the form,

$$A_t = M_t + R_{t-1}B_{t-1} - B_t \tag{III.6}$$

The constraint implies that households must make their bond investment decisions prior to purchasing the consumption good. In this way, the relevant real money balances are given by the cash-on-hand less the net change in bond investment.

The Bellman formulation of the problem is then given by,

$$\begin{aligned} V(M,B) = \max_{C,M',B'} &\left[U\left(C,\frac{A}{P}\right) + \beta\mathbb{E}V(M',B')\right] \\ \text{s.t.} \quad & PC + M' + B' = RB + M + Y \\ & A = M + RB - B' \end{aligned}$$

First order conditions are given by,[7]

$$\frac{1}{P}U_C = \beta \widetilde{V}_{M'} \tag{III.7}$$

$$\frac{1}{P}(U_C + U_a) = \beta \widetilde{V}_{B'} \tag{III.8}$$

$$V_M = \frac{1}{P}(U_C + U_a) \tag{III.9}$$

$$V_B = \frac{R}{P}(U_C + U_a) \tag{III.10}$$

Substituting (III.9) into (III.10), the result into (III.7) and then the result into (III.8) yields,

$$R' = \frac{U_C + U_a}{U_C} \tag{III.11}$$

Converting the gross rate such that $R = 1 + r$, we have,

$$r' = \frac{U_a}{U_C} \tag{III.12}$$

At this stage, I assume a functional form which nests a constant elasticity of substitution setup in the near universal constant relative risk aversion utility function so that,

$$U(C, A/P) = \frac{X^{\frac{1}{1-\sigma}} - 1}{1-\sigma} \quad \text{where} \quad X = \left[(1-\omega)C^{1-\phi} + \omega\left(\frac{A}{P}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{III.13}$$

Then our equation for the interest rate is given by,

$$r_t = \frac{\omega}{1-\omega}\left(\frac{C_t}{A_t/P_t}\right)^{\phi} \tag{III.14}$$

an expression relating the interest rate to the relative balance of consumption to real balances. Notice that this term is almost money velocity. Velocity is defined as $V = PY/M$. Then simple algebra yields the equation which relates short term rates to money velocity, specifically,

$$r_t = \frac{\omega}{1-\omega}\left(V_t \frac{C_t}{Y_t}\right)^{\phi} \tag{III.15}$$

[7]Note that $a = A/P$ for convenience.

Assuming that the central bank can control velocity, by changing money supply, we can translate the target interest rate to target velocity as,

$$r_t^* = \frac{\omega}{1-\omega}\left(V_t^* \frac{C_t}{Y_t}\right)^{\phi} \tag{III.16}$$

In the same way that the central bank is assumed to not set the interest rate to its target at all times, I assume a velocity setting process which has some inertia, so that,

$$V_t = \psi(L)V_{t-1} + (1-\psi)V_t^* \tag{III.17}$$

where $\psi(L)$ is a polynomial function analogous to $\rho(L)$ though not necessarily of the same order.

Assuming a Cobb-Douglas form for the sub-utility relationship between consumption and real balances, so that $\phi = 1$, we can rearrange the velocity equation to yield velocity at time t in terms of the target interest rate,

$$V_t = \psi(L)V_{t-1} + \frac{(1-\psi)(1-\omega)}{\omega}\frac{Y_t}{C_t}r_t^* \tag{III.18}$$

This structural equation should explain money velocity movements and provides a link between the interest rate target of the central bank and observed movements in velocity. Underlying both velocity and interest rates is central bank control of money supply, allowing the central bank to manipulate either, although the central bank faces no zero lower bound in setting velocity.

3. Data and Estimation

3.1 Structural Regression Model

I estimate (III.5) and (III.18) using an efficient GMM procedure. The reduced form of equation (III.5) is,

$$r_t = X_t\Lambda + \varepsilon_t \tag{III.19}$$

where X_t contains lags of r_t, the remaining covariates and a vector of ones. In order to follow CGG, I use two lags of the interest rate so that X_t has dimensions $t \times 5$. This implies 5 reduced form parameters however, we have 6 structural parameters that need to be recovered. The structural parameters can be expressed as,

$$\rho_1 = \lambda_1 \tag{III.20}$$

$$\rho_2 = \lambda_2 \tag{III.21}$$

$$\beta = \frac{\lambda_3}{1 - \lambda_1 - \lambda_2} \tag{III.22}$$

$$\gamma = \frac{\lambda_4}{1 - \lambda_1 - \lambda_2} \tag{III.23}$$

The reduced form parameter λ_5 is structurally related to the long-term real interest rate target and the long-run inflation target. Being that their are two structural parameters with only one reduced form parameter, we cannot recover both. Following CGG, I set the real interest rate target to equal the long-run average real rate observed over the sample and I can then recover the inflation target.

The task now is to obtain the reduced form parameter vector Λ. This is estimated via efficient GMM. Define a matrix Z_t of instrument variables. These must be time $t-1$ or further lagged variables as the structural error term is a linear combination of forecast errors. In order to satisfy the orthogonality conditions, we require information contained in the information set Ω_t. Lagged variables satisfy this criteria. Given the instrument matrix Z_t and covariates matrix X_t, the reduced form parameter vector is then obtained by,

$$\Lambda = \left((Z_t'X_t)'\Sigma_t Z_t'X_t\right)^{-1}(Z_t'X_t)'\Sigma_t Z_t' r_t \tag{III.24}$$

In order to estimate the model efficiently, the two step procedure is followed where two stage least squares is estimated first, the estimated variance-covariance matrix $\widehat{\Gamma}_t$ is obtained, and then efficient GMM can be performed setting $\Sigma_t = \widehat{\Gamma}^{-1}$.

Finally, in order to obtain standard errors for the structural parameters, I define a Jacobian matrix of the structural parameter system and standard errors are obtained using the Delta Method.

In order to estimate (III.18), I follow a similar procedure. The reduced form to be estimated is,

$$V_t = \alpha_1 V_{t-1} + \alpha_2 V_{t-2} + \alpha_3 \frac{Y_t}{C_t} \widehat{r}_t^* + \mu_t \tag{III.25}$$

where $\widehat{r}_t^*$ is the fitted interest rate target obtained from the earlier estimation. Again there are more structural parameters than available reduced form parameters. Specifically, α_3 is a function of both $(1-\psi)$, the structural component of velocity not owed to the autoregressive form and ω the elasticity parameter from consumer preferences. In order to recover $(1-\psi)$, I assume equal elasticities in utility between consumption and real balances, so that $\omega = 0.5$. This assumption potentially introduces a bias into the estimation of $1-\psi$. Specifically, $1-\psi$ is increasing in the size of ω, so as the share of real balances increases, so too does the magnitude of the interest rate target in affecting velocity. Further work will focus on estimates of ω which are obtained in the literature estimating money demand functions.

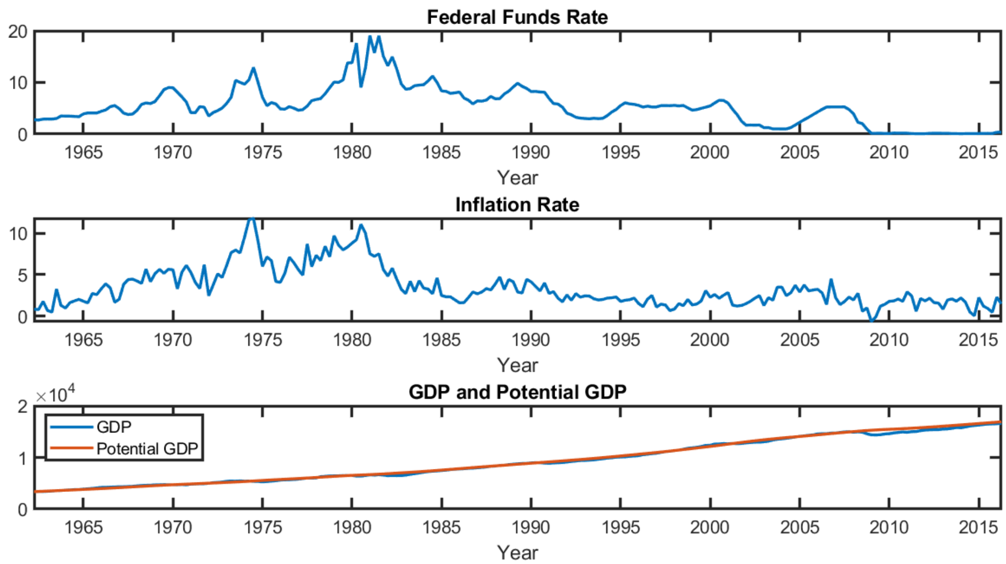
Estimation of (III.25) again proceeds via Efficient GMM analogous to the procedure performed earlier. The instrument matrix is taken to be the same as was used to estimate (III.19).

3.2 Data

The data is obtained from the St. Louis Fed FRED database. The sample period is from 1962:1 to 2016:3. The sample is then divided into 3 periods, these are 1963:3 to 1979:2 which corresponds to the pre-Volcker period of monetary policy, 1979:3 to 2008:2 which corresponds to the Volcker-Greenspan-Bernanke period of monetary policy where the zero lower bound was nonbinding and lastly 2008:3 to 2016:2, which corresponds to the zero lower bound binding period and some extra quarters at the end where the zero lower bound did not bind, corresponding to Fed tapering of quantitative easing.

The interest rate instrument of the Fed is taken to be the Federal Funds Rate. Inflation is constructed as the annualized change from one quarter to the next in the GDP deflator. These are consistent with CGG. However, where CGG used the Congressional Budget Office's (CBO) measure of the output gap, I construct the output gap as the difference between actual and potential nominal GDP, measured in 2009 chain dollars. This change, as well as some loss of years prior to 1962 means that the sample, nor the variables are exactly as in CGG. Lastly, k and q, the leads that indicate how far into the future the Fed bases its inflation and output gap forecasts are both set to 1 quarter as in CGG.

The instrument vector Z_t is constructed as in CGG. 4 lags of each variable are used, with Z_t containing lags of the Federal Funds rate, inflation, output gap, the spread between the 10 year Treasury Bond and 3 month Treasury Bill, the annualized growth rate of M2 quarter on quarter and commodity inflation. I construct this last variable as the annualized rate of change in the Producer Price Index (PPI) quarter on quarter.

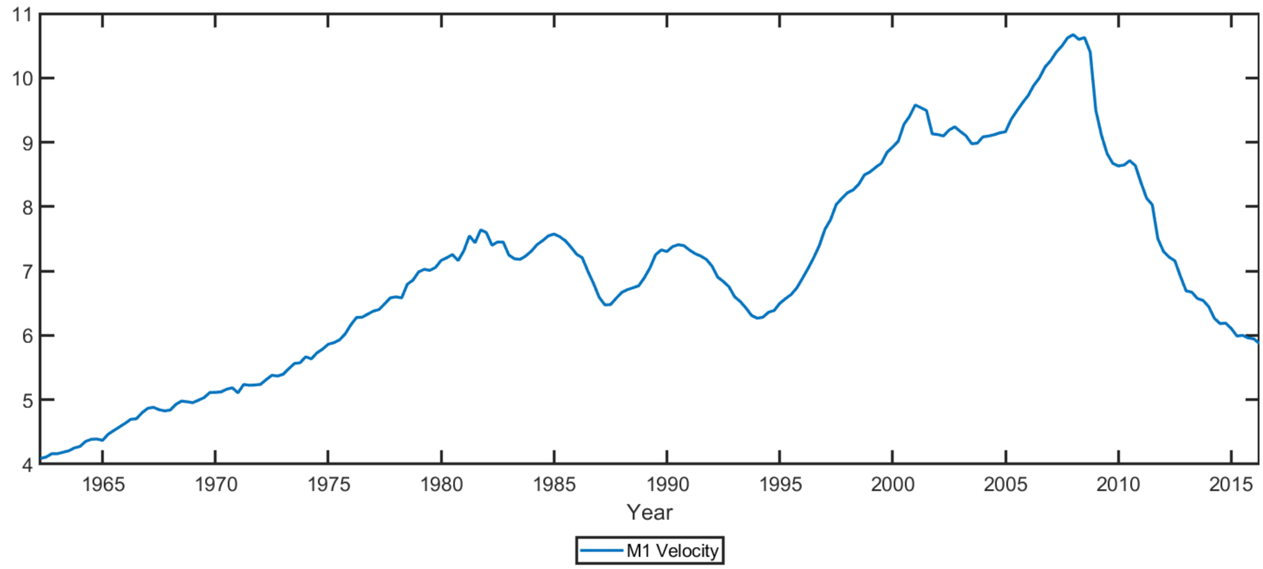


* Source: Federal Reserve Bank of St. Louis FRED database.

Figure 18. Aggregate Data

Figure 18 shows the evolution of the Federal Funds rate over the entire sample period. Of note is the consistency over the period 2008:3 to 2016:3, where the zero lower bound is taken to be binding. Additionally, we see the inflation experience over the sample, of note here is the moderation in inflation since 1982, where inflation never reaches the peaks experienced prior and is also much less volatile. Lastly, the third panel shows the output gap over the sample, we observe that potential GDP is well above actual GDP over the zero lower bound binding period. If the central bank is constrained in its monetary policy setting abilities, this could explain that slow recovery, although that question is left to future work.

The dataset contains 3 money aggregates, M1, M2 and Zero Maturity Money (MZM). A cor-



* Source: Federal Reserve Bank of St. Louis FRED database.

Figure 19. M1 Velocity

responding velocity measure must be chosen from these aggregates in order to estimate (III.25). The natural measure based on the Carlstrom and Fuerst (2003) model is M1. This is because the cash-in-advance term which appears in the money demand equation corresponds to money net of short term bonds. This suggests the use of a relatively more disaggregated measure, of which M1 is more disaggregated than the other two available measures. As such, the velocity measure chosen is M1 velocity. Figure 19 tracks the evolution of M1 Velocity over the sample. Of note is the sharp decline over the zero lower bound period. This is a result of money growing faster than GDP over the period. Of interest then is whether this phenomenon is purely a random walk, which would be the case if velocity were a non-stationary AR(1) or if the interest rate target of the Fed plays some role. A desire for negative rates would put downward pressure on velocity.

4. Results

I first estimate (III.19) and recover the structural parameters. The results are presented in Table 9. Column 1 presents results over the Pre-Volcker period. We see that the results are numerically very similar to those obtained by CGG, suggesting that the replication exercise is a success. Importantly, the $\beta < 1$ result is preserved, indicating that the Fed was not fighting

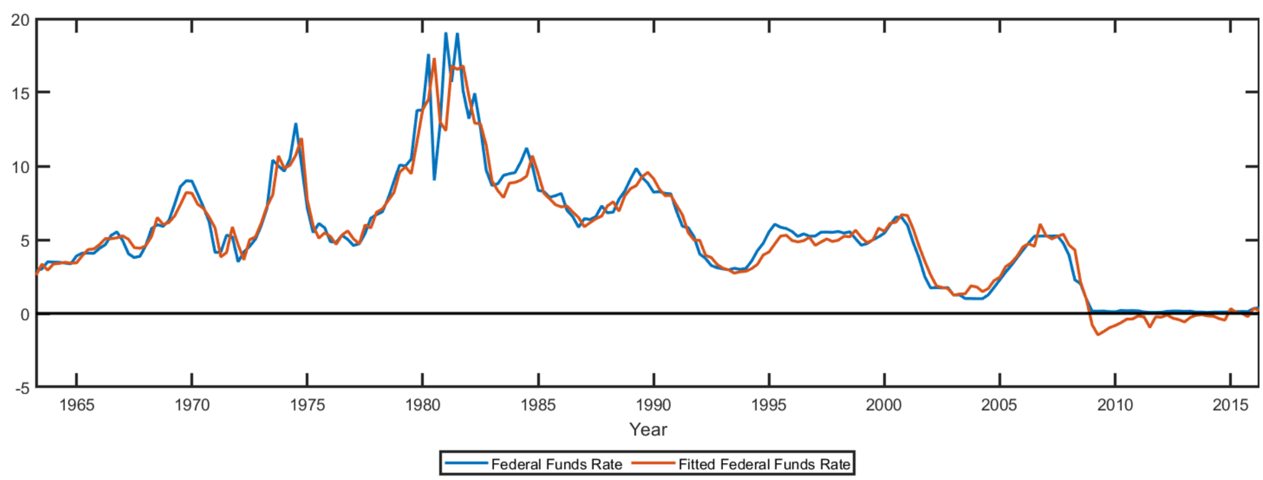
Table 9. Interest Rate Inertia Estimation

	Pre-Volcker	Full post 79 sample	79:2 to 08:2
Results:			
π^*	4.15	1.79	2.58
	(3.98)	(0.81)	(0.88)
β	0.79	1.90	2.22
	(0.08)	(0.35)	(0.45)
γ	0.70	0.67	1.21
	(0.24)	(0.10)	(0.31)
ρ	0.68	0.83	0.85
	Pre-Volcker		79:2 to 96:4
CGG Baseline:			
π^*	4.24		3.58
	(1.09)		(0.50)
β	0.83		2.15
	(0.07)		(0.40)
γ	0.27		0.93
	(0.08)		(0.42)
ρ	0.68		0.79
	(0.05)		(0.04)

inflation with monetary policy prior to Volcker's chairmanship. The inflation target is not as precisely estimated, this is probably explained by the exclusion in my sample of years prior to 1962, qualitatively though, the inflation target is not of interest over this period as the Fed was not targeting inflation. The gamma parameter capturing the Fed's reaction to the output gap is quite different, although qualitatively, an estimate of $\gamma > 0$ should be obtained. This is probably due to the different output gap measure. As an indicator of the success of replication, the results obtained appear to broadly concord with the key findings in CGG.

Consider column 3. We expect broadly similar results here despite the fact that my sample has more observations than CGG. The reason for this is that the CGG finding that the Fed is fighting inflation and output gaps post 79 should continue to hold through 2007. This period corresponds to the time of unconstrained monetary policy setting. The inflation target is lower under my estimation, probably owing to the lower inflation experienced post 96. Importantly, the beta parameters are almost identical, suggesting that the Fed is fighting inflation and that this estimate is robust to adding the 97 to 07 period. The gamma estimates are different but not qualitatively so, the Fed is more aggressively fighting output gaps in my sample, probably owing to having inflation better contained.

Comparing the results of column 3 to column 2, we see that adding the zero lower bound period implies that the inflation target is lower and that the Fed is not as strongly fighting inflation, implied by the lower beta and at the same time not as strongly fighting output gaps, implied by the lower gamma parameter. This would strongly suggest a change in objective by the Fed given no further analysis. However, an exercise which would demonstrate that rather than change its objective, the Fed is constrained in meeting its objectives would be to extrapolate what the interest rate would have been over the zero lower bound period if rates were able to be negative.



* Source: Federal Reserve Bank of St. Louis FRED database and estimation results.

Figure 20. Actual vs Fitted Interest Rate Values

To do this, I obtain fitted interest rate values over the zero lower bound period by fitting the 08:3 to 16:2 data into the parameter estimates obtained in column 3. Figure 20 shows the results of the interest rate path over the full sample period. Until 79, the pre-Volcker column 1 estimates are used, between 79:3 and 08:2, the column 3 estimates are used and from 08:3 and on, the values obtained by the extrapolation method are used. We see very clearly that the extrapolated values over the zero lower bound period are negative, and in the early part substantially so. This strongly suggests that the Fed is constrained by the zero lower bound.

Additionally, for the periods prior to 2008, we see that the fitted interest rate under the CGG rule tracks the actual Federal Funds rate quite well. The period 79 to 82 obtains the poorest fit as in CGG, although the poorness of fit over that period is not as pronounced. This period corresponds to the early Volcker years, where CGG suggest that inflation targeting was not the objective but rather some form of reserve management.

Table 10. Regression Results for Velocity Autoregressive Process

	Pre-Volcker	79:2 to 08:2	08:2 to 16:2
ψ_1	0.89	1.53	1.04
	(0.12)	(0.09)	(0.05)
ψ_2	0.12	-0.53	-0.05
	(0.12)	(0.09)	(0.05)
$1-\psi$	-0.001	0.0006	0.01
	(0.003)	(0.002)	(0.003)

Table 10 presents the results for the estimation of the velocity process. The null hypothesis is that velocity is a non-stationary AR(1). That is $\psi_1 = 1$. Alternatively, $\psi_2 = 0$ and $1-\psi = 0$. We see that for the pre-Volcker period that velocity is AR(1), non-stationary and the role for the interest rate target is not significant. The Fed is not using money velocity management as a monetary policy instrument. Over the 79:3 to 08:2 period, where the zero lower bound was nonbinding, we see that velocity is AR(2), non-stationary and the interest rate target is not significant. Again, the Fed is not using velocity as an instrument of monetary policy.

However, over the period where the zero lower bound binds, the interest rate target becomes significant, as $1-\psi$ is statistically significant. As the interest rate target is negative over the period, we see that this negativity is dragging down money velocity. The size of the coefficient for $1-\psi$ is also economically significant, accounting for a substantial portion of the cyclical component of velocity, obtained via HP filtering techniques.

5. Conclusion

The results obtained support the CGG rule in times where the zero lower bound is non-binding. Where the zero lower bound binds, alternative monetary policy instruments need to be found. This work suggests that money velocity has been such an instrument for the Fed. Alternatively, the argument that the Fed is simultaneously responding less to output gaps and inflation would need to be accepted. The path of the extrapolated interest rate suggests this is not a plausible argument.

Further work at this point would seek to make the result more robust. Within the current framework, alternative leads in monetary policy could be examined, alternative output gap or even unemployment gap measures could be used, as well as alternative inflation measures. An international extension where interest parity conditions are examined could also prove fruitful, particularly when considering the zero lower bound in other countries, like to Eurozone, as opposed to countries like Australia where the zero lower bound was never binding. The exchange rate fixing practices in China would also be an element of money demand which should be accounted for as the results are sensitive to how to the money demand function is specified and parametrized.

REFERENCES

Adam, Klaus, and Roberto M. Billi. 2006. "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates." *Journal of Money, Credit and Banking*, 38(7): 1877–1905.

Ajello, Andrea. 2016. "Financial Intermediation, Investment Dynamics, and Business Cycle Fluctuations." *American Economic Review*, 106(8): 2256–2303.

Akinci, Ozge, and Albert Queralto. 2016. "Credit Spreads, Financial Crises, and Macroprudential Policy." Federal Reserve Bank of New York Staff Report No. 802.

Albonico, Alice, Sarantis Kalyvitis, and Evi Pappa. 2014. "Capital Maintenance and Depreciation over the Business Cycle." *Journal of Economic Dynamics and Control*, 39: 273–286.

Alpanda, Sami, and Sarah Zubairy. 2017. "Addressing Household Indebtedness: Monetary, Fiscal or Macroprudential Policy?" *European Economic Review*, 92: 47–73.

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2–4): 113–172. Research Collection School of Economics.

Anzoategui, Diego, Diego Comin, Mark Gertler, and Joseba Martinez. 2019. "Edogenous Technology Adoption and R&D as Sources of Business Cycle Persistence." *American Economic Journal: Macroeconomics*, 11(3): 67–110.

Atkeson, Andrew, Varadarajan V. Chari, and Patrick J. Kehoe. 2010. "Sophisticated Monetary Policies." *The Quarterly Journal of Economics*, 125(1): 47–89.

Ausubel, Lawrence M. 1991. "The Failure of Competition in the Credit Card Market." *The American Economic Review*, 81(1): 50–81.

Bachmann, Rüdiger, and Lin Ma. 2016. "Lumpy Investment, Lumpy Inventories." *Journal of Money, Credit and Banking*, 48(5): 821–855.

Bachmann, Rüdiger, Ricardo J. Caballero, and Eduardo M. R. A. Engel. 2013. "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model." *American Economic Journal: Macroeconomics*, 5(4): 29–67.

Baxter, Marianne, and Mario J. Crucini. 1993. "Explaining Saving-Investment Correlations." *The American Economic Review*, 83(3): 416–436.

Bazdresch, Santiago. 2013. "The Role of Non-Convex Costs in Firms' Investment and Financial Dynamics." *Journal of Economic Dynamics and Control*, 37: 929–950.

Bernanke, Ben S. 1993. "Credit in the Macroeconomy." *Federal Reserve Bank of New York Quarterly Review*, 18(1): 50–70.

Bernanke, Ben S. 2020. "The New Tools of Monetary Policy." *American Economic Review*, 110(4): 943–983.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. “The Financial Accelerator in a Quantitative Business Cycle Framework.” In *Handbook of Macroeconomics.* Vol. 1, , ed. John Taylor and Mark Woodford, 1341–1393. Elsevier Science.

Bigio, Saki. 2015. “Endogenous Liquidity and the Business Cycle.” *American Economic Review*, 105(6): 1883–1927.

Caballero, Ricardo J., and Eduardo M. R. A. Engel. 1999. “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach.” *Econometrica*, 67(4): 783–826.

Caldara, Dario, and Edward Herbst. 2019. “Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVARs.” *American Economic Journal: Macroeconomics*, 11(1): 157–192.

Canova, Fabio, and Luca Sala. 2009. “Back to Square One: Identification Issues in DSGE Models.” *Journal of Monetary Economics*, 56: 431–449.

Canova, Fabio, Filippo Ferroni, and Christian Matthes. 2014. “Choosing the Variables to Estimate Singular DSGE Models.” *Journal of Applied Econometrics*, 29: 1099–1117.

Carlstrom, Charles T., and Timothy S. Fuerst. 2003. “Money Growth Rules and Price Level Determinacy.” *Review of Economic Dynamics*, 6: 263–275.

Chari, Varadarajan V., Patrick Kehoe, and Ellen R. McGrattan. 2008. “New Keynesian Models: Not Yet Useful for Policy Analysis.” Federal Reserve Bank of Minneapolis Working Paper 664.

Chetty, Raj, and Adam Szeidl. 2016. “Consumption Commitments and Habit Formation.” *Econometrica*, 84(2): 855–890.

Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans. 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy*, 113(1): 1–45.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. “Unemployment and Business Cycles.” *Econometrica*, 84(4): 1523–1569.

Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. 2011. “DSGE Models for Monetary Policy Analysis.” In *Handbook of Monetary Economics.* Vol. 3A, , ed. Benjamin Friedman and Michael Woodford, 285–367. Elsevier Science.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. “Risk Shocks.” *American Economic Review*, 104(1): 27–65.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *The Quarterly Journal of Economics*, 115(1): 147–180.

Comin, Diego, and Mark Gertler. 2006. “Medium-Term Business Cycles.” *The American Economic Review*, 96(3): 523–551.

Consolo, Agostino, and Carlo A. Favero. 2009. “Monetary Policy Inertia: More Fiction than Fact?” *Journal of Monetary Economics*, 56: 900–906.

Cooper, Russell W., and John C. Haltiwanger. 2006. "On the Nature of Capital Adjustment Costs." *Review of Economic Studies*, 73: 611–633.

Cooper, Russell W., John C. Haltiwanger, and Laura Power. 1999. "Machine Replacement and the Business Cycle: Lumps and Bumps." *The American Economic Review*, 89(4): 921–946.

Cuba-Borda, Pablo, Luca Guerrieri, Matteo Iacoviello, and Molin Zhong. Forthcoming. "Likelihood Evaluation of Models with Occasionally Binding Constraints." *Journal of Applied Econometrics.*

Cúrdia, Vasco, and Michael Woodford. 2016. "Credit Frictions and Optimal Monetary Policy." *Journal of Monetary Economics*, 84: 30–65.

Debortoli, Davide, Jordi Galí, and Luca Gambetti. 2019. "On the Empirical (Ir)relevance of the Zero Lower Bound Constraint." NBER Working Paper Series No. 25820.

Dey, Jaya. 2014. "Evaluating Monetary Policy Under Preferences with Zero Wealth Effect: A Bayesian Approach." *Journal of Economic Dynamics and Control*, 38: 209–234.

Diamond, Douglas W., and Philip H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

Doms, Mark, and Timothy Dunne. 1998. "Capital Adjustment Patterns in Manufacturing Plants." *Review of Economic Dynamics*, 1: 409–429.

Fazzari, Steven M., R. Glenn Hubbard, and Bruce C. Petersen. 1988. "Financing Constraints and Corporate Investment." *Brookings Papers on Economic Activity*, 1: 141–206.

Fiori, Giuseppe. 2012. "Lumpiness, Capital Adjustment Costs and Investment Dynamics." *Journal of Monetary Economics*, 59: 381–392.

Gathergood, John, Neale Mahoney, Neil Stewart, and Jörg Weber. 2019. "How do Individuals Repay Their Debt? The Balance-Matching Heuristic." *American Economic Review*, 109(3): 844–875.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58: 17–34.

Gertler, Mark, Luca Sala, and Antonella Trigari. 2008. "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining." *Journal of Money, Credit and Banking*, 40(8): 1713–1764.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrew Prestipino. 2018. "A Macroeconomic Model with Financial Panics." NBER Working Paper 24126.

Gomes, João F. 2001. "Financing Investment." *American Economic Review*, 91(5): 1263–1285.

Gomes, João F., Urbann Jermann, and Lukas Schmid. 2016. "Sticky Leverage." *American Economic Review*, 106(12): 3800–3828.

Gourio, François, and Anil K. Kashyap. 2007. "Investment Spikes: New Facts and a General Equilibrium Exploration." *Journal of Monetary Economics*, 54: 1–22.

Greenwood, Jeremy, and Boyan Jovanovic. 1990. "Financial Development, Growth, and the Distribution of Income." *Journal of Political Economy*, 98(5): 1076–1107.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *The American Economic Review*, 78(3): 402–417.

Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E. Smith. 2017. "The Empirical Implications of the Interest-Rate Lower Bound." *American Economic Review*, 107(7): 1971–2006.

Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica*, 50(1): 213–224.

Hennessy, Christopher A., and Toni M. Whited. 2005. "Debt Dynamics." *The Journal of Finance*, 60(3): 1129–1165.

Herbst, Edward, and Frank Schorfheide. 2015. *Bayesian Estimation of DSGE Models.* Princeton University Press.

Herbst-Murphy, Susan. 2012. "Small Business Use of Credit Cards in the U.S. Market." Federal Reserve Bank of Philadelphia, Payment Cards Center Discussion Paper.

Holmstrom, Bengt, and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *The Quarterly Journal of Economics*, 112(3): 663–691.

Jermann, Urbann, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–271.

Jovanovic, Boyan, and Chung-Yi Tse. 2010. "Entry and Exit Echoes." *Review of Economic Dynamics*, 13: 514–536.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2011. "Investment Shocks and the Relative Price of Investment." *Review of Economic Dynamics*, 14: 102–121.

Kashyap, Anil K., and Luigi Zingales. 2010. "The 2007-8 Financial Crisis: Lessons from Corporate Finance." *Journal of Financial Economics*, 97: 303–305.

Keen, Benjamin, and Evan F. Koenig. 2018. "How Robust Are Popular Models of Nominal Frictions?" *Journal of Money, Credit and Banking*, 50(6): 1299–1342.

Khan, Aubhik, and Julia K. Thomas. 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics." *Econometrica*, 76(2): 395–436.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. "Production, Growth and Business Cycles." *Journal of Monetary Economics*, 21: 195–232.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211–248.

Koenig, Evan F., Robert Leeson, and George A. Khan, ed. 2012. *The Taylor Rule and the Transformation of Monetary Policy.* . 1 ed., Hoover Institute Press.

Letterie, Wilko A., and Gerard A. Pfann. 2007. "Structural Identification of High and Low Investment Regimes." *Journal of Monetary Economics*, 54: 797–819.

McGrattan, Ellen R., and James A. Schmitz. 1999. "Maintenance and Repair: Too Big to Ignore." Federal Reserve Bank of Minneapolis Quarterly Review Fall 1999.

Nuño, Galo, and Carlos Thomas. 2017. "Bank Leverage Cycles." *American Economic Journal: Macroeconomics*, 9(2): 32–72.

Quint, Dominic, and Pau Rabanal. 2017. "Should Unconventional Monetary Policies Become Conventional?" *IMF Working Papers* 17/85.

Reiter, Michael, Tommy Sveen, and Lutz Weinke. 2013. "Lumpy Investment and the Monetary Transmission Mechanism." *Journal of Monetary Economics*, 60: 821–834.

Rouwenhorst, Geert. 1995. "Asset Pricing Implications of Equilibrium Business Cycle Models." In *Frontiers of Business Cycle Research.* , ed. Thomas F. Cooley, Chapter 10, 294–330. Princeton University Press.

Saglam, Cagri, and Vladimir Veliov. 2008. "Role of Endogenous Vintage Specific Depreciation in the Optimal Behavior of Firms." *International Journal of Economic Theory*, 4(3): 381–410.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Working Paper Series 11417.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2012. "What's News in Business Cycles." *Econometrica*, 80(6): 2733–2764.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in the US Business Cycles: A Bayesian DSGE Approach." *The American Economic Review*, 97(3): 586–606.

Stiglitz, Joseph E., and Andrew Weiss. 1981. "Credit Rationing in Markets with Imperfect Information." *The American Economic Review*, 71(3): 393–410.

Šustek, Roman. 2011. "Plant-Level Nonconvex Output Adjustment and Aggregate Fluctuations." *Journal of Monetary Economics*, 58: 400–414.

Thomas, Julia K. 2002. "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy*, 110(3): 508–534.

Winberry, Thomas. 2020. "Lumpy Investment, Business Cycles, and Stimulus Policy." Unpublished manuscript.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Wu, Jing C., and Fan D. Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit and Banking*, 48(2–3): 253–291.

Wu, Jing C., and Ji Zhang. 2019. "A Shadow Rate New Keynesian Model." *Journal of Economic Dynamics and Control*, 107: 1–29.

APPENDICES

A.1. Appendix for Chapter I

A.1 Data Construction

Table A.1. Sources of Raw Data

#	Series	Source	Table	Frequency	SA
(1)	Bank Credit, All Commercial Banks	FRB	H.8	Monthly	Yes
(2)	Cash Assets, All Commercial Banks	FRB	H.8	Monthly	Yes
(3)	Civilian Noninstitutional Population 16 Year and Over	BLS	ES	Monthly	No
(4)	Effective Federal Funds Rate	FRB	H.15	Monthly	No
(5)	Excess Reserves of Depository Institutions	FRSTL	H.3	Monthly	No
(6)	Government Consumption Expenditures and Gross Investment	BEA	1.1.5.	Quarterly	Yes
(7)	Gross Domestic Product	BEA	1.1.5.	Quarterly	Yes
(8)	Gross Domestic Product: Implicit Price Deflator	BEA	1.1.9.	Quarterly	Yes
(9)	Gross Private Domestic Investment	BEA	1.1.5.	Quarterly	Yes
(10)	Loans and Leases in Bank Credit, All Commercial Banks	FRB	H.8	Monthly	Yes
(11)	Loans to Commercial Banks, All Commercial Banks	FRB	H.8	Monthly	Yes
(12)	Moody's Seasoned Aaa Corporate Bond Yield	Moody's	N/A	Monthly	No
(13)	Moody's Seasoned Baa Corporate Bond Yield	Moody's	N/A	Monthly	No
(14)	Nonfarm Business Sector: Compensation per Hour	BLS	PC	Quarterly	Yes
(15)	Nonfarm Business Sector: Hours of All Persons	BLS	PC	Quarterly	Yes
(16)	Other Assets, All Commercial Banks	FRB	H.8	Monthly	Yes
(17)	Personal Consumption Expenditures: Nondurable Goods	BEA	1.1.5.	Quarterly	Yes
(18)	Personal Consumption Expenditures: Durable Goods	BEA	1.1.5.	Quarterly	Yes
(19)	Personal Consumption Expenditures: Services	BEA	1.1.5.	Quarterly	Yes
(20)	Personal Consumption Expenditures: Financial Services and Insurance	BEA	1.5.5.	Quarterly	Yes
(21)	Required Reserves of Depository Institutions	FRB	H.3	Monthly	No
(22)	Securities in Bank Credit, All Commercial Banks	FRB	H.8	Monthly	Yes
(23)	Total Liabilities, All Commercial Banks	FRB	H.8	Monthly	Yes

[*] 10/1/1985 to 1/1/2019. All data are accessed from the St. Louis FRED database. BEA—Bureau of Economic Analysis. BLS—Bureau of Labor Statistics. FRB—Board of Governors of the Federal Reserve System. FRSTL—Federal Reserve Bank of St Louis. H.3—Aggregate Reserves of Depository Institutions and the Monetary Base. H.8—Assets and Liabilities of Commercial Banks in the United States. H.15—Selected Interest Rates. 1.1.5.—Gross Domestic Product: Quarterly. 1.1.9.—Implicit Price Deflators for Gross Domestic Product: Quarterly. 1.5.5.—Gross Domestic Product, Expanded Detail: Quarterly. ES—Employment Statistics. PC—Productivity and Costs.

The eight series included in the estimation are functions of the raw data presented in Table A.1. The construction of each is presented in Table A.2. All variables are then demeaned and the growth

Table A.2. Construction of Estimation Variables

Variable Name	Construction
Output	(7)/(8)/(3)
Consumption	[(17)+(19)]/(8)/(3)
Investment	[(9)+(18)]/(8)/(3)
Labor Hours	(15)/(3)
Real Wages	(14)/(8)
Price Level	(8)
Marketable Securities	[(2)+(11)+(16)+(22)-(21)]/(8)/(3)
10 Year Loan to Risk Free Rate Spread	(13)/(4)

rate of each is calculated, with the exception of the credit spread variable which is only demeaned. The remaining series included in Table A.1 are used to calculate sample averages for the purposes of calibrating parameters. The construction of the calibration targets is presented in Table A.3.

Table A.3. Construction of Variables Used for Calibration

Variable Name	Construction	Target	Description
Real Risk Free Rate	(4)/(8)	β	Discount Factor
Government Spending to Output Ratio	(6)/(7)	G^y	Steady State of Government Spending Shock
Loan to Output Ratio	(10)/(7)	ζ	Collateral Proportion of Capital
Bank Assets to Output Ratio	(1)/(7)	τ^b	Bank Tax Rate
Firm Searches to Output Ratio	(20)/(7)	Ξ	Matching Efficiency
Leverage Ratio	(1)/[(1)-(23)]	ω	Proportion of Divertible Bank Assets

A.2 Technical

Households: The setup to introduce sticky wages follows Schmitt-Grohé and Uribe (2005). Workers are indexed by j and distributed uniformly on the interval $[0, 1]$. Individual worker hours are complementary, with the aggregate level of hours given by h_t. This is expressed as a Dixit-Stiglitz composite of the hours of each member of the household,

$$h_t = \left(\int_0^1 h_{jt}^{1-\frac{1}{\eta_w}} dj \right)^{\frac{1}{1-\frac{1}{\eta_w}}} \tag{A.1}$$

where $\eta_w > 1$ is the elasticity of substitution across worker types. We can relate this parameter to the estimated markup parameter λ_w as $\eta_w = \frac{1+\lambda_w}{\lambda_w}$. Each worker is paid according to their type, and paid the hourly wage w_{jt}. A hypothetical firm solves a cost minimization problem, minimizing the total wage expense $\int_0^1 w_{jt}h_{jt}dj$ subject to (A.1) by choosing demand for each worker type. The demand function arising from that problem for a worker of type j is then given by,

$$h_{jt} = \left(\frac{w_{jt}}{w_t} \right)^{-\eta_w} h_t \tag{A.2}$$

Households provide insurance to its members such that wage income is pooled and distributed evenly. The household assigns the task of wage setting subject to Calvo type wage setting frictions to a representative union that they own. The union must set the wage in a manner that renders any individual household member indifferent from deviating from the union mandated work plan. We derive the participation constraint of the individual member by solving the individual's lifetime utility maximization problem under the assumption that they are selecting their work plan and wage rather than the union. As consumption decisions do not interact with the work decision, I present only the parts of the utility maximization problem affected by the wage and labor supply decisions. Denote the period utility of member j as $U(c_t, h_t)$. This is not a function of j as the individual would continue to receive household insurance. The member is therefore concerned with their individual wage w_{jt} and the overall amount of work hours completed by the household h_t, as this will determine the consumption level. The member must additionally respect the demand constraint of firms, (A.2).

Assigning multipliers λ_{t1} and λ_{t2} to the budget and firm labor demand constraints respectively and denoting firm demanded labor as h_t^d, the relevant parts of the Lagrangian, $\mathcal{L}$, for our purposes is given by,

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{ & U(c_t, h_t) \\
& + \lambda_{t1} \left[h_t^d w_t \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{1-\eta_w} dj + \dots \right] \\
& + \lambda_{t2} \left[h_t - h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\eta_w} dj \right] \Bigg\} \\
& + \dots
\end{aligned}
$$

The first order conditions with respect to w_{jt} and h_t are then,

$$[w_{jt}]: \quad \lambda_{t1} \frac{\eta_w - 1}{\eta_w} w_{jt} = \lambda_{t2} \tag{A.3}$$

$$[h_t]: \quad \lambda_{t2} = U_h(c_t, h_t) \tag{A.4}$$

These conditions are then taken as additional constraints by the union in its optimization problem. As these must hold for all j, we must have symmetry in that $w_{jt} = w_t$. For convenience, I define $\mu_w = \frac{\eta_w}{\eta_w - 1}$.

Unions are faced with Calvo style wage adjustment frictions. With probability $\phi_w \in (0, 1)$, unions are unable to set a new wage in any particular period. Denote the optimal wage in periods where readjustment occurs as, $\widetilde{w}_t$. In periods where optimization is disallowed, wages are indexed to inflation at the rate $\iota_w \in (0, 1)$. The wage, that prevails at time t is then,

$$
w_t = \begin{cases} \widetilde{w}_t & \text{if} \quad \text{optization is permitted} \\ w_{t-1} \frac{(1+\pi_{t-1})^{\iota_w}}{1+\pi_t} & \text{otherwise} \end{cases}
$$

The drift in an optimally set wage s periods after the last reoptimization at time t are given by,[8]

$$\widetilde{w}_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{(1 + \pi_{t+k-1})^{\iota_w}}{1 + \pi_{t+k}}$$

[8]For convenience, we define $\Pi_{t+s} = \prod_{k=1}^{s} \frac{(1+\pi_{t+k-1})^{\iota_w}}{1+\pi_{t+k}}$.

The problem of the union is to maximize the portion of the household's utility maximization problem directly affected by changes in the real wage, subject to the participation constraints (A.3) and (A.4). This is done at period t, when without loss of generality, the union receives a positive Calvo signal, taking into account the expected value to the household under all future periods where the wage set at t is maintained. The union selects one wage $\widetilde{w}_t$ that will apply to each member, The Lagrangian of the union, $\mathcal{L}_t^u$, with all constraints already substituted is,[9]

$$\begin{aligned}
\mathcal{L}_t^u &= \mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d \left[w_{t+s}^{\eta_w} \int_0^1 \widetilde{w}_{t+s}^{1-\eta_w} dj - \int_0^1 \widetilde{w}_{t+s}^{-\eta_w} dj \frac{w_{t+s}^{1+\eta_w}}{\mu_w} \right] \\
&= \mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d w_{t+s}^{\eta_w} \left[\widetilde{w}_{t+s}^{1-\eta_w} - \widetilde{w}_{t+s}^{-\eta_w} \frac{w_{t+s}}{\mu_w} \right] \\
&= \mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d w_{t+s}^{\eta_w} \Pi_{t+s}^{-\eta_w} \left[\widetilde{w}_t^{1-\eta_w} \Pi_{t+s} - \widetilde{w}_t^{-\eta_w} \frac{w_{t+s}}{\mu_w} \right]
\end{aligned}$$

The above is now a function of $\widetilde{w}_t$, which is the sole variable under the control of the union and able to be set at time t. The first order condition with respect to $\widetilde{w}_t$ is then,

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d \left(\frac{\widetilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \Pi_{t+s}^{-\eta_w} \left[\frac{\eta_w - 1}{\eta_w} \widetilde{w}_t \Pi_{t+s} - \frac{w_{t+s}}{\mu_w} \right] = 0 \tag{A.5}$$

Using the first order conditions of the individual member that are treated as participation constraints, we can replace the μ_w term.[10] We then have,

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d \left(\frac{\widetilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \Pi_{t+s}^{-\eta_w} \left[\frac{\eta_w - 1}{\eta_w} \widetilde{w}_t \Pi_{t+s} + \frac{U_{h,t+s}}{\lambda_{t+s}} \right] = 0 \tag{A.6}$$

We can define the variables f_t^1 and f_t^2 as,

$$f_t^1 = \frac{\eta_w - 1}{\eta_w} \mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s \lambda_{t+s} h_{t+s}^d \left(\frac{\widetilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \Pi_{t+s}^{1-\eta_w} \widetilde{w}_t \tag{A.7}$$

$$f_t^2 = -\mathbb{E}_t \sum_{s=0}^{\infty} (\phi_w \beta)^s h_{t+s}^d \left(\frac{\widetilde{w}_t}{w_{t+s}} \right)^{-\eta_w} \Pi_{t+s}^{-\eta_w} U_{h,t+s} \tag{A.8}$$

[9] I replace λ_{t1} with λ_t for simplicity.

[10] We recognize that h_t enters the utility function negatively, so denote $U_h(c_t, h_t) = -U_{h,t}$.

Finally, each of these equations can be expressed recursively as,

$$f_t^1 = \left(\frac{\eta_w - 1}{\eta_w}\right) \widetilde{w}_t \lambda_t \left(\frac{w_t}{\widetilde{w}_t}\right)^{\eta_w} h_t^d + \psi_w \beta \mathbb{E}_t \left(\frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_w}}\right)^{\eta_w - 1} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t}\right)^{\eta_w - 1} f_{t+1}^1 \tag{A.9}$$

$$f_t^2 = -U_{h,t} \left(\frac{w_t}{\widetilde{w}_t}\right)^{\eta_w} h_t^d + \psi_w \beta \mathbb{E}_t \left(\frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_w}}\right)^{\eta_w - 1} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t}\right)^{\eta_w - 1} f_{t+1}^2 \tag{A.10}$$

The optimal wage is then determined implicitly by the equation,

$$f_t^1 = f_t^2 \tag{A.11}$$

We can define the wage index as $w_t = \left(\int_0^1 w_{jt}^{1-\eta_w} dj\right)^{(1-\eta_w)^{-1}}$. Then we can derive the law of motion for the real wage as,

$$\begin{aligned} w_t^{1-\eta_w} &= \int_0^1 w_{jt}^{1-\eta_w} dj \\ &= \int_0^{\phi_w} w_{jt}^{1-\eta_w} dj + \int_{\phi_w}^1 w_{jt}^{1-\eta_w} dj \\ &= \phi_w \widetilde{w}_t^{1-\eta_w} + (1-\phi_w) w_{t-1}^{1-\eta_w} \left(\frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_w}}\right)^{\eta_w - 1} \end{aligned} \tag{A.12}$$

Having established how wages are determined and evolve over time, we now direct attention to solving the household problem. Let $\mathcal{V}_t^h$ denote the value function of households. It is to be maximized by choosing sequences of $\{V_t\}$, and the endogenous states, $\{b_t\}, \{S_t\}, \{c_t\}$, while taking the sequences $\{tax_t\}, \{d_t\}, \{r_{t-1}\}, \{w_t\}, \{h_t\}$, as given. The optimal solution is characterized by a set of policy functions for each choice variable and endogenous state, as a function of the state variables. We can express the Bellman equation for the household discounted lifetime utility maximization problem as,

$$\begin{aligned} \mathcal{V}_t^h(c_{t-1}, S_{t-1}, b_{t-1}) = \max \ & \frac{V_t^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}_t \mathcal{V}_{t+1}^h(c_t, S_t, b_t) \\ \text{s.t.} \quad & V_t = c_t - Hc_{t-1} - \Psi_t h_t^{1+\frac{1}{\varepsilon}} S_t \\ & S_t = (c_t - Hc_{t-1})^{\gamma_t} S_{t-1}^{1-\gamma_t} \\ & c_t + tax_t + b_t = w_t h_t + b_{t-1}(1 + r_{t-1}) + d_t \end{aligned}$$

I assign the Lagrange multipliers λ_t, Π_t, and Λ_t to the constraints respectively. The first order and envelope conditions are then given by,

$$
\begin{aligned}
&[V_t]: && V_t^{-\sigma} = \lambda_t \\
&[c_t]: && \beta \mathbb{E}_t \mathcal{V}^h_{t+1,c} + \lambda_t = \gamma_t \Pi_t \frac{S_t}{c_t - Hc_{t-1}} + \Lambda_t \\
&[c_{t-1}]: && \mathcal{V}^h_{t,c} = \gamma_t H \Pi_t \frac{S_t}{c_t - Hc_{t-1}} - H\lambda_t \\
&[S_t]: && \beta \mathbb{E}_t \mathcal{V}^h_{t+1,S} + \Pi_t = \Psi_t h_t^{1+\frac{1}{\varepsilon}} \lambda_t \\
&[S_{t-1}]: && \mathcal{V}^h_{t,S} = -(1-\gamma_t) \frac{S_t}{S_{t-1}} \Pi_t \\
&[b_t]: && \beta \mathbb{E}_t \mathcal{V}^h_{t+1,b} = \Lambda_t \\
&[b_{t-1}]: && \mathcal{V}^h_{t,b} = \Lambda_t (1 + r_{t-1})
\end{aligned}
$$

Substituting away the value function derivatives, I obtain the household optimal conditions as,

$$V_t^{-\sigma} - \gamma_t \Pi_t \frac{S_t}{c_t - Hc_{t-1}} - \beta H \mathbb{E}_t \left[V_{t+1}^{-\sigma} - \gamma_{t+1} \Pi_{t+1} \frac{S_{t+1}}{c_{t+1} - Hc_t} \right] = \Lambda_t \tag{A.13}$$

$$\Psi_t h_t^{1+\frac{1}{\varepsilon}} V_t^{-\sigma} + \beta \mathbb{E}_t (1 - \gamma_{t+1}) \Pi_{t+1} \frac{S_{t+1}}{S_t} = \Pi_t \tag{A.14}$$

$$\Lambda_t = \beta (1 + r_t) \mathbb{E}_t \Lambda_{t+1} \tag{A.15}$$

It should be noted that (A.15) is the Euler Equation under preferences with habit formation. The stochastic discount factor in the model is then given as referenced in the main text, as a function of the Lagrange multipliers in the Euler Equation.

In addition to the necessary first order conditions, I also obtain the condition for h_t. This is to set the steady state value of Ψ, the labor disutility parameter that evolves exogenously. Substituting the function V_t into the utility function and differentiating with respect to h_t will yield the marginal utility of labor, which is necessary in the wage setting function (A.11).

Firms: The representative firm outlined in the main paper is the production arm of a larger firm. Once produced, goods are transferred to a separate pricing unit within the overall firm. The real transfer price is an at-cost transfer price denoted mc_t.[11] The pricing unit costlessly damages each good such that they are able to sell differentiated versions. They recognize the monopolistic competition market structure that ensues and price each good accordingly. The aggregate bundle of goods is a Dixit-Stiglitz aggregate of individual varieties i. The aggregate bundle is given by,

$$y_t = \left(\int_0^1 y_{it}^{1-\frac{1}{\eta_p}} di \right)^{\frac{1}{1-\frac{1}{\eta_p}}} \tag{A.16}$$

where $\eta_p > 1$ is the elasticity of substitution among varieties. A representative household solves a hypothetical cost minimization problem whereby the total cost $\int_0^1 p_{it} y_{it} di$ is minimized subject to (A.16), by choosing optimal levels of each variety. The demand function for variety i arising from this is,

$$y_{it} = y_t \left(\frac{p_{it}}{P_t} \right)^{-\eta_p} \tag{A.17}$$

The pricing unit sets the price while needing to satisfy demand at whatever price is chosen, and subject to Rotemberg adjustment costs that allow for price indexation at the rate $\iota_p \in (0,1)$. The parameter $\phi_p > 0$ captures price adjustment costs and thus the degree of price stickiness. The objective of the pricing unit is to maximize nominal profits. The relevant discount factor is thus the nominal version of the firm stochastic discount factor, which I denote by $\beta\Theta^n_{t,t+1}$.[12] The profit maximization problem is then,

$$\begin{aligned} \max_{p_{it}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Theta^n_{t,t+1} \left\{ (p_{it} - MC_t) y_{it} - \frac{\phi_p}{2} \left[\frac{p_{it}}{(1+\pi_{t-1})^{\iota_p} p_{i,t-1}} - 1 \right]^2 P_t y_t \right\} \\ \text{s.t.} \quad & y_{it} = y_t \left(\frac{p_{it}}{P_t} \right)^{-\eta_p} \end{aligned}$$

The resulting first order condition is,

$$\begin{aligned} & (1-\eta_p) y_t \left(\frac{p_{it}}{P_t} \right)^{-\eta_p} - y_t \frac{P_t}{p_{i,t-1}} \frac{\phi_p}{(1+\pi_{t-1})^{\iota_p}} \left[\frac{p_{it}}{(1+\pi_{t-1})^{\iota_p} p_{i,t-1}} - 1 \right] \\ & + \eta_p y_t \frac{MC_t}{p_{it}} \left(\frac{p_{it}}{P_t} \right)^{-\eta_p} + \beta \Theta^n_{t,t+1} \frac{P_{t+1}}{p_{it}} \frac{p_{i,t+1}}{p_{i,t}} \frac{y_{t+1}\phi_p}{(1+\pi_t)^{\iota_p}} \left[\frac{p_{i,t+1}}{(1+\pi_t)^{\iota_p} p_{it}} - 1 \right] = 0 \end{aligned} \tag{A.18}$$

[11]The notation for the nominal marginal cost is MC_t.

[12]The relationship between the nominal and real stochastic discount factors is given by $\Theta_{t,t+1} = \frac{\Theta^n_{t,t+1}}{1+\pi_{t+1}}$.

Imposing symmetry by setting $p_{it} = P_t$ for all i results in the pricing formula,

$$1 - \eta_p + \eta_p mc_t - \phi_p \frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} \left[\frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} - 1 \right] + \beta \Theta_{t,t+1} \phi_p \frac{y_{t+1}}{y_t} \frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_p}} \left[\frac{1+\pi_{t+1}}{(1+\pi_t)^{\iota_p}} - 1 \right] = 0 \tag{A.19}$$

The above fully specifies the economic decisions made by the pricing unit. We now consider a full description of the production unit's problem, where the problem is as specified in the main text, with some details added that were omitted in the main text. The production unit receives a lump sum transfer of real profits, Π_t^p, from the pricing unit that is given by,

$$\Pi_t^p = (1 - mc_t) y_t - \frac{\phi_p}{2} \left[\frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} - 1 \right]^2 y_t \tag{A.20}$$

Pricing unit profits are taken as given by the production unit. The detailed budget constraint of the production unit is therefore,

$$mc_t y_t + \Pi_t^p = w_t h_t + inv_t(1+\tau) - \tau \delta k_t + \Psi(u_t) k_t + \varphi(d_t) + r_t^f L_t + \frac{\kappa_L}{2} x_t^2 L_t \tag{A.21}$$

I motivate the borrowing constraint as in Jermann and Quadrini (2012). Firms receive a zero interest working capital loan that is paid back intra-temporally once profits are realized. The working capital loan covers all budget constraint expenses on the right-hand side, net of the tax shield on depreciation. In addition, a lump sum amount is included in the intra-temporal loan that covers the price adjustment costs incurred by the pricing unit. Denote $N_t^p = \Pi_t^p + \frac{\phi_p}{2} \left[\frac{1+\pi_t}{(1+\pi_{t-1})^{\iota_p}} - 1 \right]^2 y_t$, as grossed up pricing unit profits where price adjustment costs are excluded. The borrowing constraint of the production unit is then given by,

$$\zeta_t k_{t+1} - L_{t+1} = mc_t y_t + N_t^p \tag{A.22}$$

The remainder of the production unit's constraints are as specified in the main text. The firm value optimization problem is given by,

$$
\begin{aligned}
&FV(inv_{t-1}, k_t, L_t) = \max d_t + (1 - z_t + r_t^f)L_t + \beta \mathbb{E}_t \Theta_{t,t+1} FV(inv_t, k_{t+1}, L_{t+1}) \\
\text{s.t.}\quad & mc_t y_t + \Pi_t^p = w_t h_t + inv_t(1+\tau) - \tau\delta k_t + \Psi(u_t)k_t + \varphi(d_t) + r_t^f L_t + \frac{\kappa_L}{2}\frac{(Q_t S_t)^2}{L_t} \\
& \zeta_t k_{t+1} - L_{t+1} = mc_t y_t + N_t^p \\
& (x_t + z_t)L_t = L_{t+1} \\
& \frac{Q_t S_t}{L_t} = x_t \\
& k_{t+1} = (1-\delta)k_t + \zeta_t^I \Upsilon\left(\frac{inv_t}{inv_{t-1}}\right) \\
& y_t = A_t (u_t k_t)^\alpha h_t^{1-\alpha}
\end{aligned}
$$

The technology constraint is substituted for y_t and I assign the vector of Lagrange multipliers $\langle \lambda_{t1}, \mu_t, \lambda_{t2}, \lambda_{t3}, q_t \rangle$ to each respective constraint. In particular, the multiplier μ_t measures the tightness of the borrowing constraint, q_t is Tobin's q, and λ_{t2} is the shadow value of loosening the law of motion of loans. This captures the marginal value of acquiring additional loans to firms and represents their surplus in the bargaining game that determines the nominal long term interest rate. The first order and envelope conditions are given by,

$$[d_t]: \qquad 1 = \lambda_{t1}\varphi'(d_t) \tag{A.23}$$

$$[h_t]: \qquad \lambda_{t1} w_t = (1-\alpha) mc_t \frac{y_t}{h_t}(\lambda_{t1} - \mu_t) \tag{A.24}$$

$$[u_t]: \qquad \lambda_{t1}\Psi'(u_t) u_t = \alpha mc_t \frac{y_t}{k_t}(\lambda_{t1} - \mu_t) \tag{A.25}$$

$$[inv_t]: \qquad \beta \mathbb{E}_t \Theta_{t,t+1} FV_{t+1,inv} + q_t \zeta_t^I \Upsilon_{\uparrow,t} = \lambda_{t1}(1+\tau) \tag{A.26}$$

$$[inv_{t-1}]: \quad FV_{t,inv} = q_t \zeta_t^I \Upsilon_{\downarrow,t} \tag{A.27}$$

$$[k_{t+1}]: \qquad \beta \mathbb{E}_t \Theta_{t,t+1} FV_{t+1,k} = q_t - \mu_t \zeta_t \tag{A.28}$$

$$[k_t]: \qquad FV_{t,k} = \alpha mc_t \frac{y_t}{k_t}(\lambda_{t-1} - \mu_t) + \lambda_{t1}[\tau\delta - \Psi(u_t)] + q_t(1-\delta) \tag{A.29}$$

$$[x_t]: \qquad \lambda_{t2} L_t = \lambda_{t3} \tag{A.30}$$

$$[S_t]: \qquad \lambda_{t1} \kappa_L x_t L_t = \lambda_{t3} \tag{A.31}$$

$$[L_{t+1}]: \qquad \beta \mathbb{E}_t \Theta_{t,t+1} FV_{t+1,L} = \lambda_{t,2} + \mu_t \tag{A.32}$$

$$[L_t]: \qquad FV_{t,L} = 1 - z_t + r_t^f - \lambda_{t1}\left(r_t^f - \frac{\kappa_L}{2} x_t^2\right) + (x_t + z_t)\lambda_{t2} - \frac{x_t}{L_t}\lambda_{t3} \tag{A.33}$$

Banks: The expected terminal net worth of a bank is given as in (I.40). To proceed, this must be expressed recursively to obtain the bankers' objective in (I.41). The derivation follows as,

$$\begin{aligned}
\mathcal{V}_{t-1} &= (1-\tau^b)(1-\theta^B)\mathbb{E}_{t-1}\sum_{j=0}^{\infty}(\theta^B)^j \beta^{j+1}\frac{\Lambda_{t+j}}{\Lambda_{t-1}} nw_{t+j} \qquad \text{(A.34)} \\
&= (1-\tau^b)(1-\theta^B)\left[\mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}} nw_t + \mathbb{E}_{t-1}\sum_{j=1}^{\infty}(\theta^B)^j \beta^{j+1}\frac{\Lambda_{t+j}}{\Lambda_{t-1}} nw_{t+j}\right] \\
&= (1-\tau^b)(1-\theta^B)\left[\mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}} nw_t + \mathbb{E}_{t-1}\sum_{j=0}^{\infty}(\theta^B)^{j+1} \beta^{j+2}\frac{\Lambda_{t+1+j}}{\Lambda_{t-1}} nw_{t+1+j}\right] \\
&= (1-\tau^b)(1-\theta^B)\left[\mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}} nw_t + \theta^B \mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}}\mathbb{E}_t\sum_{j=0}^{\infty}(\theta^B)^j \beta^{j+1}\frac{\Lambda_{t+1+j}}{\Lambda_t} nw_{t+1+j}\right] \\
&= \mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}}\left[(1-\tau^b)(1-\theta^B) nw_t + \theta^B \mathcal{V}_t\right]
\end{aligned}$$

The endogenous state variables of the problem are given by the vector $\langle G_{t-1}, b_{t-1}\rangle$, bank assets and deposits respectively. At time t, the objective of the banker is to maximize the value function $\mathcal{V}_t$, subject to the participation, balance sheet, and net worth evolution constraints. The full statement of the terminal net worth optimization problem is then,

$$\begin{aligned}
\mathcal{V}(G_{t-1},b_{t-1}) &= \mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}}\left[(1-\tau^b)(1-\theta^B)nw_t + \theta^B \max \mathcal{V}(G_t,b_t)\right] \\
\text{s.t.} \quad & G_t = nw_t + (1-req)b_t \\
& nw_t = (1-\tau^b)\left(R^G_{t-1}G_{t-1} - R_{t-1}(1-req)b_{t-1}\right) \\
& \mathcal{V}(G_t,b_t) \geq \omega G_t
\end{aligned}$$

We proceed by way of the method of undetermined coefficients. The guess of the value function is $\mathcal{V}(G_t,b_t) = \mathcal{V}^G_t G_t - \mathcal{V}^b_t(1-req)b_t$, where $\mathcal{V}^G_t, \mathcal{V}^b_t > 0$. I first solve the maximization of the value function in time t, contained in the bracketed term of the aforementioned bankers' problem. In so doing, I assume that the participation constraint is always binding as in Gertler and Karadi (2011). I substitute the balance sheet constraint for b_t, the guess of the value function, and assign the Lagrange multiplier λ_t to the participation constraint. The Lagrangian $\mathcal{L}_t$ is then,

$$\mathcal{L}_t = \mathcal{V}^G_t G_t - \mathcal{V}^b_t(G_t - nw_t) + \lambda_t\left[\mathcal{V}^G_t G_t - \mathcal{V}^b_t(G_t - nw_t) - \omega G_t\right] \tag{A.35}$$

The first order condition with respect to bank assets is then,

$$\mathcal{V}^G_t = \mathcal{V}^b_t + \frac{\omega\lambda_t}{1+\lambda_t} \tag{A.36}$$

Substituting the FOC into the Value Function yields,

$$\begin{aligned}
\mathcal{V}_t &= G_t\left(\mathcal{V}^b_t + \frac{\omega\lambda_t}{1+\lambda_t}\right) - \mathcal{V}^b_t(G_t - nw_t) \\
&= \mathcal{V}^b_t nw_t + \omega G_t \frac{\lambda_t}{1+\lambda_t}
\end{aligned} \tag{A.37}$$

and substituting the FOC into the participation constraint yields,

$$V^b_t nw_t = \omega G_t\left(1 - \frac{\lambda_t}{1+\lambda_t}\right) \tag{A.38}$$

Then substituting for bank assets between (A.37) and (A.38),

$$\mathcal{V}_t = \mathcal{V}_t^b nw_t(1+\lambda_t) \tag{A.39}$$

We can now substitute (A.39) into the Bellman equation which leaves,

$$\mathcal{V}_{t-1} = \mathbb{E}_{t-1}\beta\frac{\Lambda_t}{\Lambda_{t-1}}\left[(1-\tau^b)(1-\theta^B)nw_t + \theta^B\mathcal{V}_t^b(1+\lambda_t)nw_t\right] \tag{A.40}$$

At this point, we define the coefficient $\Omega_t = (1-\tau^b)(1-\theta^B) + \theta^B\mathcal{V}_t^b(1+\lambda_t)$. This coefficient leaves the value function as,

$$\mathcal{V}_t = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}nw_{t+1} \tag{A.41}$$

We see that under this formulation, the effective discount factor of the bank is a multiple of the term Ω and the household's discount factor. This is a function of taxes, the survival probability, the tightness of the participation constraint, and the risk free short term rate.

In order to verify the guess of the value function, we substitute the evolution of net worth constraint into (A.41) yielding,

$$\mathcal{V}_t = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}(1-\tau^b)\left[R_t^G G_t - R_t(1-req)b_t\right] \tag{A.42}$$

Our guess is verified as a result, and we can see that the coefficient $\mathcal{V}_t^G = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}(1-\tau^b)R_t^G$ and $\mathcal{V}_t^b = \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}(1-\tau^b)R_t$, so that the coefficients represent the after-tax gross return on bank assets and the after-tax gross cost of deposits, respectively.

We can now use the participation constraint of the household to determine the optimal leverage ratio for banks. Define the leverage ratio as $lev_t := G_t/nw_t$. Then we have,

$$\begin{aligned} lev_t &= \frac{\mathcal{V}_t^b}{\omega - (\mathcal{V}_t^G - \mathcal{V}_t^b)} \\ &= \frac{\mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}(1-\tau^b)R_t}{\omega - \mathbb{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}(1-\tau^b)(R_t^G - R_t)} \end{aligned} \tag{A.43}$$

Steady State Determination: The steady state is determined numerically. In this section I describe how the calibrated targets in Table 7 are implemented by targeting the values of certain parameters and give a broad outline of how the steady state is calculated by isolating particular sections of the model.

Some steady state values are immediately obvious from particular model equations. The Euler equation (A.15) yields the long term real interest rate, which I target to the sample average real federal funds rate. The model is stationary, so I set the inflation target in the Taylor Rule (I.47) to a rate of zero. Using the Fischer equation, the short term nominal interest rate is then equal to the real rate in steady state. Also, the steady state marginal cost and Tobin's q are immediately determined by the pricing unit's FOC (A.19) and the investment demand condition (I.32). The steady state values of the above are summarized as,

$$r = 0.0035 \quad \pi^* = 0 \qquad i = r \quad mc = \frac{\eta_p - 1}{\eta_p} \quad q = 1$$

I set the level of capital utilization $u = 1$, so that capital is fully utilized in steady state. This is achieved by targeting the parameter ϑ_u. Consider the steady state capital demand (I.31) and utilization demand (I.33). Imposing $u = 1$ necessitates that,

$$\vartheta_u = \frac{1 - \zeta\mu}{\beta z} - \tau\delta - (1 - \delta) \tag{A.44}$$

I first consider the steady state in the firm block of the model. I impose a steady state loan to output ratio as in Table 7 by targeting the steady state proportion of capital that can be used as collateral ζ in the borrowing constraint (A.22). At the posterior mean, this corresponds to $\zeta = 0.2622$. I also impose a steady state level of work hours as $h = 0.3$, which corresponds to a roughly 50 hour work week. This requires targeting the steady state value of Ψ, the labor disutility term in the wage setting equation ($A.11$). I also target the steady state ratio of firm searches to output S/y. This is mapped to the data as the sample average of per capita real financial services consumption to GDP. The matching efficiency parameter Ξ is targeted to achieve this steady state outcome. The remaining variables in the firm block of the model are determined simultaneously using a numerical solver.

In the bank block, I target the steady state ratio of bank assets to output G/y. The mapping to the data is the sample average of per capita real bank credit to GDP. This is achieved by targeting the tax rate τ^b using the net worth evolution equation (I.38). Additionally, the steady state leverage ratio is imposed by targeting the proportion of bank assets that can be diverted using the optimal leverage condition (A.43). All steady state quantities in the block are then found using a numerical solver.

Lastly, the values from the bank and firm blocks are input into the household block and the relevant household equations are solved simultaneously in steady state by a numerical solver to obtain to the remaining levels.

A.3 Other Impulse Response Functions

The impulse response functions for technology, investment efficiency, monetary policy, and collateral quality shocks were presented in Section 6.2. The responses for the remaining four shocks are presented in this section.

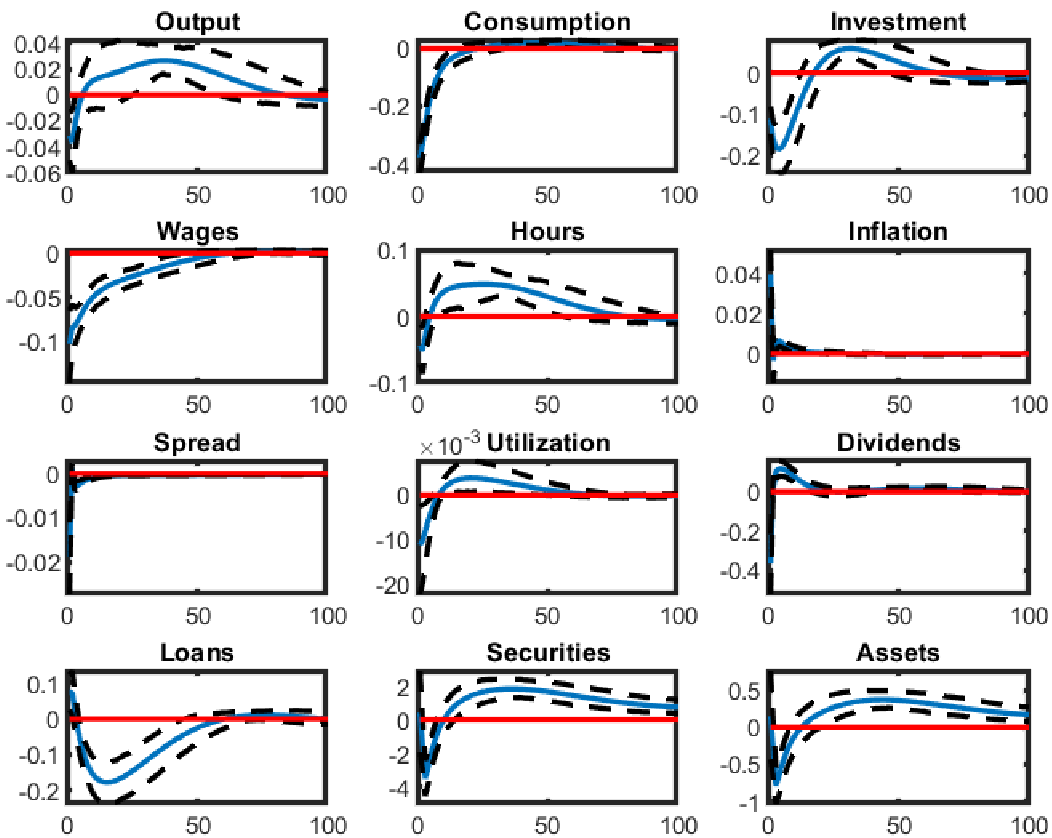


Figure A.1. Impulse Responses—Government Spending Shocks

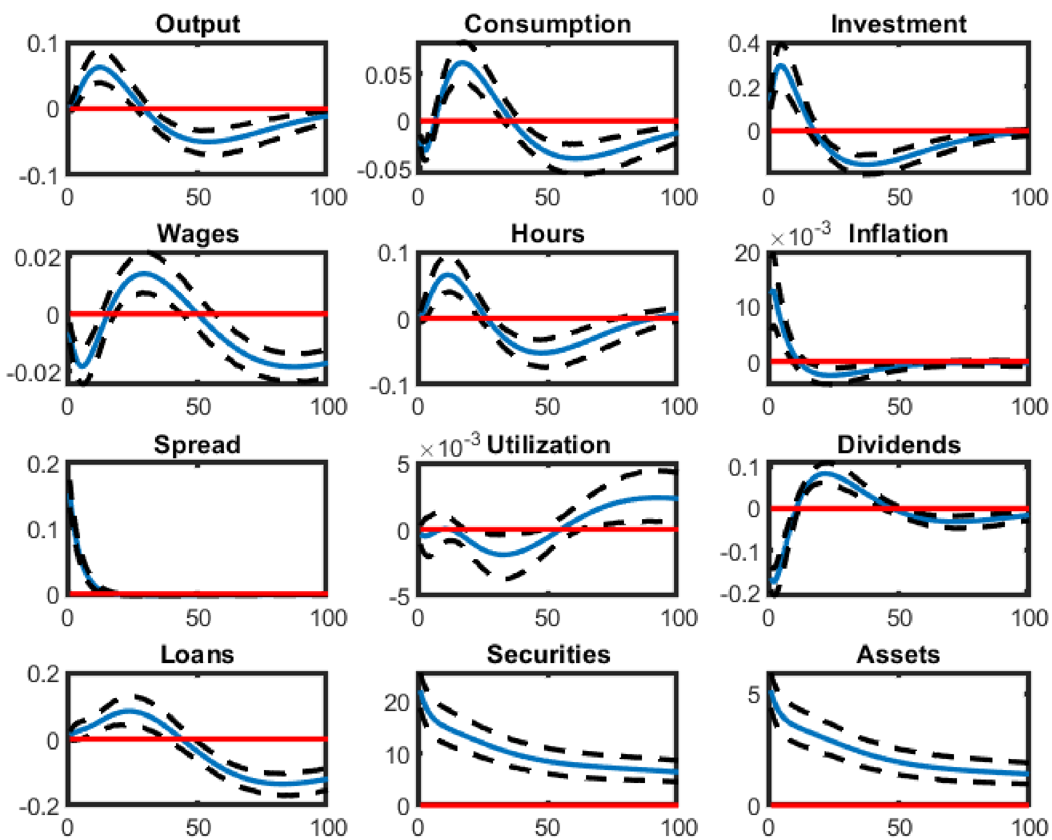


Figure A.2. Impulse Responses—Bank Bargaining Weight Shocks

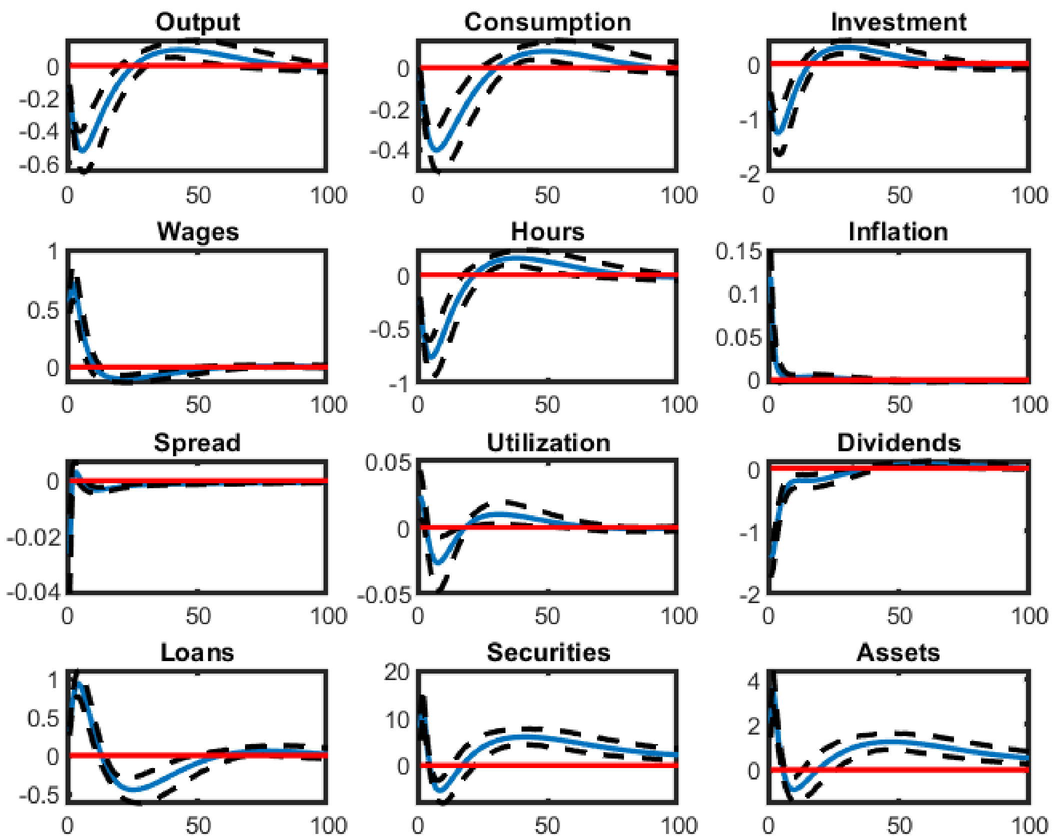


Figure A.3. Impulse Responses—Wage Markup/Labor Disutility Shocks

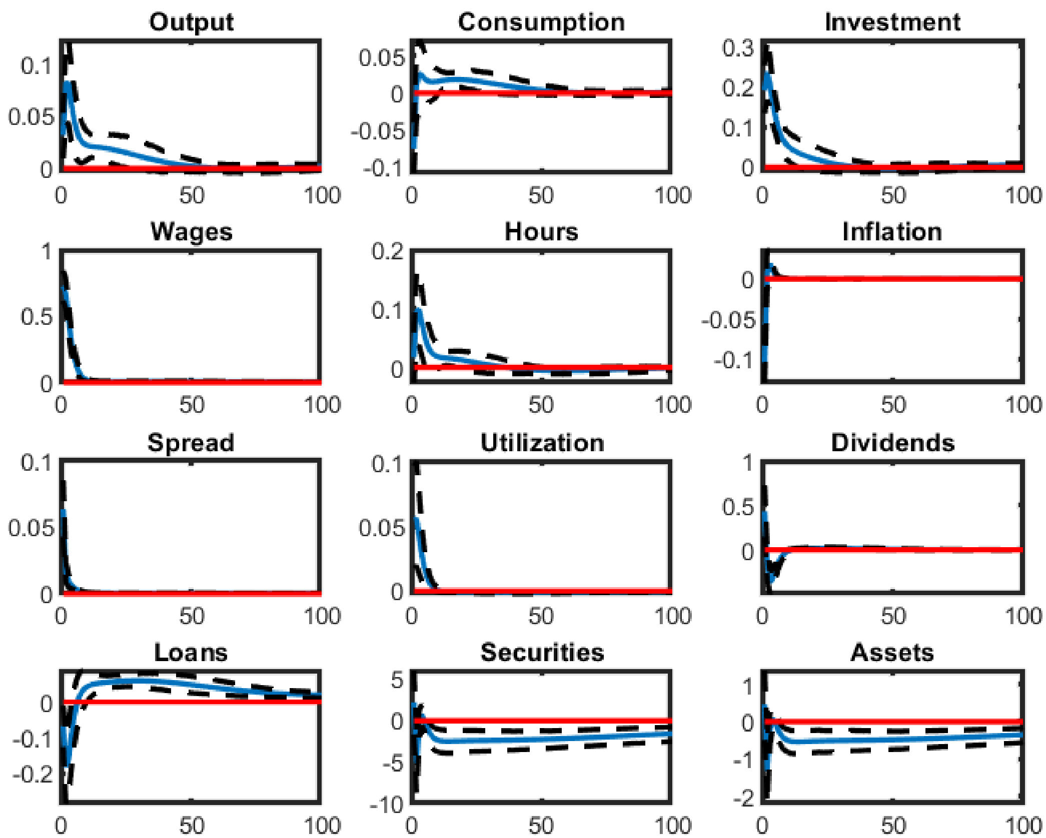


Figure A.4. Impulse Responses—Income Effect in Labor Supply Shocks

A.2. Appendix for Chapter II

A.1 The Model of Cooper, Haltiwanger and Power (1999)

The state space is three dimensional and a function of (k, ε, A). I discretize the capital and productivity states as in Appendix A.2. The size of the grid is therefore $70 \times 10 \times 5$. Firms incur fixed cost ϕ if a replacement decision is made. If not, capital depreciates. The sub-value functions W^R and W^D are given by,

$$W^R(k, \varepsilon, A) = \max_{h>0} \varepsilon A \lambda k^\alpha h^{1-\alpha} - wh - F + \beta \mathbb{E} W(1, \varepsilon', A') \tag{A.45}$$

$$W^D(k, \varepsilon, A) = \max_{h>0} \varepsilon A k^\alpha h^{1-\alpha} - wh + \beta \mathbb{E} W((1-\delta)k, \varepsilon', A') \tag{A.46}$$

$$W = \max\{W^D, W^R\} \tag{A.47}$$

The resulting policy function $R(k, \varepsilon, A) \in \{0, 1\}$ is recovered based on which sub-value function is chosen.

A.2 Solution Method

The model is solved by Value Function Iteration. I define 4 grids covering each state. The credit card grid cc^g is a $J = 250$ point grid. The capital grid K^g is a $K = 70$ point grid. The aggregate state is discretized as described in Section 2.5. The idiosyncratic state is discretized as a tensor grid with dimensions $(M \times 2) \times 2$ with $M = 10$ being the number of grid points for idiosyncratic productivity. There are 2 credit limit states. I continue to denote the idiosyncratic state as s.

The algorithm proceeds as follows,

1. Calculate the first order condition for labor and recover the demand function for labor. Substitute the demand function for h.

2. Pre-allocate the following,

   - The cash proportion of payments under each possible decision.
   - Output less expenses under each possible decision.
   - The credit card balance in the following period under each possible strategy.

- The probabilities of forbearance for each combination of idiosyncratic and aggregate productivity state.

3. Choose an initial guess for the value function, V^0.

4. Set a convergence tolerance. I choose 10^{-5}.

5. Enter the main loop beginning at,

   (a) For each decision, interpolate on the value function with the sample points being the next period credit card balance.

   (b) Calculate the expected value function.

   (c) Calculate the value function today.

   (d) Derive the policy functions for each decision.

   (e) Check convergence. If convergence is satisfied, exit the loop, if not, update the guess to the newly calculated value function and repeat the loop.