Everyday Mathematics and *Investigations in Number, Data and Space:* Spiral or Scaffold?

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Abstract

This literature review is designed to investigate the similarities and differences in *Investigations in Number, Data and Space and Everyday Mathematics* and to analyze the ways in which the curricula support the learner and learning as it relates to the two, separate second-grade units or "content strands" on "How Many Tens? How Many Ones?" and "Place Value, Money, and Time." It will also discuss the ways in which each curriculum either supports or inhibits learner equity in the elementary classroom and how to effectively employ one spiraled curriculum (*Everyday Mathematics*) through the strengths of the scaffolded curriculum (*Investigations in Number, Data and Space*).

Both curricula engage students and support an environment of active learning in mathematics. However, questions reviewed in this essay include: How does the implementation of either of these units in *Investigations in Number, Data and Space* or *Everyday Mathematics* enable *all* students to retain and understand material? What role does the teacher play in relation to each curricular unit, in the learning of elementary math concepts and how does the curriculum support that teacher? What types of assessments does each curricular unit utilize and how do these contribute to learner equity or make the content in each unit accessible for students of varying ability levels? In what ways are these curricular units alike and different and which curriculum supports mastery learning of these second grade elementary mathematical concepts? Why is learner equity a driving force behind mastery learning?

By looking at learners and learning, the role of the teacher, curriculum and instructional strategies and assessment, this essay will investigate the benefits and limitations of implementing units in *Everyday Mathematics* and *Investigations in Number, Data and Space* in second-grade

elementary classrooms. The results of this essay will provide educators with an understanding of the similarities and differences in the two curricula. It will enable them to make an appropriate curricular choice for their student demographic as well as provide support to their instruction, curriculum, assessments, and learning environments, showing them ways to maximize learner equity in second-grade elementary mathematics instruction.

Essay Introduction and Justification

This comprehensive essay analyzes the curricular strengths and weaknesses of *Everyday Mathematics* and *Investigations in Number, Data and Space* from a "learner equity" perspective. When using the term "learner equity" what is meant is the ability of all learners of all ability levels to appropriately and effectively access learning objectives. More specifically, this essay analyzes the accessibility and equity found in tens-counting lessons in *Everyday Mathematics*, a spiraled curriculum, and uses *Investigations in Number, Data and Space* to shed light on how to better scaffold students of diverse learning levels. Learner equity asks many questions: how accessible are the learning material and elementary mathematical concepts via the curriculum, activities, instructional tactics of the teacher and the classroom environment in which the students learn? Does either curriculum play favoritism toward gifted, special needs, or other segments of student demographics? Investigating both of these curricula through a "learner equity" lens results in a corresponding guide for teachers of elementary mathematics to support their implementation of *Everyday Mathematics* or *Investigations in Number, Data and Space.* It also serves as a tool for a more in-depth understanding of how to implement either unit/curriculum with the intention of all students reaching a mastery level of learning at their respective ability level.

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What is the importance of "learner equity" as it relates to elementary mathematics instruction? Campbell and Rowan (1997) discuss learner equity in that "Mathematical power for all cannot be fully realized if the classroom environment limits any child's access to challenging mathematics instruction." (pp. 61). It would also be appropriate to add "and appropriately scaffold mathematics instruction" to Campbell and Rowan's philosophy of learner equity to be more inclusive.

In order to implement a balanced and differentiated elementary mathematics curriculum, learner equity is must be achieved. This means that all students need to be met at their individual ability level and instructed on their appropriate level.

Croom (1997) explains the push for equity in mathematics education:

> During the 1980s, numerous national reports documented the under-achievement in mathematics of students in American schools. The general sentiment shared by most of these reports was a call for change [which included a new vision for mathematics education.] The underlying assumption on which this new vision was based is that changes and improvements in teaching and learning will afford every child access to a substantive mathematics education. (pp. 1)

Croom adds that both minorities and females need more equitable mathematics education in an increasingly competitive and technologically driven workforce.

The skills needed to be competitive in such an environment are obtained both directly and indirectly from skills learned early on in a child's mathematics education, such as those of logical reasoning, problem-solving and other higher-order thinking skills. "Researchers stress the importance of offering early intervention programs for underrepresented groups. These programs would… improve mathematical skills, and develop interest and positive attitudes. (Oakes 1990)"

As seen in the example of "Benny" in Erlwanger's piece "His teacher could encourage him to inquire, to discuss and reflect upon his experiences in mathematics only if she has a close, personal relationship with him and understands his ideas and feelings about mathematics." This kind of discourse could change student attitudes regarding mathematics and in turn increase equity in the classroom as a whole.

Looking forward on this problem, Burke and Curcio state, "As we enter the twenty-first century, we carry some unsolved problems with us. Many children from all backgrounds do not understand mathematics enough to use it or cannot even do many tasks accurately. This problem is especially acute for children of poverty or from homes where Standard English is not spoken. Many children leave third grade hopelessly behind, especially in urban schools." (pp. 198).

I. *Everyday Mathematics* Unit 3 - "Place Value, Money and Time" Analysis

Through analyzing the Teacher's Edition of Everyday Mathematics unit on "Place Value, Money, and Time," it appears that this resource is rather user-friendly, particularly in the way it prepares the teacher gives the teacher an overview of what the unit will focus upon, discussing how the skills taught through the unit will be "developed and expanded over time" noting that "few children have mastered them (the skills of numeration and place value, money, time and data collection and analysis) as a result of their first-grade experiences" and urges the teacher to "keep the activities lively and set a brisk pace" (Everyday Mathematics, Vol. 1 pp. 156.) This unit overview continues to provide valuable information for the teacher, as after the table of contents is a section entitled "learning goals in perspective." This section highlights learning goals for entire unit in terms of "developing" or "secure," as well as discusses why these learning goals are relevant in "links to the past" and "links to the future." Not only does this section of the teacher's edition give direction and relevancy in teaching of the unit, but also provides good

talking points for connecting with parents and students at home. These kinds of "home-school connections" are also provided for when the curriculum alerts the teacher to "Portfolio Ideas" where students have opportunities and assignments that allow them to show what they know at home.

This introduction to the unit also sets the stage for "Kid Watching" techniques, a form of ongoing, informal assessment strategies for the teacher to be watching for in their classroom. These points, in particular, are one of the key strengths of *Everyday Mathematics* as it relates to learner equity. They alert the teacher to what concepts students may potentially struggle with and provide for pre-emptive notification and time to design curricular strategies to meet these students when/if they struggle. However, when these "Kid-Watching " notifications occur in the unit, there are often not enough diverse strategies to amend students' lack of understanding, as will be discussed in more detail later.

It is apparent that *Everyday Mathematics* focuses on taking real-life situations that children may encounter, then turning those situations into "math language" or number sentences. In this way, the curriculum supports children on a mastery level in that they are instructed through meaningful mathematical problems, rather than having them complete repetitive problem sets. It also discusses strategies children may use in the unit, such as: "acting out the problem, working backward, using manipulatives, using information to write a number story, making and using a graph, and using information from a picture." (pp. 157). A strength of *Everyday Mathematics* lies in its use of manipulatives and, as Ball (1992) points out "Manipulatives – and the underlying notion that understanding comes through the fingertips – have become part of educational dogma: Using them helps students; not using them hinders students." (pp. 29) Highlighting these student strategies at the beginning of the unit also

coincidentally assists the teacher in noticing quality thinking strategies within his or her classroom to inform and design instructional tactics, a highly valuable tool for planning the daily lessons within the unit.

Another strength of this section is that the edition continuously refers to the corresponding "Math Journal" pages for students to receive practice and be assessed cumulatively throughout the unit. This journal affords students many opportunities to show what they know through verbal, pictorial, symbolic and concrete formats of mathematical representations. The journal provides cumulative assessments throughout the unit as well as creates opportunities to play games involving learned mathematical concepts, however some of these games assume certain knowledge or predicted cooperation and full student participation, not allowing for as much differentiation on the struggling student's end as on the advanced student's. *Everyday Mathematics* gives the teacher notes on "Options for Individualizing" and adjusting the activity to help he or she "reteach, enrich, and to provide for language diversity and multiage classrooms." Some of these "options" are helpful but oftentimes cater more to providing a challenge for advanced students rather than those of diversity in age, gender or language.

Everyday Mathematics **- Lesson 3.1**

The unit begins with "Mental Math and Reflexes" and a "math message" for students to read upon beginning the lesson. It scripts the lesson for the teacher, which may be helpful for instruction, but not as fluid or student-centered. The teacher should be mindful of student needs at the beginning of the lesson and be careful to explain everything to its fullest, rather than be self-focused on his or her instruction of the script. Regardless, the curriculum supports the

teacher through this scripting giving the guided support he or she may need when introducing the lesson on the base-ten system.

Students begin with blocks for hundreds, tens and ones when their teacher asks them to display "352" with those blocks using the "Place Value Mat," a grid with three sections for hundreds, tens and ones. It may be wise for students to have a bit of practice saying "three hundred and fifty-two" aloud while looking at the number "352" *before* trying to represent this with manipulatives, or to have a real-life example of something that comes in groups given to them by their teacher. The lesson prompts students to say this after representing it with their manipulatives before repeating the process with two-digit numbers. . Perhaps a good introduction to this lesson would have started with single, double and concluded with the representation of triple digits for the purpose of a quick review, then a discussion of things students know of that already come in groups, such as the wheels on a bike, pairs of shoes, and other similar examples.

Next, the lesson asks children to display with the "5" means and what the "2" means in the three-digit number, obviously searching for students to come up with "five tens and two ones." It may be better to pose questions such as "Is the five just like the two?" "Is it different? Why?" hopefully leading into a discussion of why we say "three hundred and fifty-two" instead of "three, five, two."

Lastly, children are then given random groups of hundreds, tens and ones and asked to display the numbers the teacher writes on the board, followed by the reverse of displaying blocks to a partner and having the partner display the correct number. Then the teacher puts "zero" in the tens or ones place, forcing students to make a decision as to how to show "502," asking "Is it 52, 502 or 520?" This kind of backward and forward procedure in mathematics helps children understand concepts on deeper levels, being able to use the method and manipulate numbers and

blocks to show what they know. However, for a child with processing difficulties, understanding hundreds, tens and ones may take a good bit of one-on-one time with plenty of practice time of manipulating blocks and saying and writing numbers. *Everyday Mathematics* does not provide for "adjusting the activity" here to slow the pace of the lesson or provide adequate differentiation methods for slower processers, it only recommends reteaching (in introduction of the unit.) Doing this activity with a partner should be carefully planned, perhaps pairing advanced students with slower processers or even arranging a diverse small group activity with slates for each child to show what they know and learn from others in their small group.

The corresponding Math Journal pages 51, 52 and 53 along with the enrichment activity (for advanced learners) differentiate this lesson, providing much practice and an opportunity to even move into thousands counting units. This continues the trend of having students know hundreds, tens and ones backward and forward and even offers a short writing section asking students to explain why they know what they know, a good activity to activate their schema of place value early in the unit. They then connect their new knowledge "at home" with Math Masters p. 265, which emulate more practice of concepts learned in this lesson. For students with little home support, this cumulative review may prove difficult and require scaffolded assistance from the teacher.

***Everyday Mathematics* Lesson 3.2- "Using Coins to Buy Things"**

The second lesson, Lesson 3.2 in *Everyday Mathematics*, is titled "Using Coins to Buy Things" and discusses money-specific vocabulary students need to know to be able to move forward in this unit. This is a great, schema-activating opportunity for the teacher to incorporate a bit of language arts into the math lesson, perhaps by having students practice with real coins what a penny, nickel, dime, quarter, and one-dollar bill represent. Transferring the student's

knowledge of hundreds, tens and ones-counting manipulatives to ones, fives, tens, twenty-fives and hundreds may be a bit tricky at first. Starting this activity with a discussion of what the word "cents" means would also be valuable.

The transition to a money activity directly after the beginning of base-ten counting seems like a presumptive and large jump by *Everyday Mathematics*, as students who may not have mastered previous concepts may have a hard time understanding the value of single coins when they previously had counters in the form of blocks. The Math Message on page 174 says "Count by 5s, 10s and 25s. Begin at 5s and count by 5s to 100. Begin at 300 and count by 10s to 150. Begin at 25 and count by 25s to 100. If children are having difficulty, allow them to use coins as they count." This seems like a difficult task without a prior discussion of the value of each individual coin, which person is on each coin, and how students can tell the difference. Perhaps instead of giving students coins to help them, giving them counters as they had in their previous lesson may provide the appropriate level of help (scaffolding.)

The "Table of Equivalencies" provided by *Everyday Mathematics* (pp. 174) is a helpful illustration to children of each coin's individual value, but to fully understand the dollar value, a child must have either had experience paying with money or with the concept of "part to whole." As Witherspoon (1993) points out, "We should be careful to assume that students 'understand' fractions merely because they are able to carry out an algorithm or recite definitions." (pp. 484). The actual size of a dollar bill and quarter may help children understand that quarters are smaller parts of a dollar, but this method does not hold true with dimes, nickels and pennies as dimes are greater in value but smaller in size. Also, children may have trouble understanding what the word "equivalencies" means and may need this "Table of Equivalencies" put into more kid-friendly terms. The lesson then leads into a teacher-led discussion on what the values of each

piece of money as well as how to trade out certain smaller values for larger. This discussion follows with the "fruit stand" activity where children take turns buying and selling fruit at various prices, giving them opportunities to pay for their items when buying and make change when selling. This activity is helpful and may appeal to many children though an exciting activity of play and working with money. It also has multiple solutions as there are many combinations a child could make to make a payment or change for their "purchase" and Math Journal page 55 allows children to keep a record of their "transactions," drawing pictures of the coins they used. A "kid-watching" opportunity arises for the teacher - do the children begin counting coins with the highest value? Or do they start off counting by pennies (ones)? However, again *Everyday Mathematics* does not provide a suggestion for the teacher on what to do if children are stuck counting by ones or have not fully grasped how to begin with the coin of highest value for efficiency's sake. The advanced children may begin to make change at this point in the unit, but will have more opportunities to do so in lessons 3.7 and 3.8. The activity of buying and selling fruit, making change and paying with the highest value first is a good segway into skip counting and would require more practice perhaps with manipulatives or by setting up an actual "fruit stand" or store front in the classroom where children could "play" during down time or after other lessons to reinforce the math concept. This may provide for experience and give children more confidence making change rather than just completing a worksheet.

Games are also a component of this lesson through the "Digit Game" where student flip cards to create two and three-digit numbers, with the highest number collecting the cards – a game similar to "War" that provides added practice of recognizing place value. Another game, "Spinning for Money" involves children using a spinner then taking money from the "bank" and exchanging coins for a single coin or bill of an equivalent value. Students complete Math Journal

page 57 to begin practicing skip-counting on their own, though it introduces less-than, greater-than, and equal signs which may also prove to be a difficult concept for many students to understand, let alone be able to use in context and would require additional support and explanation of this kind of "math language."

Everyday Mathematics **Lesson 3.3 – "Telling Time"**

The third lesson in unit three of *Everyday Mathematics* continues in this fashion of jumping to an entirely new concept – telling time. Although the lesson is strong as a whole, it seems too much– tens-counting, using money and telling time – if trying to be accomplished all within one elementary math unit. While each of these concepts and desired learning objectives are important and obviously related, each is a difficult concept to teach by itself and often encompass entire units on their own (unit on time, unit on money etc.) in other curricula, providing much practice and daily reinforcement for students on all levels. Regardless, suggestions can be made within this lesson to assist teaching and learning.

The lesson begins with a discussion of time-specific vocabulary, such as "digital, analog, minute/hour hand and clock face" as well as asks students to show tally marks for several numbers, encouraging counting by fives. Then students create paper clocks to use and manipulate to show what time they do certain things, such as begin school in the mornings. This may be a difficult task, again, unless children have had experience with an analog clock and may require prior discussion and teacher modeling of how to use such a clock. The curricula also suggests a discussion of A.M. and P.M., explaining what "ante-meridium" and "post-meridium" mean, a discussion that seems far above second-grade learner's ability levels. Not only is it not necessary for this lesson, it may only confuse students who have not even learned about prefixes

yet. Instead of this discussion being integrated into the lesson itself, perhaps it may better serve as an enrichment activity for advanced learners, separately.

Despite this, the discussion of estimation in regard to time is a valuable one that many students may be excited about. Children often hear their parents say "quarter 'til" or other estimations of time and may respond well to a lesson that encourages estimation. Children, at this stage of their brain development and sense of identity, also enjoy discussing their daily routines and at what time they complete those routines, providing for a fun incorporation of the "What Time Is It?" where clock faces are shown and children also get to show the times they complete certain tasks, such as eat dinner or feed their pets. This activity is rather inclusive and may require the teacher to take a good bit of time to listen to children's answers and allow them to "show what they know."

This is followed by a discussion of estimating more precise time, such as a "quarter of an hour" which when studied closely is a precursor to multiplication and division, asking students to essentially divide sixty by four and understanding that fifteen is one quarter of sixty. If the children have successfully mastered skip counting by fives, this may be a sign they are ready for estimating more precise measurements of time, but the skip-counting math journal assignments (page 58, 59 and 60) are mixed in with this activity instead of given beforehand. It might be wise to even create an extra practice sheet of one's own to mimic those math journal pages for students who may be struggling at this point in the unit. It may also be wise to encourage watch wearing and to occasionally ask children what time it is throughout the day using an analog clock or ask them to draw an analog clock with the current time on their slates.

Everyday Mathematics **Lesson 3.4 – "Exploring Numbers, Time, and Geoboards"**

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The objectives of this lesson are "to represent and rename numbers with base-10 blocks; to review writing and telling time and to make, describe, and compare shapes on a geoboard" according to the teacher's edition, volume 2. It is apparent that *Everyday Mathematics* wants to instill pre-geometry thinking patterns in second-grade students through this lesson and through the use of the geoboards. The lesson itself, however, begins with students practicing number "complements of 100" by giving a response of ten when ninety is said by their teacher and so forth. Then students are asked to put this into "math language" such as a subtraction problem like 100-90=10 or an addition problem like 90+10=100. This type of its backward and forward method of instruction is effective in promoting mastery learning of the concept and can be seen as an equity promoting activity within the classroom. However, the lesson then challenges students to use multiples of 10 and 25, which appears to be a large jump in student expectation at this point in the lesson, before it creates a whole class activity of going back to tens and ones counting through the use of slates by asking students to create multiple drawings of "36" (ie. other than three tens and six ones.) This option within the activity is good for encouraging different learning styles, but the beginning of the lesson seems a bit choppy and un-streamlined in the way it attempts to teach the concepts. The lesson follows with more practice in creating and saying three-digit numbers, using manipulatives on their "place-value mats" as well as by having students create a "clock booklet" where they make up a story about something that occurs at a specific time they've shown on their clock face. This activity, from the mindset of meaningful and equitable instruction, fails the test of providing an accessible activity for all learners and does not provide for those who may be English Language Learners, struggling to have full-participation in such an activity that requires the ability to write coherent sentences in English.

Lastly, two hands-on activities conclude the lesson; using geoboard paper (or actual boards) to compare and create shapes and creating the game entitled "Dollar Rummy." Upon first examination, the use of geoboards seems unrelated to telling time and tens-counting lessons. It is unclear whether this activity is directly linked to drawing the hands on a clock face or designed to encourage visual and special awareness, but regardless seems arbitrarily placed within the unit. The teacher's edition discusses that "Identifying likenesses and differences of shapes will prepare children for geometry in Unit 5." It seems that this is a less-important activity as it will be discussed and developed fully in a later unit and should not be placed randomly within a tens-counting unit. Regardless, it is an important activity at this stage of development as Battista (1999) suggests, "Spatial structuring is a crucial process in student's construction of geometric knowledge." (pp. 17). Perhaps it will serve the teacher and students better to use the geoboards as an "early finishers" activity or as a less-emphasized portion of the current unit to avoid confusion. Despite this, the second activity, "Dollar Rummy", seems a good activity to practice complements of 100 and reinforce counting up and counting down (backward and forward knowledge of concepts.)

Everyday Mathematics **Lesson 3.5 – Data Day: Pockets**

This lesson may be one of the strongest examples of "revealing mathematical truths through meaningful tasks" as it relates to gathering and entering data, drawing graphs and identifying the middle value or average of a group of numbers. This prepares children to discover mean, median and mode – good concepts to understand *after* being able to multiply, skip count and divide. Again, *Everyday Mathematics* jumps to a new objective of learning rather than focusing on mastery of previous objectives within the unit. The activity itself however, is strong

in accomplishing its desired goals of students understanding how to accurately predict a "middle number."

The "number of pockets" activity asks children to tell how many pockets they have on their clothes. The students with the greatest/fewest number of pockets are then asked to stand. "How many more pockets would Student A need to have as many pockets as Student B?" the teacher asks? This question in itself provides good practice in "counting up" just as the "complements of 100" activity did. Students then discuss number sentences that fit this situation. In order to find the "middle number" or mean number of pockets, students line up in order of number of pockets and gradually find the middle by having students on each end sit down one-by-one. In this way, *Everyday Mathematics* succeeds to meaningfully display the mathematical truth of median to children, regardless of its being ill-placed within the unit. The lesson is followed with a graphing activity in which children employ their understandings of how to make tallies, which is beneficial in graphing. Lesson 3.6 introduces skip counting with frames and arrows, which perhaps should have been interspersed within the unit, as should the following two lessons on counting up by making change. Lesson 3.7 seems to hit on important concepts from the previous lesson involving the fruit stand transactions, but seems to be oddly placed at the end of the unit as a review, rather than a strategy to employ when the information was first introduced.

Assessment in *Everyday Mathematics*

There are various forms of assessment in *Everyday Mathematics* that provide both cumulative assessments in the "Math Masters" book and ongoing assessments, called "kid-watching" techniques through the use of slates, tips for the teacher as well as other assignments in the children's math journals. The curriculum also helps the teacher assist parents in making

home-school connections by alerting when certain assignments may make for a good portfolio piece. This is important for parents, but depending on the student demographic, may not prove effective in providing support the child needs outside of school. In this way, *Everyday Mathematics* appears to cater to students with strong parental or home support and not make accommodations for other children, the exact demographic that needs to be targeted according to Croom aforementioned in the introduction. These particular students need extra support that they are not receiving as a result of the implementation of *Everyday Mathematics*, leaving much of that burden on the teacher or learning specialist.

The teacher is also the figurehead of assessment creation with the added resource of the Math Masters Assessment tool. There are many "Assessment Options" used as Lesson 3.9 in the unit, giving the teacher ideas for how to assess the whole class, and individually as well as through oral, slate and written assessments. In this way, *Everyday Mathematics* provides for diversity and equity in the way children are assessed.

Continued Suggestion for *Everyday Mathematics* in Promoting Learner Equity

In terms of a spiraled curriculum, continuous practice and assessment is needed in *Everyday Mathematics*, particularly for students who struggle at first to understand mathematical concepts. Much practice and reinforcement is needed to instill a concept, more so than with a gifted or advanced child, as in a spiraled curriculum such as this, concepts are introduced, a bit of practice is given, and then the concept is hit on again during a later lesson in the unit.

Individually the lessons are strong, but as a lesson sequence *Everyday Mathematics*, in its spiraled nature, makes large leaps from lesson to lesson in concepts and presumed student understandings that it hits on later in the unit. The leaps may lead to certain demographics of

students falling through the cracks without proper scaffolding techniques and does not promote mastery learning for students of all levels.

II. Investigations in Number, Data and Space – "How Many Tens, How Many Ones?"

Overview and Strengths in Comparison

Upon examination of *Investigations in Number, Data and Space,* the curriculum is organized by "Curriculum Units" broken down in to smaller components, rather than one, large volume of units as in *Everyday Mathematics*. The other components of this curriculum are "Implementing Investigations in Grade 2," the "Resources Binder" which contains transparencies and reproducible to assist the teacher in instruction as well as the student components of the "Student Activity Book," which contains the consumables of homework, practice and lastly the items in the "Student Math Handbook," a guide which contains math vocabulary and directions for games employed by the curriculum.

First stated in the "Overview of Program Components" is the goal of "engage the range of learners about mathematical ideas" which preludes the concept of learner equity in this curriculum. The very existence of the "Student Math Handbook" on math vocabulary and other ideas as a student resource, not just for the teacher, is a revolutionary concept in mathematics education as it relates to equity. However, making sure that children are adequately trained in how to make use of his resource would be the challenge.

In comparison to *Everyday Mathematics,* and through the analysis of several lessons in *Investigations in Number, Data and Space,* the second curriculum contains more opportunities for discussion on one, big mathematical idea, or concept. The curriculum then proceeds with each activity only to come full circle again within the lesson, not within a later lesson in the same

unit. In this way, each lesson builds off of one another. For example, lesson 1.1 entitled "Story Problems with Stickers" begins with a revisitation of a concept from a previous unit that will be necessary to master before undertaking the first lesson in the current. In this way, *Investigations…* ensures that students will be able to undertake the current lesson with familiarity. "Today's Plan" gives a suggested time for instruction as well as a suggested size for class instruction, the time component missing from *Everyday Mathematics.* Each portion of the "Today's Plan" has a set of vocabulary and materials that correspond as well as "Math Focus Points" which state the desired student understandings at the end of the lesson. This backward planning method assists the teacher in letting he or she know what the students should know and be able to do by the end of the day, not just by the end of the unit or section of the unit.

Another point of note on *Investigations in Number, Data and Space* is the direct and concise suggestions for ELL students, such as on page 25 in the teacher's edition of the unit, where "Differentiation" is noted for the particular lesson. In this section, the teacher is encouraged to meet with any English Language Learners in his or her class ahead of time and to give them a chance to practice verbally responding to questions individually before the questions are posed to the group, directing the teacher to help them phrase their answers by writing them out, then saying them to their teacher and to each other. In terms of learner equity, this not only gives ELL students an opportunity to know what is coming for them that day in mathematics, but also to be made to feel comfortable and confident to approach their math lesson that day, making the playing field much more equal.

Investigations… also includes differentiation techniques as they relate to intervention, or when students are truly struggling with the concepts. On page 27 of the teacher's edition, such a tactic is employed and asks the teacher to allow those struggling students to retell the "tens-

counting story" or to "tell what happened in the math story" in lesson 1.1 in their own words, helping them feel comfortable to show what they know, but to their teacher and not in front of the whole class. *Investigations in Number, Data and Space* truly employs mathematical discourse in the classroom, more so than in *Everyday Mathematics*, and as Ball (1991) points out "An unfamiliar term to many, discourse is used to highlight the ways in which knowledge is exchanged in classrooms… Without explicit attention to the patterns of discourse in the classroom, the long-established norms of school are likely to dominate – competitiveness, an emphasis on the right answers…" (pp. 1) There are also opportunities for more advanced students to show what they know when one of them has a clever method. The teacher would ask this student, in a small group or large group setting, how she figured it out, allowing this student's correct method of computation to be praised and also to serve as a model for other students in the class other than the modeling done by the teacher.

Investigations in Number, Data and Space excels in the way it helps students "reveal mathematical realities through tangible, meaningful activities that present big, mathematical ideas in hands on tasks" as stated by the curriculum, even from an objective standpoint. Each lesson starts with a mathematical problem, then encourages students to retell the problem in "math language" as well as provides for multiple solutions given by a wide-range of diversity of learners in the classroom. This not only makes each lesson more equitable, but also more accessible in the way differences in computation are encouraged. The curriculum aids the teacher in "Math Notes" where it discusses the multiple strategies the teacher might be able to expect from students throughout the instruction of the lesson as well as notes which strategies will be employed later within the unit. For the instructor to provide the most equitable and accessible form of instruction, it is important to simply be aware of all of the possible algorithms to solve a

problem, so as to keep him or her from searching for one student strategy above all others and to assist students in finding the strategy that fits *them* best, not the one that fits the teacher or the curriculum best. As stated in Phillip's (1996) piece on multicultural mathematics, "The fact that algorithms are a convention is often lost on our students, who come to think of a particular algorithm as *the* way, instead of as *a* way, to compute" *Investigations in Number, Data and Space* also echo Russell (1999) in that flexibility leads to computational fluency stating that "Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible to be able to choose an appropriate strategy for the problem at hand and to… double-check the results." (pp 154.)

Investigations in Number, Data and Space cumulatively assesses students at the end of every "Investigation" lesson, providing valuable feedback for both the teacher and the student as to whether he or she has mastered those concepts in order to move on or be "scaffolded" to the next layer of the unit.

Closing Thoughts

In summation of this analysis and suggestions for implementation, It is apparent that both a spiraled and scaffolded curriculum have particular strengths in certain classroom. Ultimately, it is up to the teachers and administrative officials in schools to decide which curriculum to implement and how. However, when one notes the obvious equity issues occurring within American classrooms, particularly in the realm of mathematics, a solution and strategy need to be employed to make mathematics more equitable for all. Much of this stems from giving students flexibility in the way they approach problems, not having one way to do something "correctly" but encouraging ideas and innovation – teaching for creativity. As Hiebert says in relation to flexibility (and essentially equity) "…today's students need flexibile approaches for

defining and solving problems. They need problem-solving methods that can be adapted to new situations, and they need the know-how to develop new methods for new kinds of problems." (pp. 1) and "…only teachers can make the specific interpretations necessary to ensure that all children learn with understanding." (pp. 73).

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