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# CORRESPONDENCE

## A TOUR OF MISTAKES

*Paul H. Edelman**

In these pages,[1] Steven Lubet recently reviewed *A Tour of the Calculus*, by David Berlinski.[2] Inspired by both the beauty of calculus and Berlinski's description of it, Lubet waxes poetic on the many parallels between the law and calculus. It is completely understandable—even admirable—that one might be led to ruminations on the relationship between calculus and one's own discipline. There is little doubt that the subject of calculus stands as one of the great intellectual feats of Western thought. It has had profound implications for physics, engineering, economics and many other disciplines—so why not law? Alas, these philosophical musings would be more persuasive had Professor Lubet better understood what it was that he was writing about.

Lubet's errors come in two types. The first is just a misunderstanding of history, but it is a misunderstanding that unfortunately forms the basis for an entire section of his review. The second type of error is more fundamentally mathematical: he does not distinguish between a definition and a theorem. Just as Lubet draws legal lessons from calculus, we can draw legal parallels from his mistakes. While some of these might be comforting, others will be more unsettling.

As noted by Lubet, the development of calculus was done more or less simultaneously in the mid-17th century by Sir Isaac Newton and Gottfried Wilhelm Leibniz. Leibniz based much of his development of the subject on the idea of an *infinitesimal*, a class of numbers that are smaller than any other number. According to Lubet, "the 'infinitesimals' turn out to be a futile fiction, notwithstanding Liebnitz's [sic] own endorsement of them. In 1734, Bishop Berkeley proved conclusively that they do not and cannot exist."[3] Lubet goes on in Part III to draw a number of legal parallels to this discrediting of the idea of infinitesimals. While the legal conclusions he draws from these events may well be true, Lubet cannot base them on the invalidity of infinitesimals: the fact of the matter is that Leibniz was right.

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* Professor of Mathematics, University of Minnesota, <edelman@math.umn.edu>. I would like to thank Steven Lubet and Suzanna Sherry for their helpful comments.

[1] *See* Steven Lubet, *A Tour of the Calculus of Justice*, 92 NW. U. L. REV. 1035 (1998).

[2] DAVID BERLINSKI, A TOUR OF THE CALCULUS (1995).

[3] Lubet, *supra* note 1, at 1037 (footnote omitted).

To be fair to Lubet, the ultimate vindication of Leibniz's belief in infinitesimals is hidden in a footnote by Berlinski: "The development of [non-Archimedean] fields by the logician Abraham Robinson in the twentieth century has made possible the development of the calculus entirely along the lines anticipated by Leibnitz [sic]."[4] Nevertheless, anyone with serious mathematical training would not have needed Berlinski's footnote; Lubet's error highlights the danger of relying on secondhand knowledge of a field quite different from one's own. Moreover, culpability aside, Lubet has lost the foundation for the legal insights he draws from the purported invalidity of infinitesimals.

And what of the supposed "proof" of Bishop Berkeley? Berlinski writes that "[w]riting in 1734, Bishop Berkeley wasted no time in attacking the very idea of infinitesimals," and later says that "[l]ooking backward, *we* can see that Berkeley was entirely correct,"[5] but never claims that Bishop Berkeley "proved conclusively" anything about the existence of infinitesimals. Indeed, he couldn't have, since by appropriately generalizing the idea of a number, Abraham Robinson was able to define them. Lubet should be more careful in using the term "proof" in the context of mathematics.

Lubet sees more parallels between the computation of the area under a curve and the way that legal trials "proceed by means of accretion of detail."[6] Surprisingly, Lubet doesn't draw the obvious parallel, that just as the sum of more and more rectangles gives better and better approximations for the area under a curve, as a trial proceeds the evidence presented gives a better and better approximation of the truth. He instead focuses on the error in the mathematical approximation:

> An integral combines rectangles until the limit of the error approaches zero, but the error-zone never actually becomes zero. . . . There is always a gap. In fact, there is always a boundless gap, because two numbers, no matter how close, are always separated by an infinite interval.
>
> Between the knowledge that we gain at trial, and real events as they occurred in the world, there is also always an infinite gap. We can *never*, not ever, know all of the rich, textured, complex, measureless, particulars that comprise actual experience.[7]

There is one obvious thing wrong with Lubet's description of the integral. It is simply not true that "there is always a boundless gap, because

[4] BERLINSKI, *supra* note 2, at 117 n.2. As Lubet observes in his reply, Berlinski asserts that a development of calculus using infinitesimals is less plausible than one using limits. *See* Steven Lubet, *Lubet Replies to Edelman: Mistakes? What Mistakes?*, 93 Nw. U. L. Rev. 347, 348 (1998). This only means that such a derivation is less intuitive and somewhat harder to understand, not that it is incorrect. There is no "dissension" among mathematicians as to the rigor of such a development. *Id.*

[5] BERLINSKI, *supra* note 2, at 114.

[6] Lubet, *supra* note 1, at 1043.

[7] *Id.* at 1043-44.

two numbers, no matter how close, are always separated by an infinite interval." Just because there are an infinite number of numbers between any two other numbers does not make the gap between them boundless. Such an assertion is equivalent to saying the distance between Evanston and Chicago is the same as the distance between Minneapolis and Chicago.

More important than this mistake is Lubet's fundamental confusion about what it *means* to compute the area under a curve. "Area" is a mathematically sophisticated notion. Except for rectangles, it is difficult to be precise about what exactly area is. In fact, mathematicians *define* the area under the curve to be the limit of certain sums. That is, calculus defines precisely what the "area" under a curve is and then offers a theorem that allows one to compute its value.[8] Berlinski is careful to make this point,[9] although Lubet apparently overlooks it.

What lesson from law should we draw from this more accurate description of what integral calculus does? The closest parallel of the sort that Lubet makes would be to claim that the outcome of the trial is not an approximation of the truth but is, in fact, the definition of the truth. Maybe lawyers should not learn too much from calculus.

Many disciplines, other than the traditional scientific ones, have attempted to use mathematics to give a firm foundation for their research. In some cases utilizing mathematics has led to some real insight. In other cases it has produced, at best, a veneer of respectability.

Given the growth of "law and" disciplines, it would not be surprising if mathematics were to creep into legal academia. Perhaps this commentary will sound a cautionary note before Law and Mathematics joins these other mongrels. Unlike the writing of French philosophers, the theorems of mathematics are not subject to wide interpretation. They mean something. And they should be understood before they are invoked.

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8 Thus, contrary to Lubet's continued insistence, there is no gap between the actual area under a curve and what calculus computes. *See* Lubet, *supra* note 4, at 347.

9 *See* BERLINSKI, *supra* note 2, at 256.