LieART 2.0 – An Improved Way to Compute Branching Rules
Undergraduate Honors Thesis in Physics
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Abstract

In this thesis, we present LieART 2.0 which contains substantial extensions to the Mathematica application LieART (Lie Algebras and Representation Theory) for computations frequently encountered in Lie algebras and representation theory, such as tensor product decomposition and subalgebra branching of irreducible representations. LieART 2.0 can now handle all classical and exceptional Lie algebras up through rank 15. The basic procedure is unchanged–it computes root systems of Lie algebras, weight systems and several other properties of irreducible representations, but new features and procedures have been included to allow the extensions to be seamless. The new version of LieART continues to be user friendly. Some extended tables of branching rules of irreducible representations are included in the supplementary material for use without Mathematica software.

Keywords: Lie algebra; Lie group; representation theory; irreducible representation; branching rule; GUT; model building; Mathematica

Contents

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PROGRAM SUMMARY

Authors: Robert Feger, Thomas W. Kephart and Robert J. Saskowski
Program Title: LieART
Licensing provisions: GNU Lesser General Public License (LGPL)
Programming language: Mathematica
Computer: x86, x86_64, PowerPC
Operating system: cross-platform
RAM: $\geq$ 4 GB recommended. Memory usage depends strongly on the Lie algebra's rank and type, as well as the dimensionality of the representations in the computation.
Keywords: Lie algebra; Lie group; representation theory; irreducible representation; tensor product; branching rule; GUT; model building; Mathematica
Classification: 4.2, 11.1
External routines/libraries: Wolfram Mathematica 8–11
Nature of problem:
The use of Lie algebras and their representations is widespread in physics, especially in particle physics. The description of nature in terms of gauge theories requires the assignment of fields to representations of compact Lie groups and their Lie algebas. Mass and interaction terms in the Lagrangian give rise to the need for computing tensor products of representations of Lie algebras. The mechanism of spontaneous symmetry breaking leads to the application of subalgebra decomposition. This computer code was designed for the purpose of Grand Unified Theory (GUT) model building, (where compact Lie groups beyond the U(1), SU(2) and SU(3) of the Standard Model of particle physics are needed), but it has found use in a variety of other applications. Tensor product decomposition and subalgebra decomposition have been implemented for all classical Lie groups SU(N), SO(N) and Sp($2N$) and all the exceptional groups E_6, E_7, E_8, F_4 and G_2. This includes both regular and irregular (special) subgroup decomposition of Lie groups up through rank 15.
Solution method:
LieART generates the weight system of an irreducible representation (irrep) of a Lie algebra by exploiting the Weyl reflection groups, which is inherent in all simple Lie algebras. Tensor products are computed by the application of Klimyk's formula, except for SU(N)'s, where the Young-tableaux algorithm is used. Subalgebra decomposition of SU(N)'s are performed by projection matrices, which are generated from an algorithm to determine maximal subalgebras as originally developed by Dynkin [1, 2].
Restrictions:
Internally irreps are represented by their unique Dynkin label. LieART's default behavior in `TraditionalForm` is to print the dimensional name, which is the labeling preferred by physicist. Most Lie algebras can have more than one irrep of the same dimension and different irreps with the same dimension are usually distinguished by one or more primes (e.g. **175** and **175**$'$ of A_4). To determine the need for one or more primes of an irrep a brute-force loop over other irreps must be performed to search for irreps with the same dimensionality. Since Lie algebras have an infinite number of irreps, this loop must be cut off, which is done by limiting the maximum Dynkin digit in the loop. In rare cases for irreps of high dimensionality in high-rank algebras the used cutoff is too low and the assignment of primes is incorrect. However, this only affects the display of the irrep. All computations involving this irrep are correct, since the internal unique representation of Dynkin labels is used.
Running time:
From less than a second to hours depending on the Lie algebra's rank and type and/or the dimensionality of the representations in the computation.

1. Introduction

The application of group theory to symmetries associated with of physical systems is one of the most important tools of modern science. Examples abound in physics, chemistry and beyond into biology and engineering. Continuous groups in the form of Lie groups are all important in physics, with smaller Lie groups entering quantum mechanics, physical chemistry and condensed matter physics, while the full spectrum of Lie groups, i.e., the classical groups SU(N), SO(N) and Sp($2N$) and the exceptional groups E_6, E_7, E_8, F_4 and G_2, have all appeared in particle physics. Lie groups have had many other application in theoretical physics from gravity, to string theory, M-theory, F-theory, and beyond. In LieART [3] the focus is on Lie algebras of compact Lie groups that are most useful for particle physics. However, most of our results could easily be extended to the non-compact forms.

LieART is a convenient easy to use Mathematica application to explore Lie groups and their algebras. It was designed to provide most of the Lie group information needed for particle physics model building. In particular it can find tensor product rules and irreducible representation (irrep) decompositions for Lie groups found in Grand Unified Theories (GUTs) and other common extensions of the standard model. But LieART has also found application in many other areas of theoretical physics. LieART 2.0 is a response to the need for easy access to a more complete list of decompositions from Lie group to their maximal subgroups. The remaining regular maximal subgroup decompositions not contained in LieART 1.0 have been added as well as the full list of irregular maximal subgroup decompositions. The complete set of maximal subgroup decompositions are now included through rank 15 of the initial group.

The gauge group of the Standard Model of particle physics is $SU(3)_C\times SU(2)_L\times U(1)_Y$. Although the Standard Model is in agreement with most data, it is thought to be incomplete with many unexplained free parameters. To determine these parameters, many extensions of the Standard-Model gauge group have been proposed. Low energy flavor models usually extend the symmetry, often with a discrete factor group but sometimes with a continuous–either global or local continuous group.

To understand high energy behavior, Grand Unified Theories were proposed, where extensions are via higher gauge symmetry, typically SU(5) [4], SO(10) [5, 6] or E_6 [7], although many other choices have been explored. Reviews of Lie algebras can be found in [8, 9], including tables of irreducible representations (irreps) and their invariants. There are also several textbooks on the topic, e.g., see [10, 11, 12].

While extensive tables already exist for building GUT models [8, 9], (for recent comprehensive results see,and [74]) it has sometimes been necessary to go beyond what is tabulated in the literature [3]. Our purpose here is to provide the software to handle any situation that may arise. In describing this software we will incorporate a short review of much of the necessary group-theory background including root and weight systems, the associated Weyl groups for Lie algebras, orthogonal basis systems, and group orbits, which we use to calculate tensor products and irrep decompositions. More details can be found in [3].

LieART's code exploits the Weyl reflection group to make computations fast and economical with respect to memory. We focus on the usability of LieART with the user in mind: Irreps can be entered by their dimensional name, or the more unique Dynkin label. LieART can display results in the form of LaTeX commands for easy inclusion in publications (see the supplemental LaTeX style file).

The paper is organized as follows: In Section 2 we give updated instructions for downloading and installing LieART, as well as locating its documentation integrated in Mathematica's help system. Section 3 contains a quick-start tutorial for LieART, introducing the most important functions for the most common tasks in an example-based fashion. Section 4 presents a self-contained overview of the Lie algebra theory used in LieART and gives notes on its implementation. A list of all LieART commands is provided. Section 5 gives details on the implementation of the new features of LieART 2.0. In Section 6, we disuss the results of the extended LieART package. Section 7 gives benchmarks for a few subalgebra decompositions of large irreps. In Section 8 we conclude and give an outlook on future versions. The appendix contains a collection of tables with branching rules for many maximal subalgebras of semisimple Lie algebras. These tables follow [8] in style, but extend most of the results. The tables can be used to conveniently lookup many frequently used results and do not require the direct use of LieART.

2. Download and Installation

2.1. Download

LieART is hosted by Hepforge, IPPP Durham. The LieART project home page is

`http://lieart.hepforge.org/`

and the LieART Mathematica application can be downloaded as tar.gz archive from

`http://www.hepforge.org/downloads/lieart/`

As of right now, the new 2.0 version of LieART is not yet available for download on Hepforge, but will be in the near future.

2.2. Automatic Installation

Start Mathematica and in the front end select the menu entry

`File` → `Install...`

In the appearing dialog select `Application` as `Type of Item to Install` and the tar.gz file in the open file dialog from `Source`. (It is not necessary to extract the tar.gz archive since Mathematica does this automatically.) Choose whether you want to install LieART for an individual user or system wide. For a system-wide installation you might be asked for the superuser password.

2.3. Manual Installation

The above procedure in Mathematica 7 only allows you to automatically install the Mathematica package file (`LieART.m`) of LieART without the documentation. We therefore suggest a manual installation of the LieART application in Mathematica 7 and in Mathematica 8 through 10 if problems with the automatic installation occur.

Extract the archive to the subdirectory `AddOns/Applications` of the directory to which `$UserBaseDirectory` is set for a user-only installation. For a system-wide installation place it in the according subdirectory of `$InstallationDirectory`. Restart Mathematica to allow it to integrate LieART's documentation in its help system.

2.4. Documentation

The documentation of LieART is integrated in Mathematica's help system. After restarting Mathematica the following path should lead to LieART's documentation:

`Help`
→ `Documentation Center`
  → `Add-Ons & Packages` (at the bottom)
    → `LieART`, Button labeled "`Documentation`"

(Alternatively, a search for "LieART" (with the correct case) in the Documentation Center leads to the same page.) The displayed page serves as the documentation home of LieART and includes links to the descriptions of its most important functions.

The documentation of LieART includes a `Quick Start Tutorial` for the impatient, which can be found near the bottom of LieART's documentation home under the section `Tutorials`.

Tables of representation properties, tensor products and branching rules generated by LieART can be found in the section `Tables` at the bottom of LieART's documentation home.

2.5. LaTeX Package

LieART comes with a LaTeX package that defines commands to display irreps, roots and weights properly. The style file `lieart.sty` can be found in the subdirectory `latex/` of the LieART project tree. Please copy it to a location where your LaTeX installation can find it.

3. Quick Start

This section provides a tutorial introducing the most important and frequently used functions of LieART for Lie-algebra and representation-theory related calculations. The functions are introduced based on simple examples that can easily be modified and extended to the user's desired application. Most examples use irreducible representations (irreps) of SU(5), which most textbooks use in examples since is less trivial than SU(3), but small enough to return results almost instantly on any recent computer. Also, SU(5) frequently appears in unified model building since the Standard-Model gauge group is one of its maximal subgroups. This tutorial can also be found in the LieART documentation integrated into the Mathematica Documentation Center as "Quick Start Tutorial" under the section "Tutorials" on the LieART documentation home.

This loads the package:

```
In[1]:= «LieART`
```

3.1. Entering Irreducible Representations

Irreps are internally described by their Dynkin label with a combined head of `Irrep` and the Lie algebra.

`Irrep[`*algebraClass*`][`*label*`]`	irrep described by its *algebraClass* and Dynkin *label*.

Entering irreps by Dynkin label.

The *algebraClass* follows the Dynkin classification of simple Lie algebras and can only be `A`, `B`, `C`, `D` for the classical algebras and `E6`, `E7`, `E8`, `F4` and `G2` for the exceptional algebras. The precise classical algebra is determined by the length of the Dynkin label.

Entering the $\overline{\mathbf{10}}$ of SU(5) by its Dynkin label and algebra class:

```
In[2]:= Irrep[A][0,0,1,0]//FullForm

Out[2]:= Irrep[A][0,0,1,0]
```

In `StandardForm` the irrep is displayed in the textbook notation of Dynkin labels:

```
In[3]:= Irrep[A][0,0,1,0]//StandardForm

Out[3]:= (0010)
```

In `TraditionalForm` (default) the irrep is displayed by its dimensional name:

```
In[4]:= Irrep[A][0,0,1,0]
```

Out[4]:= $\overline{\mathbf{10}}$

The default output format type of LieART is `TraditionalForm`. The associated user setting is overwritten for the notebook LieART is loaded in. For `StandardForm` as output format type please set the global variable `$DefaultOutputForm=StandardForm`.

As an example for entering an irrep of an exceptional algebra, consider the **27** of E_6:

```
In[5]:= Irrep[E6][1,0,0,0,0,0]

Out[5]:= 27
```

Irreps may also be entered by their dimensional name. The package transforms the irrep into its Dynkin label. Since the algebra of an irrep of a classical Lie algebra becomes ambiguous with only the dimensional name, it has to be specified.

`Irrep[`*algebra*`][`*dimname*`]`	irrep entered by its *algebra* and dimensional name *dimname*.

Entering irreps by dimensional name.

Entering the $\overline{\mathbf{10}}$ of SU(5) by its dimensional name specifying the algebra by its Dynkin classification A_4:

```
In[6]:= Irrep[A4][Bar[10]]//InputForm
Out[6]:= Irrep[A][0,0,1,0]
```

The traditional name of the algebra SU(5) may also be used:

```
In[7]:= Irrep[SU5][Bar[10]]//InputForm
Out[7]:= Irrep[A][0,0,1,0]
```

Irreps of product algebras like SU(3)⊗SU(2)⊗U(1) are specified by `ProductIrrep` with the individual irreps of simple Lie algebras as arguments.

`ProductIrrep[`*irreps*`]`	head of product *irreps*, gathering irreps of simple Lie algebras.

Product irreps.

The product irrep $(\mathbf{3},\overline{\mathbf{3}})$ of SU(3)⊗SU(3):

```
In[8]:= ProductIrrep[Irrep[SU3][3],Irrep[SU3][Bar[3]]]
```

Out[8]:= $(\mathbf{3},\overline{\mathbf{3}})$

```
In[9]:= %//InputForm
Out[9]:= ProductIrrep[Irrep[A][1,0],Irrep[A][0,1]]
```

```
In[10]:= ProductIrrep[Irrep[A][1,0],Irrep[A][0,1]]
```

Out[10]:= $(\mathbf{3},\overline{\mathbf{3}})$

Take for example the left-handed quark doublet in the Standard-Model gauge group SU(3)⊗SU(2)⊗U(1) (The U(1) charge is not typeset in bold face):

```
In[11]:= ProductIrrep[Irrep[SU3][3],Irrep[SU2][2],Irrep[U1][1/3]]
```

Out[11]:= $(\mathbf{3},\mathbf{2})(1/3)$

```
In[12]:= %//InputForm
Out[12]:= ProductIrrep[Irrep[A][1,0],Irrep[A][1],Irrep[U][1/3]]
```

3.2. Decomposing Tensor Products

`DecomposeProduct[`*irreps*`]`	decomposes the tensor product of several *irreps*.

Tensor product decomposition.

Decompose the tensor product $\mathbf{3}\otimes\overline{\mathbf{3}}$ of SU(3):

```
In[13]:= DecomposeProduct[Irrep[SU3][3],Irrep[SU3][Bar[3]]]
```

Out[13]:= $\mathbf{1}+\mathbf{8}$

Decompose the tensor product $\mathbf{27}\otimes\overline{\mathbf{27}}$ of E_6:

```
In[14]:= DecomposeProduct[Irrep[E6][27],Irrep[E6][Bar[27]]]
```

Out[14]:= $\mathbf{1}+\mathbf{78}+\mathbf{650}$

Decompose the tensor product $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$ of SU(3):

```
In[15]:= DecomposeProduct[Irrep[SU3][3],Irrep[SU3][3],Irrep[SU3][3]]
```

Out[15]:= $\mathbf{1}+2(\mathbf{8})+\mathbf{10}$

Decompose the tensor product $\mathbf{8}\otimes\mathbf{8}$ of SU(3):

```
In[16]:= DecomposeProduct[Irrep[SU3][8],Irrep[SU3][8]]
```

Out[16]:= $\mathbf{1} + 2(\mathbf{8}) + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$

Internally a sum of irreps is represented by `IrrepPlus` and `IrrepTimes`, an analog of the built-in functions `Plus` and `Times`:

```
In[17]:= %//InputForm

Out[17]:= IrrepPlus[Irrep[A][0,0], IrrepTimes[2, Irrep[A][1,1]],
           Irrep[A][3, 0], Irrep[A][0,3], Irrep[A][2,2]]
```

Results can be transformed into a list of irreps with `IrrepList`, suitable for further processing with Mathematica built-in functions like `Select` or `Cases`:

```
In[18]:= %//IrrepList
```

Out[18]:= $\{\mathbf{1},\mathbf{8},\mathbf{8},\mathbf{10},\overline{\mathbf{10}},\mathbf{27}\}$

Decompose the tensor product $\mathbf{4}\otimes\mathbf{4}\otimes\mathbf{6}\otimes\mathbf{15}$ of SU(4):

```
In[19]:= DecomposeProduct[Irrep[SU4][4],Irrep[SU4][4],Irrep[SU4][6],Irrep[SU4][15]]
```

Out[19]:= $2(\mathbf{1}) + 7(\mathbf{15}) + 4(\mathbf{20'}) + \mathbf{35} + 5(\mathbf{45}) + 3(\overline{\mathbf{45}}) + 3(\mathbf{84}) + 2(\mathbf{175}) + \mathbf{256}$

The Mathematica built-in command `Times` for products is replaced by `DecomposeProduct` for irreps as arguments. E.g., decompose the tensor product $\overline{\mathbf{10}}\otimes\mathbf{24}\otimes\mathbf{45}$ of SU(5):

```
In[20]:= Irrep[SU5][Bar[10]]*Irrep[SU5][24]*Irrep[SU5][45]
```

Out[20]:= $3(\overline{\mathbf{5}})+6(\overline{\mathbf{45}})+3(\overline{\mathbf{50}})+5(\overline{\mathbf{70}})+2(\overline{\mathbf{105}})+\overline{\mathbf{175}}''+6(\overline{\mathbf{280}})+2(\overline{\mathbf{280}}')+\overline{\mathbf{420}}+\overline{\mathbf{450}}'+3(\overline{\mathbf{480}})+2(\overline{\mathbf{720}})+\overline{\mathbf{1120}}+\overline{\mathbf{2520}}$

For powers of irreps the Mathematica built-in command `Power` may be used. E.g., decompose the tensor product $\mathbf{27}\otimes\mathbf{27}\otimes\mathbf{27}$ of E_6:

```
In[21]:= Irrep[E6][27]^3
```

Out[21]:= $\mathbf{1} + 2(\mathbf{78}) + 3(\mathbf{650}) + \mathbf{2925} + \mathbf{3003} + 2(\mathbf{5824})$

Decompose tensor products of product irreps $(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1})\otimes(\overline{\mathbf{3}}, \mathbf{3}, \mathbf{1})$ of SU(3)⊗SU(3)⊗SU(3):

```
In[22]:= DecomposeProduct[
           ProductIrrep[Irrep[SU3][3],Irrep[SU3][Bar[3]],Irrep[SU3][1]],
           ProductIrrep[Irrep[SU3][Bar[3]],Irrep[SU3][3],Irrep[SU3][1]]]
```

Out[22]:= $(\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{8},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{8},\mathbf{1}) + (\mathbf{8},\mathbf{8},\mathbf{1})$

Decompose the tensor products $(\mathbf{3}, \mathbf{2})\otimes(\overline{\mathbf{3}}, \mathbf{1})$ of SU(3)⊗SU(2):

```
In[23]:= DecomposeProduct[
           ProductIrrep[Irrep[SU3][3],Irrep[SU2][2]],
           ProductIrrep[Irrep[SU3][Bar[3]],Irrep[SU2][1]]]
```

Out[23]:= $(\mathbf{1},\mathbf{2}) + (\mathbf{8},\mathbf{2})$

3.3. Decomposition to Subalgebras

`DecomposeIrrep[`*irrep*`,` *subalgebra*`]`	decomposes *irrep* to the specified *subalgebra*.
`DecomposeIrrep[`*pirrep*`,` *subalgebra*`,` *pos*`]`	decomposes the product irrep *pirrep* at position *pos*.

Decompose irreps and product irreps.

Decompose the $\overline{\mathbf{10}}$ of SU(5) to SU(3)⊗SU(2)⊗U(1):

```
In[24]:= DecomposeIrrep[Irrep[SU5][Bar[10]],ProductAlgebra[SU3,SU2,U1]]
```

Out[24]:= $(\mathbf{1},\mathbf{1})(6) + (\mathbf{3},\mathbf{1})(-4) + (\overline{\mathbf{3}},\mathbf{2})(1)$

Decompose the **10** and the $\overline{\mathbf{5}}$ of SU(5) to SU(3)⊗SU(2)⊗U(1) (`DecomposeIrrep` is `Listable`):

```
In[25]:= DecomposeIrrep[{Irrep[SU5][10],Irrep[SU5][Bar[5]]},ProductAlgebra[SU3,SU2,U1]]
```

Out[25]:= $\{(\overline{\mathbf{3}},\mathbf{1})(4)+(\mathbf{3},\mathbf{2})(-1)+(\mathbf{1},\mathbf{1})(-6),\ (\overline{\mathbf{3}},\mathbf{1})(-2)+(\mathbf{1},\mathbf{2})(3)\}$

Decompose the **16** of SO(10) to SU(5)⊗U(1):

```
In[26]:= DecomposeIrrep[Irrep[SO10][16],ProductAlgebra[SU5,U1]]
```

Out[26]:= $(\mathbf{1})(-5)+(\overline{\mathbf{5}})(3)+(\mathbf{10})(-1)$

Decompose the **27** of E_6 to SU(3)⊗SU(3)⊗SU(3):

```
In[27]:= DecomposeIrrep[Irrep[E6][27],ProductAlgebra[SU3,SU3,SU3]]
```

Out[27]:= $(\mathbf{3},\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{3},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{3}},\mathbf{1})$

Decompose the SU(3) irrep **3** in $(\mathbf{24},\mathbf{3})(-3)$ of SU(5)⊗SU(3)⊗U(1) to SU(2)⊗U′(1),
i.e., SU(5)⊗SU(3)⊗U(1) → SU(5)⊗SU(2)⊗U′(1)⊗U(1):

```
In[28]:= DecomposeIrrep[ProductIrrep[Irrep[SU5][24],Irrep[SU3][3],Irrep[U1][-3]],
           ProductAlgebra[SU2,U1],2]
```

Out[28]:= $(\mathbf{24},\mathbf{1})(-2)(-3)+(\mathbf{24},\mathbf{2})(1)(-3)$

The same decomposition as above displayed as branching rule:

```
In[29]:= IrrepRule[#,DecomposeIrrep[#,ProductAlgebra[SU2,U1],2]]&@
           ProductIrrep[Irrep[SU5][24],Irrep[SU3][3],Irrep[U1][-3]]
```

Out[29]:= $(\mathbf{24},\mathbf{3})(-3)\to(\mathbf{24},\mathbf{1})(-2)(-3)+(\mathbf{24},\mathbf{2})(1)(-3)$

Branching rules for all totally antisymmetric irreps, so-called basic irreps, of SU(6) to SU(3)⊗SU(3)⊗U(1):

```
In[30]:= IrrepRule[#,DecomposeIrrep[#,ProductAlgebra[SU3,SU3,U1]]]&/@
           BasicIrreps[SU6]//TableForm
```

Out[30]:=
$\mathbf{6}\to(\mathbf{3},\mathbf{1})(1)+(\mathbf{1},\mathbf{3})(-1)$
$\mathbf{15}\to(\overline{\mathbf{3}},\mathbf{1})(2)+(\mathbf{1},\overline{\mathbf{3}})(-2)+(\mathbf{3},\mathbf{3})(0)$
$\mathbf{20}\to(\mathbf{1},\mathbf{1})(3)+(\mathbf{1},\mathbf{1})(-3)+(\mathbf{3},\overline{\mathbf{3}})(-1)+(\overline{\mathbf{3}},\mathbf{3})(1)$
$\overline{\mathbf{15}}\to(\mathbf{3},\mathbf{1})(-2)+(\mathbf{1},\mathbf{3})(2)+(\overline{\mathbf{3}},\overline{\mathbf{3}})(0)$
$\overline{\mathbf{6}}\to(\overline{\mathbf{3}},\mathbf{1})(-1)+(\mathbf{1},\overline{\mathbf{3}})(1)$

3.4. Young Tableaux

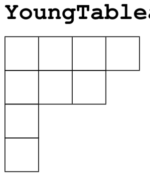
The irreps of SU(N) have a correspondence to Young tableaux, which can be displayed by `YoungTableau`.

`YoungTableau[`*irrep*`]`	Displays the Young tableau associated with an SU(N) *irrep*.

Young tableaux.

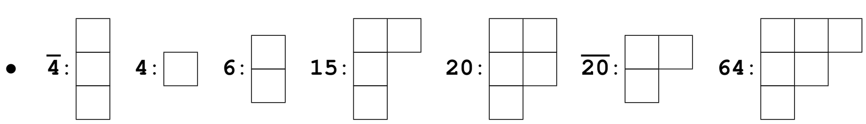
Young tableau of the **720** of SU(5):

```
In[31]:= YoungTableau[Irrep[A][1,2,0,1]]
```

Out[31]:=


Display Young tableaux of SU(4) irreps with a maximum of one column per box count:

```
In[32]:= Row[Row[{#,": ",YoungTableau[#]}]&/@
           SortBy[Irrep[A]@@@Tuples[{0,1},3],Dim],Spacer[10]]
```

Out[32]:= **1**: • $\overline{\mathbf{4}}$: **4**: **6**: **15**: **20**: $\overline{\mathbf{20}}$: **64**:


3.5. Table of LieART Commands

While various LieART commands are scattered throughout the text, we provide the complete set here for easy future reference.

`Rank[`*expr*`]`	gives the rank of the algebra of *expr*, which can be an irrep, a weight a root or an algebra itself.
`Algebra[`*algebraClass*`][`*rank*`]`	represents a classical algebra of the type *algebraClass*, which can only be `A`, `B`, `C` or `D`, with rank *rank*.
`Algebra[`*expr*`]`	gives the algebra (classical or exceptional) of *expr*, which may be an irrep, a weight or a root in any basis.
`OrthogonalSimpleRoots[`*algebra*`]`	gives the simple roots of *algebra* in the orthogonal basis.
`CartanMatrix[`*algebra*`]`	gives the Cartan matrix of *algebra*.
`OmegaMatrix[`*algebra*`]`	gives the matrix of fundamental weights of *algebra* as rows.
`OrthogonalFundamentalWeights[`*algebra*`]`	gives the fundamental weights of *algebra* in the orthogonal basis.
`OrthogonalBasis[`*expr*`]`	transforms *expr* from any basis into the orthogonal basis.
`OmegaBasis[`*expr*`]`	transforms *expr* from any basis into the ω-basis.
`AlphaBasis[`*expr*`]`	transforms *expr* from any basis into the α-basis.
`DMatrix[`*algebra*`]`	gives a matrix with inverse length factors of simple roots on the main diagonal.
`ScalarProduct[`*weight1*`,`*weight2*`]`	gives the scalar product of *expr1* and *expr2* in any basis. *expr1* and *expr2* may be weights or roots.
`MetricTensor[`*algebra*`]`	gives the metric tensor or quadratic-form matrix of *algebra*.

Basic Algebra Properties.

`Reflect[`*weightOrRoot*`,`*simpleroots*`]`	reflects *weightOrRoot* at the hyperplanes orthogonal to the specified *simpleroots*.
`Reflect[`*weightOrRoot*`]`	reflects *weightOrRoot* at the hyperplanes orthogonal to all simple roots of the Lie algebra of *weightOrRoot*.
`ReflectionMatrices[`*algebra*`]`	gives the reflection matrices of the Weyl group of *algebra*.
`Orbit[`*weightOrRoot*`,`*simpleroots*`]`	generates the Weyl group orbit of *weightOrRoot* using only the specified *simpleroots*.
`Orbit[`*weightOrRoot*`]`	generates the full Weyl group orbit of *weightOrRoot* using all simple roots of the Lie algebra of *weightOrRoot*.
`DimOrbit[`*weightOrRoot*`,`*simpleroots*`]`	gives the size of the orbit of *weightOrRoot* using only the *simpleroots*.
`DimOrbit[`*weightOrRoot*`]`	gives the size of the orbit of *weightOrRoot* using all simple roots of the Lie algebra of *weightOrRoot*.

Weyl Group Orbits

`RootSystem[`*algebra*`]`	root system of *algebra*
`ZeroRoots[`*algebra*`]`	zero roots associated with the Cartan subalgebra of *algebra*
`Height[`*root*`]`	height of a *root* within the root system
`HighestRoot[`*algebra*`]`	highest root of the root system of *algebra*
`PositiveRootQ[`*root*`]`	gives `True` if *root* is a positive root and `False` otherwise
`NumberOfPositiveRoots[`*algebra*`]`	number of positive roots of *algebra*
`PositiveRoots[`*algebra*`]`	gives only the positive roots of *algebra*

Roots

`WeightOrthogonal[`*algebraClass*`][`*label*`]`	weight in the orthogonal basis defined by its algebra *algebraClass* and Dynkin *label*
`WeightAlpha[`*algebraClass*`][`*label*`]`	weight in the α-basis defined by its algebra *algebraClass* and Dynkin *label*
`Weight[`*algebraClass*`][`*label*`]`	weight in the ω-basis defined by its algebra *algebraClass* and Dynkin *label*
`Irrep[`*algebraClass*`][`*label*`]`	irrep described by its algebra *algebraClass* and Dynkin *label*
`DynkinLabel[`*irrep*`]`	gives the Dynkin label of *irrep*
`WeightLevel[`*weight*`,`*irrep*`]`	Level of the *weight* within the *irrep*
`Height[`*irrep*`]`	height of *irrep*
`SingleDominantWeightSystem[`*irrep*`]`	dominant weights of *irrep* without their multiplicities
`WeightMultiplicity[`*weight*`,`*irrep*`]`	computes the multiplicity of *weight* within *irrep*
`DominantWeightSystem[`*irrep*`]`	dominant weights of *irrep* with their multiplicities
`WeightSystem[`*irrep*`]`	full weight system of *irrep*
`Irrep[`*algebra*`][`*dimname*`]`	irrep entered by its *algebra* and *dimname*
`ProductIrrep[`*irreps*`]`	head of product *irreps*
`Delta[`*algebra*`]`	half the sum of positive roots of *algebra* (δ=$(1,1,\ldots)$)
`WeylDimensionFormula[`*algebra*`]`	explicit Weyl dimension formula for *algebra*
`Dim[`*irrep*`]`	dimension of *irrep*
`DimName[`*irrep*`]`	dimensional name of *irrep*
`Index[`*irrep*`]`	index of *irrep*
`CongruencyClass[`*irrep*`]`	congruency class number of *irrep*

Basic Properties of Irreps

`DecomposeProduct[`*irreps*`]`	decomposes the tensor product of *irreps*
`DominantWeightsAndMul[`*weights*`]`	filters and tallies dominant weights of *weights* by multiplicities
`SortOutIrrep[`*dominantWeightsAndMul*`]`	sorts out the irrep of largest height from the collection of dominant weights *dominantWeightsAndMul*
`WeightSystemWithMul[`*irrep*`]`	full weight system of *irrep* with multiplicities
`TrivialStabilizerWeights[`*weights*`]`	drops weights that lie on a chamber wall
`ReflectToDominantWeightWithMul[`*weightAndMul*`]`	reflects *weightAndMul* to the dominant chamber and multiplies the parity of the reflection to the multiplicity

Tensor product decomposition.

`DecomposeIrrep[`*irrep*`, `*subalgebra*`]`	decomposes *irrep* to the specified *subalgebra*.
`DecomposeIrrep[`*irrep*`, `*subalgebra*`, `*index*`]`	decomposes *irrep* to the specified *subalgebra*. *index* specifies which branching rule to use, for algebra-subalgebra pairs with multiple branching rules. Only applicable for SU(15) $\to$ SU(3), $E_7 \to$ SU(2), and $E_8 \to$ SU(2).
`DecomposeIrrep[`*productIrrep*`, `*subalgebra*`, `*pos*`]`	decomposes *productIrrep* at position *pos*.
`DecomposeIrrep[`*productIrrep*`, `*subalgebra*`, `*pos*`, `*index*`]`	decomposes *productIrrep* at position *pos*. *index* specifies which branching rule to use, for algebra-subalgebra pairs with multiple branching rules. Only applicable for ...×SU(15)×... $\to$ ...×SU(3)×..., ...×E_7×... $\to$ ...×SU(2)×..., and ...×E_8×... $\to$ ...×SU(2)×....
`ProjectionMatrix[`*origin*`,`*target*`]`	defines the projection matrix for the algebra-subalgebra pair specified by *origin* and *target*
`Project[`*projectionMatrix*`,`*weights*`]`	applies the *projectionMatrix* to the *weights*
`GroupProjectedWeights[`*projectedWeights*`,`*target*`]`	groups the projected weights according to the subalgebra specified in *target*
`NonSemiSimpleSubalgebra[`*origin*`,`*simpleRootToDrop*`]`	computes the projection matrix of a maximal non-semi-simple subalgebra by dropping one dot of the Dynkin diagram *simpleRootToDrop* and turning it into a U(1) charge
`SemiSimpleSubalgebra[`*origin*`,`*simpleRootToDrop*`]`	computes the projection matrix of a maximal semi-simple subalgebra by dropping one dot from the extended Dynkin diagram.
`ExtendedWeightScheme[`*algebra*`,`*simpleRootToDrop*`]`	adds the Dynkin label associated with the extended simple root $-\gamma$ to each weight of the lowest orbit of *algebra* and drops the simple root *simpleRootToDrop*
`SpecialSubalgebra[`*origin*`,`*targetirreps*`]`	computes the projection matrix of a maximal special subalgebra by specifying the branching rule of the generating irrep.

Subalgebra decomposition of irreps and product algebra irreps.

`BranchingRulesTable[`*algebra*`,` *branchingRules*`]`	constructs a table of branching rules for the decomposition of representations of *algebra* to *subalgebras*.
`BranchingRulesTable[`*algebra*`,` *branchingRules*`,` *index*`]`	constructs a table of branching rules for the decomposition of representations of *algebra* to *subalgebras*; *index* specifies which branching rule to use, for algebra-subalgebra pairs with multiple branching rules. Only applicable for SU(15) $\to$ SU(3), $E_7 \to$ SU(2), and $E_8 \to$ SU(2).

Branching rules tables.

`YoungTableau[`*irrep*`]`	Displays the Young tableau associated with an SU(N) *irrep*.

Young tableaux.

4. Theoretical Background and Original Implementation

In this section we give a self-contained overview of the Lie algebra theory used and implemented in LieART. It is subdivided into parts discussing basic properties of Lie algebras, roots, weights, Weyl orbits, representations and decompositions. Every subsection begins with a list of the relevant LieART functions followed by text that introduces the necessary theory with reference to the functions and notes on their implementation. This section is not intended as a pedagogical introduction to Lie algebras and we refer the reader to the excellent literature serving this purpose [8, 10, 12].

4.1. Algebras

`Rank[`*expr*`]`	gives the rank of the algebra of *expr*, which can be an irrep, a weight a root or an algebra itself.
`Algebra[`*algebraClass*`][`*rank*`]`	represents a classical algebra of the type *algebraClass*, which can only be `A`, `B`, `C` or `D`, with rank *rank*.
`Algebra[`*expr*`]`	gives the algebra (classical or exceptional) of *expr*, which may be an irrep, a weight or a root in any basis.
`OrthogonalSimpleRoots[`*algebra*`]`	gives the simple roots of *algebra* in the orthogonal basis.
`CartanMatrix[`*algebra*`]`	gives the Cartan matrix of *algebra*.
`OmegaMatrix[`*algebra*`]`	gives the matrix of fundamental weights of *algebra* as rows.
`OrthogonalFundamentalWeights[`*algebra*`]`	gives the fundamental weights of *algebra* in the orthogonal basis.
`OrthogonalBasis[`*expr*`]`	transforms *expr* from any basis into the orthogonal basis.
`OmegaBasis[`*expr*`]`	transforms *expr* from any basis into the ω-basis.
`AlphaBasis[`*expr*`]`	transforms *expr* from any basis into the α-basis.
`DMatrix[`*algebra*`]`	gives a matrix with inverse length factors of simple roots on the main diagonal.
`ScalarProduct[`*weight1*`,`*weight2*`]`	gives the scalar product of *expr1* and *expr2* in any basis. *expr1* and *expr2* may be weights or roots.
`MetricTensor[`*algebra*`]`	gives the metric tensor or quadratic-form matrix of *algebra*.

Basic Algebra Properties.

4.1.1. Definition

A *Lie Algebra* is a vector space g over a field F with the *Lie bracket* $[\cdot,\cdot]$ as binary operation, which is bilinear, alternating and fulfills the Jacoby identity. The Lie bracket is often referred to as the commutator. The Lie brackets of the generators t_i of the Lie algebra are

$$[t_i, t_j] = f_{ijk} t_k \tag{1}$$

with the so-called *structure constants* f_{ijk}, that fully determine the algebra. A Lie algebra is called *simple* when it contains no non-trivial ideals. A *semi-simple* Lie algebra is a sum of simple ones.

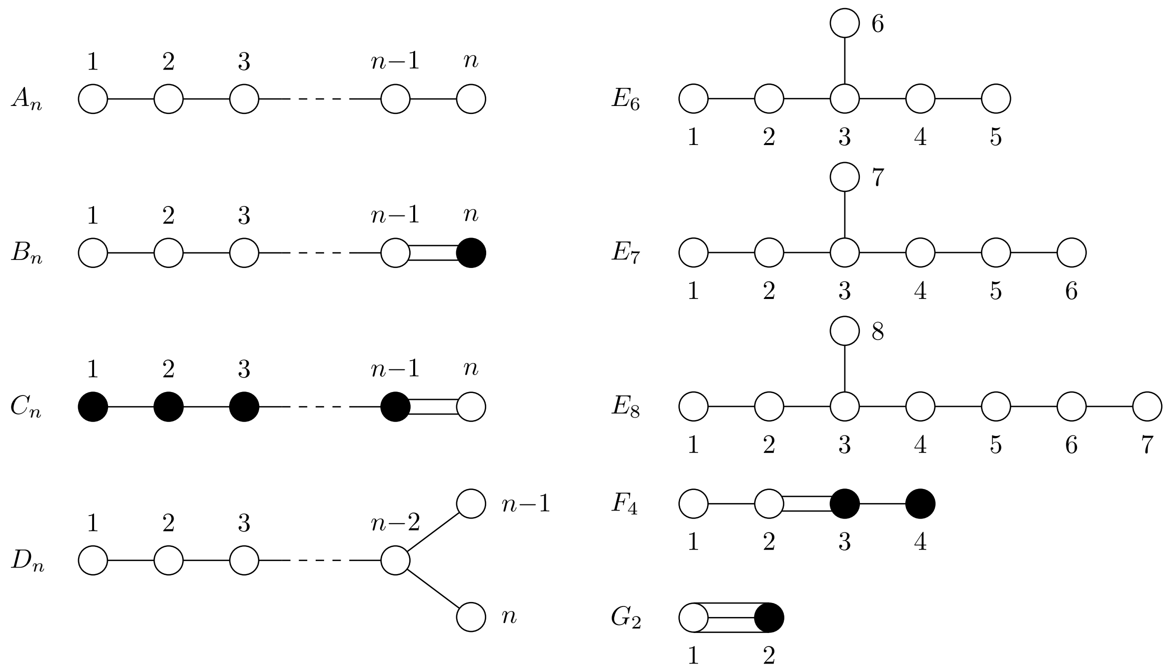


Figure 1: Dynkin Diagrams of classical and exceptional simple Lie algebras.

4.1.2. Roots

The generators t_i of a simple Lie algebra in the Cartan-Weyl basis fall into two sets: The so-called *Cartan subalgebra*, H, contains all simultaneously diagonalizable generators h_i, i.e., the generators are Hermitian and mutually commute (the Cartan subalgebra is abelian):

$$h_i = h_i^\dagger, \qquad [h_i, h_j] = 0, \qquad i, j = 1, \ldots, n. \tag{2}$$

The number of simultaneously diagonalizable generators n is called the *rank* of the algebra, and can be determined by the function `Rank[`*expr*`]` in LieART. We denote all other generators as e_α. They satisfy n eigenvalue equations with the generators of the Cartan subalgebra h_i:

$$[h_i, e_\alpha] = \alpha_i \, e_\alpha, \qquad i = 1, \ldots, n, \tag{3}$$

which is a subset of (1) and thus the α_i are structure constants, which are real numbers due to the hermiticity of the h_i's. Since the α_i are the solutions to the eigenvalue equation (3) the vectors $\alpha{=}(\alpha_1, \ldots, \alpha_n)$ are called the *root vectors*, which lie in an n-dimensional euclidian space, called the *root space*. *Roots* are functionals mapping the Cartan subalgebra H onto the real numbers (the eigenvalues), for all generators t_i, which also includes the h_i where the eigenvalues are zero. Thus, a Lie algebra has as many roots as generators. The roots are labeled by the root vectors, which we will use in its place from now on.

A zero root with an n-fold degeneracy is associated with the Cartan subalgebra. In the Cartan-Weyl basis the other generators come in conjugated pairs $e_\alpha^\dagger{=}e_{-\alpha}$ and correspond to the ladder operators of SU(2). So-called *positive roots* correspond to the raising operator e_α and negative roots to the lowering operators $e_{-\alpha}$. If α is a root so is $-\alpha$.

Some of the positive roots can be written as sum of others. Those for which this is not possible are called *simple roots* and a Lie algebra has as many simple roots as its rank. It is clear that specifying the simple roots fully determines a Lie algebra and thus can be used to replace (1), because all structure constants can be derived therefrom.

Type	Cartan	Name	Rank	Description
classical	A_n	SU(n+1)	$n \geq 1$	Special unitary algebras of n+1 complex dimension
	B_n	SO(2n+1)	$n \geq 3$	Special orthogonal algebras of odd (2n+1) real dimension
	C_n	Sp(2n)	$n \geq 2$	Symplectic algebras of even (2n) complex dimension
	D_n	SO(2n)	$n \geq 4$	Special orthogonal algebras of even (2n) real dimension
exceptional	E_6	E_6	6	Exceptional algebra of rank 6
	E_7	E_7	7	Exceptional algebra of rank 7
	E_8	E_8	8	Exceptional algebra of rank 8
	F_4	F_4	4	Exceptional algebra of rank 4
	G_2	G_2	2	Exceptional algebra of rank 2

Table 4.1: Classification of simple Lie algebras.

4.1.3. Classification of Lie Algebras

Using the commutation relations and the Jacoby identity to analyze the generators, constraints on the roots can be derived and eventually all possible root systems found, which is identical to identifying all allowed Lie algebras. It turns out that simple roots can only come in at most two lengths in one Lie algebra and at four different angles between any pair of them. The simple roots are in particular not orthogonal. The so-called *Dynkin diagrams* are an ingenious way to depict these relations: simple roots are represented by dots, which are open, ○, for the longer roots or for all roots if they only come in one length, and filled, ●, for the shorter roots. Angles between two simple roots are represented by lines connecting the dots: no line for an angle of 90°, one line for 120°, two lines for 135° and three for 150°. Figure 1 shows the Dynkin diagrams for all simple Lie algebras. Semi-simple Lie algebras have disjoint parts and can thus be reduced to two Dynkin diagrams of simple Lie algebras.

The simple Lie algebras fall into two types: four families of infinite series algebras, A_n, B_n, C_n and D_n, also called the *classical Lie algebras* and five so-called *exceptional algebras*, E_6, E_7, E_8, F_4 and G_2, with their rank as subscript (see Table 4.1). The labels are according to the classification by Cartan. The classical Lie algebras are internally represented (i.e., in `FullForm`) in LieART by `Algebra[`*algebraClass*`][`*n*`]`, with *algebraClass* being either `A`, `B`, `C` or `D` and n being the rank. In `StandardForm` the Cartan classification is explicitly displayed and in `TraditionalForm` it is written by its conventional name.

4.1.4. Bases

With respect to the Weyl reflection group, inherent in all compact Lie algebras, as we will explain later, it is convenient to express the root space in an orthogonal coordinate system, which is a subspace of Euclidian space. The specific subspace varies with the Lie algebra. For A_n it is a subspace of $\mathbb{R}^{n+1}$, where the coordinates sum to one. As the simple roots define the Lie algebra, they are explicitly specified in LieART using orthogonal coordinates and can be retrieved by `OrthogonalSimpleRoots[`*algebra*`]`. E.g., the four simple roots of A_4 (SU(5)) in orthogonal coordinates are:

```
In[33]:= OrthogonalSimpleRoots[A4]//Column
         (1, −1, 0, 0, 0)
         (0, 1, −1, 0, 0)
Out[33]:= (0, 0, 1, −1, 0)
         (0, 0, 0, 1, −1)
```

The so-called *Cartan matrix* exhibits the non-orthogonality of the simple roots. It is defined as

$$A_{ij} = \frac{2\,(\alpha_i, \alpha_j)}{(\alpha_j, \alpha_j)} \qquad i, j = 1, \ldots, n \tag{4}$$

where the scalar product $(\cdot,\cdot)$ is the ordinary scalar product of $\mathbb{R}^{n+1}$ in the case of A_n. Most textbooks translate the Dynkin diagrams to the corresponding Cartan matrix as a starting point. And in fact, the rows of the Cartan matrix are the simple roots in the so-called *ω-basis*, which is the bases of *fundamental weights*, also called the *Dynkin basis*. (Weights will be introduced later in the context of representations.) The Cartan matrix is implemented in LieART as the function `CartanMatrix[`*algebra*`]` following the definition of (4). The Cartan matrix for A_4 reads:

```
In[34]:= CartanMatrix[A4]
```

$$\text{Out[34]:=} \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

Besides the orthogonal basis, and the ω-basis, the α-basis is also useful. As the name indicates it is the basis of simple roots and it explicitly shows how, e.g., a root is composed out of simple roots. Neither the ω-basis nor the α-basis is orthogonal. The Cartan matrix mediates between the ω- and α-bases:

$$\alpha_i = \sum_{j=1}^{n} A_{ij}\omega_j, \qquad \omega_i = \sum_{j=1}^{n} (A^{-1})_{ij}\alpha_j. \tag{5}$$

where the ω_i are the fundamental weights, which we will define later. These bases are dual to each other in the sense that

$$\frac{2\,(\alpha_i, \omega_j)}{(\alpha_i, \alpha_i)} \equiv (\alpha_i^\vee, \omega_j) = \delta_{ij}, \qquad i, j = 1, \ldots, n \tag{6}$$

where $\alpha_i^\vee$ is the so-called *coroot* of α_i defined as

$$\alpha^\vee = \frac{2\alpha}{(\alpha, \alpha)}. \tag{7}$$

The transformation to the orthogonal basis can be derived from (6): Expressing α_i and ω_j in orthogonal coordinates as $\hat{\alpha}_i$ and $\hat{\omega}_j$(6) reads

$$\frac{2\hat{\alpha}_i \cdot \hat{\omega}_j}{\hat{\alpha}_i \cdot \hat{\alpha}_i} \equiv \hat{\alpha}_i^\vee \cdot \hat{\omega}_j = \delta_{ij}, \qquad i, j = 1, \ldots, n \tag{8}$$

using the ordinary scalar product of $\mathbb{R}^m$, where m is the dimension of the orthogonal subspace. Using the matrices $\hat{A}$ and $\hat{\Omega}$ with the simple *co*roots $\hat{\alpha}_i^\vee$ and the fundamental weights $\hat{\omega}_j$ as rows, we can write (8) as the matrix equation:

$$\hat{A}\hat{\Omega}^{\mathrm{T}} = I_n \tag{9}$$

where both $\hat{A}$ and $\hat{\Omega}$ are $n{\times}m$ matrices. Please note that the dimension of the orthogonal space m is not necessarily the same as the rank of the algebra n. These exceptions are: A_n with $m{=}n{+}1$, E_6 with $m{=}8$, E_7 with $m{=}8$ and G_2 with $m{=}3$. For all others $m{=}n$ holds. The matrix of the simple coroots in the orthogonal basis $\hat{A}$ is easily calculated from the simple roots given in LieART, but the matrix of fundamental weights in the orthogonal basis $\hat{\Omega}$ must be determined by (9). In the cases where $\hat{A}$ is not a square matrix its inverse does not exist. Because the rows of $\hat{A}$, which are the simple coroots in the orthogonal basis, are linear independent, $\hat{A}\hat{A}^{\mathrm{T}}$ is invertible and the so-called right-inverse $\hat{A}^{+}$ can be found via

$$\hat{A}^{+} = \hat{A}^{\mathrm{T}}(\hat{A}\hat{A}^{\mathrm{T}})^{-1} \tag{10}$$

which satisfies: $\hat{A}\hat{A}^{+}{=}I_n$, i.e., by comparing with (9) the matrix $\hat{\Omega}^{\mathrm{T}}$ can be identified with $\hat{A}^{+}$, in other words the fundamental weights as rows of $\hat{\Omega}$ in terms of simple coroots as rows of $\hat{A}$ are

$$\hat{\Omega} = (\hat{A}^{+})^{\mathrm{T}} = (\hat{A}\hat{A}^{\mathrm{T}})^{-1}\hat{A} \tag{11}$$

The Mathematica built-in function `PseudoInverse[`*matrix*`]` yields the right-inverse for our case of a *matrix* with linear independent rows, i.e., the implementation of the second equality in (11) is not needed. The matrix of the fundamental weights $\hat{\Omega}$ is implemented as `OmegaMatrix[`*algebra*`]`, e.g., for A_4:

```
In[35]:= OmegaMatrix[A4]
```

Out[35]:= $\begin{pmatrix} \frac{4}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{3}{5} & -\frac{2}{5} & -\frac{2}{5} & -\frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & -\frac{4}{5} \end{pmatrix}$

and the function `OrthogonalFundamentalWeights[`*algebra*`]` adds the proper heads to the rows of $\hat{\Omega}$, to identify them as weights is the orthogonal basis. We will discuss (fundamental) weights in Section 4.1.6 in more detail.

The matrix of the fundamental weights in the orthogonal basis $\hat{\Omega}$ mediates between the ω-basis and the orthogonal basis:

$$\omega_i = \sum_{j=1}^{n} \hat{\Omega}_{ij} e_j, \qquad e_i = \sum_{j=1}^{n} (\hat{\Omega}^{-1})_{ij} \omega_j. \tag{12}$$

The LieART functions `AlphaBasis[`*weightOrRoot*`]`, `OmegaBasis[`*weightOrRoot*`]` and `OrthogonalBasis[`*weightOrRoot*`]` transform *weightOrRoot* from any basis into the α-basis, the ω-basis and the orthogonal basis, respectively. It is obvious how the simple roots in the α-basis look:

```
In[36]:= AlphaBasis[OrthogonalSimpleRoots[A4]]//Column
         (1, 0, 0, 0)
Out[36]:= (0, 1, 0, 0)
         (0, 0, 1, 0)
         (0, 0, 0, 1)
```

and likewise the fundamental weights in the ω-basis:

```
In[37]:= OmegaBasis[OrthogonalFundamentalWeights[A4]]//Column
         [1 0 0 0]
         [0 1 0 0]
Out[37]:= [0 0 1 0]
         [0 0 0 1]
```

A root in LieART is represented by three different heads: `RootOrthogonal[`*algebraClass*`][`*label*`]` for a root in the orthogonal basis, `RootOmega[`*algebraClass*`][`*label*`]` in the ω-basis and in the α-basis by `RootAlpha[`*algebraClass*`][`*label*`]`. The *algebraClass* can only be `A`, `B`, `C` or `D` to indicate a classical Lie algebra or `E6`, `E7`, `E8`, `F4` or `G2` for the exceptionals. The *label* stands for the comma-separated coordinates. This form of the roots is displayed in `InputForm` and `FullForm`. E.g., the first simple root of A_4 in all three bases reads:

```
In[38]:= {#,OmegaBasis[#],AlphaBasis[#]}&@First[OrthogonalSimpleRoots[A4]]//InputForm

Out[38]:= {RootOrthogonal[A][1,-1,0,0,0], RootOmega[A][2,-1,0,0], RootAlpha[A][1,0,0,0]}
```

4.1.5. Scalar Product

The standard choice for the length factors (α_j, α_j) in (4) is 2 for the longer roots, if there are two root lengths. The factors $2/(\alpha_j, \alpha_j)$ can only take three different values which are: 1 for all roots of A_n, D_n, E_6, E_7, E_8 and for the long roots of B_n, C_n, F_4 and G_2; 2 for the short roots of B_n, C_n and F_4 and 3 for the short root of G_2. Their implementation in LieART is in the form of diagonal matrices with the inverse factors for the simple roots corresponding to the row on the main diagonal, i.e.,

$$D = \mathrm{diag}\left(\frac{1}{2}(\alpha_1, \alpha_1), \ldots, \frac{1}{2}(\alpha_n, \alpha_n)\right) \tag{13}$$

as defined in [21]. E.g., for F_4, to avoid a trivial example, we have:

```
In[39]:= DMatrix[F4]
```

Out[39]:= $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$

In the ω-basis the scalar product used in (4) becomes:

$$(x, y) = \sum_{i,j}^{n} x_i (A^{-1})_{ij} D_j y_j = \sum_{i,j}^{n} x_i G_{ij} y_j \tag{14}$$

where the x_i and y_j are coordinates of x and y in the ω-basis. The matrix

$$G_{ij} = (A^{-1})_{ij} \frac{(\alpha_j, \alpha_j)}{2} = (A^{-1})_{ij} D_j \tag{15}$$

is called *quadratic-form matrix* or *metric tensor* of the Lie algebra. The scalar product is available in LieART as `ScalarProduct[`*weightOrRoot1*`,`*weightOrRoot2*`]`, where *weightOrRoot1* and *weightOrRoot2* may be roots or weights in the orthogonal basis, the α-basis or the ω-basis. The function recognizes the basis by the heads of *weightOrRoot1* and *weightOrRoot2*. The LieART function for the metric tensor G is `MetricTensor[`*algebra*`]`, e.g., for A_4:

```
In[40]:= MetricTensor[A4]
```

Out[40]:= $\begin{pmatrix} \frac{4}{5} & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{6}{5} & \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{pmatrix}$

4.1.6. Representation

A *representation* is a linear map of the Lie algebra into the general linear group, i.e., the matrix group, that preserves the Lie bracket relations. It is a homomorphism that maps the generators t_i onto invertible matrices T_i, that satisfy the same "commutation" relations as the Lie algebra, namely

$$[T_i, T_j] = f_{ijk} T_k, \tag{16}$$

where the $[\cdot, \cdot]$ is now the commutator.

Points in the vector space that the matrices act on can be labeled by the set of eigenvalues of the matrices representing the generators of the Cartan subalgebra. Such a set of eigenvalues is called a *weight vector*, and the associated functional *weight*, denoted by λ. They are defined in root space which is called *weight space* in this context. The weights and weight vectors of a representation correspond to roots and root vectors of the algebra. In fact, weights can be expressed as rational linear combinations of roots, and, as pointed out in this section, eventually by simple roots. In particular, the structure functions themselves form a representation of the algebra: the *adjoint representation*, which has the same dimension as the algebra, namely the number of roots.

`Reflect[`*weightOrRoot*`,`*simpleroots*`]`	reflects *weightOrRoot* at the hyperplanes orthogonal to the specified *simpleroots*.
`Reflect[`*weightOrRoot*`]`	reflects *weightOrRoot* at the hyperplanes orthogonal to all simple roots of the Lie algebra of *weightOrRoot*.
`ReflectionMatrices[`*algebra*`]`	gives the reflection matrices of the Weyl group of *algebra*.
`Orbit[`*weightOrRoot*`,`*simpleroots*`]`	generates the Weyl group orbit of *weightOrRoot* using only the specified *simpleroots*.
`Orbit[`*weightOrRoot*`]`	generates the full Weyl group orbit of *weightOrRoot* using all simple roots of the Lie algebra of *weightOrRoot*.
`DimOrbit[`*weightOrRoot*`,`*simpleroots*`]`	gives the size of the orbit of *weightOrRoot* using only the *simpleroots*.
`DimOrbit[`*weightOrRoot*`]`	gives the size of the orbit of *weightOrRoot* using all simple roots of the Lie algebra of *weightOrRoot*.

Weyl Group Orbits

The finite group $W(L)$, called the Weyl group of the Lie algebra L, is a reflection group inherent in the root systems of all simple Lie algebras. The Coxeter groups are an abstraction of reflection groups and the so-called *Coxeter-Dynkin diagram* describing Coxeter groups are closely related to the Dynkin diagrams presented here. In fact the Coxeter-Dynkin diagram corresponding to the Dynkin diagram describes the Weyl group of the Lie algebra.

The transformations r_i generating the Weyl group are reflections of a vector x in root space at the hyperplanes orthogonal to the simple roots α_i of the Lie algebra defined by

$$r_i x = x - \frac{2\,(x, \alpha_i)}{(\alpha_i, \alpha_i)}\alpha_i, \qquad i = 1, \ldots, n, \qquad x \in \mathbb{R}^n. \tag{17}$$

The LieART function `Reflect[`*weightOrRoot*`,`*simpleroots*`]` implements the reflections r_i with *weightOrRoot* as x and *simpleroots* as a list of simple roots α_i. The result is a list of weights, because the reflection is preformed with several roots simultaneously.

If *weightOrRoots* are in the orthogonal basis and ought to be reflected using all roots, the function pattern is `Reflect[`*weightOrRoot*`]`, without the simple roots as second argument. Instead of the definition with scalar products following (17), the implementation multiplies the orthogonal coordinates with precomputed reflection matrices, which have a simple form in the orthogonal basis. The function computing the reflection matrices is `ReflectionMatrices[`*algebra*`]` and simply applies the built-in Mathematica command `ReflectionMatrix` to all simple roots and saves the result as `DownValues` of `ReflectionMatrices[`*algebra*`]`. E.g., the reflection matrices for A_4 (in the 5-dimensional orthogonal basis) are:

```
In[41]:= Row[MatrixForm /@ ReflectionMatrices[A4]]
```

Out[41]:=

$$\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&1&0\\0&0&1&0&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&0&1\\0&0&0&1&0\end{pmatrix}$$

The Weyl group of A_n is particularly simple in the orthogonal basis: It is the symmetric group S_{n+1}. The reflection matrices for A_4 above represent the generators of S_5, i.e., the coordinate permutations (12), (23), (34) and (45), respectively.

Acting on a vector x in root space by all elements of the Weyl group gives a set of points, of which some may coincide. The subset of distinct points is called the *orbit* of x and denoted as $O(x)$. The LieART function `Orbit[`*weightOrRoot*`,`*simpleroots*`]` gives the orbit of *weightOrRoot* using the *simpleroots*. If the second argument is omitted, all simple roots of the algebra associated with *weightOrRoot* are used. The function applies `Reflect` in a nested fashion and removes duplicate points in every step. The orbit of an A_n root or weight is constructed in a special way for performance reasons: The *weightOrRoot* is transformed to the orthogonal basis and the other points of its orbit are constructed by permuting its coordinates using the built-in Mathematica function `Permutations`. For example, the orbit of the first simple root of A_4 is

```
In[42]:= Orbit[First[OrthogonalSimpleRoots[A4]]]
Out[42]:= {(−1, 0, 0, 0, 1), (−1, 0, 0, 1, 0), (−1, 0, 1, 0, 0), (−1, 1, 0, 0, 0), (0, −1, 0, 0, 1),
  (0, −1, 0, 1, 0), (0, −1, 1, 0, 0), (0, 0, −1, 0, 1), (0, 0, −1, 1, 0), (0, 0, 0, −1, 1),
  (0, 0, 0, 1, −1), (0, 0, 1, −1, 0), (0, 0, 1, 0, −1), (0, 1, −1, 0, 0), (0, 1, 0, −1, 0),
  (0, 1, 0, 0, −1), (1, −1, 0, 0, 0), (1, 0, −1, 0, 0), (1, 0, 0, −1, 0), (1, 0, 0, 0, −1)}
```

which is in fact the A_4 root system without the zero roots.

With the same set of Weyl group generators, defined by the roots used, every vector is uniquely associated with only one orbit. In turn every element of an orbit allows us to generate the entire orbit by reflecting at the hyperplanes defined by the roots. The hyperplanes divide the space into so-called *Weyl chambers*. An orbit has no more than one distinct element in every chamber and the Weyl group permutes the chambers. The so-called *dominant chamber* has elements with only positive coordinates in the ω-basis, which serves us as a definite element for the orbits associated with them. The test function `DominantQ[`*weightOrRoot*`]` gives `True` if *weightOrRoot* is in the dominant chamber and `False` otherwise. The dominant root of `Out[42]` in the ω-basis is

```
In[43]:= OmegaBasis[Select[%, DominantQ]]
Out[43]:= {[1 0 0 1]}
```

(Roots and weights in the ω-basis are displayed with framed boxes following the notation of most textbooks.) If an orbit is created by LieART it is saved as a `DownValue` of `Orbit` associated with its dominant root or weight. Whenever an orbit of a non-dominant weight or root is needed, LieART first seeks the `DownValues` of `Orbit` for the weight or root, to see if the orbit has already been generated. Reusing computed orbits saves CPU time especially for Lie algebras other than A_n and the described procedure avoids saving the same orbit multiple times as `DownValue` involving different roots or weights.

The size of the orbit, i.e., its numbers of elements, denoted by $|O(x)|$, is implemented as the function `DimOrbit[`*weightOrRoot*`,`*simpleroots*`]` or `DimOrbit[`*weightOrRoot*`]` if all simple roots of the associated Lie algebras should be used. The size of the orbit in `Out[42]` is

```
In[44]:= DimOrbit[First[OrthogonalSimpleRoots[A4]]]
Out[44]:= 20
```

4.3. Roots

`RootSystem[`*algebra*`]`	root system of *algebra*
`ZeroRoots[`*algebra*`]`	zero roots associated with the Cartan subalgebra of *algebra*
`Height[`*root*`]`	height of a *root* within the root system
`HighestRoot[`*algebra*`]`	highest root of the root system of *algebra*
`PositiveRootQ[`*root*`]`	gives `True` if *root* is a positive root and `False` otherwise
`NumberOfPositiveRoots[`*algebra*`]`	number of positive roots of *algebra*
`PositiveRoots[`*algebra*`]`	gives only the positive roots of *algebra*

Roots

The roots of a Lie algebra can be built from the simple roots. There are two traditional approaches: (a) building the roots from linear combinations of simple roots. Since not all linear combinations of simple roots are roots, the difficulty lies in filtering out combinations that are roots. (b) Starting from a *highest root* the roots can be constructed by subtracting simple roots. LieART uses yet another approach: It builds the orbits of the simple roots by applying the Weyl group of the Lie algebra and adds the n-fold degenerated zero roots corresponding to the Cartan subalgebra. The simple roots of the same length belong to the same orbit, e.g., for A_n there is only one orbit besides the zero orbit (see Out[42]). Nevertheless, the orbits of all simple roots are generated and then united. The fact that non-zero roots are non-degenerate allows us to remove duplicate roots obtained by the described procedure.

The function RootSystem[*algebra*] constructs the root system by the procedure described above. As a non-trivial example we demonstrate the procedure on G_2, which has two non-trivial orbits and the zero orbit: The two simple roots of G_2

```
In[45]:= OmegaBasis[OrthogonalSimpleRoots[G2]]
```

$$\text{Out[45]:=}\quad \left\{\boxed{2\ -1},\boxed{-3\ 2}\right\}$$

have different lengths:

```
In[46]:= DMatrix[G2]
```

$$\text{Out[46]:=}\quad \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$$

Generating the Weyl group orbits of each of the simple roots

```
In[47]:= Orbit /@ OmegaBasis[OrthogonalSimpleRoots[G2]]
```

$$\text{Out[47]:=}\quad \begin{pmatrix} \boxed{-2\ 1} & \boxed{-1\ 0} & \boxed{-1\ 1} & \boxed{1\ -1} & \boxed{1\ 0} & \boxed{2\ -1} \\ \boxed{-3\ 1} & \boxed{-3\ 2} & \boxed{0\ -1} & \boxed{0\ 1} & \boxed{3\ -2} & \boxed{3\ -1} \end{pmatrix}$$

and adding the twofold degenerated zero roots constructed by ZeroRoots[*algebra*]

```
In[48]:= ZeroRoots[G2]
```

$$\text{Out[48]:=}\quad \left\{\boxed{0\ 0},\boxed{0\ 0}\right\}$$

yields the full G_2 root system, displayed in spindle shape

```
In[49]:= RootSystem[G2, SpindleShape -> True]
```

Out[49]:=

$$\begin{array}{c} \boxed{0\ 1} \\ \boxed{3\ -1} \\ \boxed{1\ 0} \\ \boxed{-1\ 1} \\ \boxed{-3\ 2}\boxed{2\ -1} \\ \boxed{0\ 0}\boxed{0\ 0} \\ \boxed{-2\ 1}\boxed{3\ -2} \\ \boxed{1\ -1} \\ \boxed{-1\ 0} \\ \boxed{-3\ 1} \\ \boxed{0\ -1} \end{array}$$

where a row stands for the same height of the roots. The *height* of a root is defined as the sum of coefficients in its linear combination of simple roots, i.e., the sum of coordinates in the α-basis. It is implemented by Height[*root*]. The *highest root* has the largest height, implemented in LieART as HighestRoot[*algebra*], which simply returns the first root of the root system, since the latter is sorted by the height of the roots decreasingly. E.g. for G_2:

```
In[50]:= HighestRoot[G2]
```

$$\text{Out[50]:=}\quad \boxed{0\ 1}$$

The *positive roots* are the roots that are only positive linear combinations of simple roots, i.e., the coordinates in the α-basis are all positive, with at least one being non-zero, to exclude the zero roots. The function PositiveRootQ[*root*] tests if *root* is positive. The *root* may be in any basis and will be transformed into the α-basis, where its coordinates are tested accordingly. The number of positive roots are explicitly stated as `NumberOfPositiveRoots[`*algebra*`]` in LieART. It serves as a limiter to the nested reflections for the generation of Weyl group orbits. There is a theorem stating that the maximum number of reflections building an element of the Weyl group is equal to the number of positive roots of the corresponding Lie algebra.

Since the root system is sorted by height, the positive roots come first. `PositiveRoots[`*algebra*`]` extracts only those with the use of `NumberOfPositiveRoots[`*algebra*`]`. E.g., for G_2:

```
In[51]:= PositiveRoots[G2]
Out[51]:= {[0 1],[3 -1],[1 0],[-1 1],[-3 2],[2 -1]}
```

4.4. Representations

`WeightOrthogonal[`*algebraClass*`][`*label*`]`	weight in the orthogonal basis defined by its algebra *algebraClass* and Dynkin *label*
`WeightAlpha[`*algebraClass*`][`*label*`]`	weight in the α-basis defined by its algebra *algebraClass* and Dynkin *label*
`Weight[`*algebraClass*`][`*label*`]`	weight in the ω-basis defined by its algebra *algebraClass* and Dynkin *label*
`Irrep[`*algebraClass*`][`*label*`]`	irrep described by its algebra *algebraClass* and Dynkin *label*
`WeightLevel[`*weight*`,`*irrep*`]`	Level of the *weight* within the *irrep*
`Height[`*irrep*`]`	height of *irrep*
`SingleDominantWeightSystem[`*irrep*`]`	dominant weights of *irrep* without their multiplicities
`WeightMultiplicity[`*weight*`,`*irrep*`]`	computes the multiplicity of *weight* within *irrep*
`DominantWeightSystem[`*irrep*`]`	dominant weights of *irrep* with their multiplicities
`WeightSystem[`*irrep*`]`	full weight system of *irrep*
`Irrep[`*algebra*`][`*dimname*`]`	irrep entered by its *algebra* and *dimname*
`ProductIrrep[`*irreps*`]`	head of product *irreps*
`Delta[`*algebra*`]`	half the sum of positive roots of *algebra* (δ=$(1,1,\ldots)$)
`WeylDimensionFormula[`*algebra*`]`	explicit Weyl dimension formula for *algebra*
`Dim[`*irrep*`]`	dimension of *irrep*
`DimName[`*irrep*`]`	dimensional name of *irrep*
`Index[`*irrep*`]`	index of *irrep*
`CongruencyClass[`*irrep*`]`	congruency class number of *irrep*

Basic Properties of Irreps

As explained in Section 4.1.6 a *representation* is a set of matrices that satisfies the same commutation relations as the algebra. Each of the matrices can be labeled by the *weight vector* with the eigenvalues of the matrices corresponding to the generators of the Cartan subalgebra, and we will refer to the weight vector simply as *weight*. The weight vector has the dimension of the Cartan subalgebra, i.e., the rank of the algebra, and not the dimension of the space the matrices act on. The latter depends on the particular representation.

The weights λ can be written as linear combination of simple roots and a crucial theorem states that the so-called *Dynkin labels* a_i defined as

$$a_i = \frac{2\,(\lambda,\alpha_i)}{(\alpha_i,\alpha_i)}, \qquad i=1,\ldots,n \tag{18}$$

are integers for all simple roots α_i. (Please note that this is also true if λ is replaced by any simple root, since this constitutes an element of the Cartan matrix as defined in (4).) The Dynkin labels are in particular used to label weights (and roots). The smallest non-zero weights with $a_i \geq 0$ are called the *fundamental weights* ω_i. They define the ω-basis or Dynkin basis already introduced. They are implemented in LieART as `OrthogonalFundamentalWeights[`*algebra*`]` in the orthogonal basis and we have given an example for A_4 in `Out[37]`. The Dynkin labels a_i of a weight λ are the coefficients of its linear combination of fundamental weights, i.e., the a_i are the coordinates in the ω-basis, which can be displayed as a row vector with comma separated entries or as a framed box following the convention of some textbooks:

$$\lambda = \sum_{i=1}^{n} a_i \omega_i = (a_1, a_2, \ldots, a_n) = \boxed{\mathtt{a_1\ a_2\ \ldots\ a_n}}. \quad (19)$$

A weight in LieART is represented by three different heads, depending on its basis, in analogy with the roots: `WeightOrthogonal[`*algebraClass*`][`*label*`]` for a weight in the orthogonal basis, in the α-basis `WeightAlpha[`*algebraClass*`][`*label*`]` and simply `Weight[`*algebraClass*`][`*label*`]` in the ω-basis, where we omit the explicit "`Omega`" for brevity, because the ω-basis is the natural basis for weights. (The same can be said for the α-basis for roots, favoring the shorter head `Root` instead of `RootAlpha` in the α-basis. Unfortunately this would clash with the built-in Mathematica function `Root[`*f*`,`*k*`]` representing the kth root of a polynomial equation defined by $f[x] = 0$.) The *algebraClass* can only be `A`, `B`, `C` or `D` to indicate a classical Lie algebra or `E6`, `E7`, `E8`, `F4` or `G2` for the exceptionals. The *label* stands for the comma-separated coordinates. This form of the weight is displayed in `InputForm` and `FullForm`. E.g., the first fundamental weight of A_4 in all three bases reads:

```
In[52]:= {#,AlphaBasis[#],OmegaBasis[#]}&
          @First[OrthogonalFundamentalWeights[A4]]//InputForm
Out[52]:= {WeightOrthogonal[A][4/5,-1/5,-1/5,-1/5,-1/5], WeightAlpha[A][4/5,3/5,2/5,1/5],
           Weight[A][1,0,0,0]}
```

Since weights are linear combinations of roots, many properties of roots translate to weights. The Weyl group also applies to weights and the weight space is also divided into Weyl chambers. A weight with only positive coordinates lies in the dominant Weyl chamber and is called a *dominant weight*. In analogy with the highest root, every irreducible representation (irrep) has a non-degenerate *highest weight*, denoted as Λ, which is also a dominant weight, but not necessarily the only dominant weight of the irrep. The weight system of the irrep can be computed from the highest weight Λ by subtracting simple roots. Thus, a highest weight Λ uniquely defines the irrep, and since a particular Lie algebra has infinitely many irreps, it serves as a label for the irrep itself using the same denotation, Λ.

In LieART an irrep is represented by `Irrep[`*algebraClass*`][`*label*`]`, where *algebraClass* defines the Lie algebra class in the same manner as for weights and roots, and *label* is the comma-separated label of the highest weight of the irrep. E.g., the 10 dimensional irrep of A_4 has the highest weight $(0, 1, 0, 0)$ and thus the irrep can be entered as `Irrep[A][0,1,0,0]`.

The so-called *Dynkin label* of an irrep is similar to the notation of a weight, but since the highest weight has only positive label entries the commas between them can be ommitted, as long as this is unambiguous. The Dynkin label in LieART is displayed in `StandardForm`, e.g., the **10** of A_4:

```
In[53]:= Irrep[A][0,1,0,0]//StandardForm
Out[53]:= (0100)
```

If at least one of the entries in the Dynkin labels has more than a single digit, all entries are separated by commas to avoid ambiguities, which is the standard textbook convention:

```
In[54]:= Irrep[A][0, 10, 3, 1] // StandardForm
Out[54]:= (0, 10, 3, 1)
```

4.4.1. Weight System

The conventional approach to computing all weights of an irrep is to subtract simple roots from the highest weight Λ that defines the irrep. The Dynkin label of the highest weight $\Lambda = (a_1, a_2, \ldots, a_n)$ reveals how many times each simple root can be subtracted: the ith root can be subtracted a_i times. The *level* of a weight is the number of simple root that need to be subtracted from the highest weight to obtain it. A weight may be obtained by different subtraction routes, but it always involves the same number of simple roots, thus its level is unique. As explained earlier, the α-basis exhibits the coefficients of the linear combination of simple roots, which are rational numbers in general. The difference between these coefficients of the weight and the highest weight show how many times each simple roots has been subtracted from the latter. The sum over these differences, for each simple root, gives the level of the weight:

$$L(\lambda, \Lambda) = \sum_{i=1}^{n} \left(\bar{\Lambda}_i - \bar{\lambda}_i\right) \tag{20}$$

where $\lambda = (\bar{\lambda}_1, \ldots, \bar{\lambda}_n)$ and $\Lambda = (\bar{\Lambda}_1, \ldots, \bar{\Lambda}_n)$ is the weight and highest weight in the α-basis, respectively. The LieART function `WeightLevel[`*weight*`,`*irrep*`]` implements this procedure. The highest level of an irrep is called its *height*. Please note that the highest weight has the lowest level, which is zero. The weight with the highest level has the coefficients of the highest weight in the alpha basis, with negative sign and rearranged (if the irrep is complex), i.e., the sum over them is the negative of the sum of the highest weight. Thus, the height of an irrep with highest weight Λ is

$$T(\lambda, \Lambda) = 2 \sum_{i=1}^{n} \bar{\Lambda}_i, \tag{21}$$

which is available in LieART as `Height[`*irrep*`]`.

The algorithm to compute the weight system used in LieART is an implementation of the scheme developed in [23]. It deviates from the traditional procedure described above for performance reasons. Some weights of an irrep may be degenerated and the procedure with subtracting simple roots only yields an upper limit of this degeneracy, which is the number of subtraction routes that lead to a weight. To compute the *multiplicity* m_λ of a weight λ of the irrep with highest weight Λ, the so-called *Freudenthal recursion formula* is usually used:

$$2 \sum_{\alpha \in \Delta^+} \sum_{k \geq 0} (\lambda{+}k\alpha, \alpha)\, m_{\lambda+k\alpha} = \left[(\Lambda{+}\delta, \Lambda{+}\delta) - (\lambda{+}\delta, \lambda{+}\delta)\right] m_\lambda \tag{22}$$

where Δ^+ denotes the positive root system, $\delta{=}(1, 1, \ldots, 1)$ is half the sum of all positive roots in the ω-basis and $m_{\lambda+k\alpha}$ is the already computed multiplicity of a weight $\lambda + k\alpha$ that is higher than λ. The sum over k is finite because the weight $\lambda + k\alpha$ must be a member of the weight system of the irrep under consideration.

The recursive nature of the Freudenthal formula makes the computation of weight multiplicities the most CPU time consuming procedure in the determination of the weight system of an irrep. The algorithm developed in [23] exploits the Weyl group in both the weight and the root system. The weight system of an irrep is a collection of Weyl group orbits, represented by their unique dominant weight. The multiplicity of the dominant weight is the same for all weights of the associated orbit. Thus, a weight system can be constructed by (a) determining the dominant weights of the irrep, (b) computing their multiplicity and (c) generating the orbits of the dominants weights with the same multiplicity by application of the Weyl group of the associated algebra.

In LieART the function `SingleDominantWeightSystem[`*irrep*`]` determines the dominant weights of *irrep* by successively subtracting positive roots starting from the highest weight and keeping only the dominant weight of the result in every step. This process terminates, because there are smallest dominant weights, i.e., the fundamental weights, constituting a lower boundary. E.g., the **40** of A_4 has two distinct dominant weights:

```
In[55]:= SingleDominantWeightSystem[Irrep[A][0, 0, 1, 1]]
Out[55]:= {[0 0 1 1], [0 1 0 0]}
```

Thus, an improved version of the Freudenthal formula considers only dominant weights λ. The second exploitable property is the existence of a stabilizer of the weight, a subgroup of its Weyl group W that fixes the weight:

$$\mathrm{Stab}_W(\lambda) = W_T := \{w \in W \mid w\lambda = \lambda\}\,. \tag{23}$$

The stabilizer W_T reduces the number of independent scalar products and previous computed multiplicities, because for $w \in W_T$, $(\lambda+k\alpha,\alpha)=(w(\lambda+k\alpha_i),w\alpha_i)=(\lambda+kw\alpha_i),w\alpha_i)$ and $m_{\lambda+k\alpha_i}=m_{w(\lambda+k\alpha_i)}=m_{\lambda+kw\alpha_i}$. Since the elements of the Weyl group W are reflections at simple roots, the stabilizer group is defined by the reflection at simple roots that map λ onto itself. Because of (17) this is the case, when $(\lambda,\alpha_i)=0$. If λ is expressed in the ω-basis as $\lambda=\sum n_i\omega_i$ the scalar product with the ith simple root is zero, when $n_i=0$. Let T be a set of these indices, i.e., $T=\{i\,|\,n_i=0\}$, and let Δ_T be the root system based on the simple roots labeled by T.

The group $\hat{W}_T$, which is the inclusion of W_T and the negative identity element $w=-1$ as $\hat{W}_T=\langle W_T,-1\rangle$, decomposes the root system into orbits $o_1,\ldots,o_r$, defined by $\hat{W}_T\alpha_i$. Each orbit has a unique representative ξ_i in the positive roots ($\xi_i\in\Delta^+$). The ξ_i's are those positive roots, that have non-zero coefficients in the ω-basis at the positions, where λ has zeros, i.e., $\xi_i=\sum m_i\omega_i$ with $m_i\geq 0$ for $i\in T$.

The computation of the multiplicity m_λ of the dominant weight λ is then accomplished by the *modified Freudenthal formula*:

$$\sum_{i=1}^{n}|o_i|\sum_{k=1}^{\infty}(\lambda+k\xi_i,\xi_i)\,m_{\lambda+k\xi_i}=[(\Lambda+\delta,\Lambda+\delta)-(\lambda+\delta,\lambda+\delta)]\,m_\lambda \tag{24}$$

where $|o_i|$ are the sizes of the orbits. It is important to note that these sizes are $|o_i|=|W_T\xi_i|$ if $\xi_i\in\Delta_T$ and $|o_i|=2\,|W_T\xi_T|$ if $\xi_i\notin\Delta_T$, because in the former case the negative roots are included in $W_T\xi_i$, i.e., the -1, while only positive roots are in the orbit $W_T\xi_i$ if $\xi_i\notin\Delta_T$, requiring a factor of 2 for the same reason as on the left-hand side of (22). It is $\xi_i\in\Delta_T$ if $(\lambda,\xi_i)=0$.

The higher weight $\lambda+k\xi_i$ is not necessarily a dominant weight, but can always be reflected to the dominant chamber to obtain the corresponding multiplicity that is already computed.

The computation of weight multiplicities is implemented in LieART as `WeightMultiplicity[`*weight*`,`*irrep*`]` following the above algorithm, using several helper functions: `T[`*weight*`]` gives the set T, the positions of zeros of the coefficients of *weight* in the ω-basis, `Xis[`*algebra*`,`*t*`]` determines the ξ's based on the set T which should be supplied via the argument t, `Alphas[`*algebra*`,`*t*`]` gives α_i with $i\in T$ to construct the orbit $W_T\xi_i$. `XisAndMul[`*algebra*`,`*t*`]` yields a list of the ξ_i's together with their associated orbit size $|o_i|$. Since the possible subsets T of zeros in the weight coefficients for a specific algebra are limited, we follow the suggestion of [23] and save this in list form as `XisAndMul` for reuse upon first evaluation in the course of a calculation. Saved values of `XisAndMul` can be retrieved by `Definition[XisAndMul]`.

Take for example the dominant weight `0 1 0 0` of the **40** of A_4 from `Out[55]`. The set of indices T is

```
In[56]:= T[Weight[A][0,1,0,0]]
```

$$\texttt{Out[56]:=}\quad\begin{pmatrix}1\\3\\4\end{pmatrix}$$

or in `InputForm`: `{{1},{3},{4}}`. (The structure as "list of lists" is due to the use of the Mathematica built-in functions `Position` and `Extract`.) The ξ_i's and the sizes of their associated orbits $|o_i|$ are

```
In[57]:= XisAndMul[A4, T[Weight[A][0,1,0,0]]]
```

$$\texttt{Out[57]:=}\quad\begin{pmatrix}\boxed{1\ 0\ 0\ 1} & 12\\ \boxed{0\ {-1}\ 1\ 1} & 6\\ \boxed{2\ {-1}\ 0\ 0} & 2\end{pmatrix}$$

and the weight multiplicity of the dominant weight in the **40** of A_4 is:

```
In[58]:= WeightMultiplicity[Weight[A][0,1,0,0], Irrep[A][0,0,1,1]]
Out[58]:= 2
```

The LieART function `DominantWeightSystem[`*irrep*`]` gives a list of the dominant weights of *irrep* along with their multiplicities:

```
In[59]:= DominantWeightSystem[Irrep[A][0,0,1,1]]
```

Out[59]:=

0 0 1 1	1
0 1 0 0	2

Thus, the weight system of the **40** consists of the Weyl group orbit of `0 0 1 1` and `0 1 0 0`, where every weight of the latter has a multiplicity of two. The function `WeightSystem[`*irrep*`]` generates these orbits based on `DominantWeightSystem` and explicitly duplicates weights according to the multiplicity. If no further processing is intended the option `SpindleShape->True` turns the output into a spindle shape display, with weights of the same level in one row. E.g., the weight system of the **40** of A_4 in spindle shape is

In[60]:= **WeightSystem[Irrep[A][0,0,1,1]]**

Out[60]:=

```
[0 0 1 1]
[0 0 2 −1][0 1 −1 2]
[0 1 0 0][0 1 0 0][1 −1 0 2]
[−1 0 0 2][0 1 1 −2][0 2 −2 1][1 −1 1 0][1 −1 1 0]
[−1 0 1 0][−1 0 1 0][0 2 −1 −1][1 −1 2 −2][1 0 −1 1][1 0 −1 1]
[−1 0 2 −2][−1 1 −1 1][−1 1 −1 1][1 0 0 −1][1 0 0 −1][2 −2 0 1]
[−1 1 0 −1][−1 1 0 −1][0 −1 0 1][0 −1 0 1][1 1 −2 0][2 −2 1 −1]
[−2 0 0 1][−1 2 −2 0][0 −1 1 −1][0 −1 1 −1][2 −1 −1 0]
[−2 0 1 −1][0 0 −1 0][0 0 −1 0]
[−2 1 −1 0][1 −2 0 0]
[−1 −1 0 0]
```

4.4.2. Properties of Irreducible Representations

Dimension. The dimension of an irrep, i.e., the number of its weights, can be calculated without explicitly generating the weight system. The *Weyl dimension formula*, which is a special case of Weyl's character formula, gives the dimension of an irrep in terms of its highest weight Λ, positive roots $\alpha\in\Delta^+$ and $\delta=(1,1,\ldots,1)$:

$$\dim(\Lambda) = \prod_{\alpha\in\Delta^+} \frac{(\alpha,\Lambda+\delta)}{(\alpha,\delta)}. \tag{25}$$

It is implemented in LieART as `Dim[`*irrep*`]`. E.g., the dimension of the **40** of A_4 can be obtained by

In[61]:= **Dim[Irrep[A][0,0,1,1]]**

Out[61]:= 40

The Dynkin label of *irrep* does not need to be numerical. By using variables the simple structure of the formula becomes explicit, e.g., for a general irrep of A_4:

In[62]:= **Dim[Irrep[A][a,b,c,d]]**

Out[62]:= $\frac{1}{288}(a+1)(b+1)(c+1)(d+1)(a+b+2)(b+c+2)(c+d+2)(a+b+c+3)(b+c+d+3)(a+b+c+d+4)$

Internally, the Weyl dimension formula is computed by `WeylDimensionFormula[`*algebra*`]` as a pure function with the digits of the Dynkin label as parameters. E.g. for A_4:

In[63]:= **WeylDimensionFormula[A4]//InputForm**

Out[63]:=

```
Function[{a1,a2,a3,a4},((1+a1)*(1+a2)*(1+a3)*(1+a4)*(2+a1+a2)*(2+a2+a3)
  *(2+a3+a4)*(3+a1+a2+a3)*(3+a2+a3+a4)*(4+a1+a2+a3+a4))/288]
```

The function `Dim[`*irrep*`]` acts only as a wrapper applying the pure function to the specified Dynkin label in the argument of `Dim`.

Index. Another important property of an irrep Λ is its *index*, denoted as $l(\Lambda)$, which is an eigenvalue of the *Casimir invariant* normalized to be an integer:

$$l(\Lambda) = \frac{\dim(\Lambda)}{\mathrm{ord}(L)} (\Lambda, \Lambda + 2\delta) , \tag{26}$$

where $\mathrm{ord}(L)$ is the order of the Lie algebra L, which is equivalent to the number of roots or the dimension of the adjoint irrep. The index is related to the length of the weights and has applications in renormalization group equations and elsewhere. The corresponding LieART function is `Index[`*irrep*`]`. E.g., the index of the **40** of A_4 is:

```
In[64]:= Index[Irrep[A][0,0,1,1]]
Out[64]:= 22
```

The label of *irrep* does not need to be numerical, similar to `Dim`.

Congruency Class. The *congruency class* expands the concept of n-ality of $\mathrm{SU}(N)$, which in turn is a generalization of SU(3) triality, to all other simple Lie algebras. LieART uses congruency classes to characterize irreps, especially for the distinction of irreps of the same dimension and with the same index. We follow the definitions of [9, 24], labeling the congruency class by the *congruency number*, which is a single number for A_n, B_n, C_n, E_6, E_7, E_8, F_4 and G_2 and a two component vector for D_n. For an irreducible representation $(a_1 a_2 \dots a_n)$ the congruency number (vector) c is:

$$\mathrm{A}_n: \quad c = \sum_{k=1}^{n} a_n \pmod{n+1} \tag{27}$$

$$\mathrm{B}_n: \quad c = a_n \pmod 2 \tag{28}$$

$$\mathrm{C}_n: \quad c = a_1 + a_3 + a_5 + \dots \pmod 2 \tag{29}$$

$$\mathrm{D}_n: \quad c = \begin{pmatrix} a_{n-1} + a_n \pmod 2 \\ 2a_1 + 2a_3 + \ldots + 2a_{n-2} + (n-2)a_{n-1} + na_n \pmod 4 \end{pmatrix} \text{ for } n \text{ odd} \tag{30}$$

$$c = \begin{pmatrix} a_{n-1} + a_n \pmod 2 \\ 2a_1 + 2a_3 + \ldots + 2a_{n-3} + (n-2)a_{n-1} + na_n \pmod 4 \end{pmatrix} \text{ for } n \text{ even} \tag{31}$$

$$\mathrm{E}_6: \quad c = a_1 - a_2 + a_4 - a_5 \pmod 3 \tag{32}$$

$$\mathrm{E}_7: \quad c = a_4 + a_6 + a_7 \pmod 2 \tag{33}$$

$$\mathrm{E}_8, \mathrm{F}_4, \mathrm{G}_2: \quad c = 0 \tag{34}$$

Please note that the congruency class definitions of [9, 24], which we use, differ from [8] for the D_n's : For D_4, i.e. SO(8), the second component of [8] is half of the definition above. The congruency class for D_5 is only a single number in [8], which is the same as the second component of the D_5 congruency class vector of our definition from [9, 24] (with no factor of 2).

These congruency numbers (vectors) c are implemented as `CongruencyClass[`*irrep*`]` in LieART. E.g., the congruency numbers of the two eight dimensional irreps of SO(8) are all distiguished by their congruency number vector:

```
In[65]:= CongruencyClass[{Irrep[D][1,0,0,0], Irrep[D][0,0,1,0], Irrep[D][0,0,0,1]}]
Out[65]:= {(02),(12),(10)}
```

The head of a congruency vector is `CongruencyVector` and it is displayed as $(c_1 c_2)$, i.e. without commas separating the two components similar to the Dynkin label of irreps.

4.4.3. Representation Names

The Dynkin label of an irreducible representation together with its Lie algebra uniquely specifies it, e.g., (0011) of A_4. An irrep in LieART is also represented (`FullForm`) by the Dynkin label and a Mathematica head that indicates the algebra class. E.g., the irrep (0011) of A_4 is represented by `Irrep[A][0,0,1,1]` in LieART.

However, it is common practice to name representations by their dimension, the *dimensional name*, which is often times shorter. The dimension of a representation is not unique, i.e., there are different irreps with the same dimensions, which might be accidental or because of a relation between them. If it is accidental, irreps with the same dimension have primes ($\mathbf{dim'}$) in their dimensional name, e.g., the $\mathbf{175'}$ of A_4 is unrelated to the $\mathbf{175}$. Irreps can be related by conjugation, when they are complex. One of the irreps is written with an overbar ($\overline{\mathbf{dim}}$). E.g., the $\overline{\mathbf{10}}$ of A_4 is the conjugate of the $\mathbf{10}$. Due to the high symmetry of SO(8) irrep, more than two related irreps of the same dimension exist. In the case of SO(8) subscripts specify the irreps completely.

The introduced properties of representations, the dimension, the index and the congruency class serve us well to discriminate between irreps with the same dimension. LieART has an algorithm implemented that determines the dimensional name of an irrep, following the naming conventions of [8]:

1. To determine the dimensional name of a specific irrep, LieART collects other irreps of the same dimensionality by brute-force scanning through a generated set of irreps.
2. Irreps that are related by conjugation or the symmetry of SO(8) not only have the same dimension, but also the same index. *Un*related irreps of equal dimension have different indices and can be organized and labeled by their indices. They are sorted by ascending index and labeled with primes ($\mathbf{dim'}$) accordingly, starting with no prime. E.g., the names of the two unrelated 70 dimensional irreps of A_4 are (the congruency class of A_4 is called "Quintality"):

Dynkin	**Dimension**	**Index**	**Quintality**	**Name**
(2001)	70	49	1	$\mathbf{70}$
(0004)	70	84	1	$\mathbf{70'}$

3. Related irreps of the same dimensionality have the same index, but mostly (see below) different congruency class numbers. For Lie algebras other than SO(8), only the conjugate of complex irreps are related. The convention here is that the irrep with *higher* congruency class number of the conjugated pair is labeled with an overbar ($\overline{\mathbf{dim}}$). Since e.g. the $\mathbf{70'}$ is a complex irrep it has a related conjugated irrep, the $\overline{\mathbf{70}}'$, i.e., overbars and primes may both appear in the labeling of an irrep. The above table for the determination of the primes involves only the lower congruency class number irreps of same dimensional and same index irreps. Consider the $\mathbf{70'}$ and its conjugate, the $\overline{\mathbf{70}}'$:

Dynkin	**Dimension**	**Index**	**Quintality**	**Name**
(0004)	70	84	1	$\mathbf{70'}$
(4000)	70	84	4	$\overline{\mathbf{70}}'$

If the congruency class number of a complex irrep is zero, it conjugate also has a congruency class number of zero. In this case, where all three, the dimension, the index and the congruency class number are the same, the structure of the Dynkin labels are consulted: With the Dynkin label interpreted as digits of an integer number, the *smaller* "number" is labeled with the overbar. E.g., the 126 dimensional irreps of A_4 are

Dynkin	**Dimension**	**Index**	**Quintality**	**Name**
(2010)	126	105	0	$\mathbf{126}$
(0102)	126	105	0	$\overline{\mathbf{126}}$
(5000)	126	210	0	$\mathbf{126'}$
(0005)	126	210	0	$\overline{\mathbf{126}}'$

Observe that this rule only applies for zero congruency class number: The $\overline{\mathbf{70}}'$ has a "larger" number (4000) than the $\mathbf{70'}$ with (0004).

4. For SO(8) irreps the convention for the labeling with primes are the same as for all other Lie algebras. Due to the three-fold symmetry most irreps come in sets of three with the same dimension and index. If only one digit of the Dynkin label is non-zero it is called the spinor, vector or conjugate irrep, depending on the dot in the Dynkin diagram it corresponds to. Usually they can be distinguished by the congruency class number, which is a two component vector for SO(8): (01) for a vector irrep, (10) for a spinor and (11) for the conjugate. The irrep is then labeled by the subscripts "v", "s" and "c", resp. E.g., the three 8 dimensional irreps of SO(8) are

Dynkin	**Dimension**	**Index**	**Congruency vector**	**Name**
(1000)	8	1	(01)	$\mathbf{8_v}$
(0001)	8	1	(10)	$\mathbf{8_s}$
(0010)	8	1	(11)	$\mathbf{8_c}$

Some irreps with more than one non-zero digit of the Dynkin label with the same congruency vectors as above are labeled the same way if they are unique. However, if there is more than one irrep with the same dimension, index and also congruency vector, there is more than one digit of the Dynkin label non-zero. The subscript label is then a mixture like "sv", and the ordering is determined by the Dynkin digit beginning with the largest. E.g. the 224 dimensional irreps of SO(8):

Dynkin	**Dimension**	**Index**	**Congruency vector**	**Name**
(2010)	224	100	(12)	$\mathbf{224_{vc}}$
(2001)	224	100	(10)	$\mathbf{224_{vs}}$
(1020)	224	100	(02)	$\mathbf{224_{cv}}$
(1002)	224	100	(02)	$\mathbf{224_{sv}}$
(0021)	224	100	(10)	$\mathbf{224_{cs}}$
(0012)	224	100	(12)	$\mathbf{224_{sc}}$

There are also cases where the congruency vector is zero in both components for all irreps of the same dimension and index. In this case subtracting the same integer from every Dynkin digit to obtain irreps with non-zero congruency class vector has proven to be a reliable way to label the irreps. E.g., the 35 dimensional irreps can be related to the 8 dimensional ones and, e.g., the primed 840 dimensional irreps to the 56 dimensional ones:

Dynkin	**Dimension**	**Index**	**Congruency vector**	**Name**
(1000)	8	1	(02)	$\mathbf{8_v}$
(0010)	8	1	(12)	$\mathbf{8_c}$
(0001)	8	1	(10)	$\mathbf{8_s}$
(2000)	35	10	(00)	$\mathbf{35_v}$
(0020)	35	10	(00)	$\mathbf{35_c}$
(0002)	35	10	(00)	$\mathbf{35_s}$
(1010)	56	15	(10)	$\mathbf{56_s}$
(1001)	56	15	(12)	$\mathbf{56_c}$
(0011)	56	15	(02)	$\mathbf{56_v}$
(2020)	840	540	(00)	$\mathbf{840'_s}$
(2002)	840	540	(00)	$\mathbf{840'_c}$
(0022)	840	540	(00)	$\mathbf{840'_v}$

In LieART the function `DimName[`*irrep*`]` determines the dimensional name according to the algorithm described above, which is automatically displayed if an irrep is displayed in `TraditionalForm`. Several internal helper functions are called by `DimName`. The function `GetIrrepByDim[`*algebra*`,` *dim*`,` *maxDynkinDigit*`]` provides irreps with the same dimension, which are then gathered into sublists by `DimName`. The function `SortSameDimAndIndex` sorts the irreps of same dimension and index by their congruency class, and automatically by the Dynkin label viewed as "numbers", if the congruency class numbers are the same. The positions of the lists of same-index irreps determines the number of primes to apply and the position of the irrep within the same-index list, whether it should be labeled by an overbar. In case of an SO(8) irrep `DimName` branches to the function `SO8Label[`*irrep*`]`, which uses `SimpleSO8Label` to give a subscript of "v", "s" and "c" in the case where the congruency vector in unique. If the congruency vector is not unique, but non-zero, `ConcatSO8Label` concatenates the mixed subscripts like "sv" in the correct ordering. If the congruency vector is zero in both components the irrep is related to irreps with non-zero congruency vector by `ReducedDynkinLabel`.

Limitations. The determination of the primes has one limitation, which requires explanation: The function `GetIrrepByDim[`*algebra*`,` *dim*`,` *maxDynkinDigit*`]` determines irreps of the same dimension. In a brute-force fashion it generates "all" irreps and extracts only those that have the dimension *dim*. Since there are infinite many irreps of any Lie algebra, it must be constrained. This is done by imposing a maximum Dynkin digit to use for the generation of Dynkin labels. Since the numbers of possible Dynkin labels grow rapidly with the maximum Dynkin digit allowed, the limit should be very low. To compare the irrep in question with others it should be at least its maximum Dynkin digit, e.g., for (2031) it is "3". The related irreps only have a permutation of the Dynkin label, thus they are included in the generated list of irreps up to a Dynkin digit of "3" in the example. However, for the determination of the primes for the unrelated irreps it may not suffice to generate irreps only up to the maximum Dynkin digit of the irrep in question: The number of primes are determined by the position in a list of same-dimensional irreps sorted by the index. If there is an irrep with a higher maximal Dynkin digit, e.g., "4" in our example, but at the same time has a lower index than the irrep it question, this procedure would give the irrep in question too few primes. This situation rarely happens, especially in A_n's, but e.g. for G_2 it happens as early as the 77 dimensional irrep:

Dynkin	Dimension	Index	Congruency number	Name
(30)	77	44	0	$\mathbf{77}$
(02)	77	55	0	$\mathbf{77'}$

When determining the name of (02) of G_2 the Dynkin labels would only be generated up to a maximum Dynkin digit of "2", the (30) would not appear and thus the (02) would be labeled without any prime. The determination of the name of (30) would "see" the (02), but would determine no prime for (30) because of its lower index compared to (02). For these two irreps the problem can be solved by generating irreps up to the maximum Dynkin digit *plus one* for the irrep in question, i.e., up to "3" for (02). Because the Dynkin label of a G_2 irrep is small, this is easily manageable. In fact, we have implemented the addition of three to the maximum Dynkin digit for G_2, because for some higher dimensions the problem will reappear. However, for Lie algebras with long Dynkin labels, the number of generated Dynkin labels becomes large and its construction slows LieART down and consumes a large amount of memory. We have found a balance between accuracy and efficiency, which pushes this problem to very high dimensional irreps, by defining the following number to add to the maximum Dynkin digit of the irrep in question: 1 for A_n,B_n,C_n and D_n with $n \leq 4$, 0 for A_n,B_n,C_n and D_n with $n \geq 5$, 1 for E_6 and F_4, 0 for E_7 and E_8 and 3 for G_2. Please note that this limitation is only connected to the labeling of irreps with primes. Computations in LieART are always performed using the Dynkin label as in the `FullForm`. If in doubt one can always use the Dynkin label displayed in `StandardForm`, `InputForm` and `FullForm` which serves as the unique identifier of an irrep.

Besides its Dynkin label, the alternative definition of `Irrep` as `Irrep[`*algebra*`][`*dimname*`]` can be used to specify an irrep by its dimensional name as *dimname* and its algebra as *algebra*. The *algebra* must be fully specified, such as `A4`, `SU5`, `E6`, not only the algebra class such as `A`. The *dimname* is an integer for the dimension with a `Bar[`*dim*`]` wrapped around it for a conjugated irrep or an `IrrepPrime[`*dim*`,` *n*`]` for an irrep with *n* primes. If only one prime is needed the second argument *n* may be omitted. The `Bar` and `IrrepPrime` can be combined in any sequence. E.g., the $\overline{\mathbf{175}}'$ can be entered by

```
In[66]:= Irrep[A4][IrrepPrime[Bar[175]]]//InputForm
```

```
Out[66]:= Irrep[A][0, 0, 2, 1]
```

Alternatives are `Irrep[SU5][IrrepPrime[Bar[175]]]`, `Irrep[SU5][Bar[IrrepPrime[175]]]` and `Irrep[A4][IrrepPrime[Bar[175],1]]`. Internally the function `GetIrrepByDimName[`*algebra*`,` *dimname*`]` determines the corresponding Dynkin label. It uses the function `GetIrrepByDim` mentioned above to find all irreps with the same dimension and then extract the irrep with the identical dimensional name. If the user specifies an irrep that does not exist, e.g. an $\overline{\mathbf{11}}$ of A_4, the comparison must stop at some point. It has been chosen that `GetIrrepByDim` generates only irreps with a maximum Dynkin digit as set by the global variable `$MaxDynkinDigit`. The default is `$MaxDynkinDigit`=3. The consequence is that the determination of the correct Dynkin label of the entered irrep may abort, because the irrep does not exist or that it involves a Dynkin digit higher that `$MaxDynkinDigit`=3. The latter is the case for the $\mathbf{70}'$ with a Dynkin label of (0004). LieART prints an error message indicating the two possible scenarios:

```
In[67]:= Irrep[A4][IrrepPrime[70]]//InputForm
         Irrep::noirrep: Either an irrep with the dimension name 70′ does not exist
Out[67]:=  in SU(5) or it has at least one Dynkin digit higher than 3.  Try with
           $MaxDynkinDigit set to a higher value than 3.  »
```

Increasing `$MaxDynkinDigit` to 4 resolves the issue:

```
In[68]:= $MaxDynkinDigit=4;
         Irrep[A4][IrrepPrime[70]]//InputForm
Out[68]:= Irrep[A][0,0,0,4]
```

4.5. Tensor Product Decomposition

`DecomposeProduct[`*irreps*`]`	decomposes the tensor product of *irreps*
`DominantWeightsAndMul[`*weights*`]`	filters and tallies dominant weights of *weights* by multiplicities
`SortOutIrrep[`*dominantWeightsAndMul*`]`	sorts out the irrep of largest height from the collection of dominant weights *dominantWeightsAndMul*
`WeightSystemWithMul[`*irrep*`]`	full weight system of *irrep* with multiplicities
`TrivialStabilizerWeights[`*weights*`]`	drops weights that lie on a chamber wall
`ReflectToDominantWeightWithMul[`*weightAndMul*`]`	reflects *weightAndMul* to the dominant chamber and multiplies the parity of the reflection to the multiplicity

Tensor product decomposition.

Tensor products of irreps can be decomposed into a direct sum of irreps. The product of two irreps R_1 and R_2 can be decomposed as

$$R_1 \otimes R_2 = \sum_i m_i R_i \tag{35}$$

with the following dimension and index sum rules:

$$\dim(R_1 \otimes R_2) = \dim(R_1) \cdot \dim(R_2) = \sum_i m_i \dim(R_i) \tag{36}$$

$$l(R_1 \otimes R_2) = l(R_1)\dim(R_2) + \dim(R_1) l(R_2) = \sum_i m_i\, l(R_i). \tag{37}$$

4.5.1. Generic Algorithm

A straight-forward method to compute the right-side of (35) is the following: Add all weights of R_2 to each weight of R_1. The resulting $\dim(R_1) \cdot \dim(R_2)$ weights belong to the different irreps R_i, which must be sorted out. Instead of all weights, one can consider just the dominant weights in the product, as each of the dominant weights represents an orbit in the irreps R_i. As an irrep is a collection of orbits, some of the dominant weights in the product represent the highest weight of an irrep in the decomposition. There is a unique dominant weight that represents the irrep of largest height in the decomposition. The sorting procedure should start with this dominant weight viewed as the highest weight of an irrep and then construct the dominant weight system of the corresponding irrep. The dominant weight system of the irrep with largest height should then be subtracted from the combined dominant weights of the product to filter it out. The same procedure is applied recursively to the remaining set of dominant weights until it is empty, i.e., all irreps have been filtered out.

LieART provides the function `DecomposeProduct[`*irreps*`]` for the decomposition of the tensor product of arbitrary many *irreps* of any classical or exceptional Lie algebra as argument. As a demonstration of generic algorithm we consider the decomposition of the SU(3) tensor product $\mathbf{8}\otimes\mathbf{8}$, which is

In[69]:= **`DecomposeProduct[Irrep[SU3][8], Irrep[SU3][8]]`**

Out[69]:= $\mathbf{1} + 2(\mathbf{8}) + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$

In the straight forward approach one adds all weights of the second $\mathbf{8}$ to each weight of the first $\mathbf{8}$, using the built-in Mathematica function `Outer`. One filters out only the dominant weights and tallies multiple occurrences thereof using the LieART function `DominantWeightsAndMul[`*weights*`]`, which also sorts the weights according to their height, when viewed as a highest weight of an irrep. For the SU(3) tensor product $\mathbf{8}\otimes\mathbf{8}$ the dominant weights along with their multiplicities are

In[70]:= **`DominantWeightsAndMul[Flatten[Outer[Plus, WeightSystem[Irrep[SU3][8]], WeightSystem[Irrep[SU3][8]]]]]`**

Out[70]:= $\begin{pmatrix} \boxed{2\ 2} & 1 \\ \boxed{0\ 3} & 2 \\ \boxed{3\ 0} & 2 \\ \boxed{1\ 1} & 6 \\ \boxed{0\ 0} & 10 \end{pmatrix}$

The dominant weight with the largest height $\boxed{2\ 2}$ must be the highest weight of an irrep. The dominant weight system of the (22) (the $\mathbf{27}$) of SU(3) is

In[71]:= **`DominantWeightSystem[Irrep[A][2, 2]]`**

Out[71]:= $\begin{pmatrix} \boxed{2\ 2} & 1 \\ \boxed{0\ 3} & 1 \\ \boxed{3\ 0} & 1 \\ \boxed{1\ 1} & 2 \\ \boxed{0\ 0} & 3 \end{pmatrix}$

It contains all dominant weights appearing in the tensor product, but with mostly smaller multiplicities. The irrep (22) can be filtered out by subtracting the multiplicities in `Out[71]` from `Out[70]`. The LieART function `SortOutIrrep[`*dominantWeightsAndMul*`]` performs the task of computing the dominant weight system of the irrep corresponding to the largest height weight and subtracting it from the tensor product. It returns the dominant weights of the tensor product with the ones of the irrep removed and passes the latter using the `Sow` and `Reap` mechanism of Mathematica:

In[72]:= **`Reap[SortOutIrrep[%%]]`**

Out[72]:= $\left\{ \begin{pmatrix} \boxed{0\ 3} & 1 \\ \boxed{3\ 0} & 1 \\ \boxed{1\ 1} & 4 \\ \boxed{0\ 0} & 7 \end{pmatrix}, (\mathbf{27}) \right\}$

The function `SortOutIrrep` is applied recursively until the list of dominant weights with multiplicities of the tensor product is empty. E.g., applying `SortOutIrrep` to the dominants weights of `Out[72]` filters out the (03) (the $\overline{\mathbf{10}}$) of SU(3):

In[73]:= **`Reap[SortOutIrrep[First[%]]]`**

Out[73]:= $\left\{\begin{pmatrix} \boxed{3\ 0} & 1 \\ \boxed{1\ 1} & 3 \\ \boxed{0\ 0} & 6 \end{pmatrix}, (\overline{\mathbf{10}})\right\}$

The irreps filtered out by `SortOutIrrep` are collected by the LieART function `GetIrreps` and can be displayed as the result of the decomposition. However, LieART computes the tensor product decomposition by an implementation of Klimyk's formula, which is far more efficient than the straight-forward procedure described above. However, the sorting-out algorithm is used for subalgebra decomposition in Section 4.6.

4.5.2. Algorithm Based on Klimyk's Formula

Adding all weights of R_2 to each weight of R_1 is costly for large irreps. LieART's algorithm to decompose tensor-product implements Klimyk's formula [20, 21, 16], which improves the runtime of tensor product decompositions considerably: Let λ_1 and λ_2 be weights and Λ_1 and Λ_2 the highest weights of R_1 and R_2, respectively. Instead of adding all weights λ_1 to each weight λ_2, the weights λ_1 are added only to the highest weight Λ_2 of R_2 together with half the sum of positive simple roots, δ=(1, ..., 1), building the set of weights

$$\mu = \lambda_1 + \Lambda_2 + \delta. \tag{38}$$

Each μ is reflected to the dominant chamber, yielding a highest weight denoted as $\{\mu\}$, with $\mathrm{sgn}(\mu)$ as the parity of the reflection. Of these dominant weights all that lie on a chamber wall are dropped. The irreps in the decomposition are $R(\{\mu\} - \delta)$. Klimyk's formula reads

$$R_1(\Lambda_1) \otimes R_2(\Lambda_2) = \sum_{\lambda_1} m_{1\lambda_1} t(\lambda_1 + \Lambda_2 + \delta) R(\{\lambda_1 + \Lambda_2 + \delta\} - \delta), \tag{39}$$

where $m_{1\lambda_1}$ denotes the multiplicity of λ_1 in R_1. We define $t(\mu)$ to be $\mathrm{sgn}(\mu)$ if μ has a trivial-stabilizer subgroup $\mathrm{Stab}(\mu) = \{1\}$ (see (23)), and zero if the stabilizer subgroup is non-trivial, i.e. the weight lies on a chamber wall:

$$t(\mu) = \begin{cases} \mathrm{sgn}(\mu) & : \quad \mathrm{Stab}(\mu) = \{1\} \\ 0 & : \quad \text{else} \end{cases}. \tag{40}$$

We will demonstrate this algorithm with LieART in the following paragraphs.

As a demonstration of the algorithm implemented we consider the decomposition of the SU(3) tensor product **6**⊗**3**, which is

In[74]:= **`DecomposeProduct[Irrep[SU3][6], Irrep[SU3][3]]`**

Out[74]:= **8** + **10**

LieART normally reorders the arguments of `DecomposeProduct` by their dimension, which usually simplifies the application of Klimyk's formula. But for didactical reasons we assume in the following that the irreps are not reordered. LieART generates the weight system with weight multiplicities of the **6**:

In[75]:= **`WeightSystemWithMul[Irrep[SU3][6]]`**

Out[75]:= $\begin{pmatrix} \boxed{2\ 0} & 1 \\ \boxed{-2\ 2} & 1 \\ \boxed{0\ -2} & 1 \\ \boxed{0\ 1} & 1 \\ \boxed{1\ -1} & 1 \\ \boxed{-1\ 0} & 1 \end{pmatrix}$

We add all weights of the **6** to the highest weight of the **3**, i.e. `HighestWeight[Irrep[SU3][3]]` = $\boxed{1\ 0}$, and `Delta[SU3]`=$\boxed{1\ 1}$, according to $\mu = \lambda_1 + \Lambda_2 + \delta$. The highest weight Λ_1 and δ can be added directly, but to add the sum to all weights in the weight system with multiplicities LieART provides the command `Add`:

```
In[76]:= mu = Add[WeightSystemWithMul[Irrep[SU3][6]], HighestWeight[Irrep[SU3][3]] +
           Delta[SU3]]
```

Out[76]:=
$$\begin{pmatrix} \boxed{4\ \ 1} & 1 \\ \boxed{0\ \ 3} & 1 \\ \boxed{2\ -1} & 1 \\ \boxed{2\ \ 2} & 1 \\ \boxed{3\ \ 0} & 1 \\ \boxed{1\ \ 1} & 1 \end{pmatrix}$$

LieART reflects these weights to the dominant chamber yielding the corresponding dominant weights and the parity of the reflection, i.e. $\mathrm{sgn}(\mu) = (-1)^l$, where l is the number of reflections needed to reach the dominant chamber:

```
In[77]:= ReflectToDominantWeightWithMul /@ mu
```

Out[77]:=
$$\begin{pmatrix} \boxed{4\ \ 1} & 1 \\ \boxed{0\ \ 3} & 1 \\ \boxed{1\ \ 1} & -1 \\ \boxed{2\ \ 2} & 1 \\ \boxed{3\ \ 0} & 1 \\ \boxed{1\ \ 1} & 1 \end{pmatrix}$$

The weights $\boxed{4\ \ 1}$, $\boxed{0\ \ 3}$, $\boxed{2\ \ 2}$, $\boxed{3\ \ 0}$ and $\boxed{1\ \ 1}$ were already in the dominant chamber, while $\boxed{2\ -1}$ needed one reflection to become the dominant weight $\boxed{1\ \ 1}$ with a parity of -1.

Weights on walls of the dominant chamber do not contribute in Klimyk's formula and must be dropped. Weights not lying on a chamber wall have a trivial-stabilizer subgroup. To keep only these weights LieART applies the command `TrivialStabilizerWeights`, which drops all weights containing at least one zero anywhere in their Dynkin label, which corresponds to lying on a chamber wall:

```
In[78]:= TrivialStabilizerWeights[ReflectToDominantWeightWithMul /@ mu]
```

Out[78]:=
$$\begin{pmatrix} \boxed{4\ \ 1} & 1 \\ \boxed{1\ \ 1} & -1 \\ \boxed{2\ \ 2} & 1 \\ \boxed{1\ \ 1} & 1 \end{pmatrix}$$

LieART subtracts δ yielding highest weights and constructs the irreps in the decomposition $R(\{\lambda_1 + \Lambda_2 + \delta\} - \delta)$:

```
In[79]:= ToIrrep/@Add[TrivialStabilizerWeights[ReflectToDominantWeightWithMul/@mu],
           -delta]
```

Out[79]:=
$$\begin{pmatrix} \mathbf{10} & 1 \\ \mathbf{1} & -1 \\ \mathbf{8} & 1 \\ \mathbf{1} & 1 \end{pmatrix}$$

While the irreps in the left column correspond to $R(\{\lambda_1 + \Lambda_2 + \delta\} - \delta)$ in Klimyks formula, the multiplicities in the right column correspond to $m_{1\lambda_1}\,\mathrm{sgn}(\lambda_1 + \Lambda_2 + \delta)$. Summing up the decomposition accordingly we see that the $\mathbf{1}$ drops out and we obtain the result $\mathbf{8} + \mathbf{10}$ as already stated in `Out[74]`.

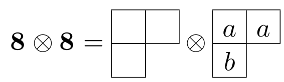
4.5.3. SU(N) Decomposition via Young Tableaux

A correspondence of SU(N) irreps and Young tableaux is very useful for the calculation of tensor products and subalgebra decomposition by hand. We have found that the algorithm for the SU(N) tensor product decomposition via Young tableaux also performs better on the computer, with respect to CPU time and memory consumption, than the procedure described in the previous section. Thus, LieART uses the Young tableaux algorithm for the tensor-product decomposition of SU(N) irreps and the procedure of adding weights and filtering out irreps for all other classical and exceptional Lie algebras.

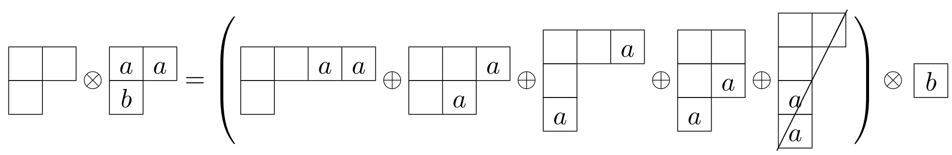
A *Young tableau* is a left-aligned set of boxes, with successive rows having an equal or smaller number of boxes. Young tableaux correspond to the symmetry of the tensors of SU(N) irreps, by first writing each index of the tensor into one box of a Young tableau and the prescription that they ought to be first symmetrized in the rows and then antisymmetrized in the columns. Please see `Out[31]` in Section 3 for a non-trivial example for a Young tableau displayed by LieART.

To demonstrate the algorithm for the tensor product decomposition via Young tableaux we use the same SU(3) tensor product as in Section 4.5.1, $\mathbf{8}\otimes\mathbf{8}$.

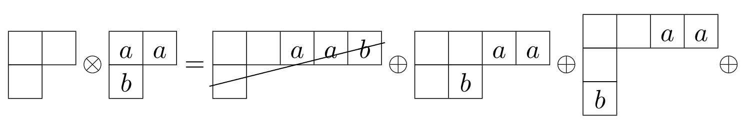
The construction principle (by hand) is to put the Young tableau with the most boxes to the left and bump all boxes of the right Young tableau row by row to the left one following certain rules. To understand these rules label the boxes of each row of the right Young tableau alphabetically:

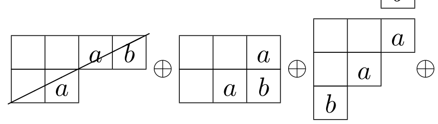

(41)

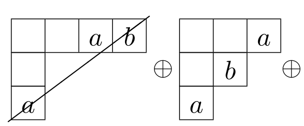
Bump the boxes of the first row of the right tableau (labeled with a's) to the ends of the left tableau to form valid Young tableaux and the additional condition that no a's are in the same column, as this would change symmetric indices to antisymmetric ones. (see crossed out tableau, which is also not a valid SU(3) tableau, because it has four (or $N+1$ in general for SU(N)) boxes in a column):

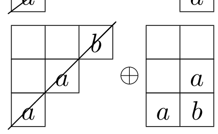

(42)

As the next step bump the boxes of the next row to all tableaux obtained by the first step obeying the same rules, but keep only tableaux with an *admissible* sequence of letters, when reading the letters from right to left row by row. A sequence of letters is admissible if at any point at least as many a's have occurred as b's and likewise for b's and c's, etc. The sequences $abcd$, $aaabc$,$abab$ are admissible, but $abba$, acb are not. In our example only the sequence baa is not admissible, which appears in the four crossed-out tableaux:

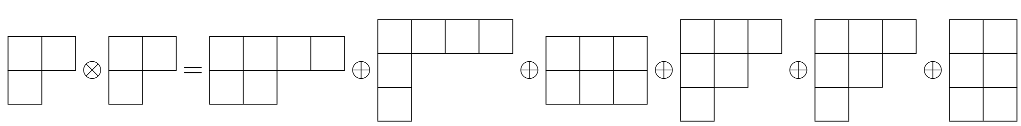

(43)


(44)

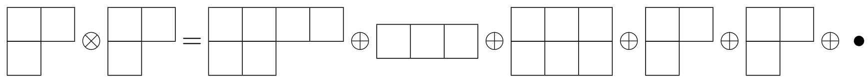

(45)


(46)

Removing the labeling with a's and b's, we obtain the SU(3) tensor product decomposition of $\mathbf{8}\otimes\mathbf{8}$ in terms of Young tableaux:


(47)

Finally we knock out triples (full columns with three boxes) to find:



$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1} \tag{48}$$

In LieART the Young tableau algorithm is automatically applied to tensor products of SU(N) irreps using `DecomposeProduct[`*irreps*`]`. After sorting the irrep with fewer boxes to the right (which we will call *irrep2* opposed to the first one named *irrep1*), the function processes through *irrep2*'s rows to bump boxes to *irrep1*. The function `BoxesToBump[`*irrep2*`,` *row*`]` gives the number of boxes in the current *row* to bump to the tableau of *irrep1*. The function `AllowedRows[`*irrep1*`,` *nboxes*`]` determines the rows of *irrep1* that are allowed to bump boxes to yielding a valid young tableau with an admissible sequence. The latter is checked by the helper function `AllowedCombination`. The function `AddToTableau[`*irrep1*`,` *rowcombinations*`]` performs the bumping of the boxes of one row of *irrep2* in all allowed combinations (*rowcombinations*) to *irrep1*. The result of the bumping is directly expressed in terms of a changed Dynkin label.

4.6. Subalgebra Decomposition

`DecomposeIrrep[`*irrep*`,` *subalgebra*`]`	decomposes *irrep* to the specified *subalgebra*.
`DecomposeIrrep[`*productIrrep*`,` *subalgebra*`,` *pos*`]`	decomposes *productIrrep* at position *pos*.
`ProjectionMatrix[`*origin*`,` *target*`]`	defines the projection matrix for the algebra-subalgebra pair specified by *origin* and *target*
`Project[`*projectionMatrix*`,` *weights*`]`	applies the *projectionMatrix* to the *weights*
`GroupProjectedWeights[`*projectedWeights*`,` *target*`]`	groups the projected weights according to the subalgebra specified in *target*
`NonSemiSimpleSubalgebra[`*origin*`,` *simpleRootToDrop*`]`	computes the projection matrix of a maximal non-semi-simple subalgebra by dropping one dot of the Dynkin diagram *simpleRootToDrop* and turning it into a U(1) charge
`SemiSimpleSubalgebra[`*origin*`,` *simpleRootToDrop*`]`	computes the projection matrix of a maximal semi-simple subalgebra by dropping one dot from the extended Dynkin diagram.
`ExtendedWeightScheme[`*algebra*`,` *simpleRootToDrop*`]`	adds the Dynkin label associated with the extended simple root $-\gamma$ to each weight of the lowest orbit of *algebra* and drops the simple root *simpleRootToDrop*
`SpecialSubalgebra[`*origin*`,` *targetirreps*`]`	computes the projection matrix of a maximal special subalgebra by specifying the branching rule of the generating irrep.

Subalgebra decomposition of irreps and product algebra irreps.

The LieART function `DecomposeIrrep[`*irrep*`,` *subalgebra*`]` decomposes an irrep of a simple Lie algebra into a maximal subalgebra specified by *subalgebra*, which can be simple, semi-simple or non-semi-simple. To decompose an irrep of a semi-simple or non-semi-simple irrep, a third argument *pos* allows one to specify which part of *productIrrep* should be decomposed into the *subalgebra*.

The implementation of `DecomposeIrrep` in LieART uses so-called projection matrices. These matrices project the weights of an irrep into the specified subalgebra. The resulting weights are further processed in the same manner as the in Section 4.6: Only the dominant weights of the decomposed weights are kept, because they uniquely define the orbits of the subalgebra and thus its irreps. In the next step the irreps comprised in the collection of dominant weights are sorted out using the same LieART functions as for the generic tensor product decomposition, discussed in section 4.5.1. It is clear that the major task is the determination of the projection matrices. They are different for each algebra-maximal-subalgebra pair and are not unique. (An extensive collection of projection matrices can be found in [25] for the Lie algebra A_n and in [26] for the Lie algebras B_n, C_n and D_n.) Once a projection matrix is known it can be used for the decomposition of all irreps of the algebra-maximal-subalgebra pair. E.g., the projection matrix for the branching SU(5) → SU(3)⊗SU(2)⊗U(1) is

```
In[80]:= ProjectionMatrix[SU5,ProductAlgebra[SU3,SU2,U1]]
```

Out[80]:= $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 4 & 6 & 3 \end{pmatrix}$

The determination of the projection matrices is closely connected to the problem of finding maximal subalgebras and we defer the description of its implementation in LieART to the next section. Taking the projection matrix `Out[80]` as given we demonstrate the algorithm of `DecomposeIrrep` to find the branching rule for the **10** of SU(5) to SU(3)⊗SU(2)⊗U(1), which is

```
In[81]:= DecomposeIrrep[Irrep[SU5][10],ProductAlgebra[SU3,SU2,U1]]
```

Out[81]:= $(\mathbf{1},\mathbf{1})(-6)+(\overline{\mathbf{3}},\mathbf{1})(4)+(\mathbf{3},\mathbf{2})(-1)$

The LieART function `Project[`*projectionMatrix*`,`*weights*`]` applies the *projectionMatrix* to each of the *weights* and a subsequent `GroupProjectedWeights[`*projectedWeights*`,`*target*`]` groups the Dynkin label of each of the *projectedWeights* according to the subalgebra specified by *target*. In the case of our example each weight of the **10** of SU(5) decomposes to SU(3)⊗SU(2)⊗U(1) as:

```
In[82]:= IrrepRule @@@ Transpose[{WeightSystem[Irrep[SU5][10]],
          Row/@GroupProjectedWeights[Project[ProjectionMatrix[SU5,
          ProductAlgebra[SU3,SU2,U1]],WeightSystem[Irrep[SU5][10]]],
          ProductAlgebra[SU3,SU2,U1]]}]
```

Out[82]:=

```
[0 1 0 0]    → [0 1][0][4]
[1 −1 1 0]   → [1 −1][0][4]
[−1 0 1 0]   → [−1 0][0][4]
[1 0 −1 1]   → [1 0][1][−1]
[−1 1 −1 1]  → [−1 1][1][−1]
[1 0 0 −1]   → [1 0][−1][−1]
[−1 1 0 −1]  → [−1 1][−1][−1]
[0 −1 0 1]   → [0 −1][1][−1]
[0 −1 1 −1]  → [0 −1][−1][−1]
[0 0 −1 0]   → [0 0][0][−6]
```

The algorithm of `DecomposeIrrep` differs slightly and keeps only the dominant weights after projection and groups only them yielding

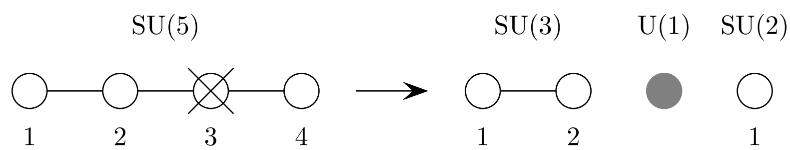
$$\begin{pmatrix} \boxed{0\ 1}\boxed{0}\boxed{4} \\ \boxed{1\ 0}\boxed{1}\boxed{-1} \\ \boxed{0\ 0}\boxed{0}\boxed{-6} \end{pmatrix} \tag{49}$$

for our example. A combination of the functions `GetAllProductIrrep` and `GetProductIrrep` filter out the product irreps, $(\overline{\mathbf{3}},\mathbf{1})(4)$, $(\mathbf{1},\mathbf{1})(-6)$ and $(\mathbf{3},\mathbf{2})(-1)$ in our case, by applying the function `GetIrrep` known from section 4.5.1 to the weights.

4.6.1. Branching Rules and Maximal Subalgebras

To determine the projection matrices we start with the algorithm to find maximal subalgebras. Subalgebras fall into two classes: *regular* and *special* subalgebras, with the first one being further categorized into non-semisimple and semisimple subalgebras. In the following we describe the derivation of the three types of maximal subalgebras: regular non-semisimple, regular semisimple and special subalgebras, originally developed by Dynkin [1, 2] and demonstrate how it is utilized by LieART to determine the projection matrices.

Non-Semisimple Subalgebras. A non-semisimple subalgebra is a semisimple subalgebra times a U(1) factor, e.g. SU(3)⊗SU(2)⊗U(1). A subalgebra of this type is obtained by removing a dot from the Dynkin diagram. The resulting two or more disconnected Dynkin diagrams symbolize the semisimple subalgebra and the removed dot, i.e., simple root, becomes the U(1) generator. E.g., the non-semisimple subalgebra SU(3)⊗SU(2)⊗U(1) can be obtained from SU(5) by removing the third dot from its Dynkin diagram:



Since the Dynkin label of a weight represents it composition of simple roots (explicitly in the α-basis), dropping a simple root (dot) from the Dynkin diagram corresponds to dropping the associated digit from the Dynkin label. The U(1) charge is the coefficient of the dropped simple root in the weight's linear combination of simple roots, i.e., the associated digit of the Dynkin label in the α-basis, which is often normalized to give integer values. Accordingly, in `Out[82]` the third Dynkin digit of the weight of the **10** has been removed after the projection and by expressing the weight system in the α-basis

```
In[83]:= AlphaBasis[WeightSystem[Irrep[SU5][10]]]//Column
         (3/5, 6/5, 4/5, 2/5)
         (3/5, 1/5, 4/5, 2/5)
         (-2/5, 1/5, 4/5, 2/5)
         (3/5, 1/5, -1/5, 2/5)
Out[83]:= (-2/5, 1/5, -1/5, 2/5)
         (3/5, 1/5, -1/5, -3/5)
         (-2/5, 1/5, -1/5, -3/5)
         (-2/5, -4/5, -1/5, 2/5)
         (-2/5, -4/5, -1/5, -3/5)
         (-2/5, -4/5, -6/5, -3/5)
```

we see that the U(1) charge at the end are the third coordinate of the weight in the α-basis multiplied by 5 to give integer values.

Writing the weights of the **10** as *columns* of a matrix $\hat{W}$ and the weights with the third digit expressed in non-normalized α-basis coordinates moved to the end as rows of a matrix $\hat{W}'$, the projection matrix $\hat{P}$ can be determined from

$$\hat{P}\hat{W} = \hat{W}' \tag{50}$$

with the right-inverse $\hat{W}^{+}$ of $\hat{W}$ (see section 4.1.4), since $\hat{W}$ is in general not a rectangular matrix:

$$\hat{P} = \hat{W}'\hat{W}^{+}. \tag{51}$$

As mentioned above the projection matrix found by this procedure can now be used to decompose any SU(5) irrep into SU(3)⊗SU(2)⊗U(1). The **10** is actually not the smallest irrep needed for the determination of the projection matrix. The **10** as well as all other irreps can be built from tensor products of the **5**, which we call the *generating irrep* of SU(5). In the orthogonal algebras only tensor products of the so-called *spinor representations* can construct all other irreps of the algebra. Thus, they must be used for the determination of the projection matrices. The generating irreps of representative Lie algebras are listed in Table 4.2.

Algebra	**Irrep (Dynkin)**	**Irrep (Name)**
A_4 (SU(5))	(1000)	**5**
B_4 (SO(9))	(0001)	**16**
C_4 (Sp(8))	(1000)	**8**
D_4 (SO(8))	(0001)	$\mathbf{8_s}$
E_6	(100000)	**27**
E_7	(0000010)	**56**
E_8	(00000010)	**248**
F_4	(0001)	**26**
G_2	(10)	**7**

Table 4.2: Generating Irreps of representative Lie algebras

In fact LieART excludes the zero-weights from the generating irreps, if any, i.e., only the lowest non-trivial orbit is needed for the determination of the projection matrices.

The calculation of a projection requires the knowledge of the simple root to drop from the Dynkin diagram for a specified algebra-subalgebra pair. LieART provides an extra package file called `BranchingRules.m`, listing this information for the implemented branching rules along with special embeddings to be discussed later. The file will be extended to encompass more branching rules in future versions of LieART, but may also be extended by the user. The definition for the more general branching rule $\mathrm{SU}(n) \to \mathrm{SU}(N{-}k)\otimes\mathrm{SU}(k)\otimes\mathrm{U}(1)$, including the demonstrated case $\mathrm{SU}(5) \to \mathrm{SU}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$, reads:

```
ProjectionMatrix[origin:Algebra[A][n_],
   ProductAlgebra[Algebra[A][m_],Algebra[A][k_],Algebra[U][1]]] :=
   NonSemiSimpleSubalgebra[origin,-k-1] /; m==(n-k-1)
```

Semisimple Subalgebras. To obtain a semisimple subalgebra without a U(1) generator, a root from the so-called *extended Dynkin diagram* is removed. The extended Dynkin diagram is constructed by adding the most negative root to the set of simple roots. (The negative of the highest root γ gives the most negative root $-\gamma$ to form the extended Dynkin diagram.) The resulting set of roots is linearly dependent, but removing one root restores the linear independence yielding a valid system of simple root of a subalgebra, which in general is semisimple. The highest roots γ and the according extended root $-\gamma$ for representative Lie algebras are listed in Table 4.3. The non-zero entries in the Dynkin label of $-\gamma$ prescribe to which existing dot in the Dynkin diagram it should connect, since the Dynkin label in the ω-basis encode the angle between two simple roots. A "1" is an angle of 120°, symbolized by a single connected line in the Dynkin diagram. A "2" is an angle of 135°, expressed by a double line in the Dynkin diagram. The minus sign gives negative angles or reverses the order of roots. The extended Dynkin diagrams for all classical and exceptional Lie Algebras are shown in Figure 2. Please note that the double line connecting the extended root $-\gamma$ for C_n is according to the "-2" in its Dynkin label.

To demonstrate the determination of the projection matrix from using the generating irrep, we cannot use an irrep of SU(N), because dropping a root from the extended Dynkin diagram of SU(N) returns SU(N). Thus, SU(N) has no *regular maximal* semisimple subalgebra. (Please note that some SU(N)'s have *special* maximal semisimple subalgebras, e.g., $\mathrm{SU}(4) \to \mathrm{SU}(2)\otimes\mathrm{SU}(2)$.) Instead we consider the subalgebra branching of $\mathrm{SO}(7)\to\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ ($B_3\to A_1\otimes A_1\otimes A_1$). The maximal subalgebra $\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ is obtained from the extended Dynkin diagram of SO(7) (B_3) by removing the second dot:

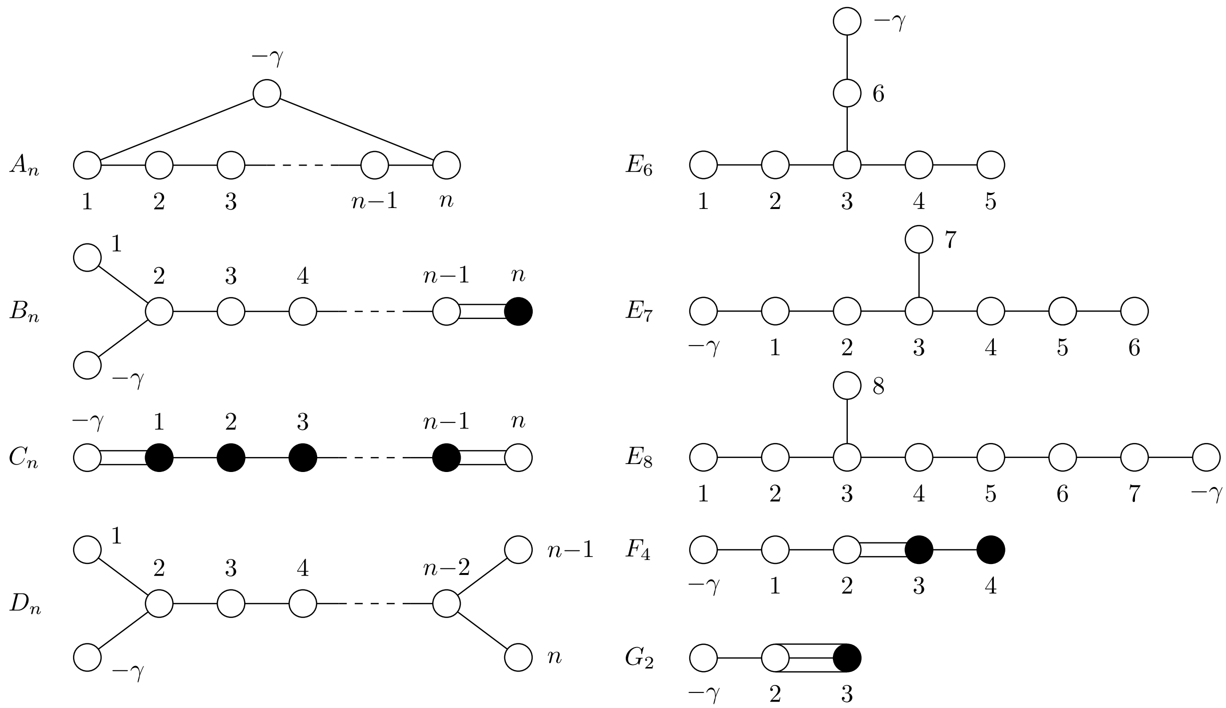
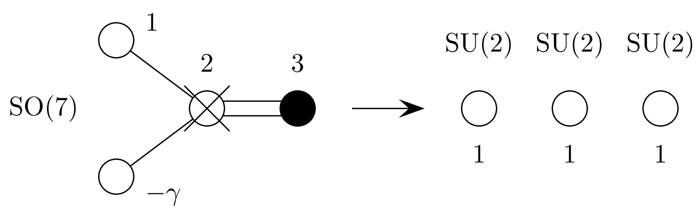


Figure 2: Extended Dynkin Diagrams of classical and exceptional simple Lie algebras.



To derive the projection matrix, we investigate the decomposition of the SO(7) generating irrep (the **8**) into three SU(2). Extending the Dynkin diagram with $-\gamma$ has the effect that each weight w gets extended by one entry with the coefficient of the weight relative to $-\gamma$, obtained by their scalar product: $(w, -\gamma)$. The so-called *extended weight scheme* of the lowest non-trivial orbit of a generating irrep is determined by the LieART function `ExtendedWeightScheme[`*algebra*`,`*simpleRootToDrop*`]`, which directly removes the Dynkin digits associated to the simple root to drop, specified by *simpleRootToDrop*. For the lowest non-trivial orbit of the generating irrep of SO(7) these two steps are:

$$
\begin{array}{c}
\boxed{0\ 0\ 1}\\
\boxed{0\ 1\ {-1}}\\
\boxed{1\ {-1}\ 1}\\
\boxed{{-1}\ 0\ 1}\\
\boxed{1\ 0\ {-1}}\\
\boxed{{-1}\ 1\ {-1}}\\
\boxed{0\ {-1}\ 1}\\
\boxed{0\ 0\ {-1}}
\end{array}
\xrightarrow{\text{insert } -\gamma}
\begin{array}{c}
\boxed{0\ {-1}\ 0\ 1}\\
\boxed{0\ {-1}\ 1\ {-1}}\\
\boxed{1\ 0\ {-1}\ 1}\\
\boxed{{-1}\ 0\ 0\ 1}\\
\boxed{1\ 0\ 0\ {-1}}\\
\boxed{{-1}\ 0\ 1\ {-1}}\\
\boxed{0\ 1\ {-1}\ 1}\\
\boxed{0\ 1\ 0\ {-1}}
\end{array}
\xrightarrow{\text{drop } 2}
\begin{array}{c}
\boxed{0\ {-1}\ 1}\\
\boxed{0\ {-1}\ {-1}}\\
\boxed{1\ 0\ 1}\\
\boxed{{-1}\ 0\ 1}\\
\boxed{1\ 0\ {-1}}\\
\boxed{{-1}\ 0\ {-1}}\\
\boxed{0\ 1\ 1}\\
\boxed{0\ 1\ {-1}}
\end{array}
\tag{52}
$$

With the weight of the SO(7) generating irrep as columns of the matrix $\hat{W}$ and the weights in the $3\,\mathrm{SU}(2)$ decomposition (right-hand side of (52)) as columns of $\hat{W}'$ the projection matrix $\hat{P}$ is computed as described for non-semisimple regular subalgebras as $\hat{P}{=}\hat{W}'\hat{W}^{+}$ with the right-inverse $\hat{W}^{+}$ of $\hat{W}$.

Algebra	Highest Root (γ)	Extended Root ($-\gamma$)
A_4 (SU(5))	1 0 0 1	−1 0 0 −1
B_4 (SO(9))	0 1 0 0	0 −1 0 0
C_4 (Sp(8))	2 0 0 0	−2 0 0 0
D_4 (SO(8))	0 1 0 0	0 −1 0 0
E_6	0 0 0 0 0 1	0 0 0 0 0 −1
E_7	1 0 0 0 0 0 0	−1 0 0 0 0 0 0
E_8	0 0 0 0 0 0 1 0	0 0 0 0 0 0 −1 0
F_4	1 0 0 0	−1 0 0 0
G_2	0 1	0 −1

Table 4.3: Highest roots γ and most negative roots $-\gamma$ of representative Lie algebras.

The definition for the branching rule SO(7)→SU(2)⊗SU(2)⊗SU(2) in the file `BranchingRules.m` reads:

```
ProjectionMatrix[origin:Algebra[B][3],
  ProductAlgebra[Algebra[A][1],Algebra[A][1],Algebra[A][1]]]:=
  SemiSimpleSubalgebra[origin,2]
```

Special Subalgebras. Special maximal subalgebras cannot be derived from the root system. The embedding of a special subalgebra does not follow a general pattern and must be derived for every algebra-subalgebra pair individually. Generating irreps are also used to derive the subalgebra embedding, which may be simple or semisimple and can involve more than one irrep of the subalgebra. LieART was previously not equipped with an algorithm to determine the maximal special subalgebras, but provided an interface to declare the embeddings (`BranchingRules.m`), which can be taken from the literature [8, 9].

As an example we consider SO(7) (B_3) again, which has G_2 as special maximal subalgebra. The generating spinor irrep of SO(7), the **8**, decomposes to the G_2 singlet plus the **7**. The weights of the **8** of SO(7) and the weights of both the **1** and **7** of G_2 are brought into lexicographical order to define the projection matrix:

$$\begin{array}{lcl}
\boxed{1\ 0\ {-1}} & \to & \boxed{2\ {-1}} \\
\boxed{1\ {-1}\ 1} & \to & \boxed{1\ 0} \\
\boxed{0\ 1\ {-1}} & \to & \boxed{1\ {-1}} \\
\boxed{0\ 0\ 1} & \to & \boxed{0\ 0} \\
\boxed{0\ 0\ {-1}} & \to & \boxed{0\ 0} \\
\boxed{0\ {-1}\ 1} & \to & \boxed{{-1}\ 1} \\
\boxed{{-1}\ 1\ {-1}} & \to & \boxed{{-1}\ 0} \\
\boxed{{-1}\ 0\ 1} & \to & \boxed{{-2}\ 1}
\end{array} \qquad (53)$$

Arranging the weights on the left-hand side of (53) as columns of $\hat{W}$ and the weights of the right-hand side as columns of $\hat{W}'$, the projection matrix $\hat{P}$ is again computed via $\hat{P}=\hat{W}'\hat{W}^+$ with the right-inverse $\hat{W}^+$ of $\hat{W}$.

These procedures are performed by the LieART function `SpecialSubalgebra[`*origin*`,` *targetirreps*`]`. The definition for the branching rule SO(7)→G_2 in the file `BranchingRules.m` reads:

```
ProjectionMatrix[origin:Algebra[B][3],ProductAlgebra[G2]]:=
  SpecialSubalgebra[origin,
  {ProductIrrep[Irrep[G2][0,0]],ProductIrrep[Irrep[G2][1,0]]}]
```

Please note that irreps of the subalgebra must be gathered in a list ({...}), even if it is a single irrep. The projection matrix for SO(7)→G_2 is

```
In[84]:= ProjectionMatrix[B3,ProductAlgebra[G2]]
```

Out[84]:= $\begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$

5. Implementation 2.0

The maximal subalgebras of simple Lie algebras are thoroughly categorized by Dynkin [1, 2] and Yamatsu [74], hence the main body of this work has been on finding the projection matrices for these branchings. These projection matrices are different for each algebra-maximal-subalgebra pair and are not unique. However, once a projection matrix is known it can be used for the decomposition of all irreps of the algebra-subalgebra pair.

LieART did already have some built in functions for calculating projection matrices: `NonSemiSimpleSubalgebra[ algebra, simpleRootToDrop]` computes the projection matrix of a maximal non-semisimple subalgebra by dropping one dot of the Dynkin diagram, `simpleRootToDrop`, and turning it into a U(1) charge. `SemiSimpleSubalgebra[algebra, simpleRootToDrop]` computes the projection matrix of a maximal semisimple subalgebra by dropping one dot, `simpleRootToDrop`, from the extended Dynkin diagram. `SpecialSubalgebra[algebra, targetirreps]` computes the projection matrix of a maximal special subalgebra by specifying the branching rule of the generating irrep [3]. However, `NonSemiSimpleSubalgebra` and `SemiSimpleSubalgebra` are only applicable to regular subalgebras, and `SpecialSubalgebra` requires exact knowledge of the branching rules for the generating irrep. Thus, many projection matrices, or rather their general form, were explicitly put into LieART, with the form of the projection matrices being found in [74] and [109]. One issue, however, was that, based on the way LieART works, some of these projection matrices do not properly map between weight systems; this was also a problem already present for `NonSemiSimpleSubalgebra` and `SemiSimpleSubalgebra`. However, this issue can be (and was) solved by permuting some of the rows of the projection matrices to ensure that the representation decomposition works properly.

Additionally, LieART already had some functionality for finding branching rules. Using projection matrices, the LieART function `DecomposeIrrep[irrep, subalgebra]` decomposes an irrep of a simple Lie algebra into a maximal subalgebra specified by `subalgebra`, which can be simple, semisimple, or non-semisimple. To decompose an irrep of a semisimple or non-semisimple irrep, a third argument, `pos`, allows one to specify which part of `productIrrep` should be decomposed into the subalgebra [3]. However, some subalgebra branchings have multiple branching rules, namely, $E_7 \to SU(2)$ has two different sets of branching rules, $E_8 \to SU(2)$ has three different sets of branching rules, and $SU(15) \to SU(3)$ has two different sets of branching rules. Thus, we had to modify `DecomposeIrrep` to take an `index` parameter, which determines which branching rules to display. An example is shown below. If no `index` is specified, `DecomposeIrrep` defaults to a `index` of 1.

```
In[85]:= DecomposeIrrep[Irrep[E7][56], ProductAlgebra[SU2]]
Out[85]:= (10)+(18)+(28)

In[86]:= DecomposeIrrep[Irrep[E7][56], ProductAlgebra[SU2], 1]
Out[86]:= (10)+(18)+(28)

In[87]:= DecomposeIrrep[Irrep[E7][56], ProductAlgebra[SU2], 2]
Out[87]:= (6)+(12)+(16)+(22)

In[88]:= DecomposeIrrep[ProductIrrep[Irrep[E7][56], Irrep[SU2][1]], ProductAlgebra[SU2],
           1, 1]
Out[88]:= (10,1)+(18,1)+(28,1)

In[89]:= DecomposeIrrep[ProductIrrep[Irrep[E7][56], Irrep[SU2][1]], ProductAlgebra[SU2],
           1, 2]
Out[89]:= (6,1)+(12,1)+(16,1)+(22,1)
```

LieART also already had a function for constructing branching rules tables. `BranchingRulesTable[algebra, subalgebras]` constructs a series of tables of the branching rules from `algebra` to each element of `subalgebras` [3]. However, because of the aforementioned multiple branching rules for some algebra-subalgebra pairs, we modified `BranchingRulesTable` to take an `index` parameter as well, which specifies which table of branching rules to display. If no `index` is specified for `BranchingRulesTable`, then it displays branching rules tables for all of the branchings. An example is shown below.

```
In[90]:= BranchingRulesTable[E8, ProductAlgebra[SU2], 1, MaxDim -> 147250]
```

Out[90]:=

Table 5.1: E_8 Branching Rules

E_8	→	SU(2)
248	=	(**3**) + (**11**) + (**15**) + (**19**) + (**23**) + (**27**) + (**29**) + (**35**) + (**39**) + (**47**)
3875	=	2(**1**) + 3(**5**) + (**7**) + 4(**9**) + 2(**11**) + 6(**13**) + 3(**15**) + 6(**17**) + 4(**19**) + 7(**21**) + 4(**23**) + 7(**25**) + 5(**27**) + 7(**29**) + 5(**31**) + 6(**33**) + 4(**35**) + 7(**37**) + 4(**39**) + 5(**41**) + 3(**43**) + 5(**45**) + 3(**47**) + 4(**49**) + 2(**51**) + 3(**53**) + 2(**55**) + 2(**57**) + (**59**) + 2(**61**) + (**63**) + (**65**) + (**69**) + (**73**)
27000	=	7(**1**) + (**3**) + 13(**5**) + 7(**7**) + 19(**9**) + 14(**11**) + 25(**13**) + 19(**15**) + 29(**17**) + 23(**19**) + 33(**21**) + 26(**23**) + 35(**25**) + 28(**27**) + 36(**29**) + 28(**31**) + 35(**33**) + 28(**35**) + 34(**37**) + 27(**39**) + 31(**41**) + 24(**43**) + 28(**45**) + 22(**47**) + 25(**49**) + 18(**51**) + 21(**53**) + 15(**55**) + 18(**57**) + 12(**59**) + 14(**61**) + 9(**63**) + 11(**65**) + 7(**67**) + 8(**69**) + 5(**71**) + 6(**73**) + 3(**75**) + 4(**77**) + 2(**79**) + 3(**81**) + (**83**) + 2(**85**) + (**89**) + (**93**)
30380	=	10(**3**) + 6(**5**) + 17(**7**) + 14(**9**) + 24(**11**) + 22(**13**) + 30(**15**) + 26(**17**) + 35(**19**) + 31(**21**) + 37(**23**) + 34(**25**) + 40(**27**) + 34(**29**) + 40(**31**) + 34(**33**) + 38(**35**) + 34(**37**) + 36(**39**) + 30(**41**) + 33(**43**) + 27(**45**) + 29(**47**) + 24(**49**) + 25(**51**) + 19(**53**) + 21(**55**) + 16(**57**) + 16(**59**) + 13(**61**) + 13(**63**) + 9(**65**) + 10(**67**) + 6(**69**) + 7(**71**) + 5(**73**) + 5(**75**) + 2(**77**) + 3(**79**) + 2(**81**) + 2(**83**) + (**85**) + (**87**) + (**91**)
147250	=	8(**1**) + 22(**3**) + 41(**5**) + 49(**7**) + 69(**9**) + 80(**11**) + 93(**13**) + 102(**15**) + 118(**17**) + 121(**19**) + 133(**21**) + 138(**23**) + 144(**25**) + 147(**27**) + 153(**29**) + 149(**31**) + 153(**33**) + 151(**35**) + 149(**37**) + 144(**39**) + 144(**41**) + 134(**43**) + 132(**45**) + 124(**47**) + 118(**49**) + 110(**51**) + 105(**53**) + 94(**55**) + 89(**57**) + 81(**59**) + 73(**61**) + 66(**63**) + 61(**65**) + 51(**67**) + 47(**69**) + 41(**71**) + 36(**73**) + 30(**75**) + 27(**77**) + 21(**79**) + 19(**81**) + 16(**83**) + 12(**85**) + 10(**87**) + 9(**89**) + 6(**91**) + 5(**93**) + 4(**95**) + 3(**97**) + 2(**99**) + 2(**101**) + (**105**) + (**107**)

```
In[91]:= BranchingRulesTable[E8, ProductAlgebra[SU2], 2, MaxDim -> 147250]
```

Out[91]:=

Table 5.2: E_8 Branching Rules

E_8	→	SU(2)
248	=	(**3**) + (**7**) + (**11**) + (**15**) + (**17**) + (**19**) + 2(**23**) + (**27**) + (**29**) + (**35**) + (**39**)
3875	=	3(**1**) + 5(**5**) + 3(**7**) + 6(**9**) + 4(**11**) + 9(**13**) + 6(**15**) + 9(**17**) + 7(**19**) + 10(**21**) + 7(**23**) + 10(**25**) + 7(**27**) + 9(**29**) + 6(**31**) + 8(**33**) + 5(**35**) + 7(**37**) + 5(**39**) + 5(**41**) + 3(**43**) + 4(**45**) + 2(**47**) + 3(**49**) + (**51**) + 2(**53**) + (**55**) + (**57**) + (**61**)
27000	=	9(**1**) + 4(**3**) + 20(**5**) + 16(**7**) + 31(**9**) + 25(**11**) + 40(**13**) + 33(**15**) + 47(**17**) + 39(**19**) + 50(**21**) + 43(**23**) + 51(**25**) + 42(**27**) + 50(**29**) + 40(**31**) + 46(**33**) + 37(**35**) + 41(**37**) + 32(**39**) + 35(**41**) + 26(**43**) + 29(**45**) + 20(**47**) + 22(**49**) + 16(**51**) + 16(**53**) + 11(**55**) + 12(**57**) + 7(**59**) + 8(**61**) + 4(**63**) + 5(**65**) + 2(**67**) + 3(**69**) + (**71**) + 2(**73**) + (**77**)
30380	=	(**1**) + 15(**3**) + 13(**5**) + 29(**7**) + 26(**9**) + 39(**11**) + 38(**13**) + 48(**15**) + 45(**17**) + 55(**19**) + 50(**21**) + 57(**23**) + 52(**25**) + 57(**27**) + 49(**29**) + 54(**31**) + 46(**33**) + 48(**35**) + 41(**37**) + 42(**39**) + 34(**41**) + 35(**43**) + 27(**45**) + 26(**47**) + 21(**49**) + 21(**51**) + 14(**53**) + 15(**55**) + 10(**57**) + 9(**59**) + 7(**61**) + 6(**63**) + 3(**65**) + 4(**67**) + (**69**) + 2(**71**) + (**73**) + (**75**)
147250	=	14(**1**) + 39(**3**) + 71(**5**) + 88(**7**) + 118(**9**) + 138(**11**) + 158(**13**) + 173(**15**) + 194(**17**) + 199(**19**) + 213(**21**) + 217(**23**) + 222(**25**) + 221(**27**) + 223(**29**) + 213(**31**) + 212(**33**) + 202(**35**) + 192(**37**) + 180(**39**) + 172(**41**) + 154(**43**) + 144(**45**) + 130(**47**) + 117(**49**) + 103(**51**) + 93(**53**) + 78(**55**) + 69(**57**) + 59(**59**) + 49(**61**) + 40(**63**) + 35(**65**) + 26(**67**) + 22(**69**) + 17(**71**) + 13(**73**) + 10(**75**) + 8(**77**) + 4(**79**) + 4(**81**) + 3(**83**) + (**85**) + (**87**) + (**89**)

In[92]:= `BranchingRulesTable[E8, ProductAlgebra[SU2], 3, MaxDim -> 147250]`

Table 5.3: E_8 Branching Rules

Out[92]:=

E_8	→	SU(2)
248	=	(**3**) + (**15**) + (**23**) + (**27**) + (**35**) + (**39**) + (**47**) + (**59**)
3875	=	2(**1**)+2(**5**)+2(**9**)+4(**13**)+2(**15**)+3(**17**)+(**19**)+4(**21**)+2(**23**)+5(**25**)+3(**27**)+4(**29**)+2(**31**)+ 4(**33**)+3(**35**)+5(**37**)+3(**39**)+4(**41**)+2(**43**)+4(**45**)+3(**47**)+4(**49**)+2(**51**)+3(**53**)+2(**55**)+ 3(**57**)+2(**59**)+3(**61**)+(**63**)+2(**65**)+(**67**)+2(**69**)+(**71**)+2(**73**)+(**77**)+(**79**)+(**81**)+(**85**)+(**93**)
27000	=	5(**1**)+8(**5**)+2(**7**)+10(**9**)+5(**11**)+14(**13**)+9(**15**)+16(**17**)+10(**19**)+18(**21**)+13(**23**)+21(**25**)+ 15(**27**) + 21(**29**) + 15(**31**) + 22(**33**) + 17(**35**) + 23(**37**) + 17(**39**) + 22(**41**) + 16(**43**) + 22(**45**) + 17(**47**) + 21(**49**) + 15(**51**) + 19(**53**) + 14(**55**) + 18(**57**) + 14(**59**) + 17(**61**) + 11(**63**) + 14(**65**) + 10(**67**) + 13(**69**) + 9(**71**) + 11(**73**) + 7(**75**) + 9(**77**) + 6(**79**) + 8(**81**) + 5(**83**) + 6(**85**) + 3(**87**) + 5(**89**)+3(**91**)+4(**93**)+2(**95**)+3(**97**)+(**99**)+2(**101**)+(**103**)+2(**105**)+(**109**)+(**113**)+(**117**)
30380	=	7(**3**)+2(**5**)+9(**7**)+5(**9**)+12(**11**)+11(**13**)+17(**15**)+12(**17**)+18(**19**)+15(**21**)+21(**23**)+19(**25**)+ 24(**27**) + 18(**29**) + 23(**31**) + 20(**33**) + 25(**35**) + 22(**37**) + 25(**39**) + 20(**41**) + 24(**43**) + 20(**45**) + 24(**47**) + 20(**49**) + 22(**51**) + 17(**53**) + 20(**55**) + 17(**57**) + 19(**59**) + 16(**61**) + 16(**63**) + 12(**65**) + 15(**67**)+11(**69**)+13(**71**)+10(**73**)+10(**75**)+7(**77**)+9(**79**)+7(**81**)+7(**83**)+5(**85**)+5(**87**)+ 3(**89**)+5(**91**)+3(**93**)+3(**95**)+2(**97**)+2(**99**)+(**101**)+2(**103**)+(**105**)+(**107**)+(**111**)+(**115**)
147250	=	5(**1**) + 11(**3**) + 21(**5**) + 22(**7**) + 33(**9**) + 39(**11**) + 47(**13**) + 52(**15**) + 60(**17**) + 60(**19**) + 69(**21**) + 73(**23**) + 79(**25**) + 81(**27**) + 86(**29**) + 83(**31**) + 90(**33**) + 92(**35**) + 93(**37**) + 92(**39**) + 95(**41**) + 90(**43**) + 93(**45**) + 92(**47**) + 91(**49**) + 87(**51**) + 87(**53**) + 81(**55**) + 82(**57**) + 79(**59**) + 75(**61**) + 70(**63**) + 69(**65**) + 63(**67**) + 62(**69**) + 58(**71**) + 54(**73**) + 49(**75**) + 47(**77**) + 42(**79**) + 41(**81**) + 37(**83**) + 33(**85**) + 29(**87**) + 28(**89**) + 24(**91**) + 23(**93**) + 20(**95**) + 17(**97**) + 15(**99**) + 14(**101**) + 11(**103**)+11(**105**)+9(**107**)+7(**109**)+6(**111**)+6(**113**)+4(**115**)+4(**117**)+3(**119**)+2(**121**)+ 2(**123**) + 2(**125**) + (**127**) + (**129**) + (**131**) + (**137**)

In[93]:= `BranchingRulesTable[E8, ProductAlgebra[SU2], MaxDim -> 147250]`

Table 5.4: E_8 Branching Rules

E_8	→	SU(2)
248	=	(3) + (11) + (15) + (19) + (23) + (27) + (29) + (35) + (39) + (47)
3875	=	2(1) + 3(5) + (7) + 4(9) + 2(11) + 6(13) + 3(15) + 6(17) + 4(19) + 7(21) + 4(23) + 7(25) + 5(27) + 7(29) + 5(31) + 6(33) + 4(35) + 7(37) + 4(39) + 5(41) + 3(43) + 5(45) + 3(47) + 4(49) + 2(51) + 3(53) + 2(55) + 2(57) + (59) + 2(61) + (63) + (65) + (69) + (73)
27000	=	7(1) + (3) + 13(5) + 7(7) + 19(9) + 14(11) + 25(13) + 19(15) + 29(17) + 23(19) + 33(21) + 26(23) + 35(25) + 28(27) + 36(29) + 28(31) + 35(33) + 28(35) + 34(37) + 27(39) + 31(41) + 24(43) + 28(45) + 22(47) + 25(49) + 18(51) + 21(53) + 15(55) + 18(57) + 12(59) + 14(61) + 9(63) + 11(65) + 7(67) + 8(69) + 5(71) + 6(73) + 3(75) + 4(77) + 2(79) + 3(81) + (83) + 2(85) + (89) + (93)
30380	=	10(3) + 6(5) + 17(7) + 14(9) + 24(11) + 22(13) + 30(15) + 26(17) + 35(19) + 31(21) + 37(23) + 34(25) + 40(27) + 34(29) + 40(31) + 34(33) + 38(35) + 34(37) + 36(39) + 30(41) + 33(43) + 27(45) + 29(47) + 24(49) + 25(51) + 19(53) + 21(55) + 16(57) + 16(59) + 13(61) + 13(63) + 9(65) + 10(67) + 6(69) + 7(71) + 5(73) + 5(75) + 2(77) + 3(79) + 2(81) + 2(83) + (85) + (87) + (91)
147250	=	8(1) + 22(3) + 41(5) + 49(7) + 69(9) + 80(11) + 93(13) + 102(15) + 118(17) + 121(19) + 133(21) + 138(23) + 144(25) + 147(27) + 153(29) + 149(31) + 153(33) + 151(35) + 149(37) + 144(39) + 144(41) + 134(43) + 132(45) + 124(47) + 118(49) + 110(51) + 105(53) + 94(55) + 89(57) + 81(59) + 73(61) + 66(63) + 61(65) + 51(67) + 47(69) + 41(71) + 36(73) + 30(75) + 27(77) + 21(79) + 19(81) + 16(83) + 12(85) + 10(87) + 9(89) + 6(91) + 5(93) + 4(95) + 3(97) + 2(99) + 2(101) + (105) + (107)

Out[93]:=

E_8	→	SU(2)
248	=	(3) + (7) + (11) + (15) + (17) + (19) + 2(23) + (27) + (29) + (35) + (39)
3875	=	3(1) + 5(5) + 3(7) + 6(9) + 4(11) + 9(13) + 6(15) + 9(17) + 7(19) + 10(21) + 7(23) + 10(25) + 7(27) + 9(29) + 6(31) + 8(33) + 5(35) + 7(37) + 5(39) + 5(41) + 3(43) + 4(45) + 2(47) + 3(49) + (51) + 2(53) + (55) + (57) + (61)
27000	=	9(1) + 4(3) + 20(5) + 16(7) + 31(9) + 25(11) + 40(13) + 33(15) + 47(17) + 39(19) + 50(21) + 43(23) + 51(25) + 42(27) + 50(29) + 40(31) + 46(33) + 37(35) + 41(37) + 32(39) + 35(41) + 26(43) + 29(45) + 20(47) + 22(49) + 16(51) + 16(53) + 11(55) + 12(57) + 7(59) + 8(61) + 4(63) + 5(65) + 2(67) + 3(69) + (71) + 2(73) + (77)
30380	=	(1) + 15(3) + 13(5) + 29(7) + 26(9) + 39(11) + 38(13) + 48(15) + 45(17) + 55(19) + 50(21) + 57(23) + 52(25) + 57(27) + 49(29) + 54(31) + 46(33) + 48(35) + 41(37) + 42(39) + 34(41) + 35(43) + 27(45) + 26(47) + 21(49) + 21(51) + 14(53) + 15(55) + 10(57) + 9(59) + 7(61) + 6(63) + 3(65) + 4(67) + (69) + 2(71) + (73) + (75)
147250	=	14(1) + 39(3) + 71(5) + 88(7) + 118(9) + 138(11) + 158(13) + 173(15) + 194(17) + 199(19) + 213(21) + 217(23) + 222(25) + 221(27) + 223(29) + 213(31) + 212(33) + 202(35) + 192(37) + 180(39) + 172(41) + 154(43) + 144(45) + 130(47) + 117(49) + 103(51) + 93(53) + 78(55) + 69(57) + 59(59) + 49(61) + 40(63) + 35(65) + 26(67) + 22(69) + 17(71) + 13(73) + 10(75) + 8(77) + 4(79) + 4(81) + 3(83) + (85) + (87) + (89)

E_8	→	SU(2)
248	=	(3) + (15) + (23) + (27) + (35) + (39) + (47) + (59)
3875	=	2(1) + 2(5) + 2(9) + 4(13) + 2(15) + 3(17) + (19) + 4(21) + 2(23) + 5(25) + 3(27) + 4(29) + 2(31) + 4(33) + 3(35) + 5(37) + 3(39) + 4(41) + 2(43) + 4(45) + 3(47) + 4(49) + 2(51) + 3(53) + 2(55) + 3(57) + 2(59) + 3(61) + (63) + 2(65) + (67) + 2(69) + (71) + 2(73) + (77) + (79) + (81) + (85) + (93)
27000	=	5(1) + 8(5) + 2(7) + 10(9) + 5(11) + 14(13) + 9(15) + 16(17) + 10(19) + 18(21) + 13(23) + 21(25) + 15(27) + 21(29) + 15(31) + 22(33) + 17(35) + 23(37) + 17(39) + 22(41) + 16(43) + 22(45) + 17(47) + 21(49) + 15(51) + 19(53) + 14(55) + 18(57) + 14(59) + 17(61) + 11(63) + 14(65) + 10(67) + 13(69) + 9(71) + 11(73) + 7(75) + 9(77) + 6(79) + 8(81) + 5(83) + 6(85) + 3(87) + 5(89) + 3(91) + 4(93) + 2(95) + 3(97) + (99) + 2(101) + (103) + 2(105) + (109) + (113) + (117)
30380	=	7(3) + 2(5) + 9(7) + 5(9) + 12(11) + 11(13) + 17(15) + 12(17) + 18(19) + 15(21) + 21(23) + 19(25) + 24(27) + 18(29) + 23(31) + 20(33) + 25(35) + 22(37) + 25(39) + 20(41) + 24(43) + 20(45) + 24(47) + 20(49) + 22(51) + 17(53) + 20(55) + 17(57) + 19(59) + 16(61) + 16(63) + 12(65) + 15(67) + 11(69) + 13(71) + 10(73) + 10(75) + 7(77) + 9(79) + 7(81) + 7(83) + 5(85) + 5(87) + 3(89) + 5(91) + 3(93) + 3(95) + 2(97) + 2(99) + (101) + 2(103) + (105) + (107) + (111) + (115)
147250	=	5(1) + 11(3) + 21(5) + 22(7) + 33(9) + 39(11) + 47(13) + 52(15) + 60(17) + 60(19) + 69(21) + 73(23) + 79(25) + 81(27) + 86(29) + 83(31) + 90(33) + 92(35) + 93(37) + 92(39) + 95(41) + 90(43) + 93(45) + 92(47) + 91(49) + 87(51) + 87(53) + 81(55) + 82(57) + 79(59) + 75(61) + 70(63) + 69(65) + 63(67) + 62(69) + 58(71) + 54(73) + 49(75) + 47(77) + 42(79) + 41(81) + 37(83) + 33(85) + 29(87) + 28(89) + 24(91) + 23(93) + 20(95) + 17(97) + 15(99) + 14(101) + 11(103) + 11(105) + 9(107) + 7(109) + 6(111) + 6(113) + 4(115) + 4(117) + 3(119) + 2(121) + 2(123) + 2(125) + (127) + (129) + (131) + (137)

6. Results

LieART now has additional functionality for finding the branching rules of regular and special maximal subalgebras of Lie algebras. We have tested that LieART can now correctly reproduce the branching rules for all maximal subalgebras, both regular and special, of Lie algebras up to rank 15. While special subalgebras cannot be simply obtained from manipulation of an algebra's Dynkin diagram, they follow the patterns given in Tables 6.1 and 6.2 below, with some abnormal exceptions. Table 6.3 gives all regular and special subalgebras up to rank 15, labeled (R) and (S), respectively. Examples of branching rules tables are given in the Appendix.

Table 6.1: Maximal Special Subalgebras of Classical Algebras

Rank	Algebra	Maximal Special Subalgebra
$mn-1$	SU(mn)	SU(m)⊗SU(n)
$\left[\frac{mn}{2}\right]$	SO(mn)	SO(m)⊗SO(n)
$2mn$	SO($4mn$)	Sp($2m$)⊗Sp($2n$)
mn	Sp($2mn$)	SO(m)⊗ Sp($2n$)

Table 6.2: Maximal Special Subalgebras of Classical Algebras

Rank	Algebra	Maximal Special Subalgebra	
$mn-1$	A_{mn-1}	$A_{m-1}\otimes A_{n-1}$	$m,n\geq 2$
$2n$	A_{2n}	B_n	$n\geq 2$
$2n-1$	A_{2n-1}	$C_n; D_n$	$n\geq 2$
$\frac{(n-1)(n+2)}{2}$	$A_{\frac{(n-1)(n+2)}{2}}$	A_n	$n\geq 3$
$\frac{n(n+3)}{2}$	$A_{\frac{n(n+3)}{2}}$	A_n	$n\geq 2$
n	B_n	A_1	$n\geq 4$
n	C_n	A_1	$n\geq 2$
$n+1$	D_{n+1}	B_n	$n\geq 3$
$n+2$	D_{n+2}	$A_1\otimes B_n$	$n\geq 4$
$n+3$	D_{n+3}	$C_2\otimes B_n$	$n\geq 4$
$m+n+1$	D_{m+n+1}	$B_m\otimes B_n$	$m+n\geq 4$
$2mn$	D_{2mn}	$C_m\otimes C_n$	

Table 6.3: Maximal Subalgebras

Rank	Algebra		Maximal subalgebras	Type
1	SU(2)	⊃	U(1)	(R)
	(SU(2), SO(3), and Sp(2) are all isomorphic.)			
2	SU(3)	⊃	SU(2)⊗U(1)	(R)
		⊃	SU(2)	(S)
	Sp(4)	⊃	SU(2)⊗SU(2); SU(2)⊗U(1)	(R)
		⊃	SU(2)	(S)
	(SO(5) is isomorphic to Sp(4), and SO(4) is isomorphic to SU(2)⊗SU(2).)			
	G_2	⊃	SU(3); SU(2)⊗SU(2)	(R)
		⊃	SU(2)	(S)
3	SU(4)	⊃	SU(3)⊗U(1); SU(2)⊗SU(2)⊗U(1)	(R)
		⊃	Sp(4); SU(2)⊗SU(2)	(S)
	SO(7)	⊃	SU(4); SU(2)⊗SU(2)⊗SU(2); Sp(4)⊗U(1)	(R)
		⊃	G_2	(S)
	Sp(6)	⊃	SU(3)⊗U(1); SU(2)⊗Sp(4)	(R)

Table 6.3 (continued)

Rank	Algebra		Maximal subalgebras	Type
		$\supset$	SU(2); SU(2)$\otimes$SU(2)	(S)
	(SO(6) is isomorphic to SU(4).)			
4	SU(5)	$\supset$	SU(4)$\otimes$U(1); SU(3)$\otimes$SU(2)$\otimes$U(1)	(R)
		$\supset$	Sp(4)	(S)
	SO(9)	$\supset$	SO(8); SU(2) $\otimes$SU(2)$\otimes$Sp(4); SU(4)$\otimes$SU(2); SO(7)$\otimes$U(1)	(R)
		$\supset$	SU(2); SU(2)$\otimes$SU(2)	(S)
	Sp(8)	$\supset$	SU(4)$\otimes$U(1); SU(2)$\otimes$Sp(6); Sp(4)$\otimes$Sp(4)	(R)
		$\supset$	SU(2); SU(2)$\otimes$SU(2)$\otimes$SU(2)	(S)
	SO(8)	$\supset$	SU(2)$\otimes$SU(2)$\otimes$SU(2)$\otimes$SU(2); SU(4)$\otimes$U(1)	(R)
		$\supset$	SU(3); SO(7); SU(2)$\otimes$Sp(4)	(S)
	F_4	$\supset$	SO(9); SU(3)$\otimes$SU(3); SU(2)$\otimes$Sp(6)	(R)
		$\supset$	SU(2); SU(2)$\otimes G_2$	(S)
5	SU(6)	$\supset$	SU(5)$\otimes$U(1); SU(4)$\otimes$SU(2)$\otimes$U(1); SU(3)$\otimes$SU(3)$\otimes$U(1)	(R)
		$\supset$	SU(3); SU(4); Sp(6); SU(3)$\otimes$SU(2)	(S)
	SO(11)	$\supset$	SO(10); SO(8)$\otimes$SU(2); SU(4)$\otimes$Sp(4); SU(2)$\otimes$SU(2)$\otimes$SO(7); SO(9)$\otimes$U(1)	(R)
		$\supset$	SU(2)	(S)
	Sp(10)	$\supset$	SU(5)$\otimes$U(1); SU(2)$\otimes$Sp(8); Sp(4)$\otimes$Sp(6)	(R)
		$\supset$	SU(2); SU(2)$\otimes$Sp(4)	(S)
	SO(10)	$\supset$	SU(5)$\otimes$U(1); SU(2)$\otimes$SU(2)$\otimes$SU(4); SO(8)$\otimes$U(1)	(R)
		$\supset$	Sp(4); SO(9); SU(2)$\otimes$SO(7); Sp(4)$\otimes$Sp(4)	(S)
6	SU(7)	$\supset$	SU(6)$\otimes$U(1); SU(5)$\otimes$SU(2)$\otimes$U(1); SU(4)$\otimes$SU(3)$\otimes$U(1)	(R)
		$\supset$	SO(7)	(S)
	SO(13)	$\supset$	SO(12); SO(10)$\otimes$SU(2); SO(8)$\otimes$Sp(4); SU(4)$\otimes$SO(7); SU(2)$\otimes$SU(2)$\otimes$SO(9); SO(11)$\otimes$U(1)	(R)
		$\supset$	SU(2)	(S)
	Sp(12)	$\supset$	SU(6)$\otimes$U(1); SU(2)$\otimes$Sp(10); Sp(4)$\otimes$Sp(8); Sp(6)$\otimes$Sp(6)	(R)
		$\supset$	SU(2); SU(2)$\otimes$SU(4); SU(2)$\otimes$Sp(4)	(S)
	SO(12)	$\supset$	SU(6)$\otimes$U(1); SU(2)$\otimes$SU(2)$\otimes$SO(8); SU(4)$\otimes$SU(4); SO(10)$\otimes$U(1)	(R)
		$\supset$	SU(2)$\otimes$Sp(6); SU(2)$\otimes$SU(2)$\otimes$SU(2); SO(11); SU(2)$\otimes$SO(9); Sp(4)$\otimes$SO(7)	(S)
	E_6	$\supset$	SO(10)$\otimes$U(1); SU(6)$\otimes$SU(2); SU(3)$\otimes$SU(3)$\otimes$SU(3)	(R)
		$\supset$	F_4; SU(3)$\otimes G_2$; Sp(8); G_2; SU(3)	(S)
7	SU(8)	$\supset$	SU(7)$\otimes$U(1); SU(6)$\otimes$SU(2)$\otimes$U(1); SU(5)$\otimes$SU(3)$\otimes$U(1); SU(4)$\otimes$SU(4)$\otimes$U(1)	(R)
		$\supset$	SO(8); Sp(8); SU(4)$\otimes$SU(2)	(S)
	SO(15)	$\supset$	SO(14); SO(12)$\otimes$SU(2); SO(10)$\otimes$Sp(4); SO(8)$\otimes$SO(7); SU(4)$\otimes$SO(9); SU(2)$\otimes$SU(2)$\otimes$SO(11); SO(13)$\otimes$U(1)	(R)
		$\supset$	SU(2); SU(4); SU(2)$\otimes$Sp(4)	(S)
	Sp(14)	$\supset$	SU(7)$\otimes$U(1); SU(2)$\otimes$Sp(12); Sp(4)$\otimes$Sp(10); Sp(6)$\otimes$Sp(8)	(R)
		$\supset$	SU(2); SU(2)$\otimes$SO(7)	(S)
	SO(14)	$\supset$	SU(7)$\otimes$U(1); SU(2)$\otimes$SU(2)$\otimes$SO(10); SU(4)$\otimes$SO(8); SO(12)$\otimes$U(1)	(R)
		$\supset$	Sp(4); Sp(6); G_2; SO(13); SU(2)$\otimes$SO(11); Sp(4)$\otimes$SO(9); SO(7)$\otimes$SO(7)	(S)
	E_7	$\supset$	$E_6\otimes$U(1); SU(8); SO(12)$\otimes$SU(2); SU(6)$\otimes$SU(3)	(R)
		$\supset$	SU(2)$\otimes F_4$; $G_2\otimes$Sp(6); SU(2)$\otimes G_2$; SU(3); SU(2)$\otimes$SU(2); SU(2); SU(2)	(S)
8	SU(9)	$\supset$	SU(8)$\otimes$U(1); SU(7)$\otimes$SU(2)$\otimes$U(1); SU(6)$\otimes$SU(3)$\otimes$U(1); SU(5)$\otimes$SU(4)$\otimes$U(1)	(R)
		$\supset$	SO(9); SU(3)$\otimes$SU(3)	(S)

Table 6.3 (continued)

Rank	Algebra		Maximal subalgebras	Type
	SO(17)	⊃	SO(16); SO(14)⊗SU(2); SO(12)⊗Sp(4); SO(10)⊗SO(7); SO(8)⊗SO(9); SU(4)⊗SO(11); SU(2)⊗SU(2)⊗SO(13); SO(15)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(16)	⊃	SU(8)⊗U(1); SU(2)⊗Sp(14); Sp(4)⊗Sp(12); Sp(6)⊗Sp(10); Sp(8)⊗Sp(8)	(*R*)
		⊃	SU(2); Sp(4); SU(2)⊗SO(8)	(*S*)
	SO(16)	⊃	SU(8)⊗U(1); SU(2)⊗SU(2)⊗SO(12); SU(4)⊗SO(10); SO(8)⊗SO(8); SO(14)⊗U(1)	(*R*)
		⊃	SO(9); SU(2)⊗Sp(8); Sp(4)⊗Sp(4); SO(15); SU(2)⊗SO(13); Sp(4)⊗SO(11); SO(7)⊗SO(9)	(*S*)
	E_8	⊃	SO(16); SU(5)⊗SU(5); E_6⊗SU(3); E_7⊗SU(2); SU(9)	(*R*)
		⊃	G_2⊗F_4; SU(2)⊗SU(3); Sp(4); SU(2); SU(2); SU(2)	(*S*)
9	SU(10)	⊃	SU(9)⊗U(1); SU(8)⊗SU(2)⊗U(1); SU(7)⊗SU(3)⊗U(1); SU(6)⊗SU(4)⊗U(1); SU(5)⊗SU(5)⊗U(1)	(*R*)
		⊃	SU(3); SU(4); SU(5); Sp(4); SO(10); Sp(10); SU(5)⊗SU(2)	(*S*)
	SO(19)	⊃	SO(18); SO(16)⊗SU(2); SO(14)⊗Sp(4); SO(12)⊗SO(7); SO(10)⊗SO(9); SO(8)⊗SO(11); SU(4)⊗SO(13); SU(2)⊗SU(2)⊗SO(15); SO(17)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(18)	⊃	SU(9)⊗U(1); SU(2)⊗Sp(16); Sp(4)⊗Sp(14); Sp(6)⊗Sp(12); Sp(8)⊗Sp(10)	(*R*)
		⊃	SU(2); SU(2)⊗SO(9); SU(2)⊗Sp(6)	(*S*)
	SO(18)	⊃	SU(9)⊗U(1); SU(2)⊗SU(2)⊗SO(14); SU(4)⊗S(12); SO(8)⊗SO(10); SO(16)⊗U(1)	(*R*)
		⊃	SU(2)⊗SU(4); SO(17); SU(2)⊗SO(15); Sp(4)⊗SO(13); SO(7)⊗SO(11); SO(9)⊗SO(9)	(*S*)
10	SU(11)	⊃	SU(10)⊗U(1); SU(9)⊗SU(2)⊗U(1); SU(8)⊗SU(3)⊗U(1); SU(7)⊗SU(4)⊗U(1); SU(6)⊗SU(5)⊗U(1)	(*R*)
		⊃	SO(11)	(*S*)
	SO(21)	⊃	SO(20); SO(18)⊗SU(2); SO(16)⊗Sp(4); SO(14)⊗SO(7); SO(12)⊗SO(9); SO(10)⊗SO(11); SO(8)⊗SO(13); SU(4)⊗SO(15); SU(2)⊗SU(2)⊗SO(17); SO(19)⊗U(1)	(*R*)
		⊃	SU(2); SU(2)⊗SO(7); SO(7); Sp(6)	(*S*)
	Sp(20)	⊃	SU(10)⊗U(1); SU(2)⊗Sp(18); Sp(4)⊗Sp(16); Sp(6)⊗Sp(14); Sp(8)⊗Sp(12); Sp(10)⊗Sp(10)	(*R*)
		⊃	SU(2); Sp(4)⊗Sp(4); SU(2)⊗SO(10); SU(6)	(*S*)
	SO(20)	⊃	SU(10)⊗U(1); SU(2)⊗SU(2)⊗SO(16); SU(4)⊗SO(14); SO(8)⊗SO(12); SO(10)⊗SO(10); SO(18)⊗U(1)	(*R*)
		⊃	SU(2)⊗Sp(10); SO(19); SU(2)⊗SO(17); Sp(4)⊗SO(15); SO(7)⊗SO(13); SO(9)⊗SO(11); SU(2)⊗SU(2)⊗Sp(4); SU(4)	(*S*)
11	SU(12)	⊃	SU(11)⊗U(1); SU(10)⊗SU(2)⊗U(1); SU(9)⊗SU(3)⊗U(1); SU(8)⊗SU(4)⊗U(1); SU(7)⊗SU(5)⊗U(1); SU(6)⊗SU(6)⊗U(1)	(*R*)
		⊃	SO(12); Sp(12); SU(6)⊗SU(2); SU(4)⊗SU(3)	(*S*)
	SO(23)	⊃	SO(22); SO(20)⊗SU(2); SO(18)⊗Sp(4); SO(16)⊗SO(7); SO(14)⊗SO(9); SO(12)⊗SO(11); SO(10)⊗SO(13); SO(8)⊗SO(15); SU(4)⊗SO(17); SU(2)⊗SU(2) SO(19); SO(21)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(22)	⊃	SU(11)⊗U(1); SU(2)⊗Sp(20); Sp(4)⊗Sp(18); Sp(6)⊗Sp(16); Sp(8)⊗Sp(14); Sp(10)⊗Sp(12)	(*R*)
		⊃	SU(2)	(*S*)

Table 6.3 (continued)

Rank	Algebra		Maximal subalgebras	Type
	SO(22)	⊃	SU(11)⊗U(1); SU(2)⊗SU(2)⊗SO(18); SU(4)⊗SO(16); SO(8)⊗SO(14); SO(10)⊗SO(12); SO(20)⊗U(1)	(R)
		⊃	SO(21); SU(2)⊗SO(19); Sp(4)⊗SO(17); SO(7)⊗SO(15); SO(9)⊗SO(13); SO(11)⊗SO(11)	(S)
12	SU(13)	⊃	SU(12)⊗U(1); SU(11)⊗SU(2)⊗U(1); SU(10)⊗SU(3)⊗U(1); SU(9)⊗SU(4)⊗U(1); SU(8)⊗SU(5)⊗U(1); SU(7)⊗SU(6)⊗U(1)	(R)
		⊃	SO(13)	(S)
	SO(25)	⊃	SO(24); SO(22)⊗SU(2); SO(20)⊗Sp(4); SO(18)⊗SO(7); SO(16)⊗SO(9); SO(14)⊗SO(11); SO(12)⊗SO(13); SO(10)⊗SO(15); SO(8)⊗SO(17); SU(4)⊗SO(19); SU(2)⊗SU(2)⊗SO(21); SO(23)⊗U(1)	(R)
		⊃	SU(2); Sp(4)⊗Sp(4)	(S)
	Sp(24)	⊃	SU(12)⊗U(1); SU(2)⊗Sp(22); Sp(4)⊗Sp(20); Sp(6)⊗Sp(18); Sp(8)⊗Sp(16); Sp(10)⊗Sp(14); Sp(12)⊗Sp(12)	(R)
		⊃	SU(2); SU(2)⊗SU(2)⊗Sp(6); SU(2)⊗Sp(8); SU(4)⊗Sp(4)	(S)
	SO(24)	⊃	SU(12)⊗U(1); SU(2)⊗SU(2)⊗SO(20); SU(4)⊗SO(18); SO(8)⊗SO(16); SO(10)⊗SO(14); SO(12)⊗SO(12); SO(22)⊗U(1)	(R)
		⊃	SO(23); SU(2)⊗SO(21); Sp(4)⊗SO(19); SO(7)⊗SO(17); SO(9)⊗SO(15); SO(11)⊗SO(13); Sp(6)⊗Sp(4); SU(2)⊗SO(8); SU(5)	(S)
13	SU(14)	⊃	SU(13)⊗U(1); SU(12)⊗SU(2)⊗U(1); SU(11)⊗SU(3)⊗U(1); SU(10)⊗SU(4)⊗U(1); SU(9)⊗SU(5)⊗U(1); SU(8)⊗SU(6)⊗U(1); SU(7)⊗SU(7)⊗U(1)	(R)
		⊃	SO(14); Sp(14); SU(7)⊗SU(2)	(S)
	SO(27)	⊃	SO(26); SO(24)⊗SU(2); SO(22)⊗Sp(4); SO(20)⊗SO(7); SO(18)⊗SO(9); SO(16)⊗SO(11); SO(14)⊗SO(13); SO(12)⊗SO(15); SO(10)⊗SO(17); SO(8)⊗SO(19); SU(4)⊗SO(21); SU(2)⊗SU(2)⊗SO(23); SO(25)⊗U(1)	(R)
		⊃	SU(2); SU(3); SO(7); SU(2)⊗SO(9)	(S)
	Sp(26)	⊃	SU(13)⊗U(1); SU(2)⊗Sp(24); Sp(4)⊗Sp(22); Sp(6)⊗Sp(20); Sp(8)⊗Sp(18); Sp(10)⊗Sp(16); Sp(12)⊗Sp(14)	(R)
		⊃	SU(2)	(S)
	SO(26)	⊃	SU(13)⊗U(1); SU(2)⊗SU(2)⊗SO(22); SU(4)⊗SO(20); SO(8)⊗SO(18); SO(10)⊗SO(16); SO(12)⊗SO(14);SO(24)⊗U(1)	(R)
		⊃	SO(25); SU(2)⊗SO(23); Sp(4)⊗SO(21); SO(7)⊗SO(19); SO(9)⊗SO(17); SO(11)⊗SO(15); SO(13)⊗SO(13); F_4	(S)
14	SU(15)	⊃	SU(14)⊗U(1); SU(13)⊗SU(2)⊗U(1); SU(12)⊗SU(3)⊗U(1); SU(11)⊗SU(4)⊗U(1); SU(10)⊗SU(5)⊗U(1); SU(9)⊗SU(6)⊗U(1); SU(8)⊗SU(7)⊗U(1)	(R)
		⊃	SO(15); SU(5)⊗SU(3); SU(3); SU(3); SU(5); SU(6)	(S)
	SO(29)	⊃	SO(28); SO(26)⊗SU(2); SO(24)⊗Sp(4); SO(22)⊗SO(7); SO(20)⊗SO(9); SO(18)⊗SO(11); SO(16)⊗SO(13); SO(14)⊗SO(15); SO(12)⊗SO(17); SO(10)⊗SO(19); SO(8)⊗SO(21); SU(4)⊗SO(23); SU(2)⊗SU(2)x SO(25); SO(27)⊗U(1)	(R)
		⊃	SU(2)	(S)
	Sp(28)	⊃	SU(14)⊗U(1); SU(2)⊗Sp(26); Sp(4)⊗Sp(24); Sp(6)⊗Sp(22); Sp(8)⊗Sp(20); Sp(10)⊗Sp(18); Sp(12)⊗Sp(16); Sp(14)⊗Sp(14)	(R)
		⊃	SU(2); SO(7)⊗Sp(4)	(S)
	SO(28)	⊃	SU(14)⊗U(1); SU(2)⊗SU(2)⊗SO(24); SU(4)⊗SO(22); SO(8)⊗SO(20); SO(10)⊗SO(18); SO(12)⊗SO(16); SO(14)⊗SO(14); SO(26)⊗U(1)	(R)

Table 6.3 (continued)

Rank	Algebra		Maximal subalgebras	Type
		⊃	SO(27); SU(2)⊗SO(25); Sp(4)⊗SO(23); SO(7)⊗SO(21); SO(9)⊗SO(19); SO(11)⊗SO(17); SO(13)⊗SO(15); SU(2)⊗SU(2)⊗SO(7)	(*S*)
15	SU(16)	⊃	SU(15)⊗U(1); SU(14)⊗SU(2)⊗U(1); SU(13)⊗SU(3)⊗U(1); SU(12)⊗SU(4)⊗U(1); SU(11)⊗SU(5)⊗U(1); SU(10)⊗SU(6)⊗U(1); SU(9)⊗SU(7)⊗U(1); SU(8)⊗SU(8)⊗U(1)	(*R*)
		⊃	SO(16); Sp(16); SO(10); SU(8)⊗SU(2); SU(4)⊗SU(4)	(*S*)
	SO(31)	⊃	SO(30); SO(28)⊗SU(2); SO(26)⊗Sp(4); SO(24)⊗SO(7); SO(22)⊗SO(9); SO(20)⊗SO(11); SO(18)⊗SO(13); SO(16)⊗SO(15); SO(14)⊗SO(17); SO(12)⊗SO(19); SO(10)⊗SO(21); SO(8)⊗SO(23); SU(4)⊗SO(25); SU(2)⊗SU(2)⊗SO(27); SO(29)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(30)	⊃	SU(15)⊗U(1); SU(2)⊗Sp(28); Sp(4)⊗Sp(26); Sp(6)⊗Sp(24); Sp(8)⊗Sp(22); Sp(10)⊗Sp(20); Sp(12)⊗Sp(18); Sp(14)⊗Sp(16)	(*R*)
		⊃	SU(2); SU(2)⊗Sp(10); Sp(4)⊗Sp(6)	(*S*)
	SO(30)	⊃	SU(15)⊗U(1); SU(2)⊗SU(2)⊗SO(26); SU(4)⊗SO(24); SO(8)⊗SO(22); SO(10)⊗SO(20); SO(12)⊗SO(18); SO(14)⊗SO(16); SO(28)⊗U(1)	(*R*)
		⊃	SO(29); SU(2)⊗SO(27); Sp(4)⊗SO(25); SO(7)⊗SO(23); SO(9)⊗SO(21); SO(11)⊗SO(19); SO(13)⊗SO(17); SO(15)⊗SO(15); SU(2)⊗SO(10); Sp(4)⊗SU(4)	(*S*)

Additionally, we fixed a bug wherein `DecomposeIrrep[Irrep[SO10][16], ProductAlgebra[SO8, U1]]` would fail to compute the branching rules and throw an error. Similar bugs for other SO(m) → SO($m-2$)⊗U(1) branchings were found and corrected. These functions now correctly compute the branching rules and no longer throw errors.

Over the course of assembling the modified program, several patterns in subalgebra branchings were observed. The first of these is that for SU(mn) → SU(m)⊗SU(n), the **mn** irrep of SU(mn) branches to the $(\mathbf{m},\mathbf{n})$ irrep of SU(m)⊗SU(n). Likewise, for SO(mn) → SO(m)⊗SO(n), the **mn** irrep of SO(mn) branches to the $(\mathbf{m},\mathbf{n})$ irrep of SO(m)⊗SO(n). Similarly, for SO($m+n$) → SO(m)⊗SO(n), the **m+n** irrep of SO($m+n$) branches to the $(\mathbf{m},\mathbf{1})+(\mathbf{1},\mathbf{n})$ irrep of SO(m)⊗SO(n). Lastly, for Sp($2m+2n$) → Sp($2m$)⊗Sp($2n$), the **2m+2n** irrep of Sp($2m+2n$) branches to the $(\mathbf{2m},\mathbf{1})+(\mathbf{1},\mathbf{2n})$ irrep of Sp($2m$)⊗Sp($2n$).

7. Benchmarks

In this section, we give runtime benchmarks for subalgebra decomposition. Since exceptional algebras have rather complicated Weyl reflection groups of high orders, computations involving them are significantly more CPU and memory demanding than with classical algebras of equal rank. In the following discussion, we give runtime benchmarks for the subalgebra decomposition of irreps of exceptional algebras.

We use the Mathematica command `Timing[]`, which gives the CPU time spent in the Mathematica kernel in seconds. It does not include the time needed for the display of results in the front end. Note that each of the following timings are taken with a newly launched Kernel to avoid speedup due to caching of intermediate results from previous computations.

The following timings were taken with Mathematica 11.1.1. on a HP® Omen™ with an Intel® Core™ i7-6700HQ (2.60 GHz) processor and 16 GB RAM.

As an example for subalgebra decomposition of a large irrep we decompose the **6696000** of E_8 to $G_2 \otimes F_4$:

```
In[94]:= Timing[DecomposeIrrep[Irrep[E8][6696000], ProductAlgebra[G2, F4]]]
```

Out[94]:=

$\{1066.14, 2(\mathbf{7},\mathbf{1}) + 2(\mathbf{14},\mathbf{1}) + (\mathbf{1},\mathbf{26}) + (\mathbf{27},\mathbf{1}) + 6(\mathbf{7},\mathbf{26}) + 5(\mathbf{14},\mathbf{26}) + 2(\mathbf{1},\mathbf{52}) + 6(\mathbf{27},\mathbf{26}) + 3(\mathbf{7},\mathbf{52}) +$
$2(\mathbf{64},\mathbf{1}) + 3(\mathbf{14},\mathbf{52}) + 2(\mathbf{77},\mathbf{1}) + 5(\mathbf{27},\mathbf{52}) + 5(\mathbf{64},\mathbf{26}) + 4(\mathbf{77},\mathbf{26}) + (\mathbf{77'},\mathbf{26}) + 3(\mathbf{64},\mathbf{52}) + 2(\mathbf{77},\mathbf{52}) +$
$(\mathbf{77'},\mathbf{52}) + (\mathbf{189},\mathbf{1}) + (\mathbf{182},\mathbf{26}) + 2(\mathbf{189},\mathbf{26}) + (\mathbf{182},\mathbf{52}) + (\mathbf{189},\mathbf{52}) + 3(\mathbf{1},\mathbf{273}) + 6(\mathbf{7},\mathbf{273}) +$
$4(\mathbf{14},\mathbf{273}) + 8(\mathbf{27},\mathbf{273}) + (\mathbf{1},\mathbf{324}) + 6(\mathbf{7},\mathbf{324}) + 5(\mathbf{64},\mathbf{273}) + 5(\mathbf{14},\mathbf{324}) + 3(\mathbf{77},\mathbf{273}) + (\mathbf{77'},\mathbf{273}) +$
$4(\mathbf{27},\mathbf{324}) + 3(\mathbf{64},\mathbf{324}) + 3(\mathbf{77},\mathbf{324}) + (\mathbf{182},\mathbf{273}) + (\mathbf{189},\mathbf{273}) + (\mathbf{189},\mathbf{324}) + 2(\mathbf{1},\mathbf{1053}) + 5(\mathbf{7},\mathbf{1053}) +$
$(\mathbf{7},\mathbf{1053'}) + 3(\mathbf{14},\mathbf{1053}) + (\mathbf{14},\mathbf{1053'}) + 5(\mathbf{27},\mathbf{1053}) + 3(\mathbf{64},\mathbf{1053}) + 2(\mathbf{77},\mathbf{1053}) + (\mathbf{77},\mathbf{1053'}) +$
$(\mathbf{189},\mathbf{1053}) + 2(\mathbf{1},\mathbf{1274}) + 2(\mathbf{7},\mathbf{1274}) + 2(\mathbf{14},\mathbf{1274}) + 3(\mathbf{27},\mathbf{1274}) + (\mathbf{64},\mathbf{1274}) + (\mathbf{77},\mathbf{1274}) +$
$(\mathbf{77'},\mathbf{1274}) + 2(\mathbf{7},\mathbf{2652}) + (\mathbf{14},\mathbf{2652}) + (\mathbf{27},\mathbf{2652}) + 2(\mathbf{1},\mathbf{4096}) + 5(\mathbf{7},\mathbf{4096}) + 3(\mathbf{14},\mathbf{4096}) +$
$4(\mathbf{27},\mathbf{4096}) + 2(\mathbf{64},\mathbf{4096}) + (\mathbf{77},\mathbf{4096}) + 2(\mathbf{7},\mathbf{8424}) + (\mathbf{14},\mathbf{8424}) + (\mathbf{27},\mathbf{8424}) + (\mathbf{64},\mathbf{8424}) +$
$(\mathbf{1},\mathbf{10829}) + (\mathbf{7},\mathbf{10829}) + (\mathbf{14},\mathbf{10829}) + (\mathbf{27},\mathbf{10829}) + (\mathbf{1},\mathbf{19278}) + (\mathbf{7},\mathbf{19278}) + (\mathbf{27},\mathbf{19278}) +$
$(\mathbf{7},\mathbf{19448}) + (\mathbf{14},\mathbf{19448}) + (\mathbf{1},\mathbf{34749}) + (\mathbf{7},\mathbf{34749})\}$

or we can decompose to $\mathrm{SU}(2) \otimes \mathrm{SU}(3)$:

```
In[95]:= Timing[DecomposeIrrep[Irrep[E8][6696000], ProductAlgebra[SU2, SU3]]]
```

Out[95]:=

$\{569.313, 19(\mathbf{1},\mathbf{1}) + 66(\mathbf{3},\mathbf{1}) + 78(\mathbf{5},\mathbf{1}) + 87(\mathbf{7},\mathbf{1}) + 121(\mathbf{1},\mathbf{8}) + 64(\mathbf{9},\mathbf{1}) + 322(\mathbf{3},\mathbf{8}) + 103(\mathbf{1},\mathbf{10}) + 103(\mathbf{1},\overline{\mathbf{10}}) +$
$48(\mathbf{11},\mathbf{1}) + 253(\mathbf{3},\mathbf{10}) + 253(\mathbf{3},\overline{\mathbf{10}}) + 428(\mathbf{5},\mathbf{8}) + 24(\mathbf{13},\mathbf{1}) + 349(\mathbf{5},\mathbf{10}) + 349(\mathbf{5},\overline{\mathbf{10}}) + 426(\mathbf{7},\mathbf{8}) +$
$13(\mathbf{15},\mathbf{1}) + 333(\mathbf{7},\mathbf{10}) + 333(\mathbf{7},\overline{\mathbf{10}}) + 342(\mathbf{9},\mathbf{8}) + 4(\mathbf{17},\mathbf{1}) + 272(\mathbf{9},\mathbf{10}) + 272(\mathbf{9},\overline{\mathbf{10}}) + 228(\mathbf{11},\mathbf{8}) +$
$2(\mathbf{19},\mathbf{1}) + 173(\mathbf{11},\mathbf{10}) + 173(\mathbf{11},\overline{\mathbf{10}}) + 126(\mathbf{13},\mathbf{8}) + 97(\mathbf{13},\mathbf{10}) + 97(\mathbf{13},\overline{\mathbf{10}}) + 57(\mathbf{15},\mathbf{8}) + 40(\mathbf{15},\mathbf{10}) +$
$40(\mathbf{15},\overline{\mathbf{10}}) + 20(\mathbf{17},\mathbf{8}) + 15(\mathbf{17},\mathbf{10}) + 15(\mathbf{17},\overline{\mathbf{10}}) + 5(\mathbf{19},\mathbf{8}) + 185(\mathbf{1},\mathbf{27}) + 36(\mathbf{1},\mathbf{28}) + 36(\mathbf{1},\overline{\mathbf{28}}) +$
$3(\mathbf{19},\mathbf{10}) + 3(\mathbf{19},\overline{\mathbf{10}}) + (\mathbf{21},\mathbf{8}) + 526(\mathbf{3},\mathbf{27}) + 115(\mathbf{3},\mathbf{28}) + 115(\mathbf{3},\overline{\mathbf{28}}) + (\mathbf{21},\mathbf{10}) + (\mathbf{21},\overline{\mathbf{10}}) + 675(\mathbf{5},\mathbf{27}) +$
$137(\mathbf{5},\mathbf{28}) + 137(\mathbf{5},\overline{\mathbf{28}}) + 673(\mathbf{7},\mathbf{27}) + 136(\mathbf{7},\mathbf{28}) + 136(\mathbf{7},\overline{\mathbf{28}}) + 138(\mathbf{1},\mathbf{35}) + 138(\mathbf{1},\overline{\mathbf{35}}) + 520(\mathbf{9},\mathbf{27}) +$
$95(\mathbf{9},\mathbf{28}) + 95(\mathbf{9},\overline{\mathbf{28}}) + 361(\mathbf{3},\mathbf{35}) + 361(\mathbf{3},\overline{\mathbf{35}}) + 342(\mathbf{11},\mathbf{27}) + 61(\mathbf{11},\mathbf{28}) + 61(\mathbf{11},\overline{\mathbf{28}}) + 476(\mathbf{5},\mathbf{35}) +$
$476(\mathbf{5},\overline{\mathbf{35}}) + 176(\mathbf{13},\mathbf{27}) + 25(\mathbf{13},\mathbf{28}) + 25(\mathbf{13},\overline{\mathbf{28}}) + 455(\mathbf{7},\mathbf{35}) + 455(\mathbf{7},\overline{\mathbf{35}}) + 77(\mathbf{15},\mathbf{27}) + 10(\mathbf{15},\mathbf{28}) +$
$10(\mathbf{15},\overline{\mathbf{28}}) + 353(\mathbf{9},\mathbf{35}) + 353(\mathbf{9},\overline{\mathbf{35}}) + 23(\mathbf{17},\mathbf{27}) + 2(\mathbf{17},\mathbf{28}) + 2(\mathbf{17},\overline{\mathbf{28}}) + 220(\mathbf{11},\mathbf{35}) + 220(\mathbf{11},\overline{\mathbf{35}}) +$
$6(\mathbf{19},\mathbf{27}) + 112(\mathbf{13},\mathbf{35}) + 112(\mathbf{13},\overline{\mathbf{35}}) + 44(\mathbf{15},\mathbf{35}) + 44(\mathbf{15},\overline{\mathbf{35}}) + 13(\mathbf{17},\mathbf{35}) + 13(\mathbf{17},\overline{\mathbf{35}}) + 2(\mathbf{19},\mathbf{35}) +$
$2(\mathbf{19},\overline{\mathbf{35}}) + 5(\mathbf{1},\mathbf{55}) + 5(\mathbf{1},\overline{\mathbf{55}}) + 6(\mathbf{3},\mathbf{55}) + 6(\mathbf{3},\overline{\mathbf{55}}) + 10(\mathbf{5},\mathbf{55}) + 10(\mathbf{5},\overline{\mathbf{55}}) + 7(\mathbf{7},\mathbf{55}) + 7(\mathbf{7},\overline{\mathbf{55}}) +$
$5(\mathbf{9},\mathbf{55}) + 5(\mathbf{9},\overline{\mathbf{55}}) + 156(\mathbf{1},\mathbf{64}) + (\mathbf{11},\mathbf{55}) + (\mathbf{11},\overline{\mathbf{55}}) + 412(\mathbf{3},\mathbf{64}) + (\mathbf{13},\mathbf{55}) + (\mathbf{13},\overline{\mathbf{55}}) + 533(\mathbf{5},\mathbf{64}) +$
$509(\mathbf{7},\mathbf{64}) + 385(\mathbf{9},\mathbf{64}) + 235(\mathbf{11},\mathbf{64}) + 116(\mathbf{13},\mathbf{64}) + 43(\mathbf{15},\mathbf{64}) + 25(\mathbf{1},\mathbf{80}) + 25(\mathbf{1},\overline{\mathbf{80}}) + 11(\mathbf{17},\mathbf{64}) +$
$90(\mathbf{1},\mathbf{81}) + 90(\mathbf{1},\overline{\mathbf{81}}) + 65(\mathbf{3},\mathbf{80}) + 65(\mathbf{3},\overline{\mathbf{80}}) + 2(\mathbf{19},\mathbf{64}) + 224(\mathbf{3},\mathbf{81}) + 224(\mathbf{3},\overline{\mathbf{81}}) + 81(\mathbf{5},\mathbf{80}) + 81(\mathbf{5},\overline{\mathbf{80}}) +$
$294(\mathbf{5},\mathbf{81}) + 294(\mathbf{5},\overline{\mathbf{81}}) + 72(\mathbf{7},\mathbf{80}) + 72(\mathbf{7},\overline{\mathbf{80}}) + 269(\mathbf{7},\mathbf{81}) + 269(\mathbf{7},\overline{\mathbf{81}}) + 49(\mathbf{9},\mathbf{80}) + 49(\mathbf{9},\overline{\mathbf{80}}) +$
$200(\mathbf{9},\mathbf{81}) + 200(\mathbf{9},\overline{\mathbf{81}}) + 26(\mathbf{11},\mathbf{80}) + 26(\mathbf{11},\overline{\mathbf{80}}) + 114(\mathbf{11},\mathbf{81}) + 114(\mathbf{11},\overline{\mathbf{81}}) + 10(\mathbf{13},\mathbf{80}) + 10(\mathbf{13},\overline{\mathbf{80}}) +$
$53(\mathbf{13},\mathbf{81}) + 53(\mathbf{13},\overline{\mathbf{81}}) + 2(\mathbf{15},\mathbf{80}) + 2(\mathbf{15},\overline{\mathbf{80}}) + 17(\mathbf{15},\mathbf{81}) + 17(\mathbf{15},\overline{\mathbf{81}}) + 4(\mathbf{17},\mathbf{81}) + 4(\mathbf{17},\overline{\mathbf{81}}) +$
$63(\mathbf{1},\mathbf{125}) + 179(\mathbf{3},\mathbf{125}) + 216(\mathbf{5},\mathbf{125}) + 204(\mathbf{7},\mathbf{125}) + 140(\mathbf{9},\mathbf{125}) + 80(\mathbf{11},\mathbf{125}) + 32(\mathbf{13},\mathbf{125}) +$
$10(\mathbf{15},\mathbf{125}) + (\mathbf{17},\mathbf{125}) + (\mathbf{1},\mathbf{143}) + (\mathbf{1},\overline{\mathbf{143}}) + (\mathbf{3},\mathbf{143}) + (\mathbf{3},\overline{\mathbf{143}}) + (\mathbf{5},\mathbf{143}) + (\mathbf{5},\overline{\mathbf{143}}) + (\mathbf{7},\mathbf{143}) +$
$(\mathbf{7},\overline{\mathbf{143}}) + 28(\mathbf{1},\mathbf{154}) + 28(\mathbf{1},\overline{\mathbf{154}}) + 70(\mathbf{3},\mathbf{154}) + 70(\mathbf{3},\overline{\mathbf{154}}) + 88(\mathbf{5},\mathbf{154}) + 88(\mathbf{5},\overline{\mathbf{154}}) + 75(\mathbf{7},\mathbf{154}) +$
$75(\mathbf{7},\overline{\mathbf{154}}) + 51(\mathbf{9},\mathbf{154}) + 51(\mathbf{9},\overline{\mathbf{154}}) + 4(\mathbf{1},\mathbf{162}) + 4(\mathbf{1},\overline{\mathbf{162}}) + 25(\mathbf{11},\mathbf{154}) + 25(\mathbf{11},\overline{\mathbf{154}}) + 16(\mathbf{3},\mathbf{162}) +$
$16(\mathbf{3},\overline{\mathbf{162}}) + 9(\mathbf{13},\mathbf{154}) + 9(\mathbf{13},\overline{\mathbf{154}}) + 15(\mathbf{5},\mathbf{162}) + 15(\mathbf{5},\overline{\mathbf{162}}) + 14(\mathbf{7},\mathbf{162}) + 14(\mathbf{7},\overline{\mathbf{162}}) + 2(\mathbf{15},\mathbf{154}) +$
$2(\mathbf{15},\overline{\mathbf{154}}) + 7(\mathbf{9},\mathbf{162}) + 7(\mathbf{9},\overline{\mathbf{162}}) + 3(\mathbf{11},\mathbf{162}) + 3(\mathbf{11},\overline{\mathbf{162}}) + 14(\mathbf{1},\mathbf{216}) + 36(\mathbf{3},\mathbf{216}) + 43(\mathbf{5},\mathbf{216}) +$
$36(\mathbf{7},\mathbf{216}) + 22(\mathbf{9},\mathbf{216}) + 10(\mathbf{11},\mathbf{216}) + 3(\mathbf{13},\mathbf{216}) + 4(\mathbf{1},\mathbf{260}) + 4(\mathbf{1},\overline{\mathbf{260}}) + 8(\mathbf{3},\mathbf{260}) + 8(\mathbf{3},\overline{\mathbf{260}}) +$
$11(\mathbf{5},\mathbf{260}) + 11(\mathbf{5},\overline{\mathbf{260}}) + 7(\mathbf{7},\mathbf{260}) + 7(\mathbf{7},\overline{\mathbf{260}}) + 4(\mathbf{9},\mathbf{260}) + 4(\mathbf{9},\overline{\mathbf{260}}) + (\mathbf{11},\mathbf{260}) + (\mathbf{11},\overline{\mathbf{260}}) +$
$(\mathbf{3},\mathbf{280}) + (\mathbf{3},\overline{\mathbf{280}}) + (\mathbf{5},\mathbf{280}) + (\mathbf{5},\overline{\mathbf{280}}) + (\mathbf{1},\mathbf{343}) + 3(\mathbf{3},\mathbf{343}) + (\mathbf{5},\mathbf{343}) + 2(\mathbf{7},\mathbf{343})\}$

or we can decompose to $\mathrm{Sp}(4)$:

```
In[96]:= Timing[DecomposeIrrep[Irrep[E8][6696000], ProductAlgebra[Sp4]]]
```

Out[96]:=

$\{1218.64, 4(\mathbf{1}) + 27(\mathbf{5}) + 64(\mathbf{10}) + 61(\mathbf{14}) + 104(\mathbf{30}) + 161(\mathbf{35}) + 127(\mathbf{35'}) + 127(\mathbf{55}) + 278(\mathbf{81}) + 201(\mathbf{84}) +$
$134(\mathbf{91}) + 297(\mathbf{105}) + 116(\mathbf{140''}) + 355(\mathbf{154}) + 175(\mathbf{165}) + 86(\mathbf{204}) + 417(\mathbf{220}) + 355(\mathbf{231}) + 373(\mathbf{260}) +$
$52(\mathbf{285}) + 132(\mathbf{286}) + 29(\mathbf{385}) + 465(\mathbf{390}) + 327(\mathbf{405}) + 287(\mathbf{429}) + 434(\mathbf{455}) + 59(\mathbf{455'}) + 10(\mathbf{506}) +$
$244(\mathbf{595}) + 412(\mathbf{625}) + 3(\mathbf{650}) + 24(\mathbf{680}) + 169(\mathbf{715}) + 406(\mathbf{770}) + 292(\mathbf{810}) + (\mathbf{819}) + 154(\mathbf{836}) +$
$314(\mathbf{935}) + 3(\mathbf{969}) + 69(\mathbf{1105}) + 81(\mathbf{1134}) + 313(\mathbf{1190}) + 151(\mathbf{1309}) + 238(\mathbf{1326}) + 194(\mathbf{1330}) + (\mathbf{1330'}) +$
$35(\mathbf{1495}) + 18(\mathbf{1615}) + 197(\mathbf{1729}) + 101(\mathbf{1820'}) + 11(\mathbf{1925}) + 46(\mathbf{1976}) + 149(\mathbf{1995}) + 98(\mathbf{2090}) +$
$2(\mathbf{2261}) + 104(\mathbf{2401}) + 41(\mathbf{2415}) + 3(\mathbf{2430}) + 78(\mathbf{2835}) + 10(\mathbf{2835'}) + 50(\mathbf{3080'}) + 25(\mathbf{3094}) + 13(\mathbf{3125}) +$
$42(\mathbf{3220}) + 29(\mathbf{3864}) + (\mathbf{3960}) + 13(\mathbf{4200}) + 19(\mathbf{4301}) + 3(\mathbf{4370}) + 8(\mathbf{4485}) + 8(\mathbf{5100}) + 3(\mathbf{5355}) +$
$5(\mathbf{5775}) + 2(\mathbf{6175}) + (\mathbf{6561})\}$

Another example is given by decomposing the **1801371** of F_4 to $\mathrm{SU}(2) \otimes \mathrm{G}_2$:

```
In[97]:= Timing[DecomposeIrrep[Irrep[F4][1801371],ProductAlgebra[SU2,G2]]]
```

Out[97]:= {127.594, (1,1) + 4(3,1) + 6(5,1) + 6(1,7) + 6(7,1) + 17(3,7) + 5(9,1) + 24(5,7) + 4(11,1) + 25(7,7) +
2(13,1) + 7(1,14) + 23(9,7) + (15,1) + 20(3,14) + 17(11,7) + 27(5,14) + 10(13,7) + 30(7,14) +
5(15,7) + 26(9,14) + 2(17,7) + 20(11,14) + 13(13,14) + 11(1,27) + 7(15,14) + 31(3,27) + 2(17,14) +
43(5,27) + (19,14) + 47(7,27) + 41(9,27) + 32(11,27) + 20(13,27) + 11(15,27) + 4(17,27) +
(19,27) + 13(1,64) + 37(3,64) + 50(5,64) + 55(7,64) + 48(9,64) + 37(11,64) + 23(13,64) + 11(1,77) +
6(1,77′) + 13(15,64) + 31(3,77) + 17(3,77′) + 5(17,64) + 43(5,77) + 23(5,77′) + 2(19,64) + 46(7,77) +
25(7,77′) + 40(9,77) + 21(9,77′) + 31(11,77) + 16(11,77′) + 19(13,77) + 10(13,77′) + 10(15,77) +
5(15,77′) + 4(17,77) + 2(17,77′) + (19,77) + (19,77′) + 6(1,182) + 18(3,182) + 24(5,182) +
25(7,182) + 11(1,189) + 21(9,182) + 32(3,189) + 15(11,182) + 43(5,189) + 8(13,182) + 46(7,189) +
4(15,182) + 39(9,189) + (17,182) + 29(11,189) + 17(13,189) + 9(15,189) + 3(17,189) + (19,189) +
(1,273) + 3(3,273) + 4(5,273) + 4(7,273) + 3(9,273) + 2(11,273) + (13,273) + 5(1,286) + 15(3,286) +
20(5,286) + 21(7,286) + 17(9,286) + 12(11,286) + 6(13,286) + 3(15,286) + (17,286) + 2(1,378) +
6(3,378) + 7(5,378) + 7(7,378) + 5(9,378) + 3(11,378) + (13,378) + 5(1,448) + 15(3,448) +
19(5,448) + 20(7,448) + 15(9,448) + 10(11,448) + 5(13,448) + 2(15,448) + (3,714) + (5,714) +
(7,714) + 2(1,729) + 6(3,729) + 7(5,729) + 7(7,729) + 5(9,729) + 3(11,729) + (13,729) + (3,896) +
(5,896) + (7,896) + (9,896) + (1,924) + 4(3,924) + 4(5,924) + 4(7,924) + 2(9,924) + (11,924) +
(3,1547) + (5,1547) + (7,1547) + (3,1728)}

One last example is the decomposition of the **2282280** of E_7 to $SU(2) \otimes G_2$:

```
In[98]:= Timing[DecomposeIrrep[Irrep[E7][2282280],ProductAlgebra[SU2,G2]]]
```

Out[98]:= {367.625, 15(2,1) + 21(4,1) + 21(6,1) + 54(2,7) + 15(8,1) + 80(4,7) + 7(10,1) + 75(6,7) + 3(12,1) +
50(8,7) + (14,1) + 57(2,14) + 24(10,7) + 87(4,14) + 8(12,7) + 78(6,14) + 2(14,7) + 50(8,14) +
23(10,14) + 7(12,14) + (14,14) + 92(2,27) + 137(4,27) + 124(6,27) + 79(8,27) + 36(10,27) +
11(12,27) + 2(14,27) + 102(2,64) + 151(4,64) + 132(6,64) + 80(8,64) + 34(10,64) + 9(12,64) +
(14,64) + 90(2,77) + 45(2,77′) + 129(4,77) + 65(4,77′) + 113(6,77) + 55(6,77′) + 67(8,77) +
31(8,77′) + 27(10,77) + 12(10,77′) + 7(12,77) + 3(12,77′) + (14,77) + 54(2,182) + 76(4,182) +
62(6,182) + 34(8,182) + 88(2,189) + 11(10,182) + 125(4,189) + 2(12,182) + 104(6,189) +
58(8,189) + 21(10,189) + 4(12,189) + 7(2,273) + 9(4,273) + 6(6,273) + 3(8,273) + 39(2,286) +
53(4,286) + 42(6,286) + 21(8,286) + 6(10,286) + (12,286) + 19(2,378) + 25(4,378) + 19(6,378) +
8(8,378) + 2(10,378) + 42(2,448) + 58(4,448) + 44(6,448) + 21(8,448) + 6(10,448) + 4(2,714) +
4(4,714) + 3(6,714) + (8,714) + 15(2,729) + 19(4,729) + 13(6,729) + 5(8,729) + (10,729) +
2(2,896) + 2(4,896) + (6,896) + 10(2,924) + 14(4,924) + 9(6,924) + 3(8,924) + 2(2,1547) +
2(4,1547) + (6,1547) + (2,1728) + (4,1728)}

8. Conclusions and Outlook

We have extended the Mathematica application LieART to find branching rules of maximal subalgebras of Lie algebras, covering all regular and special algebra-subalgebra pairs up to rank 15, as well as many beyond that. Such maximal subalgebras and branching rules are of great interest in particle physics, especially in unified model building. We have successfully reproduced existing data on irreducible representations and branching rules, and have the functionality to tabulate beyond the preexisting data.

Future work will focus on extending LieART to handle Lie superalgebras, wich are an important mathematical tool used to study supersymmetry, superstrings, supergravity, and M-theory. Additional possible directions for future research with LieART include computation of group characters, computation of explicit matrix representations of Lie groups (like the Gell-Mann matrices for SU(3)), and division of the tensor product into the symmetric and the antisymmetric part.

9. Acknowledgments

We thank the many users of LieART for their feedback and suggestions. We would also like to thank Robert Feger for many helpful discussions throughout the course of this research.

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10. Tables

We present here tables of various subalgebra branching rules in Section ?? for many classical and all exceptional Lie algebras. In presentation style, selection of subalgebra branching we closely follow [8], which has been the definitive reference for unified model building since its publication. The tables were created by the supplemental package `Tables.m`, which uses LieART for the computation. Since LieART comes with the functions that generate the tables, the user may extend them to the limit of his or her computer power. We note that these tables are a representative sample of implemented branching rules and do not cover all subalgebras within any given rank. We have denoted regular subalgebra branchings with an (R) label and special subalgebra branchings with an (S) label.

10.0.1. SU(N)

Table 10.1: SU(3) Branching Rules

SU(3)	→	SU(2) (S)
$\mathbf{3}$	=	$(\mathbf{3})$
$\mathbf{6}$	=	$(\mathbf{1})+(\mathbf{5})$
$\mathbf{8}$	=	$(\mathbf{3})+(\mathbf{5})$
$\mathbf{10}$	=	$(\mathbf{3})+(\mathbf{7})$
$\mathbf{15}$	=	$(\mathbf{3})+(\mathbf{5})+(\mathbf{7})$
$\mathbf{15'}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{9})$
$\mathbf{21}$	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})$
$\mathbf{24}$	=	$(\mathbf{3})+(\mathbf{5})+(\mathbf{7})+(\mathbf{9})$
$\mathbf{27}$	=	$(\mathbf{1})+2(\mathbf{5})+(\mathbf{7})+(\mathbf{9})$
$\mathbf{28}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{9})+(\mathbf{13})$
$\mathbf{35}$	=	$(\mathbf{3})+(\mathbf{5})+(\mathbf{7})+(\mathbf{9})+(\mathbf{11})$
$\mathbf{36}$	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})+(\mathbf{15})$
$\mathbf{42}$	=	$(\mathbf{3})+(\mathbf{5})+2(\mathbf{7})+(\mathbf{9})+(\mathbf{11})$
$\mathbf{45}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{9})+(\mathbf{13})+(\mathbf{17})$
$\mathbf{48}$	=	$(\mathbf{3})+(\mathbf{5})+(\mathbf{7})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})$
$\mathbf{55}$	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})+(\mathbf{15})+(\mathbf{19})$

Table 10.2: SU(4) Branching Rules

SU(4)	→	Sp(4) (S)
$\mathbf{4}$	=	$(\mathbf{4})$
$\mathbf{6}$	=	$(\mathbf{1})+(\mathbf{5})$
$\mathbf{10}$	=	$(\mathbf{10})$
$\mathbf{15}$	=	$(\mathbf{5})+(\mathbf{10})$
$\mathbf{20}$	=	$(\mathbf{4})+(\mathbf{16})$
$\mathbf{20'}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{14})$
$\mathbf{20''}$	=	$(\mathbf{20})$
$\mathbf{35}$	=	$(\mathbf{35'})$
$\mathbf{36}$	=	$(\mathbf{16})+(\mathbf{20})$
$\mathbf{45}$	=	$(\mathbf{10})+(\mathbf{35})$
$\mathbf{50}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{14})+(\mathbf{35})$
$\mathbf{56}$	=	$(\mathbf{56})$
$\mathbf{60}$	=	$(\mathbf{4})+(\mathbf{16})+2(\mathbf{40})$
$\mathbf{64}$	=	$(\mathbf{5})+(\mathbf{10})+(\mathbf{14})+(\mathbf{35})$
$\mathbf{70}$	=	$(\mathbf{35})+(\mathbf{35'})$

Table 10.2: SU(4) Branching Rules (continued)

$\mathbf{84}$	=	$(\mathbf{14}) + (\mathbf{35}) + (\mathbf{35'})$
$\mathbf{84'}$	=	$(\mathbf{20}) + (\mathbf{64})$
$\mathbf{84''}$	=	$(\mathbf{84})$

SU(4)	→	SU(2)×SU(2) (S)
$\mathbf{4}$	=	$(\mathbf{2},\mathbf{2})$
$\mathbf{6}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$
$\mathbf{10}$	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3})$
$\mathbf{15}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{3})$
$\mathbf{20}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{2}) + (\mathbf{2},\mathbf{4})$
$\mathbf{20'}$	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{1}) + (\mathbf{1},\mathbf{5})$
$\mathbf{20''}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{4})$
$\mathbf{35}$	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{5})$
$\mathbf{36}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{2}) + (\mathbf{2},\mathbf{4})$
$\mathbf{45}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{3}) + (\mathbf{3},\mathbf{5})$
$\mathbf{50}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{5},\mathbf{3}) + (\mathbf{3},\mathbf{5}) + (\mathbf{7},\mathbf{1}) + (\mathbf{1},\mathbf{7})$
$\mathbf{56}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{4}) + (\mathbf{6},\mathbf{6})$
$\mathbf{60}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{2}) + (\mathbf{2},\mathbf{4}) + (\mathbf{4},\mathbf{4}) + (\mathbf{6},\mathbf{2}) + (\mathbf{2},\mathbf{6})$
$\mathbf{64}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + 2(\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{1}) + (\mathbf{1},\mathbf{5}) + (\mathbf{5},\mathbf{3}) + (\mathbf{3},\mathbf{5})$
$\mathbf{70}$	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{3}) + (\mathbf{3},\mathbf{5}) + (\mathbf{5},\mathbf{5})$
$\mathbf{84}$	=	$(\mathbf{1},\mathbf{1}) + 2(\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{1}) + (\mathbf{1},\mathbf{5}) + (\mathbf{5},\mathbf{3}) + (\mathbf{3},\mathbf{5}) + (\mathbf{5},\mathbf{5})$
$\mathbf{84'}$	=	$(\mathbf{2},\mathbf{2}) + (\mathbf{4},\mathbf{2}) + (\mathbf{2},\mathbf{4}) + (\mathbf{4},\mathbf{4}) + (\mathbf{6},\mathbf{4}) + (\mathbf{4},\mathbf{6})$
$\mathbf{84''}$	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\mathbf{5},\mathbf{5}) + (\mathbf{7},\mathbf{7})$

Table 10.3: SU(5) Branching Rules

SU(5)	→	Sp(4) (S)
$\mathbf{5}$	=	$(\mathbf{5})$
$\mathbf{10}$	=	$(\mathbf{10})$
$\mathbf{15}$	=	$(\mathbf{1}) + (\mathbf{14})$
$\mathbf{24}$	=	$(\mathbf{10}) + (\mathbf{14})$
$\mathbf{35}$	=	$(\mathbf{5}) + (\mathbf{30})$
$\mathbf{40}$	=	$(\mathbf{5}) + (\mathbf{35})$
$\mathbf{45}$	=	$(\mathbf{10}) + (\mathbf{35})$
$\mathbf{50}$	=	$(\mathbf{1}) + (\mathbf{14}) + (\mathbf{35'})$
$\mathbf{70}$	=	$(\mathbf{5}) + (\mathbf{30}) + (\mathbf{35})$
$\mathbf{70'}$	=	$(\mathbf{1}) + (\mathbf{14}) + (\mathbf{55})$
$\mathbf{75}$	=	$(\mathbf{5}) + (\mathbf{35}) + (\mathbf{35'})$
$\mathbf{105}$	=	$(\mathbf{10}) + (\mathbf{14}) + (\mathbf{81})$
$\mathbf{126}$	=	$(\mathbf{10}) + (\mathbf{35}) + (\mathbf{81})$
$\mathbf{126'}$	=	$(\mathbf{5}) + (\mathbf{30}) + (\mathbf{91})$
$\mathbf{160}$	=	$(\mathbf{10}) + (\mathbf{14}) + (\mathbf{55}) + (\mathbf{81})$
$\mathbf{175}$	=	$(\mathbf{10}) + (\mathbf{14}) + (\mathbf{35}) + (\mathbf{35'}) + (\mathbf{81})$
$\mathbf{175'}$	=	$(\mathbf{5}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{105})$
$\mathbf{175''}$	=	$(\mathbf{10}) + (\mathbf{81}) + (\mathbf{84})$

Table 10.4: SU(6) Branching Rules

SU(6)	→	SU(3) (S)
$\mathbf{6}$	=	$(\overline{\mathbf{6}})$
$\mathbf{15}$	=	$(\overline{\mathbf{15}})$
$\mathbf{20}$	=	$(\mathbf{10})+(\overline{\mathbf{10}})$
$\mathbf{21}$	=	$(\mathbf{6})+(\overline{\mathbf{15}}')$
$\mathbf{35}$	=	$(\mathbf{8})+(\mathbf{27})$
$\mathbf{56}$	=	$(\mathbf{1})+(\mathbf{27})+(\overline{\mathbf{28}})$
$\mathbf{70}$	=	$(\mathbf{8})+(\mathbf{27})+(\overline{\mathbf{35}})$
$\mathbf{84}$	=	$(\mathbf{3})+(\mathbf{15})+(\mathbf{24})+(\mathbf{42})$
$\mathbf{105}$	=	$(\overline{\mathbf{3}})+(\overline{\mathbf{15}})+(\overline{\mathbf{21}})+(\overline{\mathbf{24}})+(\overline{\mathbf{42}})$
$\mathbf{105}'$	=	$(\mathbf{6})+(\overline{\mathbf{15}}')+(\overline{\mathbf{24}})+(\overline{\mathbf{60}})$
$\mathbf{120}$	=	$(\overline{\mathbf{6}})+(\mathbf{15})+(\mathbf{15}')+(\mathbf{24})+(\mathbf{60})$
$\mathbf{126}$	=	$(\mathbf{6})+(\overline{\mathbf{15}}')+(\overline{\mathbf{45}})+(\overline{\mathbf{60}})$
$\mathbf{175}$	=	$(\mathbf{1})+2(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+(\mathbf{64})$
$\mathbf{189}$	=	$(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})$
$\mathbf{210}$	=	$(\overline{\mathbf{6}})+(\mathbf{15})+(\mathbf{15}')+(\mathbf{24})+(\mathbf{42})+(\mathbf{48})+(\mathbf{60})$
$\mathbf{210}'$	=	$(\mathbf{6})+(\overline{\mathbf{15}})+(\overline{\mathbf{24}})+(\overline{\mathbf{42}})+(\overline{\mathbf{60}})+(\overline{\mathbf{63}})$
$\mathbf{280}$	=	$(\mathbf{8})+(\mathbf{10})+2(\overline{\mathbf{10}})+(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})+(\overline{\mathbf{81}})$
$\mathbf{315}$	=	$(\mathbf{6})+(\overline{\mathbf{15}})+(\overline{\mathbf{15}}')+(\overline{\mathbf{24}})+(\overline{\mathbf{42}})+(\overline{\mathbf{48}})+(\overline{\mathbf{60}})+(\overline{\mathbf{105}})$

SU(6)	→	Sp(6) (S)
$\mathbf{6}$	=	$(\mathbf{6})$
$\mathbf{15}$	=	$(\mathbf{1})+(\mathbf{14})$
$\mathbf{20}$	=	$(\mathbf{6})+(\mathbf{14}')$
$\mathbf{21}$	=	$(\mathbf{21})$
$\mathbf{35}$	=	$(\mathbf{14})+(\mathbf{21})$
$\mathbf{56}$	=	$(\mathbf{56})$
$\mathbf{70}$	=	$(\mathbf{6})+(\mathbf{64})$
$\mathbf{84}$	=	$(\mathbf{6})+(\mathbf{14}')+(\mathbf{64})$
$\mathbf{105}$	=	$(\mathbf{14})+(\mathbf{21})+(\mathbf{70})$
$\mathbf{105}'$	=	$(\mathbf{1})+(\mathbf{14})+(\mathbf{90})$
$\mathbf{120}$	=	$(\mathbf{56})+(\mathbf{64})$
$\mathbf{126}$	=	$(\mathbf{126}')$
$\mathbf{175}$	=	$(\mathbf{21})+(\mathbf{70})+(\mathbf{84})$
$\mathbf{189}$	=	$(\mathbf{1})+2(\mathbf{14})+(\mathbf{70})+(\mathbf{90})$
$\mathbf{210}$	=	$(\mathbf{6})+(\mathbf{14}')+(\mathbf{64})+(\mathbf{126})$
$\mathbf{210}'$	=	$(\mathbf{21})+(\mathbf{189})$
$\mathbf{252}$	=	$(\mathbf{252})$
$\mathbf{280}$	=	$(\mathbf{21})+(\mathbf{70})+(\mathbf{189})$
$\mathbf{315}$	=	$(\mathbf{126}')+(\mathbf{189})$

SU(6)	→	SU(3)×SU(2) (S)
$\mathbf{6}$	=	$(\overline{\mathbf{3}},\mathbf{2})$
$\mathbf{15}$	=	$(\mathbf{3},\mathbf{3})+(\overline{\mathbf{6}},\mathbf{1})$
$\mathbf{20}$	=	$(\mathbf{1},\mathbf{4})+(\mathbf{8},\mathbf{2})$
$\mathbf{21}$	=	$(\mathbf{3},\mathbf{1})+(\overline{\mathbf{6}},\mathbf{3})$
$\mathbf{35}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{8},\mathbf{1})+(\mathbf{8},\mathbf{3})$
$\mathbf{56}$	=	$(\mathbf{8},\mathbf{2})+(\overline{\mathbf{10}},\mathbf{4})$
$\mathbf{70}$	=	$(\mathbf{1},\mathbf{2})+(\mathbf{8},\mathbf{2})+(\mathbf{8},\mathbf{4})+(\overline{\mathbf{10}},\mathbf{2})$

Table 10.4: SU(6) Branching Rules (continued)

84	=	$(\overline{\mathbf{3}},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{4}) + (\mathbf{6},\mathbf{2}) + (\mathbf{6},\mathbf{4}) + (\overline{\mathbf{15}},\mathbf{2})$
105	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\mathbf{3},\mathbf{5}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{15},\mathbf{1}) + (\mathbf{15},\mathbf{3})$
105′	=	$(\mathbf{3},\mathbf{3}) + (\overline{\mathbf{6}},\mathbf{1}) + (\overline{\mathbf{6}},\mathbf{5}) + (\mathbf{15'},\mathbf{1}) + (\mathbf{15},\mathbf{3})$
120	=	$(\overline{\mathbf{3}},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{4}) + (\mathbf{6},\mathbf{2}) + (\overline{\mathbf{15}},\mathbf{2}) + (\overline{\mathbf{15}},\mathbf{4})$
126	=	$(\overline{\mathbf{6}},\mathbf{1}) + (\mathbf{15},\mathbf{3}) + (\mathbf{15'},\mathbf{5})$
175	=	$(\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{7}) + (\mathbf{8},\mathbf{3}) + (\mathbf{10},\mathbf{1}) + (\overline{\mathbf{10}},\mathbf{1}) + (\mathbf{8},\mathbf{5}) + (\mathbf{27},\mathbf{3})$
189	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{5}) + (\mathbf{8},\mathbf{1}) + 2(\mathbf{8},\mathbf{3}) + (\mathbf{10},\mathbf{3}) + (\overline{\mathbf{10}},\mathbf{3}) + (\mathbf{8},\mathbf{5}) + (\mathbf{27},\mathbf{1})$
210	=	$(\overline{\mathbf{3}},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{4}) + (\mathbf{6},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{6}) + (\mathbf{6},\mathbf{4}) + (\overline{\mathbf{15}},\mathbf{2}) + (\overline{\mathbf{15}},\mathbf{4}) + (\overline{\mathbf{24}},\mathbf{2})$
210′	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{15},\mathbf{1}) + (\mathbf{15},\mathbf{3}) + (\mathbf{15'},\mathbf{3}) + (\mathbf{15},\mathbf{5})$
252	=	$(\overline{\mathbf{15}},\mathbf{2}) + (\overline{\mathbf{21}},\mathbf{6}) + (\overline{\mathbf{24}},\mathbf{4})$
280	=	$(\mathbf{1},\mathbf{3}) + (\mathbf{8},\mathbf{1}) + 2(\mathbf{8},\mathbf{3}) + (\mathbf{10},\mathbf{1}) + (\overline{\mathbf{10}},\mathbf{1}) + (\overline{\mathbf{10}},\mathbf{3}) + (\mathbf{8},\mathbf{5}) + (\overline{\mathbf{10}},\mathbf{5}) + (\mathbf{27},\mathbf{3})$
315	=	$(\mathbf{3},\mathbf{3}) + (\overline{\mathbf{6}},\mathbf{1}) + (\overline{\mathbf{6}},\mathbf{3}) + (\overline{\mathbf{6}},\mathbf{5}) + (\mathbf{15},\mathbf{1}) + (\mathbf{15},\mathbf{3}) + (\mathbf{24},\mathbf{3}) + (\mathbf{24},\mathbf{5})$

Table 10.5: SU(8) Branching Rules

SU(8)	→	SO(8) (S)
8	=	$(\mathbf{8_v})$
28	=	$(\mathbf{28})$
36	=	$(\mathbf{1}) + (\mathbf{35_v})$
56	=	$(\mathbf{56_v})$
63	=	$(\mathbf{28}) + (\mathbf{35_v})$
70	=	$(\mathbf{35_s}) + (\mathbf{35}c)$
120	=	$(\mathbf{8_v}) + (\mathbf{112_v})$
168	=	$(\mathbf{8_v}) + (\mathbf{160_v})$
216	=	$(\mathbf{56_v}) + (\mathbf{160_v})$
280	=	$(\mathbf{8_v}) + (\mathbf{112_v}) + (\mathbf{160_v})$
330	=	$(\mathbf{1}) + (\mathbf{35_v}) + (\mathbf{294_v})$
336	=	$(\mathbf{1}) + (\mathbf{35_v}) + (\mathbf{300})$
378	=	$(\mathbf{28}) + (\mathbf{350})$
420	=	$(\mathbf{35_s}) + (\mathbf{35_c}) + (\mathbf{350})$
504	=	$(\mathbf{56_v}) + (\mathbf{224_{sv}}) + (\mathbf{224_{cv}})$
630	=	$(\mathbf{28}) + (\mathbf{35_v}) + (\mathbf{567_v})$
720	=	$(\mathbf{35_s}) + (\mathbf{35_c}) + (\mathbf{300}) + (\mathbf{350})$

SU(8)	→	Sp(8) (S)
8	=	$(\mathbf{8})$
28	=	$(\mathbf{1}) + (\mathbf{27})$
36	=	$(\mathbf{36})$
56	=	$(\mathbf{8}) + (\mathbf{48})$
63	=	$(\mathbf{27}) + (\mathbf{36})$
70	=	$(\mathbf{1}) + (\mathbf{27}) + (\mathbf{42})$
120	=	$(\mathbf{120})$
168	=	$(\mathbf{8}) + (\mathbf{160})$
216	=	$(\mathbf{8}) + (\mathbf{48}) + (\mathbf{160})$
280	=	$(\mathbf{120}) + (\mathbf{160})$
330	=	$(\mathbf{330})$
336	=	$(\mathbf{1}) + (\mathbf{27}) + (\mathbf{308})$
378	=	$(\mathbf{27}) + (\mathbf{36}) + (\mathbf{315})$
420	=	$(\mathbf{27}) + (\mathbf{36}) + (\mathbf{42}) + (\mathbf{315})$
504	=	$(\mathbf{8}) + (\mathbf{48}) + (\mathbf{160})$

Table 10.5: SU(8) Branching Rules (continued)

630	=	$(36)+(594)$
720	=	$(1)+2(27)+(42)+(308)+(315)$

SU(8)	→	SU(4) × SU(2) (S)
8	=	$(4,2)$
28	=	$(6,3)+(10,1)$
36	=	$(6,1)+(10,3)$
56	=	$(\overline{4},4)+(\overline{20},2)$
63	=	$(1,3)+(15,1)+(15,3)$
70	=	$(1,5)+(15,3)+(20',1)$
120	=	$(\overline{20},2)+(\overline{20}'',4)$
168	=	$(\overline{4},2)+(\overline{20},2)+(\overline{20}'',2)+(\overline{20},4)$
216	=	$(4,2)+(4,4)+(20,2)+(20,4)+(36,2)$
280	=	$(4,2)+(4,4)+(20,2)+(36,2)+(36,4)$
330	=	$(20',1)+(35,5)+(45,3)$
336	=	$(1,1)+(15,3)+(20',1)+(20',5)+(35,1)+(45,3)$
378	=	$(1,3)+(15,1)+(15,3)+(15,5)+(20',3)+(45,1)+(45,3)$
420	=	$(6,1)+(6,3)+(6,5)+(10,3)+(\overline{10},3)+(\overline{10},5)+(64,1)+(64,3)$
504	=	$(\overline{4},2)+(\overline{4},4)+(\overline{4},6)+(\overline{20},2)+(\overline{20},4)+(\overline{36},2)+(\overline{36},4)+(\overline{60},2)$
630	=	$(15,1)+(15,3)+(20',3)+(35,3)+(45,1)+(45,3)+(45,5)$
720	=	$(1,1)+(1,5)+(15,1)+2(15,3)+(15,5)+(20',1)+(20',3)+(20',5)+(45,3)+(\overline{45},3)+(84,1)$

Table 10.6: SU(9) Branching Rules

SU(9)	→	SO(9) (S)
9	=	(9)
36	=	(36)
45	=	$(1)+(44)$
80	=	$(36)+(44)$
84	=	(84)
126	=	(126)
165	=	$(9)+(156)$
240	=	$(9)+(231)$
315	=	$(84)+(231)$
396	=	$(9)+(156)+(231)$
495	=	$(1)+(44)+(450)$
540	=	$(1)+(44)+(495)$
630	=	$(36)+(594)$
720	=	$(126)+(594)$
990	=	$(36)+(44)+(910)$
1008	=	$(84)+(924)$

SU(9)	→	SU(3) × SU(3) (S)
9	=	$(3,3)$
36	=	$(6,\overline{3})+(\overline{3},6)$
45	=	$(\overline{3},\overline{3})+(6,6)$
80	=	$(8,1)+(1,8)+(8,8)$
84	=	$(10,1)+(1,10)+(8,8)$
126	=	$(\overline{6},\overline{6})+(15,3)+(3,15)$

Table 10.6: SU(9) Branching Rules (continued)

165	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{8},\mathbf{8}) + (\mathbf{10},\mathbf{10})$
240	=	$(\mathbf{8},\mathbf{1}) + (\mathbf{1},\mathbf{8}) + (\mathbf{8},\mathbf{8}) + (\mathbf{10},\mathbf{8}) + (\mathbf{8},\mathbf{10})$
315	=	$(\mathbf{3},\mathbf{3}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{15})$
396	=	$(\mathbf{3},\mathbf{3}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\overline{\mathbf{6}},\overline{\mathbf{6}}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15},\mathbf{15})$
495	=	$(\mathbf{3},\mathbf{3}) + (\overline{\mathbf{6}},\overline{\mathbf{6}}) + (\mathbf{15},\mathbf{15}) + (\mathbf{15'},\mathbf{15'})$
540	=	$(\mathbf{3},\mathbf{3}) + (\overline{\mathbf{6}},\overline{\mathbf{6}}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15'},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{15'}) + (\mathbf{15},\mathbf{15})$
630	=	$(\mathbf{3},\mathbf{3}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15'},\mathbf{3}) + (\mathbf{3},\mathbf{15'}) + (\mathbf{15},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{15}) + (\mathbf{15},\mathbf{15})$
720	=	$(\overline{\mathbf{3}},\overline{\mathbf{3}}) + (\mathbf{6},\overline{\mathbf{3}}) + (\overline{\mathbf{3}},\mathbf{6}) + (\mathbf{6},\mathbf{6}) + (\overline{\mathbf{15}},\overline{\mathbf{3}}) + (\overline{\mathbf{3}},\overline{\mathbf{15}}) + (\mathbf{6},\overline{\mathbf{15}}) + (\overline{\mathbf{15}},\mathbf{6}) + (\overline{\mathbf{24}},\overline{\mathbf{3}}) + (\overline{\mathbf{3}},\overline{\mathbf{24}}) + (\overline{\mathbf{15}},\overline{\mathbf{15}})$
990	=	$(\mathbf{3},\mathbf{3}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{15}) + (\mathbf{15},\mathbf{15}) + (\mathbf{15'},\mathbf{15}) + (\mathbf{15},\mathbf{15'})$
1008	=	$(\mathbf{3},\mathbf{3}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\overline{\mathbf{6}},\overline{\mathbf{6}}) + (\mathbf{15},\mathbf{3}) + (\mathbf{3},\mathbf{15}) + (\mathbf{15},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{15}) + (\mathbf{24},\mathbf{3}) + (\mathbf{3},\mathbf{24}) + (\mathbf{15},\mathbf{15}) + (\mathbf{24},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{24})$

Table 10.7: SU(10) Branching Rules

SU(10)	$\to$	SU(3) (S)
10	=	$(\mathbf{10})$
45	=	$(\overline{\mathbf{10}}) + (\mathbf{35})$
55	=	$(\mathbf{27}) + (\mathbf{28})$
99	=	$(\mathbf{8}) + (\mathbf{27}) + (\mathbf{64})$
120	=	$(\mathbf{1}) + (\mathbf{27}) + (\mathbf{28}) + (\mathbf{64})$
210	=	$(\mathbf{10}) + (\mathbf{27}) + (\overline{\mathbf{28}}) + (\mathbf{64}) + (\mathbf{81})$
220	=	$(\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{55}) + (\mathbf{64}) + (\mathbf{81})$
252	=	$(\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{81}) + (\overline{\mathbf{81}})$
330	=	$(\mathbf{8}) + (\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{80}) + (\mathbf{81})$
440	=	$(\mathbf{8}) + (\mathbf{10}) + 2(\mathbf{27}) + (\overline{\mathbf{28}}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{81}) + (\mathbf{125})$
540	=	$(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27}) + 2(\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{81}) + (\overline{\mathbf{81}}) + (\mathbf{154})$
715	=	$(\mathbf{1}) + (\mathbf{27}) + (\mathbf{28}) + (\overline{\mathbf{28}}) + (\mathbf{35}) + (\mathbf{64}) + (\mathbf{91}) + (\mathbf{125}) + (\mathbf{154}) + (\mathbf{162})$
825	=	$(\mathbf{8}) + 2(\mathbf{27}) + (\mathbf{28}) + (\overline{\mathbf{28}}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{80}) + (\mathbf{81}) + 2(\mathbf{125}) + (\mathbf{162})$

SU(10)	$\to$	SU(4) (S)
10	=	$(\mathbf{10})$
45	=	$(\mathbf{45})$
55	=	$(\mathbf{20'}) + (\mathbf{35})$
99	=	$(\mathbf{15}) + (\mathbf{84})$
120	=	$(\mathbf{50}) + (\mathbf{70})$
210	=	$(\mathbf{35}) + (\mathbf{175})$
220	=	$(\overline{\mathbf{10}}) + (\mathbf{84''}) + (\mathbf{126})$
252	=	$(\mathbf{126}) + (\overline{\mathbf{126}})$
330	=	$(\mathbf{64}) + (\mathbf{126}) + (\mathbf{140''})$
440	=	$(\mathbf{6}) + (\mathbf{64}) + (\mathbf{70}) + (\mathbf{300})$
540	=	$(\mathbf{10}) + (\mathbf{64}) + (\mathbf{70}) + (\overline{\mathbf{126}}) + (\mathbf{270})$
715	=	$(\mathbf{1}) + (\mathbf{84}) + (\mathbf{105}) + (\mathbf{165}) + (\mathbf{360'})$
825	=	$(\mathbf{20'}) + (\mathbf{84}) + (\mathbf{105}) + (\mathbf{256}) + (\mathbf{360'})$

SU(10)	$\to$	SU(5) (S)
10	=	$(\mathbf{10})$
45	=	$(\mathbf{45})$
55	=	$(\overline{\mathbf{5}}) + (\overline{\mathbf{50}})$
99	=	$(\mathbf{24}) + (\mathbf{75})$

Table 10.7: SU(10) Branching Rules (continued)

120	=	$(\mathbf{50}) + (\mathbf{70})$
210	=	$(\overline{\mathbf{35}}) + (\overline{\mathbf{175}})$
220	=	$(\mathbf{45}) + (\mathbf{175''})$
252	=	$(\mathbf{126}) + (\overline{\mathbf{126}})$
330	=	$(\mathbf{5}) + (\mathbf{45}) + (\mathbf{280})$
440	=	$(\mathbf{15}) + (\mathbf{40}) + (\mathbf{175}) + (\mathbf{210})$
540	=	$(\mathbf{10}) + (\mathbf{40}) + (\mathbf{175}) + (\mathbf{315})$
715	=	$(\overline{\mathbf{15}}) + (\overline{\mathbf{210}}) + (\overline{\mathbf{490}})$
825	=	$(\overline{\mathbf{15}}) + (\overline{\mathbf{40}}) + (\overline{\mathbf{210}}) + (\overline{\mathbf{560}})$

SU(10)	→	Sp(4) (S)
10	=	$(\mathbf{10})$
45	=	$(\mathbf{10}) + (\mathbf{35})$
55	=	$(\mathbf{1}) + (\mathbf{5}) + (\mathbf{14}) + (\mathbf{35'})$
99	=	$(\mathbf{5}) + (\mathbf{10}) + (\mathbf{14}) + (\mathbf{35}) + (\mathbf{35'})$
120	=	$(\mathbf{1}) + (\mathbf{5}) + (\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{35'})$
210	=	$(\mathbf{5}) + (\mathbf{10}) + (\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{35'}) + (\mathbf{81})$
220	=	$2(\mathbf{10}) + (\mathbf{35}) + (\mathbf{81}) + (\mathbf{84})$
252	=	$2(\mathbf{10}) + 2(\mathbf{35}) + 2(\mathbf{81})$
330	=	$(\mathbf{5}) + 2(\mathbf{10}) + (\mathbf{14}) + 2(\mathbf{35}) + (\mathbf{35'}) + (\mathbf{81}) + (\mathbf{105})$
440	=	$(\mathbf{1}) + 2(\mathbf{5}) + (\mathbf{10}) + 2(\mathbf{14}) + (\mathbf{30}) + 3(\mathbf{35}) + 2(\mathbf{35'}) + (\mathbf{81}) + (\mathbf{105})$
540	=	$(\mathbf{5}) + 3(\mathbf{10}) + (\mathbf{14}) + 3(\mathbf{35}) + (\mathbf{35'}) + 2(\mathbf{81}) + (\mathbf{84}) + (\mathbf{105})$
715	=	$2(\mathbf{1}) + (\mathbf{5}) + 2(\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + 2(\mathbf{35'}) + (\mathbf{55}) + (\mathbf{105}) + (\mathbf{165}) + (\mathbf{220})$
825	=	$2(\mathbf{1}) + 2(\mathbf{5}) + 3(\mathbf{14}) + (\mathbf{30}) + 2(\mathbf{35}) + 3(\mathbf{35'}) + (\mathbf{55}) + (\mathbf{81}) + 2(\mathbf{105}) + (\mathbf{220})$

SU(10)	→	SO(10) (S)
10	=	$(\mathbf{10})$
45	=	$(\mathbf{45})$
55	=	$(\mathbf{1}) + (\mathbf{54})$
99	=	$(\mathbf{45}) + (\mathbf{54})$
120	=	$(\mathbf{120})$
210	=	$(\mathbf{210})$
220	=	$(\mathbf{10}) + (\mathbf{210'})$
252	=	$(\mathbf{126}) + (\overline{\mathbf{126}})$
330	=	$(\mathbf{10}) + (\mathbf{320})$
440	=	$(\mathbf{120}) + (\mathbf{320})$
540	=	$(\mathbf{10}) + (\mathbf{210'}) + (\mathbf{320})$
715	=	$(\mathbf{1}) + (\mathbf{54}) + (\mathbf{660})$
825	=	$(\mathbf{1}) + (\mathbf{54}) + (\mathbf{770})$

SU(10)	→	Sp(10) (S)
10	=	$(\mathbf{10})$
45	=	$(\mathbf{1}) + (\mathbf{44})$
55	=	$(\mathbf{55})$
99	=	$(\mathbf{44}) + (\mathbf{55})$
120	=	$(\mathbf{10}) + (\mathbf{110})$
210	=	$(\mathbf{1}) + (\mathbf{44}) + (\mathbf{165})$
220	=	$(\mathbf{220})$
252	=	$(\mathbf{10}) + (\mathbf{110}) + (\mathbf{132})$
330	=	$(\mathbf{10}) + (\mathbf{320})$
440	=	$(\mathbf{10}) + (\mathbf{110}) + (\mathbf{320})$

Table 10.7: SU(10) Branching Rules (continued)

$\mathbf{540}$	=	$(\mathbf{220})+(\mathbf{320})$
$\mathbf{715}$	=	$(\mathbf{715})$
$\mathbf{825}$	=	$(\mathbf{1})+(\mathbf{44})+(\mathbf{780})$

SU(10)	→	SU(5) × SU(2) (S)
$\mathbf{10}$	=	$(\mathbf{5},\mathbf{2})$
$\mathbf{45}$	=	$(\mathbf{10},\mathbf{3})+(\mathbf{15},\mathbf{1})$
$\mathbf{55}$	=	$(\mathbf{10},\mathbf{1})+(\mathbf{15},\mathbf{3})$
$\mathbf{99}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{24},\mathbf{1})+(\mathbf{24},\mathbf{3})$
$\mathbf{120}$	=	$(\overline{\mathbf{10}},\mathbf{4})+(\overline{\mathbf{40}},\mathbf{2})$
$\mathbf{210}$	=	$(\overline{\mathbf{5}},\mathbf{5})+(\overline{\mathbf{45}},\mathbf{3})+(\overline{\mathbf{50}},\mathbf{1})$
$\mathbf{220}$	=	$(\overline{\mathbf{35}},\mathbf{4})+(\overline{\mathbf{40}},\mathbf{2})$
$\mathbf{252}$	=	$(\mathbf{1},\mathbf{6})+(\mathbf{24},\mathbf{4})+(\mathbf{75},\mathbf{2})$
$\mathbf{330}$	=	$(\overline{\mathbf{10}},\mathbf{2})+(\overline{\mathbf{35}},\mathbf{2})+(\overline{\mathbf{40}},\mathbf{2})+(\overline{\mathbf{40}},\mathbf{4})$
$\mathbf{440}$	=	$(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{45},\mathbf{2})+(\mathbf{45},\mathbf{4})+(\mathbf{70},\mathbf{2})$
$\mathbf{540}$	=	$(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{45},\mathbf{2})+(\mathbf{70},\mathbf{2})+(\mathbf{70},\mathbf{4})$
$\mathbf{715}$	=	$(\overline{\mathbf{50}},\mathbf{1})+(\overline{\mathbf{70}}',\mathbf{5})+(\overline{\mathbf{105}},\mathbf{3})$
$\mathbf{825}$	=	$(\overline{\mathbf{5}},\mathbf{1})+(\overline{\mathbf{45}},\mathbf{3})+(\overline{\mathbf{50}},\mathbf{1})+(\overline{\mathbf{50}},\mathbf{5})+(\overline{\mathbf{70}}',\mathbf{1})+(\overline{\mathbf{105}},\mathbf{3})$

Table 10.8: SU(11) Branching Rules

SU(11)	→	SO(11) (S)
$\mathbf{11}$	=	$(\mathbf{11})$
$\mathbf{55}$	=	$(\mathbf{55})$
$\mathbf{66}$	=	$(\mathbf{1})+(\mathbf{65})$
$\mathbf{120}$	=	$(\mathbf{55})+(\mathbf{65})$
$\mathbf{165}$	=	$(\mathbf{165})$
$\mathbf{286}$	=	$(\mathbf{11})+(\mathbf{275})$
$\mathbf{330}$	=	$(\mathbf{330})$
$\mathbf{440}$	=	$(\mathbf{11})+(\mathbf{429})$
$\mathbf{462}$	=	$(\mathbf{462})$
$\mathbf{594}$	=	$(\mathbf{165})+(\mathbf{429})$

Table 10.9: SU(15) Branching Rules

SU(15)	→	SU(3) (S)
$\mathbf{15}$	=	$(\mathbf{15})$
$\mathbf{105}$	=	$(\overline{\mathbf{3}})+(\overline{\mathbf{15}})+(\overline{\mathbf{21}})+(\overline{\mathbf{24}})+(\overline{\mathbf{42}})$
$\mathbf{120}$	=	$(\mathbf{6})+(\overline{\mathbf{15}})+(\overline{\mathbf{15}}')+(\overline{\mathbf{24}})+(\overline{\mathbf{60}})$
$\mathbf{224}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+2(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})$
$\mathbf{455}$	=	$(\mathbf{1})+(\mathbf{8})+2(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+(\mathbf{35})+(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})$
$\mathbf{680}$	=	$(\mathbf{1})+(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+2(\mathbf{27})+(\mathbf{28})+2(\mathbf{35})+(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{154})$
$\mathbf{1120}$	=	$4(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+4(\mathbf{35})+3(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{80})+2(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1365}$	=	$3(\overline{\mathbf{6}})+4(\mathbf{15})+3(\mathbf{15}')+(\mathbf{21})+4(\mathbf{24})+4(\mathbf{42})+3(\mathbf{48})+4(\mathbf{60})+(\mathbf{63})+2(\mathbf{90})+2(\mathbf{105})+(\mathbf{120})$

SU(15)	→	SU(3) (S)
$\mathbf{15}$	=	$(\mathbf{15}')$

Table 10.9: SU(15) Branching Rules (continued)

105	=	($\overline{\mathbf{42}}$) + ($\overline{\mathbf{63}}$)
120	=	($\overline{\mathbf{15}'}$) + ($\overline{\mathbf{45}}$) + ($\overline{\mathbf{60}}$)
224	=	(**8**) + (**27**) + (**64**) + (**125**)
455	=	(**10**) + ($\overline{\mathbf{10}}$) + (**55**) + (**64**) + (**81**) + ($\overline{\mathbf{81}}$) + (**154**)
680	=	(**1**) + (**27**) + (**28**) + ($\overline{\mathbf{28}}$) + (**64**) + (**91**) + (**125**) + (**154**) + (**162**)
1120	=	(**8**) + (**27**) + (**35**) + ($\overline{\mathbf{35}}$) + (**64**) + (**80**) + (**81**) + ($\overline{\mathbf{81}}$) + 2(**125**) + (**143**) + (**154**) + (**162**)
1365	=	(**15**) + (**24**) + (**36**) + (**42**) + (**48**) + (ß $rrep$60) + (**63**) + (**90**) + (**105**) + (**120**) + (ß $rrep$165) + (**192**) + (**195**) + (**210**)

10.0.2. SO(N)

Table 10.10: SO(7) Branching Rules

SO(7)	→	Sp(4)×U(1) (R)
7	=	(**1**)(2) + (**1**)(−2) + (**5**)(0)
8	=	(**4**)(1) + (**4**)(−1)
21	=	(**1**)(0) + (**5**)(2) + (**5**)(−2) + (**10**)(0)
27	=	(**1**)(4) + (**1**)(0) + (**1**)(−4) + (**5**)(2) + (**5**)(−2) + (**14**)(0)
35	=	(**5**)(0) + (**10**)(2) + (**10**)(0) + (**10**)(−2)
48	=	(**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**16**)(1) + (**16**)(−1)
77	=	(**1**)(6) + (**1**)(2) + (**1**)(−2) + (**1**)(−6) + (**5**)(4) + (**5**)(0) + (**5**)(−4) + (**14**)(2) + (**14**)(−2) + (**30**)(0)
105	=	(**1**)(2) + (**1**)(−2) + (**5**)(4) + 2(**5**)(0) + (**5**)(−4) + (**10**)(2) + (**10**)(−2) + (**14**)(2) + (**14**)(−2) + (**35**)(0)
112	=	(**4**)(1) + (**4**)(−1) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**20**)(1) + (**20**)(−1)
112′	=	(**16**)(1) + (**16**)(−1) + (**20**)(3) + (**20**)(1) + (**20**)(−1) + (**20**)(−3)
168	=	(**4**)(5) + (**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**4**)(−5) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**40**)(1) + (**40**)(−1)
168′	=	(**1**)(0) + (**5**)(2) + (**5**)(−2) + (**10**)(0) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**35**)(2) + (**35**)(−2) + (**35′**)(0)
182	=	(**1**)(8) + (**1**)(4) + (**1**)(0) + (**1**)(−4) + (**1**)(−8) + (**5**)(6) + (**5**)(2) + (**5**)(−2) + (**5**)(−6) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**30**)(2) + (**30**)(−2) + (**55**)(0)
189	=	(**5**)(2) + (**5**)(−2) + (**10**)(4) + (**10**)(2) + 2(**10**)(0) + (**10**)(−2) + (**10**)(−4) + (**14**)(0) + (**35**)(2) + (**35**)(0) + (**35**)(−2)
294	=	(**14**)(0) + (**35**)(2) + (**35**)(0) + (**35**)(−2) + (**35′**)(4) + (**35′**)(2) + (**35′**)(0) + (**35′**)(−2) + (**35′**)(−4)
330	=	(**1**)(4) + (**1**)(0) + (**1**)(−4) + (**5**)(6) + 2(**5**)(2) + 2(**5**)(−2) + (**5**)(−6) + (**10**)(4) + (**10**)(0) + (**10**)(−4) + (**14**)(4) + 2(**14**)(0) + (**14**)(−4) + (**30**)(2) + (**30**)(−2) + (**35**)(2) + (**35**)(−2) + (**81**)(0)
378	=	(**5**)(0) + (**10**)(2) + (**10**)(0) + (**10**)(−2) + (**14**)(2) + (**14**)(−2) + (**35**)(4) + (**35**)(2) + 2(**35**)(0) + (**35**)(−2) + (**35**)(−4) + (**35′**)(2) + (**35′**)(0) + (**35′**)(−2)
378′	=	(**1**)(10) + (**1**)(6) + (**1**)(2) + (**1**)(−2) + (**1**)(−6) + (**1**)(−10) + (**5**)(8) + (**5**)(4) + (**5**)(0) + (**5**)(−4) + (**5**)(−8) + (**14**)(6) + (**14**)(2) + (**14**)(−2) + (**14**)(−6) + (**30**)(4) + (**30**)(0) + (**30**)(−4) + (**55**)(2) + (**55**)(−2) + (**91**)(0)
448	=	(**4**)(7) + (**4**)(5) + (**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**4**)(−5) + (**4**)(−7) + (**16**)(5) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**16**)(−5) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**80**)(1) + (**80**)(−1)
512	=	(**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**16**)(5) + (**16**)(3) + 2(**16**)(1) + 2(**16**)(−1) + (**16**)(−3) + (**16**)(−5) + (**20**)(3) + (**20**)(1) + (**20**)(−1) + (**20**)(−3) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**64**)(1) + (**64**)(−1)
560	=	(**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**20**)(5) + (**20**)(3) + 2(**20**)(1) + 2(**20**)(−1) + (**20**)(−3) + (**20**)(−5) + (**40**)(1) + (**40**)(−1) + (**64**)(3) + (**64**)(1) + (**64**)(−1) + (**64**)(−3)
616	=	(**5**)(4) + (**5**)(0) + (**5**)(−4) + (**10**)(6) + (**10**)(4) + 2(**10**)(2) + (**10**)(0) + 2(**10**)(−2) + (**10**)(−4) + (**10**)(−6) + (**14**)(2) + (**14**)(−2) + (**30**)(0) + (**35**)(4) + (**35**)(2) + 2(**35**)(0) + (**35**)(−2) + (**35**)(−4) + (**81**)(2) + (**81**)(0) + (**81**)(−2)

672	=	(**40**)(1) + (**40**)(−1) + (**56**)(5) + (**56**)(3) + (**56**)(1) + (**56**)(−1) + (**56**)(−3) + (**56**)(−5) + (**64**)(3) + (**64**)(1) + (**64**)(−1) + (**64**)(−3)
693	=	(**1**)(2) + (**1**)(−2) + (**5**)(4) + 2(**5**)(0) + (**5**)(−4) + (**10**)(2) + (**10**)(−2) + (**14**)(6) + 2(**14**)(2) + 2(**14**)(−2) + (**14**)(−6) + (**30**)(4) + (**30**)(0) + (**30**)(−4) + (**35**)(4) + 2(**35**)(0) + (**35**)(−4) + (**35′**)(2) + (**35′**)(−2) + (**81**)(2) + (**81**)(−2) + (**105**)(0)
714	=	(**1**)(12) + (**1**)(8) + (**1**)(4) + (**1**)(0) + (**1**)(−4) + (**1**)(−8) + (**1**)(−12) + (**5**)(10) + (**5**)(6) + (**5**)(2) + (**5**)(−2) + (**5**)(−6) + (**5**)(−10) + (**14**)(8) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**14**)(−8) + (**30**)(6) + (**30**)(2) + (**30**)(−2) + (**30**)(−6) + (**55**)(4) + (**55**)(0) + (**55**)(−4) + (**91**)(2) + (**91**)(−2) + (**140″**)(0)
720	=	(**4**)(1) + (**4**)(−1) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**20**)(1) + (**20**)(−1) + (**40**)(5) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**40**)(−5) + (**56**)(1) + (**56**)(−1) + (**64**)(3) + (**64**)(1) + (**64**)(−1) + (**64**)(−3)
819	=	(**1**)(6) + (**1**)(2) + (**1**)(−2) + (**1**)(−6) + (**5**)(8) + 2(**5**)(4) + 2(**5**)(0) + 2(**5**)(−4) + (**5**)(−8) + (**10**)(6) + (**10**)(2) + (**10**)(−2) + (**10**)(−6) + (**14**)(6) + 2(**14**)(2) + 2(**14**)(−2) + (**14**)(−6) + (**30**)(4) + 2(**30**)(0) + (**30**)(−4) + (**35**)(4) + (**35**)(0) + (**35**)(−4) + (**55**)(2) + (**55**)(−2) + (**81**)(2) + (**81**)(−2) + (**154**)(0)
825	=	(**1**)(0) + (**5**)(2) + (**5**)(−2) + (**10**)(0) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**30**)(6) + (**30**)(2) + (**30**)(−2) + (**30**)(−6) + (**35**)(2) + (**35**)(−2) + (**35′**)(0) + (**81**)(4) + (**81**)(0) + (**81**)(−4) + (**84**)(0) + (**105**)(2) + (**105**)(−2)
1008	=	(**16**)(1) + (**16**)(−1) + (**20**)(3) + (**20**)(1) + (**20**)(−1) + (**20**)(−3) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**56**)(3) + (**56**)(1) + (**56**)(−1) + (**56**)(−3) + (**64**)(5) + (**64**)(3) + 2(**64**)(1) + 2(**64**)(−1) + (**64**)(−3) + (**64**)(−5)
1008′	=	(**4**)(9) + (**4**)(7) + (**4**)(5) + (**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**4**)(−5) + (**4**)(−7) + (**4**)(−9) + (**16**)(7) + (**16**)(5) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**16**)(−5) + (**16**)(−7) + (**40**)(5) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**40**)(−5) + (**80**)(3) + (**80**)(1) + (**80**)(−1) + (**80**)(−3) + (**140′**)(1) + (**140′**)(−1)
1254	=	(**1**)(14) + (**1**)(10) + (**1**)(6) + (**1**)(2) + (**1**)(−2) + (**1**)(−6) + (**1**)(−10) + (**1**)(−14) + (**5**)(12) + (**5**)(8) + (**5**)(4) + (**5**)(0) + (**5**)(−4) + (**5**)(−8) + (**5**)(−12) + (**14**)(10) + (**14**)(6) + (**14**)(2) + (**14**)(−2) + (**14**)(−6) + (**14**)(−10) + (**30**)(8) + (**30**)(4) + (**30**)(0) + (**30**)(−4) + (**30**)(−8) + (**55**)(6) + (**55**)(2) + (**55**)(−2) + (**55**)(−6) + (**91**)(4) + (**91**)(0) + (**91**)(−4) + (**140″**)(2) + (**140″**)(−2) + (**204**)(0)
1386	=	(**14**)(2) + (**14**)(−2) + (**30**)(0) + (**35**)(4) + (**35**)(2) + 2(**35**)(0) + (**35**)(−2) + (**35**)(−4) + (**35′**)(6) + (**35′**)(4) + 2(**35′**)(2) + 2(**35′**)(0) + 2(**35′**)(−2) + (**35′**)(−4) + (**35′**)(−6) + (**81**)(2) + (**81**)(0) + (**81**)(−2) + (**105**)(4) + (**105**)(2) + (**105**)(0) + (**105**)(−2) + (**105**)(−4)
1386′	=	(**30**)(0) + (**81**)(2) + (**81**)(0) + (**81**)(−2) + (**84**)(6) + (**84**)(4) + (**84**)(2) + (**84**)(0) + (**84**)(−2) + (**84**)(−4) + (**84**)(−6) + (**105**)(4) + (**105**)(2) + (**105**)(0) + (**105**)(−2) + (**105**)(−4)
1512	=	(**4**)(5) + (**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**4**)(−5) + (**16**)(7) + (**16**)(5) + 2(**16**)(3) + 2(**16**)(1) + 2(**16**)(−1) + 2(**16**)(−3) + (**16**)(−5) + (**16**)(−7) + (**20**)(5) + (**20**)(3) + (**20**)(1) + (**20**)(−1) + (**20**)(−3) + (**20**)(−5) + (**40**)(5) + (**40**)(3) + 2(**40**)(1) + 2(**40**)(−1) + (**40**)(−3) + (**40**)(−5) + (**64**)(3) + (**64**)(1) + (**64**)(−1) + (**64**)(−3) + (**80**)(3) + (**80**)(1) + (**80**)(−1) + (**80**)(−3) + (**140**)(1) + (**140**)(−1)
1560	=	(**5**)(6) + (**5**)(2) + (**5**)(−2) + (**5**)(−6) + (**10**)(8) + (**10**)(6) + 2(**10**)(4) + (**10**)(2) + 2(**10**)(0) + (**10**)(−2) + 2(**10**)(−4) + (**10**)(−6) + (**10**)(−8) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**30**)(2) + (**30**)(−2) + (**35**)(6) + (**35**)(4) + 2(**35**)(2) + (**35**)(0) + 2(**35**)(−2) + (**35**)(−4) + (**35**)(−6) + (**55**)(0) + (**81**)(4) + (**81**)(2) + 2(**81**)(0) + (**81**)(−2) + (**81**)(−4) + (**154**)(2) + (**154**)(0) + (**154**)(−2)
1617	=	(**5**)(2) + (**5**)(−2) + (**10**)(4) + (**10**)(2) + 2(**10**)(0) + (**10**)(−2) + (**10**)(−4) + (**14**)(4) + 2(**14**)(0) + (**14**)(−4) + (**30**)(2) + (**30**)(−2) + (**35**)(6) + (**35**)(4) + 3(**35**)(2) + 2(**35**)(0) + 3(**35**)(−2) + (**35**)(−4) + (**35**)(−6) + (**35′**)(4) + (**35′**)(2) + 2(**35′**)(0) + (**35′**)(−2) + (**35′**)(−4) + (**81**)(4) + (**81**)(2) + 2(**81**)(0) + (**81**)(−2) + (**81**)(−4) + (**105**)(2) + (**105**)(0) + (**105**)(−2)
1728	=	(**16**)(5) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**16**)(−5) + (**20**)(7) + (**20**)(5) + 2(**20**)(3) + 2(**20**)(1) + 2(**20**)(−1) + 2(**20**)(−3) + (**20**)(−5) + (**20**)(−7) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**64**)(5) + (**64**)(3) + 2(**64**)(1) + 2(**64**)(−1) + (**64**)(−3) + (**64**)(−5) + (**80**)(1) + (**80**)(−1) + (**140**)(3) + (**140**)(1) + (**140**)(−1) + (**140**)(−3)

Table 10.10: SO(7) Branching Rules (continued)

1750	=	(**1**)(8) + (**1**)(4) + (**1**)(0) + (**1**)(−4) + (**1**)(−8) + (**5**)(10) + 2(**5**)(6) + 2(**5**)(2) + 2(**5**)(−2) + 2(**5**)(−6) + (**5**)(−10) + (**10**)(8) + (**10**)(4) + (**10**)(0) + (**10**)(−4) + (**10**)(−8) + (**14**)(8) + 2(**14**)(4) + 2(**14**)(0) + 2(**14**)(−4) + (**14**)(−8) + (**30**)(6) + 2(**30**)(2) + 2(**30**)(−2) + (**30**)(−6) + (**35**)(6) + (**35**)(2) + (**35**)(−2) + (**35**)(−6) + (**55**)(4) + 2(**55**)(0) + (**55**)(−4) + (**81**)(4) + (**81**)(0) + (**81**)(−4) + (**91**)(2) + (**91**)(−2) + (**154**)(2) + (**154**)(−2) + (**260**)(0)
1911	=	(**1**)(4) + (**1**)(0) + (**1**)(−4) + (**5**)(6) + 2(**5**)(2) + 2(**5**)(−2) + (**5**)(−6) + (**10**)(4) + (**10**)(0) + (**10**)(−4) + (**14**)(8) + 2(**14**)(4) + 3(**14**)(0) + 2(**14**)(−4) + (**14**)(−8) + (**30**)(6) + 2(**30**)(2) + 2(**30**)(−2) + (**30**)(−6) + (**35**)(6) + 2(**35**)(2) + 2(**35**)(−2) + (**35**)(−6) + (**35′**)(4) + (**35′**)(0) + (**35′**)(−4) + (**55**)(4) + (**55**)(0) + (**55**)(−4) + (**81**)(4) + 2(**81**)(0) + (**81**)(−4) + (**105**)(2) + (**105**)(−2) + (**154**)(2) + (**154**)(−2) + (**220**)(0)
2016	=	(**4**)(11) + (**4**)(9) + (**4**)(7) + (**4**)(5) + (**4**)(3) + (**4**)(1) + (**4**)(−1) + (**4**)(−3) + (**4**)(−5) + (**4**)(−7) + (**4**)(−9) + (**4**)(−11) + (**16**)(9) + (**16**)(7) + (**16**)(5) + (**16**)(3) + (**16**)(1) + (**16**)(−1) + (**16**)(−3) + (**16**)(−5) + (**16**)(−7) + (**16**)(−9) + (**40**)(7) + (**40**)(5) + (**40**)(3) + (**40**)(1) + (**40**)(−1) + (**40**)(−3) + (**40**)(−5) + (**40**)(−7) + (**80**)(5) + (**80**)(3) + (**80**)(1) + (**80**)(−1) + (**80**)(−3) + (**80**)(−5) + (**140′**)(3) + (**140′**)(1) + (**140′**)(−1) + (**140′**)(−3) + (**224**)(1) + (**224**)(−1)
2079	=	(**5**)(0) + (**10**)(2) + (**10**)(0) + (**10**)(−2) + (**14**)(2) + (**14**)(−2) + (**30**)(4) + (**30**)(0) + (**30**)(−4) + (**35**)(4) + (**35**)(2) + 2(**35**)(0) + (**35**)(−2) + (**35**)(−4) + (**35′**)(2) + (**35′**)(0) + (**35′**)(−2) + (**81**)(6) + (**81**)(4) + 2(**81**)(2) + (**81**)(0) + 2(**81**)(−2) + (**81**)(−4) + (**81**)(−6) + (**84**)(2) + (**84**)(0) + (**84**)(−2) + (**105**)(4) + (**105**)(2) + 2(**105**)(0) + (**105**)(−2) + (**105**)(−4)
2079′	=	(**1**)(16) + (**1**)(12) + (**1**)(8) + (**1**)(4) + (**1**)(0) + (**1**)(−4) + (**1**)(−8) + (**1**)(−12) + (**1**)(−16) + (**5**)(14) + (**5**)(10) + (**5**)(6) + (**5**)(2) + (**5**)(−2) + (**5**)(−6) + (**5**)(−10) + (**5**)(−14) + (**14**)(12) + (**14**)(8) + (**14**)(4) + (**14**)(0) + (**14**)(−4) + (**14**)(−8) + (**14**)(−12) + (**30**)(10) + (**30**)(6) + (**30**)(2) + (**30**)(−2) + (**30**)(−6) + (**30**) (−10) + (**55**)(8) + (**55**)(4) + (**55**)(0) + (**55**)(−4) + (**55**)(−8) + (**91**)(6) + (**91**)(2) + (**91**)(−2) + (**91**)(−6) + (**140″**)(4) + (**140″**)(0) + (**140″**)(−4) + (**204**)(2) + (**204**)(−2) + (**285**)(0)
2310	=	(**14**)(0) + (**30**)(2) + (**30**)(−2) + (**35**)(2) + (**35**)(0) + (**35**)(−2) + (**35′**)(4) + (**35′**)(2) + (**35′**)(0) + (**35′**)(−2) + (**35′**)(−4) + (**81**)(4) + (**81**)(2) + 2(**81**)(0) + (**81**)(−2) + (**81**)(−4) + (**84**)(4) + (**84**)(2) + (**84**)(0) + (**84**)(−2) + (**84**)(−4) + (**105**)(6) + (**105**)(4) + 2(**105**)(2) + 2(**105**)(0) + 2(**105**)(−2) + (**105**)(−4) + (**105**)(−6)

Table 10.11: SO(8) Branching Rules

SO(8)	→	SU(2)×SU(2)×SU(2)×SU(2) (R)
$\mathbf{8_s}$	=	(**2, 1, 2, 1**) + (**1, 2, 1, 2**)
$\mathbf{8_v}$	=	(**2, 2, 1, 1**) + (**1, 1, 2, 2**)
$\mathbf{8_c}$	=	(**2, 1, 1, 2**) + (**1, 2, 2, 1**)
28	=	(**3, 1, 1, 1**) + (**1, 3, 1, 1**) + (**1, 1, 3, 1**) + (**1, 1, 1, 3**) + (**2, 2, 2, 2**)
$\mathbf{35_v}$	=	(**1, 1, 1, 1**) + (**2, 2, 2, 2**) + (**3, 3, 1, 1**) + (**1, 1, 3, 3**)
$\mathbf{35_c}$	=	(**1, 1, 1, 1**) + (**2, 2, 2, 2**) + (**3, 1, 1, 3**) + (**1, 3, 3, 1**)
$\mathbf{35_s}$	=	(**1, 1, 1, 1**) + (**2, 2, 2, 2**) + (**3, 1, 3, 1**) + (**1, 3, 1, 3**)
$\mathbf{56_s}$	=	(**2, 1, 2, 1**) + (**1, 2, 1, 2**) + (**3, 2, 1, 2**) + (**2, 3, 2, 1**) + (**2, 1, 2, 3**) + (**1, 2, 3, 2**)
$\mathbf{56_v}$	=	(**2, 2, 1, 1**) + (**1, 1, 2, 2**) + (**3, 1, 2, 2**) + (**2, 2, 3, 1**) + (**2, 2, 1, 3**) + (**1, 3, 2, 2**)
$\mathbf{56_c}$	=	(**2, 1, 1, 2**) + (**1, 2, 2, 1**) + (**3, 2, 2, 1**) + (**2, 3, 1, 2**) + (**2, 1, 3, 2**) + (**1, 2, 2, 3**)
$\mathbf{112_s}$	=	(**2, 1, 2, 1**) + (**1, 2, 1, 2**) + (**3, 2, 3, 2**) + (**2, 3, 2, 3**) + (**4, 1, 4, 1**) + (**1, 4, 1, 4**)
$\mathbf{112_v}$	=	(**2, 2, 1, 1**) + (**1, 1, 2, 2**) + (**3, 3, 2, 2**) + (**2, 2, 3, 3**) + (**4, 4, 1, 1**) + (**1, 1, 4, 4**)
$\mathbf{112_c}$	=	(**2, 1, 1, 2**) + (**1, 2, 2, 1**) + (**3, 2, 2, 3**) + (**2, 3, 3, 2**) + (**4, 1, 1, 4**) + (**1, 4, 4, 1**)
$\mathbf{160_s}$	=	(**2, 1, 2, 1**) + (**1, 2, 1, 2**) + (**3, 2, 1, 2**) + (**2, 3, 2, 1**) + (**2, 1, 2, 3**) + (**1, 2, 3, 2**) + (**4, 1, 2, 1**) + (**2, 1, 4, 1**) + (**1, 4, 1, 2**) + (**1, 2, 1, 4**) + (**3, 2, 3, 2**) + (**2, 3, 2, 3**)
$\mathbf{160_v}$	=	(**2, 2, 1, 1**) + (**1, 1, 2, 2**) + (**3, 1, 2, 2**) + (**2, 2, 3, 1**) + (**2, 2, 1, 3**) + (**1, 3, 2, 2**) + (**4, 2, 1, 1**) + (**2, 4, 1, 1**) + (**1, 1, 4, 2**) + (**1, 1, 2, 4**) + (**3, 3, 2, 2**) + (**2, 2, 3, 3**)
$\mathbf{160_c}$	=	(**2, 1, 1, 2**) + (**1, 2, 2, 1**) + (**3, 2, 2, 1**) + (**2, 3, 1, 2**) + (**2, 1, 3, 2**) + (**1, 2, 2, 3**) + (**4, 1, 1, 2**) + (**2, 1, 1, 4**) + (**1, 4, 2, 1**) + (**1, 2, 4, 1**) + (**3, 2, 2, 3**) + (**2, 3, 3, 2**)

Table 10.11: SO(8) Branching Rules (continued)

$\mathbf{224_{vs}}$	=	$\mathbf{(2,1,2,1)} + \mathbf{(1,2,1,2)} + \mathbf{(3,2,1,2)} + \mathbf{(2,3,2,1)} + \mathbf{(2,1,2,3)} + \mathbf{(1,2,3,2)} + \mathbf{(3,2,3,2)} + \mathbf{(2,3,2,3)} + \mathbf{(4,3,2,1)} + \mathbf{(3,4,1,2)} + \mathbf{(2,1,4,3)} + \mathbf{(1,2,3,4)}$
$\mathbf{224_{cs}}$	=	$\mathbf{(2,1,2,1)} + \mathbf{(1,2,1,2)} + \mathbf{(3,2,1,2)} + \mathbf{(2,3,2,1)} + \mathbf{(2,1,2,3)} + \mathbf{(1,2,3,2)} + \mathbf{(3,2,3,2)} + \mathbf{(2,3,2,3)} + \mathbf{(4,1,2,3)} + \mathbf{(3,2,1,4)} + \mathbf{(2,3,4,1)} + \mathbf{(1,4,3,2)}$
$\mathbf{224_{cv}}$	=	$\mathbf{(2,2,1,1)} + \mathbf{(1,1,2,2)} + \mathbf{(3,1,2,2)} + \mathbf{(2,2,3,1)} + \mathbf{(2,2,1,3)} + \mathbf{(1,3,2,2)} + \mathbf{(3,3,2,2)} + \mathbf{(2,2,3,3)} + \mathbf{(4,2,1,3)} + \mathbf{(3,1,2,4)} + \mathbf{(2,4,3,1)} + \mathbf{(1,3,4,2)}$
$\mathbf{224_{sv}}$	=	$\mathbf{(2,2,1,1)} + \mathbf{(1,1,2,2)} + \mathbf{(3,1,2,2)} + \mathbf{(2,2,3,1)} + \mathbf{(2,2,1,3)} + \mathbf{(1,3,2,2)} + \mathbf{(3,3,2,2)} + \mathbf{(2,2,3,3)} + \mathbf{(4,2,3,1)} + \mathbf{(3,1,4,2)} + \mathbf{(2,4,1,3)} + \mathbf{(1,3,2,4)}$
$\mathbf{224_{vc}}$	=	$\mathbf{(2,1,1,2)} + \mathbf{(1,2,2,1)} + \mathbf{(3,2,2,1)} + \mathbf{(2,3,1,2)} + \mathbf{(2,1,3,2)} + \mathbf{(1,2,2,3)} + \mathbf{(3,2,2,3)} + \mathbf{(2,3,3,2)} + \mathbf{(4,3,1,2)} + \mathbf{(3,4,2,1)} + \mathbf{(2,1,3,4)} + \mathbf{(1,2,4,3)}$
$\mathbf{224_{sc}}$	=	$\mathbf{(2,1,1,2)} + \mathbf{(1,2,2,1)} + \mathbf{(3,2,2,1)} + \mathbf{(2,3,1,2)} + \mathbf{(2,1,3,2)} + \mathbf{(1,2,2,3)} + \mathbf{(3,2,2,3)} + \mathbf{(2,3,3,2)} + \mathbf{(4,1,3,2)} + \mathbf{(3,2,4,1)} + \mathbf{(2,3,1,4)} + \mathbf{(1,4,2,3)}$
$\mathbf{294_{v}}$	=	$\mathbf{(1,1,1,1)} + \mathbf{(2,2,2,2)} + \mathbf{(3,3,1,1)} + \mathbf{(1,1,3,3)} + \mathbf{(3,3,3,3)} + \mathbf{(4,4,2,2)} + \mathbf{(2,2,4,4)} + \mathbf{(5,5,1,1)} + \mathbf{(1,1,5,5)}$
$\mathbf{294_{c}}$	=	$\mathbf{(1,1,1,1)} + \mathbf{(2,2,2,2)} + \mathbf{(3,1,1,3)} + \mathbf{(1,3,3,1)} + \mathbf{(3,3,3,3)} + \mathbf{(4,2,2,4)} + \mathbf{(2,4,4,2)} + \mathbf{(5,1,1,5)} + \mathbf{(1,5,5,1)}$
$\mathbf{294_{s}}$	=	$\mathbf{(1,1,1,1)} + \mathbf{(2,2,2,2)} + \mathbf{(3,1,3,1)} + \mathbf{(1,3,1,3)} + \mathbf{(3,3,3,3)} + \mathbf{(4,2,4,2)} + \mathbf{(2,4,2,4)} + \mathbf{(5,1,5,1)} + \mathbf{(1,5,1,5)}$
$\mathbf{300}$	=	$\mathbf{(1,1,1,1)} + \mathbf{(2,2,2,2)} + \mathbf{(3,3,1,1)} + \mathbf{(3,1,3,1)} + \mathbf{(3,1,1,3)} + \mathbf{(1,3,3,1)} + \mathbf{(1,3,1,3)} + \mathbf{(1,1,3,3)} + \mathbf{(5,1,1,1)} + \mathbf{(1,5,1,1)} + \mathbf{(1,1,5,1)} + \mathbf{(1,1,1,5)} + \mathbf{(4,2,2,2)} + \mathbf{(2,4,2,2)} + \mathbf{(2,2,4,2)} + \mathbf{(2,2,2,4)} + \mathbf{(3,3,3,3)}$
$\mathbf{350}$	=	$\mathbf{(3,1,1,1)} + \mathbf{(1,3,1,1)} + \mathbf{(1,1,3,1)} + \mathbf{(1,1,1,3)} + 3\mathbf{(2,2,2,2)} + \mathbf{(3,3,1,1)} + \mathbf{(3,1,3,1)} + \mathbf{(3,1,1,3)} + \mathbf{(1,3,3,1)} + \mathbf{(1,3,1,3)} + \mathbf{(1,1,3,3)} + \mathbf{(3,3,3,1)} + \mathbf{(3,3,1,3)} + \mathbf{(3,1,3,3)} + \mathbf{(1,3,3,3)} + \mathbf{(4,2,2,2)} + \mathbf{(2,4,2,2)} + \mathbf{(2,2,4,2)} + \mathbf{(2,2,2,4)}$
$\mathbf{567_{v}}$	=	$\mathbf{(3,1,1,1)} + \mathbf{(1,3,1,1)} + \mathbf{(1,1,3,1)} + \mathbf{(1,1,1,3)} + 2\mathbf{(2,2,2,2)} + \mathbf{(3,3,1,1)} + \mathbf{(1,1,3,3)} + \mathbf{(3,3,3,1)} + \mathbf{(3,3,1,3)} + \mathbf{(3,1,3,3)} + \mathbf{(1,3,3,3)} + \mathbf{(4,2,2,2)} + \mathbf{(2,4,2,2)} + \mathbf{(2,2,4,2)} + \mathbf{(2,2,2,4)} + \mathbf{(5,3,1,1)} + \mathbf{(3,5,1,1)} + \mathbf{(1,1,5,3)} + \mathbf{(1,1,3,5)} + \mathbf{(3,3,3,3)} + \mathbf{(4,4,2,2)} + \mathbf{(2,2,4,4)}$
$\mathbf{567_{c}}$	=	$\mathbf{(3,1,1,1)} + \mathbf{(1,3,1,1)} + \mathbf{(1,1,3,1)} + \mathbf{(1,1,1,3)} + 2\mathbf{(2,2,2,2)} + \mathbf{(3,1,1,3)} + \mathbf{(1,3,3,1)} + \mathbf{(3,3,3,1)} + \mathbf{(3,3,1,3)} + \mathbf{(3,1,3,3)} + \mathbf{(1,3,3,3)} + \mathbf{(4,2,2,2)} + \mathbf{(2,4,2,2)} + \mathbf{(2,2,4,2)} + \mathbf{(2,2,2,4)} + \mathbf{(5,1,1,3)} + \mathbf{(3,1,1,5)} + \mathbf{(1,5,3,1)} + \mathbf{(1,3,5,1)} + \mathbf{(3,3,3,3)} + \mathbf{(4,2,2,4)} + \mathbf{(2,4,4,2)}$
$\mathbf{567_{s}}$	=	$\mathbf{(3,1,1,1)} + \mathbf{(1,3,1,1)} + \mathbf{(1,1,3,1)} + \mathbf{(1,1,1,3)} + 2\mathbf{(2,2,2,2)} + \mathbf{(3,1,3,1)} + \mathbf{(1,3,1,3)} + \mathbf{(3,3,3,1)} + \mathbf{(3,3,1,3)} + \mathbf{(3,1,3,3)} + \mathbf{(1,3,3,3)} + \mathbf{(4,2,2,2)} + \mathbf{(2,4,2,2)} + \mathbf{(2,2,4,2)} + \mathbf{(2,2,2,4)} + \mathbf{(5,1,3,1)} + \mathbf{(3,1,5,1)} + \mathbf{(1,5,1,3)} + \mathbf{(1,3,1,5)} + \mathbf{(3,3,3,3)} + \mathbf{(4,2,4,2)} + \mathbf{(2,4,2,4)}$
$\mathbf{672_{vc}}$	=	$\mathbf{(2,1,2,1)} + \mathbf{(1,2,1,2)} + \mathbf{(3,2,1,2)} + \mathbf{(2,3,2,1)} + \mathbf{(2,1,2,3)} + \mathbf{(1,2,3,2)} + \mathbf{(3,2,3,2)} + \mathbf{(2,3,2,3)} + \mathbf{(4,3,2,1)} + \mathbf{(3,4,1,2)} + \mathbf{(2,1,4,3)} + \mathbf{(1,2,3,4)} + \mathbf{(4,3,2,3)} + \mathbf{(3,4,3,2)} + \mathbf{(3,2,3,4)} + \mathbf{(2,3,4,3)} + \mathbf{(5,4,1,2)} + \mathbf{(4,5,2,1)} + \mathbf{(2,1,4,5)} + \mathbf{(1,2,5,4)}$
$\mathbf{672_{cv}}$	=	$\mathbf{(2,1,2,1)} + \mathbf{(1,2,1,2)} + \mathbf{(3,2,1,2)} + \mathbf{(2,3,2,1)} + \mathbf{(2,1,2,3)} + \mathbf{(1,2,3,2)} + \mathbf{(3,2,3,2)} + \mathbf{(2,3,2,3)} + \mathbf{(4,1,2,3)} + \mathbf{(3,2,1,4)} + \mathbf{(2,3,4,1)} + \mathbf{(1,4,3,2)} + \mathbf{(4,3,2,3)} + \mathbf{(3,4,3,2)} + \mathbf{(3,2,3,4)} + \mathbf{(2,3,4,3)} + \mathbf{(5,2,1,4)} + \mathbf{(4,1,2,5)} + \mathbf{(2,5,4,1)} + \mathbf{(1,4,5,2)}$
$\mathbf{672_{cs}}$	=	$\mathbf{(2,2,1,1)} + \mathbf{(1,1,2,2)} + \mathbf{(3,1,2,2)} + \mathbf{(2,2,3,1)} + \mathbf{(2,2,1,3)} + \mathbf{(1,3,2,2)} + \mathbf{(3,3,2,2)} + \mathbf{(2,2,3,3)} + \mathbf{(4,2,1,3)} + \mathbf{(3,1,2,4)} + \mathbf{(2,4,3,1)} + \mathbf{(1,3,4,2)} + \mathbf{(4,2,3,3)} + \mathbf{(3,3,4,2)} + \mathbf{(3,3,2,4)} + \mathbf{(2,4,3,3)} + \mathbf{(5,1,2,4)} + \mathbf{(4,2,1,5)} + \mathbf{(2,4,5,1)} + \mathbf{(1,5,4,2)}$
$\mathbf{672_{sc}}$	=	$\mathbf{(2,2,1,1)} + \mathbf{(1,1,2,2)} + \mathbf{(3,1,2,2)} + \mathbf{(2,2,3,1)} + \mathbf{(2,2,1,3)} + \mathbf{(1,3,2,2)} + \mathbf{(3,3,2,2)} + \mathbf{(2,2,3,3)} + \mathbf{(4,2,3,1)} + \mathbf{(3,1,4,2)} + \mathbf{(2,4,1,3)} + \mathbf{(1,3,2,4)} + \mathbf{(4,2,3,3)} + \mathbf{(3,3,4,2)} + \mathbf{(3,3,2,4)} + \mathbf{(2,4,3,3)} + \mathbf{(5,1,4,2)} + \mathbf{(4,2,5,1)} + \mathbf{(2,4,1,5)} + \mathbf{(1,5,2,4)}$
$\mathbf{672_{vs}}$	=	$\mathbf{(2,1,1,2)} + \mathbf{(1,2,2,1)} + \mathbf{(3,2,2,1)} + \mathbf{(2,3,1,2)} + \mathbf{(2,1,3,2)} + \mathbf{(1,2,2,3)} + \mathbf{(3,2,2,3)} + \mathbf{(2,3,3,2)} + \mathbf{(4,3,1,2)} + \mathbf{(3,4,2,1)} + \mathbf{(2,1,3,4)} + \mathbf{(1,2,4,3)} + \mathbf{(4,3,3,2)} + \mathbf{(3,4,2,3)} + \mathbf{(3,2,4,3)} + \mathbf{(2,3,3,4)} + \mathbf{(5,4,2,1)} + \mathbf{(4,5,1,2)} + \mathbf{(2,1,5,4)} + \mathbf{(1,2,4,5)}$

Table 10.11: SO(8) Branching Rules (continued)

$\mathbf{672_{sv}}$	=	(2,1,1,2) + (1,2,2,1) + (3,2,2,1) + (2,3,1,2) + (2,1,3,2) + (1,2,2,3) + (3,2,2,3) + (2,3,3,2) + (4,1,3,2) + (3,2,4,1) + (2,3,1,4) + (1,4,2,3) + (4,3,3,2) + (3,4,2,3) + (3,2,4,3) + (2,3,3,4) + (5,2,4,1) + (4,1,5,2) + (2,5,1,4) + (1,4,2,5)
$\mathbf{672'_{s}}$	=	(2,1,2,1) + (1,2,1,2) + (3,2,3,2) + (2,3,2,3) + (4,1,4,1) + (1,4,1,4) + (4,3,4,3) + (3,4,3,4) + (5,2,5,2) + (2,5,2,5) + (6,1,6,1) + (1,6,1,6)
$\mathbf{672'_{v}}$	=	(2,2,1,1) + (1,1,2,2) + (3,3,2,2) + (2,2,3,3) + (4,4,1,1) + (1,1,4,4) + (4,4,3,3) + (3,3,4,4) + (5,5,2,2) + (2,2,5,5) + (6,6,1,1) + (1,1,6,6)
$\mathbf{672'_{c}}$	=	(2,1,1,2) + (1,2,2,1) + (3,2,2,3) + (2,3,3,2) + (4,1,1,4) + (1,4,4,1) + (4,3,3,4) + (3,4,4,3) + (5,2,2,5) + (2,5,5,2) + (6,1,1,6) + (1,6,6,1)
$\mathbf{840_{s}}$	=	(2,1,2,1) + (1,2,1,2) + 2(3,2,1,2) + 2(2,3,2,1) + 2(2,1,2,3) + 2(1,2,3,2) + (4,1,2,1) + (2,1,4,1) + (1,4,1,2) + (1,2,1,4) + 2(3,2,3,2) + 2(2,3,2,3) + (4,3,2,1) + (4,1,2,3) + (3,4,1,2) + (3,2,1,4) + (2,3,4,1) + (2,1,4,3) + (1,4,3,2) + (1,2,3,4) + (5,2,1,2) + (2,5,2,1) + (2,1,2,5) + (1,2,5,2) + (4,3,2,3) + (3,4,3,2) + (3,2,3,4) + (2,3,4,3)
$\mathbf{840_{v}}$	=	(2,2,1,1) + (1,1,2,2) + 2(3,1,2,2) + 2(2,2,3,1) + 2(2,2,1,3) + 2(1,3,2,2) + (4,2,1,1) + (2,4,1,1) + (1,1,4,2) + (1,1,2,4) + 2(3,3,2,2) + 2(2,2,3,3) + (4,2,3,1) + (4,2,1,3) + (3,1,4,2) + (3,1,2,4) + (2,4,3,1) + (2,4,1,3) + (1,3,4,2) + (1,3,2,4) + (5,1,2,2) + (2,2,5,1) + (2,2,1,5) + (1,5,2,2) + (4,2,3,3) + (3,3,4,2) + (3,3,2,4) + (2,4,3,3)
$\mathbf{840_{c}}$	=	(2,1,1,2) + (1,2,2,1) + 2(3,2,2,1) + 2(2,3,1,2) + 2(2,1,3,2) + 2(1,2,2,3) + (4,1,1,2) + (2,1,1,4) + (1,4,2,1) + (1,2,4,1) + 2(3,2,2,3) + 2(2,3,3,2) + (4,3,1,2) + (4,1,3,2) + (3,4,2,1) + (3,2,4,1) + (2,3,1,4) + (2,1,3,4) + (1,4,2,3) + (1,2,4,3) + (5,2,2,1) + (2,5,1,2) + (2,1,5,2) + (1,2,2,5) + (4,3,3,2) + (3,4,2,3) + (3,2,4,3) + (2,3,3,4)
$\mathbf{840'_{s}}$	=	(1,1,1,1) + 2(2,2,2,2) + (3,3,1,1) + (3,1,3,1) + (3,1,1,3) + (1,3,3,1) + (1,3,1,3) + (1,1,3,3) + (3,3,3,1) + (3,3,1,3) + (3,1,3,3) + (1,3,3,3) + (4,2,2,2) + (2,4,2,2) + (2,2,4,2) + (2,2,2,4) + (3,3,3,3) + (4,4,2,2) + (4,2,2,4) + (2,4,4,2) + (2,2,4,4) + (5,3,1,3) + (3,5,3,1) + (3,1,3,5) + (1,3,5,3)
$\mathbf{840'_{c}}$	=	(1,1,1,1) + 2(2,2,2,2) + (3,3,1,1) + (3,1,3,1) + (3,1,1,3) + (1,3,3,1) + (1,3,1,3) + (1,1,3,3) + (3,3,3,1) + (3,3,1,3) + (3,1,3,3) + (1,3,3,3) + (4,2,2,2) + (2,4,2,2) + (2,2,4,2) + (2,2,2,4) + (3,3,3,3) + (4,4,2,2) + (4,2,4,2) + (2,4,2,4) + (2,2,4,4) + (5,3,3,1) + (3,5,1,3) + (3,1,5,3) + (1,3,3,5)
$\mathbf{840'_{v}}$	=	(1,1,1,1) + 2(2,2,2,2) + (3,3,1,1) + (3,1,3,1) + (3,1,1,3) + (1,3,3,1) + (1,3,1,3) + (1,1,3,3) + (3,3,3,1) + (3,3,1,3) + (3,1,3,3) + (1,3,3,3) + (4,2,2,2) + (2,4,2,2) + (2,2,4,2) + (2,2,2,4) + (3,3,3,3) + (4,2,4,2) + (4,2,2,4) + (2,4,4,2) + (2,4,2,4) + (5,1,3,3) + (3,3,5,1) + (3,3,1,5) + (1,5,3,3)
$\mathbf{1296_{s}}$	=	(2,1,2,1) + (1,2,1,2) + 2(3,2,1,2) + 2(2,3,2,1) + 2(2,1,2,3) + 2(1,2,3,2) + (4,1,2,1) + (2,1,4,1) + (1,4,1,2) + (1,2,1,4) + 3(3,2,3,2) + 3(2,3,2,3) + (4,3,2,1) + (4,1,2,3) + (3,4,1,2) + (3,2,1,4) + (2,3,4,1) + (2,1,4,3) + (1,4,3,2) + (1,2,3,4) + (4,1,4,1) + (1,4,1,4) + (4,3,2,3) + (3,4,3,2) + (3,2,3,4) + (2,3,4,3) + (4,3,4,1) + (4,1,4,3) + (3,4,1,4) + (1,4,3,4) + (5,2,3,2) + (3,2,5,2) + (2,5,2,3) + (2,3,2,5)
$\mathbf{1296_{v}}$	=	(2,2,1,1) + (1,1,2,2) + 2(3,1,2,2) + 2(2,2,3,1) + 2(2,2,1,3) + 2(1,3,2,2) + (4,2,1,1) + (2,4,1,1) + (1,1,4,2) + (1,1,2,4) + 3(3,3,2,2) + 3(2,2,3,3) + (4,2,3,1) + (4,2,1,3) + (3,1,4,2) + (3,1,2,4) + (2,4,3,1) + (2,4,1,3) + (1,3,4,2) + (1,3,2,4) + (4,4,1,1) + (1,1,4,4) + (4,2,3,3) + (3,3,4,2) + (3,3,2,4) + (2,4,3,3) + (4,4,3,1) + (4,4,1,3) + (3,1,4,4) + (1,3,4,4) + (5,3,2,2) + (3,5,2,2) + (2,2,5,3) + (2,2,3,5)
$\mathbf{1296_{c}}$	=	(2,1,1,2) + (1,2,2,1) + 2(3,2,2,1) + 2(2,3,1,2) + 2(2,1,3,2) + 2(1,2,2,3) + (4,1,1,2) + (2,1,1,4) + (1,4,2,1) + (1,2,4,1) + 3(3,2,2,3) + 3(2,3,3,2) + (4,3,1,2) + (4,1,3,2) + (3,4,2,1) + (3,2,4,1) + (2,3,1,4) + (2,1,3,4) + (1,4,2,3) + (1,2,4,3) + (4,1,1,4) + (1,4,4,1) + (4,3,3,2) + (3,4,2,3) + (3,2,4,3) + (2,3,3,4) + (4,3,1,4) + (4,1,3,4) + (3,4,4,1) + (1,4,4,3) + (5,2,2,3) + (3,2,2,5) + (2,5,3,2) + (2,3,5,2)
$\mathbf{1386_{v}}$	=	(1,1,1,1) + (2,2,2,2) + (3,3,1,1) + (1,1,3,3) + (3,3,3,3) + (4,4,2,2) + (2,2,4,4) + (5,5,1,1) + (1,1,5,5) + (4,4,4,4) + (5,5,3,3) + (3,3,5,5) + (6,6,2,2) + (2,2,6,6) + (7,7,1,1) + (1,1,7,7)

Table 10.11: SO(8) Branching Rules (continued)

$\mathbf{1386_c}$	=	(1,1,1,1) + (2,2,2,2) + (3,1,1,3) + (1,3,3,1) + (3,3,3,3) + (4,2,2,4) + (2,4,4,2) + (5,1,ß *rrep*1,5) + (1,5,5,1) + (4,4,4,4) + (5,3,3,ß *rrep*5) + (3,5,5,3) + (6,2,2,6) + (2,6,6,2) + (7,1,1,7) + (1,7,7,1)
$\mathbf{1386_s}$	=	(1,1,1,1) + (2,2,2,2) + (3,1,3,1) + (1,3,1,3) + (3,3,3,3) + (4,2,4,2) + (2,4,2,4) + (5,1,5,1) + (1,5,1,5) + (4,4,4,4) + (5,3,5,3) + (3,5,3,5) + (6,2,6,2) + (2,6,2,6) + (7,1,7,1) + (1,7,1,7)
$\mathbf{1400_s}$	=	(2,1,2,1) + (1,2,1,2) + (3,2,1,2) + (2,3,2,1) + (2,1,2,3) + (1,2,3,2) + (4,1,2,1) + (2,1,4,1) + (1,4,1,2) + (1,2,1,4) + 2(3,2,3,2) + 2(2,3,2,3) + (4,3,2,1) + (4,1,2,3) + (3,4,1,2) + (3,2,1,4) + (2,3,4,1) + (2,1,4,3) + (1,4,3,2) + (1,2,3,4) + (4,1,4,1) + (1,4,1,4) + (5,2,1,2) + (2,5,2,1) + (2,1,2,5) + (1,2,5,2) + (6,1,2,1) + (2,1,6,1) + (1,6,1,2) + (1,2,1,6) + (4,3,2,3) + (3,4,3,2) + (3,2,3,4) + (2,3,4,3) + (5,2,3,2) + (3,2,5,2) + (2,5,2,3) + (2,3,2,5) + (4,3,4,3) + (3,4,3,4)
$\mathbf{1400_v}$	=	(2,2,1,1) + (1,1,2,2) + (3,1,2,2) + (2,2,3,1) + (2,2,1,3) + (1,3,2,2) + (4,2,1,1) + (2,4,1,1) + (1,1,4,2) + (1,1,2,4) + 2(3,3,2,2) + 2(2,2,3,3) + (4,2,3,1) + (4,2,1,3) + (3,1,4,2) + (3,1,2,4) + (2,4,3,1) + (2,4,1,3) + (1,3,4,2) + (1,3,2,4) + (4,4,1,1) + (1,1,4,4) + (5,1,2,2) + (2,2,5,1) + (2,2,1,5) + (1,5,2,2) + (6,2,1,1) + (2,6,1,1) + (1,1,6,2) + (1,1,2,6) + (4,2,3,3) + (3,3,4,2) + (3,3,2,4) + (2,4,3,3) + (5,3,2,2) + (3,5,2,2) + (2,2,5,3) + (2,2,3,5) + (4,4,3,3) + (3,3,4,4)
$\mathbf{1400_c}$	=	(2,1,1,2) + (1,2,2,1) + (3,2,2,1) + (2,3,1,2) + (2,1,3,2) + (1,2,2,3) + (4,1,1,2) + (2,1,1,4) + (1,4,2,1) + (1,2,4,1) + 2(3,2,2,3) + 2(2,3,3,2) + (4,3,1,2) + (4,1,3,2) + (3,4,2,1) + (3,2,4,1) + (2,3,1,4) + (2,1,3,4) + (1,4,2,3) + (1,2,4,3) + (4,1,1,4) + (1,4,4,1) + (5,2,2,1) + (2,5,1,2) + (2,1,5,2) + (1,2,2,5) + (6,1,1,2) + (2,1,1,6) + (1,6,2,1) + (1,2,6,1) + (4,3,3,2) + (3,4,2,3) + (3,2,4,3) + (2,3,3,4) + (5,2,2,3) + (3,2,2,5) + (2,5,3,2) + (2,3,5,2) + (4,3,3,4) + (3,4,4,3)

SO(8)	→	SU(3) (S)
$\mathbf{8_s}$	=	(8)
$\mathbf{8_v}$	=	(8)
$\mathbf{8_c}$	=	(8)
$\mathbf{28}$	=	$(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}})$
$\mathbf{35_v}$	=	(8) + (27)
$\mathbf{35_c}$	=	(8) + (27)
$\mathbf{35_s}$	=	(8) + (27)
$\mathbf{56_s}$	=	$(\mathbf{1}) + (\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27})$
$\mathbf{56_v}$	=	$(\mathbf{1}) + (\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27})$
$\mathbf{56_c}$	=	$(\mathbf{1}) + (\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27})$
$\mathbf{112_s}$	=	$(\mathbf{1}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27}) + (\mathbf{64})$
$\mathbf{112_v}$	=	$(\mathbf{1}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27}) + (\mathbf{64})$
$\mathbf{112_c}$	=	$(\mathbf{1}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + (\mathbf{27}) + (\mathbf{64})$
$\mathbf{160_s}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}})$
$\mathbf{160_v}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}})$
$\mathbf{160_c}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}})$
$\mathbf{224_{vs}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{224_{cs}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{224_{cv}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{224_{sv}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{224_{vc}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{224_{sc}}$	=	$2(\mathbf{8}) + (\mathbf{10}) + (\overline{\mathbf{10}}) + 2(\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64})$
$\mathbf{294_v}$	=	$(\mathbf{8}) + (\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{125})$
$\mathbf{294_c}$	=	$(\mathbf{8}) + (\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{125})$
$\mathbf{294_s}$	=	$(\mathbf{8}) + (\mathbf{27}) + (\mathbf{35}) + (\overline{\mathbf{35}}) + (\mathbf{64}) + (\mathbf{125})$

Table 10.11: SO(8) Branching Rules (continued)

$\mathbf{300}$	=	$(\mathbf{1})+(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})$
$\mathbf{350}$	=	$(\mathbf{1})+3(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+(\mathbf{64})$
$\mathbf{567_v}$	=	$2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{567_c}$	=	$2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{567_s}$	=	$2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{672_{vc}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672_{cv}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672_{cs}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672_{sc}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672_{vs}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672_{sv}}$	=	$2(\mathbf{8})+(\mathbf{10})+(\overline{\mathbf{10}})+3(\mathbf{27})+2(\mathbf{35})+2(\overline{\mathbf{35}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{672'_s}$	=	$(\mathbf{8})+(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{216})$
$\mathbf{672'_v}$	=	$(\mathbf{8})+(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{216})$
$\mathbf{672'_c}$	=	$(\mathbf{8})+(\mathbf{27})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{216})$
$\mathbf{840_s}$	=	$(\mathbf{1})+3(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+3(\mathbf{35})+3(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{840_v}$	=	$(\mathbf{1})+3(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+3(\mathbf{35})+3(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{840_c}$	=	$(\mathbf{1})+3(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+3(\mathbf{35})+3(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})$
$\mathbf{840'_s}$	=	$(\mathbf{1})+2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+4(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{35})+2(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{840'_c}$	=	$(\mathbf{1})+2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+4(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{35})+2(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{840'_v}$	=	$(\mathbf{1})+2(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+4(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{35})+2(\overline{\mathbf{35}})+3(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1296_s}$	=	$(\mathbf{1})+4(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+6(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1296_v}$	=	$(\mathbf{1})+4(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+6(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1296_c}$	=	$(\mathbf{1})+4(\mathbf{8})+3(\mathbf{10})+3(\overline{\mathbf{10}})+6(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1386_v}$	=	$(\mathbf{1})+(\mathbf{10})+(\overline{\mathbf{10}})+(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{154})+(\overline{\mathbf{154}})+(\mathbf{216})+(\mathbf{343})$
$\mathbf{1386_c}$	=	$(\mathbf{1})+(\mathbf{10})+(\overline{\mathbf{10}})+(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{154})+(\overline{\mathbf{154}})+(\mathbf{216})+(\mathbf{343})$
$\mathbf{1386_s}$	=	$(\mathbf{1})+(\mathbf{10})+(\overline{\mathbf{10}})+(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+2(\mathbf{64})+(\mathbf{81})+(\overline{\mathbf{81}})+(\mathbf{125})+(\mathbf{154})+(\overline{\mathbf{154}})+(\mathbf{216})+(\mathbf{343})$
$\mathbf{1400_s}$	=	$3(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+(\mathbf{80})+(\overline{\mathbf{80}})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1400_v}$	=	$3(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+(\mathbf{80})+(\overline{\mathbf{80}})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$
$\mathbf{1400_c}$	=	$3(\mathbf{8})+2(\mathbf{10})+2(\overline{\mathbf{10}})+5(\mathbf{27})+(\mathbf{28})+(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+4(\mathbf{64})+(\mathbf{80})+(\overline{\mathbf{80}})+2(\mathbf{81})+2(\overline{\mathbf{81}})+(\mathbf{125})$

SO(8)	→	Sp(4)×SU(2) (S)
$\mathbf{8_s}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{1})$
$\mathbf{8_v}$	=	$(\mathbf{4},\mathbf{2})$
$\mathbf{8_c}$	=	$(\mathbf{4},\mathbf{2})$
$\mathbf{28}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{3})+(\mathbf{10},\mathbf{1})$
$\mathbf{35_v}$	=	$(\mathbf{5},\mathbf{1})+(\mathbf{10},\mathbf{3})$
$\mathbf{35_c}$	=	$(\mathbf{5},\mathbf{1})+(\mathbf{10},\mathbf{3})$
$\mathbf{35_s}$	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{5},\mathbf{3})+(\mathbf{14},\mathbf{1})$
$\mathbf{56_s}$	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{3})+(\mathbf{10},\mathbf{1})+(\mathbf{10},\mathbf{3})$
$\mathbf{56_v}$	=	$(\mathbf{4},\mathbf{2})+(\mathbf{4},\mathbf{4})+(\mathbf{16},\mathbf{2})$
$\mathbf{56_c}$	=	$(\mathbf{4},\mathbf{2})+(\mathbf{4},\mathbf{4})+(\mathbf{16},\mathbf{2})$
$\mathbf{112_s}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{7})+(\mathbf{5},\mathbf{5})+(\mathbf{14},\mathbf{3})+(\mathbf{30},\mathbf{1})$
$\mathbf{112_v}$	=	$(\mathbf{16},\mathbf{2})+(\mathbf{20},\mathbf{4})$
$\mathbf{112_c}$	=	$(\mathbf{16},\mathbf{2})+(\mathbf{20},\mathbf{4})$
$\mathbf{160_s}$	=	$(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{5},\mathbf{3})+(\mathbf{5},\mathbf{5})+(\mathbf{10},\mathbf{3})+(\mathbf{14},\mathbf{3})+(\mathbf{35},\mathbf{1})$
$\mathbf{160_v}$	=	$(\mathbf{4},\mathbf{2})+(\mathbf{4},\mathbf{4})+(\mathbf{16},\mathbf{2})+(\mathbf{16},\mathbf{4})+(\mathbf{20},\mathbf{2})$
$\mathbf{160_c}$	=	$(\mathbf{4},\mathbf{2})+(\mathbf{4},\mathbf{4})+(\mathbf{16},\mathbf{2})+(\mathbf{16},\mathbf{4})+(\mathbf{20},\mathbf{2})$
$\mathbf{224_{vs}}$	=	$(\mathbf{5},\mathbf{3})+(\mathbf{10},\mathbf{1})+(\mathbf{10},\mathbf{3})+(\mathbf{14},\mathbf{1})+(\mathbf{10},\mathbf{5})+(\mathbf{35},\mathbf{3})$

Table 10.11: SO(8) Branching Rules (continued)

224_{cs}	=	$(5,3)+(10,1)+(10,3)+(14,1)+(10,5)+(35,3)$
224_{cv}	=	$(4,2)+(16,2)+(16,4)+(20,2)+(20,4)$
224_{sv}	=	$(4,2)+(4,4)+(4,6)+(16,2)+(16,4)+(40,2)$
224_{vc}	=	$(4,2)+(16,2)+(16,4)+(20,2)+(20,4)$
224_{sc}	=	$(4,2)+(4,4)+(4,6)+(16,2)+(16,4)+(40,2)$
294_{v}	=	$(14,1)+(35,3)+(35',5)$
294_{c}	=	$(14,1)+(35,3)+(35',5)$
294_{s}	=	$(1,1)+(1,5)+(5,3)+(1,9)+(5,7)+(14,1)+(14,5)+(30,3)+(55,1)$
300	=	$(1,1)+(1,5)+(5,3)+(5,5)+(10,3)+(14,1)+(14,5)+(35',1)+(35,3)$
350	=	$(1,3)+(5,1)+(5,3)+(5,5)+(10,1)+2(10,3)+(10,5)+(14,3)+(35,1)+(35,3)$
567_{v}	=	$(5,3)+(10,1)+(10,3)+(10,5)+(14,3)+(35,1)+(35,3)+(35',3)+(35,5)$
567_{c}	=	$(5,3)+(10,1)+(10,3)+(10,5)+(14,3)+(35,1)+(35,3)+(35',3)+(35,5)$
567_{s}	=	$(1,3)+(1,5)+2(5,3)+(1,7)+(5,5)+(10,1)+(5,7)+(14,1)+(10,5)+(14,3)+(14,5)+(30,3)+(35,3)+(81,1)$
672_{vc}	=	$(5,1)+(10,3)+(14,3)+(35,1)+(35,3)+(35',3)+(35,5)+(35',5)$
672_{cv}	=	$(5,1)+(10,3)+(14,3)+(35,1)+(35,3)+(35',3)+(35,5)+(35',5)$
672_{cs}	=	$(16,2)+(16,4)+(20,2)+(20,4)+(20,6)+(40,2)+(64,4)$
672_{sc}	=	$(4,2)+(4,4)+(4,6)+(4,8)+(16,2)+(16,4)+(16,6)+(40,2)+(40,4)+(80,2)$
672_{vs}	=	$(16,2)+(16,4)+(20,2)+(20,4)+(20,6)+(40,2)+(64,4)$
672_{sv}	=	$(4,2)+(4,4)+(4,6)+(4,8)+(16,2)+(16,4)+(16,6)+(40,2)+(40,4)+(80,2)$
$672'_{s}$	=	$(1,3)+(5,1)+(1,7)+(5,5)+(1,11)+(5,9)+(14,3)+(14,7)+(30,1)+(30,5)+(55,3)+(91,1)$
$672'_{v}$	=	$(40,2)+(56,6)+(64,4)$
$672'_{c}$	=	$(40,2)+(56,6)+(64,4)$
840_{s}	=	$(1,3)+(5,1)+(5,3)+(5,5)+(10,1)+2(10,3)+(10,5)+(14,3)+(14,5)+(35,1)+(35',1)+2(35,3)+(35',3)+(35,5)$
840_{v}	=	$(4,2)+2(4,4)+(4,6)+2(16,2)+2(16,4)+(20,2)+(16,6)+(20,4)+(40,2)+(40,4)+(64,2)$
840_{c}	=	$(4,2)+2(4,4)+(4,6)+2(16,2)+2(16,4)+(20,2)+(16,6)+(20,4)+(40,2)+(40,4)+(64,2)$
$840'_{s}$	=	$(1,1)+(5,3)+(10,1)+(10,3)+(14,1)+(14,5)+(35',1)+2(35,3)+(35',3)+(35,5)+(35',5)$
$840'_{c}$	=	$(5,1)+(5,5)+2(10,3)+(10,5)+(14,3)+(10,7)+(30,1)+(35,1)+(35,3)+(35,5)+(81,3)$
$840'_{v}$	=	$(5,1)+(5,5)+2(10,3)+(10,5)+(14,3)+(10,7)+(30,1)+(35,1)+(35,3)+(35,5)+(81,3)$
1296_{s}	=	$(1,1)+(1,5)+2(5,3)+(5,5)+(10,1)+(5,7)+2(10,3)+(14,1)+2(10,5)+(14,3)+(10,7)+(14,5)+(30,3)+(35,1)+2(35,3)+(35,5)+(81,1)+(81,3)$
1296_{v}	=	$(4,2)+(4,4)+2(16,2)+2(16,4)+2(20,2)+(16,6)+2(20,4)+(20,6)+(40,2)+(40,4)+(64,2)+(64,4)$
1296_{c}	=	$(4,2)+(4,4)+2(16,2)+2(16,4)+2(20,2)+(16,6)+2(20,4)+(20,6)+(40,2)+(40,4)+(64,2)+(64,4)$
1386_{v}	=	$(30,1)+(81,3)+(84,7)+(105,5)$
1386_{c}	=	$(30,1)+(81,3)+(84,7)+(105,5)$
1386_{s}	=	$(1,1)+(1,5)+(5,3)+(1,9)+(5,7)+(1,13)+(14,1)+(5,11)+(14,5)+(14,9)+(30,3)+(30,7)+(55,1)+(55,5)+(91,3)+(140'',1)$
1400_{s}	=	$(1,3)+(5,1)+(1,5)+(5,3)+(1,7)+2(5,5)+(5,7)+(10,3)+(10,5)+2(14,3)+(14,5)+(14,7)+(30,1)+(30,5)+(35,1)+(35,3)+(35',3)+(35,5)+(81,3)+(105,1)$
1400_{v}	=	$(4,2)+(4,4)+(4,6)+(16,2)+2(16,4)+(20,2)+(16,6)+(20,4)+(40,2)+(40,4)+(40,6)+(56,2)+(64,2)+(64,4)$
1400_{c}	=	$(4,2)+(4,4)+(4,6)+(16,2)+2(16,4)+(20,2)+(16,6)+(20,4)+(40,2)+(40,4)+(40,6)+(56,2)+(64,2)+(64,4)$

Table 10.12: SO(9) Branching Rules

SO(9)	→	SU(2) (S)
9	=	(**9**)
16	=	(**5**) + (**11**)
36	=	(**3**) + (**7**) + (**11**) + (**15**)
44	=	(**5**) + (**9**) + (**13**) + (**17**)
84	=	(**3**) + 2(**7**) + (**9**) + (**11**) + (**13**) + (**15**) + (**19**)
126	=	(**1**) + 2(**5**) + (**7**) + 2(**9**) + (**11**) + 2(**13**) + (**15**) + (**17**) + (**21**)
128	=	(**3**) + (**5**) + 2(**7**) + 2(**9**) + (**11**) + 2(**13**) + (**15**) + (**17**) + (**19**)
156	=	(**1**) + (**5**) + (**7**) + (**9**) + (**11**) + 2(**13**) + (**15**) + (**17**) + (**19**) + (**21**) + (**25**)
231	=	(**3**) + 2(**5**) + 2(**7**) + 2(**9**) + 3(**11**) + 2(**13**) + 2(**15**) + 2(**17**) + (**19**) + (**21**) + (**23**)
432	=	(**1**) + 2(**3**) + 3(**5**) + 4(**7**) + 4(**9**) + 5(**11**) + 4(**13**) + 4(**15**) + 3(**17**) + 2(**19**) + 2(**21**) + (**23**) + (**25**)
450	=	(**1**) + 2(**5**) + (**7**) + 3(**9**) + 2(**11**) + 3(**13**) + 2(**15**) + 3(**17**) + 2(**19**) + 3(**21**) + (**23**) + 2(**25**) + (**27**) + (**29**) + (**33**)
495	=	2(**1**) + 4(**5**) + 2(**7**) + 5(**9**) + 3(**11**) + 5(**13**) + 3(**15**) + 4(**17**) + 2(**19**) + 3(**21**) + (**23**) + 2(**25**) + (**29**)
576	=	(**1**)+2(**3**)+3(**5**)+4(**7**)+5(**9**)+5(**11**)+5(**13**)+5(**15**)+4(**17**)+3(**19**)+3(**21**)+2(**23**)+(**25**)+(**27**)
594	=	3(**3**) + 3(**5**) + 5(**7**) + 5(**9**) + 6(**11**) + 5(**13**) + 5(**15**) + 4(**17**) + 4(**19**) + 2(**21**) + 2(**23**) + (**25**) + (**27**)
672	=	2(**1**) + 2(**3**) + 3(**5**) + 5(**7**) + 5(**9**) + 5(**11**) + 6(**13**) + 5(**15**) + 4(**17**) + 4(**19**) + 3(**21**) + 2(**23**) + 2(**25**) + (**27**) + (**31**)
768	=	3(**3**)+5(**5**)+5(**7**)+7(**9**)+7(**11**)+6(**13**)+6(**15**)+6(**17**)+4(**19**)+3(**21**)+3(**23**)+(**25**)+(**27**)+(**29**)
910	=	3(**3**) + 3(**5**) + 6(**7**) + 5(**9**) + 7(**11**) + 6(**13**) + 7(**15**) + 5(**17**) + 6(**19**) + 4(**21**) + 4(**23**) + 2(**25**) + 2(**27**) + (**29**) + (**31**)
924	=	(**1**) + 3(**3**) + 6(**5**) + 6(**7**) + 8(**9**) + 8(**11**) + 8(**13**) + 7(**15**) + 7(**17**) + 5(**19**) + 4(**21**) + 3(**23**) + 2(**25**) + (**27**) + (**29**)
1122	=	(**1**) + (**3**) + 3(**5**) + 2(**7**) + 4(**9**) + 4(**11**) + 5(**13**) + 4(**15**) + 6(**17**) + 4(**19**) + 5(**21**) + 4(**23**) + 4(**25**) + 3(**27**) + 3(**29**) + 2(**31**) + 2(**33**) + (**35**) + (**37**) + (**41**)
1650	=	3(**1**) + 4(**3**) + 8(**5**) + 9(**7**) + 12(**9**) + 11(**11**) + 13(**13**) + 11(**15**) + 11(**17**) + 9(**19**) + 8(**21**) + 5(**23**) + 5(**25**) + 3(**27**) + 2(**29**) + (**31**) + (**33**)
1920	=	(**1**) + 4(**3**) + 7(**5**) + 8(**7**) + 10(**9**) + 12(**11**) + 11(**13**) + 12(**15**) + 12(**17**) + 10(**19**) + 9(**21**) + 8(**23**) + 6(**25**) + 4(**27**) + 4(**29**) + 2(**31**) + (**33**) + (**35**)
1980	=	4(**1**) + 3(**3**) + 8(**5**) + 9(**7**) + 13(**9**) + 10(**11**) + 15(**13**) + 11(**15**) + 12(**17**) + 10(**19**) + 10(**21**) + 6(**23**) + 7(**25**) + 4(**27**) + 3(**29**) + 2(**31**) + 2(**33**) + (**37**)
2457	=	(**1**) + 7(**3**) + 8(**5**) + 13(**7**) + 13(**9**) + 17(**11**) + 15(**13**) + 17(**15**) + 14(**17**) + 14(**19**) + 11(**21**) + 10(**23**) + 7(**25**) + 6(**27**) + 3(**29**) + 3(**31**) + (**33**) + (**35**)
2508	=	2(**1**) + (**3**) + 4(**5**) + 4(**7**) + 7(**9**) + 5(**11**) + 9(**13**) + 7(**15**) + 9(**17**) + 8(**19**) + 9(**21**) + 7(**23**) + 9(**25**) + 6(**27**) + 7(**29**) + 5(**31**) + 5(**33**) + 3(**35**) + 4(**37**) + 2(**39**) + 2(**41**) + (**43**) + (**45**) + (**49**)
2560	=	3(**1**) + 7(**3**) + 11(**5**) + 15(**7**) + 17(**9**) + 18(**11**) + 19(**13**) + 18(**15**) + 16(**17**) + 15(**19**) + 12(**21**) + 9(**23**) + 7(**25**) + 5(**27**) + 3(**29**) + 2(**31**) + (**33**)
2574	=	2(**1**) + 5(**3**) + 9(**5**) + 11(**7**) + 14(**9**) + 14(**11**) + 16(**13**) + 15(**15**) + 15(**17**) + 13(**19**) + 12(**21**) + 10(**23**) + 8(**25**) + 6(**27**) + 5(**29**) + 3(**31**) + 2(**33**) + (**35**) + (**37**)
2772	=	(**1**) + 9(**3**) + 10(**5**) + 17(**7**) + 16(**9**) + 21(**11**) + 18(**13**) + 20(**15**) + 16(**17**) + 16(**19**) + 12(**21**) + 11(**23**) + 7(**25**) + 6(**27**) + 3(**29**) + 3(**31**) + (**33**) + (**35**)
2772′	=	3(**1**) + 4(**3**) + 11(**5**) + 9(**7**) + 15(**9**) + 15(**11**) + 16(**13**) + 14(**15**) + 17(**17**) + 12(**19**) + 13(**21**) + 10(**23**) + 9(**25**) + 6(**27**) + 6(**29**) + 3(**31**) + 3(**33**) + 2(**35**) + (**37**) + (**41**)
2772″	=	(**1**) + 5(**3**) + 7(**5**) + 10(**7**) + 12(**9**) + 13(**11**) + 14(**13**) + 15(**15**) + 14(**17**) + 14(**19**) + 12(**21**) + 11(**23**) + 9(**25**) + 8(**27**) + 6(**29**) + 4(**31**) + 3(**33**) + 2(**35**) + (**37**) + (**39**)
3900	=	5(**1**) + 7(**3**) + 15(**5**) + 17(**7**) + 23(**9**) + 22(**11**) + 26(**13**) + 23(**15**) + 24(**17**) + 20(**19**) + 19(**21**) + 14(**23**) + 13(**25**) + 8(**27**) + 7(**29**) + 4(**31**) + 3(**33**) + (**35**) + (**37**)
4004	=	9(**3**) + 7(**5**) + 17(**7**) + 14(**9**) + 20(**11**) + 18(**13**) + 22(**15**) + 17(**17**) + 21(**19**) + 15(**21**) + 16(**23**) + 12(**25**) + 12(**27**) + 7(**29**) + 8(**31**) + 4(**33**) + 4(**35**) + 2(**37**) + 2(**39**) + (**43**)
4158	=	2(**1**) + 10(**3**) + 14(**5**) + 20(**7**) + 22(**9**) + 25(**11**) + 25(**13**) + 26(**15**) + 23(**17**) + 22(**19**) + 18(**21**) + 16(**23**) + 12(**25**) + 10(**27**) + 7(**29**) + 5(**31**) + 3(**33**) + 2(**35**) + (**37**) + (**39**)

Table 10.12: SO(9) Branching Rules (continued)

4608	=	3(**1**) + 10(**3**) + 16(**5**) + 21(**7**) + 25(**9**) + 27(**11**) + 28(**13**) + 28(**15**) + 26(**17**) + 24(**19**) + 21(**21**) + 17(**23**) + 14(**25**) + 11(**27**) + 8(**29**) + 5(**31**) + 4(**33**) + 2(**35**) + (**37**) + (**39**)

SO(9)	→	SU(2)×SU(2) (S)
9	=	(**3**,**3**)
16	=	(**4**,**2**) + (**2**,**4**)
36	=	(**3**,**1**) + (**1**,**3**) + (**5**,**3**) + (**3**,**5**)
44	=	(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + (**5**,**5**)
84	=	(**3**,**1**) + (**1**,**3**) + (**3**,**3**) + (**5**,**3**) + (**3**,**5**) + (**7**,**1**) + (**1**,**7**) + (**5**,**5**)
126	=	(**1**,**1**) + 2(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + (**5**,**3**) + (**3**,**5**) + (**5**,**5**) + (**7**,**3**) + (**3**,**7**)
128	=	2(**2**,**2**) + (**4**,**2**) + (**2**,**4**) + 2(**4**,**4**) + (**6**,**2**) + (**2**,**6**) + (**6**,**4**) + (**4**,**6**)
156	=	(**1**,**1**) + (**3**,**3**) + (**5**,**3**) + (**3**,**5**) + (**5**,**5**) + (**7**,**3**) + (**3**,**7**) + (**7**,**7**)
231	=	(**3**,**1**) + (**1**,**3**) + 2(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + 2(**5**,**3**) + 2(**3**,**5**) + (**5**,**5**) + (**7**,**3**) + (**3**,**7**) + (**7**,**5**) + (**5**,**7**)
432	=	2(**2**,**2**) + 3(**4**,**2**) + 3(**2**,**4**) + 4(**4**,**4**) + 2(**6**,**2**) + 2(**2**,**6**) + 2(**6**,**4**) + 2(**4**,**6**) + (**8**,**2**) + (**2**,**8**) + 2(**6**,**6**) + (**8**,**4**) + (**4**,**8**)
450	=	(**1**,**1**) + (**3**,**3**) + (**5**,**1**) + (**1**,**5**) + (**5**,**3**) + (**3**,**5**) + 2(**5**,**5**) + (**7**,**3**) + (**3**,**7**) + (**9**,**1**) + (**1**,**9**) + (**7**,**5**) + (**5**,**7**) + (**7**,**7**) + (**9**,**5**) + (**5**,**9**) + (**9**,**9**)
495	=	2(**1**,**1**) + 3(**3**,**3**) + 2(**5**,**1**) + 2(**1**,**5**) + 2(**5**,**3**) + 2(**3**,**5**) + 3(**5**,**5**) + 2(**7**,**3**) + 2(**3**,**7**) + (**9**,**1**) + (**1**,**9**) + (**7**,**5**) + (**5**,**7**) + (**7**,**7**) + (**9**,**5**) + (**5**,**9**)
576	=	2(**2**,**2**) + 3(**4**,**2**) + 3(**2**,**4**) + 4(**4**,**4**) + 2(**6**,**2**) + 2(**2**,**6**) + 3(**6**,**4**) + 3(**4**,**6**) + (**8**,**2**) + (**2**,**8**) + 2(**6**,**6**) + (**8**,**4**) + (**4**,**8**) + (**8**,**6**) + (**6**,**8**)
594	=	2(**3**,**1**) + 2(**1**,**3**) + 4(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + 4(**5**,**3**) + 4(**3**,**5**) + (**7**,**1**) + (**1**,**7**) + 3(**5**,**5**) + 2(**7**,**3**) + 2(**3**,**7**) + 2(**7**,**5**) + 2(**5**,**7**) + (**9**,**3**) + (**3**,**9**) + (**7**,**7**)
672	=	2(**2**,**2**) + 2(**4**,**2**) + 2(**2**,**4**) + 4(**4**,**4**) + 2(**6**,**2**) + 2(**2**,**6**) + 3(**6**,**4**) + 3(**4**,**6**) + 2(**8**,**2**) + 2(**2**,**8**) + 2(**6**,**6**) + (**8**,**4**) + (**4**,**8**) + (**8**,**6**) + (**6**,**8**) + (**10**,**4**) + (**4**,**10**)
768	=	2(**2**,**2**) + 4(**4**,**2**) + 4(**2**,**4**) + 4(**4**,**4**) + 3(**6**,**2**) + 3(**2**,**6**) + 4(**6**,**4**) + 4(**4**,**6**) + (**8**,**2**) + (**2**,**8**) + 2(**6**,**6**) + 2(**8**,**4**) + 2(**4**,**8**) + (**10**,**2**) + (**2**,**10**) + (**8**,**6**) + (**6**,**8**)
910	=	2(**3**,**1**) + 2(**1**,**3**) + 3(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + 4(**5**,**3**) + 4(**3**,**5**) + 2(**7**,**1**) + 2(**1**,**7**) + 4(**5**,**5**) + 2(**7**,**3**) + 2(**3**,**7**) + 3(**7**,**5**) + 3(**5**,**7**) + (**9**,**3**) + (**3**,**9**) + (**7**,**7**) + (**9**,**5**) + (**5**,**9**) + (**9**,**7**) + (**7**,**9**)
924	=	(**1**,**1**) + 2(**3**,**1**) + 2(**1**,**3**) + 5(**3**,**3**) + 3(**5**,**1**) + 3(**1**,**5**) + 5(**5**,**3**) + 5(**3**,**5**) + (**7**,**1**) + (**1**,**7**) + 5(**5**,**5**) + 3(**7**,**3**) + 3(**3**,**7**) + (**9**,**1**) + (**1**,**9**) + 3(**7**,**5**) + 3(**5**,**7**) + (**9**,**3**) + (**3**,**9**) + (**7**,**7**) + (**9**,**5**) + (**5**,**9**)
1122	=	2(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + (**5**,**3**) + (**3**,**5**) + 2(**5**,**5**) + 2(**7**,**3**) + 2(**3**,**7**) + 2(**7**,**5**) + 2(**5**,**7**) + (**9**,**3**) + (**3**,**9**) + 2(**7**,**7**) + (**9**,**5**) + (**5**,**9**) + (**11**,**3**) + (**3**,**11**) + (**9**,**7**) + (**7**,**9**) + (**9**,**9**) + (**11**,**7**) + (**7**,**11**) + (**11**,**11**)
1650	=	2(**1**,**1**) + 2(**3**,**1**) + 2(**1**,**3**) + 7(**3**,**3**) + 3(**5**,**1**) + 3(**1**,**5**) + 6(**5**,**3**) + 6(**3**,**5**) + 2(**7**,**1**) + 2(**1**,**7**) + 8(**5**,**5**) + 5(**7**,**3**) + 5(**3**,**7**) + (**9**,**1**) + (**1**,**9**) + 4(**7**,**5**) + 4(**5**,**7**) + 2(**9**,**3**) + 2(**3**,**9**) + 3(**7**,**7**) + 2(**9**,**5**) + 2(**5**,**9**) + (**11**,**3**) + (**3**,**11**) + (**9**,**7**) + (**7**,**9**)
1920	=	2(**2**,**2**) + 5(**4**,**2**) + 5(**2**,**4**) + 6(**4**,**4**) + 4(**6**,**2**) + 4(**2**,**6**) + 6(**6**,**4**) + 6(**4**,**6**) + 2(**8**,**2**) + 2(**2**,**8**) + 6(**6**,**6**) + 4(**8**,**4**) + 4(**4**,**8**) + (**10**,**2**) + (**2**,**10**) + 3(**8**,**6**) + 3(**6**,**8**) + (**10**,**4**) + (**4**,**10**) + 2(**8**,**8**) + (**10**,**6**) + (**6**,**10**) + (**10**,**8**) + (**8**,**10**)
1980	=	3(**1**,**1**) + (**3**,**1**) + (**1**,**3**) + 6(**3**,**3**) + 3(**5**,**1**) + 3(**1**,**5**) + 5(**5**,**3**) + 5(**3**,**5**) + 2(**7**,**1**) + 2(**1**,**7**) + 8(**5**,**5**) + 5(**7**,**3**) + 5(**3**,**7**) + 2(**9**,**1**) + 2(**1**,**9**) + 4(**7**,**5**) + 4(**5**,**7**) + 2(**9**,**3**) + 2(**3**,**9**) + 4(**7**,**7**) + 3(**9**,**5**) + 3(**5**,**9**) + (**11**,**3**) + (**3**,**11**) + (**13**,**1**) + (**1**,**13**) + (**9**,**7**) + (**7**,**9**) + (**11**,**5**) + (**5**,**11**) + (**9**,**9**)
2457	=	(**1**,**1**) + 4(**3**,**1**) + 4(**1**,**3**) + 6(**3**,**3**) + 3(**5**,**1**) + 3(**1**,**5**) + 9(**5**,**3**) + 9(**3**,**5**) + 3(**7**,**1**) + 3(**1**,**7**) + 9(**5**,**5**) + 5(**7**,**3**) + 5(**3**,**7**) + (**9**,**1**) + (**1**,**9**) + 7(**7**,**5**) + 7(**5**,**7**) + 3(**9**,**3**) + 3(**3**,**9**) + (**11**,**1**) + (**1**,**11**) + 4(**7**,**7**) + 3(**9**,**5**) + 3(**5**,**9**) + 2(**9**,**7**) + 2(**7**,**9**) + (**11**,**5**) + (**5**,**11**) + (**9**,**9**)
2508	=	(**1**,**1**) + 2(**3**,**3**) + (**5**,**1**) + (**1**,**5**) + (**5**,**3**) + (**3**,**5**) + (**7**,**1**) + (**1**,**7**) + 4(**5**,**5**) + 2(**7**,**3**) + 2(**3**,**7**) + (**9**,**1**) + (**1**,**9**) + 2(**7**,**5**) + 2(**5**,**7**) + (**9**,**3**) + (**3**,**9**) + 3(**7**,**7**) + 3(**9**,**5**) + 3(**5**,**9**) + (**11**,**3**) + (**3**,**11**) + (**13**,**1**) + (**1**,**13**) + 2(**9**,**7**) + 2(**7**,**9**) + (**11**,**5**) + (**5**,**11**) + 2(**9**,**9**) + (**11**,**7**) + (**7**,**11**) + (**13**,**5**) + (**5**,**13**) + (**11**,**9**) + (**9**,**11**) + (**11**,**11**) + (**13**,**9**) + (**9**,**13**) + (**13**,**13**)
2560	=	6(**2**,**2**) + 8(**4**,**2**) + 8(**2**,**4**) + 12(**4**,**4**) + 7(**6**,**2**) + 7(**2**,**6**) + 10(**6**,**4**) + 10(**4**,**6**) + 4(**8**,**2**) + 4(**2**,**8**) + 8(**6**,**6**) + 5(**8**,**4**) + 5(**4**,**8**) + (**10**,**2**) + (**2**,**10**) + 4(**8**,**6**) + 4(**6**,**8**) + 2(**10**,**4**) + 2(**4**,**10**) + 2(**8**,**8**) + (**10**,**6**) + (**6**,**10**)

Table 10.12: SO(9) Branching Rules (continued)

2574	=	$(\mathbf{1,1}) + 2(\mathbf{3,1}) + 2(\mathbf{1,3}) + 7(\mathbf{3,3}) + 3(\mathbf{5,1}) + 3(\mathbf{1,5}) + 7(\mathbf{5,3}) + 7(\mathbf{3,5}) + 2(\mathbf{7,1}) + 2(\mathbf{1,7}) + 8(\mathbf{5,5}) + 6(\mathbf{7,3}) + 6(\mathbf{3,7}) + (\mathbf{9,1}) + (\mathbf{1,9}) + 6(\mathbf{7,5}) + 6(\mathbf{5,7}) + 3(\mathbf{9,3}) + 3(\mathbf{3,9}) + 5(\mathbf{7,7}) + 3(\mathbf{9,5}) + 3(\mathbf{5,9}) + (\mathbf{11,3}) + (\mathbf{3,11}) + 2(\mathbf{9,7}) + 2(\mathbf{7,9}) + (\mathbf{11,5}) + (\mathbf{5,11}) + (\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11})$
2772	=	$5(\mathbf{3,1}) + 5(\mathbf{1,3}) + 8(\mathbf{3,3}) + 3(\mathbf{5,1}) + 3(\mathbf{1,5}) + 11(\mathbf{5,3}) + 11(\mathbf{3,5}) + 4(\mathbf{7,1}) + 4(\mathbf{1,7}) + 10(\mathbf{5,5}) + 7(\mathbf{7,3}) + 7(\mathbf{3,7}) + (\mathbf{9,1}) + (\mathbf{1,9}) + 8(\mathbf{7,5}) + 8(\mathbf{5,7}) + 4(\mathbf{9,3}) + 4(\mathbf{3,9}) + (\mathbf{11,1}) + (\mathbf{1,11}) + 4(\mathbf{7,7}) + 3(\mathbf{9,5}) + 3(\mathbf{5,9}) + (\mathbf{11,3}) + (\mathbf{3,11}) + 2(\mathbf{9,7}) + 2(\mathbf{7,9}) + (\mathbf{11,5}) + (\mathbf{5,11})$
2772′	=	$(\mathbf{1,1}) + (\mathbf{3,1}) + (\mathbf{1,3}) + 6(\mathbf{3,3}) + 4(\mathbf{5,1}) + 4(\mathbf{1,5}) + 5(\mathbf{5,3}) + 5(\mathbf{3,5}) + (\mathbf{7,1}) + (\mathbf{1,7}) + 8(\mathbf{5,5}) + 6(\mathbf{7,3}) + 6(\mathbf{3,7}) + 2(\mathbf{9,1}) + 2(\mathbf{1,9}) + 6(\mathbf{7,5}) + 6(\mathbf{5,7}) + 3(\mathbf{9,3}) + 3(\mathbf{3,9}) + (\mathbf{11,1}) + (\mathbf{1,11}) + 4(\mathbf{7,7}) + 4(\mathbf{9,5}) + 4(\mathbf{5,9}) + 2(\mathbf{11,3}) + 2(\mathbf{3,11}) + 2(\mathbf{9,7}) + 2(\mathbf{7,9}) + (\mathbf{11,5}) + (\mathbf{5,11}) + (\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11}) + (\mathbf{13,5}) + (\mathbf{5,13})$
2772″	=	$2(\mathbf{3,1}) + 2(\mathbf{1,3}) + 5(\mathbf{3,3}) + 2(\mathbf{5,1}) + 2(\mathbf{1,5}) + 6(\mathbf{5,3}) + 6(\mathbf{3,5}) + 2(\mathbf{7,1}) + 2(\mathbf{1,7}) + 7(\mathbf{5,5}) + 5(\mathbf{7,3}) + 5(\mathbf{3,7}) + (\mathbf{9,1}) + (\mathbf{1,9}) + 6(\mathbf{7,5}) + 6(\mathbf{5,7}) + 3(\mathbf{9,3}) + 3(\mathbf{3,9}) + 5(\mathbf{7,7}) + 3(\mathbf{9,5}) + 3(\mathbf{5,9}) + (\mathbf{11,3}) + (\mathbf{3,11}) + 3(\mathbf{9,7}) + 3(\mathbf{7,9}) + (\mathbf{11,5}) + (\mathbf{5,11}) + (\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11}) + (\mathbf{11,9}) + (\mathbf{9,11})$
3900	=	$3(\mathbf{1,1}) + 3(\mathbf{3,1}) + 3(\mathbf{1,3}) + 12(\mathbf{3,3}) + 5(\mathbf{5,1}) + 5(\mathbf{1,5}) + 11(\mathbf{5,3}) + 11(\mathbf{3,5}) + 3(\mathbf{7,1}) + 3(\mathbf{1,7}) + 14(\mathbf{5,5}) + 10(\mathbf{7,3}) + 10(\mathbf{3,7}) + 2(\mathbf{9,1}) + 2(\mathbf{1,9}) + 9(\mathbf{7,5}) + 9(\mathbf{5,7}) + 4(\mathbf{9,3}) + 4(\mathbf{3,9}) + 8(\mathbf{7,7}) + 5(\mathbf{9,5}) + 5(\mathbf{5,9}) + 2(\mathbf{11,3}) + 2(\mathbf{3,11}) + 3(\mathbf{9,7}) + 3(\mathbf{7,9}) + (\mathbf{11,5}) + (\mathbf{5,11}) + (\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11})$
4004	=	$4(\mathbf{3,1}) + 4(\mathbf{1,3}) + 4(\mathbf{3,3}) + (\mathbf{5,1}) + (\mathbf{1,5}) + 9(\mathbf{5,3}) + 9(\mathbf{3,5}) + 4(\mathbf{7,1}) + 4(\mathbf{1,7}) + 8(\mathbf{5,5}) + 5(\mathbf{7,3}) + 5(\mathbf{3,7}) + (\mathbf{9,1}) + (\mathbf{1,9}) + 8(\mathbf{7,5}) + 8(\mathbf{5,7}) + 5(\mathbf{9,3}) + 5(\mathbf{3,9}) + (\mathbf{11,1}) + (\mathbf{1,11}) + 6(\mathbf{7,7}) + 4(\mathbf{9,5}) + 4(\mathbf{5,9}) + (\mathbf{11,3}) + (\mathbf{3,11}) + 4(\mathbf{9,7}) + 4(\mathbf{7,9}) + 3(\mathbf{11,5}) + 3(\mathbf{5,11}) + (\mathbf{13,3}) + (\mathbf{3,13}) + 2(\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11}) + (\mathbf{11,9}) + (\mathbf{9,11}) + (\mathbf{13,7}) + (\mathbf{7,13})$
4158	=	$(\mathbf{1,1}) + 4(\mathbf{3,1}) + 4(\mathbf{1,3}) + 9(\mathbf{3,3}) + 4(\mathbf{5,1}) + 4(\mathbf{1,5}) + 12(\mathbf{5,3}) + 12(\mathbf{3,5}) + 4(\mathbf{7,1}) + 4(\mathbf{1,7}) + 12(\mathbf{5,5}) + 9(\mathbf{7,3}) + 9(\mathbf{3,7}) + 2(\mathbf{9,1}) + 2(\mathbf{1,9}) + 10(\mathbf{7,5}) + 10(\mathbf{5,7}) + 6(\mathbf{9,3}) + 6(\mathbf{3,9}) + (\mathbf{11,1}) + (\mathbf{1,11}) + 7(\mathbf{7,7}) + 5(\mathbf{9,5}) + 5(\mathbf{5,9}) + 2(\mathbf{11,3}) + 2(\mathbf{3,11}) + 3(\mathbf{9,7}) + 3(\mathbf{7,9}) + 2(\mathbf{11,5}) + 2(\mathbf{5,11}) + (\mathbf{13,3}) + (\mathbf{3,13}) + (\mathbf{9,9}) + (\mathbf{11,7}) + (\mathbf{7,11})$
4608	=	$6(\mathbf{2,2}) + 9(\mathbf{4,2}) + 9(\mathbf{2,4}) + 14(\mathbf{4,4}) + 9(\mathbf{6,2}) + 9(\mathbf{2,6}) + 13(\mathbf{6,4}) + 13(\mathbf{4,6}) + 6(\mathbf{8,2}) + 6(\mathbf{2,8}) + 12(\mathbf{6,6}) + 9(\mathbf{8,4}) + 9(\mathbf{4,8}) + 3(\mathbf{10,2}) + 3(\mathbf{2,10}) + 7(\mathbf{8,6}) + 7(\mathbf{6,8}) + 4(\mathbf{10,4}) + 4(\mathbf{4,10}) + (\mathbf{12,2}) + (\mathbf{2,12}) + 4(\mathbf{8,8}) + 3(\mathbf{10,6}) + 3(\mathbf{6,10}) + (\mathbf{12,4}) + (\mathbf{4,12}) + (\mathbf{10,8}) + (\mathbf{8,10}) + (\mathbf{12,6}) + (\mathbf{6,12})$

Table 10.13: SO(10) Branching Rules

SO(10)	→	Sp(4) (S)
10	=	$(\mathbf{10})$
16	=	$(\mathbf{16})$
45	=	$(\mathbf{10}) + (\mathbf{35})$
54	=	$(\mathbf{5}) + (\mathbf{14}) + (\mathbf{35'})$
120	=	$(\mathbf{1}) + (\mathbf{5}) + (\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{35'})$
126	=	$(\mathbf{10}) + (\mathbf{35}) + (\mathbf{81})$
144	=	$(\mathbf{4}) + (\mathbf{16}) + (\mathbf{20}) + (\mathbf{40}) + (\mathbf{64})$
210	=	$(\mathbf{5}) + (\mathbf{10}) + (\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{35'}) + (\mathbf{81})$
210′	=	$(\mathbf{10}) + (\mathbf{35}) + (\mathbf{81}) + (\mathbf{84})$
320	=	$(\mathbf{5}) + (\mathbf{10}) + (\mathbf{14}) + 2(\mathbf{35}) + (\mathbf{35'}) + (\mathbf{81}) + (\mathbf{105})$
560	=	$(\mathbf{4}) + 2(\mathbf{16}) + 2(\mathbf{20}) + 2(\mathbf{40}) + (\mathbf{56}) + 2(\mathbf{64}) + (\mathbf{80}) + (\mathbf{140})$
660	=	$(\mathbf{1}) + (\mathbf{14}) + (\mathbf{30}) + (\mathbf{35}) + (\mathbf{35'}) + (\mathbf{55}) + (\mathbf{105}) + (\mathbf{165}) + (\mathbf{220})$
672	=	$(\mathbf{16}) + (\mathbf{20}) + (\mathbf{40}) + (\mathbf{56}) + (\mathbf{64}) + (\mathbf{80}) + (\mathbf{140}) + (\mathbf{256})$
720	=	$(\mathbf{4}) + 2(\mathbf{16}) + 2(\mathbf{20}) + 2(\mathbf{40}) + (\mathbf{56}) + 2(\mathbf{64}) + (\mathbf{80}) + (\mathbf{140}) + (\mathbf{160})$
770	=	$(\mathbf{1}) + (\mathbf{5}) + 2(\mathbf{14}) + (\mathbf{30}) + 2(\mathbf{35}) + 2(\mathbf{35'}) + (\mathbf{55}) + (\mathbf{81}) + 2(\mathbf{105}) + (\mathbf{220})$
945	=	$(\mathbf{5}) + 3(\mathbf{10}) + (\mathbf{14}) + (\mathbf{30}) + 4(\mathbf{35}) + (\mathbf{35'}) + 3(\mathbf{81}) + (\mathbf{84}) + 2(\mathbf{105}) + (\mathbf{154})$
1050	=	$(\mathbf{1}) + (\mathbf{5}) + (\mathbf{10}) + 2(\mathbf{14}) + (\mathbf{30}) + 3(\mathbf{35}) + 2(\mathbf{35'}) + (\mathbf{55}) + 2(\mathbf{81}) + 2(\mathbf{105}) + (\mathbf{154}) + (\mathbf{220})$
1200	=	$(\mathbf{4}) + 3(\mathbf{16}) + 2(\mathbf{20}) + 3(\mathbf{40}) + (\mathbf{56}) + 3(\mathbf{64}) + 2(\mathbf{80}) + 2(\mathbf{140}) + (\mathbf{140'}) + (\mathbf{160})$
1386	=	$(\mathbf{5}) + 2(\mathbf{10}) + (\mathbf{14}) + (\mathbf{30}) + 3(\mathbf{35}) + 2(\mathbf{35'}) + 3(\mathbf{81}) + (\mathbf{84}) + 2(\mathbf{105}) + (\mathbf{154}) + (\mathbf{220}) + (\mathbf{231})$
1440	=	$(\mathbf{4}) + 2(\mathbf{16}) + 2(\mathbf{20}) + 3(\mathbf{40}) + (\mathbf{56}) + 3(\mathbf{64}) + 2(\mathbf{80}) + 2(\mathbf{140}) + (\mathbf{140'}) + (\mathbf{160}) + (\mathbf{256})$

Table 10.13: SO(10) Branching Rules (continued)

1728	=	2(**5**) + 3(**10**) + 3(**14**) + 2(**30**) + 5(**35**) + 3(**35′**) + (**55**) + 4(**81**) + (**84**) + 3(**105**) + 2(**154**) + (**220**)
1782	=	(**10**) + (**35**) + 2(**81**) + (**84**) + (**105**) + (**154**) + (**231**) + (**260**) + (**286**) + (**455**)
2640	=	(**4**) + 3(**16**) + 2(**20**) + 3(**40**) + 2(**56**) + 4(**64**) + 2(**80**) + (**120**) + 3(**140**) + (**140′**) + 2(**160**) + (**256**) + (**320**) + (**324**)
2772	=	(**5**) + (**14**) + (**30**) + (**35**) + 2(**35′**) + (**55**) + 2(**81**) + (**91**) + 2(**105**) + (**154**) + 2(**220**) + (**231**) + (**260**) + (**390**) + (**625**)
2970	=	(**5**) + 4(**10**) + 2(**14**) + 2(**30**) + 6(**35**) + 3(**35′**) + (**55**) + 6(**81**) + 2(**84**) + 4(**105**) + 3(**154**) + 2(**220**) + (**231**) + (**260**)

SO(10)	→	Sp(4)×Sp(4) (S)
10	=	(**5**, **1**) + (**1**, **5**)
16	=	(**4**, **4**)
45	=	(**5**, **5**) + (**10**, **1**) + (**1**, **10**)
54	=	(**1**, **1**) + (**5**, **5**) + (**14**, **1**) + (**1**, **14**)
120	=	(**10**, **1**) + (**1**, **10**) + (**10**, **5**) + (**5**, **10**)
126	=	(**1**, **1**) + (**5**, **5**) + (**10**, **10**)
144	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**)
210	=	(**5**, **1**) + (**1**, **5**) + (**10**, **5**) + (**5**, **10**) + (**10**, **10**)
210′	=	(**5**, **1**) + (**1**, **5**) + (**14**, **5**) + (**5**, **14**) + (**30**, **1**) + (**1**, **30**)
320	=	(**5**, **1**) + (**1**, **5**) + (**10**, **5**) + (**5**, **10**) + (**14**, **5**) + (**5**, **14**) + (**35**, **1**) + (**1**, **35**)
560	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**) + (**20**, **4**) + (**4**, **20**) + (**16**, **16**)
660	=	(**1**, **1**) + (**5**, **5**) + (**14**, **1**) + (**1**, **14**) + (**14**, **14**) + (**30**, **5**) + (**5**, **30**) + (**55**, **1**) + (**1**, **55**)
672	=	(**4**, **4**) + (**16**, **16**) + (**20**, **20**)
720	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**) + (**16**, **16**) + (**40**, **4**) + (**4**, **40**)
770	=	(**1**, **1**) + (**5**, **5**) + (**14**, **1**) + (**1**, **14**) + (**10**, **10**) + (**14**, **14**) + (**35′**, **1**) + (**1**, **35′**) + (**35**, **5**) + (**5**, **35**)
945	=	(**5**, **5**) + (**10**, **1**) + (**1**, **10**) + (**10**, **5**) + (**5**, **10**) + (**10**, **10**) + (**10**, **14**) + (**14**, **10**) + (**35**, **1**) + (**1**, **35**) + (**35**, **5**) + (**5**, **35**)
1050	=	(**5**, **1**) + (**1**, **5**) + (**10**, **5**) + (**5**, **10**) + (**14**, **5**) + (**5**, **14**) + (**10**, **10**) + (**35**, **10**) + (**10**, **35**)
1200	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**) + (**20**, **4**) + (**4**, **20**) + (**16**, **16**) + (**20**, **16**) + (**16**, **20**)
1386	=	2(**5**, **5**) + (**10**, **1**) + (**1**, **10**) + (**14**, **1**) + (**1**, **14**) + (**10**, **14**) + (**14**, **10**) + (**14**, **14**) + (**30**, **5**) + (**5**, **30**) + (**35**, **5**) + (**5**, **35**) + (**81**, **1**) + (**1**, **81**)
1440	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**) + (**16**, **16**) + (**20**, **16**) + (**16**, **20**) + (**20**, **20**)
1728	=	2(**5**, **5**) + (**10**, **1**) + (**1**, **10**) + (**10**, **5**) + (**5**, **10**) + (**14**, **1**) + (**1**, **14**) + 2(**10**, **10**) + (**10**, **14**) + (**14**, **10**) + (**35**, **5**) + (**5**, **35**) + (**35**, **10**) + (**10**, **35**)
1782	=	(**5**, **1**) + (**1**, **5**) + (**14**, **5**) + (**5**, **14**) + (**30**, **1**) + (**1**, **30**) + (**30**, **14**) + (**14**, **30**) + (**55**, **5**) + (**5**, **55**) + (**91**, **1**) + (**1**, **91**)
2640	=	(**4**, **4**) + (**16**, **4**) + (**4**, **16**) + (**16**, **16**) + (**40**, **4**) + (**4**, **40**) + (**40**, **16**) + (**16**, **40**) + (**80**, **4**) + (**4**, **80**)
2772	=	(**1**, **1**) + (**5**, **5**) + (**10**, **10**) + (**14**, **14**) + (**35**, **35**) + (**35′**, **35′**)
2970	=	(**5**, **1**) + (**1**, **5**) + (**10**, **5**) + (**5**, **10**) + (**14**, **5**) + (**5**, **14**) + 2(**10**, **10**) + (**35**, **1**) + (**1**, **35**) + (**35′**, **1**) + (**1**, **35′**) + (**35**, **5**) + (**5**, **35**) + (**35′**, **5**) + (**5**, ß *rrep*[1]35) + (**35**, **10**) + (**10**, **35**) + (**35**, **14**) + (**14**, **35**)

Table 10.14: SO(11) Branching Rules

SO(11)	→	SO(8)×SU(2) (R)
11	=	(**1**, **3**) + ($\mathbf{8_v}$, **1**)
32	=	($\mathbf{8_s}$, **2**) + ($\mathbf{8_c}$, **2**)
55	=	(**1**, **3**) + ($\mathbf{8_v}$, **3**) + (**28**, **1**)
65	=	(**1**, **1**) + (**1**, **5**) + ($\mathbf{8_v}$, **3**) + ($\mathbf{35_v}$, **1**)
165	=	(**1**, **1**) + ($\mathbf{8_v}$, **3**) + (**28**, **3**) + ($\mathbf{56_v}$, **1**)
275	=	(**1**, **3**) + (**1**, **7**) + ($\mathbf{8_v}$, **1**) + ($\mathbf{8_v}$, **5**) + ($\mathbf{35_v}$, **3**) + ($\mathbf{112_v}$, **1**)

Table 10.14: SO(11) Branching Rules (continued)

320	=	$(8_s,2)+(8_c,2)+(8_s,4)+(8_c,4)+(56_c,2)+(56_s,2)$
330	=	$(8_v,1)+(28,3)+(35_c,1)+(35_s,1)+(56_v,3)$
429	=	$(1,3)+(1,5)+(8_v,1)+(8_v,3)+(8_v,5)+(28,3)+(35_v,3)+(160_v,1)$
462	=	$(28,1)+(35_c,3)+(35_s,3)+(56_v,1)+(56_v,3)$
935	=	$(1,1)+(1,5)+(1,9)+(8_v,3)+(8_v,7)+(35_v,1)+(35_v,5)+(112_v,3)+(294_v,1)$
1144	=	$(1,1)+(1,5)+(8_v,3)+(8_v,5)+(28,3)+(35_v,1)+(35_v,5)+(160_v,3)+(300,1)$
1408	=	$(8_s,2)+(8_c,2)+(8_s,4)+(8_c,4)+(56_c,2)+(56_s,2)+(56_c,4)+(56_s,4)+(160_s,2)+(160_c,2)$
1430	=	$(1,3)+(8_v,1)+(8_v,3)+(8_v,5)+(28,1)+(28,3)+(28,5)+(35_v,3)+(56_v,3)+(160_v,3)+(350,1)$
1760	=	$(8_s,2)+(8_c,2)+(8_s,4)+(8_c,4)+(8_s,6)+(8_c,6)+(56_c,2)+(56_s,2)+(56_c,4)+(56_s,4)+(224_{vs},2)+(224_{vc},2)$
2025	=	$(1,3)+(1,5)+(1,7)+2(8_v,3)+(8_v,5)+(8_v,7)+(28,1)+(28,5)+(35_v,1)+(35_v,3)+(35_v,5)+(112_v,3)+(160_v,3)+(567_v,1)$
2717	=	$(1,3)+(1,7)+(8_v,1)+(1,11)+(8_v,5)+(8_v,9)+(35_v,3)+(35_v,7)+(112_v,1)+(112_v,5)+(294_v,3)+(672'_v,1)$
3003	=	$(8_v,3)+(28,1)+(28,3)+(28,5)+(35_v,1)+(35_c,3)+(35_s,3)+(56_v,1)+(56_v,3)+(56_v,5)+(160_v,3)+(224_{cv},1)+(224_{sv},1)+(350,3)$

SO(11)	→	SU(4)×Sp(4) (R)
11	=	$(1,5)+(6,1)$
32	=	$(4,4)+(\overline{4},4)$
55	=	$(6,5)+(1,10)+(15,1)$
65	=	$(1,1)+(6,5)+(1,14)+(20',1)$
165	=	$(1,10)+(10,1)+(\overline{10},1)+(6,10)+(15,5)$
275	=	$(1,5)+(6,1)+(6,14)+(20',5)+(1,30)+(50,1)$
320	=	$(4,4)+(\overline{4},4)+(4,16)+(\overline{4},16)+(20,4)+(\overline{20},4)$
330	=	$(1,5)+(10,5)+(\overline{10},5)+(15,1)+(6,10)+(15,10)$
429	=	$(1,5)+(6,1)+(6,10)+(15,5)+(6,14)+(20',5)+(1,35)+(64,1)$
462	=	$(1,1)+(6,1)+(6,5)+(15,5)+(10,10)+(\overline{10},10)+(15,10)$
935	=	$(1,1)+(6,5)+(1,14)+(20',1)+(20',14)+(6,30)+(50,5)+(1,55)+(105,1)$
1144	=	$(1,1)+(6,5)+(1,14)+(20',1)+(15,10)+(20',14)+(1,35')+(6,35)+(64,5)+(84,1)$
1408	=	$(4,4)+(\overline{4},4)+(4,16)+(\overline{4},16)+(20,4)+(\overline{20},4)+(4,20)+(\overline{4},20)+(20,16)+(\overline{20},16)+(36,4)+(\overline{36},4)$
1430	=	$(6,5)+(1,10)+(10,5)+(\overline{10},5)+(15,1)+(6,10)+(15,10)+(15,14)+(20',10)+(1,35)+(6,35)+(45,1)+(\overline{45},1)+(64,5)$
1760	=	$(4,4)+(\overline{4},4)+(4,16)+(\overline{4},16)+(20,4)+(\overline{20},4)+(20,16)+(\overline{20},16)+(4,40)+(\overline{4},40)+(60,4)+(\overline{60},4)$
2025	=	$2(6,5)+(1,10)+(1,14)+(15,1)+(20',1)+(15,14)+(20',10)+(20',14)+(6,30)+(6,35)+(50,5)+(64,5)+(1,81)+(175,1)$
2717	=	$(1,5)+(6,1)+(6,14)+(20',5)+(1,30)+(20',30)+(50,1)+(6,55)+(50,14)+(1,91)+(105,5)+(196,1)$
3003	=	$(6,5)+(1,10)+(10,1)+(\overline{10},1)+(1,14)+(6,10)+2(15,5)+(10,10)+(\overline{10},10)+(10,14)+(\overline{10},14)+(15,10)+(20',10)+(6,35)+(45,5)+(15,35)+(\overline{45},5)+(64,1)+(64,10)$

SO(11)	→	SU(2) (S)
11	=	(11)
32	=	$(6)+(10)+(16)$
55	=	$(3)+(7)+(11)+(15)+(19)$
65	=	$(5)+(9)+(13)+(17)+(21)$
165	=	$(1)+(5)+(7)+2(9)+(11)+2(13)+(15)+(17)+(19)+(21)+(25)$
275	=	$(3)+2(7)+(9)+(11)+2(13)+2(15)+(17)+2(19)+(21)+(23)+(25)+(27)+(31)$

Table 10.14: SO(11) Branching Rules (continued)

320	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+3(\mathbf{8})+2(\mathbf{10})+3(\mathbf{12})+3(\mathbf{14})+2(\mathbf{16})+2(\mathbf{18})+2(\mathbf{20})+(\mathbf{22})+(\mathbf{24})+(\mathbf{26})$
330	=	$(\mathbf{1})+3(\mathbf{5})+(\mathbf{7})+3(\mathbf{9})+2(\mathbf{11})+3(\mathbf{13})+2(\mathbf{15})+3(\mathbf{17})+(\mathbf{19})+2(\mathbf{21})+(\mathbf{23})+(\mathbf{25})+(\mathbf{29})$
429	=	$(\mathbf{3})+2(\mathbf{5})+2(\mathbf{7})+3(\mathbf{9})+3(\mathbf{11})+3(\mathbf{13})+3(\mathbf{15})+3(\mathbf{17})+2(\mathbf{19})+2(\mathbf{21})+2(\mathbf{23})+(\mathbf{25})+(\mathbf{27})+(\mathbf{29})$
462	=	$2(\mathbf{3})+(\mathbf{5})+4(\mathbf{7})+2(\mathbf{9})+4(\mathbf{11})+3(\mathbf{13})+4(\mathbf{15})+2(\mathbf{17})+3(\mathbf{19})+2(\mathbf{21})+2(\mathbf{23})+(\mathbf{25})+(\mathbf{27})+(\mathbf{31})$
935	=	$(\mathbf{1})+3(\mathbf{5})+(\mathbf{7})+4(\mathbf{9})+3(\mathbf{11})+4(\mathbf{13})+3(\mathbf{15})+5(\mathbf{17})+3(\mathbf{19})+4(\mathbf{21})+3(\mathbf{23})+4(\mathbf{25})+2(\mathbf{27})+$ $3(\mathbf{29})+(\mathbf{31})+2(\mathbf{33})+(\mathbf{35})+(\mathbf{37})+(\mathbf{41})$
1144	=	$3(\mathbf{1})+5(\mathbf{5})+3(\mathbf{7})+7(\mathbf{9})+4(\mathbf{11})+8(\mathbf{13})+5(\mathbf{15})+7(\mathbf{17})+5(\mathbf{19})+6(\mathbf{21})+3(\mathbf{23})+5(\mathbf{25})+$ $2(\mathbf{27})+3(\mathbf{29})+(\mathbf{31})+2(\mathbf{33})+(\mathbf{37})$
1408	=	$2(\mathbf{2})+5(\mathbf{4})+6(\mathbf{6})+7(\mathbf{8})+9(\mathbf{10})+9(\mathbf{12})+9(\mathbf{14})+9(\mathbf{16})+8(\mathbf{18})+7(\mathbf{20})+6(\mathbf{22})+5(\mathbf{24})+$ $3(\mathbf{26})+3(\mathbf{28})+2(\mathbf{30})+(\mathbf{32})+(\mathbf{34})$
1430	=	$4(\mathbf{3})+4(\mathbf{5})+7(\mathbf{7})+7(\mathbf{9})+9(\mathbf{11})+8(\mathbf{13})+9(\mathbf{15})+8(\mathbf{17})+8(\mathbf{19})+6(\mathbf{21})+6(\mathbf{23})+4(\mathbf{25})+$ $4(\mathbf{27})+2(\mathbf{29})+2(\mathbf{31})+(\mathbf{33})+(\mathbf{35})$
1760	=	$2(\mathbf{2})+5(\mathbf{4})+6(\mathbf{6})+8(\mathbf{8})+9(\mathbf{10})+10(\mathbf{12})+10(\mathbf{14})+10(\mathbf{16})+10(\mathbf{18})+8(\mathbf{20})+8(\mathbf{22})+6(\mathbf{24})+$ $5(\mathbf{26})+4(\mathbf{28})+3(\mathbf{30})+2(\mathbf{32})+(\mathbf{34})+(\mathbf{36})$
2025	=	$4(\mathbf{3})+4(\mathbf{5})+8(\mathbf{7})+7(\mathbf{9})+10(\mathbf{11})+9(\mathbf{13})+11(\mathbf{15})+9(\mathbf{17})+10(\mathbf{19})+8(\mathbf{21})+9(\mathbf{23})+6(\mathbf{25})+$ $6(\mathbf{27})+4(\mathbf{29})+4(\mathbf{31})+2(\mathbf{33})+2(\mathbf{35})+(\mathbf{37})+(\mathbf{39})$
2717	=	$3(\mathbf{3})+3(\mathbf{5})+5(\mathbf{7})+5(\mathbf{9})+8(\mathbf{11})+6(\mathbf{13})+9(\mathbf{15})+8(\mathbf{17})+9(\mathbf{19})+8(\mathbf{21})+9(\mathbf{23})+7(\mathbf{25})+$ $8(\mathbf{27})+6(\mathbf{29})+6(\mathbf{31})+5(\mathbf{33})+5(\mathbf{35})+3(\mathbf{37})+3(\mathbf{39})+2(\mathbf{41})+2(\mathbf{43})+(\mathbf{45})+(\mathbf{47})+(\mathbf{51})$
3003	=	$(\mathbf{1})+6(\mathbf{3})+9(\mathbf{5})+12(\mathbf{7})+14(\mathbf{9})+16(\mathbf{11})+16(\mathbf{13})+17(\mathbf{15})+16(\mathbf{17})+15(\mathbf{19})+13(\mathbf{21})+$ $12(\mathbf{23})+9(\mathbf{25})+8(\mathbf{27})+6(\mathbf{29})+4(\mathbf{31})+3(\mathbf{33})+2(\mathbf{35})+(\mathbf{37})+(\mathbf{39})$

Table 10.15: SO(14) Branching Rules

SO(14)	→	Sp(4) (S)
14	=	$(\mathbf{14})$
64	=	$(\mathbf{64})$
91	=	$(\mathbf{10})+(\mathbf{81})$
104	=	$(\mathbf{14})+(\mathbf{35'})+(\mathbf{55})$
364	=	$(\mathbf{10})+(\mathbf{35})+(\mathbf{81})+(\mathbf{84})+(\mathbf{154})$
546	=	$(\mathbf{1})+(\mathbf{14})+(\mathbf{35'})+(\mathbf{55})+(\mathbf{81})+(\mathbf{140''})+(\mathbf{220})$
832	=	$(\mathbf{16})+(\mathbf{20})+(\mathbf{40})+(\mathbf{56})+(\mathbf{64})+(\mathbf{80})+(\mathbf{140})+(\mathbf{160})+(\mathbf{256})$
896	=	$(\mathbf{10})+(\mathbf{14})+(\mathbf{35})+(\mathbf{35'})+(\mathbf{55})+2(\mathbf{81})+(\mathbf{105})+(\mathbf{220})+(\mathbf{260})$
1001	=	$(\mathbf{5})+(\mathbf{14})+(\mathbf{30})+(\mathbf{35})+(\mathbf{35'})+(\mathbf{81})+(\mathbf{91})+(\mathbf{105})+(\mathbf{154})+(\mathbf{220})+(\mathbf{231})$
1716	=	$(\mathbf{10})+(\mathbf{35})+2(\mathbf{81})+(\mathbf{84})+(\mathbf{105})+(\mathbf{154})+(\mathbf{220})+(\mathbf{231})+(\mathbf{260})+(\mathbf{455})$
2002	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{14})+2(\mathbf{30})+(\mathbf{35})+2(\mathbf{35'})+(\mathbf{55})+(\mathbf{81})+(\mathbf{91})+2(\mathbf{105})+(\mathbf{154})+(\mathbf{165})+2(\mathbf{220})+$ $(\mathbf{231})+(\mathbf{390})$
2275	=	$(\mathbf{1})+2(\mathbf{14})+(\mathbf{35'})+2(\mathbf{55})+(\mathbf{81})+(\mathbf{105})+(\mathbf{140''})+(\mathbf{165})+2(\mathbf{220})+(\mathbf{260})+(\mathbf{285})+(\mathbf{625})$
3003	=	$(\mathbf{10})+(\mathbf{14})+(\mathbf{30})+2(\mathbf{35})+(\mathbf{35'})+(\mathbf{55})+2(\mathbf{81})+2(\mathbf{84})+2(\mathbf{105})+2(\mathbf{154})+(\mathbf{165})+2(\mathbf{220})+$ $(\mathbf{231})+(\mathbf{260})+(\mathbf{390})+(\mathbf{455})$
3080	=	$(\mathbf{1})+(\mathbf{5})+2(\mathbf{14})+(\mathbf{30})+(\mathbf{35})+3(\mathbf{35'})+2(\mathbf{55})+2(\mathbf{81})+2(\mathbf{105})+(\mathbf{140''})+(\mathbf{154})+(\mathbf{165})+$ $3(\mathbf{220})+(\mathbf{260})+(\mathbf{390})+(\mathbf{625})$
4004	=	$2(\mathbf{10})+(\mathbf{14})+(\mathbf{30})+4(\mathbf{35})+(\mathbf{35'})+(\mathbf{55})+4(\mathbf{81})+2(\mathbf{84})+3(\mathbf{105})+3(\mathbf{154})+2(\mathbf{220})+(\mathbf{231})+$ $2(\mathbf{260})+(\mathbf{390})+(\mathbf{405})+(\mathbf{455})$
4928	=	$(\mathbf{4})+2(\mathbf{16})+2(\mathbf{20})+3(\mathbf{40})+2(\mathbf{56})+4(\mathbf{64})+3(\mathbf{80})+(\mathbf{120})+4(\mathbf{140})+2(\mathbf{140'})+3(\mathbf{160})+$ $(\mathbf{224})+2(\mathbf{256})+(\mathbf{320})+2(\mathbf{324})+(\mathbf{420})+(\mathbf{560})$
5265	=	$2(\mathbf{10})+2(\mathbf{14})+2(\mathbf{35})+2(\mathbf{35'})+2(\mathbf{55})+5(\mathbf{81})+2(\mathbf{84})+2(\mathbf{105})+(\mathbf{140''})+2(\mathbf{154})+3(\mathbf{220})+$ $(\mathbf{231})+3(\mathbf{260})+(\mathbf{390})+(\mathbf{455})+(\mathbf{595})+(\mathbf{625})$
5824	=	$(\mathbf{4})+2(\mathbf{16})+2(\mathbf{20})+3(\mathbf{40})+2(\mathbf{56})+4(\mathbf{64})+3(\mathbf{80})+(\mathbf{120})+4(\mathbf{140})+2(\mathbf{140'})+3(\mathbf{160})+$ $(\mathbf{224})+3(\mathbf{256})+(\mathbf{320})+2(\mathbf{324})+(\mathbf{420})+(\mathbf{560})+(\mathbf{640})$

Table 10.15: SO(14) Branching Rules (continued)

SO(14)	→	Sp(6) (S)
14	=	(**14**)
64	=	(**64**)
91	=	(**21**) + (**70**)
104	=	(**14**) + (**90**)
364	=	(**21**) + (**70**) + (**84**) + (**189**)
546	=	(**1**) + (**70**) + (**90**) + (**385**)
832	=	(**6**) + (**14′**) + (**56**) + (**64**) + (**126**) + (**216**) + (**350**)
896	=	(**14**) + (**21**) + (**70**) + (**90**) + (**189**) + (**512**)
1001	=	(**14**) + (**70**) + (**90**) + (**126′**) + (**189**) + (**512**)
1716	=	(**21**) + (**70**) + (**189**) + (**512**) + (**924**)
2002	=	(**1**) + (**14**) + (**70**) + 2(**90**) + (**126′**) + (**189**) + (**385**) + (**512**) + (**525**)
2275	=	(**14**) + (**90**) + (**385**) + (**512**) + (**1274**)
3003	=	(**14**) + (**21**) + (**70**) + (**84**) + (**90**) + 2(**189**) + (**385**) + (**512**) + (**525**) + (**924**)
3080	=	(**1**) + (**14**) + (**70**) + 2(**90**) + (**126′**) + (**189**) + (**385**) + (**512**) + (**525**) + (**1078**)
4004	=	(**14**) + 2(**21**) + 2(**70**) + (**84**) + (**90**) + 3(**189**) + 2(**512**) + (**525**) + (**594**) + (**924**)
4928	=	(**6**) + (**14′**) + (**56**) + 3(**64**) + 2(**126**) + 2(**216**) + 2(**350**) + (**378**) + 2(**448**) + (**616**) + (**1386**)
5265	=	(**14**) + (**21**) + 2(**70**) + (**84**) + (**90**) + 2(**189**) + (**385**) + 2(**512**) + (**924**) + (**2205**)
5824	=	(**6**) + (**14′**) + (**56**) + 3(**64**) + 2(**126**) + 2(**216**) + 2(**350**) + (**378**) + (**448**) + (**616**) + (**1344**) + (**1386**)

SO(14)	→	G_2 (S)
14	=	(**14**)
64	=	(**64**)
91	=	(**14**) + (**77**)
104	=	(**27**) + (**77′**)
364	=	(**1**) + (**27**) + (**77**) + (**77′**) + (**182**)
546	=	(**7**) + (**77**) + (**189**) + (**273**)
832	=	(**7**) + (**27**) + (**64**) + (**77**) + (**182**) + (**189**) + (**286**)
896	=	(**14**) + (**27**) + (**64**) + (**77**) + (**77′**) + (**189**) + (**448**)
1001	=	(**14**) + (**27**) + (**64**) + (**77′**) + (**182**) + (**189**) + (**448**)
1716	=	(**27**) + (**64**) + (**77′**) + (**182**) + (**189**) + (**448**) + (**729**)
2002	=	(**7**) + (**14**) + (**64**) + 2(**77**) + 2(**189**) + (**273**) + (**286**) + (**378**) + (**448**)
2275	=	(**27**) + (**64**) + (**77′**) + (**182**) + (**448**) + (**729**) + (**748**)
3003	=	(**7**) + (**27**) + (**64**) + 2(**77**) + (**77′**) + (**182**) + 2(**189**) + (**273**) + (**286**) + (**378**) + (**448**) + (**729**)
3080	=	(**1**) + 2(**27**) + (**64**) + (**77**) + 2(**77′**) + 2(**182**) + (**189**) + (**286**) + (**448**) + (**714**) + (**729**)
4004	=	(**7**)+2(**14**)+(**27**)+2(**64**)+3(**77**)+(**77′**)+(**182**)+3(**189**)+(**273**)+(**286**)+(**378**)+2(**448**)+(**924**)
4928	=	(**7**)+(**14**)+2(**27**)+3(**64**)+2(**77**)+(**77′**)+2(**182**)+3(**189**)+2(**286**)+(**378**)+2(**448**)+(**729**)+(**924**)
5265	=	(**7**) + (**14**) + (**27**) + 2(**64**) + 2(**77**) + (**77′**) + (**182**) + 3(**189**) + (**273**) + (**286**) + (**378**) + 2(**448**) + (**729**) + (**1547**)
5824	=	(**7**) + (**14**) + 2(**27**) + 3(**64**) + 2(**77**) + (**77′**) + 2(**182**) + 3(**189**) + 2(**286**) + (**378**) + 2(**448**) + (**729**) + (**896**) + (**924**)

SO(14)	→	SO(13) (S)
14	=	(**1**) + (**13**)
64	=	(**64**)
91	=	(**13**) + (**78**)
104	=	(**1**) + (**13**) + (**90**)
364	=	(**78**) + (**286**)
546	=	(**1**) + (**13**) + (**90**) + (**442**)
832	=	(**64**) + (**768**)
896	=	(**13**) + (**78**) + (**90**) + (**715**)

Table 10.15: SO(14) Branching Rules (continued)

1001	=	(286) + ($\overline{715}$)
1716	=	(1716)
2002	=	($\overline{715}$) + (1287)
2275	=	(1) + (13) + (90) + (442) + (1729)
3003	=	(1287) + (1716)
3080	=	(90) + (715) + (2275)
4004	=	(78) + (286) + (715) + (2925)
4928	=	(768) + (4160)
5265	=	(13) + (78) + (90) + (442) + (715) + (3927)
5824	=	(64) + (768) + (4992)

SO(14)	→	SU(2)×SO(11) (S)
14	=	(3,1) + (1,11)
64	=	(2,32)
91	=	(3,1) + (3,11) + (1,55)
104	=	(1,1) + (5,1) + (3,11) + (1,65)
364	=	(1,1) + (3,11) + (3,55) + (1,165)
546	=	(3,1) + (7,1) + (1,11) + (5,11) + (3,65) + (1,275)
832	=	(2,32) + (4,32) + (2,320)
896	=	(3,1) + (5,1) + (1,11) + (3,11) + (5,11) + (3,55) + (3,65) + (1,429)
1001	=	(1,11) + (3,55) + (3,165) + (1,330)
1716	=	(1,330) + (3,462)
2002	=	(1,55) + (3,165) + (3,330) + (1,462)
2275	=	(1,1) + (5,1) + (9,1) + (3,11) + (7,11) + (1,65) + (5,65) + (3,275) + (1,935)
3003	=	(1,165) + (3,330) + (1,462) + (3,462)
3080	=	(1,1) + (5,1) + (3,11) + (5,11) + (3,55) + (1,65) + (5,65) + (3,429) + (1,1144)
4004	=	(3,1) + (1,11) + (3,11) + (5,11) + (1,55) + (3,55) + (5,55) + (3,65) + (3,165) + (3,429) + (1,1430)
4928	=	(2,32) + (4,32) + (2,320) + (4,320) + (2,1408)
5265	=	(3,1) + (5,1) + (7,1) + 2(3,11) + (5,11) + (7,11) + (1,55) + (5,55) + (1,65) + (3,65) + (5,65) + (3,275) + (3,429) + (1,2025)
5824	=	(2,32) + (4,32) + (6,32) + (2,320) + (4,320) + (2,1760)

SO(14)	→	Sp(4)×SO(9) (S)
14	=	(5,1) + (1,9)
64	=	(4,16)
91	=	(10,1) + (5,9) + (1,36)
104	=	(1,1) + (5,9) + (14,1) + (1,44)
364	=	(10,1) + (10,9) + (5,36) + (1,84)
546	=	(5,1) + (1,9) + (14,9) + (30,1) + (5,44) + (1,156)
832	=	(4,16) + (16,16) + (4,128)
896	=	(5,1) + (1,9) + (10,9) + (14,9) + (35,1) + (5,36) + (5,44) + (1,231)
1001	=	(5,1) + (10,9) + (10,36) + (5,84) + (1,126)
1716	=	(1,36) + (5,84) + (10,126)
2002	=	(1,1) + (5,9) + (10,36) + (10,84) + (1,126) + (5,126)
2275	=	(1,1) + (5,9) + (14,1) + (30,9) + (1,44) + (55,1) + (14,44) + (5,156) + (1,450)
3003	=	(1,9) + (5,36) + (1,84) + (10,84) + (5,126) + (10,126)
3080	=	(1,1) + (5,9) + (14,1) + ($35'$,1) + (35,9) + (1,44) + (10,36) + (14,44) + (5,231) + (1,495)
4004	=	(10,1) + (5,9) + (10,9) + (35,1) + (1,36) + (35,9) + (10,36) + (14,36) + (10,44) + (5,84) + (5,231) + (1,594)
4928	=	(4,16) + (16,16) + (20,16) + (4,128) + (16,128) + (4,432)

Table 10.15: SO(14) Branching Rules (continued)

5265	=	(10,1) + 2(5,9) + (14,1) + (1,36) + (30,9) + (35,9) + (1,44) + (14,36) + (10,44) + (14,44) + (81,1) + (5,156) + (5,231) + (1,910)
5824	=	(4,16) + (16,16) + (40,16) + (4,128) + (16,128) + (4,576)

SO(14)	→	SO(7)×SO(7) (S)
14	=	(7,1) + (1,7)
64	=	(8,8)
91	=	(7,7) + (21,1) + (1,21)
104	=	(1,1) + (7,7) + (27,1) + (1,27)
364	=	(7,21) + (21,7) + (35,1) + (1,35)
546	=	(7,1) + (1,7) + (27,7) + (7,27) + (77,1) + (1,77)
832	=	(8,8) + (48,8) + (8,48)
896	=	(7,1) + (1,7) + (7,21) + (21,7) + (27,7) + (7,27) + (105,1) + (1,105)
1001	=	(35,1) + (1,35) + (21,21) + (7,35) + (35,7)
1716	=	(1,1) + (7,7) + (21,21) + (35,35)
2002	=	(21,1) + (1,21) + (7,35) + (35,7) + (21,35) + (35,21)
2275	=	(1,1) + (7,7) + (27,1) + (1,27) + (27,27) + (77,7) + (7,77) + (182,1) + (1,182)
3003	=	(7,1) + (1,7) + (7,21) + (21,7) + (21,35) + (35,21) + (35,35)
3080	=	(1,1) + (7,7) + (27,1) + (1,27) + (21,21) + (27,27) + (105,7) + (7,105) + (168′,1) + (1,168′)
4004	=	(7,7) + (21,1) + (1,21) + (21,21) + (7,35) + (35,7) + (27,21) + (21,27) + (105,7) + (7,105) + (189,1) + (1,189)
4928	=	(8,8) + (48,8) + (8,48) + (48,48) + (112,8) + (8,112)
5265	=	2(7,7) + (21,1) + (1,21) + (27,1) + (1,27) + (27,21) + (21,27) + (27,27) + (77,7) + (7,77) + (105,7) + (7,105) + (330,1) + (1,330)
5824	=	(8,8) + (48,8) + (8,48) + (48,48) + (168,8) + (8,168)

Table 10.16: SO(15) Branching Rules

SO(15)	→	SU(2) (S)
15	=	(15)
105	=	(3) + (7) + (11) + (15) + (19) + (23) + (27)
119	=	(5) + (9) + (13) + (17) + (21) + (25) + (29)
128	=	(5) + (9) + (11) + (15) + (17) + (19) + (23) + (29)
455	=	(1) + (5) + (7) + 2(9) + (11) + 3(13) + 2(15) + 2(17) + 2(19) + 2(21) + (23) + 2(25) + (27) + (29) + (31) + (33) + (37)
665	=	(3) + 2(7) + (9) + 2(11) + 2(13) + 2(15) + 2(17) + 3(19) + 2(21) + 2(23) + 2(25) + 2(27) + (29) + 2(31) + (33) + (35) + (37) + (39) + (43)
1105	=	(3) + 2(5) + 2(7) + 3(9) + 4(11) + 4(13) + 4(15) + 5(17) + 4(19) + 4(21) + 4(23) + 3(25) + 3(27) + 3(29) + 2(31) + 2(33) + 2(35) + (37) + (39) + (41)
1365	=	2(1) + 4(5) + 2(7) + 5(9) + 3(11) + 6(13) + 4(15) + 6(17) + 4(19) + 6(21) + 4(23) + 5(25) + 3(27) + 4(29) + 2(31) + 3(33) + (35) + 2(37) + (39) + (41) + (45)
1792	=	(1) + 2(3) + 3(5) + 5(7) + 5(9) + 6(11) + 7(13) + 7(15) + 7(17) + 7(19) + 7(21) + 6(23) + 6(25) + 5(27) + 4(29) + 4(31) + 3(33) + 2(35) + 2(37) + (39) + (41) + (43)
2940	=	2(1) + 4(5) + 2(7) + 6(9) + 4(11) + 7(13) + 5(15) + 8(17) + 6(19) + 8(21) + 6(23) + 8(25) + 6(27) + 7(29) + 5(31) + 7(33) + 4(35) + 5(37) + 3(39) + 4(41) + 2(43) + 3(45) + (47) + 2(49) + (51) + (53) + (57)
3003	=	4(3) + 3(5) + 7(7) + 6(9) + 10(11) + 8(13) + 11(15) + 9(17) + 11(19) + 9(21) + 10(23) + 8(25) + 9(27) + 6(29) + 7(31) + 5(33) + 5(35) + 3(37) + 3(39) + 2(41) + 2(43) + (45) + (47) + (51)
4080	=	4(1) + 8(5) + 4(7) + 11(9) + 7(11) + 13(13) + 9(15) + 14(17) + 10(19) + 14(21) + 10(23) + 13(25) + 9(27) + 11(29) + 7(31) + 9(33) + 5(35) + 7(37) + 3(39) + 5(41) + 2(43) + 3(45) + (47) + 2(49) + (53)

Table 10.16: SO(15) Branching Rules (continued)

5005	=	$6(\mathbf{3})+4(\mathbf{5})+12(\mathbf{7})+9(\mathbf{9})+14(\mathbf{11})+13(\mathbf{13})+17(\mathbf{15})+13(\mathbf{17})+18(\mathbf{19})+14(\mathbf{21})+16(\mathbf{23})+13(\mathbf{25})+14(\mathbf{27})+10(\mathbf{29})+12(\mathbf{31})+8(\mathbf{33})+8(\mathbf{35})+6(\mathbf{37})+6(\mathbf{39})+3(\mathbf{41})+4(\mathbf{43})+2(\mathbf{45})+2(\mathbf{47})+(\mathbf{49})+(\mathbf{51})+(\mathbf{55})$
5355	=	$6(\mathbf{3})+6(\mathbf{5})+11(\mathbf{7})+11(\mathbf{9})+15(\mathbf{11})+14(\mathbf{13})+17(\mathbf{15})+16(\mathbf{17})+18(\mathbf{19})+16(\mathbf{21})+17(\mathbf{23})+15(\mathbf{25})+15(\mathbf{27})+12(\mathbf{29})+12(\mathbf{31})+9(\mathbf{33})+9(\mathbf{35})+6(\mathbf{37})+6(\mathbf{39})+4(\mathbf{41})+4(\mathbf{43})+2(\mathbf{45})+2(\mathbf{47})+(\mathbf{49})+(\mathbf{51})$
6435	=	$4(\mathbf{1})+3(\mathbf{3})+10(\mathbf{5})+9(\mathbf{7})+16(\mathbf{9})+14(\mathbf{11})+19(\mathbf{13})+17(\mathbf{15})+21(\mathbf{17})+18(\mathbf{19})+21(\mathbf{21})+17(\mathbf{23})+19(\mathbf{25})+15(\mathbf{27})+16(\mathbf{29})+12(\mathbf{31})+13(\mathbf{33})+9(\mathbf{35})+9(\mathbf{37})+6(\mathbf{39})+6(\mathbf{41})+4(\mathbf{43})+4(\mathbf{45})+2(\mathbf{47})+2(\mathbf{49})+(\mathbf{51})+(\mathbf{53})+(\mathbf{57})$
6916	=	$6(\mathbf{3})+6(\mathbf{5})+12(\mathbf{7})+11(\mathbf{9})+16(\mathbf{11})+15(\mathbf{13})+19(\mathbf{15})+17(\mathbf{17})+20(\mathbf{19})+18(\mathbf{21})+20(\mathbf{23})+17(\mathbf{25})+18(\mathbf{27})+15(\mathbf{29})+16(\mathbf{31})+12(\mathbf{33})+12(\mathbf{35})+9(\mathbf{37})+9(\mathbf{39})+6(\mathbf{41})+6(\mathbf{43})+4(\mathbf{45})+4(\mathbf{47})+2(\mathbf{49})+2(\mathbf{51})+(\mathbf{53})+(\mathbf{55})$
10948	=	$5(\mathbf{3})+6(\mathbf{5})+10(\mathbf{7})+10(\mathbf{9})+15(\mathbf{11})+14(\mathbf{13})+18(\mathbf{15})+17(\mathbf{17})+20(\mathbf{19})+19(\mathbf{21})+22(\mathbf{23})+19(\mathbf{25})+21(\mathbf{27})+19(\mathbf{29})+20(\mathbf{31})+17(\mathbf{33})+18(\mathbf{35})+15(\mathbf{37})+15(\mathbf{39})+13(\mathbf{41})+12(\mathbf{43})+10(\mathbf{45})+10(\mathbf{47})+7(\mathbf{49})+7(\mathbf{51})+5(\mathbf{53})+5(\mathbf{55})+3(\mathbf{57})+3(\mathbf{59})+2(\mathbf{61})+2(\mathbf{63})+(\mathbf{65})+(\mathbf{67})+(\mathbf{71})$
11520	=	$3(\mathbf{1})+9(\mathbf{3})+15(\mathbf{5})+20(\mathbf{7})+25(\mathbf{9})+29(\mathbf{11})+32(\mathbf{13})+34(\mathbf{15})+35(\mathbf{17})+36(\mathbf{19})+35(\mathbf{21})+34(\mathbf{23})+32(\mathbf{25})+30(\mathbf{27})+27(\mathbf{29})+24(\mathbf{31})+21(\mathbf{33})+18(\mathbf{35})+15(\mathbf{37})+12(\mathbf{39})+10(\mathbf{41})+7(\mathbf{43})+6(\mathbf{45})+4(\mathbf{47})+3(\mathbf{49})+2(\mathbf{51})+(\mathbf{53})+(\mathbf{55})$

SO(15)	$\rightarrow$	SU(4) (S)
15	=	$(\mathbf{15})$
105	=	$(\mathbf{15})+(\mathbf{45})+(\overline{\mathbf{45}})$
119	=	$(\mathbf{15})+(\mathbf{20'})+(\mathbf{84})$
128	=	$2(\mathbf{64})$
455	=	$(\mathbf{1})+(\mathbf{15})+(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{45})+(\overline{\mathbf{45}})+(\mathbf{84})+(\mathbf{175})$
665	=	$(\mathbf{1})+(\mathbf{15})+(\mathbf{45})+(\overline{\mathbf{45}})+(\mathbf{84})+(\mathbf{175})+(\mathbf{300'})$
1105	=	$2(\mathbf{15})+2(\mathbf{20'})+2(\mathbf{45})+2(\overline{\mathbf{45}})+2(\mathbf{84})+(\mathbf{175})+(\mathbf{256})+(\overline{\mathbf{256}})$
1365	=	$2(\mathbf{15})+2(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+(\mathbf{45})+(\overline{\mathbf{45}})+2(\mathbf{84})+(\mathbf{105})+2(\mathbf{175})+(\mathbf{256})+(\overline{\mathbf{256}})$
1792	=	$2(\mathbf{6})+2(\mathbf{10})+2(\overline{\mathbf{10}})+2(\mathbf{50})+4(\mathbf{64})+2(\mathbf{70})+2(\overline{\mathbf{70}})+2(\mathbf{126})+2(\overline{\mathbf{126}})+2(\mathbf{300})$
2940	=	$(\mathbf{1})+(\mathbf{15})+(\mathbf{20'})+(\mathbf{45})+(\overline{\mathbf{45}})+2(\mathbf{84})+(\mathbf{105})+(\mathbf{175})+(\mathbf{256})+(\overline{\mathbf{256}})+(\mathbf{300'})+(\mathbf{729})+(\mathbf{825})$
3003	=	$(\mathbf{1})+2(\mathbf{15})+(\mathbf{20'})+3(\mathbf{45})+3(\overline{\mathbf{45}})+2(\mathbf{84})+(\mathbf{105})+3(\mathbf{175})+2(\mathbf{256})+2(\overline{\mathbf{256}})+(\mathbf{280})+(\overline{\mathbf{280}})+(\mathbf{300'})$
4080	=	$(\mathbf{1})+2(\mathbf{15})+3(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+2(\mathbf{45})+2(\overline{\mathbf{45}})+4(\mathbf{84})+(\mathbf{105})+3(\mathbf{175})+2(\mathbf{256})+2(\overline{\mathbf{256}})+(\mathbf{300'})+(\mathbf{360'})+(\overline{\mathbf{360}}')+(\mathbf{729})$
5005	=	$3(\mathbf{15})+(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+3(\mathbf{45})+3(\overline{\mathbf{45}})+3(\mathbf{84})+5(\mathbf{175})+2(\mathbf{256})+2(\overline{\mathbf{256}})+2(\mathbf{280})+2(\overline{\mathbf{280}})+2(\mathbf{300'})+(\mathbf{729})$
5355	=	$(\mathbf{1})+4(\mathbf{15})+3(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+5(\mathbf{45})+5(\overline{\mathbf{45}})+4(\mathbf{84})+5(\mathbf{175})+(\mathbf{189})+(\overline{\mathbf{189}})+3(\mathbf{256})+3(\overline{\mathbf{256}})+(\mathbf{280})+(\overline{\mathbf{280}})+(\mathbf{300'})+(\mathbf{729})$
6435	=	$(\mathbf{1})+2(\mathbf{15})+3(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+3(\mathbf{45})+3(\overline{\mathbf{45}})+4(\mathbf{84})+2(\mathbf{105})+5(\mathbf{175})+3(\mathbf{256})+3(\overline{\mathbf{256}})+(\mathbf{280})+(\overline{\mathbf{280}})+(\mathbf{300'})+3(\mathbf{729})$
6916	=	$4(\mathbf{15})+2(\mathbf{20'})+(\mathbf{35})+(\overline{\mathbf{35}})+4(\mathbf{45})+4(\overline{\mathbf{45}})+4(\mathbf{84})+5(\mathbf{175})+3(\mathbf{256})+3(\overline{\mathbf{256}})+(\mathbf{280})+(\overline{\mathbf{280}})+2(\mathbf{300'})+(\mathbf{729})+(\mathbf{875})+(\overline{\mathbf{875}})$
10948	=	$2(\mathbf{15})+(\mathbf{20'})+(\mathbf{45})+(\overline{\mathbf{45}})+2(\mathbf{84})+2(\mathbf{175})+2(\mathbf{256})+2(\overline{\mathbf{256}})+(\mathbf{280})+(\overline{\mathbf{280}})+2(\mathbf{300'})+(\mathbf{729})+(\mathbf{735})+(\mathbf{825})+(\mathbf{875})+(\overline{\mathbf{875}})+(\mathbf{1911})+(\mathbf{2156})$
11520	=	$4(\mathbf{6})+4(\mathbf{10})+4(\overline{\mathbf{10}})+4(\mathbf{50})+12(\mathbf{64})+6(\mathbf{70})+6(\overline{\mathbf{70}})+6(\mathbf{126})+6(\overline{\mathbf{126}})+2(\mathbf{140''})+2(\overline{\mathbf{140}}'')+2(\mathbf{270})+2(\overline{\mathbf{270}})+8(\mathbf{300})+4(\mathbf{384})+2(\mathbf{630})+2(\overline{\mathbf{630}})$

SO(15)	$\rightarrow$	SU(2)×Sp(4) (S)
15	=	$(\mathbf{3},\mathbf{5})$
105	=	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{5},\mathbf{10})+(\mathbf{3},\mathbf{14})$
119	=	$(\mathbf{5},\mathbf{1})+(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{5},\mathbf{14})$
128	=	$(\mathbf{6},\mathbf{4})+(\mathbf{4},\mathbf{16})+(\mathbf{2},\mathbf{20})$

Table 10.16: SO(15) Branching Rules (continued)

455	=	(**1**, **5**) + (**3**, **5**) + (**5**, **5**) + (**3**, **10**) + (**7**, **10**) + (**1**, **30**) + (**3**, **35**) + (**5**, **35**)
665	=	(**3**, **5**) + (**5**, **5**) + (**1**, **10**) + (**7**, **5**) + (**3**, **30**) + (**7**, **30**) + (**3**, **35**) + (**5**, **35**)
1105	=	(**1**, **5**) + 2(**3**, **5**) + 2(**5**, **5**) + (**7**, **5**) + (**3**, **10**) + (**5**, **10**) + (**3**, **30**) + (**5**, **30**) + (**1**, **35**) + 2(**3**, **35**) + (**5**, **35**) + (**7**, **35**)
1365	=	(**1**, **1**) + (**1**, **5**) + (**5**, **1**) + (**5**, **5**) + 2(**3**, **10**) + (**9**, **5**) + (**1**, **14**) + (**5**, **10**) + (**3**, **14**) + (**7**, **10**) + (**5**, **14**) + (**1**, **35′**) + (**3**, **35**) + (**5**, **35**) + (**5**, **35′**) + (**7**, **35**) + (**3**, **81**)
1792	=	(**2**, **4**) + 2(**4**, **4**) + (**6**, **4**) + (**8**, **4**) + 2(**2**, **16**) + 2(**4**, **16**) + (**2**, **20**) + 2(**6**, **16**) + 2(**4**, **20**) + (**8**, **16**) + (**6**, **20**) + (**2**, **40**) + (**4**, **40**) + (**6**, **40**) + (**2**, **64**) + (**4**, **64**)
2940	=	(**1**, **1**) + (**5**, **1**) + (**9**, **1**) + (**3**, **10**) + (**1**, **14**) + (**5**, **10**) + (**3**, **14**) + (**7**, **10**) + 2(**5**, **14**) + (**7**, **14**) + (**9**, **14**) + (**1**, **35′**) + (**3**, **35**) + (**5**, **35′**) + (**1**, **55**) + (**5**, **55**) + (**9**, **55**) + (**3**, **81**) + (**5**, **81**) + (**7**, **81**)
3003	=	(**3**, **1**) + 2(**3**, **5**) + (**7**, **1**) + (**5**, **5**) + (**1**, **10**) + (**7**, **5**) + (**11**, **1**) + (**3**, **10**) + 2(**5**, **10**) + (**3**, **14**) + (**7**, **10**) + (**5**, **14**) + (**9**, **10**) + (**7**, **14**) + (**9**, **14**) + (**3**, **30**) + (**1**, **35**) + 2(**3**, **35**) + (**3**, **35′**) + 2(**5**, **35**) + (**5**, **35′**) + (**7**, **35**) + (**7**, **35′**) + (**1**, **81**) + (**5**, **81**) + (**3**, **105**)
4080	=	2(**1**, **1**) + (**1**, **5**) + 2(**5**, **1**) + (**5**, **5**) + (**9**, **1**) + 3(**3**, **10**) + 2(**1**, **14**) + 2(**5**, **10**) + 2(**3**, **14**) + 2(**7**, **10**) + 3(**5**, **14**) + (**7**, **14**) + (**9**, **14**) + 2(**1**, **35′**) + 2(**3**, **35**) + (**5**, **35**) + 2(**5**, **35′**) + (**7**, **35**) + (**9**, **35′**) + (**1**, **55**) + (**5**, **55**) + 2(**3**, **81**) + (**5**, **81**) + (**7**, **81**)
5005	=	(**3**, **1**) + 2(**3**, **5**) + (**7**, **1**) + (**5**, **5**) + 2(**1**, **10**) + 2(**7**, **5**) + (**3**, **10**) + (**9**, **5**) + 3(**5**, **10**) + (**11**, **5**) + 2(**3**, **14**) + (**7**, **10**) + (**5**, **14**) + (**9**, **10**) + (**7**, **14**) + (**3**, **30**) + (**1**, **35**) + (**7**, **30**) + 2(**3**, **35**) + 2(**3**, **35′**) + 3(**5**, **35**) + (**5**, **35′**) + 2(**7**, **35**) + (**7**, **35′**) + (**9**, **35**) + (**1**, **81**) + (**3**, **81**) + (**1**, **84**) + (**5**, **81**) + (**3**, **105**) + (**5**, **105**)
5355	=	2(**3**, **1**) + (**5**, **1**) + (**3**, **5**) + (**7**, **1**) + (**5**, **5**) + 2(**1**, **10**) + (**7**, **5**) + 4(**3**, **10**) + (**1**, **14**) + 4(**5**, **10**) + 4(**3**, **14**) + 2(**7**, **10**) + 3(**5**, **14**) + (**9**, **10**) + 2(**7**, **14**) + 2(**1**, **35**) + 2(**3**, **35**) + 2(**3**, **35′**) + 3(**5**, **35**) + (**5**, **35′**) + (**7**, **35**) + (**7**, **35′**) + (**9**, **35**) + (**3**, **55**) + (**1**, **81**) + 2(**3**, **81**) + 2(**5**, **81**) + (**7**, **81**)
6435	=	(**1**, **1**) + (**1**, **5**) + (**5**, **1**) + (**3**, **5**) + 2(**5**, **5**) + (**9**, **1**) + (**7**, **5**) + 3(**3**, **10**) + (**9**, **5**) + (**1**, **14**) + 2(**5**, **10**) + (**3**, **14**) + 3(**7**, **10**) + 2(**5**, **14**) + (**9**, **10**) + (**7**, **14**) + (**11**, **10**) + (**9**, **14**) + (**1**, **30**) + (**5**, **30**) + (**1**, **35**) + (**1**, **35′**) + 3(**3**, **35**) + (**3**, **35′**) + 3(**5**, **35**) + 2(**5**, **35′**) + 2(**7**, **35**) + (**7**, **35′**) + (**9**, **35**) + 2(**3**, **81**) + (**5**, **81**) + (**3**, **84**) + (**7**, **81**) + (**1**, **105**) + (**3**, **105**) + (**5**, **105**)
6916	=	2(**3**, **1**) + (**5**, **1**) + (**3**, **5**) + 2(**7**, **1**) + 2(**1**, **10**) + 3(**3**, **10**) + (**1**, **14**) + 4(**5**, **10**) + 4(**3**, **14**) + 2(**7**, **10**) + 4(**5**, **14**) + (**9**, **10**) + 3(**7**, **14**) + (**9**, **14**) + (**1**, **35**) + 2(**3**, **35**) + 2(**3**, **35′**) + 2(**5**, **35**) + (**5**, **35′**) + (**7**, **35**) + (**7**, **35′**) + (**3**, **55**) + (**5**, **55**) + (**7**, **55**) + 2(**1**, **81**) + 2(**3**, **81**) + 3(**5**, **81**) + (**7**, **81**) + (**9**, **81**)
10948	=	2(**3**, **5**) + (**5**, **5**) + 2(**7**, **5**) + (**9**, **5**) + (**5**, **10**) + (**11**, **5**) + 2(**3**, **30**) + 2(**5**, **30**) + (**1**, **35**) + 2(**7**, **30**) + 2(**3**, **35**) + (**3**, **35′**) + (**9**, **30**) + 2(**5**, **35**) + (**11**, **30**) + 2(**7**, **35**) + (**9**, **35**) + (**1**, **81**) + (**5**, **81**) + (**3**, **91**) + (**7**, **91**) + (**11**, **91**) + (**3**, **105**) + (**5**, **105**) + (**7**, **105**) + (**3**, **154**) + (**5**, **154**) + (**7**, **154**) + (**9**, **154**)
11520	=	3(**2**, **4**) + 4(**4**, **4**) + 4(**6**, **4**) + 2(**8**, **4**) + (**10**, **4**) + 5(**2**, **16**) + 7(**4**, **16**) + 4(**2**, **20**) + 6(**6**, **16**) + 5(**4**, **20**) + 3(**8**, **16**) + 4(**6**, **20**) + (**10**, **16**) + 2(**8**, **20**) + (**10**, **20**) + 3(**2**, **40**) + 5(**4**, **40**) + 3(**6**, **40**) + 2(**8**, **40**) + (**2**, **56**) + (**4**, **56**) + (**6**, **56**) + 3(**2**, **64**) + 4(**4**, **64**) + 3(**6**, **64**) + (**8**, **64**) + (**2**, **80**) + (**4**, **80**) + (**6**, **80**) + (**2**, **140**) + (**4**, **140**)

Table 10.17: SO(21) Branching Rules

SO(21)	→	SU(2) (S)
21	=	(**21**)
210	=	(**3**) + (**7**) + (**11**) + (**15**) + (**19**) + (**23**) + (**27**) + (**31**) + (**35**) + (**39**)
230	=	(**5**) + (**9**) + (**13**) + (**17**) + (**21**) + (**25**) + (**29**) + (**33**) + (**37**) + (**41**)
1024	=	(**2**) + (**6**) + 2(**8**) + (**10**) + 2(**12**) + 2(**14**) + 2(**16**) + 2(**18**) + 3(**20**) + 2(**22**) + 2(**24**) + 3(**26**) + 2(**28**) + 2(**30**) + 2(**32**) + (**34**) + 2(**36**) + 2(**38**) + (**40**) + (**42**) + (**44**) + (**46**) + (**50**) + (**56**)
1330	=	(**3**) + 2(**7**) + (**9**) + 2(**11**) + 2(**13**) + 3(**15**) + 2(**17**) + 4(**19**) + 3(**21**) + 3(**23**) + 3(**25**) + 3(**27**) + 2(**29**) + 3(**31**) + 2(**33**) + 2(**35**) + 2(**37**) + 2(**39**) + (**41**) + 2(**43**) + (**45**) + (**47**) + (**49**) + (**51**) + (**55**)
1750	=	(**1**) + (**5**) + (**7**) + 2(**9**) + (**11**) + 3(**13**) + 2(**15**) + 3(**17**) + 3(**19**) + 3(**21**) + 3(**23**) + 4(**25**) + 3(**27**) + 3(**29**) + 3(**31**) + 3(**33**) + 2(**35**) + 3(**37**) + 2(**39**) + 2(**41**) + 2(**43**) + 2(**45**) + (**47**) + 2(**49**) + (**51**) + (**53**) + (**55**) + (**57**) + (**61**)

Table 10.17: SO(21) Branching Rules (continued)

3059	=	$(\mathbf{3}) + 2(\mathbf{5}) + 2(\mathbf{7}) + 3(\mathbf{9}) + 4(\mathbf{11}) + 4(\mathbf{13}) + 5(\mathbf{15}) + 6(\mathbf{17}) + 6(\mathbf{19}) + 6(\mathbf{21}) + 7(\mathbf{23}) + 6(\mathbf{25}) + 6(\mathbf{27}) + 6(\mathbf{29}) + 5(\mathbf{31}) + 5(\mathbf{33}) + 5(\mathbf{35}) + 4(\mathbf{37}) + 4(\mathbf{39}) + 4(\mathbf{41}) + 3(\mathbf{43}) + 3(\mathbf{45}) + 3(\mathbf{47}) + 2(\mathbf{49}) + 2(\mathbf{51}) + 2(\mathbf{53}) + (\mathbf{55}) + (\mathbf{57}) + (\mathbf{59})$
5985	=	$3(\mathbf{1}) + 6(\mathbf{5}) + 3(\mathbf{7}) + 8(\mathbf{9}) + 5(\mathbf{11}) + 10(\mathbf{13}) + 7(\mathbf{15}) + 11(\mathbf{17}) + 8(\mathbf{19}) + 12(\mathbf{21}) + 9(\mathbf{23}) + 12(\mathbf{25}) + 9(\mathbf{27}) + 12(\mathbf{29}) + 9(\mathbf{31}) + 11(\mathbf{33}) + 8(\mathbf{35}) + 10(\mathbf{37}) + 7(\mathbf{39}) + 8(\mathbf{41}) + 5(\mathbf{43}) + 7(\mathbf{45}) + 4(\mathbf{47}) + 5(\mathbf{49}) + 3(\mathbf{51}) + 4(\mathbf{53}) + 2(\mathbf{55}) + 3(\mathbf{57}) + (\mathbf{59}) + 2(\mathbf{61}) + (\mathbf{63}) + (\mathbf{65}) + (\mathbf{69})$

SO(21)	→	SU(2)×SO(7) (S)
21	=	$(\mathbf{3},\mathbf{7})$
210	=	$(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{21}) + (\mathbf{5},\mathbf{21}) + (\mathbf{3},\mathbf{27})$
230	=	$(\mathbf{5},\mathbf{1}) + (\mathbf{3},\mathbf{21}) + (\mathbf{1},\mathbf{27}) + (\mathbf{5},\mathbf{27})$
1024	=	$(\mathbf{8},\mathbf{8}) + (\mathbf{6},\mathbf{48}) + (\mathbf{2},\mathbf{112'}) + (\mathbf{4},\mathbf{112})$
1330	=	$(\mathbf{1},\mathbf{7}) + (\mathbf{3},\mathbf{7}) + (\mathbf{5},\mathbf{7}) + (\mathbf{3},\mathbf{35}) + (\mathbf{7},\mathbf{35}) + (\mathbf{1},\mathbf{77}) + (\mathbf{3},\mathbf{105}) + (\mathbf{5},\mathbf{105})$
1750	=	$(\mathbf{3},\mathbf{7}) + (\mathbf{5},\mathbf{7}) + (\mathbf{7},\mathbf{7}) + (\mathbf{1},\mathbf{35}) + (\mathbf{3},\mathbf{77}) + (\mathbf{7},\mathbf{77}) + (\mathbf{3},\mathbf{105}) + (\mathbf{5},\mathbf{105})$
3059	=	$(\mathbf{1},\mathbf{7}) + 2(\mathbf{3},\mathbf{7}) + 2(\mathbf{5},\mathbf{7}) + (\mathbf{7},\mathbf{7}) + (\mathbf{3},\mathbf{35}) + (\mathbf{5},\mathbf{35}) + (\mathbf{3},\mathbf{77}) + (\mathbf{5},\mathbf{77}) + (\mathbf{1},\mathbf{105}) + 2(\mathbf{3},\mathbf{105}) + (\mathbf{5},\mathbf{105}) + (\mathbf{7},\mathbf{105})$
5985	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{5},\mathbf{1}) + 2(\mathbf{3},\mathbf{21}) + (\mathbf{5},\mathbf{21}) + (\mathbf{1},\mathbf{27}) + (\mathbf{7},\mathbf{21}) + (\mathbf{3},\mathbf{27}) + (\mathbf{5},\mathbf{27}) + (\mathbf{1},\mathbf{35}) + (\mathbf{5},\mathbf{35}) + (\mathbf{9},\mathbf{35}) + (\mathbf{1},\mathbf{168'}) + (\mathbf{5},\mathbf{168'}) + (\mathbf{3},\mathbf{189}) + (\mathbf{5},\mathbf{189}) + (\mathbf{7},\mathbf{189}) + (\mathbf{3},\mathbf{330})$

SO(21)	→	SO(7) (S)
21	=	$(\mathbf{21})$
210	=	$(\mathbf{21}) + (\mathbf{189})$
230	=	$(\mathbf{27}) + (\mathbf{35}) + (\mathbf{168'})$
1024	=	$2(\mathbf{512})$
1330	=	$(\mathbf{1}) + (\mathbf{27}) + (\mathbf{35}) + (\mathbf{168'}) + (\mathbf{189}) + (\mathbf{294}) + (\mathbf{616})$
1750	=	$(\mathbf{7}) + (\mathbf{21}) + (\mathbf{189}) + (\mathbf{330}) + (\mathbf{378}) + (\mathbf{825})$
3059	=	$(\mathbf{21}) + (\mathbf{27}) + (\mathbf{35}) + (\mathbf{105}) + (\mathbf{168'}) + 2(\mathbf{189}) + (\mathbf{330}) + (\mathbf{378}) + (\mathbf{1617})$
5985	=	$(\mathbf{21})+(\mathbf{27})+(\mathbf{35})+(\mathbf{105})+(\mathbf{168'})+(\mathbf{189})+(\mathbf{294})+(\mathbf{330})+(\mathbf{378})+(\mathbf{616})+(\mathbf{819})+(\mathbf{1386})+(\mathbf{1617})$

SO(21)	→	Sp(6) (S)
21	=	$(\mathbf{21})$
210	=	$(\mathbf{21}) + (\mathbf{189})$
230	=	$(\mathbf{14}) + (\mathbf{90}) + (\mathbf{126'})$
1024	=	$2(\mathbf{512})$
1330	=	$(\mathbf{1}) + (\mathbf{14}) + (\mathbf{90}) + (\mathbf{126'}) + (\mathbf{189}) + (\mathbf{385}) + (\mathbf{525})$
1750	=	$(\mathbf{21}) + (\mathbf{70}) + (\mathbf{84}) + (\mathbf{189}) + (\mathbf{462}) + (\mathbf{924})$
3059	=	$(\mathbf{14}) + (\mathbf{21}) + (\mathbf{70}) + (\mathbf{90}) + (\mathbf{126'}) + 2(\mathbf{189}) + (\mathbf{512}) + (\mathbf{924}) + (\mathbf{924'})$
5985	=	$(\mathbf{14}) + (\mathbf{21}) + (\mathbf{70}) + (\mathbf{90}) + (\mathbf{126'}) + (\mathbf{189}) + (\mathbf{385}) + (\mathbf{512}) + (\mathbf{525}) + (\mathbf{924}) + (\mathbf{924'}) + (\mathbf{2205})$

Table 10.18: SO(26) Branching Rules

SO(26)	→	SO(11)×SO(15) (S)
26	=	$(\mathbf{11},\mathbf{1}) + (\mathbf{1},\mathbf{15})$
325	=	$(\mathbf{11},\mathbf{15}) + (\mathbf{55},\mathbf{1}) + (\mathbf{1},\mathbf{105})$
2600	=	$(\mathbf{55},\mathbf{15}) + (\mathbf{11},\mathbf{105}) + (\mathbf{165},\mathbf{1}) + (\mathbf{1},\mathbf{455})$
4096	=	$(\mathbf{32},\mathbf{128})$
5824	=	$(\mathbf{11},\mathbf{1}) + (\mathbf{1},\mathbf{15}) + (\mathbf{55},\mathbf{15}) + (\mathbf{65},\mathbf{15}) + (\mathbf{11},\mathbf{105}) + (\mathbf{11},\mathbf{119}) + (\mathbf{429},\mathbf{1}) + (\mathbf{1},\mathbf{1105})$
14950	=	$(\mathbf{55},\mathbf{105}) + (\mathbf{165},\mathbf{15}) + (\mathbf{330},\mathbf{1}) + (\mathbf{11},\mathbf{455}) + (\mathbf{1},\mathbf{1365})$
52325	=	$(\mathbf{11},\mathbf{15}) + (\mathbf{55},\mathbf{1}) + (\mathbf{1},\mathbf{105}) + (\mathbf{55},\mathbf{105}) + (\mathbf{65},\mathbf{105}) + (\mathbf{55},\mathbf{119}) + (\mathbf{165},\mathbf{15}) + (\mathbf{429},\mathbf{15}) + (\mathbf{11},\mathbf{455}) + (\mathbf{11},\mathbf{1105}) + (\mathbf{1430},\mathbf{1}) + (\mathbf{1},\mathbf{5355})$

Table 10.18: SO(26) Branching Rules (continued)

65780	=	(165, 105) + (330, 15) + (462, 1) + (55, 455) + (11, 1365) + (1, 3003)

SO(26)	→	SO(13)×SO(13) (S)
26	=	(13, 1) + (1, 13)
325	=	(13, 13) + (78, 1) + (1, 78)
2600	=	(13, 78) + (78, 13) + (286, 1) + (1, 286)
4096	=	(64, 64)
5824	=	(13, 1) + (1, 13) + (13, 78) + (78, 13) + (90, 13) + (13, 90) + (715, 1) + (1, 715)
14950	=	(78, 78) + (13, 286) + (286, 13) + ($\overline{715}$, 1) + (1, $\overline{715}$)
52325	=	(13, 13) + (78, 1) + (1, 78) + (78, 78) + (90, 78) + (78, 90) + (13, 286) + (286, 13) + (715, 13) + (13, 715) + (2925, 1) + (1, 2925)
65780	=	(78, 286) + (286, 78) + (13, $\overline{715}$) + ($\overline{715}$, 13) + (1287, 1) + (1, 1287)

SO(26)	→	F_4 (S)
26	=	(26)
325	=	(52) + (273)
2600	=	(273) + (1053) + (1274)
4096	=	(4096)
5824	=	(26) + (52) + (273) + (324) + (1053) + (4096)
14950	=	(324) + (1053) + (1053′) + (4096) + (8424)
52325	=	(26) + (52) + 2(273) + (324) + 2(1053) + 2(1274) + 2(4096) + (8424) + (10829) + (19278)
65780	=	(324) + (1053) + (1053′) + (2652) + (4096) + (8424) + (10829) + (17901) + (19448)

10.0.3. Sp(n)

Table 10.19: Sp(4) Branching Rules

Sp(4)	→	SU(2)×SU(2) (R)
4	=	(2, 1) + (1, 2)
5	=	(1, 1) + (2, 2)
10	=	(2, 2) + (3, 1) + (1, 3)
14	=	(1, 1) + (2, 2) + (3, 3)
16	=	(2, 1) + (1, 2) + (3, 2) + (2, 3)
20	=	(3, 2) + (2, 3) + (4, 1) + (1, 4)
30	=	(1, 1) + (2, 2) + (3, 3) + (4, 4)
35	=	(2, 2) + (3, 1) + (1, 3) + (3, 3) + (4, 2) + (2, 4)
35′	=	(3, 3) + (4, 2) + (2, 4) + (5, 1) + (1, 5)
40	=	(2, 1) + (1, 2) + (3, 2) + (2, 3) + (4, 3) + (3, 4)
55	=	(1, 1) + (2, 2) + (3, 3) + (4, 4) + (5, 5)
56	=	(4, 3) + (3, 4) + (5, 2) + (2, 5) + (6, 1) + (1, 6)
64	=	(3, 2) + (2, 3) + (4, 1) + (1, 4) + (4, 3) + (3, 4) + (5, 2) + (2, 5)
80	=	(2, 1) + (1, 2) + (3, 2) + (2, 3) + (4, 3) + (3, 4) + (5, 4) + (4, 5)
81	=	(2, 2) + (3, 1) + (1, 3) + (3, 3) + (4, 2) + (2, 4) + (4, 4) + (5, 3) + (3, 5)
84	=	(4, 4) + (5, 3) + (3, 5) + (6, 2) + (2, 6) + (7, 1) + (1, 7)
91	=	(1, 1) + (2, 2) + (3, 3) + (4, 4) + (5, 5) + (6, 6)
105	=	(3, 3) + (4, 2) + (2, 4) + (5, 1) + (1, 5) + (4, 4) + (5, 3) + (3, 5) + (6, 2) + (2, 6)

Sp(4)	→	SU(2) (S)
4	=	(4)
5	=	(5)

Table 10.19: Sp(4) Branching Rules (continued)

10	=	$(\mathbf{3})+(\mathbf{7})$
14	=	$(\mathbf{5})+(\mathbf{9})$
16	=	$(\mathbf{2})+(\mathbf{6})+(\mathbf{8})$
20	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{10})$
30	=	$(\mathbf{1})+(\mathbf{7})+(\mathbf{9})+(\mathbf{13})$
35	=	$(\mathbf{3})+(\mathbf{5})+(\mathbf{7})+(\mathbf{9})+(\mathbf{11})$
$\mathbf{35'}$	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{7})+(\mathbf{9})+(\mathbf{13})$
40	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+(\mathbf{10})+(\mathbf{12})$
55	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})+(\mathbf{17})$
56	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{16})$
64	=	$(\mathbf{2})+(\mathbf{4})+(\mathbf{6})+2(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{14})$
80	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+2(\mathbf{10})+(\mathbf{12})+(\mathbf{14})+(\mathbf{16})$
81	=	$(\mathbf{3})+(\mathbf{5})+2(\mathbf{7})+(\mathbf{9})+2(\mathbf{11})+(\mathbf{13})+(\mathbf{15})$
84	=	$(\mathbf{3})+2(\mathbf{7})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})+(\mathbf{15})+(\mathbf{19})$
91	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})+(\mathbf{15})+(\mathbf{17})+(\mathbf{21})$
105	=	$(\mathbf{3})+2(\mathbf{5})+(\mathbf{7})+2(\mathbf{9})+2(\mathbf{11})+(\mathbf{13})+(\mathbf{15})+(\mathbf{17})$

Table 10.20: Sp(6) Branching Rules

Sp(6)	→	SU(2)×Sp(4) (R)
6	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{4})$
14	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{2},\mathbf{4})$
$\mathbf{14'}$	=	$(\mathbf{1},\mathbf{4})+(\mathbf{2},\mathbf{5})$
21	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{4})+(\mathbf{1},\mathbf{10})$
56	=	$(\mathbf{4},\mathbf{1})+(\mathbf{3},\mathbf{4})+(\mathbf{2},\mathbf{10})+(\mathbf{1},\mathbf{20})$
64	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{2},\mathbf{5})+(\mathbf{3},\mathbf{4})+(\mathbf{2},\mathbf{10})+(\mathbf{1},\mathbf{16})$
70	=	$(\mathbf{1},\mathbf{5})+(\mathbf{2},\mathbf{4})+(\mathbf{3},\mathbf{5})+(\mathbf{1},\mathbf{10})+(\mathbf{2},\mathbf{16})$
84	=	$(\mathbf{1},\mathbf{10})+(\mathbf{3},\mathbf{14})+(\mathbf{2},\mathbf{16})$
90	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{2},\mathbf{4})+(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{16})$
126	=	$(\mathbf{1},\mathbf{4})+(\mathbf{2},\mathbf{5})+(\mathbf{2},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{16})+(\mathbf{3},\mathbf{16})$
$\mathbf{126'}$	=	$(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{4})+(\mathbf{3},\mathbf{10})+(\mathbf{2},\mathbf{20})+(\mathbf{1},\mathbf{35'})$
189	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{4})+(\mathbf{3},\mathbf{5})+(\mathbf{4},\mathbf{4})+(\mathbf{1},\mathbf{10})+(\mathbf{3},\mathbf{10})+(\mathbf{2},\mathbf{16})+(\mathbf{2},\mathbf{20})+(\mathbf{1},\mathbf{35})$
216	=	$(\mathbf{2},\mathbf{5})+(\mathbf{3},\mathbf{4})+(\mathbf{4},\mathbf{5})+(\mathbf{2},\mathbf{10})+(\mathbf{1},\mathbf{16})+(\mathbf{3},\mathbf{16})+(\mathbf{1},\mathbf{20})+(\mathbf{2},\mathbf{35})$
252	=	$(\mathbf{6},\mathbf{1})+(\mathbf{5},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{3},\mathbf{20})+(\mathbf{2},\mathbf{35'})+(\mathbf{1},\mathbf{56})$
330	=	$(\mathbf{1},\mathbf{20})+(\mathbf{4},\mathbf{30})+(\mathbf{2},\mathbf{35})+(\mathbf{3},\mathbf{40})$
350	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{2},\mathbf{5})+(\mathbf{3},\mathbf{4})+(\mathbf{2},\mathbf{10})+(\mathbf{4},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{16})+(\mathbf{3},\mathbf{16})+(\mathbf{3},\mathbf{20})+(\mathbf{2},\mathbf{35})+(\mathbf{1},\mathbf{40})$
378	=	$(\mathbf{2},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{16})+(\mathbf{4},\mathbf{14})+(\mathbf{3},\mathbf{16})+(\mathbf{1},\mathbf{20})+(\mathbf{2},\mathbf{35})+(\mathbf{3},\mathbf{40})$
irrep385	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{2},\mathbf{4})+(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{16})+(\mathbf{4},\mathbf{20})+(\mathbf{1},\mathbf{30})+(\mathbf{3},\mathbf{35})+(\mathbf{2},\mathbf{40})$

Sp(6)	→	SU(2) (S)
6	=	$(\mathbf{6})$
14	=	$(\mathbf{5})+(\mathbf{9})$
$\mathbf{14'}$	=	$(\mathbf{4})+(\mathbf{10})$
21	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})$
56	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{16})$
64	=	$(\mathbf{2})+(\mathbf{4})+(\mathbf{6})+2(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{14})$
70	=	$(\mathbf{3})+(\mathbf{5})+2(\mathbf{7})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})+(\mathbf{15})$
84	=	$(\mathbf{3})+2(\mathbf{7})+(\mathbf{9})+(\mathbf{11})+(\mathbf{13})+(\mathbf{15})+(\mathbf{19})$
90	=	$(\mathbf{1})+2(\mathbf{5})+(\mathbf{7})+2(\mathbf{9})+(\mathbf{11})+2(\mathbf{13})+(\mathbf{17})$

Table 10.20: Sp(6) Branching Rules (continued)

126	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+2(\mathbf{8})+2(\mathbf{10})+2(\mathbf{12})+(\mathbf{14})+(\mathbf{16})+(\mathbf{18})$
126′	=	$(\mathbf{1})+2(\mathbf{5})+(\mathbf{7})+2(\mathbf{9})+(\mathbf{11})+2(\mathbf{13})+(\mathbf{15})+(\mathbf{17})+(\mathbf{21})$
189	=	$2(\mathbf{3})+2(\mathbf{5})+3(\mathbf{7})+3(\mathbf{9})+3(\mathbf{11})+2(\mathbf{13})+2(\mathbf{15})+(\mathbf{17})+(\mathbf{19})$
216	=	$(\mathbf{2})+2(\mathbf{4})+3(\mathbf{6})+3(\mathbf{8})+3(\mathbf{10})+3(\mathbf{12})+2(\mathbf{14})+2(\mathbf{16})+(\mathbf{18})+(\mathbf{20})$
252	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+2(\mathbf{8})+3(\mathbf{10})+2(\mathbf{12})+2(\mathbf{14})+2(\mathbf{16})+2(\mathbf{18})+(\mathbf{20})+(\mathbf{22})+(\mathbf{26})$
330	=	$2(\mathbf{4})+2(\mathbf{6})+2(\mathbf{8})+3(\mathbf{10})+3(\mathbf{12})+2(\mathbf{14})+3(\mathbf{16})+2(\mathbf{18})+(\mathbf{20})+2(\mathbf{22})+(\mathbf{24})+(\mathbf{28})$
350	=	$(\mathbf{2})+3(\mathbf{4})+4(\mathbf{6})+4(\mathbf{8})+5(\mathbf{10})+4(\mathbf{12})+4(\mathbf{14})+3(\mathbf{16})+2(\mathbf{18})+(\mathbf{20})+(\mathbf{22})$
378	=	$(\mathbf{2})+3(\mathbf{4})+3(\mathbf{6})+4(\mathbf{8})+4(\mathbf{10})+4(\mathbf{12})+4(\mathbf{14})+3(\mathbf{16})+2(\mathbf{18})+2(\mathbf{20})+(\mathbf{22})+(\mathbf{24})$
385	=	$2(\mathbf{1})+(\mathbf{3})+3(\mathbf{5})+3(\mathbf{7})+5(\mathbf{9})+3(\mathbf{11})+5(\mathbf{13})+3(\mathbf{15})+3(\mathbf{17})+2(\mathbf{19})+2(\mathbf{21})+(\mathbf{25})$

Sp(6)	→	SU(2)×SU(2) (S)
6	=	$(\mathbf{3},\mathbf{2})$
14	=	$(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{1})$
14′	=	$(\mathbf{1},\mathbf{4})+(\mathbf{5},\mathbf{2})$
21	=	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{3})$
56	=	$(\mathbf{3},\mathbf{2})+(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{7},\mathbf{4})$
64	=	$(\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{2})+(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{7},\mathbf{2})$
70	=	$(\mathbf{3},\mathbf{1})+(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{3})+(\mathbf{3},\mathbf{5})+(\mathbf{7},\mathbf{1})+(\mathbf{7},\mathbf{3})$
84	=	$(\mathbf{1},\mathbf{3})+(\mathbf{5},\mathbf{3})+(\mathbf{7},\mathbf{1})+(\mathbf{1},\mathbf{7})+(\mathbf{5},\mathbf{5})+(\mathbf{9},\mathbf{3})$
90	=	$(\mathbf{1},\mathbf{1})+(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{5},\mathbf{3})+(\mathbf{5},\mathbf{5})+(\mathbf{7},\mathbf{3})+(\mathbf{9},\mathbf{1})$
126	=	$(\mathbf{3},\mathbf{2})+(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{3},\mathbf{6})+(\mathbf{7},\mathbf{2})+(\mathbf{7},\mathbf{4})+(\mathbf{9},\mathbf{2})$
126′	=	$(\mathbf{1},\mathbf{1})+(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{5},\mathbf{3})+(\mathbf{5},\mathbf{5})+(\mathbf{7},\mathbf{3})+(\mathbf{9},\mathbf{5})$
189	=	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+2(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{1})+2(\mathbf{5},\mathbf{3})+(\mathbf{3},\mathbf{5})+(\mathbf{7},\mathbf{1})+(\mathbf{5},\mathbf{5})+(\mathbf{7},\mathbf{3})+(\mathbf{7},\mathbf{5})+(\mathbf{9},\mathbf{3})$
216	=	$(\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{2})+(\mathbf{1},\mathbf{4})+(\mathbf{3},\mathbf{4})+2(\mathbf{5},\mathbf{2})+(\mathbf{1},\mathbf{6})+2(\mathbf{5},\mathbf{4})+(\mathbf{7},\mathbf{2})+(\mathbf{5},\mathbf{6})+(\mathbf{7},\mathbf{4})+(\mathbf{9},\mathbf{2})+(\mathbf{9},\mathbf{4})$
252	=	$(\mathbf{3},\mathbf{2})+(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{3},\mathbf{6})+(\mathbf{7},\mathbf{2})+(\mathbf{7},\mathbf{4})+(\mathbf{7},\mathbf{6})+(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{6})$
330	=	$(\mathbf{1},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{1},\mathbf{6})+(\mathbf{5},\mathbf{4})+(\mathbf{5},\mathbf{6})+(\mathbf{7},\mathbf{4})+(\mathbf{9},\mathbf{2})+(\mathbf{1},\mathbf{10})+(\mathbf{5},\mathbf{8})+(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{2})+$ $(\mathbf{9},\mathbf{6})+(\mathbf{13},\mathbf{4})$
350	=	$2(\mathbf{3},\mathbf{2})+(\mathbf{1},\mathbf{4})+2(\mathbf{3},\mathbf{4})+2(\mathbf{5},\mathbf{2})+2(\mathbf{5},\mathbf{4})+(\mathbf{3},\mathbf{6})+2(\mathbf{7},\mathbf{2})+(\mathbf{5},\mathbf{6})+2(\mathbf{7},\mathbf{4})+(\mathbf{9},\mathbf{2})+(\mathbf{7},\mathbf{6})+$ $(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{2})$
378	=	$(\mathbf{3},\mathbf{2})+2(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{2})+(\mathbf{5},\mathbf{4})+(\mathbf{3},\mathbf{6})+2(\mathbf{7},\mathbf{2})+(\mathbf{5},\mathbf{6})+2(\mathbf{7},\mathbf{4})+(\mathbf{3},\mathbf{8})+(\mathbf{9},\mathbf{2})+(\mathbf{7},\mathbf{6})+$ $(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{2})+(\mathbf{11},\mathbf{4})$
385	=	$(\mathbf{1},\mathbf{1})+2(\mathbf{3},\mathbf{3})+(\mathbf{5},\mathbf{1})+(\mathbf{5},\mathbf{3})+(\mathbf{3},\mathbf{5})+(\mathbf{7},\mathbf{1})+2(\mathbf{5},\mathbf{5})+2(\mathbf{7},\mathbf{3})+(\mathbf{3},\mathbf{7})+(\mathbf{9},\mathbf{1})+(\mathbf{7},\mathbf{5})+$ $(\mathbf{9},\mathbf{3})+(\mathbf{7},\mathbf{7})+(\mathbf{9},\mathbf{5})+(\mathbf{11},\mathbf{3})+(\mathbf{13},\mathbf{1})$

Table 10.21: Sp(8) Branching Rules

Sp(8)	→	SU(2)×Sp(6) (R)
8	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{6})$
27	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{6})+(\mathbf{1},\mathbf{14})$
36	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{6})+(\mathbf{1},\mathbf{21})$
42	=	$(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{14'})$
48	=	$(\mathbf{1},\mathbf{6})+(\mathbf{1},\mathbf{14'})+(\mathbf{2},\mathbf{14})$
120	=	$(\mathbf{4},\mathbf{1})+(\mathbf{3},\mathbf{6})+(\mathbf{2},\mathbf{21})+(\mathbf{1},\mathbf{56})$
160	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{6})+(\mathbf{3},\mathbf{6})+(\mathbf{2},\mathbf{14})+(\mathbf{2},\mathbf{21})+(\mathbf{1},\mathbf{64})$
288	=	$(\mathbf{1},\mathbf{14'})+(\mathbf{2},\mathbf{14})+(\mathbf{3},\mathbf{14'})+(\mathbf{1},\mathbf{64})+(\mathbf{2},\mathbf{70})$
308	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{6})+(\mathbf{1},\mathbf{14})+(\mathbf{3},\mathbf{21})+(\mathbf{2},\mathbf{64})+(\mathbf{1},\mathbf{90})$
315	=	$(\mathbf{2},\mathbf{6})+(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{14'})+(\mathbf{3},\mathbf{14})+(\mathbf{1},\mathbf{21})+(\mathbf{2},\mathbf{64})+(\mathbf{1},\mathbf{70})$
330	=	$(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{6})+(\mathbf{3},\mathbf{21})+(\mathbf{2},\mathbf{56})+(\mathbf{1},\mathbf{126'})$
594	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{6})+(\mathbf{4},\mathbf{6})+(\mathbf{3},\mathbf{14})+(\mathbf{1},\mathbf{21})+(\mathbf{3},\mathbf{21})+(\mathbf{2},\mathbf{56})+(\mathbf{2},\mathbf{64})+(\mathbf{1},\mathbf{189})$
594′	=	$(\mathbf{3},\mathbf{84})+(\mathbf{1},\mathbf{90})+(\mathbf{2},\mathbf{126})$

Table 10.21: Sp(8) Branching Rules (continued)

792	=	$(\mathbf{1},\mathbf{6})+(\mathbf{1},\mathbf{14'})+(\mathbf{2},\mathbf{14})+(\mathbf{2},\mathbf{21})+(\mathbf{1},\mathbf{64})+(\mathbf{3},\mathbf{64})+(\mathbf{2},\mathbf{70})+(\mathbf{2},\mathbf{90})+(\mathbf{1},\mathbf{126})$
792′	=	$(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{14'})+(\mathbf{2},\mathbf{64})+(\mathbf{1},\mathbf{70})+(\mathbf{3},\mathbf{70})+(\mathbf{1},\mathbf{90})+(\mathbf{2},\mathbf{126})$
792″	=	$(\mathbf{6},\mathbf{1})+(\mathbf{5},\mathbf{6})+(\mathbf{4},\mathbf{21})+(\mathbf{3},\mathbf{56})+(\mathbf{2},\mathbf{126'})+(\mathbf{1},\mathbf{252})$
825	=	$(\mathbf{1},\mathbf{21})+(\mathbf{2},\mathbf{64})+(\mathbf{1},\mathbf{70})+(\mathbf{1},\mathbf{84})+(\mathbf{3},\mathbf{90})+(\mathbf{2},\mathbf{126})$
1056	=	$(\mathbf{1},\mathbf{64})+(\mathbf{2},\mathbf{70})+(\mathbf{2},\mathbf{84})+(\mathbf{2},\mathbf{90})+(\mathbf{1},\mathbf{126})+(\mathbf{3},\mathbf{126})$

Sp(8)	→	Sp(4)×Sp(4) (R)
8	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})$
27	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{4},\mathbf{4})$
36	=	$(\mathbf{4},\mathbf{4})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{10})$
42	=	$(\mathbf{1},\mathbf{1})+(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})$
48	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{4},\mathbf{5})+(\mathbf{5},\mathbf{4})$
120	=	$(\mathbf{10},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{20},\mathbf{1})+(\mathbf{1},\mathbf{20})$
160	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{4},\mathbf{5})+(\mathbf{5},\mathbf{4})+(\mathbf{10},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{16},\mathbf{1})+(\mathbf{1},\mathbf{16})$
288	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{4},\mathbf{5})+(\mathbf{5},\mathbf{4})+(\mathbf{10},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{16},\mathbf{5})+(\mathbf{5},\mathbf{16})$
308	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})+(\mathbf{14},\mathbf{1})+(\mathbf{1},\mathbf{14})+(\mathbf{10},\mathbf{10})+(\mathbf{16},\mathbf{4})+(\mathbf{4},\mathbf{16})$
315	=	$(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+2(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{10},\mathbf{5})+(\mathbf{5},\mathbf{10})+(\mathbf{16},\mathbf{4})+(\mathbf{4},\mathbf{16})$
330	=	$(\mathbf{10},\mathbf{10})+(\mathbf{20},\mathbf{4})+(\mathbf{4},\mathbf{20})+(\mathbf{35'},\mathbf{1})+(\mathbf{1},\mathbf{35'})$
594	=	$(\mathbf{4},\mathbf{4})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{10},\mathbf{5})+(\mathbf{5},\mathbf{10})+(\mathbf{10},\mathbf{10})+(\mathbf{16},\mathbf{4})+(\mathbf{4},\mathbf{16})+(\mathbf{20},\mathbf{4})+(\mathbf{4},\mathbf{20})+(\mathbf{35},\mathbf{1})+(\mathbf{1},\mathbf{35})$
594′	=	$(\mathbf{1},\mathbf{1})+(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})+(\mathbf{10},\mathbf{10})+(\mathbf{14},\mathbf{14})+(\mathbf{16},\mathbf{16})$
792	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})+2(\mathbf{4},\mathbf{5})+2(\mathbf{5},\mathbf{4})+(\mathbf{10},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{16},\mathbf{1})+(\mathbf{1},\mathbf{16})+(\mathbf{4},\mathbf{14})+(\mathbf{14},\mathbf{4})+(\mathbf{16},\mathbf{5})+(\mathbf{5},\mathbf{16})+(\mathbf{10},\mathbf{16})+(\mathbf{16},\mathbf{10})$
792′	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{5})+2(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})+(\mathbf{10},\mathbf{5})+(\mathbf{5},\mathbf{10})+(\mathbf{14},\mathbf{5})+(\mathbf{5},\mathbf{14})+(\mathbf{10},\mathbf{10})+(\mathbf{16},\mathbf{4})+(\mathbf{4},\mathbf{16})+(\mathbf{16},\mathbf{16})$
792″	=	$(\mathbf{20},\mathbf{10})+(\mathbf{10},\mathbf{20})+(\mathbf{35'},\mathbf{4})+(\mathbf{4},\mathbf{35'})+(\mathbf{56},\mathbf{1})+(\mathbf{1},\mathbf{56})$
825	=	$(\mathbf{4},\mathbf{4})+(\mathbf{5},\mathbf{5})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{10},\mathbf{5})+(\mathbf{5},\mathbf{10})+(\mathbf{16},\mathbf{4})+(\mathbf{4},\mathbf{16})+(\mathbf{10},\mathbf{14})+(\mathbf{14},\mathbf{10})+(\mathbf{16},\mathbf{16})$
1056	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{4})+(\mathbf{4},\mathbf{5})+(\mathbf{5},\mathbf{4})+(\mathbf{10},\mathbf{4})+(\mathbf{4},\mathbf{10})+(\mathbf{16},\mathbf{5})+(\mathbf{5},\mathbf{16})+(\mathbf{10},\mathbf{16})+(\mathbf{16},\mathbf{10})+(\mathbf{16},\mathbf{14})+(\mathbf{14},\mathbf{16})$

Sp(8)	→	SU(2) (S)
8	=	$(\mathbf{8})$
27	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{13})$
36	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})+(\mathbf{15})$
42	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{11})+(\mathbf{17})$
48	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{10})+(\mathbf{12})+(\mathbf{16})$
120	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+2(\mathbf{10})+(\mathbf{12})+(\mathbf{14})+(\mathbf{16})+(\mathbf{18})+(\mathbf{22})$
160	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+2(\mathbf{8})+2(\mathbf{10})+2(\mathbf{12})+2(\mathbf{14})+(\mathbf{16})+(\mathbf{18})+(\mathbf{20})$
288	=	$(\mathbf{2})+2(\mathbf{4})+2(\mathbf{6})+3(\mathbf{8})+3(\mathbf{10})+3(\mathbf{12})+3(\mathbf{14})+2(\mathbf{16})+2(\mathbf{18})+(\mathbf{20})+(\mathbf{22})+(\mathbf{24})$
308	=	$2(\mathbf{1})+3(\mathbf{5})+2(\mathbf{7})+4(\mathbf{9})+2(\mathbf{11})+4(\mathbf{13})+2(\mathbf{15})+3(\mathbf{17})+(\mathbf{19})+2(\mathbf{21})+(\mathbf{25})$
315	=	$2(\mathbf{3})+2(\mathbf{5})+4(\mathbf{7})+3(\mathbf{9})+4(\mathbf{11})+3(\mathbf{13})+3(\mathbf{15})+2(\mathbf{17})+2(\mathbf{19})+(\mathbf{21})+(\mathbf{23})$
330	=	$(\mathbf{1})+3(\mathbf{5})+(\mathbf{7})+3(\mathbf{9})+2(\mathbf{11})+3(\mathbf{13})+2(\mathbf{15})+3(\mathbf{17})+(\mathbf{19})+2(\mathbf{21})+(\mathbf{23})+(\mathbf{25})+(\mathbf{29})$
594	=	$3(\mathbf{3})+3(\mathbf{5})+5(\mathbf{7})+5(\mathbf{9})+6(\mathbf{11})+5(\mathbf{13})+5(\mathbf{15})+4(\mathbf{17})+4(\mathbf{19})+2(\mathbf{21})+2(\mathbf{23})+(\mathbf{25})+(\mathbf{27})$
594′	=	$(\mathbf{1})+(\mathbf{3})+3(\mathbf{5})+2(\mathbf{7})+5(\mathbf{9})+3(\mathbf{11})+5(\mathbf{13})+3(\mathbf{15})+4(\mathbf{17})+3(\mathbf{19})+3(\mathbf{21})+2(\mathbf{23})+2(\mathbf{25})+(\mathbf{27})+(\mathbf{29})+(\mathbf{33})$
792	=	$2(\mathbf{2})+4(\mathbf{4})+5(\mathbf{6})+7(\mathbf{8})+7(\mathbf{10})+7(\mathbf{12})+7(\mathbf{14})+6(\mathbf{16})+5(\mathbf{18})+4(\mathbf{20})+3(\mathbf{22})+2(\mathbf{24})+(\mathbf{26})+(\mathbf{28})$
792′	=	$2(\mathbf{1})+2(\mathbf{3})+5(\mathbf{5})+5(\mathbf{7})+7(\mathbf{9})+6(\mathbf{11})+7(\mathbf{13})+6(\mathbf{15})+6(\mathbf{17})+4(\mathbf{19})+4(\mathbf{21})+2(\mathbf{23})+2(\mathbf{25})+(\mathbf{27})+(\mathbf{29})$
792″	=	$(\mathbf{2})+2(\mathbf{4})+3(\mathbf{6})+4(\mathbf{8})+4(\mathbf{10})+5(\mathbf{12})+4(\mathbf{14})+5(\mathbf{16})+4(\mathbf{18})+4(\mathbf{20})+3(\mathbf{22})+3(\mathbf{24})+2(\mathbf{26})+2(\mathbf{28})+(\mathbf{30})+(\mathbf{32})+(\mathbf{36})$
825	=	$4(\mathbf{3})+2(\mathbf{5})+7(\mathbf{7})+5(\mathbf{9})+7(\mathbf{11})+6(\mathbf{13})+7(\mathbf{15})+4(\mathbf{17})+6(\mathbf{19})+3(\mathbf{21})+3(\mathbf{23})+2(\mathbf{25})+2(\mathbf{27})+(\mathbf{31})$

Table 10.21: Sp(8) Branching Rules (continued)

1056	=	3(**2**) + 3(**4**) + 6(**6**) + 7(**8**) + 7(**10**) + 8(**12**) + 8(**14**) + 7(**16**) + 6(**18**) + 6(**20**) + 4(**22**) + 3(**24**) + 3(**26**) + (**28**) + (**30**) + (**32**)

Sp(8)	→	SU(2)×SU(2)×SU(2) (S)
8	=	(**2**, **2**, **2**)
27	=	(**3**, **3**, **1**) + (**3**, **1**, **3**) + (**1**, **3**, **3**)
36	=	(**3**, **1**, **1**) + (**1**, **3**, **1**) + (**1**, **1**, **3**) + (**3**, **3**, **3**)
42	=	(**5**, **1**, **1**) + (**1**, **5**, **1**) + (**1**, **1**, **5**) + (**3**, **3**, **3**)
48	=	(**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**)
120	=	(**2**, **2**, **2**) + (**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**) + (**4**, **4**, **4**)
160	=	2(**2**, **2**, **2**) + (**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**) + (**4**, **4**, **2**) + (**4**, **2**, **4**) + (**2**, **4**, **4**)
288	=	(**2**, **2**, **2**) + (**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**) + (**4**, **4**, **2**) + (**4**, **2**, **4**) + (**2**, **4**, **4**) + (**6**, **2**, **2**) + (**2**, **6**, **2**) + (**2**, **2**, **6**) + (**4**, **4**, **4**)
308	=	2(**1**, **1**, **1**) + (**3**, **3**, **1**) + (**3**, **1**, **3**) + (**1**, **3**, **3**) + (**5**, **1**, **1**) + (**1**, **5**, **1**) + (**1**, **1**, **5**) + 2(**3**, **3**, **3**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**) + (**5**, **5**, **1**) + (**5**, **1**, **5**) + (**1**, **5**, **5**)
315	=	(**3**, **1**, **1**) + (**1**, **3**, **1**) + (**1**, **1**, **3**) + (**3**, **3**, **1**) + (**3**, **1**, **3**) + (**1**, **3**, **3**) + 2(**3**, **3**, **3**) + (**5**, **3**, **1**) + (**5**, **1**, **3**) + (**3**, **5**, **1**) + (**3**, **1**, **5**) + (**1**, **5**, **3**) + (**1**, **3**, **5**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**)
330	=	(**1**, **1**, **1**) + (**3**, **3**, **1**) + (**3**, **1**, **3**) + (**1**, **3**, **3**) + (**5**, **1**, **1**) + (**1**, **5**, **1**) + (**1**, **1**, **5**) + (**3**, **3**, **3**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**) + (**5**, **5**, **5**)
594	=	(**3**, **1**, **1**) + (**1**, **3**, **1**) + (**1**, **1**, **3**) + 2(**3**, **3**, **1**) + 2(**3**, **1**, **3**) + 2(**1**, **3**, **3**) + 3(**3**, **3**, **3**) + (**5**, **3**, **1**) + (**5**, **1**, **3**) + (**3**, **5**, **1**) + (**3**, **1**, **5**) + (**1**, **5**, **3**) + (**1**, **3**, **5**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**) + (**5**, **5**, **3**) + (**5**, **3**, **5**) + (**3**, **5**, **5**)
594′	=	(**1**, **1**, **1**) + (**5**, **1**, **1**) + (**1**, **5**, **1**) + (**1**, **1**, **5**) + (**3**, **3**, **3**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**) + (**5**, **5**, **1**) + (**5**, **1**, **5**) + (**1**, **5**, **5**) + (**9**, **1**, **1**) + (**1**, **9**, **1**) + (**1**, **1**, **9**) + (**7**, **3**, **3**) + (**3**, **7**, **3**) + (**3**, **3**, **7**) + (**5**, **5**, **5**)
792	=	2(**2**, **2**, **2**) + 2(**4**, **2**, **2**) + 2(**2**, **4**, **2**) + 2(**2**, **2**, **4**) + 2(**4**, **4**, **2**) + 2(**4**, **2**, **4**) + 2(**2**, **4**, **4**) + (**6**, **2**, **2**) + (**2**, **6**, **2**) + (**2**, **2**, **6**) + 2(**4**, **4**, **4**) + (**6**, **4**, **2**) + (**6**, **2**, **4**) + (**4**, **6**, **2**) + (**4**, **2**, **6**) + (**2**, **6**, **4**) + (**2**, **4**, **6**)
792′	=	2(**3**, **3**, **1**) + 2(**3**, **1**, **3**) + 2(**1**, **3**, **3**) + (**3**, **3**, **3**) + (**5**, **3**, **1**) + (**5**, **1**, **3**) + (**3**, **5**, **1**) + (**3**, **1**, **5**) + (**1**, **5**, **3**) + (**1**, **3**, **5**) + 2(**5**, **3**, **3**) + 2(**3**, **5**, **3**) + 2(**3**, **3**, **5**) + (**7**, **3**, **1**) + (**7**, **1**, **3**) + (**3**, **7**, **1**) + (**3**, **1**, **7**) + (**1**, **7**, **3**) + (**1**, **3**, **7**) + (**5**, **5**, **3**) + (**5**, **3**, **5**) + (**3**, **5**, **5**)
792″	=	(**2**, **2**, **2**) + (**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**) + (**4**, **4**, **2**) + (**4**, **2**, **4**) + (**2**, **4**, **4**) + (**6**, **2**, **2**) + (**2**, **6**, **2**) + (**2**, **2**, **6**) + (**4**, **4**, **4**) + (**6**, **4**, **4**) + (**4**, **6**, **4**) + (**4**, **4**, **6**) + (**6**, **6**, **6**)
825	=	(**3**, **1**, **1**) + (**1**, **3**, **1**) + (**1**, **1**, **3**) + 3(**3**, **3**, **3**) + (**5**, **3**, **1**) + (**5**, **1**, **3**) + (**3**, **5**, **1**) + (**3**, **1**, **5**) + (**1**, **5**, **3**) + (**1**, **3**, **5**) + (**7**, **1**, **1**) + (**1**, **7**, **1**) + (**1**, **1**, **7**) + (**5**, **3**, **3**) + (**3**, **5**, **3**) + (**3**, **3**, **5**) + (**5**, **5**, **1**) + (**5**, **1**, **5**) + (**1**, **5**, **5**) + (**5**, **5**, **3**) + (**5**, **3**, **5**) + (**3**, **5**, **5**) + (**7**, **3**, **3**) + (**3**, **7**, **3**) + (**3**, **3**, **7**)
1056	=	(**2**, **2**, **2**) + (**4**, **2**, **2**) + (**2**, **4**, **2**) + (**2**, **2**, **4**) + 2(**4**, **4**, **2**) + 2(**4**, **2**, **4**) + 2(**2**, **4**, **4**) + (**6**, **2**, **2**) + (**2**, **6**, **2**) + (**2**, **2**, **6**) + (**4**, **4**, **4**) + (**6**, **4**, **2**) + (**6**, **2**, **4**) + (**4**, **6**, **2**) + (**4**, **2**, **6**) + (**2**, **6**, **4**) + (**2**, **4**, **6**) + (**8**, **2**, **2**) + (**2**, **8**, **2**) + (**2**, **2**, **8**) + (**6**, **4**, **4**) + (**4**, **6**, **4**) + (**4**, **4**, **6**)

Table 10.22: Sp(10) Branching Rules

Sp(10)	→	SU(2)×Sp(8) (R)
10	=	(**2**, **1**) + (**1**, **8**)
44	=	(**1**, **1**) + (**2**, **8**) + (**1**, **27**)
55	=	(**3**, **1**) + (**2**, **8**) + (**1**, **36**)
110	=	(**1**, **8**) + (**2**, **27**) + (**1**, **48**)
132	=	(**2**, **42**) + (**1**, **48**)
165	=	(**1**, **27**) + (**1**, **42**) + (**2**, **48**)
220	=	(**4**, **1**) + (**3**, **8**) + (**2**, **36**) + (**1**, **120**)
320	=	(**2**, **1**) + (**1**, **8**) + (**3**, **8**) + (**2**, **27**) + (**2**, **36**) + (**1**, **160**)
715	=	(**5**, **1**) + (**4**, **8**) + (**3**, **36**) + (**2**, **120**) + (**1**, **330**)

Table 10.22: Sp(10) Branching Rules (continued)

780	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{8})+(\mathbf{1},\mathbf{27})+(\mathbf{3},\mathbf{36})+(\mathbf{2},\mathbf{160})+(\mathbf{1},\mathbf{308})$
891	=	$(\mathbf{2},\mathbf{8})+(\mathbf{1},\mathbf{27})+(\mathbf{3},\mathbf{27})+(\mathbf{1},\mathbf{36})+(\mathbf{2},\mathbf{48})+(\mathbf{2},\mathbf{160})+(\mathbf{1},\mathbf{315})$
1155	=	$(\mathbf{1},\mathbf{42})+(\mathbf{3},\mathbf{42})+(\mathbf{2},\mathbf{48})+(\mathbf{2},\mathbf{288})+(\mathbf{1},\mathbf{315})$
1408	=	$(\mathbf{2},\mathbf{27})+(\mathbf{2},\mathbf{42})+(\mathbf{1},\mathbf{48})+(\mathbf{3},\mathbf{48})+(\mathbf{1},\mathbf{160})+(\mathbf{1},\mathbf{288})+(\mathbf{2},\mathbf{315})$
1430	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{8})+(\mathbf{4},\mathbf{8})+(\mathbf{3},\mathbf{27})+(\mathbf{1},\mathbf{36})+(\mathbf{3},\mathbf{36})+(\mathbf{2},\mathbf{120})+(\mathbf{2},\mathbf{160})+(\mathbf{1},\mathbf{594})$
2002	=	$(\mathbf{6},\mathbf{1})+(\mathbf{5},\mathbf{8})+(\mathbf{4},\mathbf{36})+(\mathbf{3},\mathbf{120})+(\mathbf{2},\mathbf{330})+(\mathbf{1},\mathbf{792''})$
2860	=	$(\mathbf{1},\mathbf{8})+(\mathbf{2},\mathbf{27})+(\mathbf{2},\mathbf{36})+(\mathbf{1},\mathbf{48})+(\mathbf{1},\mathbf{160})+(\mathbf{3},\mathbf{160})+(\mathbf{2},\mathbf{308})+(\mathbf{2},\mathbf{315})+(\mathbf{1},\mathbf{792})$
4004	=	$(\mathbf{1},\mathbf{36})+(\mathbf{2},\mathbf{160})+(\mathbf{3},\mathbf{308})+(\mathbf{1},\mathbf{315})+(\mathbf{2},\mathbf{792})+(\mathbf{1},\mathbf{825})$
4212	=	$(\mathbf{3},\mathbf{8})+(\mathbf{2},\mathbf{27})+(\mathbf{4},\mathbf{27})+(\mathbf{2},\mathbf{36})+(\mathbf{3},\mathbf{48})+(\mathbf{1},\mathbf{120})+(\mathbf{1},\mathbf{160})+(\mathbf{3},\mathbf{160})+(\mathbf{2},\mathbf{315})+(\mathbf{2},\mathbf{594})+(\mathbf{1},\mathbf{1232})$

Sp(10)	→	Sp(4)×Sp(6) (R)
10	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{6})$
44	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{6})+(\mathbf{1},\mathbf{14})$
55	=	$(\mathbf{4},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{21})$
110	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{6})+(\mathbf{5},\mathbf{6})+(\mathbf{1},\mathbf{14'})+(\mathbf{4},\mathbf{14})$
132	=	$(\mathbf{1},\mathbf{6})+(\mathbf{4},\mathbf{14})+(\mathbf{5},\mathbf{14'})$
165	=	$(\mathbf{1},\mathbf{1})+(\mathbf{4},\mathbf{6})+(\mathbf{1},\mathbf{14})+(\mathbf{4},\mathbf{14'})+(\mathbf{5},\mathbf{14})$
220	=	$(\mathbf{10},\mathbf{6})+(\mathbf{20},\mathbf{1})+(\mathbf{4},\mathbf{21})+(\mathbf{1},\mathbf{56})$
320	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{6})+(\mathbf{5},\mathbf{6})+(\mathbf{10},\mathbf{6})+(\mathbf{16},\mathbf{1})+(\mathbf{4},\mathbf{14})+(\mathbf{4},\mathbf{21})+(\mathbf{1},\mathbf{64})$
715	=	$(\mathbf{20},\mathbf{6})+(\mathbf{10},\mathbf{21})+(\mathbf{35'},\mathbf{1})+(\mathbf{4},\mathbf{56})+(\mathbf{1},\mathbf{126'})$
780	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{6})+(\mathbf{1},\mathbf{14})+(\mathbf{14},\mathbf{1})+(\mathbf{5},\mathbf{14})+(\mathbf{16},\mathbf{6})+(\mathbf{10},\mathbf{21})+(\mathbf{4},\mathbf{64})+(\mathbf{1},\mathbf{90})$
891	=	$(\mathbf{5},\mathbf{1})+2(\mathbf{4},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{14})+(\mathbf{4},\mathbf{14'})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{21})+(\mathbf{16},\mathbf{6})+(\mathbf{10},\mathbf{14})+(\mathbf{5},\mathbf{21})+(\mathbf{4},\mathbf{64})+(\mathbf{1},\mathbf{70})$
1155	=	$(\mathbf{4},\mathbf{6})+(\mathbf{1},\mathbf{14})+(\mathbf{4},\mathbf{14'})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{21})+(\mathbf{10},\mathbf{14})+(\mathbf{16},\mathbf{14'})+(\mathbf{4},\mathbf{64})+(\mathbf{5},\mathbf{70})$
1408	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{6})+(\mathbf{5},\mathbf{6})+(\mathbf{1},\mathbf{14'})+(\mathbf{10},\mathbf{6})+2(\mathbf{4},\mathbf{14})+(\mathbf{5},\mathbf{14'})+(\mathbf{10},\mathbf{14'})+(\mathbf{4},\mathbf{21})+(\mathbf{16},\mathbf{14})+(\mathbf{1},\mathbf{64})+(\mathbf{5},\mathbf{64})+(\mathbf{4},\mathbf{70})$
1430	=	$(\mathbf{4},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{21})+(\mathbf{16},\mathbf{6})+(\mathbf{10},\mathbf{14})+(\mathbf{5},\mathbf{21})+(\mathbf{20},\mathbf{6})+(\mathbf{10},\mathbf{21})+(\mathbf{35},\mathbf{1})+(\mathbf{4},\mathbf{56})+(\mathbf{4},\mathbf{64})+(\mathbf{1},\mathbf{189})$
2002	=	$(\mathbf{20},\mathbf{21})+(\mathbf{35'},\mathbf{6})+(\mathbf{56},\mathbf{1})+(\mathbf{10},\mathbf{56})+(\mathbf{4},\mathbf{126'})+(\mathbf{1},\mathbf{252})$
2860	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{6})+2(\mathbf{5},\mathbf{6})+(\mathbf{1},\mathbf{14'})+(\mathbf{10},\mathbf{6})+(\mathbf{16},\mathbf{1})+2(\mathbf{4},\mathbf{14})+(\mathbf{5},\mathbf{14'})+(\mathbf{14},\mathbf{6})+(\mathbf{4},\mathbf{21})+(\mathbf{16},\mathbf{14})+(\mathbf{16},\mathbf{21})+(\mathbf{1},\mathbf{64})+(\mathbf{5},\mathbf{64})+(\mathbf{10},\mathbf{64})+(\mathbf{4},\mathbf{70})+(\mathbf{4},\mathbf{90})+(\mathbf{1},\mathbf{126})$
4004	=	$(\mathbf{4},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\mathbf{4},\mathbf{14'})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{21})+(\mathbf{16},\mathbf{6})+(\mathbf{10},\mathbf{14})+(\mathbf{5},\mathbf{21})+(\mathbf{14},\mathbf{21})+(\mathbf{4},\mathbf{64})+(\mathbf{1},\mathbf{70})+(\mathbf{5},\mathbf{70})+(\mathbf{16},\mathbf{64})+(\mathbf{1},\mathbf{84})+(\mathbf{10},\mathbf{90})+(\mathbf{4},\mathbf{126})$
4212	=	$(\mathbf{5},\mathbf{6})+2(\mathbf{10},\mathbf{6})+(\mathbf{16},\mathbf{1})+(\mathbf{4},\mathbf{14})+(\mathbf{20},\mathbf{1})+(\mathbf{10},\mathbf{14'})+2(\mathbf{4},\mathbf{21})+(\mathbf{16},\mathbf{14})+(\mathbf{20},\mathbf{14})+(\mathbf{16},\mathbf{21})+(\mathbf{35},\mathbf{6})+(\mathbf{1},\mathbf{56})+(\mathbf{5},\mathbf{56})+(\mathbf{1},\mathbf{64})+(\mathbf{5},\mathbf{64})+(\mathbf{10},\mathbf{64})+(\mathbf{4},\mathbf{70})+(\mathbf{4},\mathbf{189})+(\mathbf{1},\mathbf{216})$

Sp(10)	→	SU(2) (S)
10	=	$(\mathbf{10})$
44	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{13})+(\mathbf{17})$
55	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})+(\mathbf{15})+(\mathbf{19})$
110	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{14})+(\mathbf{16})+(\mathbf{18})+(\mathbf{22})$
132	=	$(\mathbf{2})+(\mathbf{6})+(\mathbf{8})+(\mathbf{10})+(\mathbf{12})+(\mathbf{14})+(\mathbf{16})+(\mathbf{18})+(\mathbf{20})+(\mathbf{26})$
165	=	$(\mathbf{1})+(\mathbf{5})+(\mathbf{7})+2(\mathbf{9})+(\mathbf{11})+2(\mathbf{13})+(\mathbf{15})+(\mathbf{17})+(\mathbf{19})+(\mathbf{21})+(\mathbf{25})$
220	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+2(\mathbf{10})+2(\mathbf{12})+(\mathbf{14})+2(\mathbf{16})+(\mathbf{18})+(\mathbf{20})+(\mathbf{22})+(\mathbf{24})+(\mathbf{28})$
320	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+3(\mathbf{8})+2(\mathbf{10})+3(\mathbf{12})+3(\mathbf{14})+2(\mathbf{16})+2(\mathbf{18})+2(\mathbf{20})+(\mathbf{22})+(\mathbf{24})+(\mathbf{26})$
715	=	$2(\mathbf{1})+3(\mathbf{5})+2(\mathbf{7})+4(\mathbf{9})+2(\mathbf{11})+5(\mathbf{13})+3(\mathbf{15})+4(\mathbf{17})+3(\mathbf{19})+4(\mathbf{21})+2(\mathbf{23})+3(\mathbf{25})+(\mathbf{27})+2(\mathbf{29})+(\mathbf{31})+(\mathbf{33})+(\mathbf{37})$
780	=	$2(\mathbf{1})+5(\mathbf{5})+2(\mathbf{7})+6(\mathbf{9})+4(\mathbf{11})+6(\mathbf{13})+4(\mathbf{15})+6(\mathbf{17})+3(\mathbf{19})+5(\mathbf{21})+2(\mathbf{23})+3(\mathbf{25})+(\mathbf{27})+2(\mathbf{29})+(\mathbf{33})$

Table 10.22: Sp(10) Branching Rules (continued)

891	=	$3(\mathbf{3})+3(\mathbf{5})+6(\mathbf{7})+5(\mathbf{9})+7(\mathbf{11})+6(\mathbf{13})+7(\mathbf{15})+5(\mathbf{17})+5(\mathbf{19})+4(\mathbf{21})+4(\mathbf{23})+2(\mathbf{25})+2(\mathbf{27})+(\mathbf{29})+(\mathbf{31})$
1155	=	$3(\mathbf{3})+4(\mathbf{5})+5(\mathbf{7})+6(\mathbf{9})+8(\mathbf{11})+6(\mathbf{13})+7(\mathbf{15})+7(\mathbf{17})+6(\mathbf{19})+5(\mathbf{21})+5(\mathbf{23})+3(\mathbf{25})+3(\mathbf{27})+2(\mathbf{29})+(\mathbf{31})+(\mathbf{33})+(\mathbf{35})$
1408	=	$2(\mathbf{2})+5(\mathbf{4})+6(\mathbf{6})+7(\mathbf{8})+9(\mathbf{10})+9(\mathbf{12})+9(\mathbf{14})+9(\mathbf{16})+8(\mathbf{18})+7(\mathbf{20})+6(\mathbf{22})+5(\mathbf{24})+3(\mathbf{26})+3(\mathbf{28})+2(\mathbf{30})+(\mathbf{32})+(\mathbf{34})$
1430	=	$4(\mathbf{3})+4(\mathbf{5})+7(\mathbf{7})+7(\mathbf{9})+9(\mathbf{11})+8(\mathbf{13})+9(\mathbf{15})+8(\mathbf{17})+8(\mathbf{19})+6(\mathbf{21})+6(\mathbf{23})+4(\mathbf{25})+4(\mathbf{27})+2(\mathbf{29})+2(\mathbf{31})+(\mathbf{33})+(\mathbf{35})$
2002	=	$(\mathbf{2})+3(\mathbf{4})+5(\mathbf{6})+5(\mathbf{8})+7(\mathbf{10})+7(\mathbf{12})+8(\mathbf{14})+8(\mathbf{16})+8(\mathbf{18})+7(\mathbf{20})+8(\mathbf{22})+6(\mathbf{24})+6(\mathbf{26})+5(\mathbf{28})+5(\mathbf{30})+3(\mathbf{32})+3(\mathbf{34})+2(\mathbf{36})+2(\mathbf{38})+(\mathbf{40})+(\mathbf{42})+(\mathbf{46})$
2860	=	$4(\mathbf{2})+7(\mathbf{4})+11(\mathbf{6})+13(\mathbf{8})+15(\mathbf{10})+16(\mathbf{12})+17(\mathbf{14})+16(\mathbf{16})+15(\mathbf{18})+14(\mathbf{20})+12(\mathbf{22})+10(\mathbf{24})+8(\mathbf{26})+6(\mathbf{28})+5(\mathbf{30})+3(\mathbf{32})+2(\mathbf{34})+(\mathbf{36})+(\mathbf{38})$
4004	=	$9(\mathbf{3})+7(\mathbf{5})+17(\mathbf{7})+14(\mathbf{9})+20(\mathbf{11})+18(\mathbf{13})+22(\mathbf{15})+17(\mathbf{17})+21(\mathbf{19})+15(\mathbf{21})+16(\mathbf{23})+12(\mathbf{25})+12(\mathbf{27})+7(\mathbf{29})+8(\mathbf{31})+4(\mathbf{33})+4(\mathbf{35})+2(\mathbf{37})+2(\mathbf{39})+(\mathbf{43})$
4212	=	$5(\mathbf{2})+10(\mathbf{4})+14(\mathbf{6})+18(\mathbf{8})+20(\mathbf{10})+22(\mathbf{12})+22(\mathbf{14})+23(\mathbf{16})+21(\mathbf{18})+20(\mathbf{20})+17(\mathbf{22})+15(\mathbf{24})+12(\mathbf{26})+10(\mathbf{28})+7(\mathbf{30})+6(\mathbf{32})+3(\mathbf{34})+3(\mathbf{36})+(\mathbf{38})+(\mathbf{40})$

Sp(10)	→	SU(2)×Sp(4) (S)
10	=	$(\mathbf{2},\mathbf{5})$
44	=	$(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})$
55	=	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{3},\mathbf{14})$
110	=	$(\mathbf{4},\mathbf{10})+(\mathbf{2},\mathbf{35})$
132	=	$(\mathbf{6},\mathbf{1})+(\mathbf{4},\mathbf{14})+(\mathbf{2},\mathbf{35'})$
165	=	$(\mathbf{5},\mathbf{5})+(\mathbf{1},\mathbf{35'})+(\mathbf{3},\mathbf{35})$
220	=	$(\mathbf{2},\mathbf{5})+(\mathbf{4},\mathbf{5})+(\mathbf{4},\mathbf{30})+(\mathbf{2},\mathbf{35})$
320	=	$(\mathbf{2},\mathbf{5})+(\mathbf{4},\mathbf{5})+(\mathbf{2},\mathbf{10})+(\mathbf{2},\mathbf{30})+(\mathbf{2},\mathbf{35})+(\mathbf{4},\mathbf{35})$
715	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{3},\mathbf{14})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{35'})+(\mathbf{5},\mathbf{55})+(\mathbf{3},\mathbf{81})$
780	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{5},\mathbf{1})+(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{3},\mathbf{14})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{35'})+(\mathbf{3},\mathbf{35})+(\mathbf{5},\mathbf{35'})+(\mathbf{1},\mathbf{55})+(\mathbf{3},\mathbf{81})$
891	=	$(\mathbf{3},\mathbf{5})+(\mathbf{1},\mathbf{10})+(\mathbf{3},\mathbf{10})+(\mathbf{5},\mathbf{10})+(\mathbf{3},\mathbf{14})+(\mathbf{1},\mathbf{35})+(\mathbf{3},\mathbf{35})+(\mathbf{3},\mathbf{35'})+(\mathbf{5},\mathbf{35})+(\mathbf{1},\mathbf{81})+(\mathbf{3},\mathbf{81})$
1155	=	$(\mathbf{3},\mathbf{5})+(\mathbf{5},\mathbf{5})+(\mathbf{7},\mathbf{5})+(\mathbf{3},\mathbf{30})+(\mathbf{5},\mathbf{30})+(\mathbf{1},\mathbf{35})+(\mathbf{3},\mathbf{35})+(\mathbf{3},\mathbf{35'})+(\mathbf{5},\mathbf{35})+(\mathbf{1},\mathbf{105})+(\mathbf{3},\mathbf{105})$
1408	=	$(\mathbf{4},\mathbf{1})+(\mathbf{2},\mathbf{10})+(\mathbf{4},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{6},\mathbf{10})+(\mathbf{4},\mathbf{14})+(\mathbf{6},\mathbf{14})+(\mathbf{2},\mathbf{35})+(\mathbf{2},\mathbf{35'})+(\mathbf{4},\mathbf{35})+(\mathbf{4},\mathbf{35'})+(\mathbf{2},\mathbf{81})+(\mathbf{4},\mathbf{81})+(\mathbf{2},\mathbf{105})$
1430	=	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{10})+2(\mathbf{3},\mathbf{10})+(\mathbf{1},\mathbf{14})+(\mathbf{5},\mathbf{10})+2(\mathbf{3},\mathbf{14})+(\mathbf{5},\mathbf{14})+(\mathbf{1},\mathbf{35})+(\mathbf{3},\mathbf{35})+(\mathbf{3},\mathbf{35'})+(\mathbf{3},\mathbf{55})+(\mathbf{1},\mathbf{81})+(\mathbf{3},\mathbf{81})+(\mathbf{5},\mathbf{81})$
2002	=	$(\mathbf{2},\mathbf{5})+(\mathbf{4},\mathbf{5})+(\mathbf{6},\mathbf{5})+(\mathbf{2},\mathbf{30})+(\mathbf{4},\mathbf{30})+(\mathbf{6},\mathbf{30})+(\mathbf{2},\mathbf{35})+(\mathbf{4},\mathbf{35})+(\mathbf{6},\mathbf{91})+(\mathbf{2},\mathbf{105})+(\mathbf{4},\mathbf{154})$
2860	=	$(\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{5})+(\mathbf{4},\mathbf{5})+(\mathbf{6},\mathbf{5})+(\mathbf{2},\mathbf{10})+(\mathbf{4},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{4},\mathbf{14})+(\mathbf{2},\mathbf{30})+(\mathbf{4},\mathbf{30})+2(\mathbf{2},\mathbf{35})+(\mathbf{2},\mathbf{35'})+2(\mathbf{4},\mathbf{35})+(\mathbf{4},\mathbf{35'})+(\mathbf{6},\mathbf{35})+(\mathbf{6},\mathbf{35'})+(\mathbf{2},\mathbf{81})+(\mathbf{4},\mathbf{81})+(\mathbf{2},\mathbf{105})+(\mathbf{4},\mathbf{105})+(\mathbf{2},\mathbf{154})$
4004	=	$(\mathbf{3},\mathbf{1})+(\mathbf{3},\mathbf{5})+(\mathbf{7},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{5},\mathbf{10})+2(\mathbf{3},\mathbf{14})+(\mathbf{5},\mathbf{14})+(\mathbf{7},\mathbf{14})+(\mathbf{3},\mathbf{30})+(\mathbf{1},\mathbf{35})+(\mathbf{3},\mathbf{35})+2(\mathbf{3},\mathbf{35'})+(\mathbf{5},\mathbf{35})+(\mathbf{5},\mathbf{35'})+(\mathbf{7},\mathbf{35'})+(\mathbf{3},\mathbf{55})+(\mathbf{1},\mathbf{81})+(\mathbf{3},\mathbf{81})+(\mathbf{1},\mathbf{84})+(\mathbf{5},\mathbf{81})+(\mathbf{3},\mathbf{105})+(\mathbf{5},\mathbf{105})+(\mathbf{1},\mathbf{154})+(\mathbf{3},\mathbf{220})$
4212	=	$(\mathbf{2},\mathbf{5})+(\mathbf{4},\mathbf{5})+2(\mathbf{2},\mathbf{10})+2(\mathbf{4},\mathbf{10})+(\mathbf{2},\mathbf{14})+(\mathbf{6},\mathbf{10})+(\mathbf{4},\mathbf{14})+(\mathbf{2},\mathbf{30})+(\mathbf{4},\mathbf{30})+3(\mathbf{2},\mathbf{35})+(\mathbf{2},\mathbf{35'})+3(\mathbf{4},\mathbf{35})+(\mathbf{4},\mathbf{35'})+(\mathbf{6},\mathbf{35})+2(\mathbf{2},\mathbf{81})+(\mathbf{4},\mathbf{81})+(\mathbf{6},\mathbf{81})+(\mathbf{2},\mathbf{105})+(\mathbf{4},\mathbf{105})+(\mathbf{2},\mathbf{154})+(\mathbf{4},\mathbf{154})$

Table 10.23: Sp(12) Branching Rules

Sp(12)	→	SU(2)×Sp(10) (R)
12	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{10})$
65	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{10})+(\mathbf{1},\mathbf{44})$

Table 10.23: Sp(12) Branching Rules (continued)

78	=	(3, 1) + (2, 10) + (1, 55)
208	=	(1, 10) + (2, 44) + (1, 110)
364	=	(4, 1) + (3, 10) + (2, 55) + (1, 220)
429	=	(1, 44) + (2, 110) + (1, 165)
429′	=	(2, 132) + (1, 165)
560	=	(2, 1) + (1, 10) + (3, 10) + (2, 44) + (2, 55) + (1, 320)
572	=	(1, 110) + (1, 132) + (2, 165)
1365	=	(5, 1) + (4, 10) + (3, 55) + (2, 220) + (1, 715)
1650	=	(1, 1) + (2, 10) + (1, 44) + (3, 55) + (2, 320) + (1, 780)
2002	=	(2, 10) + (1, 44) + (3, 44) + (1, 55) + (2, 110) + (2, 320) + (1, 891)
2925	=	(3, 1) + (2, 10) + (4, 10) + (3, 44) + (1, 55) + (3, 55) + (2, 220) + (2, 320) + (1, 1430)
4368	=	(2, 44) + (1, 110) + (3, 110) + (2, 165) + (1, 320) + (2, 891) + (1, 1408)
4368′	=	(6, 1) + (5, 10) + (4, 55) + (3, 220) + (2, 715) + (1, 2002)
4576	=	(1, 132) + (3, 132) + (2, 165) + (2, 1155) + (1, 1408)
6006	=	(2, 110) + (2, 132) + (1, 165) + (3, 165) + (1, 891) + (1, 1155) + (2, 1408)
7800	=	(1, 10) + (2, 44) + (2, 55) + (1, 110) + (1, 320) + (3, 320) + (2, 780) + (2, 891) + (1, 2860)

Sp(12)	→	Sp(4)×Sp(8) (R)
12	=	(4, 1) + (1, 8)
65	=	(1, 1) + (5, 1) + (4, 8) + (1, 27)
78	=	(10, 1) + (4, 8) + (1, 36)
208	=	(4, 1) + (1, 8) + (5, 8) + (4, 27) + (1, 48)
364	=	(10, 8) + (20, 1) + (4, 36) + (1, 120)
429	=	(1, 1) + (4, 8) + (1, 27) + (5, 27) + (1, 42) + (4, 48)
429′	=	(1, 27) + (5, 42) + (4, 48)
560	=	(4, 1) + (1, 8) + (5, 8) + (16, 1) + (10, 8) + (4, 27) + (4, 36) + (1, 160)
572	=	(1, 8) + (4, 27) + (4, 42) + (1, 48) + (5, 48)
1365	=	(20, 8) + (35′, 1) + (10, 36) + (4, 120) + (1, 330)
1650	=	(1, 1) + (5, 1) + (4, 8) + (14, 1) + (16, 8) + (1, 27) + (5, 27) + (10, 36) + (4, 160) + (1, 308)
2002	=	(5, 1) + (10, 1) + 2(4, 8) + (16, 8) + (1, 27) + (5, 27) + (1, 36) + (10, 27) + (5, 36) + (4, 48) + (4, 160) + (1, 315)
2925	=	(10, 1) + (4, 8) + (16, 8) + (20, 8) + (35, 1) + (1, 36) + (10, 27) + (5, 36) + (10, 36) + (4, 120) + (4, 160) + (1, 594)
4368	=	(4, 1) + (1, 8) + (5, 8) + (10, 8) + 2(4, 27) + (4, 36) + (16, 27) + (4, 42) + (1, 48) + (5, 48) + (10, 48) + (1, 160) + (5, 160) + (1, 288) + (4, 315)
4368′	=	(35′, 8) + (20, 36) + (56, 1) + (10, 120) + (4, 330) + (1, 792″)
4576	=	(4, 27) + (4, 42) + (1, 48) + (5, 48) + (10, 48) + (16, 42) + (1, 160) + (5, 288) + (4, 315)
6006	=	(4, 8) + (1, 27) + (5, 27) + (1, 36) + (10, 27) + (1, 42) + (5, 42) + 2(4, 48) + (10, 42) + (16, 48) + (4, 160) + (4, 288) + (1, 315) + (5, 315)
7800	=	(4, 1) + (1, 8) + 2(5, 8) + (16, 1) + (10, 8) + (14, 8) + 2(4, 27) + (4, 36) + (16, 27) + (1, 48) + (16, 36) + (5, 48) + (1, 160) + (5, 160) + (10, 160) + (4, 308) + (4, 315) + (1, 792)

Sp(12)	→	Sp(6)×Sp(6) (R)
12	=	(6, 1) + (1, 6)
65	=	(1, 1) + (6, 6) + (14, 1) + (1, 14)
78	=	(6, 6) + (21, 1) + (1, 21)
208	=	(6, 1) + (1, 6) + (14′, 1) + (1, 14′) + (6, 14) + (14, 6)
364	=	(21, 6) + (6, 21) + (56, 1) + (1, 56)
429	=	(1, 1) + (6, 6) + (14, 1) + (1, 14) + (6, 14′) + (14′, 6) + (14, 14)
429′	=	(1, 1) + (6, 6) + (14, 14) + (14′, 14′)
560	=	(6, 1) + (1, 6) + (6, 14) + (14, 6) + (21, 6) + (6, 21) + (64, 1) + (1, 64)

Table 10.23: Sp(12) Branching Rules (continued)

$\mathbf{572}$	=	$(\mathbf{6,1})+(\mathbf{1,6})+(\mathbf{6,14})+(\mathbf{14,6})+(\mathbf{14,14'})+(\mathbf{14',14})$
$\mathbf{1365}$	=	$(\mathbf{21,21})+(\mathbf{56,6})+(\mathbf{6,56})+(\mathbf{126',1})+(\mathbf{1,126'})$
$\mathbf{1650}$	=	$(\mathbf{1,1})+(\mathbf{6,6})+(\mathbf{14,1})+(\mathbf{1,14})+(\mathbf{14,14})+(\mathbf{21,21})+(\mathbf{64,6})+(\mathbf{6,64})+(\mathbf{90,1})+(\mathbf{1,90})$
$\mathbf{2002}$	=	$2(\mathbf{6,6})+(\mathbf{14,1})+(\mathbf{1,14})+(\mathbf{6,14'})+(\mathbf{14',6})+(\mathbf{21,1})+(\mathbf{1,21})+(\mathbf{14,14})+(\mathbf{21,14})+(\mathbf{14,21})+(\mathbf{64,6})+(\mathbf{6,64})+(\mathbf{70,1})+(\mathbf{1,70})$
$\mathbf{2925}$	=	$(\mathbf{6,6})+(\mathbf{21,1})+(\mathbf{1,21})+(\mathbf{21,14})+(\mathbf{14,21})+(\mathbf{21,21})+(\mathbf{56,6})+(\mathbf{6,56})+(\mathbf{64,6})+(\mathbf{6,64})+(\mathbf{189,1})+(\mathbf{1,189})$
$\mathbf{4368}$	=	$(\mathbf{6,1})+(\mathbf{1,6})+(\mathbf{14',1})+(\mathbf{1,14'})+2(\mathbf{6,14})+2(\mathbf{14,6})+(\mathbf{21,6})+(\mathbf{6,21})+(\mathbf{14,14'})+(\mathbf{14',14})+(\mathbf{21,14'})+(\mathbf{14',21})+(\mathbf{64,1})+(\mathbf{1,64})+(\mathbf{70,6})+(\mathbf{6,70})+(\mathbf{64,14})+(\mathbf{14,64})$
$\mathbf{4368'}$	=	$(\mathbf{56,21})+(\mathbf{21,56})+(\mathbf{126',6})+(\mathbf{6,126'})+(\mathbf{252,1})+(\mathbf{1,252})$
$\mathbf{4576}$	=	$(\mathbf{6,1})+(\mathbf{1,6})+(\mathbf{6,14})+(\mathbf{14,6})+(\mathbf{21,6})+(\mathbf{6,21})+(\mathbf{14,14'})+(\mathbf{14',14})+(\mathbf{64,14})+(\mathbf{14,64})+(\mathbf{70,14'})+(\mathbf{14',70})$
$\mathbf{6006}$	=	$2(\mathbf{6,6})+(\mathbf{14,1})+(\mathbf{1,14})+(\mathbf{6,14'})+(\mathbf{14',6})+(\mathbf{21,1})+(\mathbf{1,21})+2(\mathbf{14,14})+(\mathbf{14',14'})+(\mathbf{21,14})+(\mathbf{14,21})+(\mathbf{64,6})+(\mathbf{6,64})+(\mathbf{64,14'})+(\mathbf{14',64})+(\mathbf{70,14})+(\mathbf{14,70})$
$\mathbf{7800}$	=	$(\mathbf{6,1})+(\mathbf{1,6})+(\mathbf{14',1})+(\mathbf{1,14'})+2(\mathbf{6,14})+2(\mathbf{14,6})+(\mathbf{21,6})+(\mathbf{6,21})+(\mathbf{14,14'})+(\mathbf{14',14})+(\mathbf{64,1})+(\mathbf{1,64})+(\mathbf{70,6})+(\mathbf{6,70})+(\mathbf{64,14})+(\mathbf{14,64})+(\mathbf{21,64})+(\mathbf{64,21})+(\mathbf{6,90})+(\mathbf{90,6})+(\mathbf{126,1})+(\mathbf{1,126})$

Sp(12)	→	SU(2) (S)
$\mathbf{12}$	=	$(\mathbf{12})$
$\mathbf{65}$	=	$(\mathbf{5})+(\mathbf{9})+(\mathbf{13})+(\mathbf{17})+(\mathbf{21})$
$\mathbf{78}$	=	$(\mathbf{3})+(\mathbf{7})+(\mathbf{11})+(\mathbf{15})+(\mathbf{19})+(\mathbf{23})$
$\mathbf{208}$	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+2(\mathbf{10})+(\mathbf{12})+(\mathbf{14})+2(\mathbf{16})+(\mathbf{18})+(\mathbf{20})+(\mathbf{22})+(\mathbf{24})+(\mathbf{28})$
$\mathbf{364}$	=	$(\mathbf{4})+(\mathbf{6})+(\mathbf{8})+2(\mathbf{10})+2(\mathbf{12})+2(\mathbf{14})+2(\mathbf{16})+2(\mathbf{18})+(\mathbf{20})+2(\mathbf{22})+(\mathbf{24})+(\mathbf{26})+(\mathbf{28})+(\mathbf{30})+(\mathbf{34})$
$\mathbf{429}$	=	$(\mathbf{1})+2(\mathbf{5})+(\mathbf{7})+3(\mathbf{9})+2(\mathbf{11})+3(\mathbf{13})+2(\mathbf{15})+3(\mathbf{17})+2(\mathbf{19})+2(\mathbf{21})+(\mathbf{23})+2(\mathbf{25})+(\mathbf{27})+(\mathbf{29})+(\mathbf{33})$
$\mathbf{429'}$	=	$(\mathbf{1})+(\mathbf{5})+2(\mathbf{7})+2(\mathbf{9})+(\mathbf{11})+3(\mathbf{13})+2(\mathbf{15})+2(\mathbf{17})+2(\mathbf{19})+2(\mathbf{21})+(\mathbf{23})+2(\mathbf{25})+(\mathbf{27})+(\mathbf{29})+(\mathbf{31})+(\mathbf{37})$
$\mathbf{560}$	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+3(\mathbf{8})+3(\mathbf{10})+3(\mathbf{12})+4(\mathbf{14})+3(\mathbf{16})+3(\mathbf{18})+3(\mathbf{20})+2(\mathbf{22})+2(\mathbf{24})+2(\mathbf{26})+(\mathbf{28})+(\mathbf{30})+(\mathbf{32})$
$\mathbf{572}$	=	$(\mathbf{2})+(\mathbf{4})+2(\mathbf{6})+3(\mathbf{8})+2(\mathbf{10})+3(\mathbf{12})+3(\mathbf{14})+3(\mathbf{16})+3(\mathbf{18})+3(\mathbf{20})+2(\mathbf{22})+2(\mathbf{24})+2(\mathbf{26})+(\mathbf{28})+(\mathbf{30})+(\mathbf{32})+(\mathbf{36})$
$\mathbf{1365}$	=	$2(\mathbf{1})+4(\mathbf{5})+2(\mathbf{7})+5(\mathbf{9})+3(\mathbf{11})+6(\mathbf{13})+4(\mathbf{15})+6(\mathbf{17})+4(\mathbf{19})+6(\mathbf{21})+4(\mathbf{23})+5(\mathbf{25})+3(\mathbf{27})+4(\mathbf{29})+2(\mathbf{31})+3(\mathbf{33})+(\mathbf{35})+2(\mathbf{37})+(\mathbf{39})+(\mathbf{41})+(\mathbf{45})$
$\mathbf{1650}$	=	$3(\mathbf{1})+6(\mathbf{5})+3(\mathbf{7})+8(\mathbf{9})+5(\mathbf{11})+9(\mathbf{13})+6(\mathbf{15})+9(\mathbf{17})+6(\mathbf{19})+8(\mathbf{21})+5(\mathbf{23})+7(\mathbf{25})+3(\mathbf{27})+5(\mathbf{29})+2(\mathbf{31})+3(\mathbf{33})+(\mathbf{35})+2(\mathbf{37})+(\mathbf{41})$
$\mathbf{2002}$	=	$4(\mathbf{3})+4(\mathbf{5})+8(\mathbf{7})+7(\mathbf{9})+10(\mathbf{11})+9(\mathbf{13})+11(\mathbf{15})+9(\mathbf{17})+10(\mathbf{19})+8(\mathbf{21})+8(\mathbf{23})+6(\mathbf{25})+6(\mathbf{27})+4(\mathbf{29})+4(\mathbf{31})+2(\mathbf{33})+2(\mathbf{35})+(\mathbf{37})+(\mathbf{39})$
$\mathbf{2925}$	=	$5(\mathbf{3})+5(\mathbf{5})+9(\mathbf{7})+9(\mathbf{9})+12(\mathbf{11})+11(\mathbf{13})+13(\mathbf{15})+12(\mathbf{17})+13(\mathbf{19})+11(\mathbf{21})+11(\mathbf{23})+9(\mathbf{25})+9(\mathbf{27})+6(\mathbf{29})+6(\mathbf{31})+4(\mathbf{33})+4(\mathbf{35})+2(\mathbf{37})+2(\mathbf{39})+(\mathbf{41})+(\mathbf{43})$
$\mathbf{4368}$	=	$4(\mathbf{2})+8(\mathbf{4})+11(\mathbf{6})+14(\mathbf{8})+16(\mathbf{10})+18(\mathbf{12})+19(\mathbf{14})+19(\mathbf{16})+19(\mathbf{18})+18(\mathbf{20})+17(\mathbf{22})+15(\mathbf{24})+13(\mathbf{26})+11(\mathbf{28})+9(\mathbf{30})+7(\mathbf{32})+6(\mathbf{34})+4(\mathbf{36})+3(\mathbf{38})+2(\mathbf{40})+(\mathbf{42})+(\mathbf{44})$
$\mathbf{4368'}$	=	$2(\mathbf{2})+4(\mathbf{4})+6(\mathbf{6})+8(\mathbf{8})+9(\mathbf{10})+11(\mathbf{12})+11(\mathbf{14})+13(\mathbf{16})+12(\mathbf{18})+13(\mathbf{20})+12(\mathbf{22})+12(\mathbf{24})+11(\mathbf{26})+11(\mathbf{28})+9(\mathbf{30})+9(\mathbf{32})+7(\mathbf{34})+7(\mathbf{36})+5(\mathbf{38})+5(\mathbf{40})+3(\mathbf{42})+3(\mathbf{44})+2(\mathbf{46})+2(\mathbf{48})+(\mathbf{50})+(\mathbf{52})+(\mathbf{56})$
$\mathbf{4576}$	=	$3(\mathbf{2})+7(\mathbf{4})+10(\mathbf{6})+12(\mathbf{8})+15(\mathbf{10})+16(\mathbf{12})+17(\mathbf{14})+18(\mathbf{16})+18(\mathbf{18})+17(\mathbf{20})+16(\mathbf{22})+15(\mathbf{24})+13(\mathbf{26})+12(\mathbf{28})+10(\mathbf{30})+8(\mathbf{32})+7(\mathbf{34})+5(\mathbf{36})+4(\mathbf{38})+3(\mathbf{40})+2(\mathbf{42})+(\mathbf{44})+(\mathbf{46})+(\mathbf{48})$
$\mathbf{6006}$	=	$(\mathbf{1})+8(\mathbf{3})+11(\mathbf{5})+16(\mathbf{7})+18(\mathbf{9})+23(\mathbf{11})+22(\mathbf{13})+25(\mathbf{15})+24(\mathbf{17})+24(\mathbf{19})+22(\mathbf{21})+22(\mathbf{23})+18(\mathbf{25})+17(\mathbf{27})+14(\mathbf{29})+12(\mathbf{31})+9(\mathbf{33})+8(\mathbf{35})+5(\mathbf{37})+4(\mathbf{39})+3(\mathbf{41})+2(\mathbf{43})+(\mathbf{45})+(\mathbf{47})$
$\mathbf{7800}$	=	$6(\mathbf{2})+12(\mathbf{4})+17(\mathbf{6})+22(\mathbf{8})+25(\mathbf{10})+29(\mathbf{12})+30(\mathbf{14})+31(\mathbf{16})+31(\mathbf{18})+30(\mathbf{20})+28(\mathbf{22})+26(\mathbf{24})+23(\mathbf{26})+20(\mathbf{28})+17(\mathbf{30})+14(\mathbf{32})+11(\mathbf{34})+9(\mathbf{36})+6(\mathbf{38})+5(\mathbf{40})+3(\mathbf{42})+2(\mathbf{44})+(\mathbf{46})+(\mathbf{48})$

Table 10.23: Sp(12) Branching Rules (continued)

Sp(12)	→	SU(2)×SU(4) (S)
12	=	$(2,6)$
65	=	$(3,15)+(1,20')$
78	=	$(3,1)+(1,15)+(3,20')$
208	=	$(4,10)+(4,\overline{10})+(2,64)$
364	=	$(2,6)+(4,6)+(4,50)+(2,64)$
429	=	$(5,15)+(3,45)+(3,\overline{45})+(1,84)$
429′	=	$(7,1)+(5,20')+(1,35)+(1,\overline{35})+(3,84)$
560	=	$(2,6)+(4,6)+(2,10)+(2,\overline{10})+(2,50)+(2,64)+(4,64)$
572	=	$(6,6)+(4,64)+(2,70)+(2,\overline{70})$
1365	=	$(1,1)+(5,1)+(3,15)+(1,20')+(3,20')+(5,20')+(1,84)+(5,105)+(3,175)$
1650	=	$(1,1)+(5,1)+(1,15)+(3,15)+(1,20')+(3,20')+(5,20')+(3,45)+(3,\overline{45})+(1,84)+(5,84)+(1,105)+(3,175)$
2002	=	$(1,15)+2(3,15)+(5,15)+(3,20')+(1,45)+(1,\overline{45})+(3,45)+(3,\overline{45})+(5,45)+(5,\overline{45})+(3,84)+(1,175)+(3,175)$
2925	=	$(3,1)+(1,15)+2(3,15)+(5,15)+(1,20')+2(3,20')+(5,20')+(1,45)+(1,\overline{45})+(3,45)+(3,\overline{45})+(3,84)+(3,105)+(1,175)+(3,175)+(5,175)$
4368	=	$(4,6)+(2,10)+(2,\overline{10})+(4,10)+(4,\overline{10})+(6,10)+(6,\overline{10})+2(2,64)+2(4,64)+(6,64)+(2,70)+(2,\overline{70})+(4,70)+(4,\overline{70})+(2,126)+(2,\overline{126})+(4,126)+(4,\overline{126})+(2,300)$
4368′	=	$(2,6)+(4,6)+(6,6)+(2,50)+(4,50)+(6,50)+(2,64)+(4,64)+(6,196)+(2,300)+(4,384)$
4576	=	$(4,6)+(6,6)+(8,6)+(4,50)+(6,50)+(2,64)+(4,64)+(6,64)+(2,70)+(2,\overline{70})+(4,70)+(4,\overline{70})+(2,140'')+(2,\overline{140}'')+(2,300)+(4,300)$
6006	=	$(5,1)+(3,15)+(5,15)+(7,15)+(3,20')+(5,20')+(7,20')+(3,35)+(3,\overline{35})+(1,45)+(1,\overline{45})+(3,45)+(3,\overline{45})+(5,45)+(5,\overline{45})+(1,84)+2(3,84)+(5,84)+(3,175)+(5,175)+(1,256)+(1,\overline{256})+(3,256)+(3,\overline{256})$
7800	=	$2(2,6)+(4,6)+(2,10)+(2,\overline{10})+(6,6)+(4,10)+(4,\overline{10})+(2,50)+(4,50)+3(2,64)+3(4,64)+(6,64)+(2,70)+(2,\overline{70})+(4,70)+(4,\overline{70})+(6,70)+(6,\overline{70})+(2,126)+(2,\overline{126})+(4,126)+(4,\overline{126})+(2,300)+(4,300)+(2,384)$

Sp(12)	→	SU(2)×Sp(4) (S)
12	=	$(3,4)$
65	=	$(1,5)+(5,1)+(5,5)+(3,10)$
78	=	$(3,1)+(3,5)+(1,10)+(5,10)$
208	=	$(3,4)+(5,4)+(7,4)+(3,16)+(1,20)+(5,16)$
364	=	$(1,4)+(3,4)+(5,4)+(3,16)+(5,16)+(3,20)+(7,20)$
429	=	$(1,1)+(5,1)+(3,5)+(5,5)+(9,1)+(7,5)+(3,10)+(1,14)+(5,10)+(7,10)+(5,14)+(3,35)$
429′	=	$(1,5)+(7,1)+(5,5)+(3,10)+(9,5)+(3,14)+(7,10)+(1,30)+(5,35)$
560	=	$(1,4)+2(3,4)+2(5,4)+(7,4)+(1,16)+2(3,16)+(5,16)+(3,20)+(7,16)+(5,20)$
572	=	$(3,4)+(5,4)+(7,4)+(9,4)+(1,16)+(3,16)+(5,16)+(7,16)+(5,20)+(3,40)$
1365	=	$(1,1)+(1,5)+(5,1)+(3,5)+(5,5)+2(3,10)+(1,14)+(5,10)+(7,10)+(5,14)+(1,35')+(3,35)+(5,35)+(5,35')+(7,35)+(9,35')$
1650	=	$2(1,1)+(1,5)+2(5,1)+2(3,5)+2(5,5)+(9,1)+(7,5)+3(3,10)+(9,5)+2(1,14)+2(5,10)+2(7,10)+2(5,14)+(9,14)+(1,35')+2(3,35)+(5,35)+(5,35')+(7,35)$
2002	=	$2(3,1)+(1,5)+(5,1)+3(3,5)+2(7,1)+3(5,5)+2(1,10)+2(7,5)+3(3,10)+(9,5)+4(5,10)+2(3,14)+2(7,10)+(5,14)+(9,10)+(7,14)+(1,35)+2(3,35)+(3,35')+2(5,35)+(7,35)$
2925	=	$2(3,1)+(1,5)+(5,1)+3(3,5)+(7,1)+3(5,5)+2(1,10)+2(7,5)+4(3,10)+4(5,10)+2(3,14)+2(7,10)+(5,14)+(9,10)+(7,14)+2(1,35)+2(3,35)+(3,35')+3(5,35)+(5,35')+(7,35)+(7,35')+(9,35)$

Table 10.23: Sp(12) Branching Rules (continued)

4368	=	$2(\mathbf{1},\mathbf{4})+4(\mathbf{3},\mathbf{4})+5(\mathbf{5},\mathbf{4})+4(\mathbf{7},\mathbf{4})+2(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{4})+2(\mathbf{1},\mathbf{16})+5(\mathbf{3},\mathbf{16})+(\mathbf{1},\mathbf{20})+6(\mathbf{5},\mathbf{16})+3(\mathbf{3},\mathbf{20})+4(\mathbf{7},\mathbf{16})+3(\mathbf{5},\mathbf{20})+2(\mathbf{9},\mathbf{16})+2(\mathbf{7},\mathbf{20})+(\mathbf{9},\mathbf{20})+(\mathbf{1},\mathbf{40})+2(\mathbf{3},\mathbf{40})+2(\mathbf{5},\mathbf{40})+(\mathbf{7},\mathbf{40})+(\mathbf{1},\mathbf{64})+(\mathbf{3},\mathbf{64})+(\mathbf{5},\mathbf{64})$
4368′	=	$2(\mathbf{3},\mathbf{4})+(\mathbf{5},\mathbf{4})+(\mathbf{7},\mathbf{4})+(\mathbf{1},\mathbf{16})+2(\mathbf{3},\mathbf{16})+(\mathbf{1},\mathbf{20})+2(\mathbf{5},\mathbf{16})+(\mathbf{3},\mathbf{20})+(\mathbf{7},\mathbf{16})+2(\mathbf{5},\mathbf{20})+(\mathbf{7},\mathbf{20})+(\mathbf{9},\mathbf{20})+(\mathbf{3},\mathbf{40})+(\mathbf{5},\mathbf{40})+(\mathbf{7},\mathbf{40})+(\mathbf{3},\mathbf{56})+(\mathbf{7},\mathbf{56})+(\mathbf{3},\mathbf{64})+(\mathbf{11},\mathbf{56})+(\mathbf{5},\mathbf{64})+(\mathbf{7},\mathbf{64})+(\mathbf{9},\mathbf{64})$
4576	=	$(\mathbf{1},\mathbf{4})+2(\mathbf{3},\mathbf{4})+3(\mathbf{5},\mathbf{4})+3(\mathbf{7},\mathbf{4})+2(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{4})+(\mathbf{1},\mathbf{16})+4(\mathbf{3},\mathbf{16})+(\mathbf{1},\mathbf{20})+4(\mathbf{5},\mathbf{16})+2(\mathbf{3},\mathbf{20})+3(\mathbf{7},\mathbf{16})+2(\mathbf{5},\mathbf{20})+2(\mathbf{9},\mathbf{16})+2(\mathbf{7},\mathbf{20})+(\mathbf{11},\mathbf{16})+(\mathbf{9},\mathbf{20})+(\mathbf{1},\mathbf{40})+2(\mathbf{3},\mathbf{40})+2(\mathbf{5},\mathbf{40})+(\mathbf{7},\mathbf{40})+(\mathbf{3},\mathbf{64})+(\mathbf{5},\mathbf{64})+(\mathbf{7},\mathbf{64})+(\mathbf{3},\mathbf{80})$
6006	=	$2(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{5})+2(\mathbf{5},\mathbf{1})+4(\mathbf{3},\mathbf{5})+2(\mathbf{7},\mathbf{1})+4(\mathbf{5},\mathbf{5})+(\mathbf{9},\mathbf{1})+2(\mathbf{1},\mathbf{10})+4(\mathbf{7},\mathbf{5})+(\mathbf{11},\mathbf{1})+4(\mathbf{3},\mathbf{10})+2(\mathbf{9},\mathbf{5})+(\mathbf{1},\mathbf{14})+6(\mathbf{5},\mathbf{10})+(\mathbf{11},\mathbf{5})+3(\mathbf{3},\mathbf{14})+4(\mathbf{7},\mathbf{10})+3(\mathbf{5},\mathbf{14})+3(\mathbf{9},\mathbf{10})+2(\mathbf{7},\mathbf{14})+(\mathbf{11},\mathbf{10})+(\mathbf{9},\mathbf{14})+(\mathbf{3},\mathbf{30})+(\mathbf{5},\mathbf{30})+2(\mathbf{1},\mathbf{35})+4(\mathbf{3},\mathbf{35})+(\mathbf{3},\mathbf{35'})+4(\mathbf{5},\mathbf{35})+(\mathbf{5},\mathbf{35'})+3(\mathbf{7},\mathbf{35})+(\mathbf{7},\mathbf{35'})+(\mathbf{9},\mathbf{35})+(\mathbf{1},\mathbf{81})+(\mathbf{3},\mathbf{81})+(\mathbf{5},\mathbf{81})$
7800	=	$2(\mathbf{1},\mathbf{4})+6(\mathbf{3},\mathbf{4})+6(\mathbf{5},\mathbf{4})+5(\mathbf{7},\mathbf{4})+3(\mathbf{9},\mathbf{4})+(\mathbf{11},\mathbf{4})+3(\mathbf{1},\mathbf{16})+7(\mathbf{3},\mathbf{16})+2(\mathbf{1},\mathbf{20})+8(\mathbf{5},\mathbf{16})+4(\mathbf{3},\mathbf{20})+6(\mathbf{7},\mathbf{16})+5(\mathbf{5},\mathbf{20})+3(\mathbf{9},\mathbf{16})+3(\mathbf{7},\mathbf{20})+(\mathbf{11},\mathbf{16})+(\mathbf{9},\mathbf{20})+(\mathbf{1},\mathbf{40})+3(\mathbf{3},\mathbf{40})+3(\mathbf{5},\mathbf{40})+2(\mathbf{7},\mathbf{40})+(\mathbf{9},\mathbf{40})+(\mathbf{3},\mathbf{56})+(\mathbf{1},\mathbf{64})+2(\mathbf{3},\mathbf{64})+2(\mathbf{5},\mathbf{64})+(\mathbf{7},\mathbf{64})$

Table 10.24: Sp(14) Branching Rules

Sp(14)	→	SU(2)×Sp(12) (R)
14	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{12})$
90	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{12})+(\mathbf{1},\mathbf{65})$
105	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{12})+(\mathbf{1},\mathbf{78})$
350	=	$(\mathbf{1},\mathbf{12})+(\mathbf{2},\mathbf{65})+(\mathbf{1},\mathbf{208})$
560	=	$(\mathbf{4},\mathbf{1})+(\mathbf{3},\mathbf{12})+(\mathbf{2},\mathbf{78})+(\mathbf{1},\mathbf{364})$
896	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{12})+(\mathbf{3},\mathbf{12})+(\mathbf{2},\mathbf{65})+(\mathbf{2},\mathbf{78})+(\mathbf{1},\mathbf{560})$
910	=	$(\mathbf{1},\mathbf{65})+(\mathbf{2},\mathbf{208})+(\mathbf{1},\mathbf{429})$
1430	=	$(\mathbf{2},\mathbf{429'})+(\mathbf{1},\mathbf{572})$
1638	=	$(\mathbf{1},\mathbf{208})+(\mathbf{2},\mathbf{429})+(\mathbf{1},\mathbf{572})$
2002	=	$(\mathbf{1},\mathbf{429})+(\mathbf{1},\mathbf{429'})+(\mathbf{2},\mathbf{572})$
2380	=	$(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{12})+(\mathbf{3},\mathbf{78})+(\mathbf{2},\mathbf{364})+(\mathbf{1},\mathbf{1365})$
3094	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{12})+(\mathbf{1},\mathbf{65})+(\mathbf{3},\mathbf{78})+(\mathbf{2},\mathbf{560})+(\mathbf{1},\mathbf{1650})$
3900	=	$(\mathbf{2},\mathbf{12})+(\mathbf{1},\mathbf{65})+(\mathbf{3},\mathbf{65})+(\mathbf{1},\mathbf{78})+(\mathbf{2},\mathbf{208})+(\mathbf{2},\mathbf{560})+(\mathbf{1},\mathbf{2002})$

Sp(14)	→	Sp(4)×Sp(10) (R)
14	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{10})$
90	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{10})+(\mathbf{1},\mathbf{44})$
105	=	$(\mathbf{10},\mathbf{1})+(\mathbf{4},\mathbf{10})+(\mathbf{1},\mathbf{55})$
350	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{5},\mathbf{10})+(\mathbf{4},\mathbf{44})+(\mathbf{1},\mathbf{110})$
560	=	$(\mathbf{10},\mathbf{10})+(\mathbf{20},\mathbf{1})+(\mathbf{4},\mathbf{55})+(\mathbf{1},\mathbf{220})$
896	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{5},\mathbf{10})+(\mathbf{16},\mathbf{1})+(\mathbf{10},\mathbf{10})+(\mathbf{4},\mathbf{44})+(\mathbf{4},\mathbf{55})+(\mathbf{1},\mathbf{320})$
910	=	$(\mathbf{1},\mathbf{1})+(\mathbf{4},\mathbf{10})+(\mathbf{1},\mathbf{44})+(\mathbf{5},\mathbf{44})+(\mathbf{4},\mathbf{110})+(\mathbf{1},\mathbf{165})$
1430	=	$(\mathbf{1},\mathbf{110})+(\mathbf{5},\mathbf{132})+(\mathbf{4},\mathbf{165})$
1638	=	$(\mathbf{1},\mathbf{10})+(\mathbf{4},\mathbf{44})+(\mathbf{1},\mathbf{110})+(\mathbf{5},\mathbf{110})+(\mathbf{1},\mathbf{132})+(\mathbf{4},\mathbf{165})$
2002	=	$(\mathbf{1},\mathbf{44})+(\mathbf{4},\mathbf{110})+(\mathbf{4},\mathbf{132})+(\mathbf{1},\mathbf{165})+(\mathbf{5},\mathbf{165})$
2380	=	$(\mathbf{20},\mathbf{10})+(\mathbf{35'},\mathbf{1})+(\mathbf{10},\mathbf{55})+(\mathbf{4},\mathbf{220})+(\mathbf{1},\mathbf{715})$
3094	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{10})+(\mathbf{14},\mathbf{1})+(\mathbf{16},\mathbf{10})+(\mathbf{1},\mathbf{44})+(\mathbf{5},\mathbf{44})+(\mathbf{10},\mathbf{55})+(\mathbf{4},\mathbf{320})+(\mathbf{1},\mathbf{780})$
3900	=	$(\mathbf{5},\mathbf{1})+(\mathbf{10},\mathbf{1})+2(\mathbf{4},\mathbf{10})+(\mathbf{16},\mathbf{10})+(\mathbf{1},\mathbf{44})+(\mathbf{5},\mathbf{44})+(\mathbf{10},\mathbf{44})+(\mathbf{1},\mathbf{55})+(\mathbf{5},\mathbf{55})+(\mathbf{4},\mathbf{110})+(\mathbf{4},\mathbf{320})+(\mathbf{1},\mathbf{891})$

Table 10.24: Sp(14) Branching Rules (continued)

Sp(14)	→	Sp(6)×Sp(8) (R)
14	=	(**6**, **1**) + (**1**, **8**)
90	=	(**1**, **1**) + (**6**, **8**) + (**14**, **1**) + (**1**, **27**)
105	=	(**6**, **8**) + (**21**, **1**) + (**1**, **36**)
350	=	(**6**, **1**) + (**1**, **8**) + (**14′**, **1**) + (**14**, **8**) + (**6**, **27**) + (**1**, **48**)
560	=	(**21**, **8**) + (**6**, **36**) + (**56**, **1**) + (**1**, **120**)
896	=	(**6**, **1**) + (**1**, **8**) + (**14**, **8**) + (**21**, **8**) + (**6**, **27**) + (**6**, **36**) + (**64**, **1**) + (**1**, **160**)
910	=	(**1**, **1**) + (**6**, **8**) + (**14**, **1**) + (**14′**, **8**) + (**1**, **27**) + (**14**, **27**) + (**1**, **42**) + (**6**, **48**)
1430	=	(**1**, **8**) + (**6**, **27**) + (**14′**, **42**) + (**14**, **48**)
1638	=	(**6**, **1**) + (**1**, **8**) + (**14**, **8**) + (**6**, **27**) + (**14′**, **27**) + (**6**, **42**) + (**1**, **48**) + (**14**, **48**)
2002	=	(**1**, **1**) + (**6**, **8**) + (**1**, **27**) + (**14**, **27**) + (**6**, **48**) + (**14**, **42**) + (**14′**, **48**)
2380	=	(**21**, **36**) + (**56**, **8**) + (**6**, **120**) + (**126′**, **1**) + (**1**, **330**)
3094	=	(**1**, **1**) + (**6**, **8**) + (**14**, **1**) + (**1**, **27**) + (**14**, **27**) + (**21**, **36**) + (**64**, **8**) + (**90**, **1**) + (**6**, **160**) + (**1**, **308**)
3900	=	2(**6**, **8**) + (**14**, **1**) + (**14′**, **8**) + (**21**, **1**) + (**1**, **27**) + (**1**, **36**) + (**14**, **27**) + (**21**, **27**) + (**14**, **36**) + (**6**, **48**) + (**70**, **1**) + (**64**, **8**) + (**6**, **160**) + (**1**, **315**)

Sp(14)	→	SU(2) (S)
14	=	(**14**)
90	=	(**5**) + (**9**) + (**13**) + (**17**) + (**21**) + (**25**)
105	=	(**3**) + (**7**) + (**11**) + (**15**) + (**19**) + (**23**) + (**27**)
350	=	(**4**) + (**6**) + (**8**) + 2(**10**) + 2(**12**) + (**14**) + 2(**16**) + 2(**18**) + (**20**) + 2(**22**) + (**24**) + (**26**) + (**28**) + (**30**) + (**34**)
560	=	(**4**) + (**6**) + (**8**) + 2(**10**) + 2(**12**) + 2(**14**) + 3(**16**) + 2(**18**) + 2(**20**) + 2(**22**) + 2(**24**) + (**26**) + 2(**28**) + (**30**) + (**32**) + (**34**) + (**36**) + (**40**)
896	=	(**2**) + (**4**) + 2(**6**) + 3(**8**) + 3(**10**) + 4(**12**) + 4(**14**) + 4(**16**) + 4(**18**) + 4(**20**) + 3(**22**) + 3(**24**) + 3(**26**) + 2(**28**) + 2(**30**) + 2(**32**) + (**34**) + (**36**) + (**38**)
910	=	(**1**) + 3(**5**) + (**7**) + 4(**9**) + 3(**11**) + 4(**13**) + 3(**15**) + 5(**17**) + 3(**19**) + 4(**21**) + 3(**23**) + 3(**25**) + 2(**27**) + 3(**29**) + (**31**) + 2(**33**) + (**35**) + (**37**) + (**41**)
1430	=	2(**2**) + (**4**) + 2(**6**) + 4(**8**) + 3(**10**) + 4(**12**) + 5(**14**) + 4(**16**) + 5(**18**) + 5(**20**) + 4(**22**) + 4(**24**) + 5(**26**) + 3(**28**) + 3(**30**) + 3(**32**) + 2(**34**) + 2(**36**) + 2(**38**) + (**40**) + (**42**) + (**44**) + (**50**)
1638	=	(**2**) + 2(**4**) + 4(**6**) + 4(**8**) + 5(**10**) + 5(**12**) + 6(**14**) + 6(**16**) + 6(**18**) + 6(**20**) + 6(**22**) + 5(**24**) + 5(**26**) + 4(**28**) + 4(**30**) + 3(**32**) + 2(**34**) + 2(**36**) + 2(**38**) + (**40**) + (**42**) + (**46**)
2002	=	2(**1**) + (**3**) + 3(**5**) + 4(**7**) + 6(**9**) + 4(**11**) + 8(**13**) + 6(**15**) + 7(**17**) + 7(**19**) + 7(**21**) + 5(**23**) + 7(**25**) + 5(**27**) + 5(**29**) + 4(**31**) + 4(**33**) + 2(**35**) + 3(**37**) + 2(**39**) + (**41**) + (**43**) + (**45**) + (**49**)
2380	=	2(**1**) + 5(**5**) + 2(**7**) + 6(**9**) + 4(**11**) + 7(**13**) + 5(**15**) + 8(**17**) + 5(**19**) + 8(**21**) + 6(**23**) + 7(**25**) + 5(**27**) + 7(**29**) + 4(**31**) + 5(**33**) + 3(**35**) + 4(**37**) + 2(**39**) + 3(**41**) + (**43**) + 2(**45**) + (**47**) + (**49**) + (**53**)
3094	=	4(**1**) + 7(**5**) + 4(**7**) + 10(**9**) + 6(**11**) + 12(**13**) + 8(**15**) + 12(**17**) + 9(**19**) + 12(**21**) + 8(**23**) + 11(**25**) + 7(**27**) + 9(**29**) + 5(**31**) + 7(**33**) + 3(**35**) + 5(**37**) + 2(**39**) + 3(**41**) + (**43**) + 2(**45**) + (**49**)
3900	=	5(**3**) + 5(**5**) + 10(**7**) + 9(**9**) + 13(**11**) + 12(**13**) + 15(**15**) + 13(**17**) + 15(**19**) + 13(**21**) + 14(**23**) + 11(**25**) + 11(**27**) + 9(**29**) + 9(**31**) + 6(**33**) + 6(**35**) + 4(**37**) + 4(**39**) + 2(**41**) + 2(**43**) + (**45**) + (**47**)

Sp(14)	→	SU(2)×SO(7) (S)
14	=	(**2**, **7**)
90	=	(**3**, **21**) + (**1**, **27**)
105	=	(**3**, **1**) + (**1**, **21**) + (**3**, **27**)
350	=	(**4**, **35**) + (**2**, **105**)
560	=	(**2**, **7**) + (**4**, **7**) + (**4**, **77**) + (**2**, **105**)
896	=	(**2**, **7**) + (**4**, **7**) + (**2**, **35**) + (**2**, **77**) + (**2**, **105**) + (**4**, **105**)
910	=	(**5**, **35**) + (**1**, **168′**) + (**3**, **189**)
1430	=	(**8**, **1**) + (**6**, **27**) + (**4**, **168′**) + (**2**, **294**)
1638	=	(**6**, **21**) + (**4**, **189**) + (**2**, **378**)
2002	=	(**7**, **7**) + (**5**, **105**) + (**1**, **294**) + (**3**, **378**)

Table 10.24: Sp(14) Branching Rules (continued)

2380	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{3},\mathbf{21})+(\mathbf{1},\mathbf{27})+(\mathbf{3},\mathbf{27})+(\mathbf{5},\mathbf{27})+(\mathbf{1},\mathbf{168'})+(\mathbf{5},\mathbf{182})+(\mathbf{3},\mathbf{330})$
3094	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{3},\mathbf{21})+(\mathbf{1},\mathbf{27})+(\mathbf{3},\mathbf{27})+(\mathbf{5},\mathbf{27})+(\mathbf{1},\mathbf{35})+(\mathbf{1},\mathbf{168'})+(\mathbf{5},\mathbf{168'})+(\mathbf{1},\mathbf{182})+(\mathbf{3},\mathbf{189})+(\mathbf{3},\mathbf{330})$
3900	=	$(\mathbf{1},\mathbf{21})+(\mathbf{3},\mathbf{21})+(\mathbf{5},\mathbf{21})+(\mathbf{3},\mathbf{27})+(\mathbf{3},\mathbf{35})+(\mathbf{3},\mathbf{168'})+(\mathbf{1},\mathbf{189})+(\mathbf{3},\mathbf{189})+(\mathbf{5},\mathbf{189})+(\mathbf{1},\mathbf{330})+(\mathbf{3},\mathbf{330})$

Table 10.25: Sp(16) Branching Rules

Sp(16)	→	SU(2)×Sp(14) (R)
16	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{14})$
119	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{90})$
136	=	$(\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{105})$
544	=	$(\mathbf{1},\mathbf{14})+(\mathbf{2},\mathbf{90})+(\mathbf{1},\mathbf{350})$
816	=	$(\mathbf{4},\mathbf{1})+(\mathbf{3},\mathbf{14})+(\mathbf{2},\mathbf{105})+(\mathbf{1},\mathbf{560})$
1344	=	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{14})+(\mathbf{3},\mathbf{14})+(\mathbf{2},\mathbf{90})+(\mathbf{2},\mathbf{105})+(\mathbf{1},\mathbf{896})$
1700	=	$(\mathbf{1},\mathbf{90})+(\mathbf{2},\mathbf{350})+(\mathbf{1},\mathbf{910})$
3808	=	$(\mathbf{1},\mathbf{350})+(\mathbf{2},\mathbf{910})+(\mathbf{1},\mathbf{1638})$
3876	=	$(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{14})+(\mathbf{3},\mathbf{105})+(\mathbf{2},\mathbf{560})+(\mathbf{1},\mathbf{2380})$
4862	=	$(\mathbf{2},\mathbf{1430})+(\mathbf{1},\mathbf{2002})$
5320	=	$(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{14})+(\mathbf{1},\mathbf{90})+(\mathbf{3},\mathbf{105})+(\mathbf{2},\mathbf{896})+(\mathbf{1},\mathbf{3094})$

Sp(16)	→	Sp(4)×Sp(12) (R)
16	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{12})$
119	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{4},\mathbf{12})+(\mathbf{1},\mathbf{65})$
136	=	$(\mathbf{10},\mathbf{1})+(\mathbf{4},\mathbf{12})+(\mathbf{1},\mathbf{78})$
544	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{12})+(\mathbf{5},\mathbf{12})+(\mathbf{4},\mathbf{65})+(\mathbf{1},\mathbf{208})$
816	=	$(\mathbf{20},\mathbf{1})+(\mathbf{10},\mathbf{12})+(\mathbf{4},\mathbf{78})+(\mathbf{1},\mathbf{364})$
1344	=	$(\mathbf{4},\mathbf{1})+(\mathbf{1},\mathbf{12})+(\mathbf{5},\mathbf{12})+(\mathbf{16},\mathbf{1})+(\mathbf{10},\mathbf{12})+(\mathbf{4},\mathbf{65})+(\mathbf{4},\mathbf{78})+(\mathbf{1},\mathbf{560})$
1700	=	$(\mathbf{1},\mathbf{1})+(\mathbf{4},\mathbf{12})+(\mathbf{1},\mathbf{65})+(\mathbf{5},\mathbf{65})+(\mathbf{4},\mathbf{208})+(\mathbf{1},\mathbf{429})$
3808	=	$(\mathbf{1},\mathbf{12})+(\mathbf{4},\mathbf{65})+(\mathbf{1},\mathbf{208})+(\mathbf{5},\mathbf{208})+(\mathbf{4},\mathbf{429})+(\mathbf{1},\mathbf{572})$
3876	=	$(\mathbf{20},\mathbf{12})+(\mathbf{35'},\mathbf{1})+(\mathbf{10},\mathbf{78})+(\mathbf{4},\mathbf{364})+(\mathbf{1},\mathbf{1365})$
4862	=	$(\mathbf{1},\mathbf{429})+(\mathbf{5},\mathbf{429'})+(\mathbf{4},\mathbf{572})$
5320	=	$(\mathbf{1},\mathbf{1})+(\mathbf{5},\mathbf{1})+(\mathbf{14},\mathbf{1})+(\mathbf{4},\mathbf{12})+(\mathbf{16},\mathbf{12})+(\mathbf{1},\mathbf{65})+(\mathbf{5},\mathbf{65})+(\mathbf{10},\mathbf{78})+(\mathbf{4},\mathbf{560})+(\mathbf{1},\mathbf{1650})$

Sp(16)	→	Sp(6)×Sp(10) (R)
16	=	$(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{10})$
119	=	$(\mathbf{1},\mathbf{1})+(\mathbf{14},\mathbf{1})+(\mathbf{6},\mathbf{10})+(\mathbf{1},\mathbf{44})$
136	=	$(\mathbf{6},\mathbf{10})+(\mathbf{21},\mathbf{1})+(\mathbf{1},\mathbf{55})$
544	=	$(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{14'},\mathbf{1})+(\mathbf{14},\mathbf{10})+(\mathbf{6},\mathbf{44})+(\mathbf{1},\mathbf{110})$
816	=	$(\mathbf{21},\mathbf{10})+(\mathbf{56},\mathbf{1})+(\mathbf{6},\mathbf{55})+(\mathbf{1},\mathbf{220})$
1344	=	$(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{14},\mathbf{10})+(\mathbf{21},\mathbf{10})+(\mathbf{6},\mathbf{44})+(\mathbf{6},\mathbf{55})+(\mathbf{64},\mathbf{1})+(\mathbf{1},\mathbf{320})$
1700	=	$(\mathbf{1},\mathbf{1})+(\mathbf{14},\mathbf{1})+(\mathbf{6},\mathbf{10})+(\mathbf{14'},\mathbf{10})+(\mathbf{1},\mathbf{44})+(\mathbf{14},\mathbf{44})+(\mathbf{6},\mathbf{110})+(\mathbf{1},\mathbf{165})$
3808	=	$(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{14},\mathbf{10})+(\mathbf{6},\mathbf{44})+(\mathbf{14'},\mathbf{44})+(\mathbf{1},\mathbf{110})+(\mathbf{14},\mathbf{110})+(\mathbf{1},\mathbf{132})+(\mathbf{6},\mathbf{165})$
3876	=	$(\mathbf{56},\mathbf{10})+(\mathbf{21},\mathbf{55})+(\mathbf{126'},\mathbf{1})+(\mathbf{6},\mathbf{220})+(\mathbf{1},\mathbf{715})$
4862	=	$(\mathbf{1},\mathbf{44})+(\mathbf{6},\mathbf{110})+(\mathbf{14'},\mathbf{132})+(\mathbf{14},\mathbf{165})$
5320	=	$(\mathbf{1},\mathbf{1})+(\mathbf{14},\mathbf{1})+(\mathbf{6},\mathbf{10})+(\mathbf{1},\mathbf{44})+(\mathbf{14},\mathbf{44})+(\mathbf{64},\mathbf{10})+(\mathbf{21},\mathbf{55})+(\mathbf{90},\mathbf{1})+(\mathbf{6},\mathbf{320})+(\mathbf{1},\mathbf{780})$

Sp(16)	→	Sp(8)×Sp(8) (R)
16	=	$(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8})$

119	=	(**1**,**1**) + (**8**,**8**) + (**27**,**1**) + (**1**,**27**)
136	=	(**8**,**8**) + (**36**,**1**) + (**1**,**36**)
544	=	(**8**,**1**) + (**1**,**8**) + (**8**,**27**) + (**27**,**8**) + (**48**,**1**) + (**1**,**48**)
816	=	(**36**,**8**) + (**8**,**36**) + (**120**,**1**) + (**1**,**120**)
1344	=	(**8**,**1**) + (**1**,**8**) + (**8**,**27**) + (**27**,**8**) + (**36**,**8**) + (**8**,**36**) + (**160**,**1**) + (**1**,**160**)
1700	=	(**1**,**1**) + (**8**,**8**) + (**27**,**1**) + (**1**,**27**) + (**42**,**1**) + (**1**,**42**) + (**27**,**27**) + (**8**,**48**) + (**48**,**8**)
3808	=	(**8**,**1**) + (**1**,**8**) + (**8**,**27**) + (**27**,**8**) + (**48**,**1**) + (**1**,**48**) + (**8**,**42**) + (**42**,**8**) + (**27**,**48**) + (**48**,**27**)
3876	=	(**36**,**36**) + (**120**,**8**) + (**8**,**120**) + (**330**,**1**) + (**1**,**330**)
4862	=	(**1**,**1**) + (**8**,**8**) + (**27**,**27**) + (**42**,**42**) + (**48**,**48**)
5320	=	(**1**,**1**) + (**8**,**8**) + (**27**,**1**) + (**1**,**27**) + (**27**,**27**) + (**36**,**36**) + (**160**,**8**) + (**8**,**160**) + (**308**,**1**) + (**1**,**308**)

Sp(16)	→	SU(2) (S)
16	=	(**16**)
119	=	(**5**) + (**9**) + (**13**) + (**17**) + (**21**) + (**25**) + (**29**)
136	=	(**3**) + (**7**) + (**11**) + (**15**) + (**19**) + (**23**) + (**27**) + (**31**)
544	=	(**4**) + (**6**) + (**8**) + 2(**10**) + 2(**12**) + 2(**14**) + 2(**16**) + 2(**18**) + 2(**20**) + 2(**22**) + 2(**24**) + (**26**) + 2(**28**) + (**30**) + (**32**) + (**34**) + (**36**) + (**40**)
816	=	(**4**) + (**6**) + (**8**) + 2(**10**) + 2(**12**) + 2(**14**) + 3(**16**) + 3(**18**) + 2(**20**) + 3(**22**) + 2(**24**) + 2(**26**) + 2(**28**) + 2(**30**) + (**32**) + 2(**34**) + (**36**) + (**38**) + (**40**) + (**42**) + (**46**)
1344	=	(**2**) + (**4**) + 2(**6**) + 3(**8**) + 3(**10**) + 4(**12**) + 5(**14**) + 4(**16**) + 5(**18**) + 5(**20**) + 4(**22**) + 4(**24**) + 4(**26**) + 3(**28**) + 3(**30**) + 3(**32**) + 2(**34**) + 2(**36**) + 2(**38**) + (**40**) + (**42**) + (**44**)
1700	=	2(**1**) + 3(**5**) + 2(**7**) + 5(**9**) + 3(**11**) + 6(**13**) + 4(**15**) + 6(**17**) + 5(**19**) + 6(**21**) + 4(**23**) + 6(**25**) + 4(**27**) + 4(**29**) + 3(**31**) + 4(**33**) + 2(**35**) + 3(**37**) + (**39**) + 2(**41**) + (**43**) + (**45**) + (**49**)
3808	=	2(**2**) + 3(**4**) + 5(**6**) + 7(**8**) + 7(**10**) + 9(**12**) + 9(**14**) + 10(**16**) + 10(**18**) + 11(**20**) + 10(**22**) + 10(**24**) + 10(**26**) + 9(**28**) + 8(**30**) + 8(**32**) + 6(**34**) + 6(**36**) + 5(**38**) + 4(**40**) + 3(**42**) + 3(**44**) + 2(**46**) + 2(**48**) + (**50**) + (**52**) + (**56**)
3876	=	3(**1**) + 5(**5**) + 3(**7**) + 7(**9**) + 4(**11**) + 9(**13**) + 6(**15**) + 9(**17**) + 7(**19**) + 10(**21**) + 7(**23**) + 10(**25**) + 7(**27**) + 9(**29**) + 7(**31**) + 8(**33**) + 5(**35**) + 7(**37**) + 4(**39**) + 5(**41**) + 3(**43**) + 4(**45**) + 2(**47**) + 3(**49**) + (**51**) + 2(**53**) + (**55**) + (**57**) + (**61**)
4862	=	(**1**) + 2(**3**) + 5(**5**) + 4(**7**) + 8(**9**) + 8(**11**) + 9(**13**) + 9(**15**) + 12(**17**) + 10(**19**) + 11(**21**) + 11(**23**) + 11(**25**) + 10(**27**) + 11(**29**) + 8(**31**) + 9(**33**) + 8(**35**) + 7(**37**) + 6(**39**) + 6(**41**) + 4(**43**) + 4(**45**) + 3(**47**) + 3(**49**) + 2(**51**) + 2(**53**) + (**55**) + (**57**) + (**59**) + (**65**)
5320	=	4(**1**) + 9(**5**) + 4(**7**) + 12(**9**) + 8(**11**) + 14(**13**) + 10(**15**) + 16(**17**) + 11(**19**) + 16(**21**) + 12(**23**) + 15(**25**) + 11(**27**) + 14(**29**) + 9(**31**) + 12(**33**) + 7(**35**) + 9(**37**) + 5(**39**) + 7(**41**) + 3(**43**) + 5(**45**) + 2(**47**) + 3(**49**) + (**51**) + 2(**53**) + (**57**)

Sp(16)	→	Sp(4) (S)
16	=	(**16**)
119	=	(**5**) + (**14**) + (**30**) + (**35**) + (**35′**)
136	=	2(**10**) + (**35**) + (**81**)
544	=	(**4**) + (**16**) + 2(**20**) + 2(**40**) + (**56**) + 2(**64**) + (**80**) + (**140**)
816	=	(**4**) + 2(**16**) + 2(**20**) + 2(**40**) + (**56**) + 2(**64**) + (**80**) + (**140**) + (**256**)
1344	=	2(**4**) + 4(**16**) + 3(**20**) + 4(**40**) + (**56**) + 4(**64**) + 2(**80**) + 2(**140**) + (**140′**) + (**160**)
1700	=	(**1**) + (**5**) + (**10**) + 3(**14**) + (**30**) + 4(**35**) + 3(**35′**) + 2(**55**) + 3(**81**) + 4(**105**) + (**154**) + 2(**220**)
3808	=	(**4**) + 4(**16**) + 4(**20**) + 5(**40**) + 4(**56**) + 6(**64**) + 4(**80**) + (**120**) + 5(**140**) + (**140′**) + 3(**160**) + 3(**256**) + (**324**)
3876	=	(**1**) + 3(**5**) + (**10**) + 4(**14**) + 2(**30**) + 4(**35**) + 5(**35′**) + 2(**55**) + 4(**81**) + (**91**) + 4(**105**) + 2(**154**) + 3(**220**) + (**231**) + (**260**) + (**390**) + (**625**)
4862	=	(**1**) + 2(**10**) + 3(**14**) + 3(**30**) + 4(**35**) + 3(**35′**) + 3(**55**) + 4(**81**) + 3(**84**) + 3(**105**) + (**140″**) + 3(**154**) + 2(**165**) + 4(**220**) + (**231**) + 2(**260**) + (**390**) + (**455**)

Table 10.25: Sp(16) Branching Rules (continued)

5320	=	$3(\mathbf{1})+4(\mathbf{5})+3(\mathbf{10})+7(\mathbf{14})+5(\mathbf{30})+7(\mathbf{35})+8(\mathbf{35'})+4(\mathbf{55})+7(\mathbf{81})+(\mathbf{84})+(\mathbf{91})+6(\mathbf{105})+(\mathbf{140''})+4(\mathbf{154})+(\mathbf{165})+5(\mathbf{220})+(\mathbf{231})+(\mathbf{260})+(\mathbf{390})$

Sp(16)	→	SO(8)×SU(2) (S)
16	=	$(\mathbf{8_v},\mathbf{2})$
119	=	$(\mathbf{28},\mathbf{3})+(\mathbf{35_v},\mathbf{1})$
136	=	$(\mathbf{1},\mathbf{3})+(\mathbf{28},\mathbf{1})+(\mathbf{35_v},\mathbf{3})$
544	=	$(\mathbf{56_v},\mathbf{4})+(\mathbf{160_v},\mathbf{2})$
816	=	$(\mathbf{8_v},\mathbf{2})+(\mathbf{8_v},\mathbf{4})+(\mathbf{112_v},\mathbf{4})+(\mathbf{160_v},\mathbf{2})$
1344	=	$(\mathbf{8_v},\mathbf{2})+(\mathbf{8_v},\mathbf{4})+(\mathbf{56_v},\mathbf{2})+(\mathbf{112_v},\mathbf{2})+(\mathbf{160_v},\mathbf{2})+(\mathbf{160_v},\mathbf{4})$
1700	=	$(\mathbf{35_c},\mathbf{5})+(\mathbf{35_s},\mathbf{5})+(\mathbf{300},\mathbf{1})+(\mathbf{350},\mathbf{3})$
3808	=	$(\mathbf{56_v},\mathbf{6})+(\mathbf{224_{cv}},\mathbf{4})+(\mathbf{224_{sv}},\mathbf{4})+(\mathbf{840_v},\mathbf{2})$
3876	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{28},\mathbf{3})+(\mathbf{35_v},\mathbf{1})+(\mathbf{35_v},\mathbf{3})+(\mathbf{35_v},\mathbf{5})+(\mathbf{294_v},\mathbf{5})+(\mathbf{300},\mathbf{1})+(\mathbf{567_v},\mathbf{3})$
4862	=	$(\mathbf{1},\mathbf{9})+(\mathbf{35_v},\mathbf{7})+(\mathbf{294_c},\mathbf{1})+(\mathbf{294_s},\mathbf{1})+(\mathbf{300},\mathbf{5})+(\mathbf{840'_v},\mathbf{3})$
5320	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{5})+(\mathbf{28},\mathbf{3})+(\mathbf{35_v},\mathbf{1})+(\mathbf{35_c},\mathbf{1})+(\mathbf{35_s},\mathbf{1})+(\mathbf{35_v},\mathbf{3})+(\mathbf{35_v},\mathbf{5})+(\mathbf{294_v},\mathbf{1})+(\mathbf{300},\mathbf{1})+(\mathbf{300},\mathbf{5})+(\mathbf{350},\mathbf{3})+(\mathbf{567_v},\mathbf{3})$

10.0.4. Exceptional Algebras

Table 10.26: E_6 Branching Rules

E_6	→	F_4 (S)
27	=	$(\mathbf{1})+(\mathbf{26})$
78	=	$(\mathbf{26})+(\mathbf{52})$
351	=	$(\mathbf{26})+(\mathbf{52})+(\mathbf{273})$
351′	=	$(\mathbf{1})+(\mathbf{26})+(\mathbf{324})$
650	=	$(\mathbf{1})+2(\mathbf{26})+(\mathbf{273})+(\mathbf{324})$
1728	=	$(\mathbf{26})+(\mathbf{52})+(\mathbf{273})+(\mathbf{324})+(\mathbf{1053})$
2430	=	$(\mathbf{324})+(\mathbf{1053})+(\mathbf{1053'})$
2925	=	$(\mathbf{52})+2(\mathbf{273})+(\mathbf{1053})+(\mathbf{1274})$
3003	=	$(\mathbf{1})+(\mathbf{26})+(\mathbf{324})+(\mathbf{2652})$
5824	=	$(\mathbf{26})+(\mathbf{52})+(\mathbf{273})+(\mathbf{324})+(\mathbf{1053})+(\mathbf{4096})$
7371	=	$(\mathbf{26})+(\mathbf{52})+2(\mathbf{273})+(\mathbf{324})+(\mathbf{1053})+(\mathbf{1274})+(\mathbf{4096})$
7722	=	$(\mathbf{1})+2(\mathbf{26})+(\mathbf{273})+2(\mathbf{324})+(\mathbf{2652})+(\mathbf{4096})$
17550	=	$(\mathbf{273})+(\mathbf{324})+2(\mathbf{1053})+(\mathbf{1053'})+(\mathbf{1274})+(\mathbf{4096})+(\mathbf{8424})$

E_6	→	SU(3)×G_2 (S)
27	=	$(\overline{\mathbf{6}},\mathbf{1})+(\mathbf{3},\mathbf{7})$
78	=	$(\mathbf{8},\mathbf{1})+(\mathbf{8},\mathbf{7})+(\mathbf{1},\mathbf{14})$
351	=	$(\mathbf{3},\mathbf{1})+(\mathbf{3},\mathbf{7})+(\overline{\mathbf{6}},\mathbf{7})+(\mathbf{15},\mathbf{1})+(\overline{\mathbf{6}},\mathbf{14})+(\mathbf{15},\mathbf{7})+(\mathbf{3},\mathbf{27})$
351′	=	$(\overline{\mathbf{6}},\mathbf{1})+(\mathbf{3},\mathbf{7})+(\mathbf{15'},\mathbf{1})+(\mathbf{3},\mathbf{14})+(\mathbf{15},\mathbf{7})+(\overline{\mathbf{6}},\mathbf{27})$
650	=	$(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{7})+(\mathbf{8},\mathbf{1})+2(\mathbf{8},\mathbf{7})+(\mathbf{10},\mathbf{7})+(\overline{\mathbf{10}},\mathbf{7})+(\mathbf{8},\mathbf{14})+(\mathbf{1},\mathbf{27})+(\mathbf{27},\mathbf{1})+(\mathbf{8},\mathbf{27})$
1728	=	$(\mathbf{3},\mathbf{1})+(\overline{\mathbf{6}},\mathbf{1})+2(\mathbf{3},\mathbf{7})+2(\overline{\mathbf{6}},\mathbf{7})+(\mathbf{15},\mathbf{1})+(\mathbf{3},\mathbf{14})+(\overline{\mathbf{6}},\mathbf{14})+2(\mathbf{15},\mathbf{7})+(\mathbf{24},\mathbf{1})+(\mathbf{15},\mathbf{14})+(\mathbf{3},\mathbf{27})+(\mathbf{24},\mathbf{7})+(\overline{\mathbf{6}},\mathbf{27})+(\mathbf{15},\mathbf{27})+(\mathbf{3},\mathbf{64})$
2430	=	$(\mathbf{1},\mathbf{1})+(\mathbf{8},\mathbf{1})+2(\mathbf{8},\mathbf{7})+(\mathbf{10},\mathbf{7})+(\overline{\mathbf{10}},\mathbf{7})+(\mathbf{8},\mathbf{14})+(\mathbf{10},\mathbf{14})+(\overline{\mathbf{10}},\mathbf{14})+(\mathbf{1},\mathbf{27})+(\mathbf{27},\mathbf{1})+(\mathbf{27},\mathbf{7})+(\mathbf{8},\mathbf{27})+(\mathbf{27},\mathbf{27})+(\mathbf{8},\mathbf{64})+(\mathbf{1},\mathbf{77'})$
2925	=	$2(\mathbf{1},\mathbf{7})+(\mathbf{8},\mathbf{1})+2(\mathbf{10},\mathbf{1})+2(\overline{\mathbf{10}},\mathbf{1})+3(\mathbf{8},\mathbf{7})+(\mathbf{1},\mathbf{14})+(\mathbf{10},\mathbf{7})+(\overline{\mathbf{10}},\mathbf{7})+2(\mathbf{8},\mathbf{14})+2(\mathbf{27},\mathbf{7})+2(\mathbf{8},\mathbf{27})+(\mathbf{10},\mathbf{27})+(\overline{\mathbf{10}},\mathbf{27})+(\mathbf{27},\mathbf{14})+(\mathbf{8},\mathbf{64})+(\mathbf{1},\mathbf{77})$
3003	=	$(\mathbf{1},\mathbf{1})+(\mathbf{8},\mathbf{7})+(\mathbf{10},\mathbf{7})+(\overline{\mathbf{10}},\mathbf{7})+(\mathbf{8},\mathbf{14})+(\overline{\mathbf{10}},\mathbf{14})+(\mathbf{1},\mathbf{27})+(\mathbf{27},\mathbf{1})+(\overline{\mathbf{28}},\mathbf{1})+(\mathbf{8},\mathbf{27})+(\overline{\mathbf{35}},\mathbf{7})+(\mathbf{27},\mathbf{27})+(\mathbf{8},\mathbf{64})+(\mathbf{10},\mathbf{77})$

$\mathbf{5824}$	=	$\mathbf{(1,7)}+2\mathbf{(8,1)}+\mathbf{(\overline{10},1)}+4\mathbf{(8,7)}+\mathbf{(1,14)}+\mathbf{(10,7)}+2\mathbf{(\overline{10},7)}+2\mathbf{(8,14)}+\mathbf{(10,14)}+\mathbf{(\overline{10},14)}+\mathbf{(1,27)}+\mathbf{(27,1)}+2\mathbf{(27,7)}+3\mathbf{(8,27)}+\mathbf{(\overline{35},1)}+\mathbf{(10,27)}+\mathbf{(\overline{10},27)}+\mathbf{(27,14)}+\mathbf{(\overline{35},7)}+\mathbf{(27,27)}+\mathbf{(1,64)}+\mathbf{(8,64)}+\mathbf{(10,64)}+\mathbf{(8,77)}$
$\mathbf{7371}$	=	$2\mathbf{(3,1)}+\mathbf{(\overline{6},1)}+3\mathbf{(3,7)}+4\mathbf{(\overline{6},7)}+2\mathbf{(15,1)}+2\mathbf{(3,14)}+2\mathbf{(\overline{6},14)}+4\mathbf{(15,7)}+\mathbf{(15',7)}+\mathbf{(24,1)}+2\mathbf{(15,14)}+\mathbf{(15',14)}+3\mathbf{(3,27)}+2\mathbf{(24,7)}+2\mathbf{(\overline{6},27)}+\mathbf{(24,14)}+3\mathbf{(15,27)}+\mathbf{(42,1)}+\mathbf{(42,7)}+\mathbf{(24,27)}+\mathbf{(3,64)}+\mathbf{(\overline{6},64)}+\mathbf{(15,64)}+\mathbf{(3,77)}+\mathbf{(\overline{6},77)}$
$\mathbf{7722}$	=	$2\mathbf{(\overline{6},1)}+3\mathbf{(3,7)}+2\mathbf{(\overline{6},7)}+\mathbf{(15,1)}+\mathbf{(15',1)}+2\mathbf{(3,14)}+\mathbf{(\overline{6},14)}+3\mathbf{(15,7)}+\mathbf{(24,1)}+\mathbf{(21,7)}+2\mathbf{(15,14)}+2\mathbf{(3,27)}+2\mathbf{(24,7)}+3\mathbf{(\overline{6},27)}+\mathbf{(24,14)}+2\mathbf{(15,27)}+\mathbf{(15',27)}+\mathbf{(42,7)}+\mathbf{(24,27)}+\mathbf{(60,1)}+\mathbf{(3,64)}+\mathbf{(\overline{6},64)}+\mathbf{(15,64)}+\mathbf{(3,77)}+\mathbf{(15,77)}$
$\mathbf{17550}$	=	$\mathbf{(3,1)}+2\mathbf{(\overline{6},1)}+4\mathbf{(3,7)}+4\mathbf{(\overline{6},7)}+3\mathbf{(15,1)}+\mathbf{(15',1)}+3\mathbf{(3,14)}+2\mathbf{(\overline{6},14)}+6\mathbf{(15,7)}+2\mathbf{(15',7)}+\mathbf{(24,1)}+4\mathbf{(15,14)}+3\mathbf{(3,27)}+3\mathbf{(24,7)}+4\mathbf{(\overline{6},27)}+2\mathbf{(24,14)}+4\mathbf{(15,27)}+\mathbf{(15',27)}+\mathbf{(42,1)}+2\mathbf{(42,7)}+2\mathbf{(24,27)}+\mathbf{(42,14)}+2\mathbf{(3,64)}+\mathbf{(42,27)}+2\mathbf{(\overline{6},64)}+2\mathbf{(15,64)}+\mathbf{(3,77)}+\mathbf{(\overline{6},77)}+\mathbf{(\overline{6},77')}+\mathbf{(24,64)}+\mathbf{(15,77)}+\mathbf{(3,189)}$

E_6	→	Sp(8) (S)
$\mathbf{27}$	=	$\mathbf{(27)}$
$\mathbf{78}$	=	$\mathbf{(36)}+\mathbf{(42)}$
$\mathbf{351}$	=	$\mathbf{(36)}+\mathbf{(315)}$
$\mathbf{351'}$	=	$\mathbf{(1)}+\mathbf{(42)}+\mathbf{(308)}$
$\mathbf{650}$	=	$\mathbf{(27)}+\mathbf{(308)}+\mathbf{(315)}$
$\mathbf{1728}$	=	$\mathbf{(27)}+\mathbf{(315)}+\mathbf{(594)}+\mathbf{(792')}$
$\mathbf{2430}$	=	$\mathbf{(1)}+\mathbf{(42)}+\mathbf{(308)}+\mathbf{(330)}+\mathbf{(594')}+\mathbf{(1155)}$
$\mathbf{2925}$	=	$\mathbf{(36)}+\mathbf{(315)}+\mathbf{(594)}+\mathbf{(825)}+\mathbf{(1155)}$
$\mathbf{3003}$	=	$\mathbf{(27)}+\mathbf{(792')}+\mathbf{(2184)}$
$\mathbf{5824}$	=	$\mathbf{(27)}+\mathbf{(315)}+\mathbf{(594)}+\mathbf{(792')}+\mathbf{(4096)}$
$\mathbf{7371}$	=	$\mathbf{(36)}+\mathbf{(42)}+\mathbf{(308)}+\mathbf{(315)}+\mathbf{(594)}+\mathbf{(825)}+\mathbf{(1155)}+\mathbf{(4096)}$
$\mathbf{7722}$	=	$\mathbf{(27)}+\mathbf{(308)}+\mathbf{(315)}+\mathbf{(792')}+\mathbf{(2184)}+\mathbf{(4096)}$
$\mathbf{17550}$	=	$\mathbf{(27)}+\mathbf{(308)}+\mathbf{(315)}+\mathbf{(330)}+\mathbf{(594)}+\mathbf{(792')}+\mathbf{(1155)}+\mathbf{(3696)}+\mathbf{(4096)}+\mathbf{(6237)}$

E_6	→	G_2 (S)
$\mathbf{27}$	=	$\mathbf{(27)}$
$\mathbf{78}$	=	$\mathbf{(14)}+\mathbf{(64)}$
$\mathbf{351}$	=	$\mathbf{(7)}+\mathbf{(14)}+\mathbf{(64)}+\mathbf{(77)}+\mathbf{(189)}$
$\mathbf{351'}$	=	$\mathbf{(1)}+\mathbf{(27)}+\mathbf{(64)}+\mathbf{(77')}+\mathbf{(182)}$
$\mathbf{650}$	=	$\mathbf{(7)}+2\mathbf{(27)}+\mathbf{(64)}+\mathbf{(77)}+\mathbf{(77')}+\mathbf{(182)}+\mathbf{(189)}$
$\mathbf{1728}$	=	$\mathbf{(7)}+\mathbf{(14)}+2\mathbf{(27)}+2\mathbf{(64)}+2\mathbf{(77)}+\mathbf{(77')}+\mathbf{(182)}+2\mathbf{(189)}+\mathbf{(286)}+\mathbf{(448)}$
$\mathbf{2430}$	=	$\mathbf{(1)}+2\mathbf{(27)}+2\mathbf{(64)}+\mathbf{(77)}+2\mathbf{(77')}+2\mathbf{(182)}+\mathbf{(189)}+\mathbf{(286)}+\mathbf{(448)}+\mathbf{(729)}$
$\mathbf{2925}$	=	$2\mathbf{(7)}+2\mathbf{(14)}+\mathbf{(27)}+2\mathbf{(64)}+4\mathbf{(77)}+\mathbf{(182)}+3\mathbf{(189)}+\mathbf{(273)}+2\mathbf{(286)}+\mathbf{(378)}+\mathbf{(448)}$
$\mathbf{3003}$	=	$\mathbf{(1)}+2\mathbf{(27)}+\mathbf{(64)}+\mathbf{(77)}+\mathbf{(77')}+2\mathbf{(182)}+\mathbf{(189)}+\mathbf{(286)}+\mathbf{(448)}+\mathbf{(714)}+\mathbf{(729)}$
$\mathbf{5824}$	=	$\mathbf{(7)}+2\mathbf{(14)}+3\mathbf{(27)}+4\mathbf{(64)}+3\mathbf{(77)}+2\mathbf{(77')}+2\mathbf{(182)}+4\mathbf{(189)}+2\mathbf{(286)}+\mathbf{(378)}+3\mathbf{(448)}+\mathbf{(729)}+\mathbf{(924)}$
$\mathbf{7371}$	=	$2\mathbf{(7)}+3\mathbf{(14)}+3\mathbf{(27)}+5\mathbf{(64)}+5\mathbf{(77)}+2\mathbf{(77')}+3\mathbf{(182)}+5\mathbf{(189)}+\mathbf{(273)}+3\mathbf{(286)}+2\mathbf{(378)}+3\mathbf{(448)}+\mathbf{(729)}+\mathbf{(924)}$
$\mathbf{7722}$	=	$\mathbf{(1)}+\mathbf{(7)}+\mathbf{(14)}+4\mathbf{(27)}+4\mathbf{(64)}+3\mathbf{(77)}+3\mathbf{(77')}+4\mathbf{(182)}+4\mathbf{(189)}+2\mathbf{(286)}+\mathbf{(378)}+3\mathbf{(448)}+\mathbf{(714)}+2\mathbf{(729)}+\mathbf{(924)}$
$\mathbf{17550}$	=	$\mathbf{(1)}+2\mathbf{(7)}+2\mathbf{(14)}+6\mathbf{(27)}+7\mathbf{(64)}+6\mathbf{(77)}+5\mathbf{(77')}+6\mathbf{(182)}+8\mathbf{(189)}+\mathbf{(273)}+5\mathbf{(286)}+3\mathbf{(378)}+6\mathbf{(448)}+\mathbf{(714)}+4\mathbf{(729)}+\mathbf{(896)}+2\mathbf{(924)}+\mathbf{(1547)}$

E_6	→	SU(3) (S)
$\mathbf{27}$	=	$\mathbf{(27)}$
$\mathbf{78}$	=	$\mathbf{(8)}+\mathbf{(35)}+\mathbf{(\overline{35})}$

Table 10.26: Sp(10) Branching Rules (continued)

351	=	$(8)+(10)+(\overline{10})+(27)+(35)+(\overline{35})+(64)+(81)+(\overline{81})$
351′	=	$(1)+(8)+(27)+(28)+(\overline{28})+(35)+(\overline{35})+(64)+(125)$
650	=	$(8)+(10)+(\overline{10})+3(27)+(28)+(\overline{28})+(35)+(\overline{35})+2(64)+(81)+(\overline{81})+(125)$
1728	=	$2(8)+2(10)+2(\overline{10})+4(27)+(28)+(\overline{28})+3(35)+3(\overline{35})+4(64)+(80)+(\overline{80})+2(81)+2(\overline{81})+2(125)+(154)+(\overline{154})$
2430	=	$(1)+2(8)+(10)+(\overline{10})+4(27)+2(28)+2(\overline{28})+3(35)+3(\overline{35})+4(64)+(80)+(\overline{80})+2(81)+2(\overline{81})+3(125)+(154)+(\overline{154})+(162)+(\overline{162})+(216)$
2925	=	$(1)+2(8)+4(10)+4(\overline{10})+4(27)+(28)+(\overline{28})+4(35)+4(\overline{35})+(55)+(\overline{55})+6(64)+(80)+(\overline{80})+4(81)+4(\overline{81})+2(125)+2(154)+2(\overline{154})+(216)$
3003	=	$(1)+(8)+(10)+(\overline{10})+4(27)+2(28)+2(\overline{28})+2(35)+2(\overline{35})+4(64)+(80)+(\overline{80})+2(81)+2(\overline{81})+3(125)+2(154)+2(\overline{154})+(162)+(\overline{162})+(216)+(343)$
5824	=	$4(8)+3(10)+3(\overline{10})+8(27)+2(28)+2(\overline{28})+7(35)+7(\overline{35})+8(64)+3(80)+3(\overline{80})+6(81)+6(\overline{81})+6(125)+3(154)+3(\overline{154})+(162)+(\overline{162})+2(216)+(260)+(\overline{260})$
7371	=	$(1)+5(8)+5(10)+5(\overline{10})+9(27)+3(28)+3(\overline{28})+9(35)+9(\overline{35})+(55)+(\overline{55})+11(64)+3(80)+3(\overline{80})+8(81)+8(\overline{81})+7(125)+4(154)+4(\overline{154})+(162)+(\overline{162})+3(216)+(260)+(\overline{260})$
7722	=	$(1)+4(8)+3(10)+3(\overline{10})+10(27)+4(28)+4(\overline{28})+7(35)+7(\overline{35})+10(64)+3(80)+3(\overline{80})+7(81)+7(\overline{81})+8(125)+4(154)+4(\overline{154})+2(162)+2(\overline{162})+3(216)+(260)+(\overline{260})+(343)$
17550	=	$(1)+8(8)+7(10)+7(\overline{10})+18(27)+7(28)+7(\overline{28})+14(35)+14(\overline{35})+(55)+(\overline{55})+20(64)+7(80)+7(\overline{80})+15(81)+15(\overline{81})+15(125)+(143)+(\overline{143})+9(154)+9(\overline{154})+4(162)+4(\overline{162})+7(216)+3(260)+3(\overline{260})+(280)+(\overline{280})+2(343)$

Table 10.27: E_7 Branching Rules

E_7	→	$F_4\times SU(2)$ (S)
56	=	$(1,4)+(26,2)$
133	=	$(1,3)+(26,3)+(52,1)$
912	=	$(1,2)+(26,2)+(26,4)+(52,4)+(273,2)$
1463	=	$(1,3)+(1,7)+(26,3)+(26,5)+(273,1)+(324,3)$
1539	=	$(1,1)+(1,5)+(26,1)+(26,3)+(26,5)+(52,3)+(273,3)+(324,1)$
6480	=	$(1,2)+(1,4)+(1,6)+2(26,2)+2(26,4)+(26,6)+(52,2)+(52,4)+(273,2)+(273,4)+(324,2)+(324,4)+(1053,2)$
7371	=	$(1,1)+(1,5)+(26,1)+(26,3)+(26,5)+(52,3)+(273,3)+(324,1)+(324,5)+(1053',1)+(1053,3)$
8645	=	$(1,3)+(26,1)+2(26,3)+(26,5)+(52,1)+(52,3)+(52,5)+(273,1)+(273,3)+(273,5)+(324,3)+(1053,3)+(1274,1)$
24320	=	$(1,4)+(1,6)+(1,10)+(26,2)+(26,4)+(26,6)+(26,8)+(273,4)+(324,2)+(324,4)+(324,6)+(2652,4)+(4096,2)$
27664	=	$(1,4)+2(26,2)+2(26,4)+(26,6)+2(52,2)+(52,4)+(52,6)+2(273,2)+2(273,4)+(273,6)+(324,2)+(324,4)+(1053,2)+(1053,4)+(1274,4)+(4096,2)$

E_7	→	$G_2\times Sp(6)$ (S)
56	=	$(7,6)+(1,14')$
133	=	$(14,1)+(7,14)+(1,21)$
912	=	$(7,6)+(14,14')+(27,6)+(1,64)+(7,64)$
1463	=	$(7,1)+(7,14)+(1,21)+(14,14)+(27,21)+(7,70)+(1,84)$
1539	=	$(1,1)+(1,14)+(7,14)+(7,21)+(27,1)+(14,21)+(27,14)+(7,70)+(1,90)$
6480	=	$(1,6)+2(7,6)+(1,14')+(14,6)+(7,14')+(14,14')+(27,6)+(27,14')+(7,56)+(1,64)+(64,6)+2(7,64)+(14,64)+(27,64)+(7,126)+(1,216)$
7371	=	$(1,1)+(1,14)+(7,14)+(7,21)+(27,1)+(14,21)+(27,14)+(7,70)+(64,14)+(77',1)+(14,70)+(1,90)+(27,90)+(1,126')+(7,189)$

8645	=	(7,1)+(14,1)+2(7,14)+(1,21)+(7,21)+(14,14)+(14,21)+(27,14)+(27,21)+(1,70)+ (7,70)+(64,14)+(77,1)+(27,70)+(7,90)+(14,90)+(1,189)+(7,189)
24320	=	(7,6)+(1,14′)+(14,6)+(7,14′)+(27,6)+(27,14′)+(7,56)+(7,64)+(14,64)+(27,64)+ (64,64)+(7,126)+(77,56)+(14,126)+(1,216)+(27,216)+(1,330)+(7,378)
27664	=	(1,6)+2(7,6)+(1,14′)+(14,6)+2(7,14′)+(14,14′)+2(27,6)+(27,14′)+(1,56)+(7,56)+ (1,64)+(64,6)+3(7,64)+2(14,64)+(27,56)+(77,6)+2(27,64)+(77,14′)+(1,126)+ (64,64)+(7,126)+(27,126)+(7,216)+(14,216)+(1,350)+(7,350)

E_7	$\rightarrow$	SU(2)×G_2 (S)
56	=	(4,7)+(2,14)
133	=	(3,1)+(5,7)+(1,14)+(3,27)
912	=	(2,1)+(2,7)+(8,1)+(4,7)+(6,7)+(2,14)+(4,14)+(2,27)+(4,27)+(6,27)+(4,64)+(2,77)
1463	=	(3,1)+(1,7)+(7,1)+(3,7)+(5,7)+(1,14)+(5,14)+2(3,27)+(5,27)+(7,27)+(3,64)+ (5,64)+(1,77)+(3,77′)
1539	=	(1,1)+(5,1)+2(3,7)+(5,7)+(7,7)+2(3,14)+(7,14)+2(1,27)+(3,27)+2(5,27)+ (3,64)+(5,64)+(1,77′)+(3,77)
6480	=	(2,1)+(4,1)+(6,1)+3(2,7)+4(4,7)+3(6,7)+(8,7)+3(2,14)+3(4,14)+2(6,14)+ (8,14)+3(2,27)+4(4,27)+2(6,27)+(8,27)+2(2,64)+3(4,64)+2(6,64)+2(2,77)+ (2,77′)+2(4,77)+(6,77)+(2,189)+(4,189)
7371	=	2(1,1)+2(5,1)+3(3,7)+(9,1)+2(5,7)+2(7,7)+3(3,14)+(5,14)+(7,14)+3(1,27)+ 2(3,27)+4(5,27)+(7,27)+(9,27)+(1,64)+2(3,64)+2(5,64)+(7,64)+(1,77′)+2(3,77)+ (5,77)+(5,77′)+(7,77)+(1,182)+(5,182)+2(3,189)
8645	=	2(3,1)+2(1,7)+(7,1)+3(3,7)+4(5,7)+2(7,7)+2(1,14)+(9,7)+3(3,14)+3(5,14)+ (7,14)+(9,14)+(1,27)+5(3,27)+3(5,27)+2(7,27)+(1,64)+3(3,64)+3(5,64)+(7,64)+ 2(1,77)+2(3,77)+(3,77′)+2(5,77)+(7,77)+(3,182)+(1,189)+(3,189)+(5,189)
24320	=	(4,1)+2(2,7)+4(4,7)+3(6,7)+(8,7)+3(2,14)+(10,7)+3(4,14)+3(6,14)+2(8,14)+ 3(2,27)+4(4,27)+3(6,27)+(8,27)+4(2,64)+4(4,64)+3(6,64)+2(8,64)+2(2,77)+ (2,77′)+4(4,77)+(4,77′)+3(6,77)+(6,77′)+(8,77)+(10,77)+(4,182)+2(2,189)+ 3(4,189)+2(6,189)+(8,189)+(4,273)+(2,286)+(4,286)+(6,286)+(2,448)
27664	=	(2,1)+3(4,1)+2(6,1)+4(2,7)+(8,1)+7(4,7)+(10,1)+5(6,7)+2(8,7)+5(2,14)+ (10,7)+5(4,14)+4(6,14)+2(8,14)+7(2,27)+9(4,27)+7(6,27)+3(8,27)+(10,27)+ 6(2,64)+7(4,64)+5(6,64)+2(8,64)+4(2,77)+2(2,77′)+6(4,77)+2(4,77′)+3(6,77)+ (6,77′)+(8,77)+(8,77′)+(2,182)+2(4,182)+(6,182)+3(2,189)+3(4,189)+2(6,189)+ (4,286)+(2,448)

E_7	$\rightarrow$	SU(3) (S)
56	=	(28)+($\overline{28}$)
133	=	(8)+(125)
912	=	(35)+($\overline{35}$)+(80)+($\overline{80}$)+(81)+($\overline{81}$)+(260)+($\overline{260}$)
1463	=	(1)+(27)+(28)+($\overline{28}$)+(64)+(91)+($\overline{91}$)+2(125)+(162)+($\overline{162}$)+(216)+(343)
1539	=	(8)+(27)+(64)+(81)+($\overline{81}$)+(125)+(143)+($\overline{143}$)+(154)+($\overline{154}$)+(216)+(343)
6480	=	2(27)+2(28)+2($\overline{28}$)+2(35)+2($\overline{35}$)+2(64)+2(80)+2($\overline{80}$)+2(81)+2($\overline{81}$)+2(125)+ 2(154)+2($\overline{154}$)+2(162)+2($\overline{162}$)+2(216)+(260)+($\overline{260}$)+(280)+($\overline{280}$)+2(343)+(405)+ ($\overline{405}$)+(440)+($\overline{440}$)
7371	=	(1)+(8)+2(27)+(28)+($\overline{28}$)+(35)+($\overline{35}$)+2(64)+(80)+($\overline{80}$)+(81)+($\overline{81}$)+(91)+ ($\overline{91}$)+4(125)+2(154)+2($\overline{154}$)+2(162)+2($\overline{162}$)+2(216)+(260)+($\overline{260}$)+(280)+($\overline{280}$)+ (343)+(405)+($\overline{405}$)+(440)+($\overline{440}$)+(512)+(729)
8645	=	(8)+2(10)+2($\overline{10}$)+(27)+(35)+($\overline{35}$)+(55)+($\overline{55}$)+3(64)+(80)+($\overline{80}$)+3(81)+3($\overline{81}$)+ 3(125)+(143)+($\overline{143}$)+3(154)+3($\overline{154}$)+(162)+($\overline{162}$)+3(216)+2(260)+2($\overline{260}$)+(270)+ ($\overline{270}$)+(280)+($\overline{280}$)+(343)+(405)+($\overline{405}$)+(512)+(595)+($\overline{595}$)

24320	=	$2(\mathbf{1})+4(\mathbf{27})+4(\mathbf{28})+4(\overline{\mathbf{28}})+(\mathbf{35})+(\overline{\mathbf{35}})+4(\mathbf{64})+2(\mathbf{80})+2(\overline{\mathbf{80}})+2(\mathbf{81})+2(\overline{\mathbf{81}})+2(\mathbf{91})+2(\overline{\mathbf{91}})+6(\mathbf{125})+(\mathbf{143})+(\overline{\mathbf{143}})+4(\mathbf{154})+4(\overline{\mathbf{154}})+5(\mathbf{162})+5(\overline{\mathbf{162}})+(\mathbf{190})+(\overline{\mathbf{190}})+4(\mathbf{216})+2(\mathbf{260})+2(\overline{\mathbf{260}})+3(\mathbf{280})+3(\overline{\mathbf{280}})+6(\mathbf{343})+2(\mathbf{405})+2(\overline{\mathbf{405}})+(\mathbf{405'})+(\overline{\mathbf{405}}')+3(\mathbf{440})+3(\overline{\mathbf{440}})+(\mathbf{442})+(\overline{\mathbf{442}})+2(\mathbf{512})+(\mathbf{595})+(\overline{\mathbf{595}})+(\mathbf{648})+(\overline{\mathbf{648}})+2(\mathbf{729})+(\mathbf{910})+(\overline{\mathbf{910}})$
27664	=	$3(\mathbf{10})+3(\overline{\mathbf{10}})+2(\mathbf{27})+2(\mathbf{28})+2(\overline{\mathbf{28}})+4(\mathbf{35})+4(\overline{\mathbf{35}})+3(\mathbf{55})+3(\overline{\mathbf{55}})+6(\mathbf{64})+4(\mathbf{80})+4(\overline{\mathbf{80}})+7(\mathbf{81})+7(\overline{\mathbf{81}})+4(\mathbf{125})+(\mathbf{136})+(\overline{\mathbf{136}})+(\mathbf{143})+(\overline{\mathbf{143}})+7(\mathbf{154})+7(\overline{\mathbf{154}})+3(\mathbf{162})+3(\overline{\mathbf{162}})+6(\mathbf{216})+6(\mathbf{260})+6(\overline{\mathbf{260}})+2(\mathbf{270})+2(\overline{\mathbf{270}})+4(\mathbf{280})+4(\overline{\mathbf{280}})+4(\mathbf{343})+4(\mathbf{405})+4(\overline{\mathbf{405}})+(\mathbf{440})+(\overline{\mathbf{440}})+(\mathbf{442})+(\overline{\mathbf{442}})+2(\mathbf{512})+2(\mathbf{595})+2(\overline{\mathbf{595}})+(\mathbf{648})+(\overline{\mathbf{648}})+(\mathbf{836})+(\overline{\mathbf{836}})$

E_7	→	SU(2)×SU(2) (S)
56	=	(**5**,**2**) + (**3**,**6**) + (**7**,**4**)
133	=	(**3**,**1**) + (**1**,**3**) + (**5**,**3**) + (**3**,**5**) + (**7**,**5**) + (**5**,**7**) + (**9**,**3**)
912	=	(**1**,**2**) + (**3**,**2**) + 2(**3**,**4**) + 2(**5**,**2**) + (**1**,**6**) + 3(**5**,**4**) + 2(**3**,**6**) + 2(**7**,**2**) + (**1**,**8**) + 2(**5**,**6**) + 2(**7**,**4**) + (**3**,**8**) + (**9**,**2**) + 2(**7**,**6**) + (**5**,**8**) + 2(**9**,**4**) + (**7**,**8**) + (**9**,**6**) + (**5**,**10**) + (**11**,**4**) + (**13**,**2**) + (**9**,**8**) + (**11**,**6**)
1463	=	2(**3**,**1**) + 2(**1**,**3**) + (**3**,**3**) + 4(**5**,**3**) + 3(**3**,**5**) + 2(**7**,**1**) + 2(**1**,**7**) + 3(**5**,**5**) + 2(**7**,**3**) + (**3**,**7**) + 3(**7**,**5**) + 3(**5**,**7**) + 3(**9**,**3**) + (**3**,**9**) + (**11**,**1**) + (**1**,**11**) + 2(**7**,**7**) + 2(**9**,**5**) + (**5**,**9**) + (**11**,**3**) + 2(**9**,**7**) + (**7**,**9**) + 2(**11**,**5**) + (**5**,**11**) + (**13**,**3**) + (**9**,**9**) + (**13**,**7**)
1539	=	2(**1**,**1**) + 4(**3**,**3**) + 3(**5**,**1**) + 2(**1**,**5**) + 2(**5**,**3**) + 2(**3**,**5**) + 5(**5**,**5**) + 4(**7**,**3**) + 3(**3**,**7**) + 2(**9**,**1**) + (**1**,**9**) + 3(**7**,**5**) + 2(**5**,**7**) + 2(**9**,**3**) + 3(**7**,**7**) + 3(**9**,**5**) + 2(**5**,**9**) + 2(**11**,**3**) + (**3**,**11**) + (**13**,**1**) + (**9**,**7**) + (**7**,**9**) + (**11**,**5**) + (**9**,**9**) + (**11**,**7**) + (**13**,**5**)
6480	=	2(**1**,**2**) + 6(**3**,**2**) + 3(**1**,**4**) + 9(**3**,**4**) + 8(**5**,**2**) + 3(**1**,**6**) + 12(**5**,**4**) + 9(**3**,**6**) + 8(**7**,**2**) + 2(**1**,**8**) + 11(**5**,**6**) + 12(**7**,**4**) + 6(**3**,**8**) + 6(**9**,**2**) + (**1**,**10**) + 11(**7**,**6**) + 7(**5**,**8**) + 9(**9**,**4**) + 3(**3**,**10**) + 4(**11**,**2**) + 7(**7**,**8**) + 8(**9**,**6**) + 3(**5**,**10**) + 6(**11**,**4**) + (**3**,**12**) + 2(**13**,**2**) + 5(**9**,**8**) + 3(**7**,**10**) + 5(**11**,**6**) + (**5**,**12**) + 3(**13**,**4**) + (**15**,**2**) + 2(**9**,**10**) + 3(**11**,**8**) + (**7**,**12**) + 2(**13**,**6**) + (**15**,**4**) + (**11**,**10**) + (**13**,**8**) + (**15**,**6**)
7371	=	4(**1**,**1**) + 9(**3**,**3**) + 6(**5**,**1**) + 6(**1**,**5**) + 7(**5**,**3**) + 6(**3**,**5**) + 2(**7**,**1**) + (**1**,**7**) + 13(**5**,**5**) + 11(**7**,**3**) + 8(**3**,**7**) + 5(**9**,**1**) + 3(**1**,**9**) + 9(**7**,**5**) + 8(**5**,**7**) + 6(**9**,**3**) + 3(**3**,**9**) + (**11**,**1**) + 10(**7**,**7**) + 11(**9**,**5**) + 7(**5**,**9**) + 6(**11**,**3**) + 2(**3**,**11**) + 2(**13**,**1**) + (**1**,**13**) + 6(**9**,**7**) + 4(**7**,**9**) + 5(**11**,**5**) + 2(**5**,**11**) + 2(**13**,**3**) + 5(**9**,**9**) + 5(**11**,**7**) + 3(**7**,**11**) + 4(**13**,**5**) + (**5**,**13**) + 2(**15**,**3**) + (**17**,**1**) + 2(**11**,**9**) + (**9**,**11**) + 2(**13**,**7**) + (**15**,**5**) + (**11**,**11**) + 2(**13**,**9**) + (**9**,**13**) + (**15**,**7**) + (**17**,**5**)
8645	=	5(**3**,**1**) + 5(**1**,**3**) + 8(**3**,**3**) + 3(**5**,**1**) + 2(**1**,**5**) + 13(**5**,**3**) + 12(**3**,**5**) + 6(**7**,**1**) + 4(**1**,**7**) + 13(**5**,**5**) + 11(**7**,**3**) + 8(**3**,**7**) + 3(**9**,**1**) + (**1**,**9**) + 15(**7**,**5**) + 12(**5**,**7**) + 11(**9**,**3**) + 6(**3**,**9**) + 3(**11**,**1**) + (**1**,**11**) + 11(**7**,**7**) + 10(**9**,**5**) + 6(**5**,**9**) + 6(**11**,**3**) + 2(**3**,**11**) + (**13**,**1**) + 9(**9**,**7**) + 7(**7**,**9**) + 8(**11**,**5**) + 3(**5**,**11**) + 4(**13**,**3**) + (**3**,**13**) + (**15**,**1**) + 4(**9**,**9**) + 5(**11**,**7**) + 2(**7**,**11**) + 3(**13**,**5**) + (**15**,**3**) + 3(**11**,**9**) + 2(**9**,**11**) + 3(**13**,**7**) + (**7**,**13**) + 2(**15**,**5**) + (**17**,**3**) + (**11**,**11**) + (**13**,**9**) + (**15**,**7**)
24320	=	2(**1**,**2**) + 10(**3**,**2**) + 6(**1**,**4**) + 16(**3**,**4**) + 14(**5**,**2**) + 5(**1**,**6**) + 20(**5**,**4**) + 17(**3**,**6**) + 14(**7**,**2**) + 3(**1**,**8**) + 22(**5**,**6**) + 25(**7**,**4**) + 14(**3**,**8**) + 13(**9**,**2**) + 3(**1**,**10**) + 25(**7**,**6**) + 18(**5**,**8**) + 21(**9**,**4**) + 9(**3**,**10**) + 11(**11**,**2**) + (**1**,**12**) + 19(**7**,**8**) + 21(**9**,**6**) + 10(**5**,**10**) + 16(**11**,**4**) + 5(**3**,**12**) + 6(**13**,**2**) + 16(**9**,**8**) + 13(**7**,**10**) + 17(**11**,**6**) + 5(**5**,**12**) + 11(**13**,**4**) + 2(**3**,**14**) + 4(**15**,**2**) + 9(**9**,**10**) + 13(**11**,**8**) + 6(**7**,**12**) + 10(**13**,**6**) + 2(**5**,**14**) + 6(**15**,**4**) + (**3**,**16**) + 2(**17**,**2**) + 7(**11**,**10**) + 4(**9**,**12**) + 7(**13**,**8**) + 2(**7**,**14**) + 6(**15**,**6**) + 2(**17**,**4**) + 3(**11**,**12**) + 4(**13**,**10**) + (**9**,**14**) + 4(**15**,**8**) + (**7**,**16**) + 2(**17**,**6**) + (**19**,**4**) + (**13**,**12**) + (**11**,**14**) + 2(**15**,**10**) + 2(**17**,**8**) + (**19**,**6**) + (**15**,**12**) + (**19**,**10**)
27664	=	5(**1**,**2**) + 15(**3**,**2**) + 10(**1**,**4**) + 24(**3**,**4**) + 22(**5**,**2**) + 10(**1**,**6**) + 33(**5**,**4**) + 24(**3**,**6**) + 21(**7**,**2**) + 7(**1**,**8**) + 34(**5**,**6**) + 35(**7**,**4**) + 18(**3**,**8**) + 19(**9**,**2**) + 5(**1**,**10**) + 34(**7**,**6**) + 26(**5**,**8**) + 30(**9**,**4**) + 10(**3**,**10**) + 14(**11**,**2**) + 2(**1**,**12**) + 24(**7**,**8**) + 29(**9**,**6**) + 14(**5**,**10**) + 20(**11**,**4**) + 4(**3**,**12**) + 8(**13**,**2**) + 21(**9**,**8**) + 14(**7**,**10**) + 20(**11**,**6**) + 6(**5**,**12**) + 13(**13**,**4**) + (**3**,**14**) + 4(**15**,**2**) + (**1**,**16**) + 11(**9**,**10**) + 14(**11**,**8**) + 5(**7**,**12**) + 12(**13**,**6**) + 2(**5**,**14**) + 6(**15**,**4**) + 2(**17**,**2**) + 6(**11**,**10**) + 4(**9**,**12**) + 7(**13**,**8**) + (**7**,**14**) + 5(**15**,**6**) + 2(**17**,**4**) + 2(**11**,**12**) + 4(**13**,**10**) + (**9**,**14**) + 3(**15**,**8**) + 2(**17**,**6**) + (**19**,**4**) + (**13**,**12**) + (**15**,**10**) + (**17**,**8**)

E_7	→	SU(2) (S)
56	=	(**10**) + (**18**) + (**28**)

Table 10.27: E_7 Branching Rules (continued)

133	=	(**3**) + (**11**) + (**15**) + (**19**) + (**23**) + (**27**) + (**35**)
912	=	(**2**) + (**6**) + 2(**8**) + 2(**10**) + 2(**12**) + 2(**14**) + 2(**16**) + 3(**18**) + 3(**20**) + 2(**22**) + 2(**24**) + 3(**26**) + 2(**28**) + 2(**30**) + 2(**32**) + (**34**) + 2(**36**) + (**38**) + (**40**) + (**42**) + (**44**) + (**50**)
1463	=	2(**3**) + 3(**7**) + (**9**) + 4(**11**) + 2(**13**) + 4(**15**) + 2(**17**) + 5(**19**) + 3(**21**) + 4(**23**) + 3(**25**) + 4(**27**) + 2(**29**) + 4(**31**) + 2(**33**) + 3(**35**) + 2(**37**) + 2(**39**) + (**41**) + 2(**43**) + (**45**) + (**47**) + (**51**) + (**55**)
1539	=	2(**1**) + 3(**5**) + 4(**9**) + 2(**11**) + 5(**13**) + 2(**15**) + 5(**17**) + 3(**19**) + 5(**21**) + 3(**23**) + 5(**25**) + 3(**27**) + 4(**29**) + 2(**31**) + 4(**33**) + 2(**35**) + 3(**37**) + (**39**) + 2(**41**) + (**43**) + 2(**45**) + (**49**) + (**53**)
6480	=	2(**2**) + 4(**4**) + 6(**6**) + 8(**8**) + 10(**10**) + 11(**12**) + 12(**14**) + 13(**16**) + 14(**18**) + 15(**20**) + 14(**22**) + 14(**24**) + 14(**26**) + 14(**28**) + 13(**30**) + 12(**32**) + 11(**34**) + 10(**36**) + 9(**38**) + 8(**40**) + 7(**42**) + 6(**44**) + 5(**46**) + 4(**48**) + 3(**50**) + 3(**52**) + 2(**54**) + (**56**) + (**58**) + (**60**) + (**62**)
7371	=	4(**1**) + 8(**5**) + 4(**7**) + 11(**9**) + 7(**11**) + 14(**13**) + 10(**15**) + 16(**17**) + 11(**19**) + 17(**21**) + 12(**23**) + 17(**25**) + 12(**27**) + 16(**29**) + 11(**31**) + 14(**33**) + 10(**35**) + 13(**37**) + 8(**39**) + 10(**41**) + 6(**43**) + 8(**45**) + 5(**47**) + 6(**49**) + 3(**51**) + 4(**53**) + 2(**55**) + 3(**57**) + (**59**) + 2(**61**) + (**65**) + (**69**)
8645	=	6(**3**) + 4(**5**) + 10(**7**) + 9(**9**) + 14(**11**) + 13(**13**) + 17(**15**) + 15(**17**) + 19(**19**) + 16(**21**) + 19(**23**) + 17(**25**) + 19(**27**) + 15(**29**) + 17(**31**) + 14(**33**) + 15(**35**) + 12(**37**) + 12(**39**) + 9(**41**) + 10(**43**) + 7(**45**) + 7(**47**) + 5(**49**) + 5(**51**) + 3(**53**) + 3(**55**) + 2(**57**) + 2(**59**) + (**61**) + (**63**) + (**67**)
24320	=	4(**2**) + 11(**4**) + 14(**6**) + 18(**8**) + 24(**10**) + 27(**12**) + 29(**14**) + 34(**16**) + 35(**18**) + 36(**20**) + 39(**22**) + 39(**24**) + 38(**26**) + 40(**28**) + 38(**30**) + 36(**32**) + 36(**34**) + 34(**36**) + 31(**38**) + 30(**40**) + 27(**42**) + 24(**44**) + 23(**46**) + 20(**48**) + 17(**50**) + 16(**52**) + 13(**54**) + 11(**56**) + 10(**58**) + 8(**60**) + 6(**62**) + 6(**64**) + 4(**66**) + 3(**68**) + 3(**70**) + 2(**72**) + (**74**) + (**76**) + (**78**) + (**82**)
27664	=	6(**2**) + 14(**4**) + 20(**6**) + 25(**8**) + 32(**10**) + 36(**12**) + 39(**14**) + 45(**16**) + 47(**18**) + 48(**20**) + 51(**22**) + 50(**24**) + 50(**26**) + 50(**28**) + 48(**30**) + 45(**32**) + 44(**34**) + 41(**36**) + 37(**38**) + 35(**40**) + 31(**42**) + 27(**44**) + 25(**46**) + 21(**48**) + 18(**50**) + 16(**52**) + 13(**54**) + 10(**56**) + 9(**58**) + 7(**60**) + 5(**62**) + 4(**64**) + 3(**66**) + 2(**68**) + 2(**70**) + (**72**) + (**76**)

E_7	→	SU(2) (S)
56	=	(**6**) + (**12**) + (**16**) + (**22**)
133	=	(**3**) + (**7**) + 2(**11**) + (**15**) + (**17**) + (**19**) + (**23**) + (**27**)
912	=	(**2**) + (**4**) + 3(**6**) + 4(**8**) + 3(**10**) + 4(**12**) + 5(**14**) + 4(**16**) + 4(**18**) + 4(**20**) + 4(**22**) + 3(**24**) + 3(**26**) + 2(**28**) + 2(**30**) + 2(**32**) + (**36**) + (**38**)
1463	=	3(**3**) + (**5**) + 6(**7**) + 3(**9**) + 7(**11**) + 5(**13**) + 7(**15**) + 5(**17**) + 7(**19**) + 5(**21**) + 6(**23**) + 4(**25**) + 5(**27**) + 2(**29**) + 4(**31**) + 2(**33**) + 2(**35**) + (**37**) + (**39**) + (**43**)
1539	=	3(**1**) + 5(**5**) + 3(**7**) + 7(**9**) + 5(**11**) + 8(**13**) + 5(**15**) + 9(**17**) + 5(**19**) + 8(**21**) + 4(**23**) + 6(**25**) + 4(**27**) + 4(**29**) + 2(**31**) + 3(**33**) + (**35**) + 2(**37**) + (**41**)
6480	=	5(**2**) + 9(**4**) + 14(**6**) + 17(**8**) + 20(**10**) + 23(**12**) + 24(**14**) + 25(**16**) + 25(**18**) + 24(**20**) + 23(**22**) + 21(**24**) + 19(**26**) + 17(**28**) + 14(**30**) + 12(**32**) + 10(**34**) + 7(**36**) + 6(**38**) + 4(**40**) + 3(**42**) + 2(**44**) + (**46**) + (**48**)
7371	=	6(**1**) + 3(**3**) + 15(**5**) + 11(**7**) + 22(**9**) + 17(**11**) + 27(**13**) + 21(**15**) + 28(**17**) + 22(**19**) + 27(**21**) + 21(**23**) + 24(**25**) + 17(**27**) + 20(**29**) + 13(**31**) + 15(**33**) + 9(**35**) + 10(**37**) + 6(**39**) + 6(**41**) + 3(**43**) + 4(**45**) + (**47**) + 2(**49**) + (**53**)
8645	=	(**1**) + 11(**3**) + 11(**5**) + 21(**7**) + 21(**9**) + 28(**11**) + 27(**13**) + 32(**15**) + 29(**17**) + 33(**19**) + 28(**21**) + 29(**23**) + 25(**25**) + 25(**27**) + 19(**29**) + 19(**31**) + 14(**33**) + 13(**35**) + 9(**37**) + 8(**39**) + 5(**41**) + 5(**43**) + 2(**45**) + 2(**47**) + (**49**) + (**51**)
24320	=	10(**2**) + 23(**4**) + 32(**6**) + 40(**8**) + 51(**10**) + 56(**12**) + 60(**14**) + 66(**16**) + 67(**18**) + 66(**20**) + 68(**22**) + 64(**24**) + 60(**26**) + 58(**28**) + 52(**30**) + 46(**32**) + 42(**34**) + 36(**36**) + 30(**38**) + 26(**40**) + 21(**42**) + 16(**44**) + 14(**46**) + 10(**48**) + 7(**50**) + 6(**52**) + 4(**54**) + 2(**56**) + 2(**58**) + (**60**) + (**64**)
27664	=	14(**2**) + 31(**4**) + 44(**6**) + 55(**8**) + 68(**10**) + 75(**12**) + 80(**14**) + 87(**16**) + 87(**18**) + 86(**20**) + 85(**22**) + 80(**24**) + 74(**26**) + 69(**28**) + 61(**30**) + 53(**32**) + 46(**34**) + 39(**36**) + 31(**38**) + 26(**40**) + 20(**42**) + 14(**44**) + 12(**46**) + 8(**48**) + 5(**50**) + 4(**52**) + 2(**54**) + (**56**) + (**58**)

Table 10.28: E_8 Branching Rules

E_8	→	$G_2 \times F_4$ (S)
248	=	(14,1) + (7,26) + (1,52)
3875	=	(1,1) + (27,1) + (7,26) + (27,26) + (14,52) + (7,273) + (1,324)
27000	=	(1,1) + (1,26) + (27,1) + (7,26) + (27,26) + (7,52) + (14,52) + (77′,1) + (64,26) + (7,273) + (14,273) + (1,324) + (27,324) + (1,1053′) + (7,1053)
30380	=	(7,1) + (14,1) + 2(7,26) + (14,26) + (1,52) + (27,26) + (14,52) + (77,1) + (27,52) + (64,26) + (1,273) + (7,273) + (27,273) + (7,324) + (14,324) + (7,1053) + (1,1274)
147250	=	(14,1) + (1,26) + (27,1) + 2(7,26) + (14,26) + (1,52) + 2(27,26) + (7,52) + (64,1) + (14,52) + (27,52) + (64,26) + (77,26) + (77,52) + 2(7,273) + (14,273) + (27,273) + (1,324) + (7,324) + (64,273) + (14,324) + (27,324) + (7,1053) + (27,1053) + (14,1274) + (1,4096) + (7,4096)
779247	=	2(7,1) + 2(14,1) + (1,26) + (27,1) + 5(7,26) + 3(14,26) + 2(1,52) + 4(27,26) + 2(7,52) + (64,1) + 2(14,52) + (77,1) + 3(27,52) + 3(64,26) + 2(77,26) + (64,52) + (77,52) + (77′,52) + (189,1) + (189,26) + 2(1,273) + 4(7,273) + 2(14,273) + 4(27,273) + (1,324) + 3(7,324) + 2(64,273) + 3(14,324) + (77,273) + 2(27,324) + (64,324) + (77,324) + (1,1053) + 3(7,1053) + (14,1053) + (14,1053′) + 2(27,1053) + (64,1053) + (1,1274) + (7,1274) + (14,1274) + (27,1274) + (7,2652) + (1,4096) + 2(7,4096) + (14,4096) + (27,4096) + (7,8424) + (1,10829)

E_8	→	SU(2)×SU(3)
248	=	(3,1) + (1,8) + (5,10) + (5,$\overline{10}$) + (7,8) + (3,27)
3875	=	(1,1) + (5,1) + (7,1) + (1,8) + 2(3,8) + 2(3,10) + 2(3,$\overline{10}$) + 3(5,8) + (5,10) + (5,$\overline{10}$) + 2(7,8) + 2(7,10) + 2(7,$\overline{10}$) + (9,8) + (11,8) + 2(1,27) + (1,28) + (1,$\overline{28}$) + 2(3,27) + 3(5,27) + (7,27) + 2(9,27) + (3,35) + (3,$\overline{35}$) + (5,35) + (5,$\overline{35}$) + (7,35) + (7,$\overline{35}$) + (3,64) + (5,64)
27000	=	3(1,1) + (3,1) + 3(5,1) + (7,1) + 3(1,8) + 2(9,1) + 7(3,8) + 6(3,10) + 6(3,$\overline{10}$) + 8(5,8) + (13,1) + 5(5,10) + 5(5,$\overline{10}$) + 8(7,8) + 6(7,10) + 6(7,$\overline{10}$) + 5(9,8) + 2(9,10) + 2(9,$\overline{10}$) + 2(11,8) + 2(11,10) + 2(11,$\overline{10}$) + (13,8) + 6(1,27) + (1,28) + (1,$\overline{28}$) + 7(3,27) + 12(5,27) + 2(5,28) + 2(5,$\overline{28}$) + 7(7,27) + (1,35) + (1,$\overline{35}$) + 6(9,27) + (9,28) + (9,$\overline{28}$) + 5(3,35) + 5(3,$\overline{35}$) + 2(11,27) + 5(5,35) + 5(5,$\overline{35}$) + (13,27) + 4(7,35) + 4(7,$\overline{35}$) + 2(9,35) + 2(9,$\overline{35}$) + (11,35) + (11,$\overline{35}$) + 2(1,64) + 4(3,64) + 4(5,64) + 4(7,64) + 2(9,64) + 2(3,81) + 2(3,$\overline{81}$) + (5,81) + (5,$\overline{81}$) + (7,81) + (7,$\overline{81}$) + (1,125) + (5,125)
30380	=	(1,1) + 3(3,1) + (5,1) + 3(7,1) + 3(1,8) + (9,1) + 9(3,8) + 4(1,10) + 4(1,$\overline{10}$) + (11,1) + 5(3,10) + 5(3,$\overline{10}$) + 11(5,8) + 8(5,10) + 8(5,$\overline{10}$) + 9(7,8) + 6(7,10) + 6(7,$\overline{10}$) + 6(9,8) + 4(9,10) + 4(9,$\overline{10}$) + 3(11,8) + (11,10) + (11,$\overline{10}$) + (13,8) + (13,10) + (13,$\overline{10}$) + 3(1,27) + 12(3,27) + 2(3,28) + 2(3,$\overline{28}$) + 11(5,27) + 11(7,27) + (7,28) + (7,$\overline{28}$) + 2(1,35) + 2(1,$\overline{35}$) + 5(9,27) + 5(3,35) + 5(3,$\overline{35}$) + 3(11,27) + 7(5,35) + 7(5,$\overline{35}$) + 4(7,35) + 4(7,$\overline{35}$) + 3(9,35) + 3(9,$\overline{35}$) + (11,35) + (11,$\overline{35}$) + 2(1,64) + 5(3,64) + 5(5,64) + 4(7,64) + 2(9,64) + (1,81) + (1,$\overline{81}$) + (3,81) + (3,$\overline{81}$) + 2(5,81) + 2(5,$\overline{81}$) + (7,81) + (7,$\overline{81}$) + (3,125)
147250	=	(1,1) + 4(3,1) + 8(5,1) + 4(7,1) + 10(1,8) + 5(9,1) + 23(3,8) + 5(1,10) + 5(1,$\overline{10}$) + 3(11,1) + 17(3,10) + 17(3,$\overline{10}$) + 28(5,8) + 21(5,10) + 21(5,$\overline{10}$) + 27(7,8) + 18(7,10) + 18(7,$\overline{10}$) + 18(9,8) + (17,1) + 12(9,10) + 12(9,$\overline{10}$) + 10(11,8) + 7(11,10) + 7(11,$\overline{10}$) + 5(13,8) + 2(13,10) + 2(13,$\overline{10}$) + (15,8) + (15,10) + (15,$\overline{10}$) + 12(1,27) + 2(1,28) + 2(1,$\overline{28}$) + 30(3,27) + 4(3,28) + 4(3,$\overline{28}$) + 38(5,27) + 5(5,28) + 5(5,$\overline{28}$) + 33(7,27) + 3(7,28) + 3(7,$\overline{28}$) + 8(1,35) + 8(1,$\overline{35}$) + 23(9,27) + 2(9,28) + 2(9,$\overline{28}$) + 18(3,35) + 18(3,$\overline{35}$) + 11(11,27) + (11,28) + (11,$\overline{28}$) + 21(5,35) + 21(5,$\overline{35}$) + 4(13,27) + 19(7,35) + 19(7,$\overline{35}$) + (15,27) + 12(9,35) + 12(9,$\overline{35}$) + 5(11,35) + 5(11,$\overline{35}$) + 2(13,35) + 2(13,$\overline{35}$) + 6(1,64) + 18(3,64) + 22(5,64) + 16(7,64) + 10(9,64) + 5(11,64) + (13,64) + (1,80) + (1,$\overline{80}$) + 2(1,81) + 2(1,$\overline{81}$) + (3,80) + (3,$\overline{80}$) + 7(3,81) + 7(3,$\overline{81}$) + (5,80) + (5,$\overline{80}$) + 8(5,81) + 8(5,$\overline{81}$) + (7,80) + (7,$\overline{80}$) + 7(7,81) + 7(7,$\overline{81}$) + 3(9,81) + 3(9,$\overline{81}$) + (11,81) + (11,$\overline{81}$) + 3(1,125) + 4(3,125) + 4(5,125) + 3(7,125) + 2(9,125) + (3,154) + (3,$\overline{154}$) + (5,154) + (5,$\overline{154}$)

779247	=	$4(1,1)+19(3,1)+20(5,1)+22(7,1)+31(1,8)+14(9,1)+82(3,8)+26(1,10)+26(1,\overline{10})+10(11,1)+59(3,10)+59(3,\overline{10})+106(5,8)+4(13,1)+81(5,10)+81(5,\overline{10})+99(7,8)+2(15,1)+71(7,10)+71(7,\overline{10})+73(9,8)+54(9,10)+54(ß\ rrep9,\overline{10})+44(11,8)+29(11,10)+29(11,\overline{10})+20(13,8)+14(13,10)+14(13,\overline{10})+7(15,8)+4(15,10)+4(15,\overline{10})+2(17,8)+(17,10)+(17,\overline{10})+42(1,27)+6(1,28)+6(1,\overline{28})+121(3,27)+20(3,28)+20(3,\overline{28})+145(5,27)+20(5,28)+20(5,\overline{28})+137(7,27)+20(7,28)+20(7,\overline{28})+29(1,35)+29(1,\overline{35})+95(9,27)+11(9,28)+11(9,\overline{28})+73(3,35)+73(3,\overline{35})+55(11,27)+6(11,28)+6(11,\overline{28})+92(5,35)+92(5,\overline{35})+22(13,27)+(13,28)+(13,\overline{28})+82(7,35)+82(7,\overline{35})+7(15,27)+56(9,35)+56(9,\overline{35})+(17,27)+29(11,35)+29(11,\overline{35})+12(13,35)+12(13,\overline{35})+3(15,35)+3(15,\overline{35})+(5,55)+(5,\overline{55})+29(1,64)+76(3,64)+94(5,64)+81(7,64)+54(9,64)+27(11,64)+9(13,64)+2(15,64)+3(1,80)+3(1,\overline{80})+15(1,81)+15(1,\overline{81})+7(3,80)+7(3,\overline{80})+34(3,81)+34(3,\overline{81})+8(5,80)+8(5,\overline{80})+43(5,81)+43(5,\overline{81})+6(7,80)+6(7,\overline{80})+34(7,81)+34(7,\overline{81})+3(9,80)+3(9,\overline{80})+22(9,81)+22(9,\overline{81})+(11,80)+(11,\overline{80})+9(11,81)+9(11,\overline{81})+3(13,81)+3(13,\overline{81})+7(1,125)+24(3,125)+25(5,125)+21(7,125)+11(9,125)+5(11,125)+3(1,154)+3(1,\overline{154})+6(3,154)+6(3,\overline{154})+7(5,154)+7(5,\overline{154})+5(7,154)+5(7,\overline{154})+2(9,154)+2(9,\overline{154})+(3,162)+(3,\overline{162})+(1,216)+2(3,216)+2(5,216)+(7,216)$

E_8	→	Sp(4) (S)
248	=	$(10)+(84)+(154)$
3875	=	$(5)+(14)+(30)+2(35')+(55)+(81)+(91)+2(105)+(154)+(165)+2(220)+(231)+(260)+(390)+(429)+2(625)$
27000	=	$2(1)+2(5)+3(14)+3(30)+2(35)+5(35')+4(55)+3(81)+2(84)+3(91)+5(105)+3(140'')+5(154)+4(165)+(204)+8(220)+4(231)+3(260)+(285)+6(390)+2(405)+2(429)+3(455)+(455')+(595)+4(625)+3(770)+3(810)+2(935)+(1190)+(1326)+(1330)$
30380	=	$3(10)+2(30)+5(35)+2(35')+2(55)+7(81)+5(84)+2(91)+6(105)+7(154)+2(165)+6(220)+6(231)+7(260)+2(286)+6(390)+4(405)+2(429)+6(455)+2(595)+3(625)+(715)+4(770)+(810)+(836)+2(935)+2(1190)+(1326)$
147250	=	$(1)+2(5)+3(10)+5(14)+7(30)+11(35)+9(35')+9(55)+16(81)+10(84)+7(91)+19(105)+5(140'')+19(154)+10(165)+3(204)+23(220)+17(231)+18(260)+2(285)+4(286)+(385)+22(390)+13(405)+11(429)+18(455)+(455')+7(595)+17(625)+5(715)+14(770)+9(810)+3(836)+10(935)+(1105)+(1134)+8(1190)+2(1309)+5(1326)+4(1330)+3(1729)+(1820')+3(1995)+(2090)+(2401)$
779247	=	$7(5)+17(10)+14(14)+22(30)+40(35)+30(35')+27(55)+66(81)+45(84)+27(91)+68(105)+19(140'')+79(154)+31(165)+13(204)+87(220)+73(231)+76(260)+6(285)+22(286)+2(385)+90(390)+59(405)+51(429)+81(455)+6(455')+38(595)+72(625)+2(680)+23(715)+66(770)+43(810)+20(836)+46(935)+6(1105)+8(1134)+44(1190)+17(1309)+29(1326)+22(1330)+2(1495)+(1615)+22(1729)+9(1820')+2(1976)+14(1995)+8(2090)+7(2401)+2(2415)+5(2835)+3(3080')+2(3220)$

E_8	→	SU(2) (S)
248	=	$(3)+(11)+(15)+(19)+(23)+(27)+(29)+(35)+(39)+(47)$
3875	=	$2(1)+3(5)+(7)+4(9)+2(11)+6(13)+3(15)+6(17)+4(19)+7(21)+4(23)+7(25)+5(27)+7(29)+5(31)+6(33)+4(35)+7(37)+4(39)+5(41)+3(43)+5(45)+3(47)+4(49)+2(51)+3(53)+2(55)+2(57)+(59)+2(61)+(63)+(65)+(69)+(73)$
27000	=	$7(1)+(3)+13(5)+7(7)+19(9)+14(11)+25(13)+19(15)+29(17)+23(19)+33(21)+26(23)+35(25)+28(27)+36(29)+28(31)+35(33)+28(35)+34(37)+27(39)+31(41)+24(43)+28(45)+22(47)+25(49)+18(51)+21(53)+15(55)+18(57)+12(59)+14(61)+9(63)+11(65)+7(67)+8(69)+5(71)+6(73)+3(75)+4(77)+2(79)+3(81)+(83)+2(85)+(89)+(93)$

Table 10.28: E_8 Branching Rules (continued)

30380	=	10(**3**) + 6(**5**) + 17(**7**) + 14(**9**) + 24(**11**) + 22(**13**) + 30(**15**) + 26(**17**) + 35(**19**) + 31(**21**) + 37(**23**) + 34(**25**) + 40(**27**) + 34(**29**) + 40(**31**) + 34(**33**) + 38(**35**) + 34(**37**) + 36(**39**) + 30(**41**) + 33(**43**) + 27(**45**) + 29(**47**) + 24(**49**) + 25(**51**) + 19(**53**) + 21(**55**) + 16(**57**) + 16(**59**) + 13(**61**) + 13(**63**) + 9(**65**) + 10(**67**) + 6(**69**) + 7(**71**) + 5(**73**) + 5(**75**) + 2(**77**) + 3(**79**) + 2(**81**) + 2(**83**) + (**85**) + (**87**) + (**91**)
147250	=	8(**1**) + 22(**3**) + 41(**5**) + 49(**7**) + 69(**9**) + 80(**11**) + 93(**13**) + 102(**15**) + 118(**17**) + 121(**19**) + 133(**21**) + 138(**23**) + 144(**25**) + 147(**27**) + 153(**29**) + 149(**31**) + 153(**33**) + 151(**35**) + 149(**37**) + 144(**39**) + 144(**41**) + 134(**43**) + 132(**45**) + 124(**47**) + 118(**49**) + 110(**51**) + 105(**53**) + 94(**55**) + 89(**57**) + 81(**59**) + 73(**61**) + 66(**63**) + 61(**65**) + 51(**67**) + 47(**69**) + 41(**71**) + 36(**73**) + 30(**75**) + 27(**77**) + 21(**79**) + 19(**81**) + 16(**83**) + 12(**85**) + 10(**87**) + 9(**89**) + 6(**91**) + 5(**93**) + 4(**95**) + 3(**97**) + 2(**99**) + 2(**101**) + (**105**) + (**107**)
779247	=	26(**1**) + 115(**3**) + 166(**5**) + 251(**7**) + 298(**9**) + 378(**11**) + 419(**13**) + 490(**15**) + 522(**17**) + 584(**19**) + 606(**21**) + 656(**23**) + 667(**25**) + 705(**27**) + 706(**29**) + 731(**31**) + 720(**33**) + 734(**35**) + 715(**37**) + 718(**39**) + 688(**41**) + 682(**43**) + 648(**45**) + 634(**47**) + 594(**49**) + 574(**51**) + 532(**53**) + 509(**55**) + 465(**57**) + 440(**59**) + 397(**61**) + 372(**63**) + 331(**65**) + 306(**67**) + 269(**69**) + 247(**71**) + 213(**73**) + 193(**75**) + 164(**77**) + 147(**79**) + 123(**81**) + 109(**83**) + 89(**85**) + 78(**87**) + 62(**89**) + 54(**91**) + 42(**93**) + 36(**95**) + 27(**97**) + 23(**99**) + 16(**101**) + 14(**103**) + 9(**105**) + 8(**107**) + 5(**109**) + 4(**111**) + 2(**113**) + 2(**115**) + (**117**) + (**119**)

E_8	→	SU(2) (S)
248	=	(**3**) + (**7**) + (**11**) + (**15**) + (**17**) + (**19**) + 2(**23**) + (**27**) + (**29**) + (**35**) + (**39**)
3875	=	3(**1**) + 5(**5**) + 3(**7**) + 6(**9**) + 4(**11**) + 9(**13**) + 6(**15**) + 9(**17**) + 7(**19**) + 10(**21**) + 7(**23**) + 10(**25**) + 7(**27**) + 9(**29**) + 6(**31**) + 8(**33**) + 5(**35**) + 7(**37**) + 5(**39**) + 5(**41**) + 3(**43**) + 4(**45**) + 2(**47**) + 3(**49**) + (**51**) + 2(**53**) + (**55**) + (**57**) + (**61**)
27000	=	9(**1**) + 4(**3**) + 20(**5**) + 16(**7**) + 31(**9**) + 25(**11**) + 40(**13**) + 33(**15**) + 47(**17**) + 39(**19**) + 50(**21**) + 43(**23**) + 51(**25**) + 42(**27**) + 50(**29**) + 40(**31**) + 46(**33**) + 37(**35**) + 41(**37**) + 32(**39**) + 35(**41**) + 26(**43**) + 29(**45**) + 20(**47**) + 22(**49**) + 16(**51**) + 16(**53**) + 11(**55**) + 12(**57**) + 7(**59**) + 8(**61**) + 4(**63**) + 5(**65**) + 2(**67**) + 3(**69**) + (**71**) + 2(**73**) + (**77**)
30380	=	(**1**) + 15(**3**) + 13(**5**) + 29(**7**) + 26(**9**) + 39(**11**) + 38(**13**) + 48(**15**) + 45(**17**) + 55(**19**) + 50(**21**) + 57(**23**) + 52(**25**) + 57(**27**) + 49(**29**) + 54(**31**) + 46(**33**) + 48(**35**) + 41(**37**) + 42(**39**) + 34(**41**) + 35(**43**) + 27(**45**) + 26(**47**) + 21(**49**) + 21(**51**) + 14(**53**) + 15(**55**) + 10(**57**) + 9(**59**) + 7(**61**) + 6(**63**) + 3(**65**) + 4(**67**) + (**69**) + 2(**71**) + (**73**) + (**75**)
147250	=	14(**1**) + 39(**3**) + 71(**5**) + 88(**7**) + 118(**9**) + 138(**11**) + 158(**13**) + 173(**15**) + 194(**17**) + 199(**19**) + 213(**21**) + 217(**23**) + 222(**25**) + 221(**27**) + 223(**29**) + 213(**31**) + 212(**33**) + 202(**35**) + 192(**37**) + 180(**39**) + 172(**41**) + 154(**43**) + 144(**45**) + 130(**47**) + 117(**49**) + 103(**51**) + 93(**53**) + 78(**55**) + 69(**57**) + 59(**59**) + 49(**61**) + 40(**63**) + 35(**65**) + 26(**67**) + 22(**69**) + 17(**71**) + 13(**73**) + 10(**75**) + 8(**77**) + 4(**79**) + 4(**81**) + 3(**83**) + (**85**) + (**87**) + (**89**)
779247	=	51(**1**) + 197(**3**) + 297(**5**) + 435(**7**) + 523(**9**) + 647(**11**) + 720(**13**) + 824(**15**) + 875(**17**) + 956(**19**) + 984(**21**) + 1039(**23**) + 1043(**25**) + 1072(**27**) + 1054(**29**) + 1059(**31**) + 1021(**33**) + 1006(**35**) + 953(**37**) + 922(**39**) + 857(**41**) + 815(**43**) + 745(**45**) + 697(**47**) + 625(**49**) + 575(**51**) + 507(**53**) + 459(**55**) + 396(**57**) + 353(**59**) + 298(**61**) + 261(**63**) + 215(**65**) + 185(**67**) + 149(**69**) + 126(**71**) + 98(**73**) + 81(**75**) + 61(**77**) + 50(**79**) + 35(**81**) + 28(**83**) + 19(**85**) + 15(**87**) + 9(**89**) + 7(**91**) + 4(**93**) + 3(**95**) + (**97**) + (**99**)

E_8	→	SU(2) (S)
248	=	(**3**) + (**15**) + (**23**) + (**27**) + (**35**) + (**39**) + (**47**) + (**59**)
3875	=	2(**1**) + 2(**5**) + 2(**9**) + 4(**13**) + 2(**15**) + 3(**17**) + (**19**) + 4(**21**) + 2(**23**) + 5(**25**) + 3(**27**) + 4(**29**) + 2(**31**) + 4(**33**) + 3(**35**) + 5(**37**) + 3(**39**) + 4(**41**) + 2(**43**) + 4(**45**) + 3(**47**) + 4(**49**) + 2(**51**) + 3(**53**) + 2(**55**) + 3(**57**) + 2(**59**) + 3(**61**) + (**63**) + 2(**65**) + (**67**) + 2(**69**) + (**71**) + 2(**73**) + (**77**) + (**79**) + (**81**) + (**85**) + (**93**)
27000	=	5(**1**) + 8(**5**) + 2(**7**) + 10(**9**) + 5(**11**) + 14(**13**) + 9(**15**) + 16(**17**) + 10(**19**) + 18(**21**) + 13(**23**) + 21(**25**) + 15(**27**) + 21(**29**) + 15(**31**) + 22(**33**) + 17(**35**) + 23(**37**) + 17(**39**) + 22(**41**) + 16(**43**) + 22(**45**) + 17(**47**) + 21(**49**) + 15(**51**) + 19(**53**) + 14(**55**) + 18(**57**) + 14(**59**) + 17(**61**) + 11(**63**) + 14(**65**) + 10(**67**) + 13(**69**) + 9(**71**) + 11(**73**) + 7(**75**) + 9(**77**) + 6(**79**) + 8(**81**) + 5(**83**) + 6(**85**) + 3(**87**) + 5(**89**) + 3(**91**) + 4(**93**) + 2(**95**) + 3(**97**) + (**99**) + 2(**101**) + (**103**) + 2(**105**) + (**109**) + (**113**) + (**117**)

Table 10.28: E_8 Branching Rules (continued)

30380	=	7(**3**)+2(**5**)+9(**7**)+5(**9**)+12(**11**)+11(**13**)+17(**15**)+12(**17**)+18(**19**)+15(**21**)+21(**23**)+19(**25**)+ 24(**27**) + 18(**29**) + 23(**31**) + 20(**33**) + 25(**35**) + 22(**37**) + 25(**39**) + 20(**41**) + 24(**43**) + 20(**45**) + 24(**47**) + 20(**49**) + 22(**51**) + 17(**53**) + 20(**55**) + 17(**57**) + 19(**59**) + 16(**61**) + 16(**63**) + 12(**65**) + 15(**67**) + 11(**69**) + 13(**71**) + 10(**73**) + 10(**75**) + 7(**77**) + 9(**79**) + 7(**81**) + 7(**83**) + 5(**85**) + 5(**87**) + 3(**89**) + 5(**91**) + 3(**93**) + 3(**95**) + 2(**97**) + 2(**99**) + (**101**) + 2(**103**) + (**105**) + (**107**) + (**111**) + (**115**)
147250	=	5(**1**) + 11(**3**) + 21(**5**) + 22(**7**) + 33(**9**) + 39(**11**) + 47(**13**) + 52(**15**) + 60(**17**) + 60(**19**) + 69(**21**) + 73(**23**) + 79(**25**) + 81(**27**) + 86(**29**) + 83(**31**) + 90(**33**) + 92(**35**) + 93(**37**) + 92(**39**) + 95(**41**) + 90(**43**) + 93(**45**) + 92(**47**) + 91(**49**) + 87(**51**) + 87(**53**) + 81(**55**) + 82(**57**) + 79(**59**) + 75(**61**) + 70(**63**) + 69(**65**) + 63(**67**) + 62(**69**) + 58(**71**) + 54(**73**) + 49(**75**) + 47(**77**) + 42(**79**) + 41(**81**) + 37(**83**) + 33(**85**) + 29(**87**) + 28(**89**) + 24(**91**) + 23(**93**) + 20(**95**) + 17(**97**) + 15(**99**) + 14(**101**) + 11(**103**) + 11(**105**) + 9(**107**) + 7(**109**) + 6(**111**) + 6(**113**) + 4(**115**) + 4(**117**) + 3(**119**) + 2(**121**) + 2(**123**) + 2(**125**) + (**127**) + (**129**) + (**131**) + (**137**)
779247	=	11(**1**) + 60(**3**) + 78(**5**) + 123(**7**) + 140(**9**) + 187(**11**) + 206(**13**) + 249(**15**) + 260(**17**) + 298(**19**) + 309(**21**) + 347(**23**) + 355(**25**) + 386(**27**) + 387(**29**) + 412(**31**) + 413(**33**) + 437(**35**) + 433(**37**) + 449(**39**) + 438(**41**) + 450(**43**) + 440(**45**) + 450(**47**) + 434(**49**) + 436(**51**) + 417(**53**) + 418(**55**) + 399(**57**) + 398(**59**) + 375(**61**) + 368(**63**) + 344(**65**) + 338(**67**) + 316(**69**) + 308(**71**) + 283(**73**) + 272(**75**) + 249(**77**) + 241(**79**) + 219(**81**) + 209(**83**) + 187(**85**) + 177(**87**) + 157(**89**) + 150(**91**) + 133(**93**) + 124(**95**) + 107(**97**) + 100(**99**) + 86(**101**) + 81(**103**) + 69(**105**) + 63(**107**) + 52(**109**) + 48(**111**) + 40(**113**) + 37(**115**) + 30(**117**) + 27(**119**) + 20(**121**) + 19(**123**) + 15(**125**) + 14(**127**) + 10(**129**) + 9(**131**) + 6(**133**) + 6(**135**) + 4(**137**) + 4(**139**) + 2(**141**) + 2(**143**) + (**145**) + (**147**) + (**149**) + (**151**)

Table 10.29: F_4 Branching Rules

F_4	→	SO(9) (R)
26	=	(**1**) + (**9**) + (**16**)
52	=	(**16**) + (**36**)
273	=	(**9**) + (**16**) + (**36**) + (**84**) + (**128**)
324	=	(**1**) + (**9**) + (**16**) + (**44**) + (**126**) + (**128**)
1053	=	(**16**) + (**36**) + (**84**) + (**126**) + (**128**) + (**231**) + (**432**)
1053′	=	(**126**) + (**432**) + (**495**)
1274	=	(**36**) + (**84**) + (**128**) + (**432**) + (**594**)
2652	=	(**1**) + (**9**) + (**16**) + (**44**) + (**126**) + (**128**) + (**156**) + (**576**) + (**672**) + (**924**)
4096	=	(**9**) + (**16**) + (**36**) + (**44**) + (**84**) + (**126**) + 2(**128**) + (**231**) + (**432**) + (**576**) + (**594**) + (**768**) + (**924**)
8424	=	(**84**) + (**126**) + (**128**) + (**231**) + 2(**432**) + (**495**) + (**594**) + (**768**) + (**924**) + (**1650**) + (**2560**)
10829	=	(**16**) + (**36**) + (**84**) + (**126**) + (**128**) + (**231**) + (**432**) + (**576**) + (**594**) + (**672**) + (**768**) + (**910**) + (**924**) + (**2560**) + (**2772**)
12376	=	(**672**) + (**2772**) + (**4004**) + (**4928**)
16302	=	(**1**) + (**9**) + (**16**) + (**44**) + (**126**) + (**128**) + (**156**) + (**450**) + (**576**) + (**672**) + (**924**) + (**1920**) + (**2772′**) + (**3900**) + (**4608**)
17901	=	(**126**) + (**432**) + (**495**) + (**672**) + (**768**) + (**924**) + (**1650**) + (**2560**) + (**2574**) + (**2772**) + (**4928**)

F_4	→	SU(3)×SU(3) (R)
26	=	$(\mathbf{3},\mathbf{3}) + (\overline{\mathbf{3}},\overline{\mathbf{3}}) + (\mathbf{8},\mathbf{1})$
52	=	$(\mathbf{8},\mathbf{1}) + (\mathbf{1},\mathbf{8}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{6},\overline{\mathbf{3}})$
273	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\overline{\mathbf{3}},\overline{\mathbf{3}}) + (\mathbf{8},\mathbf{1}) + (\mathbf{3},\overline{\mathbf{6}}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{6},\overline{\mathbf{3}}) + (\overline{\mathbf{3}},\mathbf{6}) + (\mathbf{10},\mathbf{1}) + (\overline{\mathbf{10}},\mathbf{1}) + (\mathbf{8},\mathbf{8}) + (\mathbf{15},\mathbf{3}) + (\overline{\mathbf{15}},\overline{\mathbf{3}})$
324	=	$(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{3}) + (\overline{\mathbf{3}},\overline{\mathbf{3}}) + (\mathbf{8},\mathbf{1}) + (\mathbf{1},\mathbf{8}) + (\overline{\mathbf{6}},\mathbf{3}) + (\mathbf{6},\overline{\mathbf{3}}) + (\overline{\mathbf{6}},\overline{\mathbf{6}}) + (\mathbf{6},\mathbf{6}) + (\mathbf{8},\mathbf{8}) + (\mathbf{15},\mathbf{3}) + (\overline{\mathbf{15}},\overline{\mathbf{3}}) + (\mathbf{27},\mathbf{1})$

Table 10.29: F_4 Branching Rules (continued)

1053	=	$2(\mathbf{3},\mathbf{3})+2(\overline{\mathbf{3}},\overline{\mathbf{3}})+2(\mathbf{8},\mathbf{1})+(\mathbf{3},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{3})+(\mathbf{6},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1})+2(\mathbf{8},\mathbf{8})+(\mathbf{10},\mathbf{8})+(\overline{\mathbf{10}},\mathbf{8})+2(\mathbf{15},\mathbf{3})+(\mathbf{3},\mathbf{15})+2(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{24},\mathbf{3})+(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\mathbf{27},\mathbf{1})$
$\mathbf{1053'}$	=	$(\mathbf{1},\mathbf{1})+(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8})+(\overline{\mathbf{6}},\mathbf{3})+(\mathbf{6},\overline{\mathbf{3}})+(\overline{\mathbf{6}},\overline{\mathbf{6}})+(\mathbf{6},\mathbf{6})+(\mathbf{8},\mathbf{8})+(\mathbf{15},\mathbf{3})+(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{6}},\mathbf{15})+(\mathbf{6},\overline{\mathbf{15}})+(\mathbf{15}',\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+(\mathbf{24},\mathbf{3})+(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\mathbf{27},\mathbf{1})+(\mathbf{1},\mathbf{27})+(\mathbf{27},\mathbf{8})$
1274	=	$(\mathbf{1},\mathbf{1})+(\mathbf{3},\mathbf{3})+(\overline{\mathbf{3}},\overline{\mathbf{3}})+(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8})+2(\overline{\mathbf{6}},\mathbf{3})+2(\mathbf{6},\overline{\mathbf{3}})+(\mathbf{10},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{1},\overline{\mathbf{10}})+(\overline{\mathbf{6}},\overline{\mathbf{6}})+(\mathbf{6},\mathbf{6})+2(\mathbf{8},\mathbf{8})+(\mathbf{15},\mathbf{3})+(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\mathbf{15}',\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{15})+(\mathbf{6},\overline{\mathbf{15}})+(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{24},\mathbf{3})+(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\mathbf{27},\mathbf{1})+(\mathbf{27},\mathbf{8})$
2652	=	$(\mathbf{1},\mathbf{1})+2(\mathbf{3},\mathbf{3})+2(\overline{\mathbf{3}},\overline{\mathbf{3}})+(\mathbf{8},\mathbf{1})+(\mathbf{3},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{3})+(\mathbf{6},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1})+(\overline{\mathbf{6}},\overline{\mathbf{6}})+(\mathbf{6},\mathbf{6})+3(\mathbf{8},\mathbf{8})+(\mathbf{10},\mathbf{8})+(\overline{\mathbf{10}},\mathbf{8})+2(\mathbf{15},\mathbf{3})+(\mathbf{3},\mathbf{15})+2(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{10},\mathbf{10})+(\overline{\mathbf{10}},\overline{\mathbf{10}})+(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{24},\mathbf{3})+(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\mathbf{27},\mathbf{1})+(\mathbf{15},\mathbf{15})+(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{24}},\mathbf{6})+(\mathbf{27},\mathbf{8})+(\mathbf{42},\mathbf{3})+(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\mathbf{64},\mathbf{1})$
4096	=	$3(\mathbf{3},\mathbf{3})+3(\overline{\mathbf{3}},\overline{\mathbf{3}})+4(\mathbf{8},\mathbf{1})+2(\mathbf{1},\mathbf{8})+2(\mathbf{3},\overline{\mathbf{6}})+3(\overline{\mathbf{6}},\mathbf{3})+3(\mathbf{6},\overline{\mathbf{3}})+2(\overline{\mathbf{3}},\mathbf{6})+(\mathbf{10},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1})+2(\overline{\mathbf{6}},\overline{\mathbf{6}})+2(\mathbf{6},\mathbf{6})+4(\mathbf{8},\mathbf{8})+2(\mathbf{10},\mathbf{8})+(\mathbf{8},\mathbf{10})+(\mathbf{8},\overline{\mathbf{10}})+2(\overline{\mathbf{10}},\mathbf{8})+4(\mathbf{15},\mathbf{3})+(\mathbf{3},\mathbf{15})+4(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+2(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{15})+(\mathbf{6},\overline{\mathbf{15}})+2(\overline{\mathbf{15}},\mathbf{6})+2(\mathbf{24},\mathbf{3})+2(\overline{\mathbf{24}},\overline{\mathbf{3}})+2(\mathbf{27},\mathbf{1})+(\mathbf{15},\mathbf{15})+(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{24}},\mathbf{6})+2(\mathbf{27},\mathbf{8})+(\mathbf{35},\mathbf{1})+(\overline{\mathbf{35}},\mathbf{1})+(\mathbf{42},\mathbf{3})+(\overline{\mathbf{42}},\overline{\mathbf{3}})$
8424	=	$(\mathbf{1},\mathbf{1})+3(\mathbf{3},\mathbf{3})+3(\overline{\mathbf{3}},\overline{\mathbf{3}})+3(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8})+2(\mathbf{3},\overline{\mathbf{6}})+3(\overline{\mathbf{6}},\mathbf{3})+3(\mathbf{6},\overline{\mathbf{3}})+2(\overline{\mathbf{3}},\mathbf{6})+2(\mathbf{10},\mathbf{1})+2(\overline{\mathbf{10}},\mathbf{1})+2(\overline{\mathbf{6}},\overline{\mathbf{6}})+2(\mathbf{6},\mathbf{6})+5(\mathbf{8},\mathbf{8})+3(\mathbf{10},\mathbf{8})+(\mathbf{8},\mathbf{10})+(\mathbf{8},\overline{\mathbf{10}})+3(\overline{\mathbf{10}},\mathbf{8})+5(\mathbf{15},\mathbf{3})+2(\mathbf{3},\mathbf{15})+5(\overline{\mathbf{15}},\overline{\mathbf{3}})+2(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+(\mathbf{10},\overline{\mathbf{10}})+(\overline{\mathbf{10}},\mathbf{10})+3(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{15})+(\mathbf{6},\overline{\mathbf{15}})+3(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{15}',\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+(\mathbf{21},\mathbf{3})+(\overline{\mathbf{21}},\overline{\mathbf{3}})+3(\mathbf{24},\mathbf{3})+(\mathbf{3},\mathbf{24})+3(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{24}})+3(\mathbf{27},\mathbf{1})+2(\mathbf{15},\mathbf{15})+2(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{24}},\mathbf{6})+3(\mathbf{27},\mathbf{8})+(\mathbf{8},\mathbf{27})+(\mathbf{35},\mathbf{1})+(\overline{\mathbf{35}},\mathbf{1})+(\mathbf{24},\mathbf{15})+(\overline{\mathbf{24}},\overline{\mathbf{15}})+(\mathbf{35},\mathbf{8})+(\overline{\mathbf{35}},\mathbf{8})+2(\mathbf{42},\mathbf{3})+2(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\mathbf{42},\overline{\mathbf{6}})+(\overline{\mathbf{42}},\mathbf{6})$
10829	=	$(\mathbf{1},\mathbf{1})+2(\mathbf{3},\mathbf{3})+2(\overline{\mathbf{3}},\overline{\mathbf{3}})+3(\mathbf{8},\mathbf{1})+3(\mathbf{1},\mathbf{8})+2(\mathbf{3},\overline{\mathbf{6}})+5(\overline{\mathbf{6}},\mathbf{3})+5(\mathbf{6},\overline{\mathbf{3}})+2(\overline{\mathbf{3}},\mathbf{6})+2(\mathbf{10},\mathbf{1})+2(\overline{\mathbf{10}},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{1},\overline{\mathbf{10}})+3(\overline{\mathbf{6}},\overline{\mathbf{6}})+3(\mathbf{6},\mathbf{6})+6(\mathbf{8},\mathbf{8})+2(\mathbf{10},\mathbf{8})+(\mathbf{8},\mathbf{10})+(\mathbf{8},\overline{\mathbf{10}})+2(\overline{\mathbf{10}},\mathbf{8})+5(\mathbf{15},\mathbf{3})+(\mathbf{3},\mathbf{15})+5(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{15}})+2(\mathbf{15}',\mathbf{3})+2(\overline{\mathbf{15}}',\overline{\mathbf{3}})+3(\mathbf{15},\overline{\mathbf{6}})+3(\overline{\mathbf{6}},\mathbf{15})+3(\mathbf{6},\overline{\mathbf{15}})+3(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{15}',\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+3(\mathbf{24},\mathbf{3})+3(\overline{\mathbf{24}},\overline{\mathbf{3}})+3(\mathbf{27},\mathbf{1})+(\mathbf{1},\mathbf{27})+2(\mathbf{15},\mathbf{15})+2(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{15})+2(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{24})+2(\overline{\mathbf{24}},\mathbf{6})+(\mathbf{6},\overline{\mathbf{24}})+(\overline{\mathbf{15}}',\overline{\mathbf{15}})+5(\mathbf{27},\mathbf{8})+(\mathbf{8},\mathbf{27})+(\mathbf{35},\mathbf{1})+(\overline{\mathbf{35}},\mathbf{1})+(\mathbf{27},\mathbf{10})+(\mathbf{27},\overline{\mathbf{10}})+(\mathbf{24},\mathbf{15})+(\overline{\mathbf{24}},\overline{\mathbf{15}})+(\mathbf{35},\mathbf{8})+(\overline{\mathbf{35}},\mathbf{8})+2(\mathbf{42},\mathbf{3})+2(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\mathbf{42},\overline{\mathbf{6}})+(\overline{\mathbf{42}},\mathbf{6})+(\mathbf{60},\mathbf{3})+(\overline{\mathbf{60}},\overline{\mathbf{3}})+(\mathbf{64},\mathbf{1})$
12376	=	$(\mathbf{1},\mathbf{1})+(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8})+2(\overline{\mathbf{6}},\mathbf{3})+2(\mathbf{6},\overline{\mathbf{3}})+(\mathbf{10},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1})+(\mathbf{1},\mathbf{10})+(\mathbf{1},\overline{\mathbf{10}})+(\overline{\mathbf{6}},\overline{\mathbf{6}})+(\mathbf{6},\mathbf{6})+2(\mathbf{8},\mathbf{8})+(\mathbf{15},\mathbf{3})+(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\mathbf{15}',\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+(\mathbf{15},\overline{\mathbf{6}})+2(\overline{\mathbf{6}},\mathbf{15})+2(\mathbf{6},\overline{\mathbf{15}})+(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{15}',\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+2(\mathbf{24},\mathbf{3})+2(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\mathbf{27},\mathbf{1})+(\mathbf{1},\mathbf{27})+(\mathbf{15},\mathbf{15})+(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{15})+(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{24})+(\overline{\mathbf{24}},\mathbf{6})+(\mathbf{6},\overline{\mathbf{24}})+(\overline{\mathbf{15}}',\overline{\mathbf{15}})+3(\mathbf{27},\mathbf{8})+(\mathbf{8},\mathbf{27})+(\mathbf{27},\mathbf{10})+(\mathbf{27},\overline{\mathbf{10}})+(\mathbf{28},\overline{\mathbf{10}})+(\overline{\mathbf{28}},\mathbf{10})+(\mathbf{24},\mathbf{15})+(\overline{\mathbf{24}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{24})+(\overline{\mathbf{15}}',\overline{\mathbf{24}})+(\mathbf{35},\mathbf{8})+(\overline{\mathbf{35}},\mathbf{8})+(\mathbf{42},\mathbf{3})+(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\mathbf{42},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{42})+(\overline{\mathbf{42}},\mathbf{6})+(\mathbf{6},\overline{\mathbf{42}})+(\mathbf{27},\mathbf{27})+(\mathbf{48},\overline{\mathbf{6}})+(\overline{\mathbf{48}},\mathbf{6})+(\mathbf{60},\mathbf{3})+(\overline{\mathbf{60}},\overline{\mathbf{3}})+(\mathbf{64},\mathbf{1})+(\mathbf{1},\mathbf{64})+(\mathbf{64},\mathbf{8})+(\mathbf{60},\mathbf{15})+(\overline{\mathbf{60}},\overline{\mathbf{15}})$
16302	=	$(\mathbf{1},\mathbf{1})+2(\mathbf{3},\mathbf{3})+2(\overline{\mathbf{3}},\overline{\mathbf{3}})+2(\mathbf{8},\mathbf{1})+2(\mathbf{1},\mathbf{8})+(\mathbf{3},\overline{\mathbf{6}})+2(\overline{\mathbf{6}},\mathbf{3})+2(\mathbf{6},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\mathbf{6})+3(\overline{\mathbf{6}},\overline{\mathbf{6}})+3(\mathbf{6},\mathbf{6})+4(\mathbf{8},\mathbf{8})+2(\mathbf{10},\mathbf{8})+(\mathbf{8},\mathbf{10})+(\mathbf{8},\overline{\mathbf{10}})+2(\overline{\mathbf{10}},\mathbf{8})+3(\mathbf{15},\mathbf{3})+(\mathbf{3},\mathbf{15})+3(\overline{\mathbf{15}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+(\mathbf{10},\mathbf{10})+(\overline{\mathbf{10}},\overline{\mathbf{10}})+2(\mathbf{15},\overline{\mathbf{6}})+2(\overline{\mathbf{6}},\mathbf{15})+2(\mathbf{6},\overline{\mathbf{15}})+2(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{15}',\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+2(\mathbf{24},\mathbf{3})+2(\overline{\mathbf{24}},\overline{\mathbf{3}})+2(\mathbf{27},\mathbf{1})+(\mathbf{1},\mathbf{27})+3(\mathbf{15},\mathbf{15})+3(\overline{\mathbf{15}},\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{15})+2(\mathbf{24},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{24})+2(\overline{\mathbf{24}},\mathbf{6})+(\mathbf{6},\overline{\mathbf{24}})+(\overline{\mathbf{15}}',\overline{\mathbf{15}})+(\mathbf{15}',\mathbf{15}')+(\overline{\mathbf{15}}',\overline{\mathbf{15}}')+4(\mathbf{27},\mathbf{8})+(\mathbf{8},\mathbf{27})+(\mathbf{35},\mathbf{1})+(\overline{\mathbf{35}},\mathbf{1})+(\mathbf{27},\mathbf{10})+(\mathbf{27},\overline{\mathbf{10}})+(\mathbf{24},\mathbf{15})+(\overline{\mathbf{24}},\overline{\mathbf{15}})+(\mathbf{35},\mathbf{8})+(\overline{\mathbf{35}},\mathbf{8})+(\mathbf{35},\mathbf{10})+(\overline{\mathbf{35}},\overline{\mathbf{10}})+2(\mathbf{42},\mathbf{3})+2(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\mathbf{24},\mathbf{24})+(\overline{\mathbf{24}},\overline{\mathbf{24}})+(\mathbf{42},\overline{\mathbf{6}})+(\overline{\mathbf{42}},\mathbf{6})+(\mathbf{27},\mathbf{27})+(\mathbf{42},\mathbf{15})+(\overline{\mathbf{42}},\overline{\mathbf{15}})+(\mathbf{60},\mathbf{3})+(\overline{\mathbf{60}},\overline{\mathbf{3}})+(\mathbf{64},\mathbf{1})+(\mathbf{60},\overline{\mathbf{6}})+(\overline{\mathbf{60}},\mathbf{6})+(\mathbf{64},\mathbf{8})+(\mathbf{90},\mathbf{3})+(\overline{\mathbf{90}},\overline{\mathbf{3}})+(\mathbf{125},\mathbf{1})$

Table 10.29: F_4 Branching Rules (continued)

17901	=	$3(\mathbf{3,3})+3(\overline{\mathbf{3}},\overline{\mathbf{3}})+3(\mathbf{8,1})+2(\mathbf{3},\overline{\mathbf{6}})+2(\overline{\mathbf{6}},\mathbf{3})+2(\mathbf{6},\overline{\mathbf{3}})+2(\overline{\mathbf{3}},\mathbf{6})+2(\mathbf{10,1})+2(\overline{\mathbf{10}},\mathbf{1})+(\overline{\mathbf{6}},\overline{\mathbf{6}})+(\mathbf{6,6})+5(\mathbf{8,8})+4(\mathbf{10,8})+(\mathbf{8,10})+(\mathbf{8},\overline{\mathbf{10}})+4(\overline{\mathbf{10}},\mathbf{8})+5(\mathbf{15,3})+3(\mathbf{3,15})+5(\overline{\mathbf{15}},\overline{\mathbf{3}})+3(\overline{\mathbf{3}},\overline{\mathbf{15}})+(\mathbf{15'},\mathbf{3})+(\overline{\mathbf{15}}',\overline{\mathbf{3}})+(\mathbf{10,10})+(\mathbf{10},\overline{\mathbf{10}})+(\overline{\mathbf{10}},\mathbf{10})+(\overline{\mathbf{10}},\overline{\mathbf{10}})+4(\mathbf{15},\overline{\mathbf{6}})+(\overline{\mathbf{6}},\mathbf{15})+(\mathbf{6},\overline{\mathbf{15}})+4(\overline{\mathbf{15}},\mathbf{6})+(\mathbf{15'},\overline{\mathbf{6}})+(\overline{\mathbf{15}}',\mathbf{6})+(\mathbf{21,3})+(\overline{\mathbf{21}},\overline{\mathbf{3}})+4(\mathbf{24,3})+(\mathbf{3,24})+4(\overline{\mathbf{24}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{24}})+3(\mathbf{27,1})+3(\mathbf{15,15})+3(\overline{\mathbf{15}},\overline{\mathbf{15}})+2(\mathbf{24},\overline{\mathbf{6}})+2(\overline{\mathbf{24}},\mathbf{6})+4(\mathbf{27,8})+2(\mathbf{8,27})+(\mathbf{21,15})+(\overline{\mathbf{21}},\overline{\mathbf{15}})+(\mathbf{35,1})+(\overline{\mathbf{35}},\mathbf{1})+(\mathbf{10,27})+(\overline{\mathbf{10}},\mathbf{27})+(\mathbf{15,24})+2(\mathbf{24,15})+2(\overline{\mathbf{24}},\overline{\mathbf{15}})+(\overline{\mathbf{15}},\overline{\mathbf{24}})+2(\mathbf{35,8})+2(\overline{\mathbf{35}},\mathbf{8})+(\mathbf{35},\overline{\mathbf{10}})+(\overline{\mathbf{35}},\mathbf{10})+3(\mathbf{42,3})+(\mathbf{3,42})+3(\overline{\mathbf{42}},\overline{\mathbf{3}})+(\overline{\mathbf{3}},\overline{\mathbf{42}})+2(\mathbf{42},\overline{\mathbf{6}})+2(\overline{\mathbf{42}},\mathbf{6})+(\mathbf{48},\overline{\mathbf{6}})+(\overline{\mathbf{48}},\mathbf{6})+(\mathbf{42,15})+(\overline{\mathbf{42}},\overline{\mathbf{15}})+(\mathbf{60,3})+(\overline{\mathbf{60}},\overline{\mathbf{3}})+(\mathbf{64,1})+(\mathbf{64,8})$

F_4	→	SU(2)×Sp(6) (R)
26	=	$(\mathbf{2,6})+(\mathbf{1,14})$
52	=	$(\mathbf{3,1})+(\mathbf{2,14'})+(\mathbf{1,21})$
273	=	$(\mathbf{2,6})+(\mathbf{3,14})+(\mathbf{1,21})+(\mathbf{2,64})+(\mathbf{1,70})$
324	=	$(\mathbf{1,1})+(\mathbf{1,14})+(\mathbf{2,14'})+(\mathbf{3,21})+(\mathbf{2,64})+(\mathbf{1,90})$
1053	=	$(\mathbf{2,6})+(\mathbf{4,6})+(\mathbf{1,14})+(\mathbf{3,14})+(\mathbf{2,56})+(\mathbf{2,64})+(\mathbf{1,70})+(\mathbf{3,70})+(\mathbf{2,126})+(\mathbf{1,189})$
1053'	=	$(\mathbf{1,1})+(\mathbf{5,1})+(\mathbf{2,14'})+(\mathbf{4,14'})+(\mathbf{3,21})+(\mathbf{3,84})+(\mathbf{1,90})+(\mathbf{1,126'})+(\mathbf{2,216})$
1274	=	$(\mathbf{3,1})+(\mathbf{2,14'})+(\mathbf{4,14'})+(\mathbf{1,21})+(\mathbf{3,21})+(\mathbf{2,64})+(\mathbf{1,84})+(\mathbf{3,90})+(\mathbf{1,189})+(\mathbf{2,216})$
2652	=	$(\mathbf{2,6})+(\mathbf{1,14})+(\mathbf{4,56})+(\mathbf{2,64})+(\mathbf{1,70})+(\mathbf{3,70})+(\mathbf{1,90})+(\mathbf{2,126})+(\mathbf{3,189})+(\mathbf{2,350})+(\mathbf{1,385})$
4096	=	$(\mathbf{2,6})+(\mathbf{1,14})+(\mathbf{2,14'})+(\mathbf{3,14})+(\mathbf{1,21})+(\mathbf{3,21})+(\mathbf{2,56})+2(\mathbf{2,64})+(\mathbf{4,64})+(\mathbf{1,70})+(\mathbf{3,70})+(\mathbf{1,90})+(\mathbf{3,90})+(\mathbf{2,126})+(\mathbf{1,189})+(\mathbf{3,189})+(\mathbf{2,216})+(\mathbf{2,350})+(\mathbf{1,512})$
8424	=	$(\mathbf{2,6})+(\mathbf{4,6})+(\mathbf{1,14})+(\mathbf{3,14})+(\mathbf{5,14})+(\mathbf{3,21})+(\mathbf{2,56})+2(\mathbf{2,64})+(\mathbf{4,64})+(\mathbf{1,70})+2(\mathbf{3,70})+(\mathbf{1,90})+(\mathbf{1,126'})+(\mathbf{2,126})+(\mathbf{4,126})+(\mathbf{1,189})+(\mathbf{3,189})+(\mathbf{2,216})+(\mathbf{2,350})+(\mathbf{2,378})+(\mathbf{2,448})+(\mathbf{1,512})+(\mathbf{3,512})+(\mathbf{1,525})$
10829	=	$(\mathbf{3,1})+2(\mathbf{2,14'})+(\mathbf{3,14})+(\mathbf{4,14'})+2(\mathbf{1,21})+(\mathbf{3,21})+(\mathbf{5,21})+2(\mathbf{2,64})+(\mathbf{4,64})+(\mathbf{1,70})+(\mathbf{1,84})+(\mathbf{3,84})+(\mathbf{1,90})+2(\mathbf{3,90})+(\mathbf{2,126})+(\mathbf{3,126'})+(\mathbf{1,189})+(\mathbf{3,189})+2(\mathbf{2,216})+(\mathbf{4,216})+(\mathbf{2,350})+(\mathbf{2,448})+(\mathbf{1,512})+(\mathbf{3,512})+(\mathbf{2,616})+(\mathbf{1,924})$
12376	=	$(\mathbf{3,1})+(\mathbf{7,1})+(\mathbf{2,14'})+(\mathbf{4,14'})+(\mathbf{6,14'})+(\mathbf{1,21})+(\mathbf{5,21})+(\mathbf{1,84})+(\mathbf{3,84})+(\mathbf{5,84})+(\mathbf{3,90})+(\mathbf{3,126'})+(\mathbf{2,216})+(\mathbf{4,216})+(\mathbf{4,330})+(\mathbf{1,462})+(\mathbf{2,616})+(\mathbf{1,924})+(\mathbf{3,1078})+(\mathbf{2,1100})$
16302	=	$(\mathbf{1,1})+(\mathbf{1,14})+(\mathbf{2,14'})+(\mathbf{3,21})+(\mathbf{2,64})+(\mathbf{3,84})+2(\mathbf{1,90})+(\mathbf{2,126})+(\mathbf{5,126'})+(\mathbf{3,189})+(\mathbf{2,216})+(\mathbf{4,216})+(\mathbf{2,350})+(\mathbf{1,385})+(\mathbf{4,448})+(\mathbf{1,512})+(\mathbf{3,512})+(\mathbf{2,616})+(\mathbf{3,924})+(\mathbf{1,1274})+(\mathbf{2,1344})$
17901	=	$(\mathbf{2,6})+(\mathbf{4,6})+(\mathbf{6,6})+(\mathbf{1,14})+(\mathbf{3,14})+(\mathbf{5,14})+(\mathbf{2,56})+(\mathbf{4,56})+(\mathbf{2,64})+(\mathbf{4,64})+(\mathbf{1,70})+2(\mathbf{3,70})+(\mathbf{5,70})+2(\mathbf{2,126})+(\mathbf{4,126})+(\mathbf{1,189})+(\mathbf{3,189})+(\mathbf{2,252})+(\mathbf{2,350})+(\mathbf{2,378})+(\mathbf{4,378})+(\mathbf{1,385})+(\mathbf{2,448})+(\mathbf{1,512})+(\mathbf{3,512})+(\mathbf{1,525})+(\mathbf{3,525})+(\mathbf{3,594})+(\mathbf{1,924'})+(\mathbf{2,1386})$

F_4	→	SU(2) (S)
26	=	$(\mathbf{9})+(\mathbf{17})$
52	=	$(\mathbf{3})+(\mathbf{11})+(\mathbf{15})+(\mathbf{23})$
273	=	$(\mathbf{3})+2(\mathbf{7})+(\mathbf{9})+2(\mathbf{11})+(\mathbf{13})+2(\mathbf{15})+(\mathbf{17})+2(\mathbf{19})+(\mathbf{21})+(\mathbf{23})+(\mathbf{25})+(\mathbf{27})+(\mathbf{31})$
324	=	$(\mathbf{1})+2(\mathbf{5})+2(\mathbf{9})+(\mathbf{11})+3(\mathbf{13})+(\mathbf{15})+2(\mathbf{17})+(\mathbf{19})+2(\mathbf{21})+(\mathbf{23})+2(\mathbf{25})+(\mathbf{29})+(\mathbf{33})$
1053	=	$(\mathbf{3})+2(\mathbf{5})+4(\mathbf{7})+4(\mathbf{9})+4(\mathbf{11})+4(\mathbf{13})+5(\mathbf{15})+5(\mathbf{17})+5(\mathbf{19})+4(\mathbf{21})+4(\mathbf{23})+3(\mathbf{25})+3(\mathbf{27})+3(\mathbf{29})+2(\mathbf{31})+(\mathbf{33})+(\mathbf{35})+(\mathbf{37})+(\mathbf{39})$
1053'	=	$2(\mathbf{1})+3(\mathbf{5})+(\mathbf{7})+4(\mathbf{9})+2(\mathbf{11})+5(\mathbf{13})+3(\mathbf{15})+5(\mathbf{17})+2(\mathbf{19})+5(\mathbf{21})+3(\mathbf{23})+4(\mathbf{25})+2(\mathbf{27})+3(\mathbf{29})+2(\mathbf{31})+2(\mathbf{33})+(\mathbf{35})+2(\mathbf{37})+(\mathbf{41})+(\mathbf{45})$
1274	=	$3(\mathbf{3})+(\mathbf{5})+4(\mathbf{7})+3(\mathbf{9})+5(\mathbf{11})+5(\mathbf{13})+6(\mathbf{15})+4(\mathbf{17})+6(\mathbf{19})+4(\mathbf{21})+5(\mathbf{23})+4(\mathbf{25})+4(\mathbf{27})+2(\mathbf{29})+3(\mathbf{31})+2(\mathbf{33})+2(\mathbf{35})+(\mathbf{37})+(\mathbf{39})+(\mathbf{43})$
2652	=	$2(\mathbf{1})+(\mathbf{3})+5(\mathbf{5})+5(\mathbf{7})+8(\mathbf{9})+6(\mathbf{11})+10(\mathbf{13})+8(\mathbf{15})+10(\mathbf{17})+9(\mathbf{19})+10(\mathbf{21})+7(\mathbf{23})+9(\mathbf{25})+7(\mathbf{27})+7(\mathbf{29})+5(\mathbf{31})+5(\mathbf{33})+3(\mathbf{35})+4(\mathbf{37})+2(\mathbf{39})+2(\mathbf{41})+(\mathbf{43})+(\mathbf{45})+(\mathbf{49})$

Table 10.29: F_4 Branching Rules (continued)

4096	=	(1) + 4(3) + 8(5) + 9(7) + 11(9) + 14(11) + 14(13) + 15(15) + 16(17) + 15(19) + 15(21) + 14(23) + 12(25) + 12(27) + 10(29) + 8(31) + 7(33) + 6(35) + 4(37) + 3(39) + 3(41) + (43) + (45) + (47)
8424	=	4(1) + 6(3) + 13(5) + 14(7) + 21(9) + 21(11) + 26(13) + 25(15) + 28(17) + 26(19) + 28(21) + 25(23) + 25(25) + 21(27) + 21(29) + 17(31) + 16(33) + 12(35) + 11(37) + 8(39) + 7(41) + 5(43) + 4(45) + 2(47) + 2(49) + (51) + (53)
10829	=	(1) + 11(3) + 12(5) + 21(7) + 21(9) + 29(11) + 29(13) + 34(15) + 31(17) + 35(19) + 32(21) + 34(23) + 29(25) + 29(27) + 24(29) + 24(31) + 19(33) + 18(35) + 13(37) + 12(39) + 8(41) + 8(43) + 5(45) + 4(47) + 2(49) + 2(51) + (53) + (55)
12376	=	9(3) + 7(5) + 18(7) + 15(9) + 23(11) + 21(13) + 28(15) + 24(17) + 31(19) + 26(21) + 30(23) + 26(25) + 29(27) + 23(29) + 26(31) + 20(33) + 21(35) + 17(37) + 17(39) + 12(41) + 13(43) + 9(45) + 9(47) + 6(49) + 6(51) + 3(53) + 4(55) + 2(57) + 2(59) + (61) + (63) + (67)
16302	=	7(1) + 6(3) + 19(5) + 17(7) + 30(9) + 28(11) + 37(13) + 34(15) + 43(17) + 37(19) + 44(21) + 38(23) + 42(25) + 35(27) + 38(29) + 30(31) + 32(33) + 25(35) + 25(37) + 18(39) + 19(41) + 13(43) + 13(45) + 8(47) + 8(49) + 5(51) + 5(53) + 2(55) + 3(57) + (59) + (61) + (65)
17901	=	5(1) + 10(3) + 20(5) + 25(7) + 33(9) + 36(11) + 42(13) + 44(15) + 48(17) + 47(19) + 49(21) + 46(23) + 46(25) + 42(27) + 41(29) + 36(31) + 33(33) + 28(35) + 26(37) + 21(39) + 18(41) + 14(43) + 12(45) + 9(47) + 7(49) + 5(51) + 4(53) + 2(55) + 2(57) + (59) + (61)

F_4	→	$SU(2)\times G_2$ (R)
26	=	(5,1) + (3,7)
52	=	(3,1) + (5,7) + (1,14)
273	=	(3,1) + (1,7) + (7,1) + (3,7) + (5,7) + (7,7) + (5,14) + (3,27)
324	=	(1,1) + (5,1) + (3,7) + (9,1) + (5,7) + (7,7) + (3,14) + (1,27) + (5,27)
1053	=	(3,1) + (5,1) + (1,7) + (7,1) + 2(3,7) + 2(5,7) + (7,7) + (9,7) + (3,14) + (5,14) + (7,14) + (3,27) + (5,27) + (7,27) + (3,64)
1053'	=	(1,1) + (5,1) + (3,7) + (5,7) + (7,7) + (3,14) + (7,14) + (1,27) + (5,27) + (9,27) + (5,64) + (1,77')
1274	=	(3,1) + (1,7) + (7,1) + (3,7) + 2(5,7) + (7,7) + (1,14) + (9,7) + (3,14) + (5,14) + (9,14) + (3,27) + (5,27) + (7,27) + (5,64) + (1,77)
2652	=	(1,1) + (5,1) + (7,1) + 2(3,7) + (9,1) + (5,7) + 2(7,7) + (13,1) + (9,7) + (3,14) + (11,7) + (5,14) + (7,14) + (1,27) + (3,27) + 2(5,27) + (7,27) + (9,27) + (3,64) + (5,64) + (3,77) + (7,77)
4096	=	(3,1) + 2(5,1) + (1,7) + (7,1) + 3(3,7) + (9,1) + 4(5,7) + (11,1) + 3(7,7) + (1,14) + 2(9,7) + 2(3,14) + (11,7) + 2(5,14) + 2(7,14) + (9,14) + (1,27) + 3(3,27) + 3(5,27) + 2(7,27) + (9,27) + (1,64) + (3,64) + (5,64) + (7,64) + (3,77) + (5,77)
8424	=	(1,1) + (3,1) + 2(5,1) + (1,7) + (7,1) + 4(3,7) + (9,1) + 4(5,7) + 4(7,7) + 2(9,7) + 3(3,14) + (11,7) + 3(5,14) + 3(7,14) + (9,14) + (11,14) + 2(1,27) + 3(3,27) + 4(5,27) + 3(7,27) + 2(9,27) + (11,27) + (1,64) + 2(3,64) + 2(5,64) + 2(7,64) + (9,64) + (3,77) + (5,77) + (5,77') + (7,77) + (3,189)
10829	=	2(3,1) + (5,1) + 2(1,7) + 2(7,1) + 3(3,7) + (9,1) + 5(5,7) + (11,1) + 4(7,7) + 2(1,14) + 3(9,7) + 2(3,14) + (11,7) + 4(5,14) + (13,7) + 2(7,14) + 2(9,14) + (11,14) + (1,27) + 4(3,27) + 4(5,27) + 4(7,27) + 2(9,27) + (11,27) + (1,64) + 2(3,64) + 3(5,64) + 2(7,64) + (9,64) + (1,77) + (3,77) + (3,77') + 2(5,77) + (7,77) + (9,77) + (1,189) + (5,189)
12376	=	(3,1) + (1,7) + (7,1) + (3,7) + 2(5,7) + (7,7) + (1,14) + (9,7) + 2(5,14) + (7,14) + (9,14) + 2(3,27) + (5,27) + 2(7,27) + (9,27) + (11,27) + (3,64) + 2(5,64) + (7,64) + (9,64) + (11,64) + (1,77) + (3,77') + (5,77) + (7,77) + (7,77') + (9,77) + (13,77) + (1,189) + (5,189) + (9,189) + (1,273) + (5,286)
16302	=	(1,1) + 2(5,1) + 2(3,7) + 2(9,1) + 2(5,7) + (11,1) + 3(7,7) + (13,1) + 2(9,7) + 2(3,14) + 2(11,7) + (17,1) + (5,14) + (13,7) + 2(7,14) + (15,7) + (9,14) + (11,14) + 2(1,27) + (3,27) + 4(5,27) + 2(7,27) + 3(9,27) + (11,27) + (13,27) + (1,64) + 2(3,64) + 2(5,64) + 2(7,64) + (9,64) + (1,77') + 2(3,77) + 2(5,77) + (5,77') + 2(7,77) + (9,77) + (11,77) + (1,182) + (5,182) + (9,182) + (3,189) + (5,189) + (7,189)

Table 10.29: F_4 Branching Rules (continued)

17901	=	$(\mathbf{1},\mathbf{1})+(\mathbf{3},\mathbf{1})+2(\mathbf{5},\mathbf{1})+(\mathbf{1},\mathbf{7})+(\mathbf{7},\mathbf{1})+4(\mathbf{3},\mathbf{7})+(\mathbf{9},\mathbf{1})+4(\mathbf{5},\mathbf{7})+4(\mathbf{7},\mathbf{7})+(\mathbf{1},\mathbf{14})+2(\mathbf{9},\mathbf{7})+3(\mathbf{3},\mathbf{14})+(\mathbf{11},\mathbf{7})+3(\mathbf{5},\mathbf{14})+3(\mathbf{7},\mathbf{14})+2(\mathbf{9},\mathbf{14})+(\mathbf{11},\mathbf{14})+(\mathbf{1},\mathbf{27})+3(\mathbf{3},\mathbf{27})+5(\mathbf{5},\mathbf{27})+4(\mathbf{7},\mathbf{27})+3(\mathbf{9},\mathbf{27})+(\mathbf{11},\mathbf{27})+(\mathbf{13},\mathbf{27})+(\mathbf{1},\mathbf{64})+3(\mathbf{3},\mathbf{64})+3(\mathbf{5},\mathbf{64})+3(\mathbf{7},\mathbf{64})+2(\mathbf{9},\mathbf{64})+(\mathbf{11},\mathbf{64})+2(\mathbf{3},\mathbf{77})+(\mathbf{3},\mathbf{77'})+(\mathbf{5},\mathbf{77})+(\mathbf{5},\mathbf{77'})+2(\mathbf{7},\mathbf{77})+(\mathbf{7},\mathbf{77'})+(\mathbf{9},\mathbf{77})+(\mathbf{11},\mathbf{77})+(\mathbf{3},\mathbf{189})+(\mathbf{5},\mathbf{189})+(\mathbf{7},\mathbf{189})+(\mathbf{3},\mathbf{286})$

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